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Neutrosophic Approach to Water Quality Assessment: A Case Study of Gomati River, the Largest River in Tripura, India

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Abstract: This study addresses the complexity of assessing river water quality, a multifaceted process influenced by numerous water quality parameters (WQPs) characterized by inherent uncertainties and diverse judgment information from decision-makers. These uncertainties and diverse judgment information can be effectively represented and simulated using Neutrosophic sets. In this study, we propose an effective water pollution rating system, the Neutrosophic water quality index (NWQI), to derive a water pollution score (NWQI-score) for rating water pollution levels. We demonstrate the application of our methodology through an assessment of water quality indices for rating pollution in the Gomati River, the largest river in Tripura, India. Using the NWQI, we highlight the NWQI score as a pivotal indicator for evaluating the river's water quality. For assessing water quality, we consider six crucial parameters, namely Total hardness, Biochemical oxygen demand, Total suspended solids, Electrical conductivity, pH, and total dissolved solids, sampled at five strategic sites along the river. Sampling was conducted from January 2024 to July 2024, guided by the distribution of waste discharge points. Finally, we conclude the study, summarizing findings and suggesting future research directions.

Keywords: Neutrosophic Set; Decision-making; Water Quality Index; River; Water Pollution.

1. Introduction

Water quality assessment is a critical environmental concern, particularly in regions with significant anthropogenic activities. Traditional methods for water quality evaluation often need to address the inherent uncertainties and complexities involved in environmental data. To overcome these limitations, this paper employs a Neutrosophic Water Quality Index (NWQI), leveraging the concepts of neutrosophic logic and sets. Neutrosophic logic, introduced by Smarandache [1], extends classical logic by incorporating the degrees of truth, indeterminacy, and falsity, providing a more nuanced approach to handling imprecise information. For any subset X of S, a neutrosophic set X on S is of the form: $X = \{(\alpha, T_X(\alpha), I_X(\alpha), F_X(\alpha)) \mid \alpha \in S\}$ where $T_X, I_X, F_X: S \to [0,1]$ represents the truth, indeterminacy, and falsity membership functions respectively. These functions must satisfy: $0 \leq T_X(\alpha) + I_X(\alpha) + F_X(\alpha) \leq 3$, for all $\alpha \in S$.

Neutrosophic sets generalize several other fuzzy set theories, such as intuitionistic fuzzy sets and Pythagorean fuzzy sets, offering a comprehensive framework for dealing with uncertainty and incomplete information [2]. The utility of neutrosophic theory has been demonstrated in various fields, as explored in the edited volumes by Smarandache and Pramanik. These works provide comprehensive insights into the latest developments and applications of neutrosophic theory, highlighting its versatility and effectiveness. The first volume, "New Trends in Neutrosophic Theory and Applications" [3], encompasses a wide range of theoretical advancements and practical implementations across different disciplines, demonstrating how neutrosophic sets and logic can address complex problems characterized by indeterminacy and uncertainty. The second volume, "New Trends in Neutrosophic Theory and Applications, Vol. 2" [4], continues this exploration by presenting further advancements and case studies, showcasing the expanding influence and utility of neutrosophic theory in solving real-world issues across various sectors. A comprehensive overview of neutrosophic sets and their applications is provided by Broumi et al. [5], emphasizing the versatility and robustness of this approach in handling complex decision-making problems. Otay and Kahraman [6] further underscore the relevance of neutrosophic sets in multi-criteria decision-making, illustrating their state-of-the-art applications in various domains. A bibliometric analysis by Peng and Dai [7] reveals the growing interest and extensive research in neutrosophic sets over the past two decades, reflecting their increasing importance in scientific literature. The introduction of rough neutrosophic sets, as discussed by Pramanik [8], adds another layer of depth to the theory, enabling a more refined analysis of data with inherent vagueness. Recent advancements, such as the weighted neutrosophic soft multiset by Granados, Das, and Osu [9], demonstrate the practical applications of neutrosophic sets in decision-making processes, further validating their effectiveness in real-world scenarios. Collectively, all these research efforts underscore the growing significance and versatility of neutrosophic sets in addressing complex decision-making challenges across diverse domains, but their effectiveness in evaluating water quality for pollution rating has been limited.

On the other hand, several models of water quality index (WQI) are available to assess the consistency of surface water, with the WQI being a prominent model for evaluating river water quality, known for its accuracy and utility. Brown et al. [10] proposed a WQI as a comprehensive measure for assessing water quality. Their work aimed to provide a unified approach for evaluating water quality based on multiple parameters. Several researchers, regulatory bodies, international organizations, and scientific communities have proposed diverse equations for computing the WQI. The Indian Council of Medical Research (ICMR) [11] published the "Manual of Standards of Quality for Drinking Water Supplies," highlighting essential guidelines for ensuring the quality and safety of drinking water in India. This report serves as a foundational resource for regulatory bodies and stakeholders involved in water supply management, emphasizing the critical importance of maintaining high standards to safeguard public health. Furthermore, the 21st Edition of the "Standard Methods for the Examination of Water and Wastewater" published by the American Public Health Association (APHA) [13] serves as a comprehensive guide for analyzing water quality and waste management. Its wide adoption in the field underscores its importance as a benchmark for water testing protocols and methodologies. The rigorous standards outlined in this Edition contribute significantly to ensuring public health and environmental sustainability in water management practices. The Central Pollution Control Board (CPCB) [14] in Delhi issued the "Guidelines for Water Quality Management" as a framework for managing and maintaining water quality standards in India, addressing various aspects such as pollution control, monitoring, and sustainable management of water resources to protect human health and the environment. The WHO Guidelines for Drinking Water Quality, 4th Edition, published in Geneva, Switzerland, by the World Health Organization [15], provide comprehensive standards and recommendations for ensuring safe drinking water globally. These guidelines encompass parameters such as microbial, chemical, and radiological aspects to safeguard public health. Similarly, the Bureau of Indian Standards (BIS) [16] has laid down the specifications for drinking water quality in India through IS 10500:2012. These standards outline various parameters, such as permissible limits for physical, chemical, and microbiological contaminants, to ensure the safety and potability of drinking water across the country. These collaborative efforts of regulatory bodies, international organizations, scientific communities, and public participation are crucial for ensuring the long-term quality and sustainability of drinking water and aquatic ecosystems worldwide. Such comprehensive and coordinated efforts are necessary to safeguard public health, protect the environment, and ensure water resources are available for future generations. Furthermore, Kumar et al. [17] investigated the impact of climate change on water resources in the upper Kharun catchment of Chhattisgarh, India. Guettaf et al. [18] conducted a comprehensive assessment of the Seybouse River's water quality in Northeast Algeria, shedding light on key parameters and pollution sources, thereby aiding in the region's management and conservation efforts. Miller & Hutchins [19] provided an extensive review of the impacts of urbanization and climate change on urban flooding and water quality in the UK, synthesizing existing knowledge to identify trends and challenges. Shah and Joshi [20] evaluated the WQI for the Sabarmati River in Gujarat, India, offering valuable insights into pollution levels and environmental impacts. Hutchins et al. [21] assessed the combined impacts of future land-use changes and climate stressors on water resources and quality in groundwater and surface water bodies within the upper Thames River basin, UK. Diamantini et al. [22] investigated water quality trends in European river basins, focusing on identifying driving factors influencing water quality dynamics. Rao et al. [23] conducted a water quality assessment and trend analysis for the Min River sea-entry section in China. Their study contributes valuable insights into the variations and quality of water in this critical area, aiding in the formulation of effective management strategies for maintaining water quality and ecological balance. Patel et al. [24] assessed the impact of urbanization and industrialization on the Sabarmati River in Ahmedabad using WQI, stressing the importance of water quality monitoring in urban areas. Tripathi et al. [25] investigated the short-term dynamics of water quality parameters in the Ganga River during low-flow conditions, finding correlations with weather variations. Their findings offer valuable insights into the environmental dynamics affecting water quality in this crucial river system. Ahmed et al. [26] analyzed WQI and machine learning methods to assess pollution in the Rawal Dam, Pakistan, providing insights into effective water quality evaluation methods. Patel and Chitnis [27] applied fuzzy logic in their study of river water quality modeling to examine the impacts of industrialization and climate change on the Sabarmati River, aiding in sustainable water management.

All these research efforts underscore the growing significance and versatility of WQI in addressing water pollution levels worldwide and in assessing water quality in surface and groundwater. While the WQI has proven invaluable in assessing water pollution globally, it has limitations. In this study, we apply the neutrosophic approach to evaluate the water quality of the Gomati River, utilizing a multi-criteria decision-making model that incorporates various water quality parameters. By converting these parameters into neutrosophic membership values and applying relative weights, we derive NWQI scores that provide a comprehensive assessment of the river's health. This innovative approach highlights the critical need for immediate intervention to improve water quality, ensuring the well-being of communities relying on the Gomati River. The structure of this research work is as follows: Section 2 introduces the Weighted Aggregate Truth Value (WATV), Weighted Aggregate Indeterminacy Value (WAIV), and Weighted Aggregate Falsity Value (WAFV) to obtain the NWQI-score. Utilizing these new concepts, we propose an effective water pollution rating system, the NWQI, for rating water pollution. In Section 3, we assess the water pollution levels in the Gomati River, located in India, using the NWQI. Through comprehensive analysis and application of the NWQI, we aim to provide valuable insights into the extent and sources of water pollution in the Gomati River basin. Finally, Section 4 concludes the study, summarizing the findings and suggesting future research directions.

2. Materials and Methods

In this section, we assess water pollution levels in the Gomati River, located in Tripura, utilizing a NWQI. Through comprehensive analysis and application of the NWQI, we aim to provide valuable insights into the extent and sources of water pollution in the Gomati River basin. This assessment enables informed decision-making and the implementation of effective mitigation strategies to safeguard water quality and ecosystem health.

2.1 Study Area and Data

We demonstrate the application of our methodology through an assessment of water quality indices for rating pollution in the Gomati River, situated in Tripura, India. Notably, the Gomati River is the largest and most significant river in Tripura, India. It originates from the confluence of two rivulets, Raima and Sarma, with Raima originating from the Longtarai Hill Range and Sarma from the Atharamura Hill Range. These two rivulets merge to form the Gomati River, which then traverses westward through the Sonamura, Udaipur, and Amarpur subdivisions, almost crossing the center of Tripura before reaching the state's western border. The Gomati River boasts the largest basin among all rivers in Tripura, covering 2,492 km² within the Indian Union. The basin spans from $91^0 15'$E to $92^0 00'$E longitude and $23^0 15'$N to $23^0 45' 5''$N latitude. Of this, 22.9% (571 km²) lies in the plains, while 77.1% (1,921 km²) is in the hill catchment. The majority of the total catchment area, 2,360 km², is situated in the Udaipur, Amarpur, and Gandachara sub-divisions of the Gomati and Dhalai districts, with the remaining 132 km² in the Sepahijala district's Sonamura sub-division. The annual water flow of the Gomati River is estimated at 249.39 million cubic meters, making it a crucial water source for the local population. It serves as the primary source for drinking water, agricultural activities, fishing, cattle rearing, and inland water transportation, meeting the diverse needs of residents. Six sampling sites along the river, from Udaipur (23.537583^0N, 91.505103^0E) to Srimantapur (Downstream to Bangladesh) (23.460685^0N, 91.260328^0E), were meticulously chosen for water sampling, guided by the distribution of point sources of waste discharge as illustrated in Figures 1, 2 and Table 1.

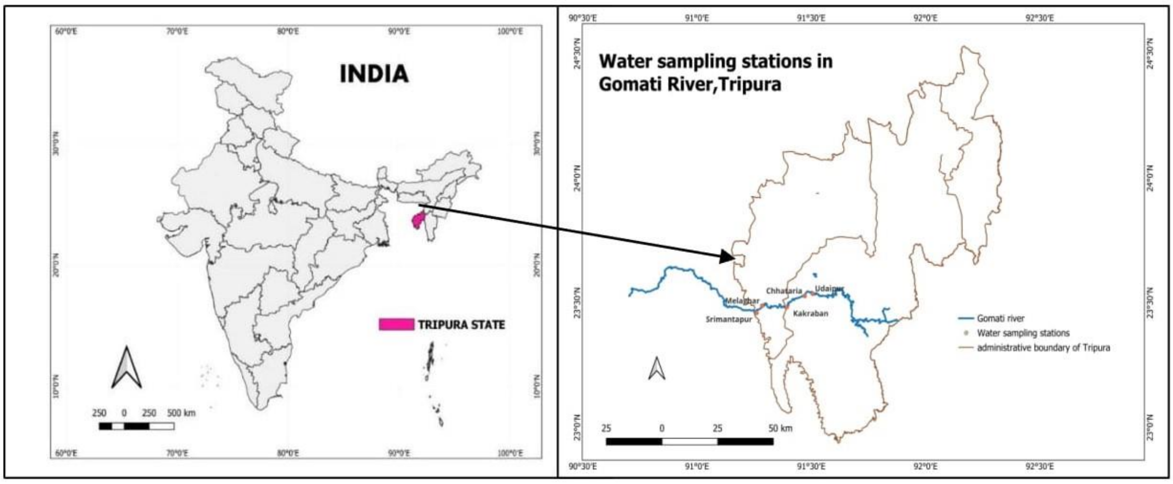

Figure 1. Study area.

Table 1. Sampling stations (S).

S	Stations	State/ Country	Location	Latitude	Longitude
α_1	Udaipur	Tripura, India	Bathing ghat, near Jogesh Chakrabarti bridge	23.537583^0N	91.505103^0E
α_2	Chhataria	Tripura, India	Bathing ghat in Jamjuri	23.528285^0 N	91.470873^0 E
α_3	Kakraban	Tripura, India	Bathing ghat, near Kakraban bridge,	23.483749^0 N	91.393142^0 E
α_4	Melaghar	Tripura, India	Near Madhuban hanging bridge	23.488407^0 N	91.284738^0 E
α_5	Srimantapur	Tripura, India	Bathing ghat, Downstream to Bangladesh	23.460685^0 N	91.260328^0 E

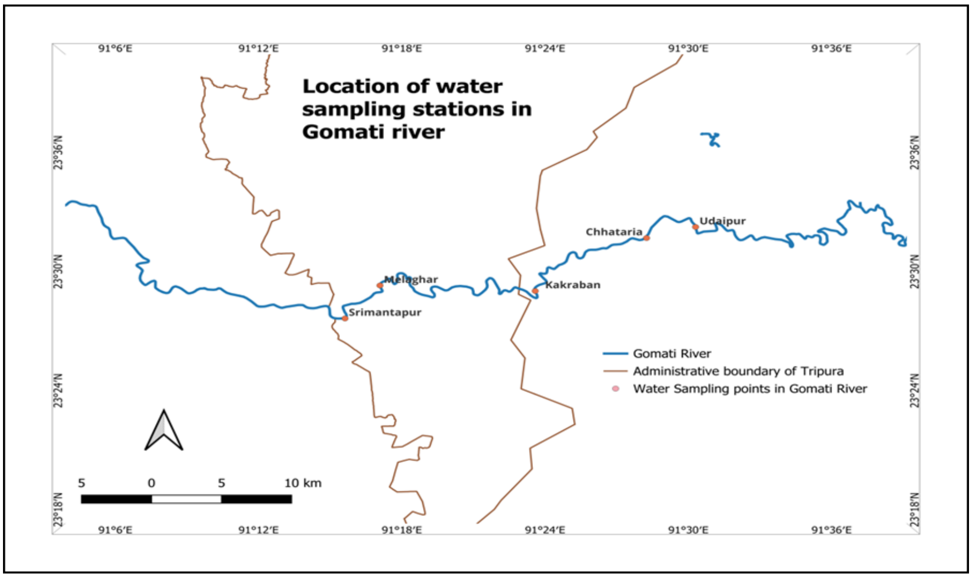

Figure 2. Sampling stations.

2.2 *Data Collection*

Water samples were collected using a Taiwan Garmin eTrex Vista Cx GPS device to record coordinates (longitude and latitude) at each sampling station. From January 2024 to July 2024, samples were taken in triplicate from each location. Thorough mixing was ensured to accurately represent the river's quality, with careful consideration of distances between sampling sites for optimal blending of discharged waste with river water. Sampling was done in 1.5-liter polypropylene bottles, washed beforehand with 10% HCl in a lab, and then rinsed with sample water from each site during collection. In this study, a total of six important WQPs were selected, namely pH (β_1), Total dissolved solid (β_2), Total suspended solid (β_3), Electrical conductivity (β_4), Biochemical oxygen demand (β_5), Total hardness (β_6), were meticulously measured at each site. The sampling and analysis followed standard methods from APHA [13]. Mean data summaries of water quality parameters (WQPs) for five selected locations from January 2024 to July 2024 are presented in Table 2. These values were compared with standards from ICMR [11], CPCB [14], WHO [15], and BIS [16] as detailed in Table 3.

Table 2. Crisp data summaries of WQPs.

S	β_1	β_2	β_3	β_4	β_5	β_6
α_1	7.28	110.05	87.65	100.50	4.3	75.25
α_2	7.33	118.75	84.65	104.45	4.6	57.85
α_3	7.43	123.55	78.45	116.67	4.4	70.25
α_4	7.56	128.35	88.55	125.35	4.9	80.20
α_5	7.59	136.85	97.45	131.25	5.3	55.75

Table 3. Standard and ideal values of the WQPs.

Parameters	**Parameter name**	**Standard value**	**Ideal value**	**Recommending agencies**
β_1	pH	6.5-8.5	7	ICMR [11]/BIS [16]
β_2	Total Dissolved Solids (mg/l)	500	0	ICMR [11]/BIS [16]
β_3	Total Suspended Solids (mg/l)	500	0	WHO [15]/CPCB [14]
β_4	Electrical Conductivity (mho/cm)	300	0	ICMR [11]
β_5	Biological Oxygen Demand (mg/l)	5.00	0	ICMR [11]
β_6	Total Hardness(mg/l)	300	0	ICMR [11]/BIS [16]

2.3 Neutrosophic Water Quality Index

In this section, we present the ideas of weighted aggregate truth value (WATV), weighted aggregate indeterminacy value (WAIV), and weighted aggregate falsity value (WAFV). Using these innovative concepts, we introduce the NWQI as an effective water pollution rating system. This NWQI facilitates the derivation of a water pollution score (NWQI-score) to evaluate the level of water pollution. This system provides a robust framework for evaluating and addressing environmental concerns about water quality.

Let us consider $S = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ be a set of sampling stations and $P = \{\beta_1, \beta_2, \beta_3, \dots, \beta_m\}$ be a set of WQPs. For each sampling station α_t of S, we consider a corresponding neutrosophic set $\boldsymbol{N_{\alpha_t}}$ on P as $N_{\alpha_t} = \left\{ \left(\beta_k, T_{\alpha_t}(\beta_k), I_{\alpha_t}(\beta_k), F_{\alpha_t}(\beta_k)\right) \mid \beta_k \in P \right\}$, where $T_{\alpha_t}, I_{\alpha_t}, F_{\alpha_t}: P \to [0,1]$ represents the truth, indeterminacy, and falsity membership functions respectively, $0 \leq T_{\alpha_t}(\beta_k) + I_{\alpha_t}(\beta_k) + F_{\alpha_t}(\beta_k) \leq 3$, for all $\beta_k \in P$, and let ϖ_k be the weight for each parameter β_k in set P, where $\varpi: P \to [0,1]$ is a weight function, such that $\varpi_k = \varpi(\beta_k)$. Then, the weighted aggregate truth value (WATV), weighted aggregate indeterminacy value (WAIV), and weighted aggregate falsity value (WAFV) for each station α_t in S are as follows:

$$WATV(\alpha_t) = \frac{\sum_{k=1}^{m}\left[\varpi(\beta_k) \times T_{(\alpha_t)}(\beta_k)\right]}{\sum_{k=1}^{m}\varpi(\beta_k)}, \beta_k \in P.$$

$$WAIV(\alpha_t) = \frac{\sum_{k=1}^{m}\left[\varpi(\beta_k) \times I_{(\alpha_t)}(\beta_k)\right]}{\sum_{k=1}^{m}\varpi(\beta_k)}, \beta_k \in P$$

$$WAFV(\alpha_t) = \frac{\sum_{k=1}^{m}\left[\varpi(\beta_k) \times F_{(\alpha_t)}(\beta_k)\right]}{\sum_{k=1}^{m}\varpi(\beta_k)}, \beta_k \in P$$

Then, the NWQI-score for each station α_t in S is defined as

$$NWQI(\alpha_t) = \frac{WATV(\alpha_t) - \mathrm{WAFV}(\alpha_t)}{1 + \mathrm{WAIV}(\alpha_t)}.$$

2.4 Algorithm (NWQI)

Step 1. Enter a set of sampling stations $S = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ and a WQP set $P = \{\beta_1, \beta_2, \beta_3, \dots, \beta_m\}$.
Step 2. Enter the resulting crisp data summaries of WQPs for all the selected sampling stations as provided by a group of experts.
Step 3. Enter standard and ideal values of the WQPs according to any recommending agencies.
Step 4. Based on the standard and ideal values of the WQPs provided in step 3, define neutrosophic membership functions for each WQP β_k in P.
Step 5. After defining neutrosophic membership functions, convert the crisp data summaries into neutrosophic membership values and present them in tabular form.
Step 6. Enter weight ϖ_k for each WQP β_k in set P, where $\varpi: P \to [0,1]$, such that $\varpi_k = \varpi(\beta_k), k = 1,2,3, \dots, m$.
Step 7. Calculate $WATV(\alpha_t), WAIV(\alpha_t)$, and $WAFV(\alpha_t)$ for each station α_t in S.
Step 8. Compute the NWQI-score for each station α_t in S using the formula

$$NWQI(\alpha_t) = \frac{WATV(\alpha_t) - \mathrm{WAFV}(\alpha_t)}{1 + \mathrm{WAIV}(\alpha_t)}.$$

Step 9. If NWQI-score $NWQI(\alpha_t)$ nears +1, it signifies excellent water quality while nearing -1 indicates poor quality.

3. Assessment of Water Pollution of Gomati River

In this section, we assess water pollution levels in the Gomati River, located in India, utilizing an NWQI. Through comprehensive analysis and application of the NWQI, we aim to provide valuable insights into the extent and sources of water pollution in the Gomati River basin. The NWQI score is a crucial indicator for assessing Gomati River's water quality. If NWQI-score nearing +1 signifies excellent water quality, while nearing -1 indicates poor quality.

Step 1. Let $S = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ be the set of sample stations along the Gomati River as in Table 1. We consider the set P of water quality parameters as follows:

$P = \{\beta_1 = pH,\ \beta_2 = \text{Total dissolved solid},\ \beta_3 = \text{Total suspended solid},\ \beta_4 = \text{Electrical conductivity},$
$\beta_5 = \text{Biochemical oxygen demand},\ \beta_6 = \text{Total hardness}\}.$

Step 2. We consider the resulting crisp data summaries of WQPs for all five selected locations, as presented in Table 2.

Step 3. We consider the standard and ideal values of the WQPs as in Table 3.

Step 4. Based on the standard and ideal values of the WQPs provided in Table 3, we define neutrosophic membership functions (Truth, Indeterminacy, and Falsity) for different WQPs as in Tables 4, 5, and 6.

Step 5. After defining neutrosophic membership functions for different water quality parameters, the mean crisp data summaries of WQPs are converted into neutrosophic membership values (Truth, Indeterminacy, and Falsity) as in Tables 7, 8, and 9, respectively.

Step 6. The relative weight ϖ_i for each WQP β_i in set P is obtained using the equation $\varpi_i = \frac{1}{S_i}$ where S_i the recommended standard value of i[th] WQP $\beta_i \in P$ (for pH (β_1) the value of S_1 is considered as 8.5). Based on the provided formula, we have determined the relative weights assigned to the WQPs in set P are as follows:

$$\varpi = \{\varpi_1 = \varpi(\beta_1) = 0.118;\ \varpi_2 = \varpi(\beta_2) = 0.002;\ \varpi_3 = \varpi(\beta_3) = 0.002;\ \varpi_4 = \varpi(\beta_4) = 0.003;$$
$$\varpi_5 = \varpi(\beta_5) = 0.2;\ \varpi_6 = \varpi(\beta_6) = 0.003\}.$$

Step 7. Based on the neutrosophic membership values as in Tables 7, 8, and 9 and relative weight ϖ as in step 7, we obtain the values of $WATV(\alpha_t), WAIV(\alpha_t)$, and $WAFV(\alpha_t)$ for each station α_t in S as in Table 10.

Step 8. From the last column of Table 10, we can see that the NWQI-score for the quality of the Gomati River's water at the stations Udaipur, Chhataria, Kakraban, Melaghar, Srimantapur were -0.12383, -0.17961, -0.17961, -0.27685, and -0.29545 respectively. Also, it is observed that the NWQI-scores (ranging from -0.29545 to -0.12383) for the water quality of the River Gomati at all sampling stations fall under the poor category, primarily due to the addition of wastewater containing higher levels of Biological Oxygen Demand (BOD).

Also, it is noted that Station Kakraban, specifically the Bathing ghat, near Kakraban bridge (23.4837490^{0}N, 91.3931420^{0}E), has an NWQI-score of -0.12383, which is higher than others, indicating better water quality at this location. Our algorithm suggests that the water quality at Station Srimantapur (Bathing ghat) downstream to Bangladesh (23.4606850^{0}N, 91.2603280^{0}E) is the most polluted compared to others. The ranking order of the sampling stations calculated from our proposed model is shown in Table 10. Through rigorous analysis of multiple parameters, we have determined that the discharge of domestic wastewater from neighboring localities has significantly impacted the river's health. After analyzing various parameters, it is evident that the quality of the Gomati River's water is unsuitable for consumption and other domestic uses. Immediate action is imperative to safeguard the health and well-being of communities reliant on the Gomati River for their water needs.

Table 4. Truth membership functions for different WQPs.

Parameters	Parameter Name	Truth Membership Functions
$\boldsymbol{\beta_1}$	pH	$\mu_{\beta_1}(x) = \begin{cases} 1; & when\ x = 7 \\ \frac{x-6.5}{7.0-6.5}; & when\ 6.5 \le x \le 7.0 \\ \frac{8.5-x}{8.5-7.0}; & when\ 7.0 \le x \le 8.5 \\ 0; & otherwise \end{cases}$
$\boldsymbol{\beta_2}$	Total dissolved solids	$\mu_{\beta_2}(x) = \begin{cases} \frac{500-x}{500-0}; & when\ 0 \le x \le 500 \\ 0; & when\ x \ge 500 \end{cases}$
$\boldsymbol{\beta_3}$	Total suspended solids	$\mu_{\beta_3}(x) = \begin{cases} \frac{500-x}{500-0}; & when\ 0 \le x \le 500 \\ 0; & when\ x \ge 500 \end{cases}$
$\boldsymbol{\beta_4}$	Electrical conductivity	$\mu_{\beta_4}(x) = \begin{cases} \frac{300-x}{300-0}; & when\ 0 \le x \le 300 \\ 1; & when\ x \ge 300 \end{cases}$
$\boldsymbol{\beta_5}$	Biochemical oxygen demand	$\mu_{\beta_5}(x) = \begin{cases} \frac{5-x}{5-0}; & when\ 0 \le x \le 5 \\ 0; & when\ x \ge 5 \end{cases}$
$\boldsymbol{\beta_6}$	Total hardness	$\mu_{\beta_7}(x) = \begin{cases} \frac{300-x}{300-0}; & when\ 0 \le x \le 300 \\ 0; & when\ x \ge 300 \end{cases}$

Table 5. Degree of indeterminacy functions for different WQPs.

Parameters	Parameter Name	Degree of Indeterminacy Functions
$\boldsymbol{\beta_1}$	pH	$\mu_{\beta_1}(x) = \begin{cases} \frac{7.0-x}{7.0-6.5}; & when\ 6.5 \le x \le 7.0 \\ \frac{x-7.0}{8.5-7.0}; & when\ 7.0 \le x \le 8.5 \\ 1; & otherwise \end{cases}$
$\boldsymbol{\beta_2}$	Total dissolved solids	$\mu_{\beta_2}(x) = \begin{cases} \frac{x-0}{500-0}; & when\ 0 \le x \le 500 \\ 1; & when\ x \ge 500 \end{cases}$
$\boldsymbol{\beta_3}$	Total suspended solids	$\mu_{\beta_3}(x) = \begin{cases} \frac{x-0}{500-0}; & when\ 0 \le x \le 500 \\ 1; & when\ x \ge 500 \end{cases}$
$\boldsymbol{\beta_4}$	Electrical conductivity	$\mu_{\beta_4}(x) = \begin{cases} \frac{x-0}{300-0}; & when\ 0 \le x \le 300 \\ 1; & when\ x \ge 300 \end{cases}$
$\boldsymbol{\beta_5}$	Biochemical oxygen demand	$\mu_{\beta_5}(x) = \begin{cases} \frac{x-0}{5-0}; & when\ 0 \le x \le 5 \\ 1; & when\ x \ge 5 \end{cases}$
$\boldsymbol{\beta_6}$	Total hardness	$\mu_{\beta_7}(x) = \begin{cases} \frac{x-0}{300-0}; & when\ 0 \le x \le 300 \\ 1; & when\ x \ge 300 \end{cases}$

Table 6. Falsity membership functions for different WQPs.

Parameters	Parameter Name	Falsity Membership Functions
$\boldsymbol{\beta_1}$	pH	$\mu_{\beta_1}(x) = \begin{cases} \frac{7.0-x}{7.0-6.5}; & when\ 6.5 \le x \le 7.0 \\ \frac{x-7.0}{8.5-7.0}; & when\ 7.0 \le x \le 8.5 \\ 1; & otherwise \end{cases}$
$\boldsymbol{\beta_2}$	Total dissolved solids	$\mu_{\beta_2}(x) = \begin{cases} \frac{x-0}{500-0}; & when\ 0 \le x \le 500 \\ 1; & when\ x \ge 500 \end{cases}$
$\boldsymbol{\beta_3}$	Total suspended solids	$\mu_{\beta_3}(x) = \begin{cases} \frac{x-0}{500-0}; & when\ 0 \le x \le 500 \\ 1; & when\ x \ge 500 \end{cases}$
$\boldsymbol{\beta_4}$	Electrical conductivity	$\mu_{\beta_4}(x) = \begin{cases} \frac{x-0}{300-0}; & when\ 0 \le x \le 300 \\ 1; & when\ x \ge 300 \end{cases}$
$\boldsymbol{\beta_5}$	Biochemical oxygen demand	$\mu_{\beta_5}(x) = \begin{cases} \frac{x-0}{5-0}; & when\ 0 \le x \le 5 \\ 1; & when\ x \ge 5 \end{cases}$
$\boldsymbol{\beta_6}$	Total hardness	$\mu_{\beta_7}(x) = \begin{cases} \frac{x-0}{300-0}; & when\ 0 \le x \le 300 \\ 1; & when\ x \ge 300 \end{cases}$

Table 7. The Truth membership values for different WQPs.

S	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\beta_3}$	$\boldsymbol{\beta_4}$	$\boldsymbol{\beta_5}$	$\boldsymbol{\beta_7}$
α1	0.813	0.780	0.825	0.665	0.14	0.749
α2	0.78	0.763	0.831	0.652	0.08	0.807
α3	0.713	0.753	0.843	0.611	0.12	0.766
α4	0.627	0.743	0.823	0.582	0.02	0.733
α5	0.607	0.726	0.805	0.563	0	0.814

Table 8. The Degree of indeterminacy values for different WQPs.

S	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\beta_3}$	$\boldsymbol{\beta_4}$	$\boldsymbol{\beta_5}$	$\boldsymbol{\beta_7}$
α1	0.187	0.220	0.175	0.335	0.86	0.251
α2	0.220	0.238	0.169	0.348	0.92	0.193
α3	0.287	0.247	0.157	0.389	0.88	0.234
α4	0.373	0.257	0.177	0.418	0.98	0.267
α5	0.393	0.274	0.195	0.438	1	0.186

Table 9. The Falsity membership values for different WQPs.

S	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\beta_3}$	$\boldsymbol{\beta_4}$	$\boldsymbol{\beta_5}$	$\boldsymbol{\beta_7}$
α1	0.187	0.220	0.175	0.335	0.86	0.251
α2	0.22	0.238	0.169	0.348	0.92	0.193
α3	0.287	0.247	0.157	0.389	0.88	0.234
α4	0.373	0.257	0.177	0.418	0.98	0.267
α5	0.393	0.274	0.195	0.436	1	0.186

Table 10. The NWQI.

S	T	I	F	NWQI
α1	0.401	0.599	0.599	-0.12383
α2	0.352	0.648	0.648	-0.17961
α3	0.352	0.648	0.648	-0.17961
α4	0.259	0.741	0.741	-0.27685
α5	0.240	0.760	0.760	-0.29545

4. Conclusions

The study delves into the complexities of assessing river water quality, dealing with various WQPs, uncertainties, and diverse decision-maker input. It introduces innovative concepts like WATV, WAIV, WAFV, and NWQI-score functions, which are integrated into the NWQI for pollution rating. The model was applied to assess Gomati River's water quality indices. The Gomati River holds immense significance in Tripura, India, as it is the largest river. The river's annual water flow of 249.39 million cubic meters plays a crucial role, serving as the primary water source for drinking, agriculture, fishing, cattle rearing, and inland water transportation for residents. In this study, we considered eighteen key parameters, with sampling conducted from January 2024 to April 2024 based on waste discharge points. The study's findings emphasize the poor water quality of the Gomati River, rendering it unsuitable for human use based on a thorough analysis of water quality parameters. The NWQI-score from the NWQI is key in assessing pollution levels, consistently indicating poor water quality across all river sites due to high levels of total coliform bacteria from wastewater. Domestic waste dumping directly impacts this contamination, stressing the river's health. Lack of sewage treatment exacerbates pollution risks, making the water unfit for consumption. Implementing regular monitoring and raising public awareness are urgent steps, alongside enforcing waste treatment laws, to safeguard the river's water quality.

In conclusion, our research has yielded promising outcomes compared to existing models, particularly through the innovative concepts of the WATV, WAIV, WAFV, and the utilization of a flexible NWQI score. These advancements make our proposed model more stable, practical, and adaptable to various real-world applications. While our efforts have addressed many challenges, there are still areas for further investigation and improvement. It's crucial to maintain a balanced perspective, recognizing both the achievements and ongoing challenges in the field of decision-making. Future studies of the strategy ought to include pertinent topics like computer science, software engineering, contemporary conditions, and others. In a future study, we will explore broader properties and operations on the Neutrosophic sets, and this proposed model will be expanded to include further practical applications in the areas of medical diagnostics and computer science.

Declarations

Ethics Approval and Consent to Participate

The results/data/figures in this manuscript have not been published elsewhere, nor are they under consideration by another publisher. All the material is owned by the authors, and/or no permissions are required.

Consent for Publication

This article does not contain any studies with human participants or animals performed by any of the authors.

Availability of Data and Materials

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Competing Interests

The authors declare no competing interests in the research.

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Author Contribution

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