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On Q^*-closed Sets in Fuzzy Neutrosophic Topology: Principles, Proofs, and Examples

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Abstract: This paper aims to present a new concept of sets known as fuzzy neutrosophic Q^*-closed sets in fuzzy neutrosophic topology. In this study, we explore and investigate more novel properties of these classes by some new definitions, theorems, and propositions. Therefore, a group of examples is presented and discussed to clarify the relationships between the new study of Q^*-closed sets with other sets.

Keywords: Fuzzy Neutrosophic Topology; Fuzzy Neutrosophic Close Set; Fuzzy Neutrosophic Q^*-Closed Sets; Fuzzy Neutrosophic Generalized Q^*-Closed Sets; Fuzzy Neutrosophic α Generalized Close Set.

1. Introduction

The notion of fuzziness has swept approximately all divisions of mathematics since the definition of the theory by Zadeh [8]. The applications of fuzzy sets have appeared in many scopes for instance the theory-concept of fuzzy topological spaces that was examined and advanced by Chang [9]. From then on several notions in general topology have been popularized. Conversely, a generalization of fuzzy topological spaces was evolved in several directions by the concept of neutrosophic sets as the expression of the neutrosophic set was defined with membership, non-membership, and indeterminacy degrees by Smarandche [10] and topological spaces of neutrosophic sets were discovered by A.Salama and S. A. Albowi [11]. Since then a survey article on the advanced areas of fuzzy neutrosophic topological spaces has been released by several writers (see, [1-15]).

For this reason, this paper aims to present the notion of Q^*- close sets in the case of fuzzy neutrosophic topology and shows all the significant definitions and theorems. Moreover, it made many detailed comparisons with many examples.

2. Preliminaries

Definition 2.1. [12]: Let F_N be a non-empty fixed set. The fuzzy neutrosophic set (FNS), λ_{N1} is an object having the form $\lambda_{N1} = \{< f_n, \mu_{\lambda_{N1}}(f_n), \sigma_{\lambda_{N1}}(f_n), v_{\lambda_{N1}}(f_n) >: f_n \in F_N\}$ where the functions $\mu_{\lambda_{N1}}, \sigma_{\lambda_{N1}}, v_{\lambda_{N1}}: F_N \rightarrow I$ where $I = [0, 1]$ labeled the degree of membership function (namely $\mu_{\lambda_{N1}}(x)$), the degree of indeterminacy function (namely $\sigma_{\lambda_{N1}}(f_n)$) and the degree of non-membership function (namely $v_{\lambda_{N1}}(fn)$) respectively of each element $f_n \in F_N$ to the set λ_{N1} and $0 \leq \mu_{\lambda_{N1}}(f_n)+\sigma_{\lambda_{N1}}(f_n)+v_{\lambda_{N1}}(f_n) \leq 3$, for each $f_n \in F_N$.

***Remark* 2.2.** [1]: FNS $\lambda_{N1} = \{< f_n, \mu_{\lambda_{N1}}(f_n), \sigma_{\lambda_{N1}}(f_n), v_{\lambda_{N1}}(f_n) >: f_n \in F_N\}$ can be identified as an ordered triple $< f_n, \mu_{\lambda_{N1}}, \sigma_{\lambda_{N1}}, v_{\lambda_{N1}} >$ in I where $I = [0, 1]$ on F_N.

***Lemma* 2.3.** [5]: *Let* F_N be a non-empty *set* and the *FNSs* λN1 *and* βN1 be in the form:

$\lambda_{N1} = \{< f_n, \mu_{\lambda_{N1}}(f_n), \sigma_{\lambda_{N1}}(f_n), v_{\lambda_{N1}}(f_n) >: f_n \in F_N\}$ and
$\beta_{N1} = \{< f_n, \mu_{\beta_{N1}}(f_n), \sigma_{\beta_{N1}}(f_n), v_{\beta_{N1}}(f_n) >: f_n \in F_N\} =$ on F_N. Then,

i. $\lambda_{N1} \subseteq \beta_{N1}$ iff $\mu_{\lambda_{N1}}(f_n) \leq \mu_{\beta_{N1}}(f_n)$, $\sigma_{\lambda_{N1}}(f_n) \leq \sigma_{\beta_{N1}}(f_n)$ and $v_{\lambda_{N1}}(f_n) \geq v_{\beta_{N1}}(f_n)$ for all $f_n \in F_N$,

ii. $\lambda_{N1} = \beta_{N1}$ iff $\lambda_{N1} \subseteq \beta_{N1}$ and $\beta_{N1} \subseteq \lambda_{N1}$,

iii. $1_{N1} - \lambda_{N1} = \{< f_n, v_{\lambda_{N1}}(f_n), 1_{N1} - \sigma_{\lambda_{N1}}(f_n), \mu_{\lambda_{N1}}(f_n) >: f_n \in F_N\}$,

iv. $\lambda_{N1} \cup \beta_{N1} = \{< f_n, \mathrm{Max}(\mu_{\lambda_{N1}}(f_n), \mu_{\beta_{N1}}(f_n)), \mathrm{Max}(\sigma_{\lambda_{N1}}(f_n), \sigma_{\beta_{N1}}(f_n)), \mathrm{Min}(v_{\lambda_{N1}}(f_n), v_{\beta_{N1}}(f_n)) >: f_n \in F_N\}$,

v. $\lambda_{N1} \cap \beta_{N1} = \{< f_n, \mathrm{Min}(\mu_{\lambda_{N1}}(f_n), \mu_{\beta_{N1}}(f_n)), \mathrm{Min}(\sigma_{\lambda_{N1}}(f_n), \sigma_{\beta_{N1}}(f_n)), \mathrm{Max}(v_{\lambda_{N1}}(f_n), v_{\beta_{N1}}(f_n)) >: f_n \in F_N\}$,

vi. $0_{N1} = < f_n, 0, 0, 1 >$ and $1_{N1} = < f_n, 1, 1, 0 >$.

***Definition* 2.4.** [9]: *Fuzzy neutrosophic* topology (FNT) *on a* non-empty set F_N is a family τ of *fuzzy neutrosophic subsets* in F_N satisfying the following.

i. $0_N, 1_N \in \tau_N$,

ii. $\lambda_{N_1} \cap \lambda_{N_2} \in \tau_N$ for any $\lambda_{N_1}, \lambda_{N_2} \in \tau_N$,

iii. $\cup \lambda_{N_j} \in \tau_N, \forall \{\lambda_{N_j}: j \in J\} \subseteq \tau_N$.

The pair (F_N, τ_N) is *c*alled *fuzzy neutrosophic topological s*pace (FNTS). Every elements of τ are called *fuzzy neutrosophic-o*pen sets ($FN - open\ set$). The complement of $FN - open\ set$ in the FN-TS (F_N, τ_N) is *c*alled *fuzzy neutrosophic* -closed set ($FN - closed\ set$).

Definition 2.5. [10]: *et* (F_N, τ_N) is $FNTS$ and $\lambda_{N1} = \{< f_n, \mu_{\lambda_{N1}}(f_n), \sigma_{\lambda_{N1}}(f_n), v_{\lambda_{N1}}(f_n) >: f_n \in F_N\}$ is FNS in F_N. Then the f*uzzy neutrosophic -closure* ($FNcl$) and the *fuzzy neutrosophic-interior* ($FNint$) of $\lambda N1$ are defined by:

i. $FNcl(\lambda_{N1}) = \cap \{\beta_{N1}: \beta_{N1}$ is FN-closed *s*et in F_N and $\lambda_{N1} \subseteq \beta_{N1}\}$,
ii. $FNint(\lambda_{N1}) = \cup \{\beta_{N1}: \beta_{N1}$ is *FN- open s*et in F_N and $\beta_{N1} \subseteq \lambda_{N1}$.

NOn the $FNcl(\lambda_N)$ is *FN-closed set* and $FNint$ t(λ_N) is $FN - open$ *s*et in F_N. And,

i. λ_{N1} is *FN- closed set* in F_N iff $FNcl(\lambda_{N1}) = \lambda_{N1}$,
ii. λ_{N1} is *FN- open set* in F_N iff $FNint(\lambda_{N1}) = \lambda_{N1}$.

Poposition 2.6. [14]: *et* (F_N, τ_N) is $FNTS$ and λ_{N1}, β_{N1} are $FNSs$ in F_N. Then the following properties are true:

i. $FNint(\lambda_{N1}) \subseteq \lambda_{N1}$ and $\lambda_{N1} \subseteq FNcl(\lambda_{N1})$,
ii. $\lambda_{N1} \subseteq \beta_{N1} \Longrightarrow FNint(\lambda_{N1}) \subseteq FNint(\beta_{N1})$ and $\lambda_{N1} \subseteq \beta_{N1} \Longrightarrow FNcl(\lambda_{N1}) \subseteq FNcl(\beta_{N1})$,
iii. $FNint(FNint(\lambda_{N1})) = FNint(\lambda_{N1})$ and $FNcl(FNcl(\lambda_{N1})) = FNcl(\lambda_{N1})$,
iv. $FNint(\lambda_{N1} \cap \beta_{N1}) = FNint(\lambda_{N1}) \cap FNint(\beta_{N1})$ and $FNcl(\lambda_{N1} \cup \beta_{N1}) = FNcl(\lambda_{N1}) \cup FNcl(\beta_{N1})$,
v. $\mathrm{FNint}(1_{N1}) = 1_{N1}$ and $\mathrm{FNcl}(0_{N1}) = 0_{N1}$.

Definition 2.7. [12]: FNS λ_{N1} in FNTS (F_N, τ_N) is called:

i. Fuzzy neutrosophic *s*emi-*closed s*et (FNS-*closed set*) if $\mathrm{FNint}(FNcl(\lambda_{N1})) \subseteq \lambda_{N1}$.
ii. Fuzzy neutrosophic Pre-closed *s*et (FNP-*closed set*) if $\mathrm{FNcl}(\mathrm{FNint}(\lambda_{N1})) \subseteq \lambda_{N1}$.
iii. Fuzzy *n*eutrosophic α-closed *s*et (FNα-*closed set*) if $\mathrm{FNcl}(\mathrm{FNint}(\mathrm{FNcl}(\lambda_{N1}))) \subseteq \lambda_{N1}$.
iv. Fuzzy neutrosophic α^m -*closed s*et (*FN* α^m-*closed set*) if $FNint(FNcl(\lambda N1)) \subseteq UN$, *w*herever $\lambda N1 \subseteq UN$ and UN is FN α -open *s*et.
v. Fuzzy neutrosophic α^g -*closed s*et (*FN* α^g-*closed set*) if $FNcl(K) \subseteq U$ whenever $K \subseteq U$ and U is *FN-* α- *open*.

The complement *o*f fuzzy neutrosophic semi-closed set is fuzzy neutrosophic semi-open *s*et, fuzzy neutrosophic pre-closed *s*et, fuzzy neutrosophic α-closed *s*et, fuzzy neutrosophic α^m -

closed set and fuzzy neutrosophic α^g -closed set is fuzzy neutrosophic semi-open set, fuzzy neutrosophic Pre-open set, fuzzy neutrosophic α-open set, fuzzy neutrosophic α^m-open set and fuzzy neutrosophic α^g -open set respectively.

Definition 2.8. A subset K of FN-TS (FN, τN) is called FN-α^g-*closed set* (briefly, FN-α^g -cs) if $FNcl$(K) ⊆ U whenever, K ⊆ U and U is FN- α- $open$.

3. Fuzzy Neutrosophic Q^*-Closed Sets

In this section, we will study a new class of sets and called it fuzzy neutrosophic Q^*- closed sets

Definition 3.1: A subset K of a $FNTS$ (F_N, τ_N) is called:

i- FN-Q^*-closed set if FN-int(K) = 0_N where, K is FN-cs.

ii- FN-Q^*-open set if FN-cl(K) = 1_N where, K is FN-os.

Example 3.2: et F = $\{f\}$, define FN-Ss and A, B in F as follows:

$$A = \{f\ (0.7, 0.6, 0.5): f \in F\}, \qquad B = \{f\ (0.8, 0.9, 0.4): f \in F\}$$

with the family τ_N = {0N, 1N, A, B} be $FNTS$.

Then, the set A is a FN-Q^*-open set because, FN-Cl(A) = 1_N

And the set B^c is an FN-Q^*-closed set because, FN-int (B^c) = 0_N.

Definition 3.3: et (F_N, τ_N) is FNTS and λ_{N1} = <x, μλN1 (x), $\sigma\lambda_{N1}$ (x), $\nu\lambda_{N1}$ (x) > is FN-S in F_N. Then, the fuzzy neutrosophic Q^*-cl (λ_{N1}) "FN-Q^*-cl (λ_{N1}) " and the fuzzy neutrosophic Q^*-interior (FN- Q^*-int) of $\lambda N1$ are defined by:

i. FN- Q^*-cl(λ_{N1}) = ∩ {β_{N1}: β_{N1} is FN- Q^*- closed set in F and λ_{N1} ⊆ β_{N1}},

ii. FN- Q^*-int(λ_{N1}) = ∪ {β_{N1}: β_{N1} is FN- Q^*- open set in F and β_{N1} ⊆ λ_{N1}}.

Theorem 3.4: Let (F_N, τ_N) is $FNTS$ and λ_{N1}, β_{N1} are FN-Ss in F_N. Then, the following properties hold:

i. FN - Q^*- cl(0_{N1}) = 0_{N1} and FN- Q^*- cl (1_{N1}) =1_{N1},

ii. λ_{N1} ⊆ FN- Q^*- cl(λ_{N1}),

iii. If λ_{N1} ⊆ β_{N1}. Then, FN Q^*- cl(λ_{N1}) ⊆ FN Q^*- cl(β_{N1}),

iv. λ_{N1} is FN- Q^*-closed set in FN iff FN- Q^*- cl (λ_{N1}) = λ_{N1},

v. FN- Q^*- cl(λ_{N1}) = FN-cl(FN- Q^*- cl(λ_{N1})).

Proof: i. By, Definition 3.3 (i). We have,

i. FN- Q^*cl(0_{N1}) = ∩ { β_{N1} : β_{N1} is FN- Q^*-closed set in F_N and $0N1$ ⊆ β_{N1}} = 0_{N1}. And,

FN- Q^*-cl(1_{N1}) = ∩ { β_{N1} : β_{N1} is FN- Q^*-closed set in F_N and 1_{N1} ⊆ β_{N1} } =1_{N1}.

ii. λ_{N1} ⊆ ∩ {β_{N1} : β_{N1} is FN- Q^*-closed set in F_N and λ_{N1} ⊆ β_{N1} } = FN- Q^*cl (λ_{N1}).

iii. Suppose that λ_{N1} ⊆ β_{N1} . Then,

∩ { β_{N1} : β_{N1} is FN- Q^*-closed set in F_N and λ_{N1} ⊆ β_{N1} }

⊆ ∩ {η_{N1}:η_{N1} is FN-Q^*-closed set in F_N and β_{N1} ⊆ ηN1}.Therefore, FN-Q^*cl(λ_{N1}) ⊆ FN-Q^*cl(β_{N1}).

iv. If, λ_{N1} is FN- Q^*-closed set. Then,

FN- Q^*cl (λ_{N1}) = ∩ { β_{N1} : β_{N1} is FN- Q^*- closed set in F_N and λ_{N1} ⊆ β_{N1} }.

And, by (ii). We get, λ_{N1} ⊆ FN- Q^*cl (λ_{N1}) but, λ_{N1} is necessarily to be the smallest set. Thus, λ_{N1} = ∩{ β_{N1} : β_{N1} is FN- Q^*- closed set in F_N and λ_{N1}⊆ β_{N1} }.Therefore, λ_{N1} = FN- Q^*cl (λ_{N1}).

Conversely; assume that v = FN-Q^*cl (λ_{N1}) by using the definition. We get, λ_{N1} is FN-Q^*- closed sets.

v. By, (iv). We get, λ_{N1} = FN- Q^*cl (λ_{N1}). Then, FN- Q^*cl (λ_{N1}) = FN- Q^*cl(FN- Q^*cl(λ_{N1})).

Theorem 3.5: Let (F_N , τ_N) is $FNTS$ and λ_{N1}, β_{N1} are FN-Ss in F_N . Then, the following Properties hold:

i. FN - Q^*- $int(0_{N1})$ = 0_{N1} and FN - Q^*- int (1_{N1}) =1_{N1},

ii. FN - Q^*- $int(\lambda_{N1}) \subseteq \lambda_{N1}$,

iii. If, $\lambda_{N1} \subseteq \beta_{N1}$. Then, FN- Q^*- int($\lambda_{N1}$) ⊆ FN- Q^*- int($\beta_{N1}$),

iv. λ_{N1} is FN- Q^*-*open* iff λ_{N1} = FN- Q^*- int (λ_{N1}),

v. FN- Q^*- int(λ_{N1}) = FN- Q^*- int(FN- Q^*- int(λ_{N1})).

Proof: *i*. By, Definition 3.3 (*ii*). We have,

FN - Q^*$int(0_{N1})$ = ∪ { β_{N1}: β_{N1} is FN- Q^*-open set in F_N and $\beta_{N1} \subseteq 0_{N1}$} = 0_{N1}.

and,

FN - Q^* $int(1N1)$ = ∪ { β_{N1}: β_{N1} is FN- Q^*-open set in F_N and $\beta_{N1} \subseteq 1_{N1}$} =$1_{N1}$.

ii. Follows from Definition.

iii. FN- Q^* int(λ_{N1}) = ∪ { β_{N1}: β_{N1} *is* FN- Q^* -open set in F_N and $\beta_{N1} \subseteq \lambda_{N1}$}.

Since, $\lambda_{N1} \subseteq \beta_{N1}$. Then, ∪ { β_{N1}: β_{N1} is FN- Q^*-open set in F_N and $\beta_{N1} \subseteq \lambda_{N1}$}
⊆ ∪ { η_{N1}: η_{N1} is FN - Q^*- *open set* in F_N and $\eta_{N1} \subseteq \beta_{N1}$}.

Therefore, FN - Q^* int(λ_{N1}) ⊆ FN - Q^* int(β_{N1}).

Suppose that λ_{N1} is FN-open set in F_N Then,

$\lambda_{N1} \subseteq$ FN - Q^* int(λ_{N1}) …… (1).

By using (*ii*). *We* get, FN- Q^* int(λ_{N1}) ⊆ λ_{N1}…….. (2).

From (1) and (2) we have, λ_{N1} = FN - Q^* int (λ_{N1}).

Conversely; assume that λ_{N1} = FN - Q^* int (λ_{N1}) by using the definition. We get, λ_{N1} is FN - Q^*-*open* set in F_N .

v. By, (*iv*). We get, λ_{N1} = FN - Q^* $int(\lambda_{N1})$. Then, FN -Q^* $int(\lambda_{N1})$ = FN- Q^* int(FN- Q^* int(λ_{N1})).

Remark 3.6: Every FN-Q^*-closed set is FN-closed set but the converse is not true.The following the example show this case.

Example 3.7: Let FN ={f} define FN -Ss A_{1N}, A_{2N}, A_{3N}, and A_{4N} in F as follows:
A_{1N} = {f, (0.3, 0.7, 0.5): f ∈ F},
A_{2N}= {f, (0.5, 0 .3, 0.3): f ∈ F},
A_{3N}= {f, (0.3, 0.3, 0.5): f ∈ F},
A_{4N}= {f, (0.5, 0.7, 0.3): f ∈ F}and the family τ_N = {0_N, 1_N, A_{1N}, A_{2N}, A_{3N} , A_{4N} } be FN-TS.
Then, Int (A $_{1N}$ ^c) = {f, (0.5, 0.3, 0.3)} ≠ 0. Then A $_{1N}$ ^cis closed set but not Q^*-cs.

Proposition 3.8: For any FN-TS, the following statements satisfy:

i. Every FN-Q^* -closed set is FN-α-closed set.
ii. Every FN-Q^* -closed set is FN-pre-closed set.
iii. Every FN-Q^* -closed set is FN-semi-closed set.
iv. Every FN-Q^* -closed set is FN-β-closed set.
v. Every FN-Q^* -closed set is FN-α^g-closed set.

Remark 3.9: The conversion of proposition 3.8 is not true. The following example shows that.

Example 3.10: Let F = {a, b}, define $FNSs$ and A, B, C, D in F as follows:

A = {f (0.3, 0.5, 0.4), (0.6, 0.2, 0.5);f ∈ F },

B = {f (0.2, 0.6, 0.7), (0.5, 0.3, 0.1); $f \in$ F },
C = {f (0.3, 0.6, 0.4), (0.6, 0.3, 0.1); $f \in$ F },
D = {f (0.2, 0.5, 0.7), (0.5, 0.2, 0.5); $f \in$ F },
And the family τ_N = {0_N, 1_N, A, B, C, D} be FN-TS.Then,
FN-int(D^c) = <f (0.2, 0.5, 0.7), (0.5, 0.2, 0.5)> $\neq 0$. So, D^c is not FN-Q^* -closed set. And,
FN -int(FN -cl(f (0.7, 0.5, 0.2), (0.5, 0.8, 0.5)>) = FN-int(f (0.7, 0.5, 0.2), (0.5, 0.8, 0.5)>) =
< f (0.2, 0.5, 0.7), (0.5, 0.2, 0.5)>, where, < f (0.2, 0.5, 0.7), (0.5, 0.2, 0.5)> ⊆ D^c

Then, D^c is FN- semi-closed set.
And, FN-cl(FN -int(FN-cl(f <(0.7, 0.5, 0.2), (0.5, 0.8, 0.5)>))) = D^c
Then, D^c is FN- α – closed set.
And, FN-int (D^c) = (<f (0.2, 0.5, 0.7), (0.5, 0.2, 0.5)>)
So, FN-cl(FN-int(<f (0.2, 0.5, 0.7), (0.5, 0.2, 0.5)>)) = D^c
Then, D^c is FN- pre – closed set.
And,
FN-int(FN-cl(FN-int(f (0.7, 0.5, 0.2), (0.5, 0.8, 0.5)>))) = (<f (0.2, 0.5, 0.7), (0.5, 0.2, 0.5)>) ⊆ D^c.
So, D^c is FN-β – closed set.

Proposition 3.11: Let (F_N , τ_N) be a $FNTS$. K is FN - Q^*- $o$pen $s$et in FN iff for each U is a FN - Q^*-$closed$ set $s$uch that U ⊆ K and U ⊆ FN-int(K).

Proof: Let K is a FN - Q^*-$o$pen, then 1_N_K is FN- Q^*-closed so, 1_N_K ⊆ U, then $FNcl$(1N_K) ⊆ U.
Put 1N_K=U and 1N _ FN -int(K) ⊆ U, for each U ⊆ K and
U ⊆ FN-int(K). Then 1N _U ⊆ FN-int(K).
⇐ To prove 1N _K is a FN- Q^*-clo$s$ed $s$et. We take K to be FN - Q^*-$o$pen.
So, for each K, is FN - Q^*-$closed$ $s$et.
Then, 1N _ FN -int(K) ⊆ 1N _U therefore FN -cl(K) ⊆ 1N _U for each
K ⊆ U, so 1N _ K is FN - Q^*-clo$s$ed.

Definition 3.12: Let (F_N , τ_N) is FN-TS. Fuzzy neutrosophic $s$et K is called:

i. Fuzzy neutrosophic generalized-Q^*-clo$s$ed $s$et (FN-G-Q^*-closed $s$et) if FN-cl (K) ⊆ UN wherever UN is FN- Q^*-$o$pen $s$et in F. K is said to be $fuzzy\ neutrosophic$ Generalized-Q^*-$open\ set$ (FN-G-Q^*-open set) in (F_N , τ_N) if the complement 1N-K is FN-G- Q^*-closed set.
ii. Fuzzy neutrosophic - Q^*-generalized- clo$s$ed $s$et (FN- Q^* -G-cs) if FN-Q^*-cl (K) wherever, K is FN-open $s$et. K is $s$aid to be Fuzzy neutrosophic- Q^*-generalized-$o$pen $s$et (FN-Q^*-G-os) if the complement 1N-K is FN- Q^* -G-cs.

Theorem 3.13: Let (F_N , τ_N) be a FNTS. A fuzzy neutrosophic $s$et K is FN- Q^*-G-os iff U_N ⊆ K and U_N is an FN- Q^*-open set, so 1_K is an FN- Q^*-G-closed set in F_N.

Proof: Let K be FN- Q^*-G-$o$pen $s$et in FN and let UN be any FN- Q^*-clo$s$ed $s$et in FN $s$uch that U_N ⊆ K and U_N is an FN- Q^*-$o$pen $s$et, so $1 - $K is an FN- Q^*-G-clo$s$ed $s$et in FN.
Therefore, for all FN-open sets ν to say $\nu = 1 - U_N$,
$1 -$ K ⊆ $1 - U_N$, then cl(1_K) ⊆ 1 - U_N.

So, 1_(1_UN) = U_N ⊆ 1- cl($1 - K$)= int(K).
⇐ Let U_N be an FN- Q^*-closed set.
So, for each $U_N \in \tau$N, such that U_N ⊆ K, U_N is an FN-open set.

Now, $U_N \subseteq$ int(K). If K is a FN- Q^*-G-open set, this implies 1_K is an FN- Q^*-G-closed set, take $\nu \in \tau_N$ such that $1_K \subseteq \nu$, since $\nu \in \tau_N$, then 1_ν is an FN-Q^*-closed set and $1_\nu \subseteq K$, so by hypothesis $1_\nu \subseteq$ int(K).

Therefore, 1_int(K) = cl(K) ⊆ 1_(1_ν) = ν
So, by Definition we get that 1_K is an FN - Q^*- G -closed set.

Definition 3.14: Let (F_N, τ_N) is FN-TS and $\lambda_{N1} = < x, \mu_{\lambda_{N1}}(x), \sigma_{\lambda_{N1}}(x), \nu_{\lambda_{N1}}(x) >$ is FN-S in F_N. T hen, the fuzzy neutrosophic Q^* generalized-cl(λ_{N1}) "FN-Q^*-G-cl (λ_{N1}) " and the fuzzy neutrosophic Q^* generalized – interior (FN - Q^*-G-int) of λ_{N1} are defined by:

i. FN- Q^*-G-$cl(\lambda_{N1})$ = ∩ {$\beta N1$: $\beta N1$ is FN - Q^*-G-$closed\ set$ in F and $\lambda N1 \subseteq \beta N1$},
ii. FN- Q^*-G-$int(\lambda_{N1})$ = ∪ {$\beta N1$: $\beta N1$ is FN- Q^*-G-open set in F and $\beta N1 \subseteq \lambda N1$}.

***Theorem* 3.15**: Let (F_N, τ_N) be a FN-TF, for each $\lambda_{N1} \in F_N$, For each $\lambda N1$, the operator FN - Q^*-G- cl satisfies the following statement:

i. FN - Q^*-G-cl(0_{N1}) = 0_{N1} , FN- Q^*-G- cl(1_{N1}) = 1_{N1},
ii. $\lambda_{N1} \subseteq$ FN- Q^*-G- cl(λ_{N1}),
iii. FN - Q^*-G- $cl(\lambda_{N1})$ ∪ FN- Q^*-G- $cl(\mu)$ ⊆ FN- Q^*-G- $cl(\lambda_{N1} \cup \mu)$,
iv. FN - Q^*-G- cl(FN- Q^*-G- cl(λ_{N1})) = FN- Q^*-G- cl(λ_{N1}),
v. If λ_{N1} is an FN - Q^*-G-$closed\ set$, then FN - Q^*-G- $cl(\lambda_{N1})$= λ_{N1},
vi. FN - Q^*-G- cl (λ_{N1}) ⊆ FN - Q^*-cl(λ_{N1}) ⊆ cl(λ_{N1}).

Proof: Directly by the definition.

Theorem 3.16: Let (F_N, τ_N) be a FN-TF, for each $\lambda_{N1} \in I^X$, For each λ_{N1}, the operator FN- Q^*-G-int, satisfies the following statement:

(i) FN - Q^*-G- int(0_{N1}) = 0_{N1}, FN- Q^*-G- int(1_{N1}) = 1_{N1},
(ii) FN- Q^*-G- int(λ_{N1}) ⊆ λ_{N1},
(iii) FN- Q^*-G- $int(\lambda_{N1} \cap \mu)$ = FN - Q^*-G- int(λ_{N1}) ∩ FN - Q^*-G- int(μ),
(iv) FN- Q^*-G- int(λ_{N1}) = FN- Q^*-G- int(FN - Q^*-G- int(λ_{N1})).

Proof: Directly by the definition.

Remark 3.17: Every FN -Q^*-cs is FN- α^g -cs but the converge is not true.
Example 3.18: Let F = {f}, define FN -Ss and A, B in F as follows:

A = {f (0.1, 0.2, 0.8): f ∈ F}and B = {f (0.7, 0.5, 0.2): f ∈ F} with the family,

τ_N = {0_N, 1_N, A, B} be FNTS. Then, the set B^c is a FT- α^g -cs because, FN -cl(B^c) ⊆ UN, where B^c ⊆ U_N and U_N = {f (0.7, 0.5, 0.2): f ∈ F}, {f (0.2, 0.5, 0.7): f ∈ F} ⊆ {f (0.7, 0.5, 0.2): f ∈ F},

And, B^c is not FN-Q^*-closed set because, FN-int(B^c) = {f (0.1, 0.2, 0.8): f ∈ F} ≠ 0_N.

Remark 3.19: The relationship between different sets in FN-TS (F_N, τ_N) can be shown in the next Figure 1.

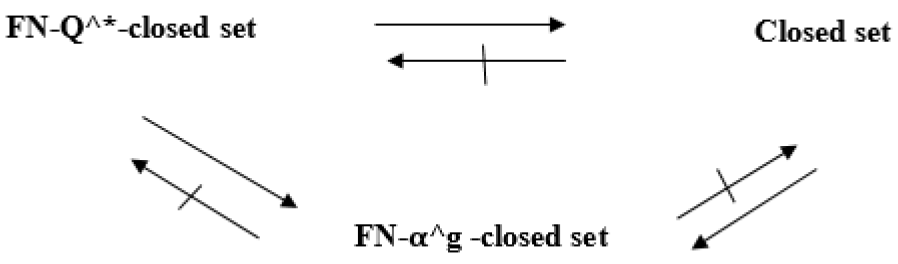

Figure 1.

4. Conclusions

In this paper, a recent notion concerning the theory of fuzzy neutrosophic sets has been defined, which is said to be fuzzy neutrosophic Q^*- closed set. The work has suggested some characteristics of the newly established concept. Some relations among the defined model with other sets have been explained by fuzzy neutrosophic topology.

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Author Contributions
All authors contributed equally to this research.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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Conflict of interest
The authors declare that there is no conflict of interest in the research.

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