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Exploring the Potential of Neutrosophic Topological Spaces in Computer Science

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Abstract: Neutrosophic topological spaces (NTS) offer a novel framework for uncertainty modeling by incorporating degrees of truth, indeterminacy, and falsity. This paper investigates the potential applications of NTS in computer science. We provide background on neutrosophic sets and their extension to topological spaces. We then explore how NTS could be used for uncertainty modeling in data analysis (e.g., handling noisy data in sensor networks), pattern recognition (e.g., improving image classification with imprecise features), and information retrieval (e.g., enhancing search results by considering relevance uncertainty). We discuss the challenges associated with applying NTS and highlight promising areas for future research, such as developing efficient algorithms for NTS operations. Overall, this paper aims to stimulate further exploration of how neutrosophic topological spaces can contribute to advancements in various computer science domains.

Keywords: Neutrosophic Sets; Topological Space; Uncertainty Modeling; Computer Science; Data Analysis; Pattern Recognition; Information Retrieval.

1. Introduction

Traditional data analysis methods struggle with inherent uncertainty in real-world data (e.g., noisy sensors [15]). Neutrosophic topological spaces (NTS) offer a promising solution. NTS extends neutrosophic sets, which incorporate degrees of truth (T), indeterminacy (I), and falsity (F), to the realm of topology [2-6]. This allows for more nuanced uncertainty modeling in computer science (e.g., data analysis, pattern recognition, and information retrieval). We explore these applications, challenges, and future research directions for utilizing NTS. To establish a common understanding, this section introduces the core concepts of neutrosophic logic. We build upon the pioneering work of Smarandache [1, 2] who introduced the neutrosophic components that are fundamental to this logic. Additionally, Salama et al. have contributed significantly to the field through various applications explored in their publications [7-32].

Here, we focus on the core elements that define neutrosophic logic:

Neutrosophic Components: These form the foundation for representing uncertainty in neutrosophic logic. Introduced by Smarandache [1, 2], they consist of three values:

- Truth (T): Represents the degree of membership an element has in a set. Values range from 0 (completely false) to 1 (completely true).
- Indeterminacy (I): Represents the degree of uncertainty associated with an element's membership in a set. Values range from 0 (completely determinate) to 1 (completely indeterminate).

- Falsity (F): Represents the degree of non-membership an element has in a set. Values range from 0 (completely true member) to 1 (completely false member).

These components allow neutrosophic logic to move beyond the limitations of classical logic (true/false) by incorporating varying degrees of truth, indeterminacy, and falsity.

1.1 Motivation

The limitations of classical methods in handling uncertainty can lead to inaccurate results and hinder progress in various computer science domains. This motivates the exploration of alternative frameworks like NTS that can capture the inherent vagueness of real-world data.

1.2 Research Questions

This paper investigates the potential of NTS in computer science by exploring the following questions:

- How can NTS be effectively utilized for uncertainty modeling in data analysis, pattern recognition, and information retrieval tasks?
- What are the challenges associated with implementing and utilizing NTS in computer science applications?
- What are promising areas for future research on developing efficient algorithms and exploring applications of NTS in other computer science subfields?

2. Neutrosophic Topological Spaces: Unveiling Uncertainty in Topology

Building on our understanding of neutrosophic sets and topological spaces, this section explores Neutrosophic Topological Spaces (NTS), a framework for incorporating uncertainty into topological structures.

2.1 Formalizing Neutrosophic Topological Spaces

An NTS (X, τ, T) is a triplet where:

- **X:** The same set of elements used in the topological space definition, representing the collection of objects under consideration.
- **τ:** The collection of open sets in X, satisfying the axioms of a topological space (as discussed earlier).
- **T:** A neutrosophic set on the collection of open sets (τ). Here, T assigns a degree of truth (T(U)), indeterminacy (I(U)), and falsity (F(U)) to each open set U in τ.

Key Idea: An NTS adds the concept of neutrosophic membership to a topological space. This allows us to represent not only whether a set is open but also the degree of certainty associated with its openness.

Notation:

- X: Universe set.
- τ: Collection of open sets in X (forming the neutrosophic topology).
- T: Neutrosophic set on τ, assigning truth (T(U)), indeterminacy (I(U)), and falsity (F(U)) to open sets U in τ.
- A: Subset of X.

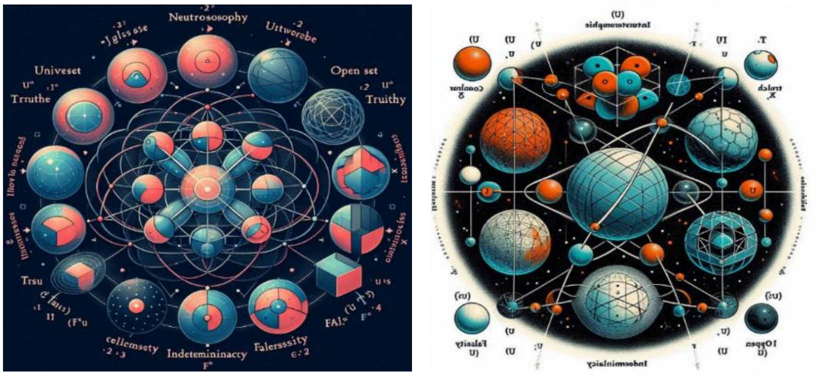

Figure 1. Key Components of a neutrosophic topological space.

Figure 1 aims to visually represent the key components of a NTS. It emphasizes that NTS builds upon the concept of traditional topological spaces (with open sets) by incorporating neutrosophic logic. This allows for assigning degrees of truth, indeterminacy, and falsity to elements and potentially to open sets within the space, enabling the modeling of uncertainty.

Formal Definitions:

- **Neutrosophic Interior (int(A))**
  - int(A) = supremum { U ∣ U ⊆ A, U ∈ τ, for all U ∈ τ }
  - Intuitively, we take the largest neutrosophic open set (in terms of truth membership and minimal indeterminacy/falsity) entirely contained within A.

Figure 2. Neutrosophic interior: capturing the "Largest Openness" within a set.

Figure 2 aims to visualize the concept of a neutrosophic interior in an NTS. It conveys that the neutrosophic interior goes beyond the simple idea of elements being "inside" a set. It incorporates neutrosophic logic by considering the degree of truth (T) associated with elements' membership. The largest shaded region captures the elements with the highest truth value of being definitively or somewhat "inside" set A.

- **Neutrosophic Closure (cl(A))**
  - cl(A) = infimum { F ∣ A ⊆ F, F is neutrosophic closed set for all F }
  - We find the smallest neutrosophic closed set (in terms of high falsity membership) containing A, encompassing all elements with some degree of "outside" certainty or indeterminacy.

Figure 3. Neutrosophic Closure (cl(A)) - Visualizing the "Smallest Closed Set".

Figure 3 aims to visualize the concept of a neutrosophic closure in an NTS. It conveys that the neutrosophic closure goes beyond the simple idea of elements being "outside" a set. It incorporates neutrosophic logic by considering the degree of falsity (F) associated with elements' exclusion. The smallest shaded region captures the elements with the highest falsity value of being definitively or somewhat "outside" set A.

- **Neutrosophic Exterior (ext(A))**
  - ext(A) = complement(int(A))
  - The neutrosophic exterior is simply the complement of the neutrosophic interior. It represents elements definitively "outside" A.

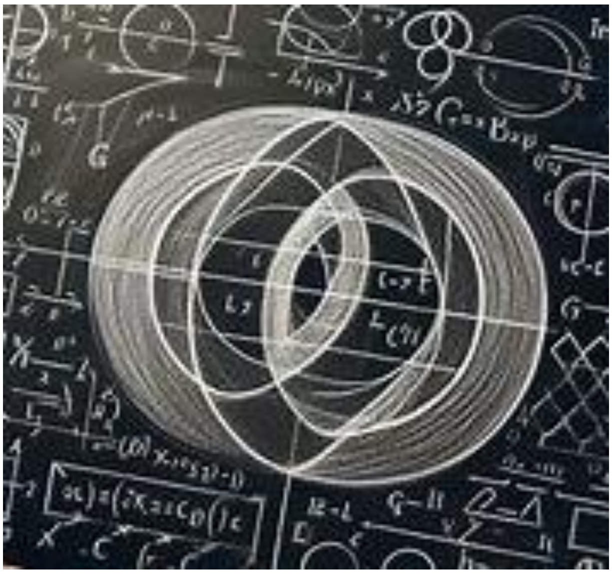

Figure 4. Neutrosophic Exterior (ext(A)): Charting the Elements definitively "Outside"

Figure 4 aims to visualize the concept of a neutrosophic exterior in an NTS. It emphasizes that the exterior goes beyond simply identifying elements that are not in set A. It incorporates neutrosophic logic by considering the degree of falsity (F) associated with elements being definitively outside the set. The exterior region captures the elements with the highest falsity value of being definitively "outside" set A.

- **Neutrosophic Boundary (nb(A))**
  - nb(A) = cl(A) ∩ int(A^c)
  - Here, A^c denotes the complement of A. The neutrosophic boundary captures elements on the "edge" by finding the intersection of the neutrosophic closure of A and the neutrosophic interior of A's complement. These elements have some level of uncertainty about belonging to A.

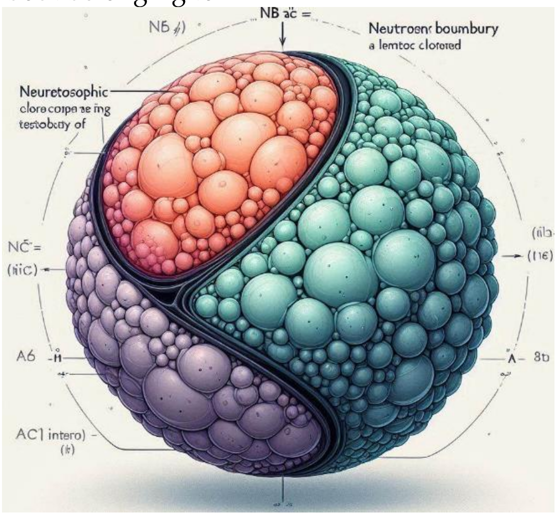

Figure 5. Neutrosophic Boundary (nb(A)) - Visualizing the "Intersection of Closure and Interior"

Figure 5 aims to visualize the concept of a neutrosophic boundary in an NTS. It highlights that the boundary arises from the intersection of two neutrosophic concepts:

2.2 Exploring Theoretical Properties of NTS

NTS introduces complexity compared to traditional spaces due to truth (T), indeterminacy (I), and falsity (F) degrees. Here is a deeper look at their theoretical properties:

1. *Mathematical Properties*
   - Continuity: NTS continuity can be defined based on the continuity of the underlying T, I, and F functions mapping elements to their neutrosophic values. This allows for studying how small changes in elements within an NTS lead to corresponding changes in their neutrosophic characteristics.

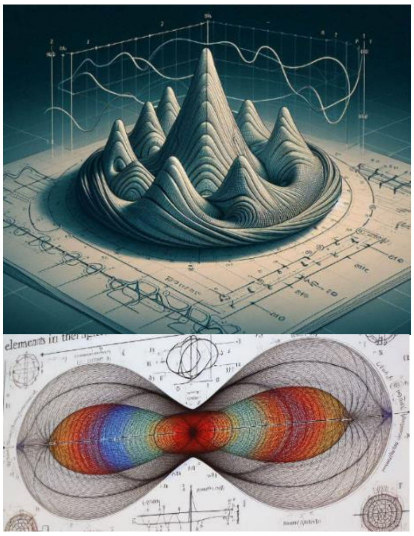

Figure 6. NTS continuity: Decomposing change through T, I, and F Functions.

Figure 6 explains how continuity in NTS is analyzed by considering the T, I, and F functions. It emphasizes that small changes in elements within the space should be reflected in corresponding, relatively small changes in their truth, indeterminacy, and falsity values assigned by these functions. This maintains a sense of "continuity" even when dealing with uncertainty in neutrosophic sets.

- Compactness: Compactness in NTS can be explored by extending existing definitions from traditional topology to consider the neutrosophic nature of sets. This involves investigating properties like finite sub-coverings for open neutrosophic sets.

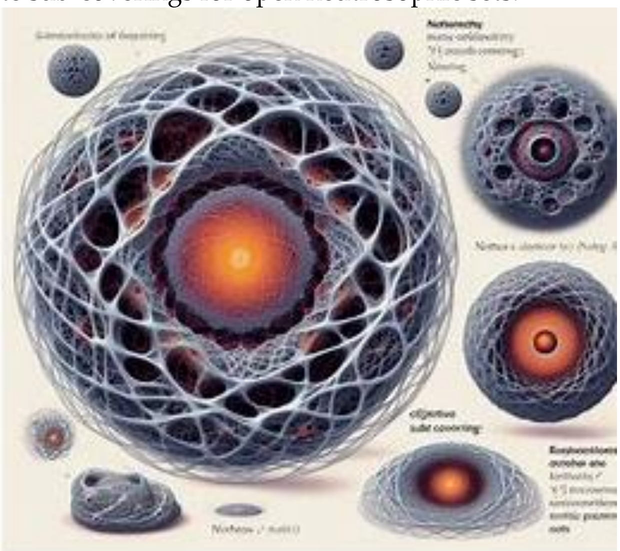

Figure 7. Compactness in NTS: Achieving "Finitude" with neutrosophic open sets.

Figure 7 explains how compactness is achieved in NTS. It emphasizes that even though we are dealing with neutrosophic open sets that have uncertainty associated with them, it's still possible to achieve compactness. The figure might illustrate how a finite number of strategically chosen neutrosophic open sets can entirely cover the neutrosophic space, ensuring a kind of "finitude" despite the inherent uncertainty.

- Separation Axioms: NTS separation axioms can be formulated to analyze the separation of distinct neutrosophic sets based on their T, I, and F values. This could involve defining axioms like T1 (separation by truth) or stronger axioms requiring separation based on combinations of T, I, and F.

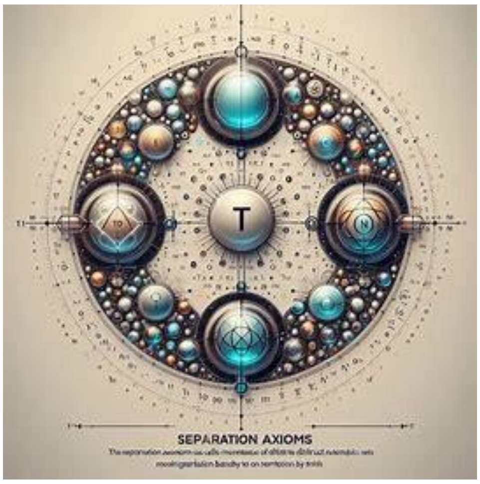

Figure 8. Degrees of Separation in Neutrosophic Topological Spaces - A Clockwise Exploration.

2. *Relationships with Other Topological Concepts*
   - Fuzzy Topology: Explore connections between NTS and fuzzy topology, which deals with degrees of membership. Investigate how neutrosophic sets can be seen as an extension of fuzzy sets, incorporating the additional concept of falsity (F) alongside truth and indeterminacy.
   - Probabilistic Topology: Investigate relationships with probabilistic topology, which utilizes probabilities to represent uncertainty. Explore how neutrosophic degrees (T, I, and F) might be related to probabilistic measures and how these frameworks can complement each other.
3. *Formal Frameworks for Reasoning about Uncertainty*
   - Neutrosophic Logic Extensions: Develop formal frameworks for reasoning. Here is an expanded section on Formal Frameworks for Reasoning about Uncertainty within NTS:
4. *Formal Frameworks for Reasoning about Uncertainty*

Neutrosophic logic extensions play a crucial role in enabling formal reasoning about uncertainty within NTS. These extensions build upon the principles of neutrosophic logic, which incorporates degrees of truth (T), indeterminacy (I), and falsity (F), to develop reasoning systems specifically tailored to handle the uncertainties captured by NTS.

Here are two key directions for developing formal frameworks for reasoning in NTS:

- Neutrosophic Propositional Logic Extensions:
  - Develop a neutrosophic propositional logic system suitable for reasoning about propositions involving elements and sets within an NTS.
  - This would involve defining neutrosophic connectives (AND, OR, NOT) that operate on propositions with truth, indeterminacy, and falsity values.
  - For example, a neutrosophic version of "AND" might consider not only the truth values of two propositions but also their indeterminacy and falsity to determine the resulting neutrosophic truth value of the combined proposition.
  - Inference rules specific to NTS would be established. These rules would allow for deriving new neutrosophic propositions based on existing ones, following logical principles while incorporating uncertainty degrees.
- Neutrosophic Predicate Logic Extensions:

- Extend neutrosophic reasoning beyond propositions to handle predicates (statements about variables).
- This would enable reasoning about properties of elements within the NTS and relationships between them, considering the inherent uncertainty.
- Quantifiers like "all" and "some" would be reinterpreted in the neutrosophic context, allowing for reasoning about the prevalence of certain properties or relationships within the NTS with their associated uncertainty degrees.

Inspiration from Existing Frameworks:

- Fuzzy Logic: Leverage concepts from fuzzy logic, which deals with degrees of truth membership. Adapt existing fuzzy logic operators and inference rules to handle the additional dimension of falsity present in neutrosophic sets [33].
- Probabilistic Logic: Explore connections with probabilistic logic, which utilizes probabilities to represent uncertainty. Investigate how neutrosophic degrees (T, I, and F) might be related to probabilistic measures and how these frameworks can inform each other. The goal is to develop a richer framework that captures both the qualitative aspects of uncertainty (T, I, F) and the quantitative aspects (probabilities).

Challenges and Considerations:

- Defining robust and efficient algorithms for automated reasoning within these neutrosophic logic extensions is crucial for practical applications. This involves developing methods to manipulate neutrosophic propositions and predicates while ensuring computational efficiency.
- Exploring the relationships between different neutrosophic logic extensions and their expressive power is important. Some extensions might be simpler but less expressive, while others might offer more nuanced reasoning capabilities but come at a higher computational cost. Identifying the right balance for specific applications is essential.

By developing and applying these formal frameworks for reasoning about uncertainty within NTS, researchers can enable a more robust and nuanced analysis of problems involving inherently ambiguous or imprecise data. This holds significant potential for various domains where uncertainty plays a key role, such as image analysis with noisy data, robot navigation in dynamic environments, or information retrieval with fuzzy user queries.

Applying Neutrosophic Topological Spaces to a Dataset Example

- Application 1:

Let us consider a dataset related to weather conditions in a specific location. We can use the concepts of neutrosophic topological spaces (NTS) to represent uncertainty associated with weather patterns.

Dataset Example: Daily High Temperatures

Imagine we have a dataset containing daily high temperatures for a year in a particular city. Each data point represents the maximum temperature recorded on a specific day.

1. Neutrosophic Universe (X):

$X = \{T \mid T$ is a temperature reading in degrees Celsius$\}$. This set represents all possible high temperatures that could occur in the city throughout the year.

2. Neutrosophic Open Sets (τ):

We can define open sets based on what is considered "typical" or "unusual" temperature ranges for different seasons. Here are some examples:

$U_1 = \{T \mid 15 \leq T \leq 25\}$ (Represents comfortable spring/fall temperatures)
$U_2 = \{T \mid 25 \leq T \leq 30\}$ (Represents typical summer temperatures)
$U_3 = \{T \mid 5 \leq T \leq 15\}$ (Represents unusually cold temperatures)
$U_4 = \{T \mid 30 \leq T \leq 35\}$ (Represents unusually hot temperatures)

These open sets satisfy the axioms of a topological space, ensuring properties like the union of sets representing valid temperature ranges.

3. Neutrosophic Set on Open Sets (T)

Now, we incorporate uncertainty by assigning degrees of truth (T(U)), indeterminacy (I(U)), and falsity (F(U)) to each open set based on how well it reflects typical weather patterns. Here is a possible assignment as shown in Table 1:

Table 1. Neutrosophic characterization of temperature ranges.

Open Set (U)	Temperature Range (°C)	Degree of Truth (T(U))	Degree of Indeterminacy (I(U))	Degree of Falsity (F(U))
U_1	$15 \leq T \leq 25$	0.8	0.1	0.1
U_2	$25 \leq T \leq 30$	0.7	0.2	0.1
U_3	$5 \leq T \leq 15$	0.3	0.4	0.3
U_4	$30 \leq T \leq 35$	0.4	0.3	0.3

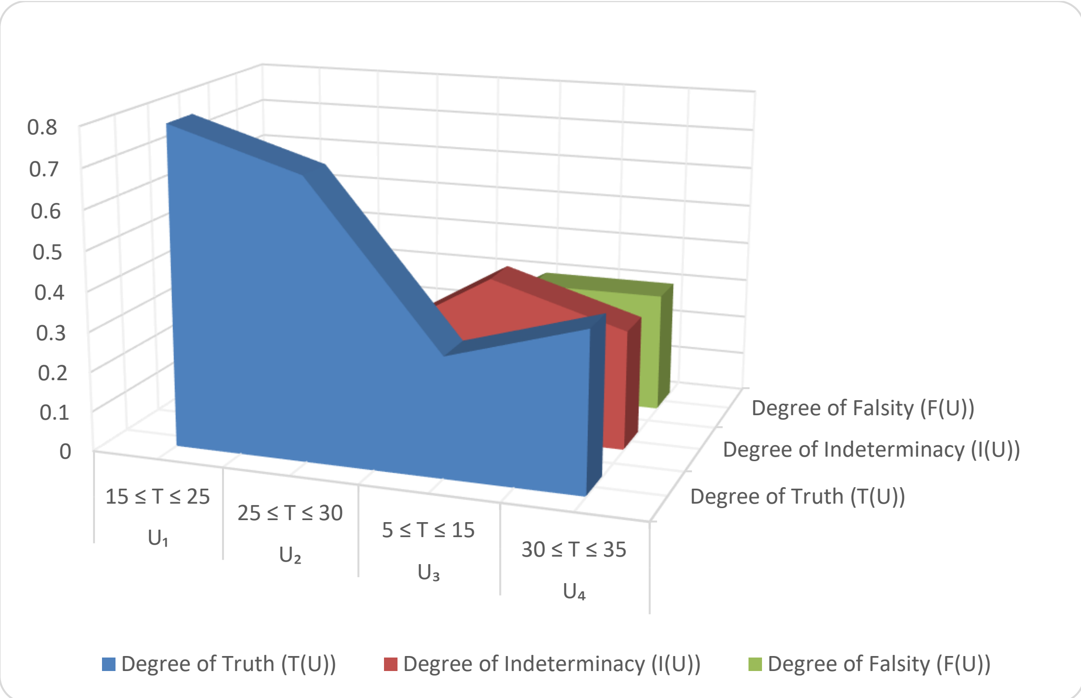

Figure 9. Neutrosophic values for open sets.

Figure 9 highlights that in an NTS, open sets are not just defined by their "openness" but also by the neutrosophic characteristics (T, I, and F) of the elements they contain. The visualization helps us understand how these neutrosophic values influence the classification of elements within open sets.

4. Neutrosophic Closed Sets Based on Weather Dataset Application

- **Concept:** Neutrosophic closed sets are the complements of neutrosophic open sets in a neutrosophic topological space (NTS).
- **Reference:** The provided information about the weather dataset application serves as the context for defining neutrosophic closed sets.
- **Table 2:** Here is a table outlining the neutrosophic closed sets based on the open sets defined in the application:

Table 2. Neutrosophic closed sets for temperature characterization.

Open Set (U)	Temperature Range (°C)	Neutrosophic Closed Set (U^c)	Temperature Range (°C)	Complement Truth ($T(U^c)$)	Complement Indeterminacy ($I(U^c)$)	Complement Falsity ($F(U^c)$)
U_1	$15 \leq T \leq 25$	U_1^c	T < 15 OR T > 25	0.2	0.1	0.8
U_2	$25 \leq T \leq 30$	U_2^c	T < 25 OR T > 30	0.3	0.2	0.7
U_3	$5 \leq T \leq 15$	U_3^c	T < 5 OR T > 15	0.7	0.4	0.3
U_4	$30 \leq T \leq 35$	U_4^c	T < 30 OR T > 35	0.6	0.3	0.4

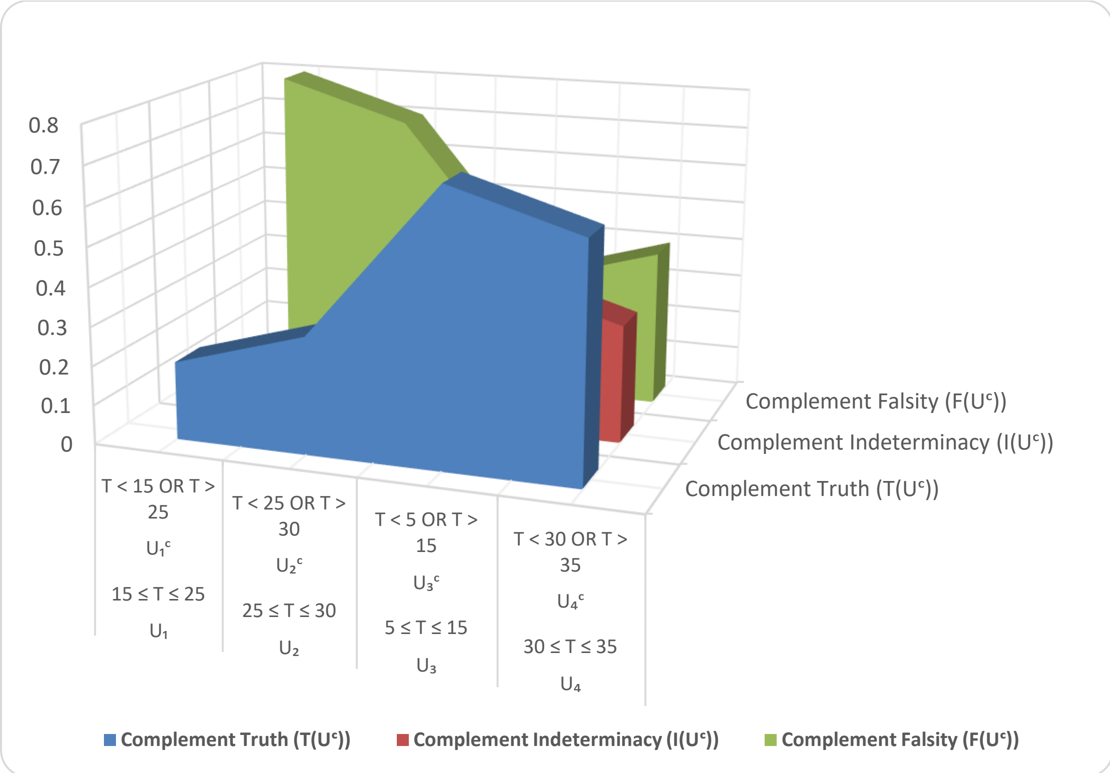

Figure 10. Neutrosophic values for closed sets.

Figure 10 emphasizes that in an NTS, closed sets are also characterized by neutrosophic values (T, I, and F) – not just the elements definitively outside them (which would be completely false with F(U^c) = 1). The visualization helps us understand how these neutrosophic values influence the classification of elements in closed sets.

Explanation:

- U^c represents the complement of the open set U.
- The temperature range for the closed set is defined by the logical OR between temperatures less than the lower bound or greater than the upper bound of the corresponding open set.
- The complement truth ($T(U^c)$), complement indeterminacy ($I(U^c)$), and complement falsity ($F(U^c)$) are assigned based on the "oppositeness" to the original open set's neutrosophic values. Higher truth values in the closed set indicate a stronger degree of membership for temperatures outside the "typical" range of the open set.

Note: These are just possible assignments for the complement neutrosophic values. Depending on the specific weather patterns in the dataset, the values might be adjusted to better reflect the uncertainty associated with non-typical temperatures.

4.1 Analyzing Daily High Temperatures

With the NTS defined, we can analyze specific temperature readings from the dataset. Here is an example:

- Suppose a day has a high temperature of 22°C. This temperature falls within the open set U_1 (comfortable spring/fall temperatures).

4.2 Neutrosophic Open Set Analysis

Following the definition of neutrosophic open sets:

- We can define a neutrosophic neighborhood N(22) = {T | 20 ≤ T ≤ 24} with a neutrosophic truth value of T(N(22)) = {0.75, 0.15, 0.1}.
- This neighborhood is entirely contained within U_1 (temperatures between 20°C and 24°C are within 15-25°C).
- Since such a neighborhood exists for 22°C within U_1, it confirms that 22°C is indeed neutrosophically open within the defined NTS.

4.3 Neutrosophic Interior, Closure, etc

We can further explore the dataset using neutrosophic concepts:

- Neutrosophic Interior: Identify the core temperature range within a specific season with the highest degree of truth value.
- Neutrosophic Closure: Find the broader temperature range encompassing days with some possibility of being within that season (considering unseasonably cold or hot days).
- Neutrosophic Exterior: Identify days with temperatures definitively outside the "typical" seasonal ranges.
- Neutrosophic Boundary: Analyze days with temperatures on the "edge" of seasons, where there is uncertainty about their classification.

By applying these neutrosophic concepts, we can gain a more nuanced understanding of the temperature patterns in the dataset, accounting for the inherent uncertainty in weather data.

4.4 Analyzing Daily High Temperatures using Neutrosophic Closed Sets

Building upon the concept of neutrosophic-closed sets derived from the weather dataset application; let's explore how they can be used to analyze daily high temperatures.

Scenario:

Imagine you have a recorded daily high temperature (T) in degrees Celsius for a specific day. We can analyze it using the defined neutrosophic closed sets (U^c) presented in the previous table.

Steps:

1. Temperature Reading:
   - Identify the recorded daily high temperature (T) in °C.
2. Neutrosophic Closed Set Analysis:
   - Check if T falls within any of the neutrosophic closed, sets (U^c) defined earlier.
     - If T falls within a closed set (e.g., U_1^c: T < 15 OR T > 25), it suggests a high degree of truth ($T(U^c)$) for temperatures outside the "typical" range associated with the corresponding open set (U_1: comfortable spring/fall temperatures).
     - If T does not fall within any closed set, it indicates a temperature within one of the defined open sets, potentially representing a "typical" range for the season.
3. Neutrosophic Degree Interpretation:
   - Even if T doesn't fall within a closed set (implying it's within an open set), consider the complement neutrosophic values (complement truth ($T(U^c)$), complement

indeterminacy ($I(U^c)$), complement falsity ($F(U^c)$)) associated with the corresponding open set.

- A high complement truth value ($T(U^c)$) for a specific open set (e.g., U_1) suggests there's still some possibility of T being an outlier within that seemingly "typical" range.

Benefits:

- Neutrosophic closed sets provide additional insights into how far a temperature deviates from the "typical" range for a season.
- The complement neutrosophic values offer a nuanced understanding of the uncertainty associated with temperatures within open sets.

Limitations:

- The effectiveness relies heavily on the chosen open sets and their neutrosophic values. These might need adjustments based on the specific weather patterns in the dataset.
- Utilizing neutrosophic closed sets requires familiarity with neutrosophic set theory for accurate interpretation.

Comparison with Open Set Analysis:

- Analyzing with open sets focuses on whether a temperature falls within a "typical" range, considering its degree of truth ($T(U)$).
- Analyzing with closed sets focuses on how far a temperature deviates from the "typical" range, considering the complement truth ($T(U^c)$) and other neutrosophic values of the corresponding open set.

By combining analysis with open and closed sets, we gain a more comprehensive understanding of daily high temperatures and the associated uncertainty within the context of the defined neutrosophic weather dataset application.

Application 2:

Imagine a scenario where you are analyzing temperature readings from sensors in a greenhouse. Here is how we can define a neutrosophic topological space (NTS) to represent the concept of "optimal temperature range" for plant growth:

- Set X: $X = \{T \mid T$ is a temperature reading in degrees Celsius$\}$. This represents the collection of all possible temperature readings.
- Open Sets (τ): We define some open sets based on what is considered favorable for plant growth. Examples could be:
  - $U_1 = \{T \mid 18 \leq T \leq 25\}$ (Represents temperatures between 18°C and 25°C).
  - $U_2 = \{T \mid 22 \leq T \leq 28\}$ (Represents temperatures between 22°C and 28°C, a slightly wider range).

These open sets satisfy the axioms of a topological space, such as the union of U_1 and U_2 being another open set (temperatures between 18°C and 28°C) and the intersection of U_1 and U_2 being another open set (temperatures between 22°C and 25°C).

- Neutrosophic Set (T): This is where we incorporate uncertainty. We assign degrees of truth ($T(U)$), indeterminacy ($I(U)$), and falsity ($F(U)$) to each open set (U) based on how well it represents the optimal temperature range. Here's a possible assignment:
  - $T(U_1) = \{0.8, 0.1, 0.1\}$
    - High truth (0.8) signifies U_1 (18-25°C) is a good temperature range for most plants.
    - Low indeterminacy (0.1) suggests there is some certainty about this range being favorable.
    - Low falsity (0.1) means it's unlikely that this range is entirely unsuitable.
  - $T(U_2) = \{0.6, 0.2, 0.2\}$

- A lower truth (0.6) indicates U_2 (22-28°C) is a less ideal but still acceptable range for some plants.
- Higher indeterminacy (0.2) reflects more uncertainty about the suitability of this wider range.
- Higher falsity (0.2) suggests there is a greater chance that temperatures outside U_1 might be unsuitable.

This example demonstrates how an NTS can represent the concept of "optimal temperature" with uncertainty. The neutrosophic set (T) allows us to capture the idea that there is a core range (U_1) with a high truth value and a broader range (U_2) with a lower truth value but some possibility of being suitable as shown in Table 3.

Table 3. Neutrosophic truth values for open sets.

Open Set (U)	Degree of Truth (T(U))	Degree of Indeterminacy (I(U))	Degree of Falsity (F(U))
U_1	0.8	0.1	0.1
U_2	0.6	0.2	0.2

Neutrosophic Open, Closed Sets, Interior, and Closure in NTS

Scenario: We continue with the greenhouse temperature analysis example. We have defined the set X of all temperature readings and open sets U_1 (18-25°C) and U_2 (22-28°C) with their neutrosophic truth values (T(U)), indeterminacy (I(U)), and falsity (F(U)).

Neutrosophic Open Set

An open set U in an NTS is neutrosophically open if, for any element x in U, there exists a neutrosophic neighborhood N(x) of x such that N(x) is entirely contained within U.

Example

Consider a temperature reading of 20°C (x = 20°C). This temperature falls within the open set U_1 (18-25°C). We can define a neutrosophic neighborhood $N(20) = \{T \mid 19 \leq T \leq 21\}$ with a neutrosophic truth value of $T(N(20)) = \{0.7, 0.2, 0.1\}$. This neighborhood is entirely contained within U_1 (all temperatures between 19°C and 21°C are within 18-25°C). Since we can find such a neighborhood for any element within U_1, the set U_1 is neutrosophically open as shown in Table 4.

Neutrosophic Open Set Analysis

Table 4. Neutrosophic temperature analysis for a specific reading.

Criteria	Description	Result
Temperature Reading (°C)	x = 20°C	-
Neutrosophic Neighborhood	N(20) = {T	$19 \leq T \leq 21$}
Temperature Range of Neighborhood (°C)	-	$19 \leq T \leq 21$
Open Set	U_1 = {T	$18 \leq T \leq 25$}
Relationship between Neighborhood and Open Set	$N(20) \subseteq U_1$	N(20) is entirely contained within U_1

Conclusion: Based on the analysis, since we can find a neutrosophic neighborhood N(20) entirely within the open set U_1, the set U_1 is neutrosophically open.

A neutrosophic closed set U in an NTS is the complement of a neutrosophic open set. In simpler terms, if U' is the complement of U (elements in X that are not in U), then U' is neutrosophically closed if it's neutrosophically open.

Let us use the provided greenhouse scenario with the neutrosophic topological space (NTS) defined for the "optimal temperature range" to explore the concepts of the neutrosophic interior, closure, exterior, and boundary.

1. ***Neutrosophic Interior (int(A))***

Imagine a specific plant species known to thrive within a narrow temperature range. Let A = {T ∣ 20 ≤ T ≤ 23}. This subset represents the ideal temperature range for this plant.

Finding the neutrosophic interior (int(A)) involves identifying the largest neutrosophic open set entirely contained within A. Since U_1 (18-25°C) encompasses A completely, int(A) would likely be a neutrosophic set derived from U_1 with a slightly lower truth value due to A being a stricter range within U_1.

For example, int(A) = {0.7, 0.2, 0.1}. This reflects a high degree of certainty (0.7) that temperatures within 20-23°C are good for the plant, with some indeterminacy (0.2) acknowledging the possibility of slightly broader ranges being suitable, and minimal falsity (0.1) indicating it's unlikely temperatures outside this range are entirely unsuitable.

2. ***Neutrosophic Closure (cl(A))***

The neutrosophic closure (cl(A)) represents the smallest neutrosophic closed set containing A. In this case, we want to find the set encompassing all temperatures with some degree of "outside" certainty or indeterminacy for the specific plant.

Since U_2 (22-28°C) is the wider open set containing A, cl(A) could be derived from U_2 with a slightly higher truth value compared to $T(U_2)$. This reflects the inclusion of some temperatures outside the ideal range (20-23°C) with some possibility of being acceptable.

For instance, cl(A) = {0.7, 0.1, 0.2}. Here, a high truth value (0.7) captures the certainty that temperatures within the broader range (22-28°C) might still be suitable, with some indeterminacy (0.1) remaining, and a slightly higher falsity (0.2) acknowledging the increased chance of unsuitable temperatures outside the ideal range.

3. ***Neutrosophic Exterior (ext(A))***

The neutrosophic exterior (ext(A)) is the complement of the neutrosophic interior. It represents elements definitively "outside" the ideal temperature range for the plant.

Therefore, ext(A) = complement({0.7, 0.2, 0.1}) = {complement(0.7), complement(0.2), complement(0.1)}. Depending on the specific neutrosophic set operations used, this could translate to values indicating high falsity (certainty of being unsuitable) and minimal truth/indeterminacy for temperatures outside the 20-23°C range.

4. ***Neutrosophic Boundary (nb(A))***

The neutrosophic boundary (nb(A)) captures elements on the "edge" of the ideal range, with some uncertainty about their suitability for the plant.

Here, nb(A) = cl(A) ∩ int(A^c). A^c represents the complement of A, which is all temperatures outside the 20-23°C range. The intersection with cl(A) captures the "edge" elements within the broader range (22-28°C) that might be suitable with some uncertainty.

The resulting neutrosophic set for nb(A) would likely have truth values indicating some possibility of being suitable (but lower than the ideal range), with higher indeterminacy reflecting the uncertainty about their exact impact on the plant, and some degree of falsity acknowledging the chance of being unsuitable.

The complement of U_1 (18-25°C) is $U'_1 = \{T \mid T < 18 \text{ or } T > 25\}$. This set includes temperatures below 18°C and above 25°C. To analyze if U'_1 is neutrosophically closed, we need to verify if its complement (which is U_1) is neutrosophically open (as discussed previously). Since we established U_1 is neutrosophically open, its complement U'_1 is neutrosophically closed.

Table 5 shows the neutrosophic values (degree of truth (T(U)), indeterminacy (I(U)), and falsity (F(U))) for the open sets (U) used in the greenhouse temperature analysis example:

Table 5. Neutrosophic values for open sets.

Open Set (U)	Temperature Range (°C)	Degree of Truth (T(U))	Degree of Indeterminacy (I(U))	Degree of Falsity (F(U))
U_1	$18 \le T \le 25$	0.8	0.1	0.1
U_2	$22 \le T \le 28$	0.6	0.2	0.2

In our case, the neutrosophic interior of U_1 (18-25°C) might be a smaller temperature range within U_1 with an even higher degree of truth-value (T(U)) and lower indeterminacy (I(U)) and falsity (F(U)) compared to $T(U_1)$. This represents the core range within 18-25°C that is ideal for plant growth with the least uncertainty. Calculating the exact neutrosophic interior might involve specific definitions for a given NTS depending on the application.

The neutrosophic closure of U_1 (18-25°C) might include temperatures outside the 18-25°C range but with lower truth-values (T(U)) and higher indeterminacy (I(U)) and falsity (F(U)) compared to $T(U_1)$. This represents a wider range encompassing the possibility that some plants might tolerate slightly higher or lower temperatures with some uncertainty about their suitability. Calculating the exact neutrosophic closure depends on the specific definitions used in the NTS.

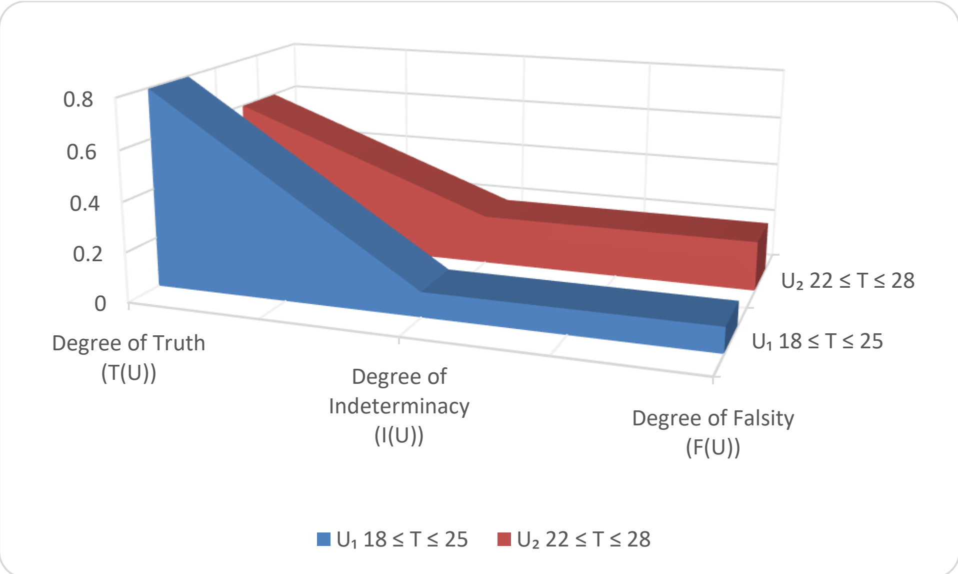

Figure 11. Neutrosophic values for open sets.

Figure 11 highlights that in an NTS, open sets are characterized by neutrosophic properties (T and I) in addition to their "openness." The visualization helps us understand how these neutrosophic values influence the classification of elements within open sets. Here, we see two open sets (U_1 and U_2) with different truth-value ranges, indicating varying degrees of certainty about element membership within these sets.

4.5 Properties and Applications in Computer Science

This section explores key properties of NTS that make them relevant for computer science applications:

Modeling Uncertainty in Functions: NTS allows the defining of continuous functions between neutrosophic topological spaces. This is valuable for situations where a function's behavior might be imprecise or have varying degrees of reliability.

Openness and Closeness with Uncertainty: Building on continuity, NTS enables defining neutrosophic open and closed sets. This allows us to represent the uncertainty associated with whether an element belongs to an open or closed set.

Fuzzy Neighborhoods: The concept of neighborhoods in topological spaces is extended to NTS. This allows us to model "fuzzy" neighborhoods where the degree of certainty about an element's proximity to another element can be incorporated.

Example:

Scenario: Imagine a system that analyzes customer reviews for a product. We can use an NTS to represent the concept of "positive sentiment" in reviews with uncertainty.

1. ***Continuity with Uncertainty:***

Consider a function f that maps review scores (numerical values) to a neutrosophic set representing sentiment. Here is how continuity with uncertainty can be demonstrated:

- Review Scores (X): X = {0, 1, 2, 3, 4, 5} (Possible scores for a review)
- Neutrosophic Set for Sentiment (T): We define a function f that maps scores to sentiment with uncertainty:
    - f(0) = {0.1, 0.8, 0.1} (Very low truth for positive sentiment, high indeterminacy, suggesting uncertainty)
    - f(1) = {0.2, 0.7, 0.1} (Low truth for positive sentiment, moderate indeterminacy)
    - f(2) = {0.4, 0.5, 0.1} (Moderate truth for positive sentiment, moderate indeterminacy)
    - f(3) = {0.6, 0.3, 0.1} (High truth for positive sentiment, low indeterminacy)
    - f(4) = {0.8, 0.1, 0.1} (Very high truth for positive sentiment, low indeterminacy)
    - f(5) = {0.9, 0.05, 0.05} (Extremely high truth for positive sentiment, very low indeterminacy)

Continuity Property: We can define continuity with uncertainty for this function f. This means that small changes in the review score (X) should result in small changes in the degrees of truth, indeterminacy, and falsity for the sentiment (T(f(x))).

For example, if a review score changes slightly from 3.5 to 4, the neutrosophic representation of sentiment (f(x)) should also change slightly, reflecting the gradual increase in positivity with the higher score. Analyzing customer reviews for a product using neutrosophic logic.

Table 6. Neutrosophic sentiment analysis of review scores.

Review Score (x)	Neutrosophic Sentiment (T(f(x)))	Degree of Truth (T)	Degree of Indeterminacy (I)	Degree of Falsity (F)	Interpretation
0	{0.1, 0.8, 0.1}	0.1	0.8	0.1	Very low truth for positive sentiment, high uncertainty.
1	{0.2, 0.7, 0.1}	0.2	0.7	0.1	Low truth for positive sentiment, moderate uncertainty.
2	{0.4, 0.5, 0.1}	0.4	0.5	0.1	Moderate truth for positive sentiment, moderate uncertainty.
3	{0.6, 0.3, 0.1}	0.6	0.3	0.1	High truth for positive sentiment, low uncertainty.

4	{0.8, 0.1, 0.1}	0.8	0.1	0.1	Very high truth for positive sentiment, low uncertainty.
5	{0.9, 0.05, 0.05}	0.9	0.05	0.05	Extremely high truth for positive sentiment, very low uncertainty.

Continuity Property:

The function f exhibits continuity with uncertainty. This means small changes in review scores (x) lead to gradual changes in the neutrosophic sentiment (T(f(x))). For example:

A slight increase from 3.5 to 4 in the review score would result in a neutrosophic sentiment with a higher degree of truth for positive sentiment (T) compared to f(3) but still lower than f(4). The indeterminacy (I) and falsity (F) might also change slightly, reflecting the nuance in sentiment with the higher score.

Table 6 demonstrates how neutrosophic logic allows us to represent the uncertainty associated with sentiment analysis. The degrees of truth, indeterminacy, and falsity capture the varying confidence levels in assigning positive sentiment to different review scores.

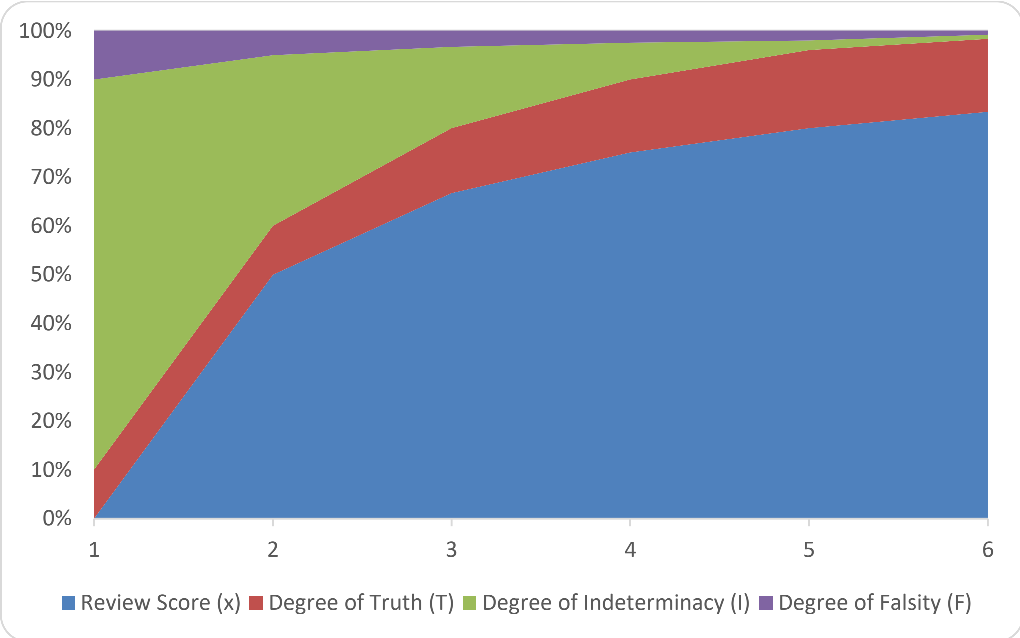

Figure 12. Neutrosophic sentiment analysis.

Figure 12 depicts sentiment analysis using neutrosophic sets. The sentiment is rated based on the degree of truth (T) and the degree of indeterminacy/falsity, providing a more nuanced representation than traditional sentiment analysis

2. ***Openness and Closeness with Uncertainty***

We can define neutrosophic versions of open and closed sets based on the sentiment neutrosophic set (T):

- Open Set (U): U = {f(x) | T(f(x)) >= 0.5 for all degrees (truth, indeterminacy, falsity)} This set represents reviews with a sentiment leaning towards positive (truth >= 0.5 for positive sentiment).
- Closed Set (C): C = {f(x) | T(f(x)) <= 0.5 for all degrees} This set represents reviews with a sentiment leaning towards negative (truth <= 0.5 for positive sentiment).

Example: Consider a review with a score of 3. Using the function f, we get f(3) = {0.6, 0.3, 0.1}. Since the truth-value (0.6) for positive sentiment is greater than 0.5, this review would be classified as belonging to the open set (U) of reviews with a positive sentiment leaning.

3. *Neighborhoods with Uncertainty*

Neighborhoods in traditional topological spaces define sets of elements "close" to a particular element. In NTS, we can extend this concept with uncertainty:

- Element (x): x = 3 (Review score)
- Neighborhood (N): N(x) = {f(y) ∣ y ∈ X, ∣y-x∣ <= 1} This neighborhood represents reviews with scores close to 3 (y values within 1 point of x).

The neutrosophic representation of sentiment for reviews in this neighborhood (N(x)) will also have varying degrees of truth, indeterminacy, and falsity. Reviews with scores closer to 3 (like 2 or 4) will have a higher degree of truth for positive sentiment compared to reviews further away (like 0 or 5).

This example demonstrates how NTS can incorporate uncertainty into properties like continuity, open/closed sets, and neighborhoods, allowing for a more nuanced representation of concepts in computer science applications.

Example:

Scenario: Analyzing customer reviews for a product using an NTS to represent "positive sentiment" with uncertainty.

a) Continuity Uncertainty

Table 7. Decoding review sentiment with neutrosophic sets.

Review Score (X)	Neutrosophic Sentiment (T(f(x)))	Truth (T)	Indeterminacy (I)	Falsity (F)
0	{0.1, 0.8, 0.1}	0.1	0.8	0.1
1	{0.2, 0.7, 0.1}	0.2	0.7	0.1
2	{0.4, 0.5, 0.1}	0.4	0.5	0.1
3	{0.6, 0.3, 0.1}	0.6	0.3	0.1
4	{0.8, 0.1, 0.1}	0.8	0.1	0.1
5	{0.9, 0.05, 0.05}	0.9	0.05	0.05

Explanation:

- Table 7 shows a function f that maps review scores (X) to neutrosophic sentiment (T(f(x))).
- Each neutrosophic sentiment value represents degrees of truth (T), indeterminacy (I), and falsity (F) for positive sentiment in the review.
- Continuity with uncertainty is demonstrated by the gradual increase in the truth value (T) for positive sentiment as the review score increases.

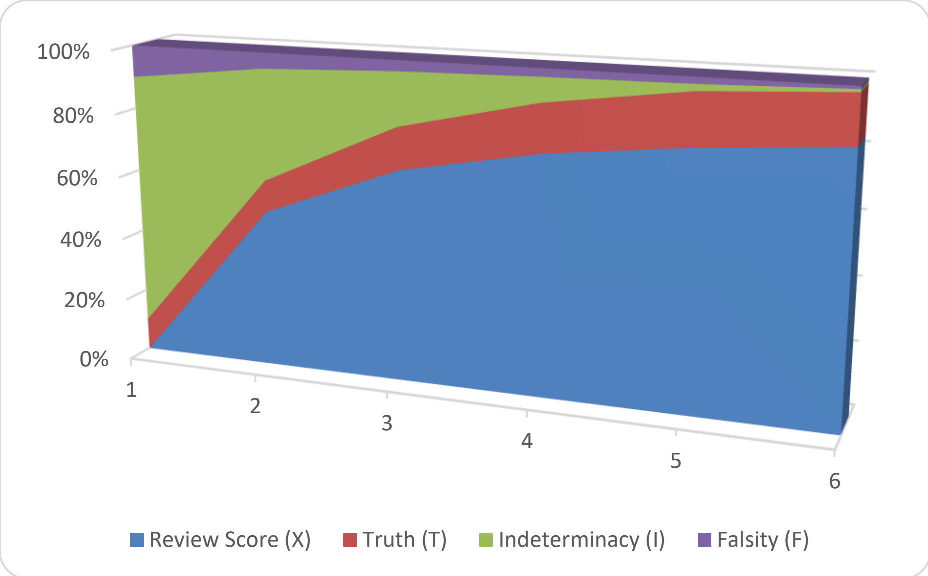

Figure 13. Continuity in sentiment analysis with neutrosophic sets.

Figure 13 illustrates how sentiment analysis with neutrosophic sets can capture the evolving nature and uncertainty associated with sentiment in text data. The curves on the graph would ideally show how the truth, indeterminacy, and falsity of sentiment classifications change over time or sentiment score.

b) Openness and Closeness with Uncertainty

We can define neutrosophic versions of open and closed sets based on the sentiment neutrosophic set (T):

Table 8. Neutrosophic sentiment classification of reviews.

Set	Description	Example (Review Score)	Neutrosophic Sentiment (T(f(x)))	Truth (T)	Indeterminacy (I)	Falsity (F)
Open Set (U)	Reviews with sentiment leaning positive	3	{0.6, 0.3, 0.1}	0.6 (>= 0.5)	0.3	0.1
Closed Set (C)	Reviews with sentiment leaning negative	1	{0.2, 0.7, 0.1}	0.2 (<= 0.5)	0.7	0.1

Explanation:

Table 8 defines open (U) and closed (C) sets based on the truth value (T) for positive sentiment.

An example review score is assigned to each set to illustrate its neutrosophic sentiment representation.

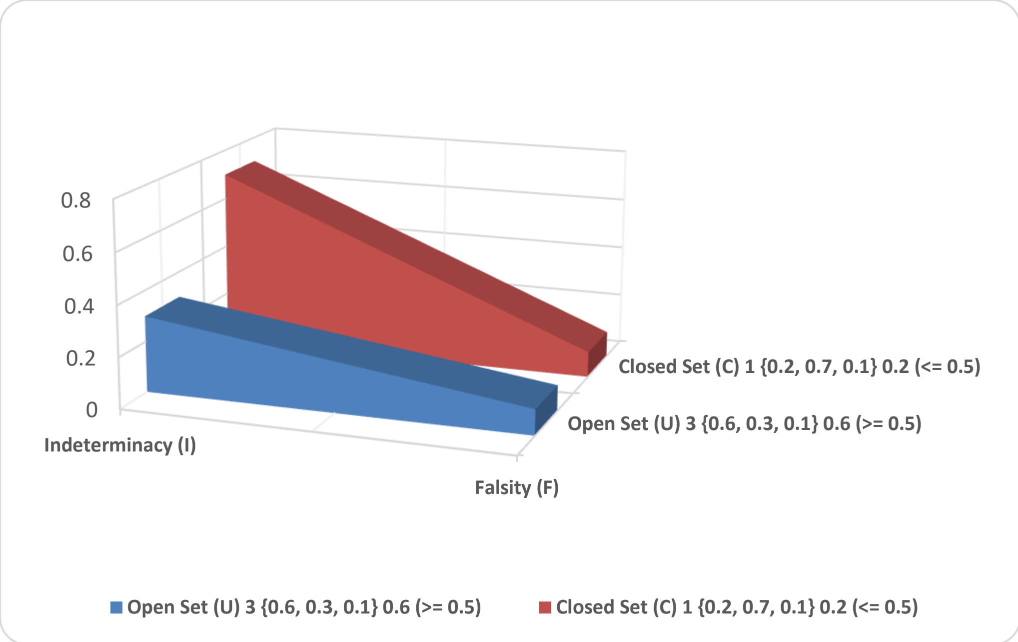

Figure 14. Distribution of review sentiment with uncertainty.

Figure 14 highlights the value of neutrosophic sets in sentiment analysis. It goes beyond simply classifying reviews as positive, neutral, or negative. By incorporating uncertainty, the graph provides a more nuanced understanding of the sentiment distribution, acknowledging that some reviews might be difficult to classify definitively.

c) Neighborhoods with Uncertainty

Table 9. Neutrosophic neighborhoods for review scores.

Element (x)	Review Score	Neighborhood (N(x))	Description
3	3	{f(2), f(3), f(4)}	Reviews with scores close to 3 (y values within 1 point of x=3)

Explanation:

- Table 9 defines a neighborhood (N(x)) for a specific review score (x).
- The neighborhood includes neutrosophic sentiment representations (f(y)) for reviews within the score range.
- These sentiment representations will have varying degrees of truth, indeterminacy, and falsity based on their proximity to the central score (x=3).

These tables display how properties and characteristics of NTS can be applied with numerical examples. The neutrosophic approach allows for a more nuanced representation of concepts like sentiment analysis in computer science applications.

5. Applications of NTS in Computer Science

5.1 Data Analysis: Modeling Uncertainty with Neutrosophic Topological Spaces

Data analysis often encounters challenges due to inherent uncertainty in the data itself. Neutrosophic topological spaces (NTS) offer a promising framework to address these challenges by explicitly representing and reasoning about uncertainty. This section explores how NTS can be applied in data analysis tasks, with specific examples:

- **Handling Noisy Sensor Data:** Sensor data can be prone to noise due to various factors like environmental conditions or limitations of the sensor itself. Traditional data analysis methods might treat this noise as outliers or errors. However, NTS can model the uncertainty associated with each data point. By assigning degrees of truth (T), indeterminacy (I), and falsity (F) to data points based on the level of noise, NTS can provide a more nuanced representation of the data. This allows for algorithms that are more robust to noise and can extract meaningful insights even from imperfect data.
    - **Example:** Imagine a sensor network monitoring temperature in a building. Some sensors might be malfunctioning or experiencing temporary interference. Using NTS, we can assign a lower degree of truth (T) and a higher degree of indeterminacy (I) to data points from these sensors. This allows the analysis to consider the data while acknowledging its potential unreliability.

Numerical Example: Handling Noisy Sensor Data with NTS

Scenario: A network of temperature sensors monitors a building. Sensor readings can be noisy due to malfunction or interference.

Traditional Approach:

- Sensor readings are treated as single, definitive values (e.g., 23°C).
- Noisy readings might be discarded as outliers or errors, potentially leading to inaccurate analysis.

NTS Approach:

i). **Define Neutrosophic Set (T):** We assign degrees of truth (T), indeterminacy (I), and falsity (F) to each sensor reading based on its potential noise level.
ii). **Sensor Readings (X):** Possible readings range from 18°C to 28°C.
iii). **Example Sensor Data:**

Table 10. Neutrosophic evaluation of sensor temperature readings.

Sensor ID	Reading (°C)	Truth (T)	Indeterminacy (I)	Falsity (F)	Reason for Uncertainty
S1	22.5	0.9	0.05	0.05	Stable sensor, likely accurate reading
S2	20.0	0.7	0.2	0.1	The sensor might be slightly cooler due to a draft
S3	27.8	0.5	0.3	0.2	Uncertain reading, possible sensor malfunction, or external heat source

Explanation:

- Sensor S1 has a high truth value (0.9) indicating a reliable reading.
- Sensor S2 has a lower truth value (0.7) due to potential coolness caused by a draft, increasing indeterminacy (0.2).
- Sensor S3 has the lowest truth value (0.5) due to high uncertainty, with high indeterminacy (0.3) and a possibility of being entirely false (0.2) due to malfunction or external heat.

Benefits of NTS:

- **Preserves Information:** All sensor readings are considered, even those with uncertainty.
- **Nuances Uncertainty:** Degrees of truth, indeterminacy, and falsity reflect the level of confidence in each reading.
- **Robust Algorithms:** Algorithms can be designed to handle uncertainty and extract meaningful insights from noisy data.

Example Usage:

An algorithm analyzing the building temperature might:

- Give more weight to readings with high truth values (e.g., S1) when calculating average temperature.
- Consider readings with lower truth values (e.g., S2) while acknowledging their potential inaccuracy.
- Identify sensors like S3 with very low truth for further investigation or potential repair.

This example demonstrates how NTS can be applied to handle noisy sensor data in a more nuanced and informative way compared to traditional approaches.

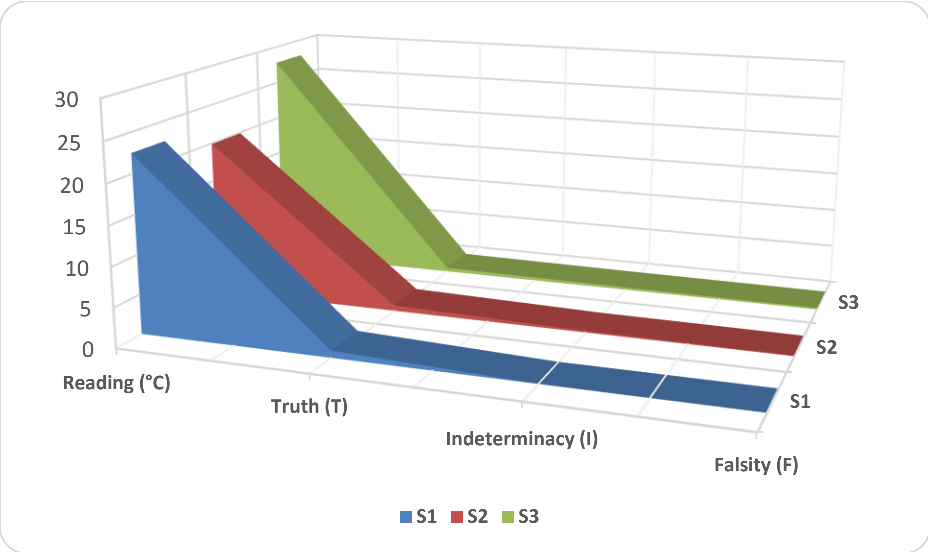

Figure 15. Neutrosophic evaluation of sensor temperature readings.

Figure 15 showcases how neutrosophic sets can be used to evaluate sensor data by incorporating uncertainty. It goes beyond just a single temperature value and reflects the confidence or uncertainty we have in each reading. By visualizing the truth, indeterminacy, and falsity values, the graph helps assess the reliability of sensor temperature readings across different temperature ranges.

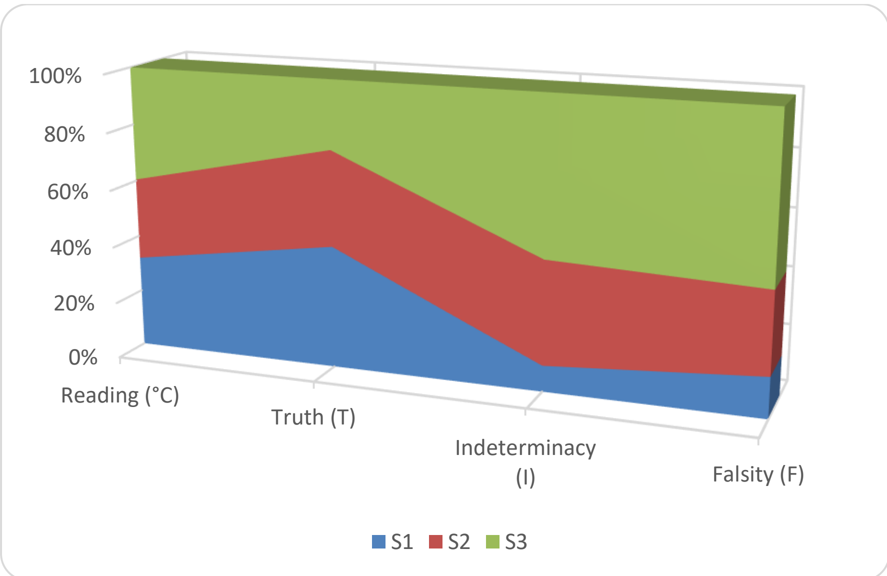

Figure 16. Neutrosophic temperature assessment with sensor uncertainty.

Figure 16 highlights the benefit of neutrosophic logic in sensor temperature assessment. By incorporating uncertainty (indeterminacy), it provides a more nuanced understanding of sensor readings than a simple temperature value. The visualization helps assess sensor reliability across different temperature ranges by considering the confidence level (truth) and uncertainty level in the readings.

- **Representing Missing Information:** Missing data is another common challenge in data analysis. Traditional methods might simply ignore missing values or impute them with average values. NTS can represent the uncertainty associated with missing information by assigning a high degree of indeterminacy (I) and low degrees of truth (T) and falsity (F). This allows algorithms to account for the missing data without introducing bias by assuming specific values.
    - **Example:** A medical dataset might have missing entries for specific patient attributes. Using NTS, these missing values can be represented with high indeterminacy, allowing statistical models to account for their absence while still analyzing the available data effectively.

Example: Representing Missing Information with NTS

Scenario: A medical dataset tracks blood pressure readings for patients. Some entries might have missing values for specific blood pressure readings (systolic or diastolic).

Traditional Approach:

- Missing values are often ignored, potentially leading to biased analysis with incomplete data.
- Alternatively, missing values might be imputed with average values, which can mask underlying variations.

NTS Approach:

i). **Define Neutrosophic Set (T):** We assign degrees of truth (T), indeterminacy (I), and falsity (F) to represent the uncertainty associated with missing data points.

ii). **Data Points (X):** Each data point represents a patient with systolic and diastolic blood pressure readings.

iii). **Example data with missing values:**

Table 11. Neutrosophic blood pressure readings for patients.

Patient ID	Systolic BP (mmHg)	Truth (T)	Indeterminacy (I)	Falsity (F)	Diastolic BP (mmHg)	Truth (T)	Indeterminacy (I)	Falsity (F)
P1	120	0.9	0.05	0.05	80	0.9	0.05	0.05
P2	-	0.1	0.8	0.1	75	0.9	0.05	0.05
P3	140	0.9	0.05	0.05	-	0.1	0.8	0.1

Explanation:

- Patient P1 has complete data with high truth values (0.9) for both readings.
- Patient P2 has a missing value for systolic BP represented by low truth (0.1), high indeterminacy (0.8), and low falsity (0.1), indicating the absence of information but not necessarily an abnormally low pressure.
- Patient P3 has a complete diastolic reading but a missing systolic BP with similar neutrosophic values as P2.

Benefits of NTS:

- **Preserves Data Integrity:** All available data points, including missing ones, are included in the analysis.

- **Explicit Uncertainty:** Missing information is explicitly represented by high indeterminacy, allowing for more robust statistical modeling.
- **Reduced Bias:** Algorithms avoid introducing bias by assuming specific values for missing data.

Example Usage:

Statistical models analyzing blood pressure can:

- Analyze the relationship between available data points (e.g., diastolic BP) and other patient attributes.
- Account for the uncertainty associated with missing values by incorporating the neutrosophic representation into the analysis.
- Identify patients with missing data points for further investigation or potential data collection efforts.

This example demonstrates how NTS can be a valuable tool for representing missing information in a medical dataset, leading to more robust and informative statistical analysis compared to traditional methods.

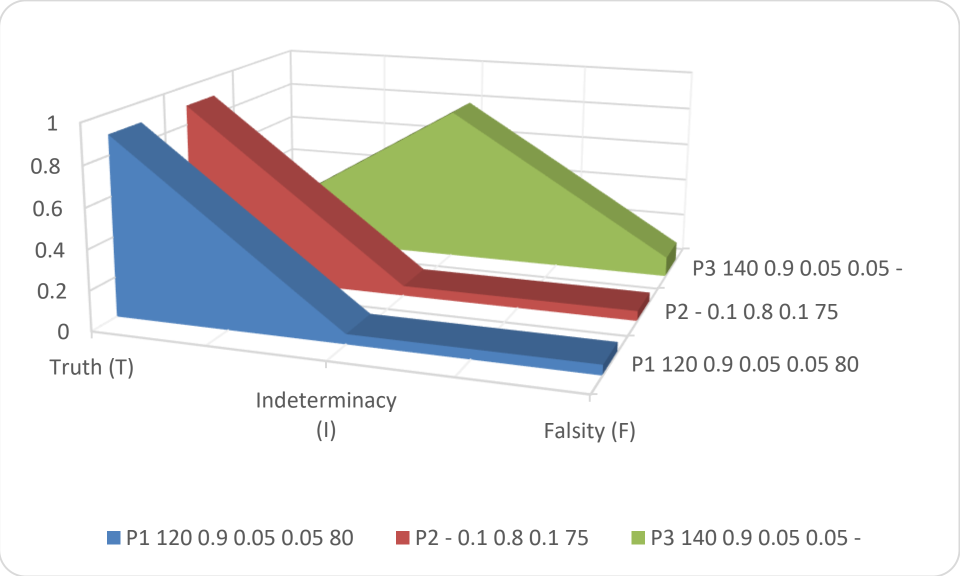

Figure 17. Neutrosophic blood pressure readings for patients

Figure 17 visualizes blood pressure readings for patients using neutrosophic sets. Neutrosophic sets allow the incorporation of uncertainty into the analysis, going beyond just a single numerical value for systolic and diastolic pressure.

- **Modeling Inherent Vagueness:** Some data is inherently vague or subjective. For example, sentiment analysis might involve classifying text as positive, negative, or neutral. However, the sentiment might not always be clear-cut. NTS can model this vagueness by assigning varying degrees of truth to different sentiment categories for a specific text sample.
    - **Example:** A social media post expressing frustration might not be entirely negative but might also contain some humor. Using NTS, the sentiment analysis algorithm could assign a moderate degree of truth to "negative" and a lower degree to "positive," capturing the nuanced sentiment of the post. These examples display how NTS can enhance data analysis by incorporating uncertainty into the modeling process. By representing the "fuzziness" of real-world data, NTS can lead to more robust and informative analysis results.

Example:

Scenario: Sentiment analysis of a social media post expressing frustration, but with a touch of humor.

Traditional Approach:

- Classifies the post as a single category (e.g., "negative" based on frustration).
- Might miss the subtle humor aspect, leading to an overly negative interpretation.

NTS Approach:

i). **Define Neutrosophic Set (T):** We assign degrees of truth (T) to different sentiment categories for the post.
ii). **Sentiment Categories (X):** Positive, Negative, Neutral
iii). **Example Social Media Post:** "This printer is driving me crazy! Maybe I should just write everything in pen and ink from now on. #firstworldproblems"
iv). **Sentiment Analysis with NTS:**

Table 12. Neutrosophic characterization of sentiment categories.

Sentiment Category	Neutrosophic Representation (T)	Truth (T)	Indeterminacy (I)	Falsity (F)
Positive	{0.3, 0.2, 0.5}	0.3 (moderate)	0.2	0.5 (high)
Negative	{0.6, 0.1, 0.3}	0.6 (high)	0.1	0.3
Neutral	{0.1, 0.7, 0.2}	0.1 (low)	0.7 (high)	0.2

Explanation:

- "Negative" has a high truth value (0.6) due to the frustration expressed.
- "Positive" has a moderate truth value (0.3) reflecting the underlying humor (emoji).
- "Neutral" has a low truth value (0.1) as the post does not strictly fall into either positive or negative.
- Both "Positive" and "Negative" have high falsity values (0.5 and 0.3) due to the presence of elements of both sentiments.
- High indeterminacy (0.7) for "Neutral" reflects the ambiguous nature of the overall sentiment.

Benefits of NTS:

- **Captures Nuance:** The model incorporates both frustration and humor, providing a more accurate representation of the post's sentiment.
- **Reduces Bias:** Avoids forcing the post into a single, potentially inaccurate category.
- **Flexibility:** The degrees of truth, indeterminacy, and falsity can be adjusted based on the specific context.

Example Usage:

- Social media sentiment analysis can consider both the negative and humorous aspects of the post for a more nuanced understanding of public opinion.
- Recommendation systems can utilize neutrosophic sentiment analysis to suggest content that aligns with the user's overall emotional response, not just a single category.

This example highlights how NTS can model the inherent vagueness of data by incorporating varying degrees of truth for different sentiment categories. This leads to a more comprehensive and informative analysis of subjective information like social media posts.

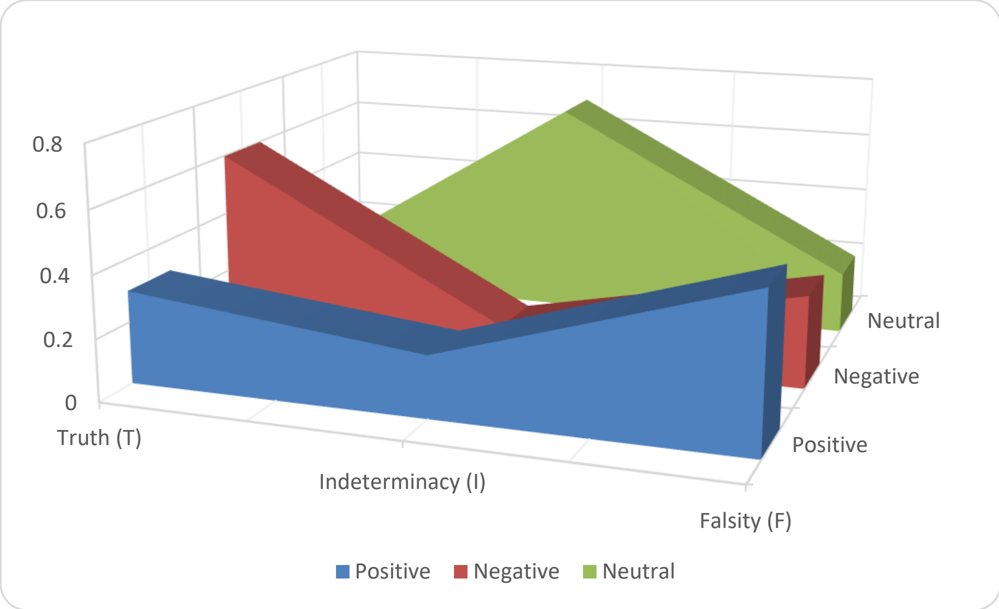

Figure 18. Sentiment analysis with neutrosophic sets: distribution of truth, indeterminacy, and falsity.

Figure 18 visualizes the distribution of sentiment analysis results using neutrosophic sets. Neutrosophic sets allow us to capture not just the positive or negative sentiment of opinions or reviews but also the level of uncertainty associated with that sentiment.

5.2 Pattern Recognition: Improving Accuracy with Neutrosophic Topological Spaces

Pattern recognition tasks, such as image classification, often face challenges due to ambiguity and noise in the data. Traditional methods rely on crisp feature extraction and classification algorithms, which might struggle with features that are imprecise or have varying degrees of reliability. NTS offers a novel approach by incorporating uncertainty into the feature representation, potentially leading to improved accuracy in pattern recognition.

Here is how NTS can be applied in pattern recognition:

- Modeling Uncertainty in Features: Image features like color, texture, or edges can be imprecise or ambiguous. For example, an image might have blurry edges or variations in color intensity due to lighting conditions. Traditional methods assign a single, definitive value to each feature. NTS, however, allows us to represent the uncertainty associated with each feature by assigning degrees of truth (T), indeterminacy (I), and falsity (F).
    - Example: Consider an image classification task that distinguishes between cats and dogs. A particular image might have a region with ambiguous features that could belong to either a cat or a dog. Using NTS, this region's "cat-like" feature can be assigned a moderate degree of truth (T) and a high degree of indeterminacy (I), reflecting the uncertainty.
- Robust Classification with Uncertainty Propagation: Pattern recognition algorithms typically make a definitive classification decision (e.g., "cat" or "dog"). NTS allows for propagating uncertainty through the classification process. By considering the neutrosophic representation of features, the algorithm can assign not only a most likely class but also degrees of truth, indeterminacy, and falsity for other possible classes.
    - Example: Building on the previous example, the image classifier might still classify the ambiguous region as "cat" with the highest T value. However, it can also report a non-negligible degree of truth (T) for the "dog" class due to the uncertainty in the features.

- Improved Handling of Ambiguous Cases: Traditional methods might struggle with borderline cases, where the features closely resemble multiple classes. NTS can provide a more nuanced representation of such ambiguity, allowing the classifier to assign higher degrees of truth to multiple possible classes, reflecting the difficulty in making a definitive decision.
    - Example: Imagine an image containing both a cat and a dog close, leading to features that might be a mix of "cat-like" and "dog-like." Using NTS, the classifier can assign moderate degrees of truth to both "cat" and "dog" classes, reflecting the challenge of distinguishing them based on ambiguous features.

By incorporating uncertainty into feature representation and classification, NTS can potentially improve the accuracy of pattern recognition tasks. This is achieved by:

- Accounting for inherent ambiguity in real-world data.
- Providing more informative classification results with uncertainty measures.
- Developing more robust algorithms that can handle borderline cases effectively.

However, it is important to acknowledge that the development and application of NTS in pattern recognition is an ongoing area of research. Further exploration is needed to establish efficient algorithms and practical implementations for utilizing NTS in this domain.

- Pattern Recognition Example with Neutrosophic Topological Spaces (NTS)

Scenario: Image classification for distinguishing between cats and dogs.

Traditional Approach:

- Image features (color, texture, edges) are represented by single, definitive values.
- Classification algorithms might misclassify images with ambiguous features.

Figure 19. Challenges in traditional image classification: ambiguous features lead to misclassification.

Figure 19 emphasizes that traditional image classification algorithms can be fooled by certain image features. By understanding these ambiguities, researchers can develop more robust algorithms that can handle the complexities of real-world images.

NTS Approach:

i). Neutrosophic Sets (T): We define neutrosophic sets to represent uncertainty in image features.

ii). Image Features (X): Features like "cat-like color," "dog-like texture," and "edge strength."

iii). Example Image: An image with a blurry region that could be part of a cat or a dog.

iv). Feature Representation with NTS:

Table 13. Neutrosophic characterization of image features with uncertainty.

Feature	Neutrosophic Representation (T)	Truth (T)	Indeterminacy (I)	Falsity (F)	Explanation
Cat-like Color	{0.6, 0.3, 0.1}	0.6 (moderate)	0.3	0.1 (low)	The color resembles a cat, but not entirely definitive
Dog-like Texture	{0.4, 0.4, 0.2}	0.4 (moderate)	0.4	0.2	The texture could belong to a dog, but also uncertain
Edge Strength	{0.8, 0.1, 0.1}	0.8 (high)	0.1	0.1 (low)	Edges are clear, but don't provide a strong class indication

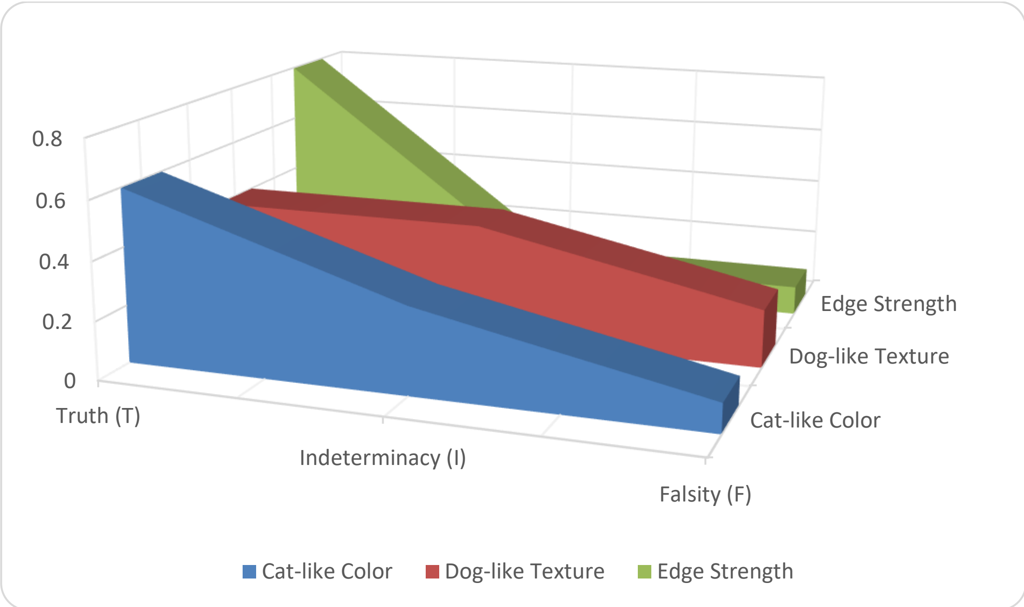

Figure 20. Neutrosophic representation of image features: color, texture, and edge strength.

Figure 20 highlights the benefit of neutrosophic sets in image representation. By incorporating uncertainty (indeterminacy) alongside truth values, it provides a more nuanced understanding of image features. This can be useful in image analysis tasks where there might be ambiguity or varying degrees of presence for these features within the image.

Figure 21. Neutrosophic feature analysis: cat-like color with uncertain dog-like texture.

Figure 21 highlights how neutrosophic sets can be used to analyze image features with uncertainty. In this case, the analysis confirms that the cat is mostly white (high Truth for white color) but avoids definitively stating the presence or absence of dog-like fur texture (low Truth, high Indeterminacy for dog-like texture). This nuanced approach reflects the complexity of image features and the potential for ambiguity in their analysis.

Classification with Uncertainty Propagation:

- The classifier analyzes the neutrosophic features.
- It assigns a class label (cat/dog) and propagates uncertainty through the process.

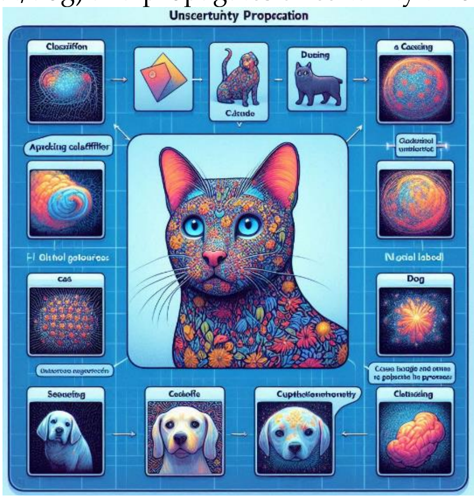

Figure 22. Classification with uncertainty propagation for cats and dogs.

Figure 22 displays the value of uncertainty propagation in image classification. By incorporating uncertainty (indeterminacy), the system avoids making absolute classifications (100% cat or 100% dog) and expresses its confidence level in the results.

Classification Result:

- Class: Cat (due to higher truth value for "cat-like color")
- Uncertainty:
  - Truth (T) for Dog: 0.3 (moderate) due to "dog-like texture"
  - Indeterminacy (I): 0.4 reflecting overall ambiguity in the region.

Figure 23. Neutrosophic classification: cat with dog-like texture.

Figure 23 highlights how neutrosophic sets can be used to refine image analysis beyond a single class label (cat). It allows for a more nuanced understanding of image regions by incorporating uncertainty about specific features (dog-like texture) in this case. This can be useful in situations where there are subtle variations within an image or where some features might be ambiguous.

Benefits of NTS:

- Uncertainty in Features: Captures the ambiguity of the blurry region.
- Informed Classification: Classifies as "cat" but acknowledges the possibility of being a dog.
- Handling Ambiguous Cases: Provides a more nuanced representation for difficult-to-classify regions.

NTS allows the classification algorithm to account for uncertainty in features, potentially leading to more accurate results, especially for borderline cases with ambiguous features.

Note: This is a simplified example. Real-world image classification might involve more complex features and higher dimensionality in the neutrosophic representation.

5.3 Information Retrieval: Enhancing Search with Neutrosophic Topological Spaces

Information retrieval (IR) systems aim to find relevant documents based on user queries. However, traditional IR methods often face challenges due to the inherent ambiguity and uncertainty in both user queries and document content. Neutrosophic topological spaces (NTS) offer a promising approach to address these limitations by incorporating uncertainty into the search process, potentially leading to enhanced search results.

Here is how NTS can be applied in information retrieval:

- **Modeling Uncertainty in User Queries:** User queries can be imprecise or ambiguous. For example, a query like "good restaurants" might have varying interpretations depending on the user's specific criteria (e.g., cuisine, price range, ambiance). Traditional IR systems treat queries as definitive statements. NTS, however, allows us to represent the uncertainty associated with a query by assigning degrees of truth (T), indeterminacy (I), and falsity (F) to different aspects of the query.
    - **Example:** A user searching for "healthy food options" might have a clear idea of "healthy" (high T) but be uncertain about specific dietary restrictions (moderate I). Using NTS, the search engine can consider both aspects while retrieving documents.
- **Neutrosophic Document Representation:** Documents themselves can also contain ambiguity or varying degrees of relevance to a query. Traditional IR systems assign a single relevance score to each document. NTS can be used to represent the uncertainty associated with a document's relevance by assigning degrees of truth (T), indeterminacy (I), and falsity (F) based on keyword presence, semantic similarity, or other factors.
    - **Example:** An article discussing vegetarian options might be highly relevant (high T) to the "healthy food options" query but might not address specific dietary needs. Using NTS, the document can be assigned a high T for general relevance and a moderate I for addressing specific dietary restrictions.
- **Uncertainty-Aware Ranking:** Traditional IR systems rank documents based on a single relevance score. NTS allows for ranking documents based on a neutrosophic representation of relevance. This ranking can consider not only the degree of truth (T) of a document being relevant but also the indeterminacy (I) associated with its relevance.
    - **Example:** The search engine might prioritize documents with a high T for general relevance to the "healthy food options" query but also consider documents with a moderate T and high I for addressing specific dietary needs, offering the user a broader range of potentially useful options.

By incorporating uncertainty into both queries and document representation, NTS can potentially improve information retrieval in several ways:

- **More nuanced search results:** Users receive results that reflect the ambiguity inherent in their queries.

- **Improved handling of borderline cases:** Documents that are somewhat relevant but might not perfectly match the query can still be surfaced for user consideration.
- **Reduced user frustration:** Users are less likely to encounter irrelevant results due to a more flexible matching process.

However, implementing NTS in IR systems presents challenges:

- **Developing efficient ranking algorithms:** Ranking algorithms need to be adapted to handle neutrosophic relevance scores effectively.
- **User interface design considerations:** Presenting uncertain information to users clearly and understandably is crucial.

Despite these challenges, NTS offers a promising approach to enhancing information retrieval systems by providing a more nuanced and flexible way to match user queries with relevant documents, even when those queries and documents contain inherent ambiguity and uncertainty.

Example:
Scenario: User searching for "healthy food options" with potential dietary restrictions.
Traditional IR Approach:

- The user query is treated definitively ("healthy food options").
- Search results might miss relevant documents due to a rigid matching process.

NTS Approach:

i). **Neutrosophic Sets (T):** Define sets to represent uncertainty in queries and documents.
ii). **Query Aspects (X):** Aspects like "healthy ingredients" and "dietary restrictions."
iii). **User Query with NTS:**

Table 14. Neutrosophic user preferences for recipe selection.

Aspect	Neutrosophic Representation (T)	Truth (T)	Indeterminacy (I)	Falsity (F)	Explanation
Healthy Ingredients	{0.8, 0.1, 0.1}	0.8 (high)	0.1	0.1 (low)	User prioritizes healthy ingredients
Dietary Restrictions	{0.7, 0.2, 0.1}	0.7 (moderate)	0.2	0.1 (low)	The user has some idea of restrictions but is unsure of the specifics

iv). **Document Representation with NTS:**

Table 15. Neutrosophic assessment of document relevance.

Document	Description	Relevance (T)	Indeterminacy (I)	Falsity (F)	Explanation
Doc 1	Article on vegetarian recipes	{0.7, 0.2, 0.1}	0.7 (high)	0.2	0.1 (low)
Doc 2	Blog post on gluten-free baking	{0.5, 0.3, 0.2}	0.5 (moderate)	0.3	0.2

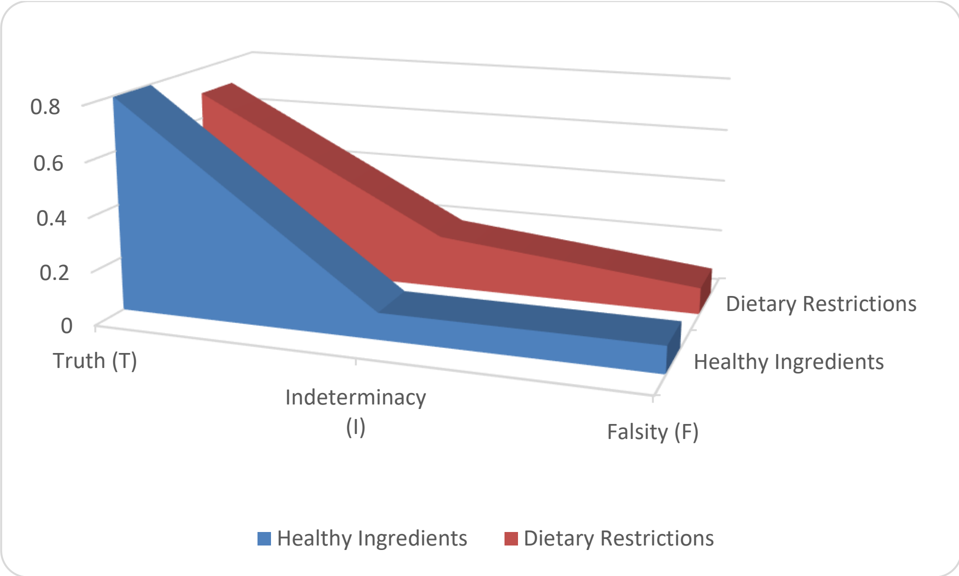

Figure 24. Neutrosophic representation of user preferences: healthy ingredients and dietary restrictions.

Figure 24 highlights the benefit of neutrosophic sets in representing user preferences for food. It goes beyond a simple classification of "healthy" or "unhealthy" and incorporates uncertainty. This can help develop recommender systems or personalized dietary plans that consider the nuances of user preferences and potential ambiguities in ingredient perception.

Uncertainty-Aware Ranking:
- Search engine ranks documents based on neutrosophic relevance scores.

Ranked Results:

i). Doc 1 (High relevance for general healthy options)
ii). Doc 2 (Moderate relevance, but addresses potential dietary restrictions)

Benefits of NTS:
- **Query Nuance:** Captures both the focus on healthy ingredients and the uncertainty about dietary needs.
- **Document Relevancy:** Identifies documents relevant to healthy options while considering potential dietary restrictions.
- **Improved Ranking:** Prioritizes highly relevant documents but also surfaces potentially useful options with some ambiguity.

Overall Improvement:

NTS allows the search engine to consider the uncertainty in both the user query and document content. This can lead to more nuanced search results that better reflect the user's intent, even when their query is not entirely specific.

6. Challenges of Implementing and Utilizing NTS in Computer Science

- Computational Complexity: Operations on neutrosophic sets can be computationally expensive, impacting algorithm efficiency, especially for large datasets.
- Limited Algorithms: Compared to traditional spaces, there's a lack of established algorithms for working with NTS.
- Theoretical Considerations: Further theoretical development is needed to fully understand the mathematical properties and limitations of NTS.

Future Research Directions

- Efficient Algorithms: Develop efficient algorithms and alternative representations for handling neutrosophic set operations in large-scale applications.
- User Interface Design: Design intuitive user interfaces to ensure user comprehension and interaction with uncertain information represented by neutrosophic sets.
- Integration with Frameworks: Integrate NTS with existing frameworks for data analysis tasks like machine learning and data mining.

7. Conclusion

Neutrosophic Topological Spaces (NTS) offer a promising framework for modeling uncertainty in computer science. NTS extends traditional topological spaces by capturing the degree of certainty associated with open sets. This opens doors for applications in data analysis, pattern recognition, information retrieval, and human-computer interaction.

Challenges include computational complexity, lack of established algorithms, and theoretical considerations. Future research should focus on efficient algorithms, exploring theoretical properties, and user interface design for uncertainty representation.

Overall, NTS has the potential to revolutionize how computers handle imprecise information. Their broader impact extends beyond computer science, potentially influencing engineering, decision-making, and artificial intelligence.

Ethics approval and consent to participate

The results and data in this manuscript have not been published elsewhere, nor are they under consideration by another publisher. All the material is owned by the authors, and/or no permissions are required.

This study includes images generated using artificial intelligence software (ChatGPT4). Please note that these images were produced for illustrative purposes and do not represent real experimental data. It shows what the AI model generated, which may not fully reflect the actual results of our research. For accurate and complete information, we encourage readers to refer to the original data sources and the methods used.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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Conflict of interest

The authors declare that there is no conflict of interest in the research.

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