Neutrosophic Systems with Applications, Vol. 20, 2024

https://doi.org/10.61356/j.nswa.2024.20346

Normed-Bifuzzy Valued-Ideals of Semigroups

Mohammad Hamidi [1,*] and Rasul Rasuli [1]

[1] Department of Mathematics, Payame Noor University (PNU), Tehran, Iran;
Emails: m.hamidi@pnu.ac.ir; rasulirasul@yahoo.com.

* Correspondence: m.hamidi@pnu.ac.ir.

Abstract. As concerning the views of T norms and T conorms, the intent of article is to define and probe the fuzzy semigroups, fuzzy ideals, fuzzy bi-ideals, bifuzzy subsemigroups, bifuzzy ideals, bifuzzy bi-ideals, fuzzy $(1,2)$-ideals and bifuzzy $(1,2)$-ideals in any given semigroup. Also, we indicate and study their basic properties of them in completely regular semigroups. Finally, we extend these concepts and so characterise (pre)image of them in semigroup homomorphisms.

Keywords: Regular semigroups, Fuzzy subsemigroups, Fuzzy bi-ideals, Bifuzzy subsemigroup, Bifuzzy ideal, Bifuzzy bi-ideals.

---

1. Introduction

Theory of semigroup with one operation in univesal algebra, initiated in the 20 th century [7]. In the real world, a purely mathematical set alone is not of much use, and having a weight for each element in this set is a necessity. Combining algebraic structures as systematic systems in the form of sets with labeled and weighted elements can be used as precise complex networks with many applications in the real world. Therefore, in addition to algebraic structures, it is necessary to have collections that can create indexes or weights in the elements of these structures. Fuzzy set theory which is inserted (in this regard) by Zadeh [28] is a generalization of crisp sets. Based on this concepts, Kuroki [12, 13], presented the fuzzy semigroup and fuzzy ideals in semigroups and delineated them and later was extended by Mordeson et al. [18]. In [25], the substructures prime, strongly prime, semiprime and irreducible fuzzy bi-ideals of a semigroup were expressed by Shabir, Jun and Bano. The related notions of fuzzy bi-ideals [11, 14, 27], intuitionistic fuzzy sets [1], intuitionistic fuzzy generalized bi-ideal of a semigroup [9] and intuitionistic fuzzy bi-ideals and intuitionistic F. I [10], are mentioned in the bibliography. Today, some research are investigated in these scoups such as hesitant bifuzzy set (an introduction): a new approach to assess the reliability of

the systems [4], B. F. I of d-algebras [5], singlevalued neutrosophic filters in EQ-algebras [6], EQ-algebras based on fuzzy hyper EQ-filters [8], F. I and F. F on topologies generated by fuzzy relations [24] and rough bipolar F. I in semigroups [17]. In this work, we inspected some assets of fuzzy algebraic structures, by using norms, defined fuzzy subsemigroups as $FST(S)$, F. I as $FIT(S)$, fuzzy bi-ideals as $FBIT(S)$, bifuzzy subsemigroup as $BFSN(S)$, B. F. I as $BFIN(S)$, bifuzzy bi-ideals as $BFBIN(S)$, of semigroup S. In addition, we by using norms, define the novel concept fuzzy (1,2)-ideal of S as $F(1,2)IT(S)$ and bifuzzy (1,2)-ideal of S as $BF(1,2)IN(S)$ and we prove that $\mho = (\xi_\mho, \partial_\mho) \in BF(1,2)IN(S)$ if and only if $\xi_\mho \in F(1,2)IT(S)$ and $\bar{\partial_\mho} \in F(1,2)IT(S)$. Also we show that $\mho = (\xi_\mho, \partial_\mho) \in BF(1,2)IN(S)$ if and only if $\triangle\mho = (\xi_\mho, \bar{\xi_\mho}) \in BF(1,2)IN(S)$ and $\nabla\mho = (\bar{\partial_\mho}, \partial_\mho) \in BF(1,2)IN(S)$. Aldo we show that for any given $\mho = (\xi_\mho, \partial_\mho) \in BF(1,2)IN(S)$ and $B = (\xi_B, \partial_B) \in BF(1,2)IN(S)$, $\mho \cap B \in BF(1,2)IN(S)$. Finally we prove that under some conditions $\mho = (\xi_\mho, \partial_\mho) \in BF(1,2)IN(S) \iff \mho = (\xi_\mho, \partial_\mho) \in BFBIN(S)$. In final, we investigate image and pre-image of $FIT(S), FBIT(S), BFSN(S), BFIN(S), BFBIN(S), F(1,2)IT(S), BF(1,2)$
$IN(S)$ under homomorphisms.

2. Peliminaries

Lemma 2.1. [15, 19] *As* S $= (S, *)$ *be a semigroup so for all* $a \in S$*,* S *is completely regular iff* $a \in a^2Sa^2$.

Definition 2.2. [3, 7] Let $\mathcal{O} \neq \emptyset$ be a set. Define

(*i*) $\mho = \{(x, \eth(x)) \;:\; x \in \mathcal{O}\}$ is a fuzzy subset of $\mathcal{O}$, which $\eth : \mathcal{O} \to [0,1]$ ($\eth \in [0,1]^{\mathcal{O}}$). For any $k \in [0,1]$, $\mathbb{U}(\eth; k) = \{x \in \mathcal{O} : \eth(x) \geq T^{nor}\}$ is a upper level cut set and $\mathbb{L}(\eth; k) = \{x \in \mathcal{O} : \eth(x) \leq T^{nor}\}$ is a lower level cut set.

(*ii*) $\mho = \{(x, \eth_\mho(x), \partial_\mho(x)) \mid x \in \mathcal{O}\}$ is a bifuzzy subset of $\mathcal{O}$, which $\eth_\mho, \partial_\mho \in [0,1]^{\mathcal{O}}$ and for all $x \in \mathcal{O}$ we get $0 \leq \eth_\mho(x) + \partial_\mho(x) \leq 1$ ($\mho \in BF(\mathcal{O})$).

Definition 2.3. [2] Let $l, m, n \in \mathcal{I} = [0,1]$.

(*i*) triangular norm is a map $T^{nor} : \mathcal{I} \times \mathcal{I} \to \mathcal{I}$, by $T^{nor}(l,1) = l$, $T^{nor}(l,m) \leq T^{nor}(l,n)$ if $m \leq n$, $T^{nor}(l,m) = T^{nor}(m,n)$ and $T^{nor}(l, T^{nor}(m,n)) = T^{nor}(T^{nor}(l,m), n)$.

(*ii*) triangular conorm is a function $C^{con} : \mathcal{I} \times \mathcal{I} \to \mathcal{I}$, by $C^{con}(l,0) = l$, $C^{con}(l,m) \leq C^{con}(l,n)$ if $m \leq n$, $C^{con}(l,m) = C^{con}(m,l)$ and $C^{con}(l, C^{con}(m,n)) = C^{con}(C^{con}(l,m), n)$.

Definition 2.4. [20, 21] Let $\eth \in [0,1]^S$ and $x, y, w \in S$.

(*i*) $\eth$ is a fuzzy subsemigroup of S regarding T^{nor}, if $\eth(xy) \geq T^{nor}(\eth(x), \eth(y))$($\eth \in FST(S)$).

(*ii*) $\eth$ is a F. I of S regarding T^{nor}, if (1) $\eth(xy) \geq T^{nor}(\eth(x), \eth(y))$, (2) $\eth(xy) \geq \eth(y)$, (3) $\eth(xy) \geq \eth(x)$ ($\eth \in FIT(S)$).

(*iii*) $\eth$ is a fuzzy bi-ideal of S regarding T^{nor}-norm T^{nor} if, (1) $\eth(xy) \geq T^{nor}(\eth(x), \eth(y))$, (2) $\eth(xwy) \geq T^{nor}(\eth(x), \eth(y))$ ($\eth \in FBIT(S)$).

Definition 2.5. [20, 21] Let $\mho = (\eth_\mho, \partial_\mho) \in BF(S)$. Then $\mho$ is a

(*i*) bifuzzy subsemigroup of S regarding T^{nor} and a C^{con}, if, (1) $\eth_\mho(xy) \geq T^{nor}(\eth_\mho(x), \eth_\mho(y))$, (2) $\partial_\mho(xy) \leq C^{con}(\partial_\mho(x), \partial_\mho(y))$ ($\eth \in BFSN(S)$).

(*ii*) B. F. I of S regarding T^{nor} and C^{con}, if (1) $\eth_\mho(xy) \geq T^{nor}(\eth_\mho(x), \eth_\mho(y))$, (2) $\eth_\mho(xy) \geq \eth_\mho(x)$, (3) $\eth_\mho(xy) \geq \eth_\mho(y)$, (4) $\partial_\mho(xy) \leq C^{con}(\partial_\mho(x), \partial_\mho(y))$, (5) $\partial_\mho(xy) \leq \partial_\mho(y)$, (6) $\partial_\mho(xy) \leq \partial_\mho(x)$ ($\mho \in BFIN(S)$).

(*iii*) bifuzzy bi-ideal of S regarding T^{nor} and C^{con}, if it satisfies: (1) $\eth_\mho(xy) \geq T^{nor}(\eth_\mho(x), \eth_\mho(y))$, (2) $\eth_\mho(xwy) \geq T^{nor}(\eth_\mho(x), \eth_\mho(y))$, (3) $\partial_\mho(xy) \leq C^{con}(\partial_\mho(x), \partial_\mho(y))$, (4) $\partial_\mho(xwy) \leq C^{con}(\partial_\mho(x), \partial_\mho(y))$ ($\mho \in BFBIN(S)$).

3. **Results On** $BFBIN(S)$

In this section, we investigate some properties of $BFBIN(S), BFIN(S)$ and obtain the relation beween of them. Let $\mho = (\eth_\mho, \partial_\mho), B = (\eth_B, \partial_B) \in BF(\mathcal{O})$. Then $\mho \cap B = (\eth_\mho, \partial_\mho) \cap (\eth_B, \partial_B) = (\eth_\mho \cap \eth_B, \partial_\mho \cap \partial_B) = (\eth_{\mho\cap B}, \partial_{\mho\cap B})$ is a bifuzzy subset, which $\mho \cap B : S \to [0,1]$ will be defined by $(\mho \cap B)(s) = (\eth_{\mho\cap B}(s), \partial_{\mho\cap B}(s)) = (T^{nor}(\eth_\mho(s), \eth_B(s)), C^{con}(\partial_\mho(s), \partial_B(s)))$ with $s \in S$.

Theorem 3.1. *Let* $\mho = (\eth_\mho, \partial_\mho) \in BFBIN(S)$ *and* $B = (\eth_B, \partial_B) \in BFBIN(S.)$ *Thus* $\mho \cap B \in BFBIN(S)$.

Proof. Let $p, q, r \in \mathcal{O}$. Then

$$\begin{aligned}\eth_{\mho\cap B}(pq) &= T^{nor}(\eth_\mho(pq), \eth_B(pq)) \geq T^{nor}(T^{nor}(\eth_\mho(p), \eth_\mho(q)), T^{nor}(\eth_B(p), \eth_B(q))) \\ &= T^{nor}(T^{nor}(\eth_\mho(p), \eth_B(p)), T^{nor}(\eth_\mho(q), \eth_B(q))) \ = T^{nor}(\eth_{\mho\cap B}(p), \eth_{\mho\cap B}(q)).\end{aligned}$$

In a similar way, one can see that

$\eth_{\mho\cap B}(prq) \geq T^{nor}(\eth_{\mho\cap B}(p), \eth_{\mho\cap B}(q))$, $\partial_{\mho\cap B}(pq) \leq C^{con}(\partial_{\mho\cap B}(p), \partial_{\mho\cap B}(q))$ and $\partial_{\mho\cap B}(prq) \leq C^{con}(\partial_{\mho\cap B}(p), \partial_{\mho\cap B}(q))$. Therefore, we get that $\mho \cap B \in BFBIN(S)$. □

Example 3.2. Let S has a zero and

$$x * y = \begin{cases} x & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$$

and assume that $|S| > 2$, where $|S|$ denotes the cardinality of S, then $(S, *)$ is a semigroup. Define $\mho = (\eth_\mho, \partial_\mho) \in BF(S)$ and $B = (\eth_B, \partial_B) \in BF(S)$ as

$$\eth_\mho(x) = \begin{cases} 0.55 & \text{if } x = 0 \\ 0.2 & \text{otherwise} \end{cases}, \partial_\mho(x) = \begin{cases} 0.35 & \text{if } x = 0 \\ 0.15 & \text{otherwise} \end{cases},$$

$$\eth_B(x) = \begin{cases} 0.45 & \text{if } x = 0 \\ 0.1 & \text{otherwise} \end{cases}, \partial_B(x) = \begin{cases} 0.25 & \text{if } x = 0 \\ 0.05 & \text{otherwise} \end{cases},$$

$T^{nor}(u, z) = T_m^{nor}(u, z) = \min\{u, z\}$ and $C^{con}(u, z) = C_m^{con}(u, z) = \max\{u, z\}$, for all $u, z \in \mathcal{I}$. Then $\mho = (\eth_\mho, \partial_\mho) \in BFBIN(S)$ and $B = (\eth_B, \partial_B) \in BFBIN(S)$ and $\mho \cap B \in BFBIN(S)$.

Corollary 3.3. (1) *If* $\{\mho_i\}_{i \in I} \subseteq BFBIN(S)$, *then* $\mho = \bigcap_{i \in I} \mho_i \in BFBIN(S)$.

(2) *If* $\{\mho_i\}_{i \in I} \subseteq BFSN(S)$, *then* $\mho = \bigcap_{i \in I} \mho_i \in BFSN(S)$.

Theorem 3.4. *Let* $\eth_\mho \in FBIT(S)$. *Then* $\boxminus\mho = (\eth_\mho, \bar{\eth}_\mho) \in BFBIN(S)$, *which* $\bar{\eth}_\mho = 1 - \eth_\mho$.

Proof. Let $r, s, t \in \mathcal{O}$. Since $\mho = (\eth_\mho, \partial_\mho) \in BFBIN(S)$, we get $\eth_\mho(rs) \geq T^{nor}(\eth_\mho(r), \eth_\mho(s))$ and $\eth_\mho(rts) \geq T^{nor}(\eth_\mho(r), \eth_\mho(s))$. Now

$$\begin{aligned} \eth_\mho(rs) \geq T^{nor}(\eth_\mho(r), \eth_\mho(s)) &\Rightarrow -\eth_\mho(rs) \leq -T^{nor}(\eth_\mho(r), \eth_\mho(s)) \\ &\Rightarrow 1 - \eth_\mho(rs) \leq 1 - T^{nor}(\eth_\mho(r), \eth_\mho(s)) \Rightarrow \bar{\eth}_\mho(rs) \leq C^{con}(1 - \eth_\mho(r), 1 - \eth_\mho(s)) \\ &\Rightarrow \bar{\eth}_\mho(rs) \leq C^{con}(\bar{\eth}_\mho(r), \bar{\eth}_\mho(s)) \end{aligned}$$

and

$$\begin{aligned} \eth_\mho(rts) \geq T^{nor}(\eth_\mho(r), \eth_\mho(s)) &\Rightarrow -\eth_\mho(rts) \leq -T^{nor}(\eth_\mho(r), \eth_\mho(s)) \\ &\Rightarrow 1 - \eth_\mho(rts) \leq 1 - T^{nor}(\eth_\mho(r), \eth_\mho(s)) \Rightarrow \bar{\eth}_\mho(rts) \leq C^{con}(1 - \eth_\mho(r), 1 - \eth_\mho(s)) \\ &\Rightarrow \bar{\eth}_\mho(rts) \leq C^{con}(\bar{\eth}_\mho(r), \bar{\eth}_\mho(s)). \end{aligned}$$

Therefore, $\boxminus\mho = (\eth_\mho, \bar{\eth}_\mho) \in BFBIN(S)$. □

We recall that T^{nor} and C^{con} are idempotent, if for any $t \in \mathcal{I}, T^{nor}(t, t) = t$ and $S(t, t) = t$.

Theorem 3.5. *Let* $\mho = (\eth_\mho, \partial_\mho) \in BFBIN(S)$. *If* T^{nor} *and* C^{con} *are idempotent which* S *is completely regular, then* $\mho(s) = \mho(s^2)$ *with* $s \in S$.

Proof. Assume $s \in S$. Since S is completely regular, using of Lemma 2.1, there exists $x \in S$ so $s = s^2 x s^2$. Now

$$\eth_\mho(s) = \eth_\mho(s^2 x s^2) \geq T^{nor}(\eth_\mho(s^2), \eth_\mho(s^2)) = \eth_\mho(s^2) = \eth_\mho(ss) \geq T^{nor}(\eth_\mho(s), \eth_\mho(s)) = \eth_\mho(s)$$

and so $\eth_{\mho}(s) = \eth_{\mho}(s^2)$. Also $\partial_{\mho}(s) = \partial_{\mho}(s^2xs^2) \leq C^{con}(\partial_{\mho}(s^2), \partial_{\mho}(s^2)) = \partial_{\mho}(s^2) = \partial_{\mho}(ss) \leq C^{con}(\partial_{\mho}(s), \partial_{\mho}(s)) = \partial_{\mho}(s)$ and so $\partial_{\mho}(s) = \partial_{\mho}(s^2)$. Thus, we get that $\mho(s) = (\eth_{\mho}(s), \partial_{\mho}(s)) = (\eth_{\mho}(s^2), \partial_{\mho}(s^2)) = \mho(s^2)$. □

Theorem 3.6. *Let $\mho = (\eth_{\mho}, \partial_{\mho}) \in BFIN(S)$. If T^{nor} and C^{con} are idempotent and S is an intra-regular, then for all $a \in S$, $\mho(a) = \mho(a^2)$,*

Proof. Suppose $a \in S$. Since S is an intra-regular, find $x, y \in S$ so $a = xa^2y$. Then $\eth_{\mho}(a) = \eth_{\mho}(xa^2y) \geq \eth_{\mho}(a^2y) \geq \eth_{\mho}(a^2) \geq T^{nor}(\eth_{\mho}(a), \eth_{\mho}(a)) = \eth_{\mho}(a)$ thus $\eth_{\mho}(a) = \eth_{\mho}(a^2)$. Also, $\partial_{\mho}(a) = \partial_{\mho}(xa^2y) \leq \partial_{\mho}(a^2y) \leq \partial_{\mho}(a^2) \leq C^{con}(\partial_{\mho}(a), \partial_{\mho}(a)) = \partial_{\mho}(a)$, so $\partial_{\mho}(a) = \partial_{\mho}(a^2)$. Therefore, we get that $\mho(a) = (\eth_{\mho}(a), \partial_{\mho}(a)) = (\eth_{\mho}(a^2), \partial_{\mho}(a^2)) = \mho(a^2)$. □

Theorem 3.7. *Let $\mho = (\eth_{\mho}, \partial_{\mho}) \in BFIN(S)$. If T^{nor} and C^{con} are idempotent and S is an intra-regular, then for all $a, b \in S$, $\mho(ab) = \mho(ba)$.*

Proof. Let $a, b \in S$. Using Theorem 3.6, we have that $\eth_{\mho}(a) = \eth_{\mho}(a^2)$ and $\partial_{\mho}(a) = \partial_{\mho}(a^2)$. It follows that $\eth_{\mho}(ab) = \eth_{\mho}((ab)^2)$ and $\partial_{\mho}(ab) = \partial_{\mho}((ab)^2)$. Thus

$$\begin{aligned}\eth_{\mho}(ab) &= \eth_{\mho}((ab)^2) = \eth_{\mho}(abab) \geq \eth_{\mho}(bab) \geq \eth_{\mho}(ba)\\ &= \eth_{\mho}((ba)^2) = \eth_{\mho}(baba) \geq \eth_{\mho}(aba) \geq \eth_{\mho}(ab)\end{aligned}$$

then $\eth_{\mho}(ab) = \eth_{\mho}(ba)$. In addition,

$$\begin{aligned}\partial_{\mho}(ab) &= \partial_{\mho}((ab)^2) = \partial_{\mho}(abab) \leq \partial_{\mho}(bab) \leq \partial_{\mho}(ba)\\ &= \partial_{\mho}((ba)^2) = \partial_{\mho}(baba) \leq \partial_{\mho}(aba) \leq \partial_{\mho}(ab),\end{aligned}$$

then $\partial_{\mho}(ab) = \partial_{\mho}(ba)$. Thus, we get that $\mho(ab) = (\eth_{\mho}(ab), \partial_{\mho}(ab)) = (\eth_{\mho}(ba), \partial_{\mho}(ba)) = \mho(ba)$.
□

Theorem 3.8. *Let $\mho = (\eth_{\mho}, \partial_{\mho}) \in BF(S)$. Then $\mho = (\eth_{\mho}, \partial_{\mho}) \in BFIN(S)$ if and only if $\eth_{\mho} \in FBIT(S)$ and $\bar{\partial}_{\mho} \in FBIT(S)$.*

Proof. Let $\mho = (\eth_{\mho}, \partial_{\mho}) \in BFIN(S)$. Then for all $f, g, h \in S$, we get that $\eth_{\mho}(fg) \geq T^{nor}(\eth_{\mho}(f), \eth_{\mho}(g))$ and $\eth_{\mho}(fhg) \geq T^{nor}(\eth_{\mho}(f), \eth_{\mho}(g))$ which mean that $\eth_{\mho} \in FBIT(S)$. Also

$$\begin{aligned}&\partial_{\mho}(fg) \leq C^{con}(\partial_{\mho}(f), \partial_{\mho}(g)) \Longleftrightarrow -\partial_{\mho}(fg) \geq -C^{con}(\partial_{\mho}(f), \partial_{\mho}(g))\\ &\qquad \Longleftrightarrow 1 - \partial_{\mho}(fg) \geq 1 - C^{con}(\partial_{\mho}(f), \partial_{\mho}(g)) \Longleftrightarrow \bar{\partial}_{\mho}(fg) \geq T^{nor}(1 - \partial_{\mho}(f), 1 - \partial_{\mho}(g))\\ &\qquad \Longleftrightarrow \bar{\partial}_{\mho}(fg) \geq T^{nor}(\bar{\partial}_{\mho}(f), \bar{\partial}_{\mho}(g)),\end{aligned}$$

thus $\bar{\partial}_{\mho}(fg) \geq T^{nor}(\bar{\partial}_{\mho}(f), \bar{\partial}_{\mho}(g))$. Also

$$\begin{aligned}
\partial_{\mho}(fhg) \leq C^{con}(\partial_{\mho}(f), \partial_{\mho}(g)) &\Longleftrightarrow -\partial_{\mho}(fhg) \geq -C^{con}(\partial_{\mho}(f), \partial_{\mho}(g)) \\
&\Longleftrightarrow 1 - \partial_{\mho}(fhg) \geq 1 - C^{con}(\partial_{\mho}(f), \partial_{\mho}(g)) \Longleftrightarrow \bar{\partial}_{\mho}(fhg) \geq T^{nor}(1 - \partial_{\mho}(f), 1 - \partial_{\mho}(g)) \\
&\Longleftrightarrow \bar{\partial}_{\mho}(fhg) \geq T^{nor}(\bar{\partial}_{\mho}(f), \bar{\partial}_{\mho}(g)),
\end{aligned}$$

then $\bar{\partial}_{\mho}(fhg) \geq T^{nor}(\bar{\partial}_{\mho}(f), \bar{\partial}_{\mho}(g))$ and so $\bar{\partial}_{\mho} \in FBIT(S)$.

Conversly, let $\eth_{\mho} \in FBIT(S)$, $\bar{\partial}_{\mho} \in FBIT(S)$ and $f, g, h \in S$. As $\eth_{\mho} \in FBIT(S)$ so $\eth_{\mho}(fg) \geq T^{nor}(\eth_{\mho}(f), \eth_{\mho}(g))$ and $\eth_{\mho}(fhg) \geq T^{nor}(\eth_{\mho}(f), \eth_{\mho}(g))$. Since $\bar{\partial}_{\mho} \in FBIT(S)$,

$$\begin{aligned}
&\bar{\partial}_{\mho}(fg) \geq T^{nor}(\bar{\partial}_{\mho}(f), \bar{\partial}_{\mho}(g)) \Longleftrightarrow -\bar{\partial}_{\mho}(fg) \leq -T^{nor}(\bar{\partial}_{\mho}(f), \bar{\partial}_{\mho}(g)) \\
&\Longleftrightarrow 1 - \bar{\partial}_{\mho}(fg) \leq 1 - T^{nor}(\bar{\partial}_{\mho}(f), \bar{\partial}_{\mho}(g)) \Longleftrightarrow \partial_{\mho}(fg) \leq C^{con}(1 - \bar{\partial}_{\mho}(f), 1 - \bar{\partial}_{\mho}(g) \\
&\Longleftrightarrow \partial_{\mho}(fg) \leq C^{con}(\partial_{\mho}(f), \partial_{\mho}(g)),
\end{aligned}$$

thus $\partial_{\mho}(fg) \leq C^{con}(\partial_{\mho}(f), \partial_{\mho}(g))$ and

$$\begin{aligned}
\bar{\partial}_{\mho}(fhg) \geq T^{nor}(\bar{\partial}_{\mho}(f), \bar{\partial}_{\mho}(g)) &\Longleftrightarrow -\bar{\partial}_{\mho}(fhg) \leq -T^{nor}(\bar{\partial}_{\mho}(f), \bar{\partial}_{\mho}(g)) \\
&\Longleftrightarrow 1 - \bar{\partial}_{\mho}(fhg) \leq 1 - T^{nor}(\bar{\partial}_{\mho}(f), \bar{\partial}_{\mho}(g)) \Longleftrightarrow \partial_{\mho}(fhg) \leq C^{con}(1 - \bar{\partial}_{\mho}(f), 1 - \bar{\partial}_{\mho}(g) \\
&\Longleftrightarrow \partial_{\mho}(fhg) \leq C^{con}(\partial_{\mho}(f), \partial_{\mho}(g)),
\end{aligned}$$

so $\partial_{\mho}(fhg) \leq C^{con}(\partial_{\mho}(f), \partial_{\mho}(g))$. Therefore, we conclude that $\mho = (\eth_{\mho}, \partial_{\mho}) \in BFIN(S)$. □

4. **Bifuzzy(1,2)-ideals of subsemigrous and norms**

In this section, we define the notation of bifuzzy(1,2)-ideals of subsemigrous regarding norms and study their properties. In [26], F. Wang, introduced the concepts of fuzzy subsemigroups and fuzzy (1,2)-ideal in semigroups an in special case. In what follows, we introduce the fuzzy (1,2)-ideal and bifuzzy (1,2)-ideal of semigroups regarding any arbitrary triangular norms and any arbitrary triangular conorms.

Definition 4.1. Let $\eth \in \mathcal{I}^S$ and $x, y, z, w \in S$. Then

- (i) $\eth$ is a fuzzy (1,2)-ideal of S regarding T^{nor} if, (1) $\eth(xy) \geq T^{nor}(\eth(x), \eth(y))$, (2) $\eth(xw(yz)) \geq T^{nor}(\eth(x), T^{nor}(\eth(y), \eth(z)))$ $(\eth \in F(1,2)IT(S))$.
- (ii) A bifuzzy set $\mho = (\eth_{\mho}, \partial_{\mho}) \in BF(S)$ is a of S regarding T^{nor} and C^{con}, if (1) $\eth_{\mho}(xy) \geq T^{nor}(\eth_{\mho}(x), \eth_{\mho}(y))$ (2) $\eth_{\mho}(xw(yz)) \geq T^{nor}(\eth_{\mho}(x), T^{nor}(\eth_{\mho}(y), \eth_{\mho}(z)))$ (3) $\partial_{\mho}(xy) \leq C^{con}(\partial_{\mho}(x), \partial_{\mho}(y))$ (4) $\partial_{\mho}(xw(yz)) \leq C^{con}(\partial_{\mho}(x), C^{con}(\partial_{\mho}(y), \partial_{\mho}(z)))$ $(\eth \in BF(1,2)IN(S))$.

Example 4.2. Let $S = \{-2, -4, -6, -8\}$. Then $(S, *)$ is a semigroup and $\eth \in \mathcal{I}^S$ as follows:

$*$	-2	-4	-6	-8
-2	-2	-2	-2	-2
-4	-2	-4	-6	-2
-6	-2	-6	-6	-4
-8	-2	-4	-8	-4

$$, \eth(b) = \begin{cases} 0.1 & \text{if } b = -2 \\ 0.2 & \text{if } b = -4 \\ 0.3 & \text{if } b = -6 \\ 0.4 & \text{if } b = -8 \end{cases}$$

and for all $u, z \in \mathcal{I}$, $T^{nor}(u, z) = T_p^{nor}(u, z) = uz$. Clearly, $\eth \in F(1,2)IT(S)$. Also define $\mho = (\eth_\mho, \partial_\mho) \in BF(S)$, which

$$\eth_\mho(b) = \begin{cases} 0.3 & \text{if } b = -2 \\ 0.5 & \text{if } b = -4 \\ 0.2 & \text{if } b = -6 \\ 0.6 & \text{if } b = -8 \end{cases}, \partial_\mho(b) = \begin{cases} 0.4 & \text{if } b = -2 \\ 0.2 & \text{if } bx = -4 \\ 0.6 & \text{if } b = -6 \\ 0.1 & \text{if } b = -8 \end{cases},$$

$T^{nor}(u, z) = T_p^{nor}(u, z) = uz$ and $C^{con}(u, z) = C_p^{con}(u, z) = u + z - uz$, for all $u, z \in \mathcal{I}$. One can see that $\mho = (\eth_\mho, \partial_\mho) \in BF(1,2)IN(S)$.

Theorem 4.3. *Assume $\mho = (\eth_\mho, \partial_\mho) \in BF(S)$. Then $\mho = (\eth_\mho, \partial_\mho) \in BF(1,2)IN(S)$ if and only if $\eth_\mho \in F(1,2)IT(S)$ and $\bar{\partial_\mho} \in F(1,2)IT(S)$.*

Proof. Let $i, j, k, \in S$ and $\mho = (\eth_\mho, \partial_\mho) \in BF(1,2)IN(S)$. Then $\eth_\mho(ij) \geq T^{nor}(\eth_\mho(i), \eth_\mho(j))$ and $\eth_\mho(ik(jm)) \geq T^{nor}(\eth_\mho(i), T^{nor}(\eth_\mho(j), \eth_\mho(m)))$ and so $\eth_\mho \in F(1,2)IT(S)$. Since $\mho = (\eth_\mho, \partial_\mho) \in BF(1,2)IN(S)$,

$$\begin{aligned} \partial_\mho(ik(jm)) \leq C^{con}(\partial_\mho(i), \partial_\mho(j)) &\Rightarrow -\partial_\mho(ij) \geq -C^{con}(\partial_\mho(i), \partial_\mho(j)) \\ &\Rightarrow 1 - \partial_\mho(ij) \geq 1 - C^{con}(\partial_\mho(i), \partial_\mho(j)) = T^{nor}(1 - \partial_\mho(i), 1 - \partial_\mho(j)) \\ &\Rightarrow \bar{\partial_\mho}(ij) \geq T^{nor}(\bar{\partial_\mho}(i), \bar{\partial_\mho}(j)) \end{aligned}$$

and

$$\begin{aligned} \partial_\mho(ij) \leq C^{con}(\partial_\mho(i), C^{con}(\partial_\mho(j), \partial_\mho(m))) &\Rightarrow -\partial_\mho(ik(jm)) \geq -C^{con}(\partial_\mho(i), C^{con}(\partial_\mho(j), \partial_\mho(m))) \\ &\Rightarrow 1 - \partial_\mho(ik(jm)) \geq 1 - C^{con}(\partial_\mho(i), C^{con}(\partial_\mho(j), \partial_\mho(m))) \\ &= T^{nor}(1 - \partial_\mho(i), 1 - C^{con}(\partial_\mho(j), \partial_\mho(m))) = T^{nor}(1 - \partial_\mho(i), T^{nor}(1 - \partial_\mho(j), 1 - \partial_\mho(m))) \\ &\Rightarrow \bar{\partial_\mho}(ik(jm)) \geq T^{nor}(\bar{\partial_\mho}(i), T^{nor}(\bar{\partial_\mho}(j), \bar{\partial_\mho}(m))). \end{aligned}$$

Therefore, $\bar{\partial_\mho} \in F(1,2)IT(S)$.

Conversly, let $\eth_\mho \in F(1,2)IT(S)$ and $\bar{\partial_\mho} \in F(1,2)IT(S)$ then $\eth_\mho(ij) \geq T^{nor}(\eth_\mho(i), \eth_\mho(j))$ and $\eth_\mho(ik(jm)) \geq T^{nor}(\eth_\mho(i), T^{nor}(\eth_\mho(j), \eth_\mho(m)))$. Also

$$\begin{aligned} \bar{\partial_\mho}(ij) \geq T^{nor}(\bar{\partial_\mho}(i), \bar{\partial_\mho}(j)) = T^{nor}(1 - \partial_\mho(i), 1 - \partial_\mho(j)) &\Rightarrow 1 - \partial_\mho(ij) \geq 1 - C^{con}(\partial_\mho(i), \partial_\mho(j)) \\ &\Rightarrow -\partial_\mho(ij) \geq -C^{con}(\partial_\mho(i), \partial_\mho(j)) \Rightarrow \partial_\mho(ij) \leq C^{con}(\partial_\mho(i), \partial_\mho(j)). \end{aligned}$$

Thus, $\partial_{\mho}(ij) \leq C^{con}(\partial_{\mho}(i), \partial_{\mho}(j))$ and

$$\begin{aligned}
\bar{\partial}_{\mho}(ik(jm)) &\geq T^{nor}(\bar{\partial}_{\mho}(i), T^{nor}(\bar{\partial}_{\mho}(j), \bar{\partial}_{\mho}(m))) \\
&= T^{nor}(1 - \partial_{\mho}(i), T^{nor}(1 - \partial_{\mho}(j), 1 - \partial_{\mho}(m))) \\
&= T^{nor}(1 - \partial_{\mho}(i), 1 - C^{con}(\partial_{\mho}(j), \partial_{\mho}(m))) \\
&\Rightarrow 1 - \partial_{\mho}(ik(jm)) \geq 1 - C^{con}(\partial_{\mho}(i), C^{con}(\partial_{\mho}(j), \partial_{\mho}(m))) \\
&\Rightarrow -\partial_{\mho}(ik(jm)) \geq -C^{con}(\partial_{\mho}(i), C^{con}(\partial_{\mho}(j), \partial_{\mho}(m))) \\
&\Rightarrow \partial_{\mho}(ik(jm)) \leq C^{con}(\partial_{\mho}(i), C^{con}(\partial_{\mho}(j), \partial_{\mho}(m))).
\end{aligned}$$

Hence, $\partial_{\mho}(ik(jm)) \leq C^{con}(\partial_{\mho}(i), C^{con}(\partial_{\mho}(j), \partial_{\mho}(m)))$ and so $\mho = (\eth_{\mho}, \partial_{\mho}) \in BF(1,2)IN(S)$.

□

Theorem 4.4. *Let* $\mho = (\eth_{\mho}, \partial_{\mho}) \in BF(1,2)IN(S)$. *Then* $\mho = (\eth_{\mho}, \partial_{\mho}) \in BF(1,2)IN(S)$ *if and only if* $\triangle\mho = (\eth_{\mho}, \bar{\eth}_{\mho}) \in BF(1,2)IN(S)$ *and* $\triangledown\mho = (\bar{\partial}_{\mho}, \partial_{\mho}) \in BF(1,2)IN(S)$.

Proof. Let $u, v, z, w \in S$. If $\mho = (\eth_{\mho}, \partial_{\mho}) \in BF(1,2)IN(S)$, then $\eth_{\mho}(uv) \geq T^{nor}(\eth_{\mho}(u), \eth_{\mho}(v))$ and $\eth_{\mho}(uw(vz)) \geq T^{nor}(\eth_{\mho}(u), T^{nor}(\eth_{\mho}(v), \eth_{\mho}(z)))$ and

$$\begin{aligned}
\eth_{\mho}(uw(vz)) &\geq T^{nor}(\eth_{\mho}(u), \eth_{\mho}(v)) \Rightarrow -\eth_{\mho}(uv) \leq -T^{nor}(\eth_{\mho}(u), \eth_{\mho}(v)) \\
&\Rightarrow 1 - \eth_{\mho}(uv) \leq 1 - T^{nor}(\eth_{\mho}(u), \eth_{\mho}(v)) = C^{con}(1 - \eth_{\mho}(u), 1 - \eth_{\mho}(v)) \\
&\Rightarrow \bar{\eth}_{\mho}(uv) \leq C^{con}(\bar{\eth}_{\mho}(u), \bar{\eth}_{\mho}(v))
\end{aligned}$$

and so $\bar{\eth}_{\mho}(uv) \leq C^{con}(\bar{\eth}_{\mho}(u), \bar{\eth}_{\mho}(v))$. Moreover,

$$\begin{aligned}
&\eth_{\mho}(uw(vz)) \geq T^{nor}(\eth_{\mho}(u), T^{nor}(\eth_{\mho}(v), \eth_{\mho}(z))) \\
&\Rightarrow -\eth_{\mho}(uw(vz)) \leq -T^{nor}(\eth_{\mho}(u), T^{nor}(\eth_{\mho}(v), \eth_{\mho}(z))) \\
&\Rightarrow 1 - \eth_{\mho}(uw(vz)) \leq 1 - T^{nor}(\eth_{\mho}(u), T^{nor}(\eth_{\mho}(v), \eth_{\mho}(z))) \\
&= C^{con}(1 - \eth_{\mho}(u), 1 - T^{nor}(\eth_{\mho}(v), \eth_{\mho}(z))) = C^{con}(1 - \eth_{\mho}(u), C^{con}(1 - \eth_{\mho}(v), 1 - \eth_{\mho}(z))) \\
&\Rightarrow \bar{\eth}_{\mho}(uw(vz)) \leq C^{con}(\bar{\eth}_{\mho}(u), C^{con}(\bar{\eth}_{\mho}(v), \bar{\eth}_{\mho}(z))),
\end{aligned}$$

thus $\bar{\eth}_{\mho}(uw(vz)) \leq C^{con}(\bar{\eth}_{\mho}(u), C^{con}(\bar{\eth}_{\mho}(v), \bar{\eth}_{\mho}(z)))$. Hence, we give that $\triangle\mho = (\eth_{\mho}, \bar{\eth}_{\mho}) \in BF(1,2)IN(S)$. Now, we prove that $\triangledown\mho = (\bar{\partial}_{\mho}, \partial_{\mho}) \in BF(1,2)IN(S)$. As $\mho = (\eth_{\mho}, \partial_{\mho}) \in BF(1,2)IN(S)$, then $\partial_{\mho}(uv) \leq C^{con}(\partial_{\mho}(u), \partial_{\mho}(v))$ and $\partial_{\mho}(xw(vz)) \leq$

$C^{con}(\partial_{\mho}(u), C^{con}(\partial_{\mho}(v), \partial_{\mho}(z)))$ and then

$$\partial_{\mho}(uv) \leq C^{con}(\partial_{\mho}(u), \partial_{\mho}(v))$$
$$\Rightarrow -\partial_{\mho}(uv) \geq -C^{con}(\partial_{\mho}(u), \partial_{\mho}(v))$$
$$\Rightarrow 1 - \partial_{\mho}(uv) \geq 1 - C^{con}(\partial_{\mho}(u), \partial_{\mho}(v)) = T^{nor}(1 - \partial_{\mho}(u), 1 - \partial_{\mho}(v))$$
$$\Rightarrow \bar{\partial_{\mho}}(uv) \geq T^{nor}(\bar{\partial_{\mho}}(u), \bar{\partial_{\mho}}(v))\ and so \bar{\partial_{\mho}}(uv) \geq T^{nor}(\bar{\partial_{\mho}}(u), \bar{\partial_{\mho}}(v))$$

Also

$$\partial_{\mho}(uw(vz)) \leq C^{con}(\partial_{\mho}(u), C^{con}(\partial_{\mho}(v), \partial_{\mho}(z)))$$
$$\Rightarrow -\partial_{\mho}(uw(vz)) \geq -C^{con}(\partial_{\mho}(u), C^{con}(\partial_{\mho}(v), \partial_{\mho}(z)))$$
$$\Rightarrow 1 - \partial_{\mho}(uw(vz)) \geq 1 - C^{con}(\partial_{\mho}(u), C^{con}(\partial_{\mho}(v), \partial_{\mho}(z))) = T^{nor}(1 - \partial_{\mho}(u), 1 - C^{con}(\partial_{\mho}(v), \partial_{\mho}(z)))$$
$$= T^{nor}(1 - \partial_{\mho}(u), T^{nor}(1 - \partial_{\mho}(v), 1 - \partial_{\mho}(z)))$$
$$\Rightarrow \bar{\partial_{\mho}}(uw(vz)) \geq T^{nor}(\bar{\partial_{\mho}}(u), T^{nor}(\bar{\partial_{\mho}}(v), \bar{\partial_{\mho}}(z))).$$

Thus, $\bar{\partial_{\mho}}(uw(vz)) \geq T^{nor}(\bar{\partial_{\mho}}(u), T^{nor}(\bar{\partial_{\mho}}(v), \bar{\partial_{\mho}}(z)))$. Hence, we get that $\triangledown\mho = (\eth_{\mho}, \bar{\eth_{\mho}}) \in BF(1,2)IN(S)$. □

Assume S be a semigroup and $\emptyset \neq B \subseteq S$. We recall that B is a $(1,2)$-ideal of S, if for every $x, y, z \in B$ and for every $w \in S, xw(yz) \in B$.

Theorem 4.5. *Let $\mho = (\eth_{\mho}, \partial_{\mho}) \in BF(1,2)IN(S)$ and T^{nor} and C^{con} be idempotent. Then for all $T^{nor} \in [0,1]$, $U(\eth_{\mho}; t)$ and $L(\partial_{\mho}; t)$ are $(1,2)$-ideal of S.*

Proof. Let $x, y \in U(\eth_{\mho}; t)$. Then, $\eth_{\mho}(xy) \geq T^{nor}(\eth_{\mho}(x), \eth_{\mho}(y)) \geq T^{nor}(t,t) = t$ and so $xy \in U(\eth_{\mho}; t)$ and $U(\eth_{\mho}; t) \neq \emptyset$. Let $x, y, z \in U(\eth_{\mho}; t)$ and $w \in S$. Then

$$\eth_{\mho}(xw(yz)) \geq T^{nor}(\eth_{\mho}(x), T^{nor}(\eth_{\mho}(y), \eth_{\mho}(z))) \geq T^{nor}(t, T^{nor}(t,t)) = T^{nor}(t,t) = t$$

and so $xw(yz) \in U(\eth_{\mho}; t)$. It follows that $U(\eth_{\mho}; t)$ is a $(1,2)$-ideal of S for all $T^{nor} \in [0,1]$.

Similarly, if $x, y \in L(\partial_{\mho}; t)$, then $\partial_{\mho}(xy) \leq C^{con}(\partial_{\mho}(x), \partial_{\mho}(y)) \leq C^{con}(t,t) = t$. Hence, $xy \in L(\partial_{\mho}; t)$ and $L(\partial_{\mho}; t) \neq \emptyset$. Let $x, y, z \in L(\partial_{\mho}; t)$ and $w \in S$. Then

$$\partial_{\mho}(xw(yz)) \leq C^{con}(\partial_{\mho}(x), C^{con}(\partial_{\mho}(y), \partial_{\mho}(z))) \leq C^{con}(t, C^{con}(t,t)) = C^{con}(t,t) = t.$$

Thus, $xw(yz) \in L(\partial_{\mho}; t)$ and so $L(\partial_{\mho}; t)$ is a $(1,2)$-ideal of S for all $T^{nor} \in [0,1]$. □

Corollary 4.6. *Let $\mho = (\eth_{\mho}, \partial_{\mho}) \in BF(1,2)IN(S)$ and $a \in S$ be a fixed element. Then $M = \{x \in \mathcal{O} : \eth_{\mho}(x) \geq \eth_{\mho}(a)\}$ and $N = \{x \in \mathcal{O} : \partial_{\mho}(x) \leq \partial_{\mho}(a)\}$ are $(1,2)$-ideal of S.*

Theorem 4.7. *Let $J \subseteq S$ and $\mho = (\eth_\mho, \partial_\mho) \in BF(S)$ defined by*

$$\eth_\mho(a) = \begin{cases} c_0 & \text{if } a \in J \\ c_1 & \text{if } a \notin J \end{cases}, \partial_\mho(a) = \begin{cases} c_0 & \text{if } a \notin J \\ c_1 & \text{if } a \in J \end{cases}$$

for all $a \in S$ and $c_0, c_1 \in [0,1]$ so $c_0 > c_1$ and T^{nor} and C^{con} be idempotent. Then $\mho = (\eth_\mho, \partial_\mho) \in BF(1,2)IN(S)$ if and only if $J = U(\eth_\mho; c_0) = L(\partial_\mho; c_0)$ be a $(1,2)$-ideal of S.

Proof. Let $J = U(\eth_\mho; c_0) = L(\partial_\mho; c_0)$ be a $(1,2)$-ideal of S. Since $c_0 > c_1$ so $c_1 = T^{nor}(c_1, c_0)$ and $c_0 = C^{con}(c_1, c_0)$ for $x, y \in S$, we have the follwing conditions:

(a) Assume $x \in J$ with $y \notin J$, so $xy \notin J$ and $\eth_\mho(xy) = c_1 \geq c_1 = T^{nor}(c_0, c_1) = T^{nor}(\eth_\mho(x), \eth_\mho(y))$ and $\partial_\mho(xy) = c_0 \leq c_0 = C^{con}(c_1, c_0) = C^{con}(\partial_\mho(x), \partial_\mho(y))$.

(b) As $x \notin J$ that $y \in J$, thus $xy \notin J$ and so $\eth_\mho(xy) = c_1 \geq c_1 = T^{nor}(c_1, c_0) = T^{nor}(\eth_\mho(x), \eth_\mho(y))$ and $\partial_\mho(xy) = c_0 \leq c_0 = C^{con}(c_0, c_1) = C^{con}(\partial_\mho(x), \partial_\mho(y))$.

(c) Assume $x \notin J$ which $y \notin J$, hence $xy \notin J$ then $\eth_\mho(xy) = c_1 \geq c_1 = T^{nor}(c_1, c_1) = T^{nor}(\eth_\mho(x), \eth_\mho(y))$ and $\partial_\mho(xy) = c_0 \leq c_0 = C^{con}(c_0, c_0) = C^{con}(\partial_\mho(x), \partial_\mho(y))$.

(d) Suppose $x \in J$ with $y \in J$, hence $xy \in J$ and so $\eth_\mho(xy) = c_0 \geq c_0 = T^{nor}(c_0, c_0) = T^{nor}(\eth_\mho(x), \eth_\mho(y))$ and $\partial_\mho(xy) = c_1 \leq c_1 = C^{con}(c_1, c_1) = C^{con}(\partial_\mho(x), \partial_\mho(y))$. Thus from (a)-(d) we get that $\eth_\mho(xy) \geq T^{nor}(\eth_\mho(x), \eth_\mho(y))$ and $\partial_\mho(xy) \leq C^{con}(\partial_\mho(x), \partial_\mho(y))$ for all $x, y \in S$. Now, let $x, y, z, w \in S$ and we investigate the follwing conditions:

(a) As $x \in J$ and $y, z \notin J$, thus, $xw(yz) \notin J$. Hence,

$$\begin{aligned}\eth_\mho(xw(yz)) &= c_1 \geq c_1 = T^{nor}(c_0, c_1) = T^{nor}(c_0, T^{nor}(c_1, c_1)) \\ &= T^{nor}(\eth_\mho(x), T^{nor}(\eth_\mho(y), \eth_\mho(z))) \textit{ and } \partial_\mho(xw(yz)) = c_0 \leq c_0 = C^{con}(c_1, c_0) \\ &= C^{con}(c_1, C^{con}(c_0, c_0)) = C^{con}(\partial_\mho(x), C^{con}(\partial_\mho(y), \partial_\mho(z))).\end{aligned}$$

(b) If $y \in J$ and $x, z \notin J$, then, $xw(yz) \notin J$. Hence

$$\begin{aligned}&\eth_\mho(xw(yz)) = c_1 \geq c_1 = T^{nor}(c_1, c_0) = T^{nor}(T^{nor}(c_1, c_1), c_0) = T^{nor}(T^{nor}(\eth_\mho(x), \eth_\mho(z)), \eth_\mho(y)) \\ &= T^{nor}(\eth_\mho(x), T^{nor}(\eth_\mho(y), \eth_\mho(z))) \textit{ and } \partial_\mho(xw(yz)) = c_0 \leq c_0 \\ &= C^{con}(c_0, c_1) = C^{con}(C^{con}(c_0, c_0), c_1) = C^{con}(C^{con}(\partial_\mho(x), \partial_\mho(z)), \partial_\mho(y)) \\ &= C^{con}(\partial_\mho(x), C^{con}(\partial_\mho(y), \partial_\mho(z))).\end{aligned}$$

(c) Assume $z \in J$ and $x, y \notin J$, so, $xw(yz) \notin J$. Thus,

$$\eth_\mho(xw(yz)) = c_1 \geq c_1 = T^{nor}(c_1, c_0) = T^{nor}(T^{nor}(c_1, c_1), c_0)$$

$$= T^{nor}(T^{nor}(\eth_\mho(x), \eth_\mho(y)), \eth_\mho(z)) = T^{nor}(\eth_\mho(x), T^{nor}(\eth_\mho(y), \eth_\mho(z))) \textit{ and}$$

$$\partial_\mho(xw(yz)) = c_0 \leq c_0 = C^{con}(c_0, c_1) = C^{con}(C^{con}(c_0, c_0), c_1)$$

$$= C^{con}(C^{con}(\partial_\mho(x), \partial_\mho(y)), \partial_\mho(z)) = C^{con}(\partial_\mho(x), C^{con}(\partial_\mho(y), \partial_\mho(z))).$$

(d) Suppose $x, y \in J$ such that $z \notin J$, then, $xw(yz) \notin J$. Hence,

$$ð_{℧}(xw(yz)) = c_1 \geq c_1 = T^{nor}(c_1, c_0) = T^{nor}(c_1, T^{nor}(c_0, c_0))$$

$$= T^{nor}(ð_{℧}(z), T^{nor}(ð_{℧}(x), ð_{℧}(y))) = T^{nor}(ð_{℧}(x), T^{nor}(ð_{℧}(y), ð_{℧}(z))) \text{ and}$$

$$\partial_{℧}(xw(yz)) = c_0 \leq c_0 = C^{con}(c_0, c_1) = C^{con}(c_0, C^{con}(c_1, c_1))$$

$$= C^{con}(\partial_{℧}(z), C^{con}(\partial_{℧}(x), \partial_{℧}(y))) = C^{con}(\partial_{℧}(x), C^{con}(\partial_{℧}(y), \partial_{℧}(z))).$$

(e) As $x, z \in J$ and $y \notin J$, hence, $xw(yz) \notin J$. It follows that

$$ð_{℧}(xw(yz)) = c_1 \geq c_1 = T^{nor}(c_1, c_0) = T^{nor}(c_1, T^{nor}(c_0, c_0))$$

$$= T^{nor}(ð_{℧}(y), T^{nor}(ð_{℧}(x), ð_{℧}(z))) = T^{nor}(ð_{℧}(x), T^{nor}(ð_{℧}(y), ð_{℧}(z))) \text{ and}$$

$$\partial_{℧}(xw(yz)) = c_0 \leq c_0 = C^{con}(c_0, c_1) = C^{con}(c_0, C^{con}(c_1, c_1))$$

$$= C^{con}(\partial_{℧}(y), C^{con}(\partial_{℧}(x), \partial_{℧}(z))) = C^{con}(\partial_{℧}(x), C^{con}(\partial_{℧}(z), \partial_{℧}(y))).$$

(f) Assume $y, z \in J$ with $x \notin J$, then, $xw(yz) \notin J$. Thus,

$$ð_{℧}(xw(yz)) = c_1 \geq c_1 = T^{nor}(c_1, c_0) = T^{nor}(c_1, T^{nor}(c_0, c_0)) = T^{nor}(ð_{℧}(x), T^{nor}(ð_{℧}(y), ð_{℧}(z))) \text{ and}$$

$$\partial_{℧}(xw(yz)) = c_0 \leq c_0 = C^{con}(c_0, c_1) = C^{con}(c_0, C^{con}(c_1, c_1)) = C^{con}(\partial_{℧}(x), C^{con}(\partial_{℧}(y), \partial_{℧}(z))).$$

(j) Suppose $x, y, z \in J$ hence, $xw(yz) \in J$. So

$$ð_{℧}(xw(yz)) = c_0 \geq c_0 = T^{nor}(c_0, c_0)$$
$$= T^{nor}(c_0, T^{nor}(c_0, c_0)) = T^{nor}(ð_{℧}(x), T^{nor}(ð_{℧}(y), ð_{℧}(z))) \text{ and}$$
$$\partial_{℧}(xw(yz)) = c_1 \leq c_1 = C^{con}(c_1, c_1)$$
$$= C^{con}(c_1, C^{con}(c_1, c_1)) = C^{con}(\partial_{℧}(x), C^{con}(\partial_{℧}(y), \partial_{℧}(z))).$$

(h) If $x, y, z \notin J$ then, $xw(yz) \notin J$. Hence,

$$ð_{℧}(xw(yz)) = c_1 \geq c_1 = T^{nor}(c_1, c_1)$$
$$= T^{nor}(c_1, T^{nor}(c_1, c_1)) = T^{nor}(ð_{℧}(x), T^{nor}(ð_{℧}(y), ð_{℧}(z))) \text{ and}$$
$$\partial_{℧}(xw(yz)) = c_0 \leq c_0 = C^{con}(c_0, c_0)$$
$$= C^{con}(c_0, C^{con}(c_0, c_0)) = C^{con}(\partial_{℧}(x), C^{con}(\partial_{℧}(y), \partial_{℧}(z))).$$

Therefore, fram (a)-(h), we get that $ð_{℧}(xw(yz)) \geq T^{nor}(ð_{℧}(x), T^{nor}(ð_{℧}(y), ð_{℧}(z)))$ *and* $\partial_{℧}(xw(yz)) \leq C^{con}(\partial_{℧}(x), C^{con}(\partial_{℧}(y), \partial_{℧}(z)))$. Thus, $℧ = (ð_{℧}, \partial_{℧}) \in BF(1,2)IN(S)$. □

Theorem 4.8. *Let* $℧ = (ð_{℧}, \partial_{℧}) \in BF(1,2)IN(S)$ *and* $B = (ð_B, \partial_B) \in BF(1,2)IN(S.)$ *Then* $℧ \cap B \in BF(1,2)IN(S)$.

Proof. Let $e, f, w \in \mathcal{O}$. Then

$$\eth_{\mho\cap B}(ef) = T^{nor}(\eth_{\mho}(ef), \eth_B(ef)) \geq T^{nor}(T^{nor}(\eth_{\mho}(e), \eth_{\mho}(f)), T^{nor}(\eth_B(e), \eth_B(f)))$$
$$= T^{nor}(T^{nor}(\eth_{\mho}(e), \eth_B(e)), T^{nor}(\eth_{\mho}(f), \eth_B(f))) = T^{nor}(\eth_{\mho\cap B}(e), \eth_{\mho\cap B}(f)),$$

thus $\eth_{\mho\cap B}(ef) \geq T^{nor}(\eth_{\mho\cap B}(e), \eth_{\mho\cap B}(f))$. Also

$$\eth_{\mho\cap B}(ew(fz)) = T^{nor}(\eth_{\mho}(ew(fz)), \eth_B(ew(fz)))$$
$$\geq T^{nor}(T^{nor}(\eth_{\mho}(e), T^{nor}(\eth_{\mho}(f), \eth_{\mho}(z))), T^{nor}(\eth_B(e), T^{nor}(\eth_B(f)), \eth_B(z)))$$
$$= T^{nor}(T^{nor}(\eth_{\mho}(e), \eth_B(e)), T^{nor}(T^{nor}(\eth_{\mho}(f), \eth_{\mho}(z)), T^{nor}(\eth_B(f)), \eth_B(z))))$$
$$= T^{nor}(T^{nor}(\eth_{\mho}(e), \eth_B(e)), T^{nor}(T^{nor}(\eth_{\mho}(f), \eth_B(f)), T^{nor}(\eth_{\mho}(z)), \eth_B(z))))$$
$$= T^{nor}(\eth_{\mho\cap B}(e), T^{nor}(\eth_{\mho\cap B}(f), \eth_{\mho\cap B}(z))),$$

so $\eth_{\mho\cap B}(ew(fz)) \geq T^{nor}(\eth_{\mho\cap B}(e), T^{nor}(\eth_{\mho\cap B}(f), \eth_{\mho\cap B}(z)))$. Since

$$\partial_{\mho\cap B}(ef) = C^{con}(\partial_{\mho}(ef), \partial_B(ef)) \leq C^{con}(C^{con}(\partial_{\mho}(e), \partial_{\mho}(f)), C^{con}(\partial_B(e), \partial_B(f)))$$
$$= C^{con}(C^{con}(\partial_{\mho}(e), \partial_B(e)), C^{con}(\partial_{\mho}(f), \partial_B(f))) = C^{con}(\partial_{\mho\cap B}(e), \partial_{\mho\cap B}(f)),$$

so $\partial_{\mho\cap B}(ef) \leq C^{con}(\partial_{\mho\cap B}(e), \partial_{\mho\cap B}(f))$. As

$$\partial_{\mho\cap B}(ew(fz)) = C^{con}(\partial_{\mho}(ew(fz)), \partial_B(ew(fz)))$$
$$\leq C^{con}(C^{con}(\partial_{\mho}(e), C^{con}(\partial_{\mho}(f), \partial_{\mho}(z))), C^{con}(\partial_B(e), C^{con}(\partial_B(f)), \partial_B(z)))$$
$$= C^{con}(C^{con}(\partial_{\mho}(e), \partial_B(e)), C^{con}(C^{con}(\partial_{\mho}(f), \partial_{\mho}(z)), C^{con}(\partial_B(f)), \partial_B(z))))$$
$$= C^{con}(C^{con}(\partial_{\mho}(e), \partial_B(e)), C^{con}(C^{con}(\partial_{\mho}(f), \partial_B(y)), C^{con}(\partial_{\mho}(z)), \partial_B(z))))$$
$$= C^{con}(\partial_{\mho\cap B}(e), C^{con}(\partial_{\mho\cap B}(f), \partial_{\mho\cap B}(z)))$$

so $\partial_{\mho\cap B}(ew(fz)) \leq C^{con}(\partial_{\mho\cap B}(x), C^{con}(\partial_{\mho\cap B}(f), \partial_{\mho\cap B}(z)))$. Therefore, we get that $\mho \cap B \in BF(1,2)IN(S)$. □

Example 4.9. Let $S = \{-10, -20, -30, -40, -50, -60\}$. Then $(S, *)$ is a semigroup and $\eth \in \mathcal{I}^S$ as follows:

$*$	-10	-20	-30	-40	-50	-60
-10	-10	-10	-10	-40	-10	-10
-20	-10	-20	-20	-40	-20	-20
-30	-10	-20	-30	-40	-50	-50
-40	-10	-10	-40	-40	-40	-40
-50	-10	-20	-30	-40	-50	-50
-60	-10	-20	-30	-40	-50	-60

$$, \eth(p) = \begin{cases} 0.1 & \text{if } p = -10 \\ 0.2 & \text{if } p = -20 \\ 0.3 & \text{if } p = -30 \\ 0.4 & \text{if } p = -40 \\ 0.5 & \text{if } p = -50 \\ 0.6 & \text{if } p = -60 \end{cases}$$

and for all $u, z \in \mathcal{I}$, $T^{nor}(u,z) = T_p^{nor}(u,z) = uz$. Clearly, $\eth \in F(1,2)IT(S)$. Define $\mho = (\eth_\mho, \partial_\mho) \in BF(S)$ and $B = (\eth_B, \partial_B) \in BF(S)$ with

$$\eth_\mho(p) = \begin{cases} 0.3 & \text{if } p = -10 \\ 0.5 & \text{if } p = -20 \\ 0.2 & \text{if } p = -30 \\ 0.6 & \text{if } p = -40 \\ 0.4 & \text{if } p = -50 \\ 0.1 & \text{if } p = -60 \end{cases}, \partial_\mho(p) = \begin{cases} 0.4 & \text{if } p = -10 \\ 0.2 & \text{if } p = -20 \\ 0.6 & \text{if } p = -30 \\ 0.1 & \text{if } p = -40 \\ 0.3 & \text{if } p = -50 \\ 0.7 & \text{if } p = -60 \end{cases},$$

$$\eth_B(p) = \begin{cases} 0.35 & \text{if } p = -10 \\ 0.25 & \text{if } p = -20 \\ 0.1 & \text{if } p = -30 \\ 0.55 & \text{if } p = -40 \\ 0.45 & \text{if } p = -50 \\ 0.2 & \text{if } p = -60 \end{cases}, \partial_B(p) = \begin{cases} 0.25 & \text{if } p = -10 \\ 0.15 & \text{if } p = -20 \\ 0.2 & \text{if } p = -30 \\ 0.35 & \text{if } p = -40 \\ 0.1 & \text{if } p = -50 \\ 0.4 & \text{if } p = -60 \end{cases},$$

$T^{nor}(u,z) = T_p^{nor}(u,z) = uz$ and $C^{con}(u,z) = C_p^{con}(u,z) = u + z - uz$, for all $u, z \in \mathcal{I}$. Thus $\mho = (\eth_\mho, \partial_\mho) \in BF(1,2)IN(S)$ and $B = (\eth_B, \partial_B) \in BF(1,2)IN(S)$ and $\mho \cap B \in BF(1,2)IN(S)$.

Corollary 4.10. *(i) If $\{\mho_i\}_{i \in I} \subseteq BF(1,2)IN(S)$, then $\mho = \cap_{i \in I} \mho_i \in BF(1,2)IN(S)$.*

(ii) If $\{\mho_i\}_{i \in I} \subseteq F(1,2)IT(S)$, then $\mho = \cap_{i \in I} \mho_i \in F(1,2)IT(S)$.

Theorem 4.11. *Every $BFBIN(S)$ is a $BF(1,2)IN(S)$.*

Proof. Let $\mho = (\eth_\mho, \partial_\mho) \in BFBIN(S)$ and $m, n, z, w \in S$. Since $\mho = (\eth_\mho, \partial_\mho) \in FBIT(S)$, we get that $\eth_\mho(mn) \geq T^{nor}(\eth_\mho(m), \eth_\mho(n))$. Also

$$\begin{aligned} \eth_\mho(mw(nz)) &= \eth_\mho((mwn)z) \geq T^{nor}(\eth_\mho(mwn), \eth_\mho(z)) \\ &\geq T^{nor}(T^{nor}(\eth_\mho(m), \eth_\mho(n)), \eth_\mho(z)) = T^{nor}(\eth_\mho(m), T^{nor}(\eth_\mho(n), \eth_\mho(z))), \end{aligned}$$

thus $\eth_\mho(mw(nz)) \geq T^{nor}(\eth_\mho(m), T^{nor}(\eth_\mho(n), \eth_\mho(z)))$. Moreover $\partial_\mho(mn) \leq C^{con}(\partial_\mho(m), \partial_\mho(n))$ and

$$\begin{aligned} \partial_\mho(mw(nz)) &= \partial_\mho((mwn)z) \leq C^{con}(\partial_\mho(mwn), \partial_\mho(z)) \\ &\leq C^{con}(C^{con}(\partial_\mho(m), \partial_\mho(n)), \partial_\mho(z)) = C^{con}(\partial_\mho(m), C^{con}(\partial_\mho(n), \partial_\mho(z))), \end{aligned}$$

then $\partial_\mho(mw(nz)) \leq C^{con}(\partial_\mho(m), C^{con}(\partial_\mho(n), \partial_\mho(z)))$. Therefore, we give that $\mho = (\eth_\mho, \partial_\mho) \in BF(1,2)IN(S)$ □

Now by additional condition on semigroup S, we prove the converse of Theorem 4.11.

Theorem 4.12. *Let S be a regular semigroup and T^{nor} and C^{con} be idempotent. Then every $BF(1,2)IN(S)$ is a $BFBIN(S)$.*

Proof. Let $\mho = (\eth_\mho, \partial_\mho) \in BF(1,2)IN(S)$ and $c, d, w, s \in S$. Because S is a regular semigroup for all $c \in S$, there exists $s \in S$ so $c = csc$. Then $cw \in (cSc)S \subseteq cSc$ and $cw = csc$. As $\mho = (\eth_\mho, \partial_\mho) \in BF(1,2)IN(S)$, then $\eth_\mho(cd) \geq T^{nor}(\eth_\mho(c), \eth_\mho(d))$. Also

$$\eth_\mho(cwd) = \eth_\mho(cs(xd)) \geq T^{nor}(\eth_\mho(c), T^{nor}(\eth_\mho(c), \eth_\mho(d)))$$
$$= T^{nor}(T^{nor}(\eth_\mho(c), \eth_\mho(c)), \eth_\mho(d)) = T^{nor}(\eth_\mho(c), \eth_\mho(d))$$

and so $\eth_\mho(cwd) \geq T^{nor}(\eth_\mho(c), \eth_\mho(d))$. Also $\partial_\mho(cd) \leq C^{con}(\partial_\mho(c), \partial_\mho(d))$. In addition,

$$\partial_\mho(cwd) = \partial_\mho(cs(cd))$$
$$\leq C^{con}(\partial_\mho(c), C^{con}(\partial_\mho(c), \partial_\mho(d)))$$
$$= C^{con}(C^{con}(\partial_\mho(c), \partial_\mho(c)), \partial_\mho(d)) = C^{con}(\partial_\mho(c), \partial_\mho(d)),$$

and so $\partial_\mho(cwd) \leq C^{con}(\partial_\mho(c), \partial_\mho(d))$. Hence, $\mho = (\eth_\mho, \partial_\mho) \in BFBIN(S)$. □

5. Homomorphisms on $F(1,2)IT(S)$ and $BF(1,2)IN(S)$.

In this section, we apply the concept of homomorphism over $FIT(S), FBIT(S)$, $BFSN(S), BFIN(S), BFBIN(S), F(1,2)IT(S), BF(1,2)IN(S)$ and extend the bifuzzy bi-ideal on semirings. Throughout this section we let that $\acute{S} = (\acute{S}, \acute{*})$ be a semigroup.

Theorem 5.1. *Assume $\eth \in FIT(S)$ and $\partial \in FIT(\acute{S})$ and $\varrho : S \to \acute{S}$ be an onto homomorphism. Hence* (1) $\varrho(\eth) \in FIT(\acute{S})$,
(2) $\varrho^{-1}(\partial) \in FIT(S)$.

Proof. (1) Let $u, v \in \acute{S}$ and $b, d \in S$ so $u = \varrho(b)$ and $v = \varrho(d)$. Then

$$\varrho(\eth)(uv) = \bigvee\{\eth(bd) \mid u = \varrho(b), v = \varrho(d)\} \geq \bigvee\{T^{nor}(\eth(b), \eth(d)) \mid u = \varrho(b), v = \varrho(d)\}$$
$$= T^{nor}(\bigvee\{\eth(b) \mid u = f(b)\}, \bigvee\{\eth(d) \mid v = \varrho(d)\}) = T^{nor}(\varrho(\eth)(u), \varrho(\eth)(v))$$

and

$$\varrho(\eth)(uv) = \bigvee\{\eth(bd) \mid u = \varrho(b), v = \varrho(d)\} \geq \bigvee\{\eth(b) \mid u = \varrho(b)\} = \varrho(\eth)(u)$$

and

$$\varrho(\eth)(uv) = \bigvee\{\eth(bd) \mid u = \varrho(b), v = \varrho(d)\} \geq \bigvee\{\eth(d) \mid v = \varrho(d)\} = \varrho(\eth)(v).$$

Then, $\varrho(\eth) \in FIT(\acute{S})$.

(2) assume $b, d \in S$. Therefore

$$\varrho^{-1}(\partial)(bd) = \partial(\varrho(bd)) = \partial(\varrho(b)\varrho(d)) \geq T^{nor}(\partial(\varrho(b)), \partial(\varrho(d))) = T^{nor}(\varrho^{-1}(\partial)(b), \varrho^{-1}(\partial)(d))$$

and $\varrho^{-1}(\partial)(bd) = \partial(\varrho(bd)) = \partial(\varrho(b)\varrho(d)) \geq \partial(\varrho(b)) = \varrho^{-1}(\partial)(b)$ and $\varrho^{-1}(\partial)(bd) = \partial(\varrho(bd)) = \partial(\varrho(b)\varrho(d)) \geq \partial(\varrho(d)) = \varrho^{-1}(\partial)(d)$. Thus, $\varrho^{-1}(\partial) \in FIT(S)$. □

Theorem 5.2. *Let $\eth \in FBIT(S)$ and $\partial \in FBIT(\acute{S})$ and $\varrho : S \to \acute{S}$ be an onto homomorphism. Then $\varrho(\eth) \in FBIT(\acute{S})$ and $\varrho^{-1}(\partial) \in FBIT(S)$.*

Proof. Suppose $u, v, z \in \acute{S}$ that $f, g, w \in S$ so $u = \varrho(f)$ and $v = \varrho(g)$ and $z = \varrho(w)$. Then

$$\begin{aligned}\varrho(\eth)(uv) &= \bigvee\{\eth(fg) \mid u = \varrho(f), v = \varrho(g)\}\\ &\geq \bigvee\{T^{nor}(\eth(f), \eth(g)) \mid u = \varrho(f), v = \varrho(g)\}\\ &= T^{nor}(\bigvee\{\eth(f) \mid u = \varrho(f)\}, \bigvee\{\eth(y) \mid v = \varrho(g)\})\\ &= T^{nor}(\varrho(\eth)(u), \varrho(\eth)(v)) \textit{ and}\end{aligned}$$

$$\begin{aligned}\varrho(\eth)(uzv) &= \bigvee\{\eth(fwg) \mid u = \varrho(f), v = \varrho(g), z = \varrho(w)\}\\ &\geq \bigvee\{T^{nor}(\eth(f), \eth(g)) \mid u = \varrho(f), v = \varrho(g)\}\\ &= T^{nor}(\bigvee\{\eth(f) \mid u = \varrho(f)\}, \bigvee\{\eth(g) \mid v = \varrho(g)\})\\ &= T^{nor}(\varrho(\eth)(u), \varrho(\eth)(v)).\end{aligned}$$

Then, $\varrho(\eth) \in FBIT(\acute{S})$. Assume $f, g, w \in S$. Since

$$\begin{aligned}&\varrho^{-1}(\partial)(fg) = \partial(\varrho(fg)) = \partial(\varrho(f)\varrho(g))\\ &\geq T^{nor}(\partial(\varrho(f)), \partial(\varrho(g))) = T^{nor}(\varrho^{-1}(\partial)(f), \varrho^{-1}(\partial)(g)) \textit{ and}\\ &\varrho^{-1}(\partial)(fwg) = \partial(\varrho(fwg)) = \partial(\varrho(f)\varrho(w)\varrho(g))\\ &\geq T^{nor}(\partial(\varrho(f)), \partial(\varrho(g))) = T^{nor}(\varrho^{-1}(\partial)(f), \varrho^{-1}(\partial)(g)),\end{aligned}$$

we get $\varrho^{-1}(\partial) \in FBIT(S)$. □

Theorem 5.3. *Let $\mho = (\eth_\mho, \partial_\mho) \in BFSN(S)$ and $B = (\eth_B, \partial_B) \in BFSN(\acute{S})$ and $\varrho : S \to \acute{S}$ be a homomorphism. Then $\varrho(\mho) \in BFSN(\acute{S})$ and $\varrho^{-1}(B) \in BFSN(S)$.*

Proof. Let $u, v \in \acute{S}$ and $j, k \in S$ so $u = \varrho(j)$ and $v = \varrho(k)$. Then

$$\varrho(\eth_{\mho})(uv) = \bigvee\{\eth_{\mho}(jk) \ : \ u = \varrho(j), v = \varrho(k)\}$$
$$\geq \bigvee\{T^{nor}(\eth_{\mho}(j), \eth_{\mho}(k)) \ : \ u = \varrho(j), v = \varrho(k)\}$$
$$= T^{nor}(\bigvee\{\eth_{\mho}(j) \ : \ u = \varrho(j)\}, \bigvee\{\eth_{\mho}(k) \ : \ v = \varrho(k)\}) = T^{nor}(\varrho(\eth_{\mho})(u), \varrho(\eth_{\mho})(v)) \ and$$
$$\varrho(\partial_{\mho})(uv) = \bigwedge\{\partial_{\mho}(jk) \ : \ u = \varrho(j), v = \varrho(k)\}$$
$$\leq \bigwedge\{C^{con}(\partial_{\mho}(j), \partial_{\mho}(k)) \ : \ u = \varrho(j), v = \varrho(k)\}$$
$$= C^{con}(\bigwedge\{\partial_{\mho}(j) \ : \ u = \varrho(j)\}, \bigwedge\{\partial_{\mho}(k) \ : \ v = \varrho(k)\}) = C^{con}(\varrho(\partial_{\mho})(u), \varrho(\partial_{\mho})(v))$$

which mean that $\varrho(\mho) = (\varrho(\eth_{\mho}), \varrho(\partial_{\mho})) \in BFSN(\acute{S})$. Assume $j, k \in S$. Therefore

$$\varrho^{-1}(\eth_B)(jk) = \eth_B(\varrho(jk)) = \eth_B(\varrho(j)\varrho(k))$$
$$\geq T^{nor}(\eth_B(\varrho(j)), \eth_B(\varrho(k)))$$
$$= T^{nor}(\varrho^{-1}(\eth_B)(j), \varrho^{-1}(\eth_B)(k)) \ and$$
$$\varrho^{-1}(\partial_B)(jk) = \partial_B(\varrho(jk)) = \partial_B(\varrho(j)\varrho(k))$$
$$\leq C^{con}(\partial_B(\varrho(j)), \partial_B(\varrho(k))) = C^{con}(\varrho^{-1}(\partial_B)(j), \varrho^{-1}(\partial_B)(k)).$$

Thus, $\varrho^{-1}(B) = (\varrho^{-1}(\eth_B), \varrho^{-1}(\partial_B)) \in BFSN(S)$. □

Theorem 5.4. *Let $\mho = (\eth_{\mho}, \partial_{\mho}) \in BFIN(S)$ and $B = (\eth_B, \partial_B) \in BFIN(\acute{S})$ and $\varrho : S \to \acute{S}$ be a homomorphism. Then, $\varrho(\mho) \in BFIN(\acute{S})$ and $\varrho^{-1}(B) \in BFIN(S)$.*

Proof. Let $u, v \in \acute{S}$ and $x, y \in S$ so $u = \varrho(x)$ and $v = \varrho(y)$. Thus

$$\varrho(\eth_{\mho})(uv) = \bigvee\{\eth_{\mho}(xy) \ : \ u = \varrho(x), v = \varrho(y)\}$$
$$\geq \bigvee\{T^{nor}(\eth_{\mho}(x), \eth_{\mho}(y)) \ : \ u = \varrho(x), v = \varrho(y)\}$$
$$= T^{nor}(\bigvee\{\eth_{\mho}(x) \ : \ u = f(x)\}, \bigvee\{\eth_{\mho}(y) \ : \ v = \varrho(y)\}) = T^{nor}(\varrho(\eth_{\mho})(u), \varrho(\eth_{\mho})(v)).$$

Also $\varrho(\eth_{\mho})(uv) = \bigvee\{\eth_{\mho}(xy) \ : \ u = \varrho(x), v = \varrho(y)\} \geq \bigvee\{\eth_{\mho}(x) \ : \ u = \varrho(x)\} = \varrho(\eth_{\mho})(u)$ and $\varrho(\eth_{\mho})(uv) = \bigvee\{\eth_{\mho}(xy) \ : \ u = \varrho(x), v = \varrho(y)\} \geq \bigvee\{\eth_{\mho}(y) \ : \ v = \varrho(y)\} = \varrho(\eth_{\mho})(v)$. Moreover

$$\varrho(\partial_{\mho})(uv) = \bigwedge\{\partial_{\mho}(xy) \ : \ u = \varrho(x), v = \varrho(y)\}$$
$$\leq \bigwedge\{C^{con}(\partial_{\mho}(x), \partial_{\mho}(y)) \ : \ u = \varrho(x), v = \varrho(y)\}$$
$$= C^{con}(\bigwedge\{\partial_{\mho}(x) \ : \ u = f(x)\}, \bigwedge\{\partial_{\mho}(y) \mid v = \varrho(y)\})$$
$$= C^{con}(\varrho(\partial_{\mho})(u), \varrho(\eth_{\mho})(v)).$$

Also

$$\begin{aligned}
&\varrho(\partial_{\mho})(uv) = \bigwedge\{\partial_{\mho}(xy) \; : \; u = \varrho(x), v = \varrho(y)\} \\
&\leq \bigwedge\{\partial_{\mho}(x) \; : \; u = \varrho(x)\} = \varrho(\partial_{\mho})(u) and \\
&\varrho(\partial_{\mho})(uv) = \bigwedge\{\partial_{\mho}(xy) \mid u = \varrho(x), v = \varrho(y)\} \\
&\leq \bigwedge\{\partial_{\mho}(y) \; : \; v = \varrho(y)\} = \varrho(\partial_{\mho})(v).
\end{aligned}$$

Thus $\varrho(\mho) = (\varrho(\eth_{\mho}), \varrho(\partial_{\mho})) \in BFIN(\acute{S})$. Let $x, y \in S$. Then

$$\begin{aligned}
&\varrho^{-1}(\eth_B)(xy) = \eth_B(\varrho(xy)) = \eth_B(\varrho(x)\varrho(y)) \\
&\geq T^{nor}(\eth_B(\varrho(x)), \eth_B(\varrho(y))) = T^{nor}(\varrho^{-1}(\eth_B)(x), \varrho^{-1}(\eth_B)(y)), \\
&\varrho^{-1}(\eth_B)(xy) = \eth_B(\varrho(xy)) = \eth_B(\varrho(x)\varrho(y)) \geq \eth_B(\varrho(x)) = \varrho^{-1}(\eth_B)(x) \\
&and \varrho^{-1}(\eth_B)(xy) = \eth_B(\varrho(xy)) = \eth_B(\varrho(x)\varrho(y)) \geq \eth_B(\varrho(y)) = \varrho^{-1}(\eth_B)(y).
\end{aligned}$$

Also

$$\begin{aligned}
&\varrho^{-1}(\partial_B)(xy) = \partial_B(\varrho(xy)) = \partial_B(\varrho(x)\varrho(y)) \leq C^{con}(\partial_B(\varrho(x)), \partial_B(\varrho(y))) \\
&= C^{con}(\varrho^{-1}(\partial_B)(x), \varrho^{-1}(\partial_B)(y)), \\
&\varrho^{-1}(\partial_B)(xy) = \partial_B(\varrho(xy)) = \partial_B(\varrho(x)\varrho(y)) \leq \partial_B(\varrho(x)) = \varrho^{-1}(\partial_B)(x) \; and \\
&\varrho^{-1}(\partial_B)(xy) = \partial_B(\varrho(xy)) = \partial_B(\varrho(x)\varrho(y)) \leq \partial_B(\varrho(y)) = \varrho^{-1}(\partial_B)(y).
\end{aligned}$$

Therefore, $\varrho^{-1}(B) = (\varrho^{-1}(\eth_B), \varrho^{-1}(\partial_B)) \in BFIN(S)$. □

Theorem 5.5. *Let $\mho = (\eth_{\mho}, \partial_{\mho}) \in BFBIN(S)$, $B = (\eth_B, \partial_B) \in BFBIN(\acute{S})$ and $\varrho : S \to \acute{S}$ be a homomorphism. Then $\varrho(\mho) \in BFBIN(\acute{S})$ and $\varrho^{-1}(B) \in BFBIN(S)$.*

Proof. Let $u, v, z \in \acute{S}$ and $x, y, w \in S$ so $u = \varrho(x)$ and $v = \varrho(y)$ and $z = \varrho(w)$. Then

$$\begin{aligned}
&\varrho(\eth_{\mho})(uv) = \bigvee\{\eth_{\mho}(xy) \mid u = \varrho(x), v = \varrho(y)\} \\
&\geq \bigvee\{T^{nor}(\eth_{\mho}(x), \eth_{\mho}(y)) \mid u = \varrho(x), v = \varrho(y)\} \\
&= T^{nor}(\bigvee\{\eth_{\mho}(x) \mid u = f(x)\}, \bigvee\{\eth_{\mho}(y) \mid v = \varrho(y)\}) = T^{nor}(\varrho(\eth_{\mho})(u), \varrho(\eth_{\mho})(v)) \; and
\end{aligned}$$

$$\begin{aligned}
\varrho(\eth_{\mho})(uzv) &= \bigvee\{\eth_{\mho}(xwy) \mid u = \varrho(x), v = \varrho(y), z = \varrho(w)\} \\
&\geq \bigvee\{T^{nor}(\eth_{\mho}(x), \eth_{\mho}(y)) \mid u = \varrho(x), v = \varrho(y)\} \\
&= T^{nor}(\bigvee\{\eth_{\mho}(x) \mid u = f(x)\}, \bigvee\{\eth_{\mho}(y) \mid v = \varrho(y)\}) = T^{nor}(\varrho(\eth_{\mho})(u), \varrho(\eth_{\mho})(v)).
\end{aligned}$$

Also

$$\begin{aligned}\varrho(\partial_{\mho})(uv) &= \bigwedge\{\partial_{\mho}(xy) \mid u = \varrho(x), v = \varrho(y)\} \\ &\leq \bigwedge\{C^{con}(\partial_{\mho}(x), \partial_{\mho}(y)) \mid u = \varrho(x), v = \varrho(y)\} \\ &= C^{con}(\bigwedge\{\partial_{\mho}(x) \mid u = f(x)\}, \bigwedge\{\partial_{\mho}(y) \mid v = \varrho(y)\}) = C^{con}(\varrho(\partial_{\mho})(u), \varrho(\partial_{\mho})(v)) \ and\end{aligned}$$

$$\begin{aligned}\varrho(\partial_{\mho})(uzv) &= \bigwedge\{\partial_{\mho}(xwy) \mid u = \varrho(x), v = \varrho(y), z = \varrho(w)\} \\ &\leq \bigwedge\{C^{con}(\partial_{\mho}(x), \partial_{\mho}(y)) \mid u = \varrho(x), v = \varrho(y)\} \\ &= C^{con}(\bigwedge\{\partial_{\mho}(x) \mid u = f(x)\}, \bigwedge\{\partial_{\mho}(y) \mid v = \varrho(y)\}) = C^{con}(\varrho(\partial_{\mho})(u), \varrho(\partial_{\mho})(v)).\end{aligned}$$

Thus $\varrho(\mho) = (\varrho(\eth_{\mho}), \varrho(\partial_{\mho})) \in BFBIN(\acute{S})$. Let $x, y, w \in S$. So

$$\begin{aligned}\varrho^{-1}(\eth_B)(xy) &= \eth_B(\varrho(xy)) = \eth_B(\varrho(x)\varrho(y)) \geq T^{nor}(\eth_B(\varrho(x)), \eth_B(\varrho(y))) \\ &= T^{nor}(\varrho^{-1}(\eth_B)(x), \varrho^{-1}(\eth_B)(y)) \ and\end{aligned}$$

$$\begin{aligned}\varrho^{-1}(\eth_B)(xwy) &= \eth_B(\varrho(xwy)) = \eth_B(\varrho(x)\varrho(w)\varrho(y)) \\ &\geq T^{nor}(\eth_B(\varrho(x)), \eth_B(\varrho(y))) = T^{nor}(\varrho^{-1}(\eth_B)(x), \varrho^{-1}(\eth_B)(y)).\end{aligned}$$

Also

$$\begin{aligned}\varrho^{-1}(\partial_B)(xy) &= \partial_B(\varrho(xy)) = \partial_B(\varrho(x)\varrho(y)) \\ &\leq C^{con}(\partial_B(\varrho(x)), \partial_B(\varrho(y))) = C^{con}(\varrho^{-1}(\partial_B)(x), \varrho^{-1}(\partial_B)(y)) \ and\end{aligned}$$

$$\begin{aligned}\varrho^{-1}(\partial_B)(xwy) &= \partial_B(\varrho(xwy)) = \partial_B(\varrho(x)\varrho(w)\varrho(y)) \\ &\leq C^{con}(\partial_B(\varrho(x)), \partial_B(\varrho(y))) = C^{con}(\varrho^{-1}(\partial_B)(x), \varrho^{-1}(\partial_B)(y)).\end{aligned}$$

Then, $\varrho^{-1}(B) = (\varrho^{-1}(\eth_B), \varrho^{-1}(\partial_B)) \in BFBIN(S)$. □

Theorem 5.6. *Let* $\eth \in F(1,2)IT(S)$, $\partial \in F(1,2)IT(\acute{S})$ *and* $\varrho : S \to \acute{S}$ *be a homomorphism. Then,* $\varrho(\eth) \in F(1,2)IT$ *and* $\varrho^{-1}(\partial) \in F(1,2)IT(S)$.

Proof. Let $m, n, p, q \in \acute{S}$ and $x, w, y, z \in S$ so $m = \varrho(x)$ and $n = \varrho(w)$ and $p = \varrho(y)$ and $q = \varrho(z)$. Now

$$\begin{aligned}\varrho(\eth)(mn) &= \bigvee\{\eth(xw) \mid m = \varrho(x), n = \varrho(y)\} \\ &\geq \bigvee\{T^{nor}(\eth(x), \eth(w)) \mid m = \varrho(x), n = \varrho(y)\} \\ &= T^{nor}(\bigvee\{\eth(x) \mid u = \varrho(x)\}, \bigvee\{\eth(y) \mid v = \varrho(y)\}) = T^{nor}(\varrho(\eth)(m), \varrho(\eth)(n)) \ and\end{aligned}$$

$$\begin{aligned}\varrho(ð)(mn(pq)) &= \bigvee\{ð(xw(yz)) \mid m=\varrho(x), n=\varrho(w), p=\varrho(y), q=\varrho(z)\}\\ &\geq \bigvee\{T^{nor}(ð(x), T^{nor}(ð(y), ð_{℧}(z))) \mid m=\varrho(x), p=\varrho(y), q=\varrho(z)\}\\ &= T^{nor}(\bigvee\{ð(x) \mid m=f(x)\}, \bigvee T^{nor}(ð(y), ð(z)) \mid p=\varrho(y), q=\varrho(z)\})\\ &= T^{nor}(\bigvee\{ð(x) \mid m=f(x)\}, T^{nor}(\bigvee\{ð(y) \mid p=\varrho(y)\}, \bigvee\{ð(z) \mid q=\varrho(z)\})\\ &= T^{nor}(\varrho(ð)(m), T^{nor}(\varrho(ð)(p), \varrho(ð)(q))).\end{aligned}$$

Then $\varrho(ð) \in F(1,2)IT(\acute{S})$. Let $x, w, y, z \in S$. Then

$$\varrho^{-1}(\partial)(xy) = \partial(\varrho(xy)) = \partial(\varrho(x)\varrho(y)) \geq T^{nor}(\partial(\varrho(x)), \partial(\varrho(y))) = T^{nor}(\varrho^{-1}(\partial)(x), \varrho^{-1}(\partial)(y))$$

and

$$\begin{aligned}&\varrho^{-1}(\partial)(xw(yz)) = \partial(\varrho(xw(yz)))\\ &= \partial(\varrho(x)\varrho(w)\varrho(y)\varrho(z)) = \partial(\varrho(x)\varrho(w)(\varrho(y)\varrho(z)))\\ &\geq T^{nor}(\partial(\varrho(x)), T^{nor}(\partial(\varrho(y)), \varrho(z)))) = T^{nor}(\varrho^{-1}(\partial)(x), T^{nor}(\varrho^{-1}(\partial)(y), \varrho^{-1}(\partial)(z))).\end{aligned}$$

Thus, $\varrho^{-1}(\partial) \in F(1,2)IT(S)$. □

Theorem 5.7. *Let* $℧ = (ð_{℧}, \partial_{℧}) \in BF(1,2)IN(S)$ *and* $B = (ð_B, \partial_B) \in BF(1,2)IN(\acute{S})$. *suppose that* $\varrho : S \to \acute{S}$ *be a homomorphism. Then*
(1) $\varrho(℧) \in BF(1,2)IN(\acute{S})$.
(2) $\varrho^{-1}(B) \in BF(1,2)IN(S)$.

Proof. (1) Let $m, n, p, q \in \acute{S}$ and $x, w, y, z \in S$ so $m = \varrho(x)$ and $n = \varrho(w)$ and $p = \varrho(y)$ and $q = \varrho(z)$. Hence

$$\begin{aligned}&\varrho(ð_{℧})(mn) = \bigvee\{ð_{℧}(xw) \mid m=\varrho(x), n=\varrho(y)\}\\ &\geq \bigvee\{T^{nor}(ð_{℧}(x), ð_{℧}(w)) \mid m=\varrho(x), n=\varrho(y)\}\\ &= T^{nor}(\bigvee\{ð_{℧}(x) \mid u=f(x)\}, \bigvee\{ð_{℧}(y) \mid v=\varrho(y)\}) = T^{nor}(\varrho(ð_{℧})(m), \varrho(ð_{℧})(n))\end{aligned}$$

and

$$\begin{aligned}\varrho(ð_{℧})(mn(pq)) &= \bigvee\{ð_{℧}(xw(yz)) \mid m=\varrho(x), n=\varrho(w), p=\varrho(y), q=\varrho(z)\}\\ &\geq \bigvee\{T^{nor}(ð_{℧}(x), T^{nor}(ð_{℧}(y), ð_{℧}(z))) \mid m=\varrho(x), p=\varrho(y), q=\varrho(z)\}\\ &= T^{nor}(\bigvee\{ð_{℧}(x) \mid m=f(x)\}, \bigvee T^{nor}(ð_{℧}(y), ð_{℧}(z)) \mid p=\varrho(y), q=\varrho(z)\})\\ &= T^{nor}(\bigvee\{ð_{℧}(x) \mid m=f(x)\}, T^{nor}(\bigvee\{ð_{℧}(y) \mid p=\varrho(y)\}, \bigvee\{ð_{℧}(z) \mid q=\varrho(z)\})\\ &= T^{nor}(\varrho(ð_{℧})(m), T^{nor}(\varrho(ð_{℧})(p), \varrho(ð_{℧})(q))).\end{aligned}$$

Also

$$\begin{aligned}\varrho(\partial_{\mho})(mn) &= \bigwedge\{\partial_{\mho}(xw) \mid m=\varrho(x), n=\varrho(y)\}\\ &\leq \bigwedge\{C^{con}(\partial_{\mho}(x),\partial_{\mho}(w)) \mid m=\varrho(x), n=\varrho(y)\}\\ &= C^{con}(\bigwedge\{\partial_{\mho}(x) \mid u=f(x)\}, \bigwedge\{\partial_{\mho}(y) \mid v=\varrho(y)\}) = C^{con}(\varrho(\partial_{\mho})(m), \varrho(\partial_{\mho})(n))\end{aligned}$$

and

$$\begin{aligned}\varrho(\partial_{\mho})(mn(pq)) &= \bigwedge\{\partial_{\mho}(xw(yz)) \mid m=\varrho(x), n=\varrho(w), p=\varrho(y), q=\varrho(z)\}\\ &\leq \bigwedge\{C^{con}(\partial_{\mho}(x), C^{con}(\partial_{\mho}(y),\partial_{\mho}(z))) \mid m=\varrho(x), p=\varrho(y), q=\varrho(z)\}\\ &= C^{con}(\bigwedge\{\partial_{\mho}(x) \mid m=f(x)\}, \bigwedge C^{con}(\partial_{\mho}(y),\partial_{\mho}(z)) \mid p=\varrho(y), q=\varrho(z)\})\\ &= C^{con}(\bigwedge\{\partial_{\mho}(x) \mid m=f(x)\}, C^{con}(\bigwedge\{\partial_{\mho}(y) \mid p=\varrho(y)\}, \bigwedge\{\partial_{\mho}(z) \mid q=\varrho(z)\})\\ &= C^{con}(\varrho(\partial_{\mho})(m), C^{con}(\varrho(\partial_{\mho})(p), \varrho(\partial_{\mho})(q))).\end{aligned}$$

Then, $\varrho(\mho) = (\varrho(\eth_{\mho}), \varrho(\partial_{\mho})) \in BF(1,2)IN(\acute{S})$. Let $x, w, y, z \in S$. Then

$$\begin{aligned}\varrho^{-1}(\eth_B)(xy) &= \eth_B(\varrho(xy)) = \eth_B(\varrho(x)\varrho(y))\\ &\geq T^{nor}(\eth_B(\varrho(x)), \eth_B(\varrho(y))) = T^{nor}(\varrho^{-1}(\eth_B)(x), \varrho^{-1}(\eth_B)(y))\end{aligned}$$

and

$$\begin{aligned}\varrho^{-1}(\eth_B)(xw(yz)) &= \eth_B(\varrho(xw(yz))) = \eth_B(\varrho(x)\varrho(w)\varrho(y)\varrho(z)) = \eth_B(\varrho(x)\varrho(w)(\varrho(y)\varrho(z)))\\ &\geq T^{nor}(\eth_B(\varrho(x)), T^{nor}(\eth_B(\varrho(y)), \varrho(z))))\\ &= T^{nor}(\varrho^{-1}(\eth_B)(x), T^{nor}(\varrho^{-1}(\eth_B)(y), \varrho^{-1}(\eth_B)(z))).\end{aligned}$$

Also

$$\begin{aligned}\varrho^{-1}(\partial_B)(xy) &= \partial_B(\varrho(xy)) = \partial_B(\varrho(x)\varrho(y))\\ &\leq C^{con}(\partial_B(\varrho(x)), \partial_B(\varrho(y))) = C^{con}(\varrho^{-1}(\partial_B)(x), \varrho^{-1}(\partial_B)(y))\end{aligned}$$

and

$$\begin{aligned}\varrho^{-1}(\partial_B)(xw(yz)) &= \partial_B(\varrho(xw(yz))) = \partial_B(\varrho(x)\varrho(w)\varrho(y)\varrho(z))\\ &= \partial_B(\varrho(x)\varrho(w)(\varrho(y)\varrho(z))) \leq C^{con}(\partial_B(\varrho(x)), C^{con}(\partial_B(\varrho(y)), \varrho(z))))\\ &= C^{con}(\varrho^{-1}(\partial_B)(x), C^{con}(\varrho^{-1}(\partial_B)(y), \varrho^{-1}(\partial_B)(z))).\end{aligned}$$

Therefore, $\varrho^{-1}(B) = (\varrho^{-1}(\eth_B), \varrho^{-1}(\partial_B)) \in BF(1,2)IN(S)$. □

6. Applications, discussion and conclusions

In this study, as using the notions of triangular norms and triangular conorms, the fuzzy semigroups, fuzzy ideals, fuzzy bi-ideals, bifuzzy subsemigroups, bifuzzy ideals, bifuzzy bi-ideals, fuzzy $(1, 2)$-ideals and bifuzzy $(1, 2)$-ideals in any given semigroup will be defined and investigated and obtained some basic properties of them. Now one can study fuzzy semirings, fuzzy ideals, fuzzy bi-ideals, bifuzzy subsemirings, bifuzzy ideals, bifuzzy bi-ideals, fuzzy $(1, 2)$-ideals and bifuzzy $(1, 2)$-ideals in any given semiring and this can be an open problem for future research directions.

Funding: This research received no external funding.

Acknowledgments: We would like to thank the reviewers for carefully reading the manuscript and making several helpful comments to increase the quality of the paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87-96.
2. J. J. Buckley and E. Eslami, *An introduction to fuzzy logic and fuzzy sets*, Springer-Verlag Berlin Heidelberg GmbH, https://doi.org/10.1007/978-3-7908-1799-7, (2002).
3. T. Gerstenkorn and J. Manko, *Bifuzzy probabilistic sets*, Fuzzy Sets and Systems, **71** (1995), 207-214.
4. S. Chaube, D. K. Joshi, C. S. Ujarari, Hesitant bifuzzy set (an introduction): a new approach to assess the reliability of the systems, Mathematics and computers in simulation, **205** (2023), 98–107.
5. T. Grace, C. Suanoom, , J. Phuto and A. Iampan, Bi-fuzzy ideals of d-algebras, ICIC Express Letters Part B: Applications, **14(2)** (2023), 133-140.
6. M. Hamidi, A. B. Saeid, F. Smarandache, Singlevalued neutrosophic filters in EQ-algebras, Journal of Intelligent & Fuzzy Systems, **36(1)** (2019), 805–818.
7. J. Howie, *Fundamentals of semigroup theory*, London Mathematical Society Monographs. New Series, 12. Oxford Science Publications. The Clarendon Press, Oxford University Press, New York, 1995.
8. M. Hamidi, A. B. Saeid, EQ-algebras based on fuzzy hyper EQ-filters, Soft Computing, **23** (2019), 5289–5305.
9. K. Hur, H. W. Kang and J. H. Ryou, *Intuitionistic fuzzy generalized bi-ideal of a semigroup*, International Mathematical Forum, **1**(2006), 257-274.
10. K. Hur, K. J. Kim and H. K. Song, *Intuitionistic fuzzy ideals and bi-ideals*, Honam Math.J, **26(3)**(2004), 309-330.
11. Y. B. Jun, S .M. Hong and J. Meng, *Fuzzy interior ideals in semigroups*,Indian J. of Pure Appl. Math., **26**(1995), 859-963.
12. N. Kuroki, *On fuzzy ideals and bi-ideals in semigroups*, Fuzzy Sets and Systems, **5** (1981), 203-215.
13. N. Kuroki, *On fuzzy semigroups*, Inform. Sci., **53** (1991), 203-236.
14. N. Kuroki, *Fuzzy generalized bi-ideals in semigroups*, Inform. Sci., **66**(1992), 235-243.
15. S. Lajos, *On generalized bi-ideals in semigroups*, Coll. Math. Soc. Janos
16. D. Mandal, *Fuzzy bi-ideals and fuzzy quasi-ideals in ordered semirings*, Gulf Journal of Mathematics, **2(4)**(2014), 60-67.
17. N. Malik, M.Shabir, T. M. Al-shami, R. Gul, M. Arar, M. Hosny, Rough bipolar fuzzy ideals in semigroups, Complex & Intelligent Systems, https://doi.org/10.1007/s40747-023-01132-1, (2023).

18. J. N. Mordeson and N. Kuroki, *Fuzzy semigroups*, Springer, Berlin., 2003.
19. M. Petrich, *Introduction to semigroups*, Columbus, Ohio (1973).
20. R. Rasuli, *Fuzzy ideals of subtraction semigroups with respect to A* T^{nor}*-norm and a* T^{nor}*-conorm*, The Journal of Fuzzy Mathematics Los Angeles, **24** (4) (2016), 881-892.
21. R. Rasuli, *Norms over bifuzzy bi-ideals with operators in semigroups*, Notes on Intuitionistic Fuzzy Sets , **25**(2019), 1-11.
22. R. Rasuli, *t-conorms over anti fuzzy subgroups on direct product of groupos*, Annals of Mathematics and Computer Science, **10**(2022), 8-18.
23. A. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl., **35** (1971), 512-517.
24. K. Saadaoui, S. Milles, L. Zedam, Fuzzy ideals and fuzzy filters on topologies generated by fuzzy relations, International Journal of Analysis and Applications, **20** (2022), 48.
25. M. Shabir, Y. B. Jun and M. Bano, *On prime fuzzy bi-ideals of semigroups*, Iranian Journal of Fuzzy Systems, **7(3)**(2010), 115-128.
26. F. Wang, Intuitionistic fuzzy (1,2)-ideals of semigroups, IOP Conf. Series: Journal of Physics: Conf. Series 1324 (2019) 012002.
27. X. Y. Xie, *Fuzzy ideal extensions of ordered semigroups*, Lobach Journal of Mathematics, **19**(2005), 29-40.
28. L. A. Zadeh, *Fuzzy sets*, Inform. Control., **8**(1965), 338-353.

Received: 02 Apr 2024, **Revised:** 25 Jun 2024,

Accepted: 26 Jul 2024, **Available online:** 04 Aug 2024.