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Waste Reduction and Recycling: Schweizer-Sklar Aggregation Operators Based on Neutrosophic Fuzzy Rough Sets and Their Application in Green Supply Chain Management

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Abstract: Green supply chain management (GSCM) is a valuable application that is used to reduce the overall environmental impact of the supply chain. Waste reduction and recycling are crucial components of sustainable technique that aims to reduce ecological impact and encourage reserve effectiveness. In this manuscript, we initiate the technique of Schweizer-Sklar (SS) operational laws based on neutrosophic fuzzy rough (NFR) values for SS t-norm (SSTN) and SS t-conorm (SSTCN). Further, we derive the NFR SS weighted averaging (NFRSSWA) operator and the NFR SS weighted geometric (NFRSSWG) operator. Some basic properties for the above-initiated techniques are derived. Additionally, we describe the application in green supply chain management, called waste reeducation and recycling based on initiated operators in multi-attribute decision-making (MADM) problems. Finally, we illustrate an example for comparing the ranking values of the proposed techniques with the ranking values of the existing technique to enhance the worth of the derived theory.

Keywords: Neutrosophic Fuzzy Rough Sets, Schweizer-Sklar Aggregation Operators, Green Supply Chain Management, Waste Reduction and Recycling, Decision-Making Problems.

1. Introduction

Green supply chain management (GSCM) or simply chain management are very reliable and effective techniques used for decreasing the environmental impact of the supply chain [1]. Further, the utilization of the GSCM in the environment of waste reduction and recycling is very complex because of uncertainty and ambiguity [2]. During the decision-making procedure, we have lost a lot of information because of complexity and uncertainty, where the terms zero and one are part of classical set theory, but in many genuine life problems, we have required more options for making a reliable and accurate decision [3]. For this, Zadeh [4] introduced the fuzzy sets (FSs). The technique of FSs contained the grade of membership function which is defined from universal set to unit interval. The technique of FSs is very effective for coping with uncertain and vague information because of their features, and due to this reason, many scholars have utilized it in many fields, for instance, visualization of fuzzy bibliometric problems [5], defuzzification based on fuzzy expert systems for possibilistic mean in control-LSD techniques [6], and two technique of generalized information measures for FSs theory [7].

In the structure of FSs theory, it is quite complex to deal with genuine life problems because of the membership function, where the membership function is not enough for coping with complex problems, because in many problems, we require the membership function and non-membership function. Therefore, Atanassov [8, 9] introduced the intuitionistic fuzzy sets (IFSs). The IFSs have covered the membership function and non-membership function with the condition that the sum of both functions will be covered in the unit interval. Further, the technique of IFSs has received a lot of attention from different scholars, for instance, Archimedean Heronian mean operators for complex IFSs [10], zero-sum decision-making matrix games based on intuitionistic fuzzy goals [11], and new aggregation operators for intuitionistic fuzzy soft sets and their application in decision-making problems [12].

In 1998, Smarandache [13] proposed the technique of neutrosophic sets (NSs) which covered three major functions, called membership function, non-membership function, and abstinence function with a condition that is the sum of the triplet will be covered in [0, 3]. The technique of NSs is very reliable because the existing techniques of FSs and IFSs are the special cases of the NSs. Further, Ye [14] presented the aggregation operators for simplified NSs. Moreover, in 1982, Pawlak [15] proposed the technique of rough sets (RSs). Further, in 1990, Dubois and Prade [16] evaluated the rough fuzzy sets and fuzzy rough sets. Moreover, Jena et al. [17] exposed the intuitionistic fuzzy rough sets (IFRSs) in 2002. In 2014, Broumi et al. [18] presented the rough neutrosophic sets (RNSs). In 2017, Yang et al. [19] derived the single-valued neutrosophic sets. In 2023, Bibi and Ali [20] introduced the technique of Aczel-Alsina aggregation operators for neutrosophic fuzzy rough sets.

The technique of SSTN and SSTCN in the statistical triangle inequality was proposed by Schweizer and Sklar [21]. Further, the aggregation operators for IFSs were invented by Xu [22]. The technique of geometric aggregation operators for IFSs was presented by Xu and Yager [23]. The Schweizer-Sklar prioritized aggregation operators for IFSs were presented by Garg et al. [24]. The Maclaurin symmetric mean operators for IFSs based on SSTN and SSTCN were derived by Wang and Liu [25]. The intuitionistic fuzzy Schweizer-Sklar operators for IFSs were presented by Khan et al. [26]. The aggregation operators for rough neutrosophic sets were proposed by Mondal et al. [27]. The single-valued neutrosophic Schweizer-Sklar Muirhead mean operators were invented by Zhang et al. [28]. The single-valued Schweizer-Sklar Hamy mean operators were derived by Yuan et al. [29]. The single-valued neutrosophic Schweizer-Sklar prioritized operators were proposed by Liu et al. [30]. The generalized interval neutrosophic Schweizer-Sklar prioritized operators were proposed by Khan et al. [31]. The major contribution of this manuscript is listed below:

- To initiate the technique of SS operational laws based on NFR values for SSTN and SSTCN.
- To derive the NFRSSWA operator and the NFRSSWG operator. Some basic properties for the above-initiated techniques are derived.
- To describe the application in green supply chain management, called waste reeducation and recycling based on initiated operators in MADM problems.
- To illustrate an example for comparing the ranking values of the proposed techniques with the ranking values of the existing technique to enhance the worth of the derived theory.

This manuscript is arranged in the following shape: In Section 2, we revised the technique of NFRSs with the concept of SSTN and SSTCN based on the unit interval. In Section 3, we initiated the technique of Schweizer-Sklar operational laws based on NFR values for SSTN and SSTCN. Further, we derived the NFRSSWA operator and the NFRSSWG operator. Some basic properties for the above-initiated techniques are derived. In Section 4, we described the application in green supply chain management, called waste reeducation and recycling based on initiated operators in MADM problems. In Section 5, we illustrated an example for comparing the ranking values of the proposed techniques with the ranking values of the existing technique. Some concluding remarks are described in Section 6.

2. Preliminaries

In this section, we revised the technique of NFRSs with the concept of SSTN and SSTCN based on unit interval.

Definition 1. [18] The upper and lower approximation space for NFRSs is described in the shape:

$$\Lambda^{NFUA}(\mathfrak{M}) = \{\mathcal{H}, \phi_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}), \sigma_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}), \kappa_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) | \mathcal{H} \in \Upsilon\}$$

$$\Lambda^{NFLA}(\mathfrak{M}) = \{\mathcal{H}, \phi_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}), \sigma_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}), \kappa_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) | \mathcal{H} \in \Upsilon\}$$

Where Υ represents the fixed set with sub-relation $\Lambda \in \Upsilon \times \Upsilon$, then the pair (Υ, Λ), called neutrosophic fuzzy space of the approximation and $\mathfrak{M} \subseteq NFS(\Upsilon)$. Further, we explained the technique of truth, falsity, and abstinence grade, such as

$$\phi_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) = \bigvee \ell \in \Upsilon \left[\phi_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \vee \phi_{\mathfrak{M}(\mathcal{H})}\right]$$

$$\sigma_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) = \bigwedge \ell \in \Upsilon \left[\sigma_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \wedge \sigma_{\mathfrak{M}(\mathcal{H})}\right]$$

$$\kappa_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) = \bigwedge \ell \in \Upsilon \left[\kappa_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \wedge \kappa_{\mathfrak{M}(\mathcal{H})}\right]$$

$$\phi_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) = \bigwedge \ell \in \Upsilon \left[\phi_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \wedge \phi_{\mathfrak{M}(\mathcal{H})}\right]$$

$$\sigma_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) = \bigvee \ell \in \Upsilon \left[\sigma_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \vee \sigma_{\mathfrak{M}(\mathcal{H})}\right]$$

$$\kappa_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) = \bigvee \ell \in \Upsilon \left[\kappa_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \vee \kappa_{\mathfrak{M}(\mathcal{H})}\right]$$

For the above technique, we have the following technique, such as $0 \le \phi_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) + \sigma_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) + \kappa_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) \le 3$ and $0 \le \phi_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) + \sigma_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) + \kappa_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) \le 3$. Finally, the pair of upper and lower space $\left(\Lambda^{NFLA}(\mathfrak{M}), \Lambda^{NFUA}(\mathfrak{M})\right)$, called NFRS, if both are not equal. Further, the NFRVs are described in the shape: $\mathfrak{T}_{q} = \left(\left(\phi_{q}^{lb}, \sigma_{q}^{lb}, \kappa_{q}^{lb}\right), \left(\phi_{q}^{ub}, \sigma_{q}^{ub}, \kappa_{q}^{ub}\right)\right), \mathcal{N} = 1,2, \dots, \mathcal{N}$.

Definition 2: [21] The mathematical shape of SSTN and SSTCN based on a unit interval are described below:

$$\Xi_{ss}(\mu, \zeta) = (\mu^{\alpha} + \zeta^{\alpha} - 1)^{\frac{1}{\alpha}}$$

$$\Xi_{ss}^{*}(\mu, \zeta) = 1 - ((1-\mu)^{\alpha} + (1-\zeta)^{\alpha} - 1)^{\frac{1}{\alpha}}$$

3. Schweizer-Sklar Aggregation Operators for NFRVs

In this section, we propose the technique of SS operational laws based on NFRVs. Further, we initiate the technique of the NFRSSWA operator and NFRSSWG operator based on the collection of a finite number of NFRVs.

Definition 3. Consider any two NFRVs $\mathfrak{T}_{q} = \left(\left(\phi_{q}^{lb}, \sigma_{q}^{lb}, \kappa_{q}^{lb}\right), \left(\phi_{q}^{ub}, \sigma_{q}^{ub}, \kappa_{q}^{ub}\right)\right), q = 1,2$, then

$$\mathfrak{T}_1 \oplus \mathfrak{T}_2 = \begin{pmatrix} \left(1 - \left((1-\phi_1^{lb})^{\beta} + (1-\phi_2^{lb})^{\beta} - 1\right)^{\frac{1}{\beta}}, \left((\sigma_1^{lb})^{\beta}, (\sigma_2^{lb})^{\beta} - 1\right)^{\frac{1}{\beta}}, \left((\kappa_1^{lb})^{\beta}, (\kappa_2^{lb})^{\beta} - 1\right)^{\frac{1}{\beta}}\right) \\ \left(1 - \left((1-\phi_1^{ub})^{\beta} + (1-\phi_2^{ub})^{\beta} - 1\right)^{\frac{1}{\beta}}, \left((\sigma_1^{ub})^{\beta}, (\sigma_2^{ub})^{\beta} - 1\right)^{\frac{1}{\beta}}, \left((\kappa_1^{ub})^{\beta}, (\kappa_2^{ub})^{\beta} - 1\right)^{\frac{1}{\beta}}\right) \end{pmatrix}$$

$$\mathfrak{T}_1 \otimes \mathfrak{T}_2 = \begin{pmatrix} \left(\left((\phi_1^{lb})^{\beta}, (\phi_2^{lb})^{\beta} - 1\right)^{\frac{1}{\beta}}, 1 - \left((1-\sigma_1^{lb})^{\beta} + (1-\sigma_2^{lb})^{\beta} - 1\right)^{\frac{1}{\beta}}, 1 - \left((1-\kappa_1^{lb})^{\beta} + (1-\kappa_2^{lb})^{\beta} - 1\right)^{\frac{1}{\beta}}\right) \\ \left(\left((\phi_1^{ub})^{\beta}, (\phi_2^{ub})^{\beta} - 1\right)^{\frac{1}{\beta}}, 1 - \left((1-\sigma_1^{ub})^{\beta} + (1-\sigma_2^{ub})^{\beta} - 1\right)^{\frac{1}{\beta}}, 1 - \left((1-\kappa_1^{ub})^{\beta} + (1-\kappa_2^{ub})^{\beta} - 1\right)^{\frac{1}{\beta}}\right) \end{pmatrix}$$

$$\vartheta \mathfrak{T}_1 = \begin{pmatrix} \left(1 - \left(\vartheta(1-\phi_1^{lb})^{\beta} - (\vartheta - 1)\right)^{\frac{1}{\beta}}, \left(\vartheta(\sigma_1^{lb})^{\beta} - (\vartheta - 1)\right)^{\frac{1}{\beta}}, \left(\vartheta(\kappa_1^{lb})^{\beta} - (\vartheta - 1)\right)^{\frac{1}{\beta}}\right) \\ \left(1 - \left(\vartheta(1-\phi_1^{ub})^{\beta} - (\vartheta - 1)\right)^{\frac{1}{\beta}}, \left(\vartheta(\sigma_1^{ub})^{\beta} - (\vartheta - 1)\right)^{\frac{1}{\beta}}, \left(\vartheta(\kappa_1^{ub})^{\beta} - (\vartheta - 1)\right)^{\frac{1}{\beta}}\right) \end{pmatrix}$$

$$\mathfrak{T}_1^{\vartheta} = \begin{pmatrix} \left(\left(\vartheta(\phi_1^{lb})^{\beta} - (\vartheta - 1)\right)^{\frac{1}{\beta}}, 1 - \left(\vartheta(1-\sigma_1^{lb})^{\beta} - (\vartheta - 1)^{\beta}\right)^{\frac{1}{\beta}}, 1 - \left(\vartheta(1-\kappa_1^{lb})^{\beta} - (\vartheta - 1)^{\beta}\right)^{\frac{1}{\beta}}\right) \\ \left(\left(\vartheta(\phi_1^{ub})^{\beta} - (\vartheta - 1)\right)^{\frac{1}{\beta}}, 1 - \left(\vartheta(1-\sigma_1^{ub})^{\beta} - (\vartheta - 1)^{\beta}\right)^{\frac{1}{\beta}}, 1 - \left(\vartheta(1-\kappa_1^{ub})^{\beta} - (\vartheta - 1)^{\beta}\right)^{\frac{1}{\beta}}\right) \end{pmatrix}$$

Definition 4. Consider the collection of NFRVs $\mathfrak{T}_{q} = \left(\left(\phi_{q}^{lb}, \sigma_{q}^{lb}, \kappa_{q}^{lb}\right), \left(\phi_{q}^{ub}, \sigma_{q}^{ub}, \kappa_{q}^{ub}\right)\right), q = 1,2, \dots, \mathcal{N}$ with weight vector $\sum_{q}^{\mathcal{N}} \mathfrak{M}_{q} = 1$. Then, we described the technique of $NFRSSWA$ operator, such as

$$NFRSSWA(\mathfrak{T}_1,\mathfrak{T}_2,\dots\mathfrak{T}_{\mathcal{N}})=\oplus_{q=1}^{\mathcal{N}}\mathfrak{M}_q\mathfrak{T}_q$$
$$=\left(\begin{array}{c}\left(1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(1-\phi_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\sigma_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\kappa_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}\right)\\\left(1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(1-\phi_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\sigma_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\kappa_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}\right)\end{array}\right)$$

Theorem 1. Consider the collection of NFRVs $\mathfrak{T}_q=\left(\left(\phi_q^{lb},\sigma_q^{lb},\kappa_q^{lb}\right),\left(\phi_q^{ub},\sigma_q^{ub},\kappa_q^{ub}\right)\right),q=1,2,\dots.,\mathcal{N}$ with weight vector $\sum_q^{\mathcal{N}}\mathfrak{M}_q=1$. Then, we prove that the aggregated value of the NFRSSWA operator is again an NFRV, such as

$$NFRSSWA(\mathfrak{T}_1,\mathfrak{T}_2,\dots\mathfrak{T}_{\mathcal{N}})=\left(\begin{array}{c}\left(1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(1-\phi_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\sigma_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\kappa_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}\right)\\\left(1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(1-\phi_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\sigma_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\kappa_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}\right)\end{array}\right)$$

Proof: Considering the technique of Mathematical induction, we derive the above theory. For this, first, we consider, $\mathcal{N}=2$, then

$$NFRSSWA(\mathfrak{T}_1,\mathfrak{T}_2)=\left(\begin{array}{c}\left(1-\left(\sum_{q=1}^{2}\mathfrak{M}_q\left(1-\phi_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{2}\mathfrak{M}_q\left(\sigma_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{2}\mathfrak{M}_q\left(\kappa_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}\right)\\\left(1-\left(\sum_{q=1}^{2}\mathfrak{M}_q\left(1-\phi_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{2}\mathfrak{M}_q\left(\sigma_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{2}\mathfrak{M}_q\left(\kappa_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}\right)\end{array}\right)$$

Noticed that the above information is still computed in the shape of NFRV. Further, we assume that the above theory holds for $\mathcal{N}=n$, then

$$NFRSSWA(\mathfrak{T}_1,\mathfrak{T}_2,\dots\mathfrak{T}_n)=\left(\begin{array}{c}\left(1-\left(\sum_{q=1}^{n}\mathfrak{M}_q\left(1-\phi_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\sigma_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\kappa_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}\right)\\\left(1-\left(\sum_{q=1}^{n}\mathfrak{M}_q\left(1-\phi_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\sigma_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\kappa_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}\right)\end{array}\right)$$

Then, we evaluate it for $\mathcal{N}=n+1$, such as

$$NFRSSWA(\mathfrak{T}_1,\mathfrak{T}_2,\dots\mathfrak{T}_{n+1})=\oplus_{q=1}^{n+1}\mathfrak{M}_q\mathfrak{T}_q=\oplus_{q=1}^{n}\mathfrak{M}_q\mathfrak{T}_q\oplus\mathfrak{M}_{n+1}\mathfrak{T}_{n+1}$$

$$= \left(\begin{pmatrix} 1-\left(\sum_{q=1}^{n}\mathfrak{M}_q\left(1-\phi_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\sigma_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\kappa_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}} \end{pmatrix} \\ \begin{pmatrix} 1-\left(\sum_{q=1}^{n}\mathfrak{M}_q\left(1-\phi_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\sigma_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\kappa_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}} \end{pmatrix} \right) \oplus \mathfrak{M}_{n+1}\mathfrak{T}_{n+1}$$

$$= \left(\begin{pmatrix} 1-\left(\sum_{q=1}^{n}\mathfrak{M}_q\left(1-\phi_q^{lb}\right)^{\beta}+\mathfrak{M}_{q+1}\left(1-\phi_{q+1}^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\sigma_q^{lb}\right)^{\beta}+\mathfrak{M}_{q+1}\left(\sigma_{q+1}^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}, \\ \left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\kappa_q^{lb}\right)^{\beta}+\mathfrak{M}_{q+1}\left(\kappa_{q+1}^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}} \end{pmatrix} \\ \begin{pmatrix} 1-\left(\sum_{q=1}^{n}\mathfrak{M}_q\left(1-\phi_q^{ub}\right)^{\beta}+\mathfrak{M}_{q+1}\left(1-\phi_{q+1}^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\sigma_q^{ub}\right)^{\beta}+\mathfrak{M}_{q+1}\left(\sigma_{q+1}^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}, \\ \left(\sum_{q=1}^{n}\mathfrak{M}_q\left(\kappa_q^{ub}\right)^{\beta}+\mathfrak{M}_{q+1}\left(\kappa_{q+1}^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}} \end{pmatrix} \right)$$

$$= \left(\begin{pmatrix} 1-\left(\sum_{q=1}^{n+1}\mathfrak{M}_q\left(1-\phi_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{n+1}\mathfrak{M}_q\left(\sigma_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{n+1}\mathfrak{M}_q\left(\kappa_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}} \end{pmatrix} \\ \begin{pmatrix} 1-\left(\sum_{q=1}^{n+1}\mathfrak{M}_q\left(1-\phi_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{n+1}\mathfrak{M}_q\left(\sigma_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{n+1}\mathfrak{M}_q\left(\kappa_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}} \end{pmatrix} \right)$$

$$= NFRSSWA(\mathfrak{T}_1, \mathfrak{T}_2, \dots \mathfrak{T}_{n+1})$$

Hence, the proposed theory holds for all positive values of n. Furthermore, we described the technique of idempotency, monotonicity, and boundedness for the collection of NFRVs.

Theorem 2. Consider the collection of NFRVs $\mathfrak{T}_q = \left(\left(\phi_q^{lb}, \sigma_q^{lb}, \kappa_q^{lb}\right), \left(\phi_q^{ub}, \sigma_q^{ub}, \kappa_q^{ub}\right)\right), q = 1,2, \dots, \mathcal{N}$, if $\mathfrak{T}_q = \left(\left(\phi_q^{lb}, \sigma_q^{lb}, \kappa_q^{lb}\right), \left(\phi_q^{ub}, \sigma_q^{ub}, \kappa_q^{ub}\right)\right) = \left((\phi^{lb}, \sigma^{lb}, \kappa^{lb}), (\phi^{ub}, \sigma^{ub}, \kappa^{ub})\right) = \mathfrak{T}, \forall\, q = 1,2, \dots, \mathcal{N}$, thus

$$NFRSSWA(\mathfrak{T}_1, \mathfrak{T}_2, \dots \mathfrak{T}_{\mathcal{N}}) = \left((\phi^{lb}, \sigma^{lb}, \kappa^{lb}), (\phi^{ub}, \sigma^{ub}, \kappa^{ub})\right) = \mathfrak{T}$$

Proof: By hypothesis, we are clear that $\mathfrak{T}_q = \left(\left(\phi_q^{lb}, \sigma_q^{lb}, \kappa_q^{lb}\right), \left(\phi_q^{ub}, \sigma_q^{ub}, \kappa_q^{ub}\right)\right) = \left((\phi^{lb}, \sigma^{lb}, \kappa^{lb}), (\phi^{ub}, \sigma^{ub}, \kappa^{ub})\right)$, then

$$NFRSSWA(\mathfrak{T}_1, \mathfrak{T}_2, \dots \mathfrak{T}_{\mathcal{N}}) = (\mathfrak{T}, \mathfrak{T}, \dots \dots, \mathfrak{T})$$

$$= \left(\begin{pmatrix} 1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(1-\phi_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\sigma_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\kappa_q^{lb}\right)^{\beta}\right)^{\frac{1}{\beta}} \end{pmatrix} \\ \begin{pmatrix} 1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(1-\phi_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\sigma_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_q\left(\kappa_q^{ub}\right)^{\beta}\right)^{\frac{1}{\beta}} \end{pmatrix} \right)$$

$$=\left(\begin{array}{c}\left(1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(1-\phi^{lb})^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(\sigma^{lb})^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(\kappa^{lb})^{\beta}\right)^{\frac{1}{\beta}}\right)\\ \left(1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(1-\phi^{ub})^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(\sigma^{ub})^{\beta}\right)^{\frac{1}{\beta}},\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(\kappa^{lb})^{\beta}\right)^{\frac{1}{\beta}}\right)\end{array}\right)$$

$$=\left(\begin{array}{c}\left(1-\left((1-\phi^{lb})^{\beta}\right)^{\frac{1}{\beta}},\left((\sigma^{lb})^{\beta}\right)^{\frac{1}{\beta}},\left((\kappa^{lb})^{\beta}\right)^{\frac{1}{\beta}}\right)\\ \left(1-\left((1-\phi^{ub})^{\beta}\right)^{\frac{1}{\beta}},\left((\sigma^{ub})^{\beta}\right)^{\frac{1}{\beta}},\left((\kappa^{lb})^{\beta}\right)^{\frac{1}{\beta}}\right)\end{array}\right)\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}=1$$

$$=\left((\phi^{lb},\sigma^{lb},\kappa^{lb}),(\phi^{ub},\sigma^{ub},\kappa^{ub})\right)=\mathfrak{T}.$$

Hence, the property of idempotency is holding.

Theorem 3. Consider the collection of NFRVs $\mathfrak{T}_{q}=\left((\phi_{q}^{lb},\sigma_{q}^{lb},\kappa_{q}^{lb}),(\phi_{q}^{ub},\sigma_{q}^{ub},\kappa_{q}^{ub})\right), q=1,2,\dots,\mathcal{N}$, if $\mathfrak{T}_{q}^{s}$ and $\mathfrak{T}_{q}^{g}$ are the smallest and greatest NFRV, then

$$\mathfrak{T}_{q}^{s}\leq NFRSSWA(\mathfrak{T}_{1},\mathfrak{T}_{2},\dots\mathfrak{T}_{\mathcal{N}})\leq\mathfrak{T}_{q}^{g}.$$

Called boundedness.

Theorem 4. Consider any two collections of NFRVs $\mathfrak{T}_{q}=\left((\phi_{q}^{lb},\sigma_{q}^{lb},\kappa_{q}^{lb}),(\phi_{q}^{ub},\sigma_{q}^{ub},\kappa_{q}^{ub})\right), q=1,2,\dots,\mathcal{N}$ and $\mathfrak{T}_{q}^{\epsilon}=\left((\phi_{q}^{\epsilon lb},\sigma_{q}^{\epsilon lb},\kappa_{q}^{\epsilon lb}),(\phi_{q}^{\epsilon ub},\sigma_{q}^{\epsilon ub},\kappa_{q}^{\epsilon ub})\right), q=1,2,\dots,\mathcal{N}$. If $\mathfrak{T}_{q}\leq\mathfrak{T}_{q}^{\epsilon}$, then

$$NFRSSWA(\mathfrak{T}_{1},\mathfrak{T}_{2},\dots\mathfrak{T}_{\mathcal{N}})\leq NFRSSWA(\mathfrak{T}_{1}^{\epsilon},\mathfrak{T}_{2}^{\epsilon},\dots\mathfrak{T}_{\mathcal{N}}^{\epsilon}).$$

Called monotonicity.

Further, we described the technique of NFRSSWG operators based on the collection of NFRVS.

Definition 5. Consider the collection of NFRVs $\mathfrak{T}_{q}=\left((\phi_{q}^{lb},\sigma_{q}^{lb},\kappa_{q}^{lb}),(\phi_{q}^{ub},\sigma_{q}^{ub},\kappa_{q}^{ub})\right), q=1,2,\dots,\mathcal{N}$, then we described the technique of the NFRSSWG operator, such as

$$NFRSSWG(\mathfrak{T}_{1},\mathfrak{T}_{2},\dots\mathfrak{T}_{\mathcal{N}})=\otimes_{q=1}^{\mathcal{N}}\mathfrak{T}_{q}^{\mathfrak{M}_{q}}$$

$$=\left(\begin{array}{c}\left(\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(\phi_{q}^{lb})^{\beta}\right)^{\frac{1}{\beta}},1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(1-\sigma_{q}^{lb})^{\beta}\right)^{\frac{1}{\beta}},1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(1-\kappa_{q}^{lb})^{\beta}\right)^{\frac{1}{\beta}}\right)\\ \left(\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(\phi_{q}^{ub})^{\beta}\right)^{\frac{1}{\beta}},1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(1-\sigma_{q}^{ub})^{\beta}\right)^{\frac{1}{\beta}},1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(1-\kappa_{q}^{ub})^{\beta}\right)^{\frac{1}{\beta}}\right)\end{array}\right)$$

Theorem 5: Consider the collection of NFRVs $\mathfrak{T}_{q}=\left((\phi_{q}^{lb},\sigma_{q}^{lb},\kappa_{q}^{lb}),(\phi_{q}^{ub},\sigma_{q}^{ub},\kappa_{q}^{ub})\right), q=1,2,\dots,\mathcal{N}$, then we prove the aggregated value of the NFRSSWG operator is again an NFRV, such as

$$NFRSSWG(\mathfrak{T}_{1},\mathfrak{T}_{2},\dots\mathfrak{T}_{\mathcal{N}})$$

$$=\left(\begin{array}{c}\left(\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(\phi_{q}^{lb})^{\beta}\right)^{\frac{1}{\beta}},1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(1-\sigma_{q}^{lb})^{\beta}\right)^{\frac{1}{\beta}},1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(1-\kappa_{q}^{lb})^{\beta}\right)^{\frac{1}{\beta}}\right)\\ \left(\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(\phi_{q}^{ub})^{\beta}\right)^{\frac{1}{\beta}},1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(1-\sigma_{q}^{ub})^{\beta}\right)^{\frac{1}{\beta}},1-\left(\sum_{q=1}^{\mathcal{N}}\mathfrak{M}_{q}(1-\kappa_{q}^{ub})^{\beta}\right)^{\frac{1}{\beta}}\right)\end{array}\right)$$

Theorem 6. Consider the collection of NFRVs $\mathfrak{T}_{q} = \left((\phi_{q}^{lb}, \sigma_{q}^{lb}, \kappa_{q}^{lb}), (\phi_{q}^{ub}, \sigma_{q}^{ub}, \kappa_{q}^{ub})\right), q = 1,2, \dots, \mathcal{N}$, if $\mathfrak{T}_{q} = \left((\phi_{q}^{lb}, \sigma_{q}^{lb}, \kappa_{q}^{lb}), (\phi_{q}^{ub}, \sigma_{q}^{ub}, \kappa_{q}^{ub})\right) = \left((\phi^{lb}, \sigma^{lb}, \kappa^{lb}), (\phi^{ub}, \sigma^{ub}, \kappa^{ub})\right) = \mathfrak{T}, \forall q = 1,2, \dots, \mathcal{N}$, then

$$NFRSSWG(\mathfrak{T}_1, \mathfrak{T}_2, \dots \mathfrak{T}_{\mathcal{N}}) = \left((\phi^{lb}, \sigma^{lb}, \kappa^{lb}), (\phi^{ub}, \sigma^{ub}, \kappa^{ub})\right) = \mathfrak{T}$$

Theorem 7. Consider the collection of NFRVs $\mathfrak{T}_{q} = \left((\phi_{q}^{lb}, \sigma_{q}^{lb}, \kappa_{q}^{lb}), (\phi_{q}^{ub}, \sigma_{q}^{ub}, \kappa_{q}^{ub})\right), q = 1,2, \dots, \mathcal{N}$, if $\mathfrak{T}_{q}^{s}$ and $\mathfrak{T}_{q}^{g}$ are the smallest and greatest NFRV, then

$$\mathfrak{T}_{q}^{s} \leq NFRSSWG(\mathfrak{T}_1, \mathfrak{T}_2, \dots \mathfrak{T}_{\mathcal{N}}) \leq \mathfrak{T}_{q}^{g}.$$

Called boundedness.

Theorem 8. Consider any two collections of NFRVs $\mathfrak{T}_{q} = \left((\phi_{q}^{lb}, \sigma_{q}^{lb}, \kappa_{q}^{lb}), (\phi_{q}^{ub}, \sigma_{q}^{ub}, \kappa_{q}^{ub})\right), q = 1,2, \dots, \mathcal{N}$ and $\mathfrak{T}_{q}^{\epsilon} = \left((\phi_{q}^{\epsilon lb}, \sigma_{q}^{\epsilon lb}, \kappa_{q}^{\epsilon lb}), (\phi_{q}^{\epsilon ub}, \sigma_{q}^{\epsilon ub}, \kappa_{q}^{\epsilon ub})\right), q = 1,2, \dots, \mathcal{N}$. If $\mathfrak{T}_{q} \leq \mathfrak{T}_{q}^{\epsilon}$, then

$$NFRSSWG(\mathfrak{T}_1, \mathfrak{T}_2, \dots \mathfrak{T}_{\mathcal{N}}) \leq NFRSSWG(\mathfrak{T}_1^{\epsilon}, \mathfrak{T}_2^{\epsilon}, \dots \mathfrak{T}_{\mathcal{N}}^{\epsilon}).$$

Called monotonicity.

4. Application in Green Supply Chain Management

In this section, we discuss the technique of green supply chain management in the environment of fuzzy set theory. For this, we consider the application of waste reduction and recycling from green supply chain management and based on initiated operators we try to evaluate our problems.

For this, we consider the collection of alternatives $\mathfrak{T}_1^{\epsilon}, \mathfrak{T}_2^{\epsilon}, \dots \mathfrak{T}_{\mathcal{N}}^{\epsilon}$ with the collection of attributes for each alternative $\mathfrak{T}_1^{AT}, \mathfrak{T}_2^{AT}, \dots \mathfrak{T}_m^{AT}$. Further, we consider the weight vector for each attribute $\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_{q} = 1$. Moreover, we compute the decision matrix by using the information of NFRVs, such as the upper and lower approximation space for NFRSs is described in the shape:

$$\Lambda^{NFUA}(\mathfrak{M}) = \left\{\mathcal{H}, \phi_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}), \sigma_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}), \kappa_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) | \mathcal{H} \in \Upsilon\right\}$$

$$\Lambda^{NFLA}(\mathfrak{M}) = \left\{\mathcal{H}, \phi_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}), \sigma_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}), \kappa_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) | \mathcal{H} \in \Upsilon\right\}$$

Where Υ represents the fixed set with sub-relation $\Lambda \in \Upsilon \times \Upsilon$, then the pair (Υ, Λ), called neutrosophic fuzzy space of the approximation and $\mathfrak{M} \subseteq NFS(\Upsilon)$. Further, we explained the technique of truth, falsity, and abstinence grade, such as

$$\phi_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) = \bigvee \ell \in \Upsilon \left[\phi_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \vee \phi_{\mathfrak{M}(\mathcal{H})}\right]$$

$$\sigma_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) = \bigwedge \ell \in \Upsilon \left[\sigma_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \wedge \sigma_{\mathfrak{M}(\mathcal{H})}\right]$$

$$\kappa_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) = \bigwedge \ell \in \Upsilon \left[\kappa_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \wedge \kappa_{\mathfrak{M}(\mathcal{H})}\right]$$

$$\phi_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) = \bigwedge \ell \in \Upsilon \left[\phi_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \wedge \phi_{\mathfrak{M}(\mathcal{H})}\right]$$

$$\sigma_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) = \bigvee \ell \in \Upsilon \left[\sigma_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \vee \sigma_{\mathfrak{M}(\mathcal{H})}\right]$$

$$\kappa_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) = \bigvee \ell \in \Upsilon \left[\kappa_{\Lambda(\mathcal{H})}(\mathcal{H}, \ell) \vee \kappa_{\mathfrak{M}(\mathcal{H})}\right]$$

For the above technique, we have the following technique, such as $0 \leq \phi_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) + \sigma_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) + \kappa_{\Lambda^{NFUA}(\mathfrak{M})}(\mathcal{H}) \leq 3$ and $0 \leq \phi_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) + \sigma_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) + \kappa_{\Lambda^{NFLA}(\mathfrak{M})}(\mathcal{H}) \leq 3$. Finally, the pair of upper and lower space $\left(\Lambda^{NFLA}(\mathfrak{M}), \Lambda^{NFUA}(\mathfrak{M})\right)$, called NFRS, if both are not equal. Further, the NFRVs are described in the shape: $\mathfrak{T}_{q} = \left((\phi_{q}^{lb}, \sigma_{q}^{lb}, \kappa_{q}^{lb}), (\phi_{q}^{ub}, \sigma_{q}^{ub}, \kappa_{q}^{ub})\right), \mathcal{N} = 1,2, \dots, \mathcal{N}$. The procedure of the decision-making technique is described below:

Step 1. Compute the matrix of information based on NFRVs, if the matrix covers all the benefit types of information then we are not required to be normalized, but in the matrix, if we have cost type of information, then we required to be normalized, such as

$$N = \begin{cases} \left((\phi_{q}^{lb}, \sigma_{q}^{lb}, \kappa_{q}^{lb}), (\phi_{q}^{ub}, \sigma_{q}^{ub}, \kappa_{q}^{ub})\right) & for\ benefit \\ \left((\phi_{q}^{lb}, \sigma_{q}^{lb}, \kappa_{q}^{lb}), (\phi_{q}^{ub}, \sigma_{q}^{ub}, \kappa_{q}^{ub})\right)^{c} & for\ cost\ type \end{cases}$$

Step 2. Aggregate the information after normalization by using the technique of NFRSSWA operator and NFRSSWG operator, such as

$$NFRSSWA(\mathfrak{T}_1, \mathfrak{T}_2, \dots \mathfrak{T}_{\mathcal{N}}) = \left(\begin{array}{c} \left(1 - \left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(1 - \phi_q^{lb}\right)^{\beta} \right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(\sigma_q^{lb}\right)^{\beta} \right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(\kappa_q^{lb}\right)^{\beta} \right)^{\frac{1}{\beta}} \right) \\ \left(1 - \left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(1 - \phi_q^{ub}\right)^{\beta} \right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(\sigma_q^{ub}\right)^{\beta} \right)^{\frac{1}{\beta}}, \left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(\kappa_q^{ub}\right)^{\beta} \right)^{\frac{1}{\beta}} \right) \end{array} \right)$$

and

$$NFRSSWG(\mathfrak{T}_1, \mathfrak{T}_2, \dots \mathfrak{T}_{\mathcal{N}}) = \left(\begin{array}{c} \left(\left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(\phi_q^{lb}\right)^{\beta} \right)^{\frac{1}{\beta}}, 1 - \left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(1 - \sigma_q^{lb}\right)^{\beta} \right)^{\frac{1}{\beta}}, 1 - \left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(1 - \kappa_q^{lb}\right)^{\beta} \right)^{\frac{1}{\beta}} \right) \\ \left(\left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(\phi_q^{ub}\right)^{\beta} \right)^{\frac{1}{\beta}}, 1 - \left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(1 - \sigma_q^{ub}\right)^{\beta} \right)^{\frac{1}{\beta}}, 1 - \left(\sum_{q=1}^{\mathcal{N}} \mathfrak{M}_q \left(1 - \kappa_q^{ub}\right)^{\beta} \right)^{\frac{1}{\beta}} \right) \end{array} \right)$$

Step 3. Describe the Score values of the aggregated values, such as

$$SV(\mathfrak{T}_q) = \frac{1}{3}\left(\left(\phi_q^{lb} - \sigma_q^{lb} - \kappa_q^{lb}\right) + \left(\phi_q^{ub} - \sigma_q^{ub} - \kappa_q^{ub}\right)\right)$$

Step 4. Find the ranking values based on the Score values for evaluating the best optimal among the collection of information.

Further, we evaluate the problems of waste reduction and recycling in the application of green supply chain management based on initiated procedures to enhance the worth of the derived theory.

4.1 Waste Reduction and Recycling: Application in Green Supply Chain Management

In this section, we implement the application of waste reduction and recycling form green supply chain management according to initiated operators, called NFRSSWA operator and NFRSSWG operator. Waste reduction and recycling are two different techniques that are used to safe or rescue environmental impact based on their criteria. For this, we consider some alternatives in the shape, such as:

1) Lean Manufacturing.
2) Education and Awareness.
3) Product Designed.
4) Source Reduction.
5) Benefits

Further, we use the collection of weight vectors (0.4,0.3,0.2,0.1) based on the following attributes, such as social impact, political impact, environmental impact, and growth analysis. The procedure of the decision-making technique is described below:

Step 1. Compute the matrix of information based on NFRVs see Table 1, if the matrix covered all the benefit types of information then we are not required to be normalized, but in the matrix, if we have cost type of information, then we required to be normalized, such as

$$N = \begin{cases} \left(\left(\phi_q^{lb}, \sigma_q^{lb}, \kappa_q^{lb}\right), \left(\phi_q^{ub}, \sigma_q^{ub}, \kappa_q^{ub}\right)\right) & for\ benefit \\ \left(\left(\phi_q^{lb}, \sigma_q^{lb}, \kappa_q^{lb}\right), \left(\phi_q^{ub}, \sigma_q^{ub}, \kappa_q^{ub}\right)\right)^c & for\ cost\ type \end{cases}$$

The information in Table 1 in not need to be normalized.

Table 1. NFR decision matrix.

	$\mathfrak{T}_1^{AT}$	$\mathfrak{T}_2^{AT}$	$\mathfrak{T}_3^{AT}$	$\mathfrak{T}_4^{AT}$
$\mathfrak{T}_1^{\epsilon}$	$\begin{pmatrix}(0.5,0.4,0.7),\\(0.2,0.4,0.6)\end{pmatrix}$	$\begin{pmatrix}(0.51,0.41,0.71),\\(0.21,0.41,0.61)\end{pmatrix}$	$\begin{pmatrix}(0.52,0.42,0.72),\\(0.22,0.42,0.62)\end{pmatrix}$	$\begin{pmatrix}(0.53,0.43,0.73),\\(0.23,0.43,0.63)\end{pmatrix}$
$\mathfrak{T}_2^{\epsilon}$	$\begin{pmatrix}(0.4,0.6,0.7),\\(0.4,0.5,0.3)\end{pmatrix}$	$\begin{pmatrix}(0.41,0.61,0.71),\\(0.41,0.51,0.31)\end{pmatrix}$	$\begin{pmatrix}(0.42,0.62,0.72),\\(0.42,0.52,0.32)\end{pmatrix}$	$\begin{pmatrix}(0.43,0.63,0.73),\\(0.43,0.53,0.33)\end{pmatrix}$
$\mathfrak{T}_3^{\epsilon}$	$\begin{pmatrix}(0.7,0.4,0.5),\\(0.2,0.3,0.8)\end{pmatrix}$	$\begin{pmatrix}(0.71,0.41,0.51),\\(0.21,0.31,0.81)\end{pmatrix}$	$\begin{pmatrix}(0.72,0.42,0.52),\\(0.22,0.32,0.82)\end{pmatrix}$	$\begin{pmatrix}(0.73,0.43,0.53),\\(0.23,0.33,0.83)\end{pmatrix}$
$\mathfrak{T}_4^{\epsilon}$	$\begin{pmatrix}(0.3,0.6,0.4),\\(0.4,0.5,0.7)\end{pmatrix}$	$\begin{pmatrix}(0.31,0.61,0.41),\\(0.41,0.51,0.71)\end{pmatrix}$	$\begin{pmatrix}(0.32,0.62,0.42),\\(0.42,0.52,0.72)\end{pmatrix}$	$\begin{pmatrix}(0.33,0.63,0.43),\\(0.43,0.53,0.73)\end{pmatrix}$
$\mathfrak{T}_5^{\epsilon}$	$\begin{pmatrix}(0.6,0.4,0.3),\\(0.5,0.6,0.5)\end{pmatrix}$	$\begin{pmatrix}(0.61,0.41,0.31),\\(0.51,0.61,0.51)\end{pmatrix}$	$\begin{pmatrix}(0.62,0.42,0.32),\\(0.52,0.62,0.52)\end{pmatrix}$	$\begin{pmatrix}(0.63,0.43,0.33),\\(0.53,0.63,0.53)\end{pmatrix}$

Step 2. Aggregate the information after normalization by using the technique of NFRSSWA operator and NFRSSWG operator, see Table 2.

Table 2. Aggregated decision matrix.

	NFRSSWA operator	NFRSSWG operator
$\mathfrak{T}_1^{\epsilon}$	$\begin{pmatrix}(0.51474,0.41477,0.71496),\\(0.21516,0.41502,0.61497)\end{pmatrix}$	$\begin{pmatrix}(0.51499,0.41502,0.71465),\\(0.21479,0.41477,0,61497)\end{pmatrix}$
$\mathfrak{T}_2^{\epsilon}$	$\begin{pmatrix}(0.41477,0.61472,0.71497),\\(0.41503,0.515,0.31508)\end{pmatrix}$	$\begin{pmatrix}(0.41503,0.61498,0.71466),\\(0.41477,0.51475,0.31479)\end{pmatrix}$
$\mathfrak{T}_3^{\epsilon}$	$\begin{pmatrix}(0.71467,0.41478,0.51501),\\(0.21518,0.31509,0.81497)\end{pmatrix}$	$\begin{pmatrix}(0.71498,0.41504,0.51476),\\(0.21481,0.3148,0.814550)\end{pmatrix}$
$\mathfrak{T}_4^{\epsilon}$	$\begin{pmatrix}(0.31478,0.6147,0.41502),\\(0.41502,0.51499,0.71495)\end{pmatrix}$	$\begin{pmatrix}(0.31506,0.61497,0.41476)\\(0.41476,0.51474,0.71495)\end{pmatrix}$
$\mathfrak{T}_5^{\epsilon}$	$\begin{pmatrix}(0.61472,0.41477,0.31508),\\(0.515,0.61498,0.515)\end{pmatrix}$	$\begin{pmatrix}(0.61498,0.41503,0.31479),\\(0.51475,0.61472,0.51475)\end{pmatrix}$

Step 3. Describe the Score values of the aggregated values, see Table 3.

Table 3. Score decision matrix.

	NFRSSWA	NFRSSWG
$\mathfrak{T}_1^{\epsilon}$	-0.34354	-0.34297
$\mathfrak{T}_2^{\epsilon}$	-0.3102	-0.30965
$\mathfrak{T}_3^{\epsilon}$	-0.2436	-0.24297
$\mathfrak{T}_4^{\epsilon}$	-0.37683	-0.37629
$\mathfrak{T}_5^{\epsilon}$	-0.3101	-0.30967

Step 4. Find the ranking values based on the Score values for evaluating the best optimal among the collection of information, see Table 4.

Table 4. Ranking values.

Operator	Score values	Best optimal
NFRSSWA Operator	$\mathfrak{T}_3^{\epsilon} > \mathfrak{T}_5^{\epsilon} > \mathfrak{T}_2^{\epsilon} > \mathfrak{T}_1^{\epsilon} > \mathfrak{T}_4^{\epsilon}$	$\mathfrak{T}_3^{\epsilon}$
NFRSSWG Operator	$\mathfrak{T}_3^{\epsilon} > \mathfrak{T}_2^{\epsilon} > \mathfrak{T}_5^{\epsilon} > \mathfrak{T}_1^{\epsilon} > \mathfrak{T}_4^{\epsilon}$	$\mathfrak{T}_3^{\epsilon}$

According to the theory of NFRSSWA operator and NFRSSWG operator, we obtained the best optimal is $\mathfrak{T}_3^{\epsilon}$. Further, we evaluate the problems of waste reduction and recycling in the application of green supply chain management based on initiated procedures to enhance the worth of the derived theory. Further, we discuss the stability or influence of the parameter by using the value of the NFRSSWA operator, see Table 5.

Table 5. NFRSSWA ranking matrix for different values of parameters.

Parameter	Score values	Ranking values
$\boldsymbol{\beta}$ =2	$-0.34354, -0.31047, -0.24393, -0.37683, -0.3101$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =3	$-0.34383, -0.30939, -0.24268, -0.37709, -0.31046$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =4	$-0.34411, -0.310475, -0.24427, -0.37734, -0.31073$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =5	$-0.3444, -0.31102, -0.24461, -0.37759, -0.311$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =6	$-0.34468, -0.31129, -0.24494, -0.37785, -0.31127$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =7	$-0.34497, -0.31157, -0.24527, -0.3781, -0.31127$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =8	$-0.34525, -0.31184, -0.2456, -0.37458, -0.3118$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =9	$-0.34553, -0.31211, -0.24592, -0.3786, -0.31207$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =10	$-0.34581, -0.31238, -0.25625, -0.37885, -0.31233$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =15	$-0.34717, -0.3137, -0.24782, -0.38009, -0.31364$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$

According to the theory of the NFRSSWA operator, we obtained the best optimal is $\mathfrak{T}_3^{\epsilon}$. Further, we discuss the stability or influence of the parameter by using the value of the NFRSSWG operator, see Table 6.

Table 6. NFRSSWG ranking matrix for different values of parameters.

Parameter	Score values	Ranking values
$\boldsymbol{\beta}$ =2	$-0.34297, -0.30965, -0.24297, -0.37629, -0.30967$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =3	$-0.34269, -0.30939, -0.24268, -0.376, -0.30941$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =4	$-0.34241, -0.30912, -0.24239, -0.37572, -0.30916$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =5	$-0.34213, -0.30886, -0.2421, -0.37543, -0.3089$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =6	$-0.34185, -0.30859, -0.24181, -0.37515, -0.30865$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =7	$-0.34158, -0.30833, -0.24153, -0.37486, -0.3084$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =8	$-0.3413, -0.30807, -0.24125, -0.37458, -0.30815$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =9	$-0.34103, -0.30781, -0.24097, -0.3743, -0.3079$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =10	$-0.34076, -0.30755, -0.24069, -0.37402, -0.30765$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
$\boldsymbol{\beta}$ =15	$-0.33942, -0.30627, -0.23935, -0.37265, -0.30641$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$

According to the theory of the NFRSSWG operator, we obtained the best optimal is $\mathfrak{T}_3^{\epsilon}$. Further, we described the geometrical interpretation of the information in Table 5 and Table 6 in the shape of Figure 1 and Figure 2.

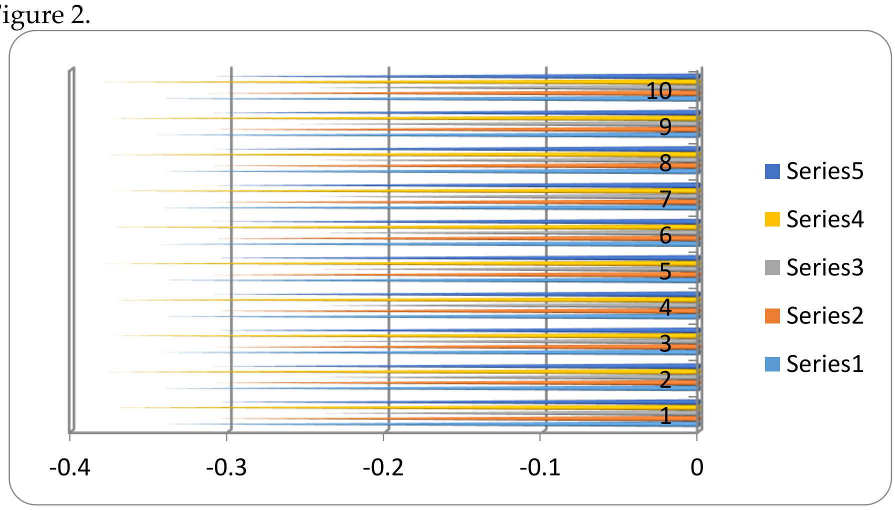

Figure 1. The geometrical shape of the data is in Table 5.

5. Comparative Analysis

In this section, we compare the proposed ranking values with the ranking values of some existing techniques to enhance the worth of the proposed theory. For comparing the proposed ranking values with some existing ranking values, we have considered the following existing techniques, such as the prorposed theory of Xu [22], Xu and Yager [23], Garg et al. [24], Wang and Liu [25], Khan et al. [26], Mondal et al. [27], Zhang et al. [28], Yuan et al. [29], Liu et al. [30], Khan et al. [31]. The comparative analysis is listed in Table 7.

Table 7. Comparative analysis based on the data in Table 1.

Methods	Score values	Ranking values
Xu [22]	n/a	n/a
Xu and Yager [23]	n/a	n/a
Garg et al. [24]	n/a	n/a
Wang and Liu [25]	n/a	n/a
Khan et al. [26]	n/a	n/a
Mondal et al. [27]	n/a	n/a
Zhang et al. [28]	n/a	n/a
Yuan et al. [29]	n/a	n/a
Liu et al. [30]	n/a	n/a
Khan et al. [31]	n/a	n/a
NFRSSWA operator	$-0.34354, -0.31047, -0.24393, -0.37683, -0.3101$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$
NFRSSWG operator	$-0.34297, -0.30965, -0.24297, -0.37629, -0.30967$	$\mathfrak{T}_3^{\epsilon} \succ \mathfrak{T}_2^{\epsilon} \succ \mathfrak{T}_5^{\epsilon} \succ \mathfrak{T}_1^{\epsilon} \succ \mathfrak{T}_4^{\epsilon}$

According to the theory of NFRSSWA operator and NFRSSWG operator, we obtained the best optimal is $\mathfrak{T}_3^{\epsilon}$. Further, we explained the limitations of the existing techniques. After all, the existing techniques have failed to evaluate the data in Table 1 because the existing techniques are the special cases of the proposed theory.

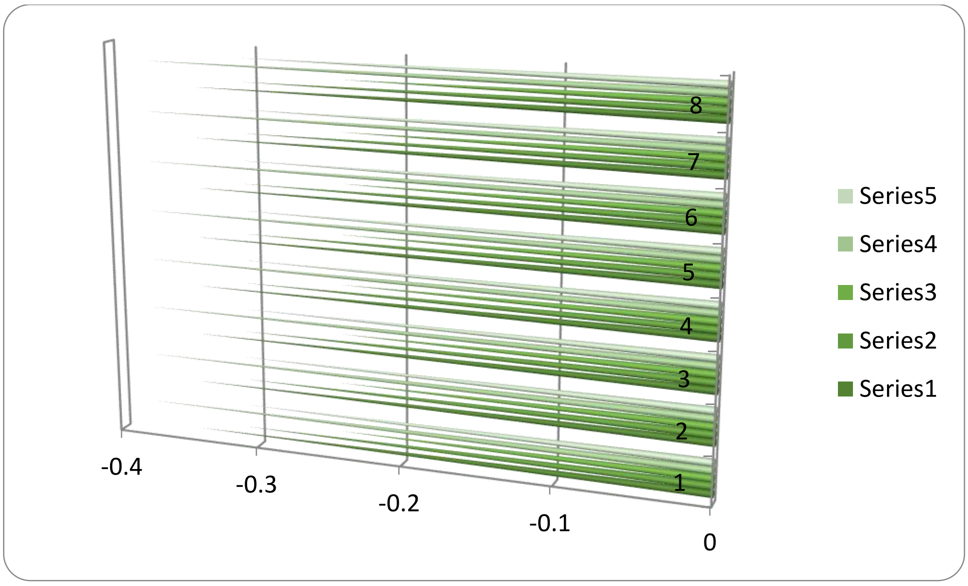

Figure 2. The geometrical shape of the data is in Table 6.

Hence, the proposed theory is very novel and effective and because of this reason, they are more effective than the prevailing information.

6. Conclusion

The major contribution of this manuscript is listed below:

i). We initiated the technique of SS operational laws based on NFR values for SSTN and SSTCN.
ii). We derived the NFRSSWA operator and the NFRSSWG operator. Some basic properties for the above-initiated techniques are derived.
iii). We described the application in green supply chain management, called waste reeducation and recycling based on initiated operators in MADM problems.
iv). We illustrated an example for comparing the ranking values of the proposed techniques with the ranking values of the existing technique.

In the future, we will expose the new theory of neutrosophic hesitant fuzzy rough sets and their extensions. Further, we will derive some methods, techniques, and measures for the above-initiated ideas. Lastly, we will utilize it in the environment of decision-making techniques, artificial intelligence, game theory, data science, and many others to enhance the worth of the proposed theory.

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Author Contributions
All authors contributed equally to this research.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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Conflict of interest
The authors declare that there is no conflict of interest in the research.

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