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Forgotten Topological Index and its Properties on Neutrosophic Graphs

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Abstract: Topological indices play a significant role in crisp, fuzzy graphs and their real-life application. But to avoid the vagueness in the final result, these indices should be dealt with in the neutrosophic environment since it consolidates the uncertain quantity or values of an event in the name of "indeterminacy membership". Except for the wiener index, no other indices are introduced under the neutrosophic graphical setting. In this article, we consider the forgotten topological index (ToI) and the Edge forgotten index in the 3-valued logic neutrosophic graph and came up with some important theorem results and applications.

Keywords: Neutrosophic Graph, Topological Index, Forgotten Topological Index, Edge Forgotten Index, Neutrosophic Forgotten Topological Index.

1. Introduction

The topological indices (ToI) occupy an irreplaceable place in the case of molecular structure, chemical compounds, communication networks, spectral graphs, etc. Most of the discussion is done with crisp graphs and now it has been extended to fuzzy graphical background. Zadeh's [1] fuzzy set theory surpassed the classical crisp set and its application since the fuzzy set has an extended range of membership values to deal with uncertainty. Later, Rosenfeld [2] came up with the fuzziness in the graphs, which is called a fuzzy graph. Numerous fuzzy graph models and detailed exploration of its structural components were done by [3, 4]. Kalathian et al. [5] initiated accomplishing the ToI in fuzzy graphs and Poulik et al. [6] accomplished the same in the case of bipolar fuzzy graphs. Also, they have a brief discussion on some indices like the Wiener absolute index and the randic index in bipolar fuzzy graphs [7, 8]. Binu et al. [9] applied the wiener index in fuzzy graphs, which is useful for learning about illegal immigration networks. The f-index in fuzzy graphs was first coined by Islam and Pal [10] and also they carried out work on edge F-index [11], where prior importance is given to the edges. Islam and Pal [12-14] jointly contributed some other ToI to fuzzy graphical settings, which include the first Zagreb index, the second Zagreb index, the hyper-wiener index, etc. ToI doesn't have a profound study in the intuitionistic fuzzy graph (Int-FG) theory, which emerged using Atanossov's [15] intuitionistic fuzzy set concept. Here, he defined a new non-membership function explicitly, which of course leads to more accuracy than a fuzzy set. Parvathi et al. [16, 17] managed to improvise the fuzzy graph model to Int-FG theory. Of the indices, only the wiener index is executed with Int-FG with an application [18].

The idea to increase the accuracy in outcome is achieved through Smarandache's [19, 20] neutrosophic set definition. This segregates the uncertain conditions in real life, so they named this membership "indeterminacy". Broumi et al. [21] applied this set theory to graphical representation and they finally came up with neutrosophic graphs. Various developments and explorations have been done with the neutrosophic graph and its components. Ghods and Rostami [22] demonstrated some types of ToI shortly and discussed the wiener index in neutrosophic graphs. In this chapter, we newly implement the Forgotten ToI concept in neutrosophic graphs to attain some important results and discuss its properties and application. Also, the Edge Forgotten index is defined and overviewed with some theorems.

2. Preliminaries

In this section, some fundamental definitions are listed which are useful to enrich our findings in the following discussions.

Definition 2.1. [19] Let X denote a universal set. A neutrosophic set $\overline{R}$ defined on X is called as $\overline{R} = \{(r, T_{\overline{R}}(r), I_{\overline{R}}(r), F_{\overline{R}}(r)): r \in X\}$, where $T_{\overline{R}}(r): X \to [0,1]$, $I_{\overline{R}}(r): X \to [0,1]$, $F_{\overline{R}}(r): X \to [0,1]$ is known to be a degree of truth membership, degree of indeterminacy membership, and degree of false membership of r on $\overline{R}$ respectively and satisfy the condition $0 \leq T_{\overline{R}} + I_{\overline{R}} + F_{\overline{R}} \leq 3, \forall r \in X$.

Definition 2.2. [21] A neutrosophic graph is of the form $G = (\Upsilon, \alpha, \mu)$, where $\alpha = (T_1, I_1, F_1)$ and $\mu = (T_2, I_2, F_2)$ with the following conditions, (i) The functions $T_1: \Upsilon \to [0,1]$, $I_1: \Upsilon \to [0,1]$ and $F_1: \Upsilon \to [0,1]$ denotes the degree of truth, indeterminacy, and false membership functions of the element $v_i \in \Upsilon$, respectively, and $0 \leq T_1(v_i) + I_1(v_i) + F_1(v_i) \leq 3$, for all $v_i \in \Upsilon$. (ii) The functions $T_2: \varepsilon \subseteq \Upsilon \times \Upsilon \to [0,1]$, $I_2: \varepsilon \subseteq \Upsilon \times \Upsilon \to [0,1]$ and $F_2: \varepsilon \subseteq \Upsilon \times \Upsilon \to [0,1]$ denote the degree of truth, indeterminacy, and false membership functions of the edge (v_i, v_j) respectively, such that

$T_2(v_i, v_j) \leq \min[T_1(v_i), T_1(v_j)]$,
$I_2(v_i, v_j) \leq \min[I_1(v_i), I_1(v_j)]$,
$F_2(v_i, v_j) \leq \max[F_1(v_i), F_1(v_j)]$ and
$0 \leq T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$, for every edge (v_i, v_j).

3. Results on Topological Indices of Neutrosophic Graphs

Definition 3.1. Let $\mathfrak{G} = (\Upsilon, \alpha, \mu)$ be a neutrosophic graph(NG). The first Zagreb index of the NG $\mathfrak{G}$ is denoted by $ZN_1(\mathfrak{G})$ and is defined by

$$ZN_1(\mathfrak{G}) = \sum_{v\in\Upsilon} (T_1(v), I_1(v), F_1(v))(d^2(v))$$
$$= \sum_{v\in\Upsilon} (T_1(v), I_1(v), F_1(v))(d^2_{T_1}(v), d^2_{I_1}(v), d^2_{F_1}(v))$$
$$= \sum_{v\in\Upsilon} (T_1(v)d^2_{T_1}(v) + I_1(v)d^2_{I_1}(v) + F_1(v)d^2_{F_1}(v))$$
$$= \sum_{v\in\Upsilon} T_1(v)d^2_{T_1}(v) + \sum_{v\in\Upsilon} I_1(v)d^2_{I_1}(v) + \sum_{v\in\Upsilon} F_1(v)d^2_{F_1}(v).$$

(i.e), $ZN_1(\mathfrak{G}) = TZN_1(\mathfrak{G}) + IZN_1(\mathfrak{G}) + FZN_1(\mathfrak{G})$

Definition 3.2. Consider $\mathfrak{G} = (\Upsilon, \alpha, \mu)$ to be a NG. Then the second Zagreb index of the NG $\mathfrak{G}$ is denoted by $ZN_2(\mathfrak{G})$ and is defined by:

$$ZN_2(\mathfrak{G}) = \frac{1}{2}\sum_{uv\in\varepsilon} [(T_1(u), I_1(u), F_1(u))d(u)][(T_1(v), I_1(v), F_1(v))d(v)].$$

Definition 3.3. Consider $\mathfrak{G} = (\Upsilon, \alpha, \mu)$ be a NG. Then Forgotten index of the NG $\mathfrak{G}$ is denoted by $FN(\mathfrak{G})$ and is defined by:

$$FN(\mathfrak{G}) = \sum_{v\in\Upsilon} [(T_1(v), I_1(v), F_1(v))d(v)]^3.$$
$$= \sum_{v\in\Upsilon} [T_1(v)(d_{T_1}(v))]^3 + \sum_{v\in\Upsilon} [I_1(v)(d_{I_1}(v))]^3 + \sum_{v\in\Upsilon} [F_1(v)(d_{F_1}(v))]^3$$

Theorem 3.4. Let $\mathfrak{G}$ be a NG with n vertices and m edges. Then

(i) $FN(\mathfrak{G}) \leq n^3(t^3 + i^3 + f^3)$
(ii) $FN(\mathfrak{G}) \leq 24n^3m^3$.

Proof:

(i) Since $T_1(v) \leq 1$, $I_1(v) \leq 1$, $F_1(v) \leq 1$, the following result is obtained:

$$FN(\mathfrak{G}) = [\sum_{v\in\Upsilon} [T_1(v)(d_{T_1}(v))]^3 + \sum_{v\in\Upsilon} [I_1(v)(d_{I_1}(v))]^3 + \sum_{v\in\Upsilon} [F_1(v)(d_{F_1}(v))]^3]$$

$$\leq [[\sum_{v\in\Upsilon} T_1(v)]^3[\sum_{v\in\Upsilon} (d_{T_1}(v))]^3 + [\sum_{v\in\Upsilon} I_1(v)]^3[\sum_{v\in\Upsilon} (d_{I_1}(v))]^3$$

$$+[\sum_{v\in\Upsilon} F_1(v)]^3[\sum_{v\in\Upsilon} (d_{F_1}(v))]^3] \leq n^3(t^3+i^3+f^3)$$

(ii) By (i) and the result $(t,i,f)(\mathfrak{G}) = (2\sum T_2, 2\sum I_2, 2\sum F_2) \leq (2m,2m,2m)$. The required inequality follows

$$FN(\mathfrak{G}) \leq n^3(t^3+i^3+f^3) \leq n^3[(2m)^3+(2m)^3+(2m)^3] \leq 24n^3m^3.$$

Definition 3.5. Let $\mathfrak{G} = (\Upsilon, \alpha, \mu)$ be a NG and $u \in \Upsilon$. Then the first ZI at the vertex u of the NG $\mathfrak{G}$ is denoted by $ZN_1(u)$:

$$ZN_1(u) = [FN(\mathfrak{G}) - FN(\mathfrak{G}_u)].$$

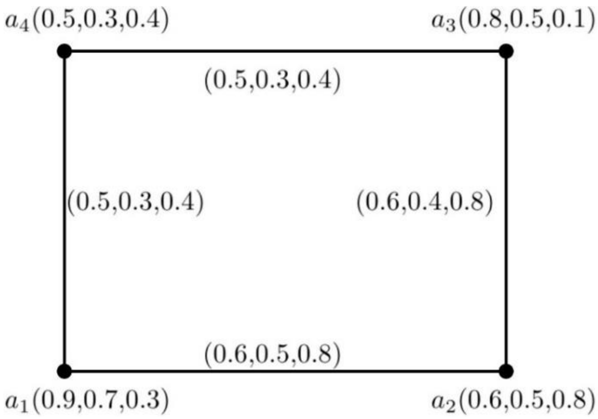

Figure 1. Neutrosophic graph with $ZN_1(\mathfrak{G}) = 7.507$

The following example is a calculation of the F-index of a neutrosophic graph.

Example 3.6. Let $\mathfrak{G}$ be an NG shown in Figure 1. Then,

$$FN(\mathfrak{G}) = \sum_{v\in\Upsilon}[T_1(v)(d_{T_1}(v))]^3 + \sum_{v\in\Upsilon}[I_1(v)(d_{I_1}(v))]^3 + \sum_{v\in\Upsilon}[F_1(v)(d_{F_1}(v))]^3$$
$$= 2.150019 + 0.315448 + 2.178304$$
$$= 4.643771$$

Example 3.7. Let H be a neutrosophic subgraph (NSG) of the NG $\mathfrak{G}$ as shown in Figure 2, which is obtained by deletion of the edge v_1v_4.

$$FN(H) = \sum_{v\in\Upsilon}[T_1(v)(d_{T_1}(v))]^3 + \sum_{v\in\Upsilon}[I_1(v)(d_{I_1}(v))]^3 + \sum_{v\in\Upsilon}[F_1(v)(d_{F_1}(v))]^3$$
$$= 1.227809 + 0.177604 + 2.1168$$
$$= 3.522213$$
$$\leq ZN_1(\mathfrak{G})$$

From the examples 3.6 and 3.7, we get $FN(H) \leq FN(\mathfrak{G})$.

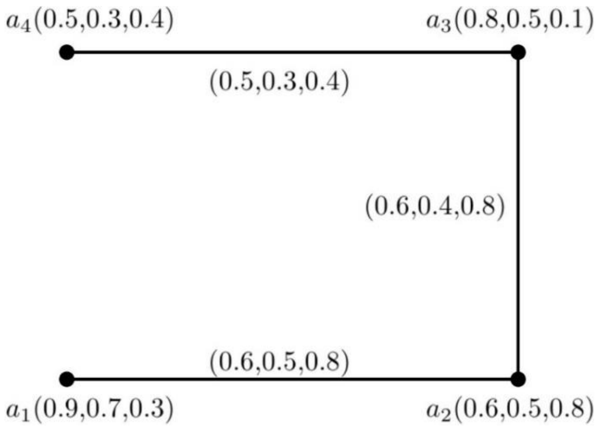

Figure 2. A NSG of the NG $\mathfrak{G}$ of Figure 1.

Proposition 3.8. Let $H = (\Upsilon', \alpha', \mu')$ be a partial neutrosophic subgraph(PNSG) of a NG $\mathfrak{G} = (\Upsilon, \alpha, \mu)$. Then $FN(H) \leq FN(\mathfrak{G})$.

Proof:

Since H is a PNSG of $\mathfrak{G}$ then for any $u, v \in \Upsilon'$, $\alpha'(u) \leq \alpha(u)$ and $\mu'(u,v) \leq \mu(u,v)$,

$d_H(u) = \sum_{v\in\Upsilon'} \mu'(u,v) \leq \sum_{v\in\Upsilon'} \mu(u,v) \leq \sum_{v\in\Upsilon} \mu(u,v) = d_{\mathfrak{G}}(u)$

Therefore,

$$FN(H) = \sum_{v\in\Upsilon'} [T_1'(v)(d_{T_1'}(v))]^3 + \sum_{v\in\Upsilon'} [I_1'(v)(d_{I_1'}(v))]^3 + \sum_{v\in\Upsilon'} [F_1'(v)(d_{F_1'}(v))]^3$$
$$\leq \sum_{v\in\Upsilon} [T_1(v)(d_{T_1}(v))]^3 + \sum_{v\in\Upsilon} [I_1(v)(d_{I_1}(v))]^3 + \sum_{v\in\Upsilon} [F_1(v)(d_{F_1}(v))]^3$$
$$= FN(\mathfrak{G})$$

This implies, $FN(H) \leq FN(\mathfrak{G})$.

Corollary 3.9. Let $H = (\Upsilon', \alpha', \mu')$ be a NSG of a NG $\mathfrak{G} = (\Upsilon, \alpha, \mu)$. Then $FN(H) \leq FN(\mathfrak{G})$.

Let $0 \leq l \leq 1$, the NG $\mathfrak{G}_l = (\Upsilon', \alpha', \mu')$ be a NSG of a NG $\mathfrak{G} = (\Upsilon, \alpha, \mu)$ and is defined as $\Upsilon' = \{v \in \Upsilon: \alpha(v) \leq l\}$ and $\alpha'(v) = \alpha(v)$, and $\mu'(uv) = \mu(uv)$ for $uv \in \mu^*$.

Theorem 3.10. Let $\mathfrak{G}$ be an NG and let $0 \leq l_1 \leq l_2 \leq 1$. Then $FN(\mathfrak{G}_{l_2}) \leq FN(\mathfrak{G}_{l_1})$

Proof: $\mathfrak{G}_{l_2}$ is PNSG of $\mathfrak{G}_{l_1}$.Then the result follows by Proposition 3.8.

Corollary 3.11. Let $\mathfrak{G}$ be an NG and let $0 \leq l_1 \leq l_2 \leq \ldots \leq l_n \leq 1$. Then

$FN(\mathfrak{G}_{l_n}) \leq FN(\mathfrak{G}_{l_{n-1}}) \leq \ldots \leq FN(\mathfrak{G}_{l_2}) \leq FN(\mathfrak{G}_{l_1})$

4. Edge Forgotten Index on Neutrosophic Graphs

Definition 4.1. Let $\mathfrak{G}$ be a neutrosophic graph(NG). Then Edge Forgotten index(EdFI) of $\mathfrak{G}$ is given by, $EdFI(\mathfrak{G}) = \sum_{uv\in\varepsilon} \{[(T_1(u), I_1(u), F_1(u))d(u)]^2 + [(T_1(v), I_1(v), F_1(v))d(v)]^2\}$.

Example 4.2. Let $\mathfrak{G}$ be an NG shown in Figure 3. Then,

$$EdFI(\mathfrak{G}) = \sum_{uv\in\varepsilon} \{[T_1(u)(d_{T_1}(u))]^2 + [I_1(u)(d_{I_1}(u))]^2 + [F_1(u)(d_{F_1}(u))]^2$$
$$+[T_1(v)(d_{T_1}(v))]^2 + [I_1(v)(d_{I_1}(v))]^2 + [F_1(v)(d_{F_1}(v))]^2\}$$
$$= 1.3738 + 0.9176 + 1.6822$$
$$= 3.9736$$

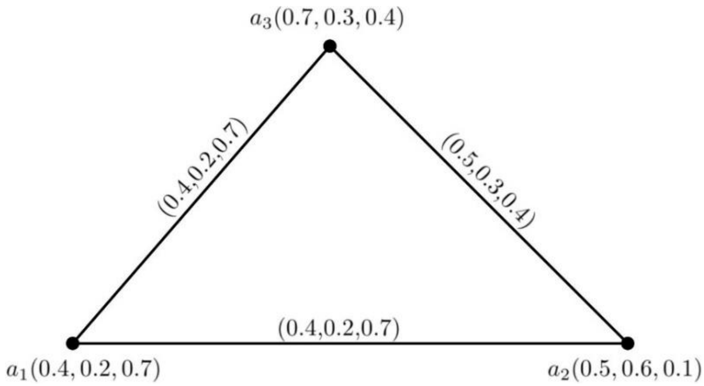

Figure 3. NG with $EdFI(\mathfrak{G}) = 3.9736$.

For Figure 3, we get $FN(\mathfrak{G}) = 1.432534$ and it is not equal to $EdFI(\mathfrak{G})$. Therefore, for a neutrosophic graph, $FN(\mathfrak{G}) \neq EdFI(\mathfrak{G})$.

Theorem 4.3. Let $H = (\Upsilon', \alpha', \mu')$ be a partial neutrosophic subgraph(PNSG) of a NG $\mathfrak{G} = (\Upsilon, \alpha, \mu)$. Then $EdFI(H) \leq EdFI(\mathfrak{G})$.

Proof:

Since H is a PNSG of $\mathfrak{G}$ then for any $u, v \in \Upsilon'$, $\alpha'(u) \leq \alpha(u)$ and $\mu'(u,v) \leq \mu(u,v)$

$d_H(u) = \sum_{v\in\Upsilon'} \mu'(u,v) \leq \sum_{v\in\Upsilon'} \mu(u,v) \leq \sum_{v\in\Upsilon} \mu(u,v) = d_{\mathfrak{G}}(u)$

Therefore,

$$EdFI(H) = \sum_{uv\in\varepsilon(H)} \{[T_1(u)(d_{T_1}(u))]^2 + [I_1(u)(d_{I_1}(u))]^2 + [F_1(u)(d_{F_1}(u))]^2 + [T_1(v)(d_{T_1}(v))]^2 + [I_1(v)(d_{I_1}(v))]^2 + [F_1(v)(d_{F_1}(v))]^2\}$$

$$\leq \sum_{uv\in\varepsilon(\mathfrak{G})} \{[T_1(u)(d_{T_1}(u))]^2 + [I_1(u)(d_{I_1}(u))]^2 + [F_1(u)(d_{F_1}(u))]^2 + [T_1(v)(d_{T_1}(v))]^2 + [I_1(v)(d_{I_1}(v))]^2 + [F_1(v)(d_{F_1}(v))]^2\}$$

$$= EdFI(\mathfrak{G})$$

This implies, $EdFI(H) \leq EdFI(\mathfrak{G})$.

Theorem 4.4. Let $\mathfrak{G}_1$ and $\mathfrak{G}_2$ be two isomorphic NGs. Then $EdFI(\mathfrak{G}_1) = EdFI(\mathfrak{G}_2)$.

Proof:

Since $\mathfrak{G}_1$ and $\mathfrak{G}_2$ be two isomorphic NGs, there exists a bijection mapping $\phi: V(\mathfrak{G}_1) \to V(\mathfrak{G}_2)$, such that $\sigma_{\mathfrak{G}_1}(v) = \sigma_{\mathfrak{G}_2}(\phi(v))$ and $\mu_{\mathfrak{G}_1}(uv) = \mu_{\mathfrak{G}_2}(\phi(u)\phi(v))$, $\forall u, v \in V(\mathfrak{G}_1)$. Then,

$$d_{\mathfrak{G}_1}(v) = \sum_{u\in V(\mathfrak{G}_1)} \mu_{\mathfrak{G}_1}(uv)$$
$$= \sum_{\phi(u)\in V(\mathfrak{G}_2)} \mu_{\mathfrak{G}_2}(\phi(u)\phi(v))$$
$$= d_{\mathfrak{G}_2}(\phi(v))$$

This implies,

$$EdFI(\mathfrak{G}_1) = \sum_{uv\in\varepsilon(\mathfrak{G}_1)} [\sigma_{\mathfrak{G}_1}(u)d_{\mathfrak{G}_1}(u)]^2 + [\sigma_{\mathfrak{G}_1}(v)d_{\mathfrak{G}_1}(v)]^2$$
$$= \sum_{\phi(u)\phi(v)\in\varepsilon(\mathfrak{G}_2)} [\sigma_{\mathfrak{G}_2}(\phi(u))d_{\mathfrak{G}_2}(\phi(u))]^2 + [\sigma_{\mathfrak{G}_2}(\phi(v))d_{\mathfrak{G}_2}(\phi(v))]^2$$
$$= EdFI(\mathfrak{G}_2)$$

5. Application of Forgotten Index in Neutrosophic Graph

Electricity supply management is very critical since electricity is a commodity that is consumed in moderate quantities by all households based on various seasons like summer, autumn, winter, etc. Recent studies summarize that both the electricity network and its distribution are essential. The distribution system must be improvised to ensure good electric current supply and maintenance. We also have a problem related to the electricity distribution system. Here, we consider a decision-making problem solved using the forgotten index. Four households are taken and finally, we come up with the household, which is the most efficient user of electricity. The vertices are considered as the households and the edges denote the electricity distribution from one household to another. Let us take the vertex memberships as household electricity usage in summer, autumn, and winter respectively. The edge membership is taken as the electricity network between the households, which is stable, uncertain, and unstable respectively. The vertex membership of the household is calculated using the expression: $\frac{\text{Usage of electricity}}{\text{Maximum electricity usage recorded}}$, for each membership. The score value and each vertex membership are tabulated in Table 1. The score of each edge is the interconnected electricity usage of households and is listed in Table 2. The edge membership value is given using the edge score divided by max electricity usage and it is found in Table 3. The main graph and its corresponding neutrosophic graph are picturized in Figures 4 and 5. The degree of each vertex is listed in Table 4, which is found using Table 3.

$$FN(\mathfrak{G}) = \sum_{v \in \Upsilon} [T_1(v)(d_{T_1}(v))]^3 + \sum_{v \in \Upsilon} [I_1(v)(d_{I_1}(v))]^3 + \sum_{v \in \Upsilon} [F_1(v)(d_{F_1}(v))]^3$$
$$= 7.547 + 2.276 + 12.155$$
$$= 21.978$$

Table 1. Calculation of vertex memberships.

Name of the house	Vertex name	Electricity(units)	Memberships
House 1	H_1	(150,55,70)	(0.91,0.69,0.74)
House 2	H_2	(125,35,55)	(0.76,0.44,0.58)
House 3	H_3	(140,65,80)	(0.85,0.81,0.84)
House 4	H_4	(165,80,95)	(1,1,1)

Table 2. Score of edge memberships.

	H_1	H_2	H_3	H_4
H_1		(95,20,70)	(0,0,0)	(130,40,85)
H_2	(95,20,70)		(100,20,80)	(0,0,0)
H_3	(0,0,0)	(100,20,80)		(120,55,90)
H_4	(130,40,85)	(0,0,0)	(120,55,90)	

Table 3. Calculation of edge memberships.

	H_1	H_2	H_3	H_4
H_1		(0.56,0.25,0.74)	(0,0,0)	(0.79,0.5,0.89)
H_2	(0.56,0.25,0.74)		(0.61,0.25,0.84)	(0,0,0)
H_3	(0,0,0)	(0.61,0.25,0.84)		(0.73,0.69,0.95)
H_4	(0.79,0.5,0.89)	(0,0,0)	(0.73,0.69,0.95)	

Table 4. Degree of membership vertex.

Vertex degree	H_1	H_2	H_3	H_4
	(1.35,0.75,1.63)	(1.17,0.5,1.58)	(1.34,0.94,1.79)	(1.52,1.19,1.84)

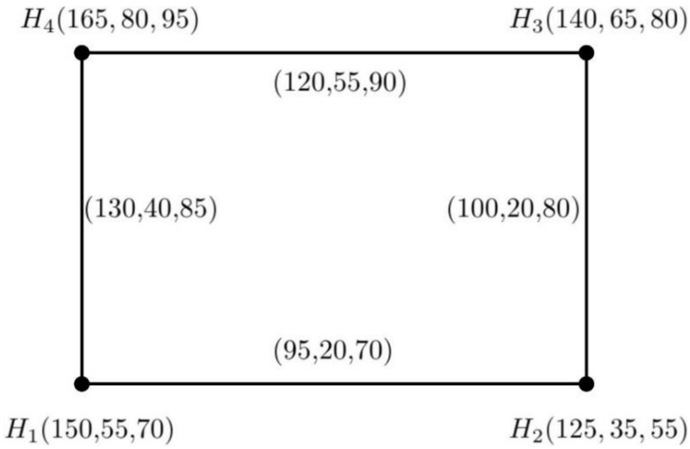

Figure 4. Main graph of crisp type.

$$\mathrm{FN}(H_1) = \sum_{v\in \Upsilon} [\mathrm{T}_1(\mathrm{v})(d_{T_1}(\mathrm{v}))]^3 + \sum_{v\in \Upsilon} [\mathrm{I}_1(\mathrm{v})(d_{I_1}(\mathrm{v}))]^3 + \sum_{v\in \Upsilon} [\mathrm{F}_1(\mathrm{v})(d_{F_1}(v))]^3$$

$= 1.854 + 0.139 + 1.755$
$= 3.748$

$$\mathrm{FN}(H_2) = \sum_{v\in \Upsilon} [\mathrm{T}_1(\mathrm{v})(d_{T_1}(\mathrm{v}))]^3 + \sum_{v\in \Upsilon} [\mathrm{I}_1(\mathrm{v})(d_{I_1}(\mathrm{v}))]^3 + \sum_{v\in \Upsilon} [\mathrm{F}_1(\mathrm{v})(d_{F_1}(\mathrm{v}))]^3$$

$= 0.703 + 0.011 + 0.77$
$= 1.484$

$$\mathrm{FN}(H_3) = \sum_{v\in \Upsilon} [\mathrm{T}_1(\mathrm{v})(d_{T_1}(\mathrm{v}))]^3 + \sum_{v\in \Upsilon} [\mathrm{I}_1(v)(d_{I_1}(\mathrm{v}))]^3 + \sum_{v\in \Upsilon} [\mathrm{F}_1(v)(d_{F_1}(\mathrm{v}))]^3$$

$= 1.478 + 0.441 + 3.4$
$= 5.319$

$$\mathrm{FN}(H_4) = \sum_{v\in \Upsilon} [\mathrm{T}_1(\mathrm{v})(d_{T_1}(\mathrm{v}))]^3 + \sum_{v\in \Upsilon} [\mathrm{I}_1(v)(d_{I_1}(\mathrm{v}))]^3 + \sum_{v\in \Upsilon} [\mathrm{F}_1(v)(d_{F_1}(\mathrm{v}))]^3$$

$= 3.512 + 1.685 + 6.23$
$= 11.427$

$$\mathrm{SC}(H_1) = \frac{\mathrm{FN}(\mathfrak{G}) - \mathrm{FN}(\mathfrak{G}_{H_1})}{\mathrm{FN}(\mathfrak{G})} = \frac{21.978 - 3.748}{21.978} = 0.829$$

$$\mathrm{SC}(H_2) = \frac{\mathrm{FN}(\mathfrak{G}) - \mathrm{FN}(\mathfrak{G}_{H_2})}{\mathrm{FN}(\mathfrak{G})} = 0.932$$

$$\mathrm{SC}(H_3) = \frac{\mathrm{FN}(\mathfrak{G}) - \mathrm{FN}(\mathfrak{G}_{H_3})}{\mathrm{FN}(\mathfrak{G})} = 0.758$$

$$\mathrm{SC}(H_4) = \frac{\mathrm{FN}(\mathfrak{G}) - \mathrm{FN}(\mathfrak{G}_{H_4})}{\mathrm{FN}(\mathfrak{G})} = 0.48$$

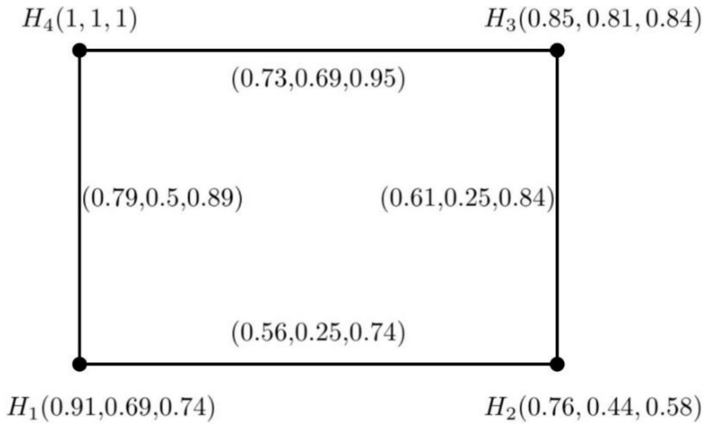

Figure 5. Neutrosophic graph based on Figure 4.

It is seen that household H_2 properly uses electricity by reducing the usage in the autumn and winter seasons. Hence, household H_2 is the most efficient user in the household group.

6. Conclusion

Topological indices play a vital role in the field of spectral and chemical graph theory, molecular structural descriptions, etc. In this article, the F-index for neutrosophic graph(NG) is defined and provided results. An application related to decision-making is structured and handled using the Forgotten index for an NG. Numerous topological indices deal with crisp graphs but they are not extended to a NG. NG provides a powerful framework for representing uncertain and vague information in graph theory. Through the incorporation of additional membership "indeterminacy", as well as hesitation degrees, NG offers a more flexible and expressive approach to modeling complex relationships than traditional crisp graphs. One of the key strengths of NG lies in their ability to handle ambiguity and uncertainty inherent in real-world data, making them particularly useful in decision-making processes, where precise information may be lacking or unreliable. Additionally, NG has been successfully applied in various fields such as pattern recognition, image processing, social network analysis, and optimization problems. Despite their advantages, it's essential to note that NG also poses challenges in terms of computation and interpretation, especially when dealing with large-scale systems. Future research efforts should focus on developing efficient algorithms and methodologies for analyzing and utilizing NG effectively.

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Author Contributions
All authors contributed equally to this research.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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Conflict of interest

The authors declare that there is no conflict of interest in the research.

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