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Some Operations on Neutrosophic Hypersoft Matrices and Their Applications

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Abstract: This paper aims to extend the concept of Neutrosophic Hypersoft Matrix (NHSM) theory. NHSM is the matrix representation of a Neutrosophic Hypersoft Set (NHSS), where NHSS is the combination of a Neutrosophic set and a Hypersoft set. An NHSS can be stored in computer memory using the matrix notion, which is very useful and applicable. Based on NHSM, we provide some new notions (operations) such as NHS-sub-matrix, Equal NHSM, Null NHSM, Universal NHSM, Complement NHSM, NH-choice matrix (NHCM), product of NHCM and combined NHCM along with examples and characterizations. Additionally, we develop an NHSM algorithm using a value matrix, grace matrix, and mean matrix to solve a decision-making problem based on NHSSs. Finally, the NHSM algorithm was used to select the critically suffering chronic kidney disease (CKD) patient.

Keywords: Soft Set, Neutrosophic Soft Set, Neutrosophic Hypersoft Set, Neutrosophic Hypersoft Matrix, Neutrosophic Hyper-choice Matrix.

1. Introduction

To deal with uncertainty, Lotfi A. Zadeh [1] 1965 introduced the concept of Fuzzy logic and Fuzzy sets. In Fuzzy logic, it represents the degree of truth as an extension of valuation.To deal with imprecise and vague information K. Atanassov [2] in 1986 introduced the concept of Intuitionistic fuzzy sets and Intuitionistic fuzzy logic. Similarly, Pythagorean fuzzy sets,Pythagorean fuzzy numbers, and several other concepts and their applications in MCDM,MADM, and MAGDM were proposed in [3-11]. Thomason [12] expanded the idea of Fuzzy sets to Fuzzy matrices (FM) and talked about the convergence of powers of Fuzzy matrices.Fuzzy matrices only take into account membership values while solving the Decision-making problems. To deal with both membership and non-membership values, Pal et al. [13]transformed the well-known Fuzzy matrix into the Intuitionistic fuzzy matrix (IFM), whose constituent parts come from the unit interval [0; 1].The parameterization of the attributes is not discussed in any of the aforementioned studies. Molodtsov [14] 1999 generalized the concept of fuzzy set theory to soft set theory which helps to deal with uncertainty. Some basic properties of soft set theory were proposed by P. K. Maji et al. [15]. Later on, several interesting results based on soft set theory were obtained by embedding the idea of Fuzzyset, Intuitionistic fuzzy set, Vague set, Rough set, Interval-valued intuitionistic fuzzy set andso on. Also, various applications of the above-mentioned sets in decision-making problems were developed in [16-22].

Naim Cagman et.al. [23] introduced the notion of the soft matrix (SM), which isthe representation of soft sets and also defined products of such matrices. They also have offered a soft max-min decision-making algorithm to solve some problems with uncertainties. However, because ofthe different product order, this method does not satisfy the commutative law, as it could lead to two different outcomes when used to solve identical decision-making problems. Further,this approach will be wholly invalid if a decision-making problem requires the perspectives of atleast three observers. To overcome such limitations Yong [24] et.al., introduceda Fuzzy soft matrix (FSM). Further, to deal with both membership and non-membership valuesin a parametric manner, Rajarajeswari et al. [25] proposed the Intuitionistic fuzzy soft matrix(IFSM). This idea handles the uncertain object more accurately with their parametrization and ensures that the sum of membership degrees and non-membership degrees does not exceed 1. Abhishek [26] et.al., proposed the idea of the Pythagorean fuzzy soft matrix (PFSM) that altered the condition MV + NMV ≤ 1 to MV 2 + NMV 2 ≤ 1.

F. Smarandache [27] 1998 introduced the concept of Neutrosophic sets and Neutrosophic logic with indeterminate data. Neutrosophic soft sets were introduced by Maji in [28]. Healso gave an application on Neutrosophic soft sets in decision-making problems. The conceptof Generalized Neutrosophic soft set theory was proposed by Said Broumi [29]. Similarly, several concepts based on Neutrosophic Soft set theory have emerged in recent days. Lateron, in 2015 Irfan Deli et al. [30] introduced the concept of Neutrosophic soft matrix (NSM)and their operators which are more functional to make theoretical studies in the Neutrosophic soft set theory. Also, Tanushree Mitra Basa et al. [31] in 2015 developed the Neutrosophic soft matrix theory by defining various operations on them. In 2017 Tuhin Bera [32] et.al.,further extended the concept of Neutrosophic soft matrix theory and presented an applicationin decision making. Following this, Sujit Das [33] et.al., gave an application in group decision-making. Similarly, Jayasudha et.al. [34] gave an application of neutrosophic soft matrices in decision-making.Smarandache [35] introduced Hypersoft sets which deal with multi-attribute functions. Following this, Ihsan et al. [36] explored the concept of Hypersoft expert sets along with an applicationin decision-making for the recruitment process. Further, Muhammad Saqlainet al. [37] developed a new concept called the Neutrosophic Hypersoft set and also studied some operations on it. Rana Muhammad Zulqarnain et al. [38] developed the generalized version of aggregate operators on Neutrosophic Hypersoft sets. In 2021 Abdul Samad [39] et.al., devised a methodthat is an extension of the TOPSIS technique using Neutrosophic hypersoft sets based on correlation coefficient to determine the effectiveness of hand sanitizer to reduce COVID-19 effects.Further, a Neutrosophic hypersoft expert set was introduced by Ihsan et al.[40]. Ihsanet al. [41] developed Single valued neutrosophic hypersoft expert set and gave an applicationin decision-making. In 2022, J. Jayasudha et al. [42] developed a new concept called Interval-valued Neutrosophic Hypersoft Expert set and developed some new notions. The same authorsin [43] introduced the concept of Interval-valued Neutrosophic Hypersoft Topological Spacesand established some notions, properties, and results, with examples. To reduce the complicated framework of Neutrosophic Hyper-soft sets, Rana Muhammad Zulqarnainet et al. [44]developed the concept of Neutrosophic Hypersoft Matrices (NHSM) and provided certain basic operators and operations on them. Further, certain new notions, operations, and properties of Neutrosophic Hypersoft Matrices (NHSM) have been explored by Naveed Jafaret et al. [45]. Neutrosophic soft matrices parametrically evaluate the attributes chosen whereas the Neutrosophic hypersoft matrix can parametrically evaluate the sub-attributes of the attributes chosen.

The present study aims to extend the concept of NHSM theory developing some basic notions and operations along with their properties. The organization of our manuscript is as follows:

In Section 2 we recall some fundamental definitions which would further be helpful to extend the NHSM theory. In Sections 3 and 4, we develop the NHSM theory by defining basic notions of classical matrix theory in NHSM has been discussed with examples. Also, NHCM, combined NHCM, and some properties of NHSM have been examined. In Section 5, an efficient methodology for evaluating NHSMs has been developed based on certain new notions such as value matrix, grace matrix, mean matrix, and total mean matrix, using which we solve NHSS-based decision-making problems.

2. Preliminaries

This section provides some basic notions of soft set theory which would be helpful to read this paper. Throughout this paper,the Neutrosophic Hypersoft Matrix is represented by NHSM.

Definition 2.1. [14]Let U be an initial universal set and E be a set of parameters. Let P(U) denote the power set of U. Consider a nonempty set B⊂E. A pair (G, B) is called a soft set of U, where G is a mapping given by G: B →P(U). A soft set over U is a parameterized family of subsets of the universe U. For ε ∈ B, G(ε) may be considered as the set of ε-approximate elements of the soft set (G, B).

Definition 2.2.[46]A neutrosophic set B on the universal set Y is attributed to three individualistic degrees namely, *truth-membership degree* (η), *indeterminacy-membership degree* (υ),and *falsity-membership degree* (φ), which is defined as;

$$B = \{< y, \eta_B(y), \upsilon_B(y), \varphi_B(y) >: y \in Y\},$$

where η_B, υ_B, φ_B: $Y \rightarrow]^{-}0,1^{+}[$ and $^{-}0 \leq \eta_B(y) + \upsilon_B(y) + \varphi_B(y) \leq 3^{+}$.

Definition 2.3. [47]A neutrosophic soft set g over Y is a neutrosophic set valued function from E to $N(Y)$. It can be written as $g = \{(e, \eta_{g(e)}(y), \upsilon_{g(e)}(y), \varphi_{g(e)}(y) >: y \epsilon Y\}): e \in E\}$ where, $N(Y)$denotes all neutrosophic sets over Y.

Definition 2.4. [37]Let ξ be the universal set and $P(\xi)$.be the power set of ξ. Consider $l^1, l^2, l^3, \dots, l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are respectively the set L^1, L^2, $L^3, \dots, L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3, \dots, n\}$, then the pair $(G, L^1 \times L^2 \times L^3 \dots L^n)$ is said to be Hypersoft set over ξ where

$$G: L^1 \times L^2 \times L^3 \dots L^n \rightarrow P(\xi)$$

Definition 2.5. [37]Let ξ be the universal set and $P(\xi)$.be the power set of ξ. Consider $l^1, l^2, l^3, \dots, l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are respectively the set L^1, L^2, $L^3, \dots, L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3, \dots, n\}$ and their relation $L^1 \times L^2 \times L^3 \dots L^n = S$, then the pair (G, S) is said to be Neutrosophic Hypersoft set over ξ where $G: L^1 \times L^2 \times L^3 \dots L^n \rightarrow P(\xi)$ and $G(L^1 \times L^2 \times L^3 \dots L^n) = \{< x, \eta(G(S)), \upsilon(G(S)), \varphi(G(S)) >, x \in \xi\}$ where η is the *truth-membership value,* υ *is the indeterminacy-membership value* and φ is the *falsity-membership value* such that $\eta, \upsilon, \varphi : \xi \rightarrow [0, 1]$ also $0 \leq \eta(G(S)) + \upsilon(G(S)) + \varphi(G(S)) \leq 3$.

Definition 2.6. [44] Let $U = \{u^1, u^2, \dots, u^\alpha\}$ and (U) be the universal set and power set of the universal set, respectively, and also consider $L_1, L_2, \dots, L_\beta$ for $\beta \geq 1, \beta$ well-defined attributes, whose corresponding attribute values are, respectively, the set $L_1^a, L_2^b, \dots, L_\beta^z$ and their relation $L_1^a \times L_2^b \times \dots \times L_\beta^z$, where $a, b, c, \dots, z = 1, 2, \dots, \beta$, then the pair $(G, L_1^a \times L_2^b \times \dots \times L_\beta^z)$ is said to be neutrosophic hypersoft set over U, where $G: L_1^a \times L_2^b \times \dots \times L_\beta^z \rightarrow P(U)$, and it is defined as

$$G(L_1^a \times L_2^b \times \dots \times L_\beta^z) = \{u, \eta_i(u), \upsilon_i(u), \varphi_i(u): u \in U, i \in L_1^a \times L_2^b \times \dots \times L_\beta^z\}.$$

Let $R_i = L_1^a \times L_2^b \times \dots \times L_\beta^z$ be the relation, and its characteristic function is

$$X_{R_i} = L_1^a \times L_2^b \times \dots \times L_\beta^z \rightarrow P(U);$$

It is defined as $X_{R_i} = \{u, \eta_i(u), \upsilon_i(u), \varphi_i(u): u \in U, i \in L_1^a \times L_2^b \times \dots \times L_\beta^z\}$ and can be a representation of R_i as given in Table 1.

Table 1. Tabular representation of the characteristic function.

U	L_1^a	L_2^b	…	L_β^z
u^1	$X_{R_i}(u^1, L_1^a)$	$X_{R_i}(u^1, L_2^b)$	…	$X_{R_i}(u^1, L_\beta^z)$
u^2	$X_{R_i}(u^2, L_1^a)$	$X_{R_i}(u^2, L_2^b)$	…	$X_{R_i}(u^2, L_\beta^z)$
⋮	⋮	⋮	⋮	⋮
u^α	$X_{R_i}(u^\alpha, L_1^a)$	$X_{R_i}(u^\alpha, L_2^b)$	…	$X_{R_i}(u^\alpha, L_\beta^z)$

If $M_{ij} = X_{R_i}(u^i, L_j^k)$, where $i = 1,2,3,\dots,\alpha, j = 1,2,3,\dots,\beta, k = a,b,c,\dots,z$, then the matrix is defined as:

$$[M_{ij}]_{\alpha\times\beta} = \begin{pmatrix} M_{11} & M_{12}\cdots & M_{1\beta} \\ M_{21} & M_{22}\cdots & M_{2\beta} \\ \vdots & \vdots\ \vdots & \vdots \\ M_{\alpha1} & M_{\alpha2}\cdots & M_{\alpha\beta} \end{pmatrix}$$

Where$M_{ij} = \left(\eta_{L_j^k}(u_i), \upsilon_{L_j^k}(u_i), \varphi_{L_j^k}(u_i), u_i \in U, L_j^k \in L_1^a \times L_2^b \times \dots \times L_\beta^z\right) = (\eta_{ijk}^M, \upsilon_{ijk}^M, \varphi_{ijk}^M)$.

Thus, we can represent any neutrosophic hypersoft set in terms of the neutrosophic hypersoft matrix (NHSM), and it means that they are interchangeable.

Definition 2.7. [44]Let $O = [O_{ij}]$ be the square NHSM of order $\varsigma \times \varsigma$, where $O_{ij} = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)$, then O^t is said to be the transpose of square NHSM if rows and columns of O are interchanged. It is denoted as

$$O^t = [O_{ij}]^t = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)^t = (\eta_{jki}^O, \upsilon_{jki}^O, \varphi_{jki}^O) = [O_{ji}].$$

Definition 2.8. [44]Let $O = [O_{ij}]$ and $M = [m_{ij}]$ be two NHSMs of order $\varsigma \times \nu$, where $O_{ij} = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)$and $M_{ij} = (\eta_{ijk}^M, \upsilon_{ijk}^M, \varphi_{ijk}^M)$. Then, their union is defined as follows:

$$O \cup M = D, where \eta_{ijk}^D = \max(\eta_{ijk}^O, \eta_{ijk}^M), \upsilon_{ijk}^D = \frac{(\upsilon_{ijk}^O + \upsilon_{ijk}^M)}{2}, \varphi_{ijk}^O = \min(\varphi_{ijk}^O, \varphi_{ijk}^M).$$

Definition 2.9. [44]Let $O = [O_{ij}]$ and $M = [m_{ij}]$ be two NHSMs of order $\varsigma \times \nu$, where $O_{ij} = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)$and $M_{ij} = (\eta_{ijk}^M, \upsilon_{ijk}^M, \varphi_{ijk}^M)$. Then, their intersection is defined as follows:

$$O \cap M = D, where \eta_{ijk}^D = \min(\eta_{ijk}^O, \eta_{ijk}^M), \upsilon_{ijk}^D = \frac{(\upsilon_{ijk}^O + \upsilon_{ijk}^M)}{2}, \varphi_{ijk}^O = \max(\varphi_{ijk}^O, \varphi_{ijk}^M).$$

Operations of NHSM [45] Let $M = [m_{ij}] = (\eta_{ijk}^m, \upsilon_{ijk}^m, \varphi_{ijk}^m)$ and $N = [n_{ij}] = (\eta_{ijk}^n, \upsilon_{ijk}^n, \varphi_{ijk}^n)$ be two NHSMs of order $\varsigma \times \nu$. Then,

- **Arithmetic mean**$M \circledast N = O$,
  where $\eta_{ijk}^o = \frac{\eta_{ijk}^m + \eta_{ijk}^n}{2}, \upsilon_{ijk}^o = \frac{\upsilon_{ijk}^m + \upsilon_{ijk}^n}{2}, \varphi_{ijk}^o = \frac{\varphi_{ijk}^m + \varphi_{ijk}^n}{2}, \forall\, i,j,k.$

- **Weighted Arithmetic mean**$M \circledast^w N = O$,
  where $\eta_{ijk}^o = \frac{w_1\eta_{ijk}^m + w_2\eta_{ijk}^n}{w_1+w_2}, \upsilon_{ijk}^o = \frac{w_1\upsilon_{ijk}^m + w_2\upsilon_{ijk}^n}{w_1+w_2}, \varphi_{ijk}^o = \frac{w_1\varphi_{ijk}^m + w_2\varphi_{ijk}^n}{w_1+w_2}, \forall\, i,j,k$and $w_1, w_2 > 0$.

- **Geometric mean**$M \circledcirc N = O$,
  where $\eta_{ijk}^o = \sqrt{\eta_{ijk}^m . \eta_{ijk}^n}, \upsilon_{ijk}^o = \sqrt{\upsilon_{ijk}^m . \upsilon_{ijk}^n}, \varphi_{ijk}^o = \sqrt{\varphi_{ijk}^m . \varphi_{ijk}^n}, \forall\, i,j,k.$

- **Weighted Geometric mean**$M \circledcirc^w N = O$,
  where
  $\eta_{ijk}^o = (w_1 + w_2)\sqrt{(\eta_{ijk}^m)^{w_1}.(\eta_{ijk}^n)^{w_2}}, \upsilon_{ijk}^o = (w_1 + w_2)\sqrt{(\upsilon_{ijk}^m)^{w_1}.(\upsilon_{ijk}^n)^{w_2}}, \varphi_{ijk}^o = (w_1 + w_2)\sqrt{(\varphi_{ijk}^m)^{w_1}.(\varphi_{ijk}^n)^{w_2}}, \forall\, i,j,k$ and $w_1, w_2 > 0$.

- **Harmonic mean**$M \boxdot N = O$,
  where
  $\eta_{ijk}^o = \frac{2\eta_{ijk}^m\eta_{ijk}^n}{\eta_{ijk}^m + \eta_{ijk}^n}, \upsilon_{ijk}^o = \frac{2\upsilon_{ijk}^m\upsilon_{ijk}^n}{\upsilon_{ijk}^m + \upsilon_{ijk}^n}, \varphi_{ijk}^o = \frac{2\varphi_{ijk}^m\varphi_{ijk}^n}{\varphi_{ijk}^m + \varphi_{ijk}^n}, \forall\, i,j,k.$

- **Weighted Harmonic mean**$M \boxdot^{w} N = O$,
  where
  $\eta_{ijk}^{o} = \frac{w_1+w_2}{\frac{w_1}{\eta_{ijk}^{m}}+\frac{w_2}{\eta_{ijk}^{n}}}, \upsilon_{ijk}^{o} = \frac{w_1+w_2}{\frac{w_1}{\upsilon_{ijk}^{m}}+\frac{w_2}{\upsilon_{ijk}^{n}}}, \varphi_{ijk}^{o} = \frac{w_1+w_2}{\frac{w_1}{\varphi_{ijk}^{m}}+\frac{w_2}{\varphi_{ijk}^{n}}}, \forall\, i,j,k$ and $w_1, w_2 > 0$.

Definition 2.10. [44] Let $B = \left[b_{ij}\right]_{\varsigma \times \nu}$ be a Neutrosophic hypersoft matrix where $b_{ij} = \left(\eta_{ijk}^{b}, \upsilon_{ijk}^{b}, \varphi_{ijk}^{b}\right)$. If an indeterminacy membership degree (υ)lies in favor of falsity-membership degree (φ)then the Value matrix of the matrix of the matrix B which is symbolized by $V(B)$ and is defined as $V(B) = \left[v_{ij}^{b}\right]_{m \times n}$, where $v_{ij}^{b} = \eta_{ijk}^{b} - \left(\upsilon_{ijk}^{b} + \varphi_{ijk}^{b}\right), \forall\, i,j,k$.

3. Neutrosophic Hypersoft Matrix

This section provides the concept of Complement NHSM, Universal NHSM, Null NHSM and Equal NHSM.

Example 3.1. Let Υ be the collection of Laptop's appeared within the Laptop showroom:
$\Upsilon = \{L_1 = HP\ Chromebook\ 11a - na0006MU, L_2 = Realme\ Book\ Slim,\ L_3 = Acer\ Aspire\ 5\ A515 - 54G, L_4 = HP\ 15s - gr0012AU\}$.

The decision-maker offers his conclusion around the choice procedure of the alternatives such as; $\daleth_1 = Inches, \daleth_2 = Processor, \daleth_3 = Memory, \daleth_4 = SSD$. Moreover, the above-mentioned attributes have advanced bifurcation and can be classified as follows: $\daleth_1^{a} = \{11.6, 14, 15.6\}, \daleth_2^{b} = \{MediaTek\ Octa - core, Intel\ Core\ i5, AMD\ Dual\ Core\}, \daleth_3^{c} = \{4GB, 8GB\}, \daleth_4^{d} = \{64GB, 512GB, 256GB\}$

Let the function be $Z : \daleth_1^{a} \times \daleth_2^{b} \times \daleth_3^{c} \times \daleth_4^{d} \rightarrow P(\Upsilon)$. Neutrosophic hypersoft set is defined as; $Z : (\daleth_1^{a} \times \daleth_2^{b} \times \daleth_3^{c} \times \daleth_4^{d}) \rightarrow P(\Upsilon)$.

Let us assume $Z\,(\daleth_1^{a} \times \daleth_2^{b} \times \daleth_3^{c} \times \daleth_4^{d}) = Z(14, Intel\ Core\ i5, 8GB, 512GB) = \{L_2, L_3\}$. Then, the neutrosophic hypersoft set of the above-expected connection is;

$$Z\,(\daleth_1^{a} \times \daleth_2^{b} \times \daleth_3^{c} \times \daleth_4^{d}) = Z(14, Intel\ Core\ i5, 8GB, 512GB)$$
$$= \begin{Bmatrix} (L_2, \{0.6, 0.3, 0.5\}, \{0.4, 0.5, 0.2\}, \{0.5, 0.7, 0.6\}, \{0.2, 0.3, 0.7\}) \\ (L_3, \{0.3, 0.6, 0.5\}, \{0.2, 0.1, 0.3\}, \{0.4, 0.7, 0.1\}, \{0.7, 0.3, 0.5\}) \end{Bmatrix}$$

Further, it can be represented in matrix form as:

$$[Z]_{2\times 4} = \begin{pmatrix} (0.6, 0.3, 0.5) & (0.4, 0.5, 0.2) & (0.5, 0.7, 0.6) & (0.2, 0.3, 0.7) \\ (0.3, 0.6, 0.5) & (0.2, 0.1, 0.3) & (0.4, 0.7, 0.1) & (0.7, 0.3, 0.5) \end{pmatrix}$$

Definition 3.2. Let $Z = \left[z_{ij}\right]$ and $Q = \left[q_{ij}\right]$ be two NHSMs with order $\varsigma \times \nu$, where $z_{ij} = \left(\eta_{ijk}^{z}, \upsilon_{ijk}^{z}, \varphi_{ijk}^{z}\right)$ and $q_{ij} = \left(\eta_{ijk}^{q}, \upsilon_{ijk}^{q}, \varphi_{ijk}^{q}\right)$. Z is said to be the Neutrosophic Hypersoft Submatrix of Q if;

$$\eta_{ijk}^{z} \leq \eta_{ijk}^{q}, \upsilon_{ijk}^{z} \leq \upsilon_{ijk}^{q}, \varphi_{ijk}^{z} \geq \varphi_{ijk}^{q}.$$

Example 3.3. Consider the NHSM $[Z]_{2\times 4}$ of example 3.1.

$$[Z]_{2\times 4} = \begin{pmatrix} (0.6, 0.3, 0.5) & (0.4, 0.5, 0.2) & (0.5, 0.7, 0.6) & (0.2, 0.3, 0.7) \\ (0.3, 0.6, 0.5) & (0.2, 0.1, 0.3) & (0.4, 0.7, 0.1) & (0.7, 0.3, 0.5) \end{pmatrix}$$

Presently consider another NHSM $[Q]$ related with the Neutrosophic hypersoft set $Q : (\daleth_1^{a} \times \daleth_2^{b} \times \daleth_3^{c} \times \daleth_4^{d}) \rightarrow P(\Upsilon)$ over the same universe and attributes as in Example 3.1.

$Q\,(\daleth_1^{a} \times \daleth_2^{b} \times \daleth_3^{c} \times \daleth_4^{d}) = Q(14, Intel\ Core\ i5, 8GB, 512GB)$
$$= \begin{Bmatrix} (L_2, \{0.8, 0.5, 0.5\}, \{0.7, 0.6, 0.1\}, \{0.6, 0.8, 0.5\}, \{0.6, 0.4, 0.5\}) \\ (L_3, \{0.5, 0.8, 0.1\}, \{0.8, 0.3, 0.2\}, \{0.5, 0.8, 0.1\}, \{0.8, 0.8, 0.4\}) \end{Bmatrix}$$

Hence the NHSM $[Q]$ is written as,

$$[Q]_{2\times 4} = \begin{pmatrix} (0.8,0.5,0.5) & (0.7,0.6,0.1) & (0.6,0.8,0.5) & (0.6,0.4,0.5) \\ (0.5,0.8,0.1) & (0.8,0.3,0.2) & (0.5,0.8,0.1) & (0.8,0.8,0.4) \end{pmatrix}$$

Therefore, we can observe that the membership value of L_2 for $8GB$ in both sets is $(0.5,0.7,0.6)$ and $(0.6,0.8,0.5)$ which satisfies the definition of Neutrosophic Hypersoft Submatrix as $0.5 < 0.6, 0.7 < 0.8, 0.6 > 0.5$. This shows that $(0.5,0.7,0.6) \subseteq (0.6,0.8,0.5)$ and the same was the case with the rest of the attributes of NHSM $[Z]$ and NHSM $[Q]$.

Definition 3.4. Let $Z = [z_{ij}]$ and $Q = [q_{ij}]$ be two NHSMs with order $\varsigma \times \nu$, where $z_{ij} = (\eta^z_{ijk}, \upsilon^z_{ijk}, \varphi^z_{ijk})$ and $q_{ij} = (\eta^q_{ijk}, \upsilon^q_{ijk}, \varphi^q_{ijk})$. Z is said to be the Equal NHSM of Q if;

$$\eta^z_{ijk} = \eta^q_{ijk}, \upsilon^z_{ijk} = \upsilon^q_{ijk}, \varphi^z_{ijk} = \varphi^q_{ijk}.$$

Example 3.5. Consider the NHSM $[Z]_{2\times 4}$ of Example 3.1.

$$[Z]_{2\times 4} = \begin{pmatrix} (0.6,0.3,0.5) & (0.4,0.5,0.2) & (0.5,0.7,0.6) & (0.2,0.3,0.7) \\ (0.3,0.6,0.5) & (0.2,0.1,0.3) & (0.4,0.7,0.1) & (0.7,0.3,0.5) \end{pmatrix}$$

Presently consider another NHSM $[Q]$ related with the Neutrosophic hypersoft set $Q : (\daleth^a_1 \times \daleth^b_2 \times \daleth^c_3 \times \daleth^d_4) \rightarrow P(\Upsilon)$ over the same universe and attributes as in Example 3.1.

$Q\ (\daleth^a_1 \times \daleth^b_2 \times \daleth^c_3 \times \daleth^d_4) = Q(14, Intel\ Core\ i5, 8GB, 512GB)$

$$= \begin{Bmatrix} (L_2, (0.6,0.3,0.5), (0.4,0.5,0.2), (0.5,0.7,0.6), (0.2,0.3,0.7)) \\ (L_3, (0.3,0.6,0.5), (0.2,0.1,0.3), (0.4,0.7,0.1), (0.7,0.3,0.5)) \end{Bmatrix}$$

Hence the NHSM $[Q]$ is written as,

$$[Q]_{2\times 4} = \begin{pmatrix} (0.6,0.3,0.5) & (0.4,0.5,0.2) & (0.5,0.7,0.6) & (0.2,0.3,0.7) \\ (0.3,0.6,0.5) & (0.2,0.1,0.3) & (0.4,0.7,0.1) & (0.7,0.3,0.5) \end{pmatrix}$$

Thus, we can observe that the membership value of L_2 for $8GB$ in both the matrices satisfy the definition of Equal NHSM as $0.5 = 0.5, 0.7 = 0.7, 0.6 = 0.6$. This shows that $(0.5,0.7,0.6) = (0.5,0.7,0.6)$ and the same was the case with the rest of the attributes of NHSM $[Z]$ and NHSM $[Q]$.

Definition 3.6. Let $Z = [z_{ij}]$ be an NHSM with order $\varsigma \times \nu$,where $z_{ij} = (\eta^z_{ijk}, \upsilon^z_{ijk}, \varphi^z_{ijk})$. Then the Neutrosophic hypersoft matrix Z is known as the Null-NHSM if;

$$\eta^z_{ijk} = 0, \upsilon^z_{ijk} = 1, \varphi^z_{ijk} = 1.$$

Example 3.7. Consider the same universe and attributes as in Example 3.1. Then a Null-NHSM Z is given by;

$$[Z]_{2\times 4} = \begin{pmatrix} (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) \\ (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) \\ (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) \\ (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) \end{pmatrix}$$

Definition 3.8. Let $Z = [z_{ij}] \in$ NHSM with order $\varsigma \times \nu$,where $z_{ij} = (\eta^z_{ijk}, \upsilon^z_{ijk}, \varphi^z_{ijk})$. The matrix Z is known as the Universal-NHSM in the case;

$$\eta^z_{ijk} = 1, \upsilon^z_{ijk} = 0, \varphi^z_{ijk} = 0.$$

Example 3.9. Consider the same universe and attributes as in Example 3.1. Then a Universal-NHSM Z is given by;

$$[Z]_{2\times 4} = \begin{pmatrix} (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) \\ (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) \\ (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) \\ (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) \end{pmatrix}$$

Definition 3.10. Let $Z = [z_{ij}] \in$ NHSM with order $\varsigma \times \nu$, where $z_{ij} = (\eta^z_{ijk}, \upsilon^z_{ijk}, \varphi^z_{ijk})$. Then $Z = [z_{ij}]^c$ is said to Complement NHSM of $Z = [z_{ij}]$ if;

$$(\eta^z_{ijk}, \upsilon^z_{ijk}, \varphi^z_{ijk})^c = (\varphi^z_{ijk}, \upsilon^z_{ijk}, \eta^z_{ijk})$$

Example 3.11. Consider the NHSM $[Z]_{2\times 4}$ of example 3.1.

$$[Z]_{2\times 4} = \begin{pmatrix} (0.6, 0.3, 0.5) & (0.4, 0.5, 0.2) & (0.5, 0.7, 0.6) & (0.2, 0.3, 0.7) \\ (0.3, 0.6, 0.5) & (0.2, 0.1, 0.3) & (0.4, 0.7, 0.1) & (0.7, 0.3, 0.5) \end{pmatrix}$$

Then,

$$[Z]^c = \begin{pmatrix} (0.5, 0.3, 0.6) & (0.2, 0.5, 0.4) & (0.6, 0.7, 0.5) & (0.7, 0.3, 0.2) \\ (0.5, 0.6, 0.3) & (0.3, 0.1, 0.2) & (0.1, 0.7, 0.4) & (0.5, 0.3, 0.7) \end{pmatrix}$$

4. NHCM and Some Properties of NHSM

This section provides the concept of NHCM, combined NHCM and product of NHCM, and some properties of NHSM.

4.1 NHCM and Combined NHCM

Definition 4.1. NHCM is a square matrix whose rows and columns both indicate attributes. If ξ is a NHCM, where $\xi = [\xi_{ij}]$ such that $\xi_{ij} = (\eta^\xi_{ijk}, \upsilon^\xi_{ijk}, \varphi^\xi_{ijk})$ is defined as,

$(\eta^\xi_{ijk}, \upsilon^\xi_{ijk}, \varphi^\xi_{ijk}) = (1, 0.5, 0)$, when both i^{th} and j^{th} attributes are under the choice of the decision maker.

$(\eta^\xi_{ijk}, \upsilon^\xi_{ijk}, \varphi^\xi_{ijk}) = (0, 0.5, 1)$, when atleast one of the attributes i or j is not under the choice of the decision maker.

Example 4.2. Consider the same universe and attributes as in Example 3.1. If Mr. X is interested in buying a laptop based on the attributes $(⅃^a_1 \times ⅃^b_2 \times ⅃^d_4) = (14, Intel\ Core\ i5, 512GB)$. Then the NHCM of Mr. X can be represented as,

$$[\xi_{ij}]_X = ⅃_X \overset{⅃_X}{\begin{pmatrix} (1.0, 0.5, 0.0) & (1.0, 0.5, 0.0) & (0.0, 0.5, 1.0) & (1.0, 0.5, 0.0) \\ (1.0, 0.5, 0.0) & (1.0, 0.5, 0.0) & (0.0, 0.5, 1.0) & (1.0, 0.5, 0.0) \\ (0.0, 0.5, 1.0) & (0.0, 0.5, 1.0) & (0.0, 0.5, 1.0) & (0.0, 0.5, 1.0) \\ (1.0, 0.5, 0.0) & (1.0, 0.5, 0.0) & (0.0, 0.5, 1.0) & (1.0, 0.5, 0.0) \end{pmatrix}}$$

Definition 4.3. Combined NHCM is a square matrix whose rows indicate the choice attributes of the single decision maker and columns indicate the combined choice attributes of remaining decision-makers, i.e., acquired from the intersection of attributes sets, which is denoted by ξ^C.

Note 4.1. Combined NHCM is applicable only for combined opinion because the attribute values of each decision maker occur from the same relation of attribute values.

Example 4.4. Consider the same universe and attributes as in Example 3.1. If Mr. X, Mr. Y, and Mr. Z wants to buy a Laptop based on their combined opinion of attribute relation $(⅃^a_1 \times ⅃^b_2 \times ⅃^c_3 \times ⅃^d_4) = (14, Intel\ Core\ i5, 8GB, 512GB)$, then attributes chosen by Mr. X is $(⅃^a_1 \times ⅃^b_2 \times ⅃^d_4) = (14, Intel\ Core\ i5, 512GB) \subset (14, Intel\ Core\ i5, 8GB, 512GB)$, the attributes chosen by Mr. Y is $(⅃^a_1 \times ⅃^d_4) = (14, 512GB) \subset (14, Intel\ Core\ i5, 8GB, 512GB)$, and the attributes chosen by Mr. Z is $(⅃^a_1 \times ⅃^c_3 \times ⅃^d_4) = (14, 8GB, 512GB) \subset (14, Intel\ Core\ i5, 8GB, 512GB)$. Then combined NHCM of Mr. X, Mr. Y, and Mr. Z is,

$$⅃_{(Y\cap Z)}$$

$$[\xi_{ij}^{C}]_{(X,Y\cap Z)} = \rfloor_X \begin{pmatrix} (1.0, 0.5, 0.0) & (0.0, 0.5, 1.0) & (0.0, 0.5, 1.0) & (1.0, 0.5, 0.0) \\ (1.0, 0.5, 0.0) & (0.0, 0.5, 1.0) & (0.0, 0.5, 1.0) & (1.0, 0.5, 0.0) \\ (0.0, 0.5, 1.0) & (0.0, 0.5, 1.0) & (0.0, 0.5, 1.0) & (0.0, 0.5, 1.0) \\ (1.0, 0.5, 0.0) & (0.0, 0.5, 1.0) & (0.0, 0.5, 1.0) & (1.0, 0.5, 0.0) \end{pmatrix}$$

Similarly, we can give the combined NHCM for $[\xi_{ij}^{C}]_{(Y,X\cap Z),}[\xi_{ij}^{C}]_{(Z,Y\cap X)}$ also.

Definition 4.5. Let $Z = [z_{ij}]$ be NHCM and $\xi^C = [\xi_{ij}^{C}]$ be combined with NHCM, where $z_{ij} = (\eta_{ijk}^{z}, \upsilon_{ijk}^{z}, \varphi_{ijk}^{z})$ and $[\xi_{ij}^{C} = (\eta_{ijk}^{\xi^C}, \upsilon_{ijk}^{\xi^C}, \varphi_{ijk}^{\xi^C})$. Then Z and ξ^C are said to be conformable for the product if number of columns of NHCM is equal to the number of rows of combined NHCM. If $Z = [z_{ij}]_{\varsigma\times\nu}$ and $\xi^C = [\xi_{ij}^{C}]_{\varsigma\times\nu}$, then $Z \otimes \xi^C = [P_{im}]_{\varsigma\times\nu}$, were

$$P_{im} = \left(max_{jk} \min\left(\eta_{ijk}^{z}, \eta_{jkm}^{\xi^C}\right), min_{jk} \max\left(\upsilon_{ijk}^{z}, \upsilon_{jkm}^{\xi^C}\right), min_{jk} \max\left(\varphi_{ijk}^{z}, \varphi_{jkm}^{\xi^C}\right)\right).$$

4.2 Properties of NHSM

Proposition 4.6. Let $Z = [z_{ij}], Q = [q_{ij}]$ and $W = [w_{ij}]$, where $z_{ij} = (\eta_{ijk}^{z}, \upsilon_{ijk}^{z}, \varphi_{ijk}^{z}), q_{ij} = (\eta_{ijk}^{q}, \upsilon_{ijk}^{q}, \varphi_{ijk}^{q}), w_{ij} = (\eta_{ijk}^{w}, \upsilon_{ijk}^{w}, \varphi_{ijk}^{w}) \in NHSMs$ of order $\varsigma \times \nu$. Then,

i). $Z \circledast (Q \cup W) = (Z \circledast Q) \cup (Z \circledast W)$, $(Z \cup Q) \circledast W = (Z \circledast W) \cup (Q \circledast W)$
ii). $Z \circledast (Q \cap W) = (Z \circledast Q) \cap (Z \circledast W)$, $(Z \cap Q) \circledast W = (Z \circledast W) \cap (Q \circledast W)$
iii). $Z \circledcirc (Q \cup W) \neq (Z \circledcirc Q) \cup (Z \circledcirc W)$, $(Z \cup Q) \circledcirc W \neq (Z \circledcirc W) \cup (Q \circledcirc W)$
iv). $Z \circledcirc (Q \cap W) \neq (Z \circledcirc Q) \cap (Z \circledcirc W)$, $(Z \cap Q) \circledcirc W \neq (Z \circledcirc W) \cap (Q \circledcirc W)$
v). $Z \boxdot (Q \cup W) \neq (Z \boxdot Q) \cup (Z \boxdot W)$, $(Z \cup Q) \boxdot W \neq (Z \boxdot W) \cup (Q \boxdot W)$
vi). $Z \boxdot (Q \cap W) \neq (Z \boxdot Q) \cap (Z \boxdot W)$, $(Z \cap Q) \boxdot W \neq (Z \boxdot W) \cap (Q \boxdot W)$

Proof:(i) $Here\ Z \circledast (Q \cup W),\ (Z \circledast Q) \cup (Z \circledast W) \in NHSM_{\varsigma\times\nu}$. Then,

$$Z \circledast (Q \cup W) = [(\eta_{ijk}^{z}, \upsilon_{ijk}^{z}, \varphi_{ijk}^{z})] \circledast \left[\left(\max(\eta_{ijk}^{q}, \eta_{ijk}^{w}), \frac{(\upsilon_{ijk}^{q} + \upsilon_{ijk}^{w})}{2}, \min(\varphi_{ijk}^{q}, \varphi_{ijk}^{w})\right)\right]$$

$$=[(\frac{\eta_{ijk}^{z}+\max(\eta_{ijk}^{q},\eta_{ijk}^{w})}{2}, \frac{\upsilon_{ijk}^{z}+\frac{(\upsilon_{ijk}^{q}+\upsilon_{ijk}^{w})}{2}}{2}, \frac{\varphi_{ijk}^{z}+\min(\varphi_{ijk}^{q},\varphi_{ijk}^{w})}{2})]$$

$$=[(\max[(\frac{\eta_{ijk}^{z}+\eta_{ijk}^{q}}{2}), (\frac{\eta_{ijk}^{z}+\eta_{ijk}^{w}}{2})], \frac{\frac{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{q}}{2}+\frac{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{w}}{2}}{2}, \min[(\frac{\varphi_{ijk}^{z}+\varphi_{ijk}^{q}}{2}), (\frac{\varphi_{ijk}^{z}+\varphi_{ijk}^{w}}{2})])]$$

$$=[(\frac{\eta_{ijk}^{z}+\eta_{ijk}^{q}}{2}, \frac{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{q}}{2}, \frac{\varphi_{ijk}^{z}+\varphi_{ijk}^{q}}{2})] \cup [(\frac{\eta_{ijk}^{z}+\eta_{ijk}^{w}}{2}, \frac{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{w}}{2}, \frac{\varphi_{ijk}^{z}+\varphi_{ijk}^{w}}{2})]$$

$$Z \circledast (Q \cup W) = (Z \circledast Q) \cup (Z \circledast W)$$

$(ii) Here\ Z \circledast (Q \cap W),\ (Z \circledast Q) \cap (Z \circledast W) \in NHSM_{\varsigma\times\nu}$. Then,

$$Z \circledast (Q \cap W) = [(\eta_{ijk}^{z}, \upsilon_{ijk}^{z}, \varphi_{ijk}^{z})] \circledast \left[\left(\min(\eta_{ijk}^{q}, \eta_{ijk}^{w}), \frac{(\upsilon_{ijk}^{q} + \upsilon_{ijk}^{w})}{2}, \max(\varphi_{ijk}^{q}, \varphi_{ijk}^{w})\right)\right]$$

$$=[(\frac{\eta_{ijk}^{z}+\min(\eta_{ijk}^{q},\eta_{ijk}^{w})}{2}, \frac{\upsilon_{ijk}^{z}+\frac{(\upsilon_{ijk}^{q}+\upsilon_{ijk}^{w})}{2}}{2}, \frac{\varphi_{ijk}^{z}+\max(\varphi_{ijk}^{q},\varphi_{ijk}^{w})}{2})]$$

$=[(\min[(\frac{\eta_{ijk}^{z}+\eta_{ijk}^{q}}{2}),(\frac{\eta_{ijk}^{z}+\eta_{ijk}^{w}}{2})],\frac{\frac{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{q}}{2}+\frac{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{w}}{2}}{2},\max[(\frac{\varphi_{ijk}^{z}+\varphi_{ijk}^{q}}{2}),(\frac{\varphi_{ijk}^{z}+\varphi_{ijk}^{w}}{2})])]$

$=[(\frac{\eta_{ijk}^{z}+\eta_{ijk}^{q}}{2},\frac{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{q}}{2},\frac{\varphi_{ijk}^{z}+\varphi_{ijk}^{q}}{2})]\cap[(\frac{\eta_{ijk}^{z}+\eta_{ijk}^{w}}{2},\frac{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{w}}{2},\frac{\varphi_{ijk}^{z}+\varphi_{ijk}^{w}}{2})]$

$Z \circledast (Q \cap W) = (Z \circledast Q) \cap (Z \circledast W)$

$(iii) Here\ (Z \cup Q) \circledcirc W, (Z \circledcirc W) \cup (Q \circledcirc W) \in NHSM_{\varsigma\times\nu}$. Then,

$$(Z \cup Q) \circledcirc W = \left[\left(\max(\eta_{ijk}^{z},\eta_{ijk}^{q}), \frac{(\upsilon_{ijk}^{z}+\upsilon_{ijk}^{q})}{2}, \min(\varphi_{ijk}^{z},\varphi_{ijk}^{q})\right)\right] \circledcirc [(\eta_{ijk}^{w},\upsilon_{ijk}^{w},\varphi_{ijk}^{w})]$$

$$= \left[\left(\sqrt{\max\left(\eta_{ijk}^{z},\eta_{ijk}^{q}\right).\eta_{ijk}^{w}}, \sqrt{\frac{(\upsilon_{ijk}^{z}+\upsilon_{ijk}^{q})}{2}.\upsilon_{ijk}^{w}}, \sqrt{\min\left(\varphi_{ijk}^{z},\varphi_{ijk}^{q}\right).\varphi_{ijk}^{w}}\right)\right]$$

$$(Z \cup Q) \circledcirc W = \left[\left(\max\left(\sqrt{\eta_{ijk}^{z}.\eta_{ijk}^{w}}, \sqrt{\eta_{ijk}^{q}.\eta_{ijk}^{w}}\right), \left(\sqrt{\frac{(\upsilon_{ijk}^{z}+\upsilon_{ijk}^{w})}{2}} + \sqrt{\frac{(\upsilon_{ijk}^{q}+\upsilon_{ijk}^{w})}{2}}\right), \min\left(\sqrt{(\varphi_{ijk}^{z},\varphi_{ijk}^{w})}, \sqrt{(\varphi_{ijk}^{q},\varphi_{ijk}^{w})}\right)\right)\right]$$

$(Z \circledcirc W) \cup (Q \circledcirc W)$

$$= \left[\left(\max\left(\sqrt{\eta_{ijk}^{z}.\eta_{ijk}^{w}}, \sqrt{\eta_{ijk}^{q}.\eta_{ijk}^{w}}\right), \left(\frac{\sqrt{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{w}} + \sqrt{\upsilon_{ijk}^{q}+\upsilon_{ijk}^{w}}}{2}\right), \min\left(\sqrt{(\varphi_{ijk}^{z},\varphi_{ijk}^{w})}, \sqrt{(\varphi_{ijk}^{q},\varphi_{ijk}^{w})}\right)\right)\right]$$

We have, $(Z \cup Q) \circledcirc W \neq (Z \circledcirc W) \cup (Q \circledcirc W)$.

$(iv) Here\ (Z \cap Q) \circledcirc W, (Z \circledcirc W) \cap (Q \circledcirc W) \in NHSM_{\varsigma\times\nu}$. Then,

$$(Z \cap Q) \circledcirc W = \left[\left(\min(\eta_{ijk}^{z},\eta_{ijk}^{q}), \frac{(\upsilon_{ijk}^{z}+\upsilon_{ijk}^{q})}{2}, \max(\varphi_{ijk}^{z},\varphi_{ijk}^{q})\right)\right] \circledcirc [(\eta_{ijk}^{w},\upsilon_{ijk}^{w},\varphi_{ijk}^{w})]$$

$$= \left[\left(\sqrt{\min\left(\eta_{ijk}^{z},\eta_{ijk}^{q}\right).\eta_{ijk}^{w}}, \sqrt{\frac{(\upsilon_{ijk}^{z}+\upsilon_{ijk}^{q})}{2}.\upsilon_{ijk}^{w}}, \sqrt{\max\left(\varphi_{ijk}^{z},\varphi_{ijk}^{q}\right).\varphi_{ijk}^{w}}\right)\right]$$

$$(Z\cap Q)\circledcirc W=\left[\left(\min\left(\sqrt{\eta^{z}_{ijk}.\eta^{w}_{ijk}},\sqrt{\eta^{q}_{ijk}.\eta^{w}_{ijk}}\right),\left(\sqrt{\frac{\left(\upsilon^{z}_{ijk}+\upsilon^{w}_{ijk}\right)}{2}}+\sqrt{\frac{\left(\upsilon^{q}_{ijk}+\upsilon^{w}_{ijk}\right)}{2}}\right),\max\left(\sqrt{\left(\varphi^{z}_{ijk},\varphi^{w}_{ijk}\right)},\sqrt{\left(\varphi^{q}_{ijk},\varphi^{w}_{ijk}\right)}\right)\right)\right]$$

$(Z\circledcirc W)\cap(Q\circledcirc W)$

$$=\left[\left(\min\left(\sqrt{\eta^{z}_{ijk}.\eta^{w}_{ijk}},\sqrt{\eta^{q}_{ijk}.\eta^{w}_{ijk}}\right),\left(\frac{\sqrt{\upsilon^{z}_{ijk}+\upsilon^{w}_{ijk}}+\sqrt{\upsilon^{q}_{ijk}+\upsilon^{w}_{ijk}}}{2}\right),\max\left(\sqrt{\left(\varphi^{z}_{ijk},\varphi^{w}_{ijk}\right)},\sqrt{\left(\varphi^{q}_{ijk},\varphi^{w}_{ijk}\right)}\right)\right)\right]$$

We have, $(Z\cap Q)\circledcirc W\neq(Z\circledcirc W)\cap(Q\circledcirc W)$.

$(v)Here\ (Z\cup Q)\boxdot W,(Z\boxdot W)\cup(Q\boxdot W)\ \in NHSM_{\varsigma\times\nu}$. Then,

$$(Z\cup Q)\boxdot W=\left[\left(\max\left(\eta^{z}_{ijk},\eta^{q}_{ijk}\right),\frac{\left(\upsilon^{z}_{ijk}+\upsilon^{q}_{ijk}\right)}{2},\min\left(\varphi^{z}_{ijk},\varphi^{q}_{ijk}\right)\right)\right]\boxdot\left[\left(\eta^{w}_{ijk},\upsilon^{w}_{ijk},\varphi^{w}_{ijk}\right)\right]$$

$$=[(\frac{\max\left(\eta^{z}_{ijk},\eta^{q}_{ijk}\right).2\eta^{w}_{ijk}}{\max\left(\eta^{z}_{ijk},\eta^{q}_{ijk}\right)+\eta^{w}_{ijk}},\frac{\frac{\left(\upsilon^{z}_{ijk}+\upsilon^{q}_{ijk}\right)}{2}.2\upsilon^{w}_{ijk}}{\frac{\left(\upsilon^{z}_{ijk}+\upsilon^{q}_{ijk}\right)}{2}+\upsilon^{w}_{ijk}},\frac{\min\left(\varphi^{z}_{ijk},\varphi^{q}_{ijk}\right).2\varphi^{w}_{ijk}}{\min\left(\varphi^{z}_{ijk},\varphi^{q}_{ijk}\right)+\varphi^{w}_{ijk}})]$$

$$(Z\cup Q)\boxdot W=[(\max\left(\frac{2\eta^{z}_{ijk}.\eta^{w}_{ijk}}{\eta^{z}_{ijk}+\eta^{w}_{ijk}},\frac{2\eta^{q}_{ijk}.\eta^{w}_{ijk}}{\eta^{q}_{ijk}+\eta^{w}_{ijk}}\right),\left(\frac{2\upsilon^{z}_{ijk}.\upsilon^{w}_{ijk}}{\upsilon^{z}_{ijk}+\upsilon^{w}_{ijk}}+\frac{2\upsilon^{q}_{ijk}.\upsilon^{w}_{ijk}}{\upsilon^{q}_{ijk}+\upsilon^{w}_{ijk}}\right),\min\left(\frac{2\varphi^{z}_{ijk}.\varphi^{w}_{ijk}}{\varphi^{z}_{ijk}+\varphi^{w}_{ijk}},\frac{2\varphi^{q}_{ijk}.\varphi^{w}_{ijk}}{\varphi^{q}_{ijk}+\varphi^{w}_{ijk}}\right))]$$

$(Z\boxdot W)\cup(Q\boxdot W)$

$$=\left[\left(\max\left(\frac{2\eta^{z}_{ijk}.\eta^{w}_{ijk}}{\eta^{z}_{ijk}+\eta^{w}_{ijk}},\frac{2\eta^{q}_{ijk}.\eta^{w}_{ijk}}{\eta^{q}_{ijk}+\eta^{w}_{ijk}}\right),\left(\frac{\frac{2\upsilon^{z}_{ijk}.\upsilon^{w}_{ijk}}{\upsilon^{z}_{ijk}+\upsilon^{w}_{ijk}}+\frac{2\upsilon^{q}_{ijk}.\upsilon^{w}_{ijk}}{\upsilon^{q}_{ijk}+\upsilon^{w}_{ijk}}}{2}\right),\min\left(\frac{2\varphi^{z}_{ijk}.\varphi^{w}_{ijk}}{\varphi^{z}_{ijk}+\varphi^{w}_{ijk}},\frac{2\varphi^{q}_{ijk}.\varphi^{w}_{ijk}}{\varphi^{q}_{ijk}+\varphi^{w}_{ijk}}\right)\right)\right]$$

We have, $(Z\cup Q)\boxdot W\neq(Z\boxdot W)\cup(Q\boxdot W)$.

$(v)Here\ (Z\cap Q)\boxdot W,(Z\boxdot W)\cap(Q\boxdot W)\ \in NHSM_{\varsigma\times\nu}$. Then,

$$(Z\cap Q)\boxdot W=\left[\left(\min\left(\eta^{z}_{ijk},\eta^{q}_{ijk}\right),\frac{\left(\upsilon^{z}_{ijk}+\upsilon^{q}_{ijk}\right)}{2},\max\left(\varphi^{z}_{ijk},\varphi^{q}_{ijk}\right)\right)\right]\boxdot\left[\left(\eta^{w}_{ijk},\upsilon^{w}_{ijk},\varphi^{w}_{ijk}\right)\right]$$

$$=[(\frac{\min\left(\eta_{ijk}^{z},\eta_{ijk}^{q}\right).2\eta_{ijk}^{w}}{\min\left(\eta_{ijk}^{z},\eta_{ijk}^{q}\right)+\eta_{ijk}^{w}},\frac{\frac{\left(\upsilon_{ijk}^{z}+\upsilon_{ijk}^{q}\right)}{2}.2\upsilon_{ijk}^{w}}{\frac{\left(\upsilon_{ijk}^{z}+\upsilon_{ijk}^{q}\right)}{2}+\upsilon_{ijk}^{w}},\frac{\max\left(\varphi_{ijk}^{z},\varphi_{ijk}^{q}\right).2\varphi_{ijk}^{w}}{\max\left(\varphi_{ijk}^{z},\varphi_{ijk}^{q}\right)+\varphi_{ijk}^{w}})]$$

$$(Z\cap Q)\boxdot W=[(\min\left(\frac{2\eta_{ijk}^{z}.\eta_{ijk}^{w}}{\eta_{ijk}^{z}+\eta_{ijk}^{w}},\frac{2\eta_{ijk}^{q}.\eta_{ijk}^{w}}{\eta_{ijk}^{q}+\eta_{ijk}^{w}}\right),\left(\frac{2\upsilon_{ijk}^{z}.\upsilon_{ijk}^{w}}{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{w}}+\frac{2\upsilon_{ijk}^{q}.\upsilon_{ijk}^{w}}{\upsilon_{ijk}^{q}+\upsilon_{ijk}^{w}}\right),\max\left(\frac{2\varphi_{ijk}^{z}.\varphi_{ijk}^{w}}{\varphi_{ijk}^{z}+\varphi_{ijk}^{w}},\frac{2\varphi_{ijk}^{q}.\varphi_{ijk}^{w}}{\varphi_{ijk}^{q}+\varphi_{ijk}^{w}}\right))]$$

$$(Z\boxdot W)\cap(Q\boxdot W)$$

$$=\left[\left(\min\left(\frac{2\eta_{ijk}^{z}.\eta_{ijk}^{w}}{\eta_{ijk}^{z}+\eta_{ijk}^{w}},\frac{2\eta_{ijk}^{q}.\eta_{ijk}^{w}}{\eta_{ijk}^{q}+\eta_{ijk}^{w}}\right),\left(\frac{\frac{2\upsilon_{ijk}^{z}.\upsilon_{ijk}^{w}}{\upsilon_{ijk}^{z}+\upsilon_{ijk}^{w}}+\frac{2\upsilon_{ijk}^{q}.\upsilon_{ijk}^{w}}{\upsilon_{ijk}^{q}+\upsilon_{ijk}^{w}}}{2}\right),\max\left(\frac{2\varphi_{ijk}^{z}.\varphi_{ijk}^{w}}{\varphi_{ijk}^{z}+\varphi_{ijk}^{w}},\frac{2\varphi_{ijk}^{q}.\varphi_{ijk}^{w}}{\varphi_{ijk}^{q}+\varphi_{ijk}^{w}}\right)\right)\right]$$

We have, $(Z\cap Q)\boxdot W\neq(Z\boxdot W)\cap(Q\boxdot W)$.

Proposition 4.7. Let $Z=[z_{ij}]$, where $z_{ij}=\left(\eta_{ijk}^{z},\upsilon_{ijk}^{z},\varphi_{ijk}^{z}\right)\in NHSMs$ of order $\varsigma\times\nu$.Then,

i). $Z\circledast^{w}Z=Z$
ii). $Z\circledcirc^{w}Z=Z$
iii). $Z\boxdot^{w}Z=Z$

Proof: *For all* i,j,k *and* $w_1,w_2>0$ *we have,*

i). $Z\circledast^{w}Z=[(\frac{w_1\eta_{ijk}^{z}+w_2\eta_{ijk}^{z}}{w_1+w_2},\frac{w_1\upsilon_{ijk}^{z}+w_2\upsilon_{ijk}^{z}}{w_1+w_2},\frac{w_1\varphi_{ijk}^{z}+w_2\varphi_{ijk}^{z}}{w_1+w_2})]=\left[\left(\eta_{ijk}^{z},\upsilon_{ijk}^{z},\varphi_{ijk}^{z}\right)\right]=Z.$

ii). $Z\circledcirc^{w}Z=\left[\left((w_1+w_2)\sqrt{(\eta_{ijk}^{z})^{w_1}.(\eta_{ijk}^{z})^{w_2}},(w_1+w_2)\sqrt{(\upsilon_{ijk}^{z})^{w_1}.(\upsilon_{ijk}^{z})^{w_2}},\ (w_1+w_2)\sqrt{(\varphi_{ijk}^{z})^{w_1}.(\varphi_{ijk}^{z})^{w_2}}\right)\right]=\left[\left(\eta_{ijk}^{z},\upsilon_{ijk}^{z},\varphi_{ijk}^{z}\right)\right]=Z.$

iii). $Z\boxdot^{w}Z=\left[\left(\frac{w_1+w_2}{\frac{w_1}{\eta_{ijk}^{z}}+\frac{w_2}{\eta_{ijk}^{z}}},\frac{w_1+w_2}{\frac{w_1}{\upsilon_{ijk}^{z}}+\frac{w_2}{\upsilon_{ijk}^{z}}},\frac{w_1+w_2}{\frac{w_1}{\varphi_{ijk}^{z}}+\frac{w_2}{\varphi_{ijk}^{z}}}\right)\right]=\left[\left(\eta_{ijk}^{z},\upsilon_{ijk}^{z},\varphi_{ijk}^{z}\right)\right]=Z.$

5. An Application of NHSM in a Decision-making Problem

Based on some of these matrix operations an efficient methodology named NHSM-algorithm can be developed to solve Neutrosophic hypersoft set-based decision-making problems.

Definition 5.1. Let $Z=[z_{ij}]_{\varsigma\times\nu}$ be a Neutrosophic hypersoft matrix where $z_{ij}=\left(\eta_{ijk}^{z},\upsilon_{ijk}^{z},\varphi_{ijk}^{z}\right)$.

i). If an indeterminacy membership degree (υ)lies in favor of truth-membership degree (η) then the Grace matrix of the matrix Zwhich is symbolized by $G(Z)$ and is defined as $G(Z)=[g_{ij}^{z}]_{\varsigma\times\nu}$, where $g_{ij}^{z}=(\eta_{ijk}^{z}+\upsilon_{ijk}^{z})-\varphi_{ijk}^{z}$, $\forall i,j,k$.
ii). From the Grace matrix $G(Z)$ and Value matrix $V(Z)$, the Mean matrix $M(Z)$ is defined as; $M(Z)=[m_{ij}^{z}]_{\varsigma\times\nu}=\frac{V(Z)+G(Z)}{2}$.
iii). The Total mean of an object (for single observer) is given by; $\sum_{j=1}^{n}m_{ij}^{z},\forall i$ where m_{ij}^{z} are entries in the mean matrix.

5.1 *Properties of Mean Function*

All properties of the real matrix are satisfied by the Value matrix and Grace matrix. Hence, the Mean matrix which is obtained from the value matrix and grace matrix is also a real matrix. So, all properties of the real matrix are obeyed by the mean function.

5.2 *Methodology*

From the given ς number of alternatives, the decision-maker needs to select the most appropriate alternative based on the attributes (ϑ) chosen. If anyone of the attributes has encouraged sub-attributes that outline an NHSM, then the decision-maker gives his inclination for each alternative conjuring to the sub-attributes of the chosen attributes in the form of NHSM. Hence, an NHSM of order $\varsigma \times \nu$ is obtained, from which the value matrix and grace matrix are computed. Then, at that point, the mean matrix and lastly the total mean of each alternative is determined.
The algorithm for the above approach is;

5.3 *NHSM-algorithm*

1) Input the Neutrosophic hypersoft set from the given situation based on the attributes chosen.
2) Using 1 construct the NHSM Z.
3) Calculate $V(Z)$ and $G(Z)$ from the result of 2.
4) Compute the Mean matrix $M(Z)$.
5) From the Mean matrix, derive the Total Mean matrix.
6) The maximum of $\sum_{j=1}^{n} m_{ij}^{z}$ will be the optimal solution.
7) Suppose if Total mean matrix has a maximum value for more than one alternative, then any one alternative can be chosen according to the decision maker.

5.4 *Application in the Medical Field*

Example 5.2. Let Υ be the set of patients suffering from chronic kidney disease (CKD):

$$\Upsilon = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}\}$$

Now, the problem of the Doctor is to identify the patient who is critically moving towards stage: 5 of CKD, so he offers his conclusion around the choice procedure of the alternatives such as;

$$\daleth = \{\daleth_1 = Creatinine\ level, \daleth_2 = Kidney\ function, \daleth_3 = GFR\ rate\ per\ 1.73m^2, \daleth_4 = Dialysis\ or\ kidney\ transplant\}.$$

Moreover, the above-mentioned attributes have advanced bifurcation and can be classified as follows:

$$\daleth_1^a = \{Low, Normal, High\}, \daleth_2^b = \{Normal, Mildloss, Mildtosevereloss, Severeloss, Failure\},$$
$$\daleth_3^c = \{> 90\, mL/min, 60 - 89\, mL/min, 30 - 59\, mL/min, 15 - 29\, mL/min, < 15mL / min\}$$
$$\daleth_4^d = \{N/A, Possiblyrequired, Required\}$$

Stage 1	Stage 2	Stage 3A	Stage 3B	Stage 4	Stage 5
$\boldsymbol{GFR \geq 90}$	$89 \geq GFR \geq 60$	$59 \geq GFR \geq 40$	$39 \geq GFR \geq 30$	$29 \geq GFR \geq 15$	$GFR \leq 15$
Normal	Mild loss	Mild	Moderately	Severe loss	Failure

Let the function be $\hat{Z} : \daleth_1^a \times \daleth_2^b \times \daleth_3^c \times \daleth_4^d \rightarrow P(\Upsilon)$. Neutrosophic hypersoft set is defined as; $Z : (\daleth_1^a \times \daleth_2^b \times \daleth_3^c \times \daleth_4^d) \rightarrow P(\Upsilon)$. Let us assume $Z\,(\daleth_1^a \times \daleth_2^b \times \daleth_3^c \times \daleth_4^d) = Z(High, Severe\ loss, 15 - 29\, mL/min, Required)$ is the actual requirement of the Doctor. On this basis, four patients are shortlisted P_1, P_2, P_5, P_6 according to the above-defined relation $(High, Severe\ loss, 15-29\ mL/min\ , Required)$.

The doctor gives his opinion for each alternative in the form of a Neutrosophic hypersoft set as follows;

$Z = Z(High, Severe\ loss, 15-29\ mL/min\ , Required)$.

$$= \begin{Bmatrix} (P_1, & \{0.6, 0.4, 0.2\}, \{0.1, 0.8, 0.2\}, \{0.2, 0.1, 0.8\}, \{0.3, 0.8, 0.2\}) \\ (P_2, & \{0.4, 0.8, 0.1\}, \{0.5, 0.3, 0.8\}, \{0.5, 0.1, 0.4\}, \{0.7, 0.5, 0.3\}) \\ (P_5, & \{0.3, 0.5, 0.7\}, \{0.7, 0.2, 0.5\}, \{0.5, 0.6, 0.7\}, \{0.8, 0.1, 0.2\}) \\ (P_6, & \{0.8, 0.1, 0.3\}, \{0.8, 0.1, 0.1\}, \{0.2, 0.5, 0.8\}, \{0.6, 0.5, 0.4\}) \end{Bmatrix}$$

The Neutrosophic hypersoft matrix derived from the above Neutrosophic hypersoft set is,

$$Z = \begin{pmatrix} (0.6, 0.4, 0.2) & (0.1, 0.8, 0.2) & (0.2, 0.1, 0.8) & (0.3, 0.8, 0.2) \\ (0.4, 0.8, 0.1) & (0.5, 0.3, 0.8) & (0.5, 0.1, 0.4) & (0.7, 0.5, 0.3) \\ (0.3, 0.5, 0.7) & (0.7, 0.2, 0.5) & (0.5, 0.6, 0.7) & (0.8, 0.1, 0.2) \\ (0.8, 0.1, 0.3) & (0.8, 0.1, 0.1) & (0.2, 0.5, 0.8) & (0.6, 0.5, 0.4) \end{pmatrix}$$

The Value matrix $V(Z) = [v_{ij}^z] = [\eta_{ijk}^z - (v_{ijk}^z + \varphi_{ijk}^z)]$ is,

$$V(Z) = \begin{pmatrix} 00.0 & -0.9 & -0.7 & -0.7 \\ -0.5 & -0.6 & 00.0 & -0.1 \\ -0.9 & 00.0 & -0.8 & 00.5 \\ 00.4 & 00.6 & -1.1 & -0.3 \end{pmatrix}$$

The Grace matrix $G(Z) = [g_{ij}^z] = [(\eta_{ijk}^z + v_{ijk}^z) - \varphi_{ijk}^z]$ is,

$$G(Z) = \begin{pmatrix} 0.8 & 0.7 & -0.5 & 0.9 \\ 1.1 & 0.0 & 00.2 & 0.9 \\ 0.1 & 0.4 & 00.4 & 0.7 \\ 0.6 & 0.8 & -0.1 & 0.7 \end{pmatrix}$$

The Mean matrix $M(Z) = \frac{V(Z)+G(Z)}{2}$is,

$$M(Z) = \begin{pmatrix} 00.4 & -0.1 & -0.6 & 0.1 \\ 00.3 & -0.3 & 00.1 & 0.4 \\ -0.4 & 00.2 & -0.2 & 0.6 \\ 00.5 & 00.7 & -0.6 & 0.2 \end{pmatrix}$$

The Total mean matrix $M(Z) = \sum_{j=1}^{n} m_{ij}^z$ is,

$$M(Z) = \begin{pmatrix} -0.2 \\ 00.5 \\ 00.2 \\ 00.8 \end{pmatrix}$$

From the above Total mean matrix, the maximum mean value is, $max_{1\le i\le 4}\{m_{ij}^z\} = P_6 = 0.8$. Therefore, according to the NHSMalgorithm, patient P_6 is in critical condition moving towards stage: 5 of CKD.

Number of patients suffering from CKD = 10

⇓

Patients shortlisted based on attributes chosen = 4

⇓

According to NHSM-algorithm patient $\boldsymbol{P_6}$ is critically suffering from CKD.

6. Discussion

It may be inferred from the current investigation that the findings obtained by the suggested methodology are more flexible when compared to the available approaches. The fundamental advantage of the suggested method is that it includes more information and addresses data uncertainty by taking into account the membership, non-membership, and indeterminacy of sub-attributes. It is also a helpful tool for the decision-making process when dealing with faulty and imprecise data. In all the existing matrix theories except the Neutrosophic hypersoft matrix the motivation for the score value assigned to one parameter will not impact the other values. This results in more information loss. On the contrary, the suggested technique does not result in any significant information loss. The advantage of the proposed approach over existing NS-matrix methods is that it not only detects the level of discrimination but also the level of similarity between observations, preventing choices from being made for unfavorable reasons. As a result, it is also an appropriate technique for drawing the right conclusions in Decision-making problems even though the information is uncertain. However, the present methodology cannot deal with the situation when the decisions are provided in Interval form. Hence for such situations, Interval-valued Neutrosophic hypersoft sets and Interval-valued Neutrosophic hypersoft expert sets [42] were developed to deal with such situations.

7. Conclusion

In this paper, a few definitions based on NHSM and NHCM have been characterized, Product of NHCM and combined NHCM have been examined with examples. Additionally, an application of NHSM in decision-making is proposed based on a value matrix, grace matrix, and mean matrix.

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Author Contributions
All authors contributed equally to this research.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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Conflict of interest
The authors declare that there is no conflict of interest in the research.

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