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Some Similarity Measures on Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert Sets and Their Applications in Medical Diagnosis

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Abstract: In recent years, the application of fuzzy sets has gained significant attraction in various fields, including medical diagnosis, due to their ability to manage uncertainties and imprecise information. This paper focuses on the comparative analysis of similarity measures within the realm of Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert Sets (GIVIFSESs) and explores their application in the domain of medical diagnosis. Most of the important topics in fuzzy set theory are the similarity measures between the generalizations of fuzzy set theory. Similarity measures are a crucial tool which was used in data science. In this process, we measure how much the data sets are related and comparable. Measures of similarity give a numerical value that reveals the strength of associations between sets or sets of variables. In this paper, we initiate a new concept of generalized interval-valued intuitionistic fuzzy soft expert sets and their fundamental operations. This new concept is more flexible than existing concepts based on their algebraic definition. Unlike fuzzy sets, the concept of generalized interval-valued intuitionistic fuzzy soft expert sets is characterized by a degree of membership and degree of non-membership along with fuzzy set theory. The proposed methodology is validated through an empirical application in medical diagnosis, where (GIV-IFSESs) are employed to model the uncertainty and imprecision inherent in expert assessments. The selected similarity measures are then applied to quantify the degree of resemblance between different medical cases, facilitating a more informed decision-making process. We introduce several types of similarity measures on generalized interval-valued intuitionistic fuzzy soft expert sets. We also discuss a similarity measure of Type-I, Type-II, and Type-III for two (GIVIFSESs) and its application in medical diagnosis problems.

Keywords: Fuzzy Sets; Soft Sets; Interval-valued Intuitionistic Fuzzy Soft Sets; Similarity Measures.

1. Introduction

The majority of real-world problems include data that is highly unclear and imprecise. To cope with uncertainties and fuzziness, classical mathematical theories such as fuzzy mathematics, probability theory, and interval mathematics have traditionally been used. However, as Molodtsov [1] pointed out, all of these theories have inherent challenges and shortcomings. He pioneered the idea of soft sets as a new mathematical technique for dealing with uncertainties that traditional mathematical tools cannot rectify. He has demonstrated multiple applications of this theory in the solution of several significant issues in economics, engineering, social science, medicine, and other domains. The soft setting is also a fascinating and well-liked subject, where various forms of decision-making issues are frequently resolved. The ideas of soft subset, soft superset, soft equality, null soft set, and absolute soft set were first developed by Maji and Biswas [2]. They also performed some soft-set operations and confirmed De Morgan's laws. Ali et al. [3] addressed some previous studies' mistakes and defined some new soft-set procedures. Following that, Ali et al. [4] examined certain algebraic structures of soft sets and studied several essential features linked with the new operations. Maji et al. and Roy [5-7] have investigated soft set theory further and applied it to solve several decision-making difficulties. They also created and researched the concept of fuzzy soft set, a more generalized concept that is a blend of fuzzy set and soft set. Maji et al. defined the soft set operations union and intersection. Soft sets and fuzzy soft sets were introduced into the incomplete environment by Zou and Xiao [8], respectively. Wajid et al. [9] proposed an enhanced tool for fuzzy sets, intuitionistic fuzzy sets, and image fuzzy sets. They also presented spherical fuzzy sets and their representations of spherical fuzzy T-norm and T-conorm. As a generalization of the soft set, Alkhazaleh et al. [10] developed the notion of soft multiset. They also defined fuzzy parameterized interval-valued fuzzy soft sets [11] and the possibility of fuzzy soft sets [12] and proposed their applications for decision-making and medical diagnosis. Alkhazaleh and Salleh [13] established the concept of a soft expert set, in which the user can obtain the opinions of all experts in a single model without performing any activities. Even after a procedure, the user can obtain the opinions of all professionals. Salleh [14] provided a quick survey from the soft set to the intuitionistic fuzzy soft set in 2011. By integrating the interval-valued fuzzy set [16, 17] and soft set models, Yang et al. [15] proposed the notion of interval-valued fuzzy soft set. Yang et al. [18] discussed the application of soft sets with interval-valued intuitionistic fuzzy soft sets. Interval-valued intuitionistic fuzzy sets are extensively used to select the best alternative in multi-attribute decision-making issues, however, how to evaluate uncertainty is a big open problem. Majumdar and Samanta [19] proposed and investigated a generalized fuzzy soft set in which the degree is associated with the parameterization of fuzzy sets for defining a fuzzy soft set. Babitha and John [20] presented generalized intuitionistic fuzzy soft sets and solved a multi-criteria decision-making problem using generalized intuitionistic fuzzy soft sets.

1.1 Motivation and Purpose of the Paper

In this paper, we simplify the concept of generalized interval-valued fuzzy soft sets as introduced by Majumdar and Samanta to generalized interval-valued intuitionistic fuzzy soft expert sets. In our generalization of interval-valued intuitionistic fuzzy soft expert sets, a degree is attached to the parameterization of fuzzy sets while defining an interval-valued intuitionistic fuzzy soft expert set. Also, we give some applications of generalized interval-valued intuitionistic fuzzy soft expert set-in decision-making problems. We have further studied the similarity between two generalized interval-valued intuitionistic fuzzy soft expert sets, and it has been applied in medical diagnosis.

In the realm of decision-making and information processing, the ability to effectively manage uncertainty and imprecision is paramount. Fuzzy set theory has emerged as a powerful tool for capturing and modeling vagueness inherent in real-world data. The integration of fuzzy sets with expert opinions further enhances decision-making processes, providing a comprehensive framework for tackling complex problems [20-29]. This research endeavors to explore and compare various similarity measures within the context of Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert Sets (GIVIFSESs) and subsequently apply these measures in the domain of medical diagnosis.

i). As medical diagnosis involves subjective expert opinions, imprecise data, and uncertainty, traditional approaches may fall short of providing accurate and reliable results. Fuzzy set theory, coupled with soft expert information, offers a promising avenue to address these challenges. The motivation for this study lies in the need for a robust methodology that not only represents the complexity of medical diagnostic scenarios but also enables a rigorous comparison of similarity measures within the (GIVIFSESs) framework.

ii). The (GIVIFSESs) framework is introduced as a comprehensive model that extends the capabilities of traditional fuzzy sets by incorporating interval-valued intuitionistic fuzzy sets and soft expert information. This hybrid approach aims to capture not only the uncertainty and imprecision associated with medical diagnoses but also the inherent hesitancy present in expert opinions. The integration of these elements enhances the fidelity of the representation and provides a more realistic model for decision-making in medical contexts.

iii). Similarity measures play a crucial role in assessing the degree of resemblance between different entities represented by (GIVIFSESs). The selection of an appropriate similarity measure is pivotal for ensuring the accuracy and reliability of decision support systems. This study seeks to compare and evaluate various similarity measures to identify the most suitable measure for capturing the subtleties of expert opinions in the context of medical diagnosis.

iv). The primary objectives of this research are twofold:

a. To conduct a comprehensive comparative analysis of similarity measures within the (GIVIFSESs) framework.
b. To demonstrate the practical application of the selected similarity measures in medical diagnosis scenarios, highlighting their efficacy in handling uncertainty and imprecision.

The organization of this paper is as follows: In section 2, some preliminary definitions and results are given which will be used in the rest of the paper. In section 3 a definition of generalized interval-valued intuitionistic fuzzy soft expert set is given and fundamental operations of GIVIFSESs namely Sum, Product, Subtraction, Max- Product, Min-Product, Complement, Power, and Scaler product. In section 4, the similarity between two generalized interval-valued intuitionistic fuzzy soft expert sets has been discussed. In this section, we defined several types of similarity measures and also discussed some of the results related to those measures. An application of these similarity measures in medical diagnosis has been shown in section 5. In this section, we also compared the results of the similarity measures which were discussed in the previous section. Section 6 concludes the paper.

2. Preliminaries

In this section, we recall some basic definitions such as soft set, soft expert set, fuzzy soft expert set, interval-valued fuzzy set, interval-valued intuitionistic fuzzy soft expert set, and generalized interval-valued fuzzy soft set which are required in the construction of new concepts.

Definition 2.1. [1] *Let* $\widehat{F}: \mathcal{A} \rightarrow P(U)$ Is a mapping, whose domain is a set of parameters, and codomain is a power set of universal setU. **Soft set** over the universal set U is represented as a pair $(\widehat{\mathrm{F}}: \mathcal{A})$. For $\propto \in \mathcal{A}$, $\widehat{\mathrm{F}}(\propto)$ gives the set of $\propto$-approximate elements of the soft set $(\widehat{\mathrm{F}}: \mathcal{A})$. It is considered a parameterized family of subsets of the set U.

Definition 2.2. [13] Let $\widehat{F}: \mathcal{A} \rightarrow P(U)$ Is a mapping, whose domain is the cartesian product of the set of parameters, the panel of experts, and the set of opinions i-e $\mathrm{Ą} = \hat{\mathrm{E}} \times \mathrm{Ķ} \times \mathrm{Ọ}$ and codomain is a power set of universal set U. **Soft expert set** over the universal set U is represented as a pair $(\widehat{\mathrm{F}}: \mathcal{A})$.

Definition 2.3. [21] Let $\widehat{F}: \mathcal{A} \rightarrow I^U$ is a mapping, whose domain is the cartesian product of a set of parameters, set of experts, and set of opinions i-e $\mathrm{Ą} = \hat{\mathrm{E}} \times \mathrm{Ķ} \times \mathrm{Ọ}$ and codomain I^Udenoted all fuzzy subsets of U is a power set of universal set U. **Fuzzy soft expert set** over the universal set U is represented as a pair $(\widehat{\mathrm{F}}: \mathcal{A})$.

Definition 2.4. [15] Let U be a non-empty set. A function $\mathrm{Ą}: \mathrm{U} \rightarrow \mathrm{Int}[0,1]$ is called an **interval-valued fuzzy set**, whose domain is a universal set and codomain is the set of all closed sub-intervals of

[0,1], the set of all interval-valued fuzzy sets on U is denoted by I^U. For every Ą $\in I^U$ and u$\in U$, Ą(u) $= [Ą^-(u), Ą^+(u)]$ is called the degree of membership of an element u to Ą, lower fuzzy set, and upper fuzzy set in U represented by the following maps $Ą^-: U \to I$ and $Ą^+: U \to I$ respectively.

Definition 2.5. [22] Let ξ: Ą × Ģ → Ķ(U) is a mapping, whose domain is a cartesian product of the set of attributes Ą and the set of experts Ģ and codomain in the set of the interval-valued intuitionistic fuzzy sets on the universe set U which is denoted as Ķ(U). **Interval-valued intuitionistic fuzzy soft expert set** (IVIFSE set) is a triplet(ξ: Ą, Ģ). For b∈ Ą and p∈ Ģ we define

$$\xi_p^b = \left\{< u, \left[\gamma^{-b}_{\ p}(u), \gamma^{+b}_{\ p}(u)\right], \left[\zeta^{-b}_{\ p}(u), \zeta^{+b}_{\ p}(u)\right] >: u \in U\right\}.$$

Definition 2.6. [23] Let $\check{U} = \{u_1, u_2, u_3 \dots\dots u_n\}$ be the universal set of elements and $\hat{E} = \{v_1, v_2, v_3 \dots\dots v_n\}$ be the universal set of parameters. The pair $(\check{U}, \hat{E})$ will be called a soft universe. Let Ϝ: $\hat{E} \to Int(\check{U})$ and ɳ be a fuzzy set of $\hat{E}$ i.e ɳ: $\hat{E} \to I = [0,1]$, where $Int(\check{U})$ is the set of all interval-valued fuzzy subsets on $\check{U}$. Let $Ϝ_ɳ$: $\hat{E} \to Int(\check{U}) \times I$ be a function defined as follows:

$$Ϝ_ɳ(v) = \big(Ϝ(v)(u), ɳ(v)\big), \forall u \in \check{U}.$$

Then $Ϝ_ɳ$ is called a **generalized interval-valued fuzzy soft set** (GIVFS set) over the soft set $(\check{U}, \hat{E})$.

Definition 2.7 [24] If m(ƒ,ƃ) is the similarity measure between two interval-valued intuitionistic fuzzy soft sets (F,A) and (G,B) then,

1. m(ƒ, ƃ) = m(ƃ, ƒ,)
2. 0 ≤ m(ƒ, ƃ) ≤ 1
3. m(ƒ, ƃ) = 1 ⇔ (F, A) = (G, B)

3. The Proposed concept of Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert sets and Their Fundamental Operations.

The terms Generalized Interval-Valued Fuzzy Soft set and Interval-Valued Intuitionistic Fuzzy Soft Expert set are combined in this section to create the new concept known as Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert sets. We define the idea of Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert sets and define some related concepts pertaining to this notion as well as their basic operations on this concept.

Definition 3.1. Let $\hat{U} = \{\xi_1, \xi_2, \xi_3, \dots\dots \xi_n\}$ be the initial universal set of elements, $\hat{A} = \{\sigma_1, \sigma_2, \sigma_3 \dots\dots \sigma_n\}$ be the set of attributes and $\hat{G} = \{\rho_1, \rho_2, \rho_3 \dots\dots \rho_n\}$ be the set of experts. The Interval-Valued Intuitionistic Fuzzy Soft Expert set is a triplet $(\hat{U}, \hat{A}, \hat{G})$ that is described by a mapping £: $\hat{A} \times \hat{G} \to$ Ķ$(\hat{U})$ where Ķ$(\hat{U})$ is the set of all Interval-Valued Intuitionistic Fuzzy subset on $\hat{U}$. Let £: $\hat{A} \times \hat{G} \to$ Ķ$(\hat{U})$ and ɳ be a fuzzy soft expert set of $\hat{A} \times \hat{G}$, i.e., ɳ: $\hat{A} \times \hat{G} \to I = [0,1]$. Let $£_ɳ$: $\hat{A} \times \hat{G} \to$ Ķ$(\hat{U}) \times$ I be a function defined as follows:

$$£_{ɳ}{}^{\rho}_{\sigma} = \left(\left\{< \xi, \left[\alpha^{-\rho}_{\ \sigma}, \alpha^{+\rho}_{\ \sigma}(u)\right], \left[\beta^{-\rho}_{\ \sigma}, \beta^{+\rho}_{\ \sigma}\right] >\right\}, ɳ\,^{\rho}_{\sigma}(\xi)\right),$$

Then $£_{ɳ}{}^{\rho}_{\sigma}$ is called a Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert set. GIVIFSEs over the soft expert set $(\hat{U}, \hat{A}, \hat{G})$.

Example 3.1. Let $\hat{U} = \{\xi_1, \xi_2, \xi_3\}$ be the set of universe, $\hat{A} = \{\sigma_1, \sigma_2, \sigma_3\}$ be the set of attributes and $\hat{G} = \{\rho_1, \rho_2\}$ be the set of expert and ɳ: $\hat{A} \times \hat{G} \to I$ define a function $£_ɳ$: $\hat{A} \times \hat{G} \to$ Ķ$(\hat{U}) \times$ I as follows:

$$£_{ɳ}{}^{\rho 1}_{\sigma 1} = \left(\left\{\frac{\xi_1}{[0.2,0,4], [0.1,0.3]}, \frac{\xi_2}{[0.1,0.5], [0.4,0.5]}, \frac{\xi_3}{[0.3,0.4], [0.4,0.5]}\right\}, 0.5\right),$$

$$£_{\eta_{\sigma 2}}^{\rho 1}=\left(\left\{\frac{\xi_1}{[0.3,0,4],[0.1,0.2]},\ \frac{\xi_2}{[0.5,0.6],[0.4,0.6]},\ \frac{\xi_3}{[0.2,0.4],[0.3,0.5]}\right\},0.6\right),$$

$$£_{\eta_{\sigma 3}}^{\rho 1}=\left(\left\{\frac{\xi_1}{[0.2,0,5],[0.1,0.3]},\ \frac{\xi_2}{[0.4,0.6],[0.5,0.7]},\ \frac{\xi_3}{[0.1,0.2],[0.3,0.4]}\right\},0.3\right),$$

$$£_{\eta_{\sigma 1}}^{\rho 2}=\left(\left\{\frac{\xi_1}{[0.1,0,3],[0.4,0.6]},\ \frac{\xi_2}{[0.5,0.6],[0.1,0.3]},\ \frac{\xi_3}{[0.7,0.8],[0.1,0.2]}\right\},0.4\right),$$

$$£_{\eta_{\sigma 2}}^{\rho 2}=\left(\left\{\frac{\xi_1}{[0.0,0.4],[0.0,0.3]},\ \frac{\xi_2}{[0.1,0.2],[0.6,0.7]},\ \frac{\xi_3}{[0.1,0.5],[0.0,0.3]}\right\},0.1\right),$$

$$£_{\eta_{\sigma 3}}^{\rho 2}=\left(\left\{\frac{\xi_1}{[0.1,0.4],[0.1,0.2]},\ \frac{\xi_2}{[0.2,0.5],[0.0,0.5]},\ \frac{\xi_3}{[0.3,0.5],[0.0,0.1]}\right\},0.7\right),$$

Then $£_{\eta}$ is called a GIVIFSEs over $(\hat{U},\ \hat{A},\hat{G})$. In matrix notation we write in Table 1 and 2,

Table 1. Opinion of expert $\boldsymbol{\rho_1}$.

$\boldsymbol{\rho_1}$	$\boldsymbol{\xi_1}$	$\boldsymbol{\xi_2}$	$\boldsymbol{\xi_3}$	$\eta_{\sigma}^{\rho}(\boldsymbol{\xi})$
$\boldsymbol{\sigma_1}$	[0.2,0,4], [0.1,0.3]	[0.1,0.5], [0.4,0.5]	[0.3,0.4], [0.4,0.5]	0.5
$\boldsymbol{\sigma_2}$	[0.3,0,4], [0.1,0.2]	[0.5,0.6], [0.4,0.6]	[0.2,0.4], [0.3,0.5]	0.6
$\boldsymbol{\sigma_3}$	[0.2,0,5], [0.1,0.3]	[0.4,0.6], [0.5,0.7]	[0.1,0.2], [0.3,0.4]	0.3

Table 2. Opinion of expert $\boldsymbol{\rho_2}$.

$\boldsymbol{\rho_2}$	$\boldsymbol{\xi_1}$	$\boldsymbol{\xi_2}$	$\boldsymbol{\xi_3}$	$\eta_{\sigma}^{\rho}(\boldsymbol{\xi})$
$\boldsymbol{\sigma_1}$	[0.1,0,3], [0.4,0.6]	[0.5,0.6], [0.1,0.3]	[0.7,0.8], [0.1,0.2]	0.5
$\boldsymbol{\sigma_2}$	[0.0,0.4], [0.0,0.3]	[0.1,0.2], [0.6,0.7]	[0.1,0.5], [0.0,0.3]	0.6
$\boldsymbol{\sigma_3}$	[0.1,0.4], [0.1,0.2]	[0.2,0.5], [0.0,0.5]	[0.3,0.5], [0.0,0.1]	0.3

Definition 3.2. Let $£_{\eta_1}$ and Γ_{η_2} be two GIVIFSEs over $(\hat{U},\ \hat{A},\hat{G})$. Then $£_{\eta_1}$ is called a *GIVIFSE* subset of Γ_{η_2} and we write $£_{\eta_1}\subseteq\Gamma_{\eta_2}$ if.

a) $\eta_1{}_{\sigma}^{\rho}(\xi)$ is a fuzzy subset of $\eta_2{}_{\sigma}^{\rho}(\xi)\ \forall\,\xi\in\hat{U}$,

b) $£_{\eta_{\sigma}}^{\rho}$ is an IVIF subset of $\Gamma_{\eta_{\sigma}}^{\rho}$, $\forall\ \sigma\in\hat{A}\ and\ \rho\in\hat{G}$.

Definition 3.3. The complement of GIVIFSEs over $(\hat{U},\ \hat{A},\hat{G})$ is denoted by $(\hat{U},\hat{A},\hat{G})^{\acute{c}}$ and is defined for all $\sigma\in\hat{A}\ and\ \rho\in\hat{G}$ as follows:

$$(\hat{U},\hat{A},\hat{G})^{\acute{c}}=\left(\left\{<\xi,\left[\beta^{-}{}_{\sigma}^{\rho},\beta^{+}{}_{\sigma}^{\rho}\right],\left[\alpha^{-}{}_{\sigma}^{\rho},\alpha^{+}{}_{\sigma}^{\rho}(u)\right]>\right\},1-\eta_{\sigma}^{\rho}(\xi)\right),$$

Where $£_{\eta_{\sigma}}^{\rho}=\left(\left\{<\xi,\left[\alpha^{-}{}_{\sigma}^{\rho},\alpha^{+}{}_{\sigma}^{\rho}(u)\right],\left[\beta^{-}{}_{\sigma}^{\rho},\beta^{+}{}_{\sigma}^{\rho}\right]>\right\},\eta_{\sigma}^{\rho}(\xi)\right)$.

Definition 3.4. Let $£_{\eta_1}$ be a GIVIFSEs over $(\hat{U},\ \hat{A},\hat{G})$. Then the complement of $£_{\eta_1}$ denoted by $£_{\eta_1}{}^{\acute{c}}$ and is defined by $£_{\eta_1}{}^{\acute{c}}=\Gamma_{\eta}$ such that $\eta_2{}_{\sigma}^{\rho}(\xi)=\acute{c}\ (\eta_1{}_{\sigma}^{\rho}(\xi))$ and $\Gamma_{\sigma}^{\rho}=\acute{c}\ (£_{\sigma}^{\rho})\ \forall\ \sigma\in\hat{A}\ and\ \rho\in\hat{G}$ where $\acute{c}$ is a fuzzy complement and $\hat{c}$ is an Interval-Valued Intuitionistic Fuzzy complement.

Definition 3.5. The sum of two GIVIFSEs $(£_1,\hat{A}_1,\hat{G}_1)$ and $(£_2,\hat{A}_2,\hat{G}_2)$ over ξ is denoted as $(£_1,\hat{A}_1,\hat{G}_1)+(£_2,\hat{A}_2,\hat{G}_2)$ and defined as:

$$£_{1\sigma}^{\rho}+£_{2\sigma}^{\rho}=\left(\left\{\begin{matrix}<\xi,\begin{bmatrix}\alpha_1{}^{-}{}_{\sigma}^{\rho}(\xi)+\alpha_2{}^{-}{}_{\sigma}^{\rho}(\xi)-\alpha_1{}^{-}{}_{\sigma}^{\rho}(\xi)\alpha_2{}^{-}{}_{\sigma}^{\rho}(\xi),\\ \alpha_1{}^{+}{}_{\sigma}^{\rho}(\xi)+\alpha_2{}^{+}{}_{\sigma}^{\rho}(\xi)-\alpha_1{}^{+}{}_{\sigma}^{\rho}(\xi)\alpha_2{}^{+}{}_{\sigma}^{\rho}(\xi),\end{bmatrix},\\ \left[\beta_1{}^{-}{}_{\sigma}^{\rho}(\xi)\beta_2{}^{-}{}_{\sigma}^{\rho}(\xi),\beta_1{}^{+}{}_{\sigma}^{\rho}(\xi)\beta_2{}^{+}{}_{\sigma}^{\rho}(\xi)\right],>\end{matrix}\right\},\ \begin{matrix}\\ \eta_1{}_{\sigma}^{\rho}(\xi)+\eta_2{}_{\sigma}^{\rho}(\xi)-\eta_1{}_{\sigma}^{\rho}(\xi)\eta_2{}_{\sigma}^{\rho}(\xi)\end{matrix}\right),$$

Where $\text{£}_{\eta_{1\sigma}}^{\rho} = \left(\{< \xi, [\alpha_1{}^{-\rho}_{\sigma}, \alpha_1{}^{+\rho}_{\sigma}(u)], [\beta_1{}^{-\rho}_{\sigma}, \beta_1{}^{+\rho}_{\sigma}] >\}, \eta_1{}^{\rho}_{\sigma}(\xi)\right)$,

and $\text{£}_{\eta_{2\sigma}}^{\rho} = \left(\{< \xi, [\alpha_2{}^{-\rho}_{\sigma}, \alpha_2{}^{+\rho}_{\sigma}(u)], [\beta_2{}^{-\rho}_{\sigma}, \beta_2{}^{+\rho}_{\sigma}] >\}, \eta_2{}^{\rho}_{\sigma}(\xi)\right)$.

Definition 3.6. The difference between two GIVIFSEs $(\text{£}_1, \hat{A}_1, \hat{G}_1)$ and $(\text{£}_2, \hat{A}_2, \hat{G}_2)$ over ξ is denoted as $(\text{£}_1, \hat{A}_1, \hat{G}_1) - (\text{£}_2, \hat{A}_2, \hat{G}_2)$ and defined as:

$$\text{£}_{1\sigma}^{\rho} - \text{£}_{2\sigma}^{\rho} = \begin{pmatrix} \left\{ \begin{matrix} < \xi, [\alpha_1{}^{-\rho}_{\sigma}(\xi) \wedge \beta_2{}^{-\rho}_{\sigma}(\xi), \alpha_1{}^{+\rho}_{\sigma}(\xi) \wedge \beta_2{}^{+\rho}_{\sigma}(\xi)], \\ [\beta_1{}^{-\rho}_{\sigma}(\xi) \vee \alpha_2{}^{-\rho}_{\sigma}(\xi), \beta_1{}^{+\rho}_{\sigma}(\xi) \vee \alpha_1{}^{+\rho}_{\sigma}(\xi)] > \end{matrix} \right\}, \\ \eta_1{}^{\rho}_{\sigma}(\xi) \wedge 1 - \eta_2{}^{\rho}_{\sigma}(\xi) \end{pmatrix},$$

Where $\text{£}_{\eta_{1\sigma}}^{\rho} = \left(\{< \xi, [\alpha_1{}^{-\rho}_{\sigma}, \alpha_1{}^{+\rho}_{\sigma}(u)], [\beta_1{}^{-\rho}_{\sigma}, \beta_1{}^{+\rho}_{\sigma}] >\}, \eta_1{}^{\rho}_{\sigma}(\xi)\right)$,

and $\text{£}_{\eta_{2\sigma}}^{\rho} = \left(\{< \xi, [\alpha_2{}^{-\rho}_{\sigma}, \alpha_2{}^{+\rho}_{\sigma}(u)], [\beta_2{}^{-\rho}_{\sigma}, \beta_2{}^{+\rho}_{\sigma}] >\}, \eta_2{}^{\rho}_{\sigma}(\xi)\right)$.

Definition 3.7. The product of two GIVIFSEs $(\text{£}_1, \hat{A}_1, \hat{G}_1)$ and $(\text{£}_2, \hat{A}_2, \hat{G}_2)$ over ξ is denoted as $(\text{£}_1, \hat{A}_1, \hat{G}_1) \times (\text{£}_2, \hat{A}_2, \hat{G}_2)$ and defined as:

$$\text{£}_{1\sigma}^{\rho}\text{£}_{2\sigma}^{\rho} = \begin{pmatrix} \left\{ \begin{matrix} < \xi, [\alpha_1{}^{-\rho}_{\sigma}(\xi)\alpha_2{}^{-\rho}_{\sigma}(\xi), \alpha_1{}^{+\rho}_{\sigma}(\xi)\alpha_2{}^{+\rho}_{\sigma}(\xi)], \\ \begin{bmatrix} \beta_1{}^{-\rho}_{\sigma}(\xi) + \beta_2{}^{-\rho}_{\sigma}(\xi) - \beta_1{}^{-\rho}_{\sigma}(\xi)\beta_2{}^{-\rho}_{\sigma}(\xi), \\ \beta_1{}^{+\rho}_{\sigma}(\xi) + \beta_2{}^{+\rho}_{\sigma}(\xi) - \beta_1{}^{+\rho}_{\sigma}(\xi)\beta_2{}^{+\rho}_{\sigma}(\xi) \end{bmatrix} > \end{matrix} \right\}, \\ \eta_1{}^{\rho}_{\sigma}(\xi)\eta_2{}^{\rho}_{\sigma}(\xi) \end{pmatrix},$$

Where $\text{£}_{\eta_{1\sigma}}^{\rho} = \left(\{< \xi, [\alpha_1{}^{-\rho}_{\sigma}, \alpha_1{}^{+\rho}_{\sigma}(u)], [\beta_1{}^{-\rho}_{\sigma}, \beta_1{}^{+\rho}_{\sigma}] >\}, \eta_1{}^{\rho}_{\sigma}(\xi)\right)$

and $\text{£}_{\eta_{2\sigma}}^{\rho} = \left(\{< \xi, [\alpha_2{}^{-\rho}_{\sigma}, \alpha_2{}^{+\rho}_{\sigma}(u)], [\beta_2{}^{-\rho}_{\sigma}, \beta_2{}^{+\rho}_{\sigma}] >\}, \eta_2{}^{\rho}_{\sigma}(\xi)\right)$

Definition 3.8. The scalar product of GIVIFSEs $(\xi, \hat{A}, \hat{G})$ with an arbitrary real number $\text{ṛ} > 0$ is denoted by $\text{ṛ}(\xi, \hat{A}, \hat{G})$ and defined as follows:

$$\text{ṛ}\text{£}_{\sigma}^{\rho} = \begin{pmatrix} \{< \xi, [1 - (1 - \alpha^{-\rho}_{\sigma})^{\text{r}}, 1 - (1 - \alpha^{+\rho}_{\sigma})^{\text{r}}], [(\beta^{-\rho}_{\sigma})^{\text{r}}, (\beta^{+\rho}_{\sigma})^{\text{r}}] >, \} \\ 1 - (1 - \eta_{\sigma}^{\rho})^{\text{r}} \end{pmatrix}$$

Definition 3.9 The power of GIVIFSEs$(\xi, \hat{A}, \hat{G})$ with an arbitrary real number $\text{ṛ} > 0$ is denoted by $(\xi, \hat{A}, \hat{G})^{\text{r}}$ over ξ and defined as follow:

$$(\text{£}_{\sigma}^{\rho})^{\text{r}} = \begin{pmatrix} \{< \xi, [(\alpha^{-\rho}_{\sigma})^{\text{r}}, (\alpha^{+\rho}_{\sigma})^{\text{r}}][1 - (1 - \beta^{-\rho}_{\sigma})^{\text{r}}, 1 - (1 - \beta^{+\rho}_{\sigma})^{\text{r}}] >, \} \\ (\eta_{\sigma}^{\rho})^{\text{r}} \end{pmatrix}.$$

4. Some Novel Similarity Measures for Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert sets

We are frequently interested in determining if two patterns, photos or designs are similar, or at least to what degree they are similar. There are different methods for calculating the similarity measures between two fuzzy sets, hesitant fuzzy sets, fuzzy soft sets, intuitionistic fuzzy sets, Interval-Valued Intuitionistic Fuzzy Soft sets, and Generalized Interval-Valued Fuzzy Soft set. In this section, we explain three distinct types of similarity measures for GIVIFSESs. Recently Majumdar and Samanta [25, 26] have studied the similarity measure of soft sets and fuzzy soft sets. Rafiq, Muhammad et al. [27] investigated the family of novel similarity measures between spherical fuzzy sets based on cosine function by considering the positive, neutral, negative and refusal grades in SFSs. We have further studied the similarity between two GIVIFSESs and it has been applied in medical diagnosis. Similarity measures have extensive application in several areas such as pattern recognition, image processing, region extraction, and coding theory. We are using the set theoretic approach because it is more straightforward to calculate and is also a commonly used technique. Let us suppose that.

$$£_{\eta_{1\sigma_1}}^{\rho_1} = \left(\left(\{< \xi, [\alpha_1{}^{-}{}_{\sigma}^{\rho}(\xi), \alpha_1{}^{+}{}_{\sigma}^{\rho}(\xi)], [\beta_1{}^{-}{}_{\sigma}^{\rho}(\xi), \beta_1{}^{+}{}_{\sigma}^{\rho}(\xi)] >\}, \eta_1{}_{\sigma}^{\rho}(\xi)\right): \xi \in \hat{U}, {}_{\sigma_1}^{\rho_1} \in \hat{A}_1 \times \hat{G}_1\right),$$
$$\Gamma_{\eta_{2\sigma_2}}^{\rho_2} = \left(\left(\{< \xi, [\alpha_2{}^{-}{}_{\sigma}^{\rho}(\xi), \alpha_2{}^{+}{}_{\sigma}^{\rho}(\xi)], [\beta_2{}^{-}{}_{\sigma}^{\rho}(\xi), \beta_2{}^{+}{}_{\sigma}^{\rho}(\xi)] >\}, \eta_2{}_{\sigma}^{\rho}(\xi)\right): \xi \in \hat{U}, {}_{\sigma_2}^{\rho_2} \in \hat{A}_2 \times \hat{G}_2\right),$$

be two GIVIFSESs over ξ.

4.1 Type-I Similarity Measures for Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert Sets

Definition 4.1. Let $\hat{U} = \{\xi_x : x = 1,2,3, \ldots\ldots, m\}$ be the finite universal set of elements ξ_x , $\hat{A} = \{\sigma_{\hat{y}} : \hat{y} = 1,2,3, \ldots\ldots, n\}$ be the set of attributes and $\hat{G} = \{\rho_{\check{z}} : \check{z} = 1.2.3, \ldots\ldots\ldots, t\}$ be the set of experts (or decision makers). Let $£_{\eta 1}$ and $\Gamma_{\eta 2}$ be two GIVIFSESs over ξ. Let $SM(£_{\eta 1}, \Gamma_{\eta 2})$ specify the similarity measures between GIVIFSESs $£_{\eta 1}$ and $\Gamma_{\eta 2}$. We must first find similarities between their $\rho_{\check{z}}$th approximations to see similar measurements $£_{\eta 1}$ and $\Gamma_{\eta 2}$. Let $S_i(£_{\eta 1}, \Gamma_{\eta 2})$ indicate the similarity measure between the two $\rho_{\check{z}}$th approximation of :

$$£_{\eta_{1\sigma}}^{\rho} = \left(\{< \xi, [\alpha_1{}^{-}{}_{\sigma}^{\rho}(\xi), \alpha_1{}^{+}{}_{\sigma}^{\rho}(\xi)], [\beta_1{}^{-}{}_{\sigma}^{\rho}(\xi), \beta_1{}^{+}{}_{\sigma}^{\rho}(\xi)] >\}, \eta_1{}_{\sigma}^{\rho}(\xi)\right),$$

$$\Gamma_{\eta_{2\sigma}}^{\rho} = \left(\{< \xi, [\alpha_2{}^{-}{}_{\sigma}^{\rho}(\xi), \alpha_2{}^{+}{}_{\sigma}^{\rho}(\xi)], [\beta_2{}^{-}{}_{\sigma}^{\rho}(\xi), \beta_2{}^{+}{}_{\sigma}^{\rho}(\xi)] >\}, \eta_2{}_{\sigma}^{\rho}(\xi)\right),$$

Then $S_i(£_{\eta 1}, \Gamma_{\eta 2})$ is defined as follows:

$$S_i(£_{\eta 1}, \Gamma_{\eta 2}) = \underset{\xi \in \hat{U}}{max}\left\{ \underset{\sigma \in \hat{A}, \rho \in \hat{G}}{min} \begin{pmatrix} [\alpha_1{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x), \alpha_1{}^{+}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x)], \\ [\beta_1{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x), \beta_1{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x)], \\ [\alpha_2{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x), \alpha_2{}^{+}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x)], \\ [\beta_2{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x), \beta_2{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x)], \\ \eta_{1(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x), \eta_{2(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x) \end{pmatrix} \right\} \quad (4.1)$$

where i=1,2,3….,.

when i=1, ŷ =1, ž=1 and x=1,2,3….,m, then by using (4.1) we get:

$$S_1(£_{\eta 1}, \Gamma_{\eta 2}) = \left\{ \begin{array}{c} \max \\ \xi \in \hat{U} \end{array} \left\{ \boxed{\begin{array}{c} min \\ \sigma_1 \in \hat{A}, \rho \in \hat{G} \end{array}} \begin{pmatrix} \left[\alpha_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_1), \alpha_1{}^+{}_{(\sigma_1,\rho_1)}(\xi_1)\right], \\ \left[\beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_1), \beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_1)\right], \\ \left[\alpha_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_1), \alpha_2{}^+{}_{(\sigma_1,\rho_2)}(\xi_1)\right], \\ \left[\beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_1), \beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_1)\right], \\ \eta_{1(\sigma_1,\rho_1)}(\xi_1), \eta_{2(\sigma_1,\rho_2)}(\xi_1) \end{pmatrix} \right\}, \\ \begin{array}{c} \max \\ \xi \in \hat{U} \end{array} \left\{ \boxed{\begin{array}{c} min \\ \sigma_1 \in \hat{A}, \rho \in \hat{G} \end{array}} \begin{pmatrix} \left[\alpha_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_2), \alpha_1{}^+{}_{(\sigma_1,\rho_1)}(\xi_2)\right], \\ \left[\beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_2), \beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_2)\right], \\ \left[\alpha_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_2), \alpha_2{}^+{}_{(\sigma_1,\rho_2)}(\xi_2)\right], \\ \left[\beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_2), \beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_2)\right], \\ \eta_{1(\sigma_1,\rho_1)}(\xi_2), \eta_{2(\sigma_1,\rho_2)}(\xi_2) \end{pmatrix} \right\}, \\ \begin{array}{c} \max \\ \xi \in \hat{U} \end{array} \left\{ \boxed{\begin{array}{c} min \\ \sigma_1 \in \hat{A}, \rho \in \hat{G} \end{array}} \begin{pmatrix} \left[\alpha_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_3), \alpha_1{}^+{}_{(\sigma_1,\rho_1)}(\xi_3)\right], \\ \left[\beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_3), \beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_3)\right], \\ \left[\alpha_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_3), \alpha_2{}^+{}_{(\sigma_1,\rho_2)}(\xi_3)\right], \\ \left[\beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_3), \beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_3)\right], \\ \eta_{1(\sigma_1,\rho_1)}(\xi_3), \eta_{2(\sigma_1,\rho_2)}(\xi_3) \end{pmatrix} \right\} \dots\dots\dots, \\ \begin{array}{c} \max \\ \xi \in \hat{U} \end{array} \left\{ \boxed{\begin{array}{c} min \\ \sigma_1 \in \hat{A}, \rho \in \hat{G} \end{array}} \begin{pmatrix} \left[\alpha_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_m), \alpha_1{}^+{}_{(\sigma_1,\rho_1)}(\xi_m)\right], \\ \left[\beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_m), \beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_m)\right], \\ \left[\alpha_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_m), \alpha_2{}^+{}_{(\sigma_1,\rho_2)}(\xi_m)\right], \\ \left[\beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_m), \beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_m)\right], \\ \eta_{1(\sigma_1,\rho_1)}(\xi_m), \eta_{2(\sigma_1,\rho_2)}(\xi_m) \end{pmatrix} \right\} \end{array} \right. \quad (4.2)$$

and when i=2, ŷ=2, ž=1 and x=1,2,3....,m, then (4.1):

$$S_2(£_{\eta 1},\Gamma_{\eta 2}) = \begin{matrix} \underset{\xi \in \hat{U}}{max}\left\{\boxed{\underset{\sigma_2 \in \hat{A}, \rho \in \hat{G}}{min}}\begin{pmatrix} \left[\alpha_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_1), \alpha_1{}^{+}{}_{(\sigma_2,\rho_1)}(\xi_1)\right], \\ \left[\beta_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_1), \beta_1{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_1)\right], \\ \left[\alpha_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_1), \alpha_2{}^{+}{}_{(\sigma_2,\rho_2)}(\xi_1)\right], \\ \left[\beta_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_1), \beta_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_1)\right], \\ \eta_{1(\sigma_2,\rho_1)}(\xi_1), \eta_{2(\sigma_2,\rho_2)}(\xi_1) \end{pmatrix}\right\}, \\ \underset{\xi \in \hat{U}}{max}\left\{\boxed{\underset{\sigma_2 \in \hat{A}, \rho \in \hat{G}}{min}}\begin{pmatrix} \left[\alpha_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_2), \alpha_1{}^{+}{}_{(\sigma_2,\rho_1)}(\xi_2)\right], \\ \left[\beta_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_2), \beta_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_2)\right], \\ \left[\alpha_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_2), \alpha_2{}^{+}{}_{(\sigma_2,\rho_2)}(\xi_2)\right], \\ \left[\beta_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_2), \beta_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_2)\right], \\ \eta_{1(\sigma_2,\rho_1)}(\xi_2), \eta_{2(\sigma_2,\rho_2)}(\xi_2) \end{pmatrix}\right\}, \\ \underset{\xi \in \hat{U}}{max}\left\{\boxed{\underset{\sigma_2 \in \hat{A}, \rho \in \hat{G}}{min}}\begin{pmatrix} \left[\alpha_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_3), \alpha_1{}^{+}{}_{(\sigma_2,\rho_1)}(\xi_3)\right], \\ \left[\beta_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_3), \beta_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_3)\right], \\ \left[\alpha_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_3), \alpha_2{}^{+}{}_{(\sigma_2,\rho_2)}(\xi_3)\right], \\ \left[\beta_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_3), \beta_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_3)\right], \\ \eta_{1(\sigma_2,\rho_1)}(\xi_3), \eta_{2(\sigma_2,\rho_2)}(\xi_3) \end{pmatrix}\right\} \dots\dots\dots, \\ \underset{\xi \in \hat{U}}{max}\left\{\boxed{\underset{\sigma_2 \in \hat{A}, \rho \in \hat{G}}{min}}\begin{pmatrix} \left[\alpha_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_m), \alpha_1{}^{+}{}_{(\sigma_2,\rho_1)}(\xi_m)\right], \\ \left[\beta_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_m), \beta_1{}^{-}{}_{(\sigma_2,\rho_1)}(\xi_m)\right], \\ \left[\alpha_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_m), \alpha_2{}^{+}{}_{(\sigma_2,\rho_2)}(\xi_m)\right], \\ \left[\beta_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_m), \beta_2{}^{-}{}_{(\sigma_2,\rho_2)}(\xi_m)\right], \\ \eta_{1(\sigma_2,\rho_1)}(\xi_m), \eta_{2(\sigma_2,\rho_2)}(\xi_m) \end{pmatrix}\right\} \end{matrix} \quad (4.3)$$

when i =l, ŷ=n, ž=1 and x=1,2,3....,m, then by using (4.1):

$$S_l(\pounds_{\eta 1},\Gamma_{\eta 2}) = \begin{Bmatrix} \underset{\xi \in \hat{U}}{max}\left\{\boxed{\underset{\sigma_n \in \hat{A}, \rho \in \hat{G}}{min}}\begin{pmatrix}\left[{\alpha_1}^-_{(\sigma_n,\rho_1)}(\xi_1), {\alpha_1}^+_{(\sigma_n,\rho_1)}(\xi_1)\right], \\ \left[{\beta_1}^-_{(\sigma_n,\rho_1)}(\xi_1), {\beta_1}^-_{(\sigma_n,\rho_1)}(\xi_1)\right], \\ \left[{\alpha_2}^-_{(\sigma_n,\rho_2)}(\xi_1), {\alpha_2}^+_{(\sigma_n,\rho_2)}(\xi_1)\right], \\ \left[{\beta_2}^-_{(\sigma_n,\rho_2)}(\xi_1), {\beta_2}^-_{(\sigma_n,\rho_2)}(\xi_1)\right], \\ \eta_{1(\sigma_n,\rho_1)}(\xi_1), \eta_{2(\sigma_n,\rho_2)}(\xi_1)\end{pmatrix}\right\}, \\ \underset{\xi \in \hat{U}}{max}\left\{\boxed{\underset{\sigma_n \in \hat{A}, \rho \in \hat{G}}{min}}\begin{pmatrix}\left[{\alpha_1}^-_{(\sigma_n,\rho_1)}(\xi_2), {\alpha_1}^+_{(\sigma_n,\rho_1)}(\xi_2)\right], \\ \left[{\beta_1}^-_{(\sigma_n,\rho_1)}(\xi_2), {\beta_1}^-_{(\sigma_n,\rho_1)}(\xi_2)\right], \\ \left[{\alpha_2}^-_{(\sigma_n,\rho_2)}(\xi_2), {\alpha_2}^+_{(\sigma_n,\rho_2)}(\xi_2)\right], \\ \left[{\beta_2}^-_{(\sigma_n,\rho_2)}(\xi_2), {\beta_2}^-_{(\sigma_n,\rho_2)}(\xi_2)\right], \\ \eta_{1(\sigma_n,\rho_1)}(\xi_2), \eta_{2(\sigma_n,\rho_2)}(\xi_2)\end{pmatrix}\right\}, \\ \underset{\xi \in \hat{U}}{max}\left\{\boxed{\underset{\sigma_n \in \hat{A}, \rho \in \hat{G}}{min}}\begin{pmatrix}\left[{\alpha_1}^-_{(\sigma_n,\rho_1)}(\xi_3), {\alpha_1}^+_{(\sigma_n,\rho_1)}(\xi_3)\right], \\ \left[{\beta_1}^-_{(\sigma_n,\rho_1)}(\xi_3), {\beta_1}^-_{(\sigma_n,\rho_1)}(\xi_3)\right], \\ \left[{\alpha_2}^-_{(\sigma_n,\rho_2)}(\xi_3), {\alpha_2}^+_{(\sigma_n,\rho_2)}(\xi_3)\right], \\ \left[{\beta_2}^-_{(\sigma_n,\rho_2)}(\xi_3), {\beta_2}^-_{(\sigma_n,\rho_2)}(\xi_3)\right], \\ \eta_{1(\sigma_n,\rho_1)}(\xi_3), \eta_{2(\sigma_n,\rho_2)}(\xi_3)\end{pmatrix}\right\} \dots\dots\dots, \\ \underset{\xi \in \hat{U}}{max}\left\{\boxed{\underset{\sigma_n \in \hat{A}, \rho \in \hat{G}}{min}}\begin{pmatrix}\left[{\alpha_1}^-_{(\sigma_n,\rho_1)}(\xi_m), {\alpha_1}^+_{(\sigma_n,\rho_1)}(\xi_m)\right], \\ \left[{\beta_1}^-_{(\sigma_n,\rho_1)}(\xi_m), {\beta_1}^-_{(\sigma_n,\rho_1)}(\xi_m)\right], \\ \left[{\alpha_2}^-_{(\sigma_n,\rho_2)}(\xi_m), {\alpha_2}^+_{(\sigma_n,\rho_2)}(\xi_m)\right], \\ \left[{\beta_2}^-_{(\sigma_n,\rho_2)}(\xi_m), {\beta_2}^-_{(\sigma_n,\rho_2)}(\xi_m)\right], \\ \eta_{1(\sigma_n,\rho_1)}(\xi_m), \eta_{2(\sigma_n,\rho_2)}(\xi_m)\end{pmatrix}\right\} \end{Bmatrix} \quad (4.4)$$

And so on. where

$$\left[{\alpha_1}^-{}^{\rho}_{\sigma}(\xi), {\alpha_1}^+{}^{\rho}_{\sigma}(\xi)\right], \left[{\beta_1}^-{}^{\rho}_{\sigma}(\xi), {\beta_1}^+{}^{\rho}_{\sigma}(\xi)\right] = \frac{1}{3}\begin{pmatrix} {\alpha_1}^-{}^{\rho}_{\sigma}(\xi) + {\alpha_1}^+{}^{\rho}_{\sigma}(\xi) - \\ {\alpha_1}^-{}^{\rho}_{\sigma}(\xi){\alpha_1}^+{}^{\rho}_{\sigma}(\xi) + \\ {\beta_1}^-{}^{\rho}_{\sigma}(\xi) + {\beta_1}^+{}^{\rho}_{\sigma}(\xi) - \\ {\beta_1}^-{}^{\rho}_{\sigma}(\xi){\beta_1}^+{}^{\rho}_{\sigma}(\xi) \end{pmatrix}, \quad (4.5)$$

$$\left[{\alpha_2}^-{}^{\rho}_{\sigma}(\xi), {\alpha_2}^+{}^{\rho}_{\sigma}(\xi)\right], \left[{\beta_2}^-{}^{\rho}_{\sigma}(\xi), {\beta_2}^+{}^{\rho}_{\sigma}(\xi)\right] = \frac{1}{3}\begin{pmatrix} {\alpha_2}^-{}^{\rho}_{\sigma}(\xi) + {\alpha_2}^+{}^{\rho}_{\sigma}(\xi) - \\ {\alpha_2}^-{}^{\rho}_{\sigma}(\xi){\alpha_2}^+{}^{\rho}_{\sigma}(\xi) + \\ {\beta_2}^-{}^{\rho}_{\sigma}(\xi) + {\beta_2}^+{}^{\rho}_{\sigma}(\xi) - \\ {\beta_2}^-{}^{\rho}_{\sigma}(\xi){\beta_2}^+{}^{\rho}_{\sigma}(\xi) \end{pmatrix}, \quad (4.6)$$

Then the similarity measures between the GIVIFSESs $\pounds_{\eta_1}$ and Γ_{η_2} is defined as,

$$SM(\pounds_{\eta_1},\Gamma_{\eta_2}) = \frac{1}{n}\Sigma S_i(\pounds_{\eta_1},\Gamma_{\eta_2}) \qquad (4.7)$$

Definition 4.2. Let us suppose that.

$$\pounds_{\eta_1}{}^{\rho_1}_{\sigma_1} = \left(\left(\{< \xi, \left[{\alpha_1}^-{}^{\rho}_{\sigma}, {\alpha_1}^+{}^{\rho}_{\sigma}\right], \left[{\beta_1}^-{}^{\rho}_{\sigma}, {\beta_1}^+{}^{\rho}_{\sigma}\right] >\}, \eta_1{}^{\rho}_{\sigma}(\xi)\right): \xi \in \hat{U}, {}^{\rho_1}_{\sigma_1} \in \hat{A}_1 \times \hat{G}_1\right),$$

$$\Gamma_{\eta_2}{}^{\rho_2}_{\sigma_2} = \left(\left(\{< \xi, \left[{\alpha_2}^-{}^{\rho}_{\sigma}, {\alpha_2}^+{}^{\rho}_{\sigma}\right], \left[{\beta_2}^-{}^{\rho}_{\sigma}, {\beta_2}^+{}^{\rho}_{\sigma}\right] >\}, \eta_2{}^{\rho}_{\sigma}(\xi)\right): \xi \in \hat{U}, {}^{\rho_2}_{\sigma_2} \in \hat{A}_2 \times \hat{G}_2\right),$$

be two GIVIFSESs over ξ. Then $£_{\eta_1}$ and Γ_{η_2} are said to be λ −similar, denoted by $£_{\eta_1} \overset{\lambda}{\cong} \Gamma_{\eta_2}$ if and only if $S(£_{\eta_1}, \Gamma_{\eta_2})$ for $\lambda \in (0,1)$. We call the two GIVIFSESs significantly similar if $S(£_{\eta_1}, \Gamma_{\eta_2}) > \frac{1}{2}$.

Theorem 4.1. If $SM(£_{\eta 1}, \Gamma_{\eta_2})$ be the similarity measures between two GIVIFSESs $£_{\eta_1}$ and Γ_{η_2} then.

i. $SM(£_{\eta_1}, \Gamma_{\eta_2}) = SM(\Gamma_{\eta_2}, £_{\eta_1})$,
ii. $0 \leq SM(£_{\eta_1}, \Gamma_{\eta_2}) \leq 1$,
iii. $SM(£_{\eta_1}, \Gamma_{\eta_2}) = 1 \Leftrightarrow £_{\eta_1} = \Gamma_{\eta_2}$.

4.2 Type-II Similarity Measures for Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert Sets

Definition 4.3. Let $\xi = \{\xi_x : x = 1,2,3, \ldots\ldots, m\}$ be the finite universal set of elements ξ_x: , $\hat{A} = \{\sigma_{\hat{y}} : \hat{y} = 1,2,3, \ldots\ldots, n\}$ be the set of attributes and $\hat{G} = \{\rho_{\check{z}} : \check{z} = 1.2.3, \ldots\ldots, t\}$ be the set of experts (or decision makers).

Let $£_{\eta_1}$ and Γ_{η_2} be two GIVIFSESs over ξ. Let $\widetilde{SM}(£_{\eta_1}, \Gamma_{\eta_2})$ specify the similarity measures between GIVIFSESs $£_{\eta_1}$ and Γ_{η_2}. We must first find similarity between their $\rho_{\check{z}}$th approximations in order to see the similarity measurements $£_{\eta_1}$ and Γ_{η_2}. Let $S_i(£_{\eta_1}, \Gamma_{\eta_2})$ indicate the similarity measure between the two $\rho_{\check{z}}$th approximation of

$$£_{\eta_1}{}^{\rho}_{\sigma} = \left(\{< \xi, [\alpha_1{}^{-\rho}_{\sigma}(\xi), \alpha_1{}^{+\rho}_{\sigma}(\xi)], [\beta_1{}^{-\rho}_{\sigma}(\xi), \beta_1{}^{+\rho}_{\sigma}(\xi)] >\}, \eta_1{}^{\rho}_{\sigma}(\xi)\right),$$

$$\Gamma_{\eta_2}{}^{\rho}_{\sigma} = \left(\{< \xi, [\alpha_2{}^{-\rho}_{\sigma}(\xi), \alpha_2{}^{+\rho}_{\sigma}(\xi)], [\beta_2{}^{-\rho}_{\sigma}(\xi), \beta_2{}^{+\rho}_{\sigma}(\xi)] >\}, \eta_2{}^{\rho}_{\sigma}(\xi)\right),$$

Then $S_i(£_{\eta_1}, \Gamma_{\eta_2})$ is defined as follow:

$$S_i(£_{\eta 1}, \Gamma_{\eta 2}) = \min_{\xi \in \hat{U}} \left\{ \boxed{\max_{\sigma \in \hat{A}, \rho \in \hat{G}}} \begin{pmatrix} [\alpha_1{}^-{}_{(\sigma_{\hat{y}}, \rho_{\check{z}})}(\xi_x), \alpha_1{}^+{}_{(\sigma_{\hat{y}}, \rho_{\check{z}})}(\xi_x)], \\ [\beta_1{}^-{}_{(\sigma_{\hat{y}}, \rho_{\check{z}})}(\xi_x), \beta_1{}^-{}_{(\sigma_{\hat{y}}, \rho_{\check{z}})}(\xi_x)], \\ [\alpha_2{}^-{}_{(\sigma_{\hat{y}}, \rho_{\check{z}+1})}(\xi_x), \alpha_2{}^+{}_{(\sigma_{\hat{y}}, \rho_{\check{z}+1})}(\xi_x)], \\ [\beta_2{}^-{}_{(\sigma_{\hat{y}}, \rho_{\check{z}+1})}(\xi_x), \beta_2{}^-{}_{(\sigma_{\hat{y}}, \rho_{\check{z}+1})}(\xi_x)], \\ \eta_{1(\sigma_{\hat{y}}, \rho_{\check{z}})}(\xi_x), \eta_{2(\sigma_{\hat{y}}, \rho_{\check{z}+1})}(\xi_x) \end{pmatrix} \right\} \quad (4.8)$$

Where i= 1, 2, 3…., l

when i=1, ŷ=1, ž=1 and x=1,2,3…,m, then by using equation (4.8) we get:

$$S_1\left(£_{\eta 1}, \Gamma_{\eta 2}\right) = \begin{matrix} \min\limits_{\xi \in \hat{U}} \left\{ \boxed{\max\limits_{\sigma_1 \in \hat{A}, \rho \in \hat{G}}} \left(\begin{matrix} \left[\alpha_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_1), \alpha_1{}^+{}_{(\sigma_1,\rho_1)}(\xi_1)\right], \\ \left[\beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_1), \beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_1)\right], \\ \left[\alpha_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_1), \alpha_2{}^+{}_{(\sigma_1,\rho_2)}(\xi_1)\right], \\ \left[\beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_1), \beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_1)\right], \\ \eta_{1(\sigma_1,\rho_1)}(\xi_1), \eta_{2(\sigma_1,\rho_2)}(\xi_1) \end{matrix} \right) \right\}, \\ \min\limits_{\xi \in \hat{U}} \left\{ \boxed{\max\limits_{\sigma_1 \in \hat{A}, \rho \in \hat{G}}} \left(\begin{matrix} \left[\alpha_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_2), \alpha_1{}^+{}_{(\sigma_1,\rho_1)}(\xi_2)\right], \\ \left[\beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_2), \beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_2)\right], \\ \left[\alpha_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_2), \alpha_2{}^+{}_{(\sigma_1,\rho_2)}(\xi_2)\right], \\ \left[\beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_2), \beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_2)\right], \\ \eta_{1(\sigma_1,\rho_1)}(\xi_2), \eta_{2(\sigma_1,\rho_2)}(\xi_2) \end{matrix} \right) \right\}, \\ \min\limits_{\xi \in \hat{U}} \left\{ \boxed{\max\limits_{\sigma_1 \in \hat{A}, \rho \in \hat{G}}} \left(\begin{matrix} \left[\alpha_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_3), \alpha_1{}^+{}_{(\sigma_1,\rho_1)}(\xi_3)\right], \\ \left[\beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_3), \beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_3)\right], \\ \left[\alpha_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_3), \alpha_2{}^+{}_{(\sigma_1,\rho_2)}(\xi_3)\right], \\ \left[\beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_3), \beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_3)\right], \\ \eta_{1(\sigma_1,\rho_1)}(\xi_3), \eta_{2(\sigma_1,\rho_2)}(\xi_3) \end{matrix} \right) \right\} \dots\dots\dots, \\ \min\limits_{\xi \in \hat{U}} \left\{ \boxed{\max\limits_{\sigma_1 \in \hat{A}, \rho \in \hat{G}}} \left(\begin{matrix} \left[\alpha_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_m), \alpha_1{}^+{}_{(\sigma_1,\rho_1)}(\xi_m)\right], \\ \left[\beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_m), \beta_1{}^-{}_{(\sigma_1,\rho_1)}(\xi_m)\right], \\ \left[\alpha_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_m), \alpha_2{}^+{}_{(\sigma_1,\rho_2)}(\xi_m)\right], \\ \left[\beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_m), \beta_2{}^-{}_{(\sigma_1,\rho_2)}(\xi_m)\right], \\ \eta_{1(\sigma_1,\rho_1)}(\xi_m), \eta_{2(\sigma_1,\rho_2)}(\xi_m) \end{matrix} \right) \right\} \end{matrix} \quad (4.9)$$

and when i=2, ŷ=2, ž=1 and x=1,2,3....,m, then equation (4.8):

$$S_2(£_{\eta 1}, \Gamma_{\eta 2}) = \boxed{\begin{array}{c} \underset{\xi \in \hat{U}}{min}\left\{\boxed{\underset{\sigma_2 \in \hat{A}, \rho \in \hat{G}}{max}}\begin{pmatrix} \left[{\alpha_1}^-_{(\sigma_2,\rho_1)}(\xi_1), {\alpha_1}^+_{(\sigma_2,\rho_1)}(\xi_1)\right], \\ \left[{\beta_1}^-_{(\sigma_2,\rho_1)}(\xi_1), {\beta_1}^-_{(\sigma_2,\rho_2)}(\xi_1)\right], \\ \left[{\alpha_2}^-_{(\sigma_2,\rho_2)}(\xi_1), {\alpha_2}^+_{(\sigma_2,\rho_2)}(\xi_1)\right], \\ \left[{\beta_2}^-_{(\sigma_2,\rho_2)}(\xi_1), {\beta_2}^-_{(\sigma_2,\rho_2)}(\xi_1)\right], \\ \eta_{1(\sigma_2,\rho_1)}(\xi_1), \eta_{2(\sigma_2,\rho_2)}(\xi_1) \end{pmatrix}\right\}, \\ \underset{\xi \in \hat{U}}{min}\left\{\boxed{\underset{\sigma_2 \in \hat{A}, \rho \in \hat{G}}{max}}\begin{pmatrix} \left[{\alpha_1}^-_{(\sigma_2,\rho_1)}(\xi_2), {\alpha_1}^+_{(\sigma_2,\rho_1)}(\xi_2)\right], \\ \left[{\beta_1}^-_{(\sigma_2,\rho_1)}(\xi_2), {\beta_1}^-_{(\sigma_2,\rho_1)}(\xi_2)\right], \\ \left[{\alpha_2}^-_{(\sigma_2,\rho_2)}(\xi_2), {\alpha_2}^+_{(\sigma_2,\rho_2)}(\xi_2)\right], \\ \left[{\beta_2}^-_{(\sigma_2,\rho_2)}(\xi_2), {\beta_2}^-_{(\sigma_2,\rho_2)}(\xi_2)\right], \\ \eta_{1(\sigma_2,\rho_1)}(\xi_2), \eta_{2(\sigma_2,\rho_2)}(\xi_2) \end{pmatrix}\right\}, \\ \underset{\xi \in \hat{U}}{min}\left\{\boxed{\underset{\sigma_2 \in \hat{A}, \rho \in \hat{G}}{max}}\begin{pmatrix} \left[{\alpha_1}^-_{(\sigma_2,\rho_1)}(\xi_3), {\alpha_1}^+_{(\sigma_2,\rho_1)}(\xi_3)\right], \\ \left[{\beta_1}^-_{(\sigma_2,\rho_1)}(\xi_3), {\beta_1}^-_{(\sigma_2,\rho_1)}(\xi_3)\right], \\ \left[{\alpha_2}^-_{(\sigma_2,\rho_2)}(\xi_3), {\alpha_2}^+_{(\sigma_2,\rho_2)}(\xi_3)\right], \\ \left[{\beta_2}^-_{(\sigma_2,\rho_2)}(\xi_3), {\beta_2}^-_{(\sigma_2,\rho_2)}(\xi_3)\right], \\ \eta_{1(\sigma_2,\rho_1)}(\xi_3), \eta_{2(\sigma_2,\rho_2)}(\xi_3) \end{pmatrix}\right\} \dots\dots\dots, \\ \underset{\xi \in \hat{U}}{min}\left\{\boxed{\underset{\sigma_2 \in \hat{A}, \rho \in \hat{G}}{max}}\begin{pmatrix} \left[{\alpha_1}^-_{(\sigma_2,\rho_1)}(\xi_m), {\alpha_1}^+_{(\sigma_2,\rho_1)}(\xi_m)\right], \\ \left[{\beta_1}^-_{(\sigma_2,\rho_1)}(\xi_m), {\beta_1}^-_{(\sigma_2,\rho_1)}(\xi_m)\right], \\ \left[{\alpha_2}^-_{(\sigma_2,\rho_2)}(\xi_m), {\alpha_2}^+_{(\sigma_2,\rho_2)}(\xi_m)\right], \\ \left[{\beta_2}^-_{(\sigma_2,\rho_2)}(\xi_m), {\beta_2}^-_{(\sigma_2,\rho_2)}(\xi_m)\right], \\ \eta_{1(\sigma_2,\rho_1)}(\xi_m), \eta_{2(\sigma_2,\rho_2)}(\xi_m) \end{pmatrix}\right\} \quad (4.10) \end{array}}$$

when i=l, ŷ=n, ž=1 and x =1,2,3….,m, then by using equation (4.8):

$$S_l(£_{\eta 1}, \Gamma_{\eta 2}) = \begin{array}{l} \min\limits_{\xi \in \hat{U}} \left\{ \boxed{\max\limits_{\sigma_n \in \hat{A}, \rho \in \hat{G}}} \begin{pmatrix} \left[\alpha_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_1), \alpha_1{}^{+}{}_{(\sigma_n,\rho_1)}(\xi_1)\right], \\ \left[\beta_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_1), \beta_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_1)\right], \\ \left[\alpha_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_1), \alpha_2{}^{+}{}_{(\sigma_n,\rho_2)}(\xi_1)\right], \\ \left[\beta_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_1), \beta_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_1)\right], \\ \eta_{1(\sigma_n,\rho_1)}(\xi_1), \eta_{2(\sigma_n,\rho_2)}(\xi_1) \end{pmatrix} \right\}, \\ \min\limits_{\xi \in \hat{U}} \left\{ \boxed{\max\limits_{\sigma_n \in \hat{A}, \rho \in \hat{G}}} \begin{pmatrix} \left[\alpha_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_2), \alpha_1{}^{+}{}_{(\sigma_n,\rho_1)}(\xi_2)\right], \\ \left[\beta_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_2), \beta_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_2)\right], \\ \left[\alpha_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_2), \alpha_2{}^{+}{}_{(\sigma_n,\rho_2)}(\xi_2)\right], \\ \left[\beta_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_2), \beta_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_2)\right], \\ \eta_{1(\sigma_n,\rho_1)}(\xi_2), \eta_{2(\sigma_n,\rho_2)}(\xi_2) \end{pmatrix} \right\}, \\ \min\limits_{\xi \in \hat{U}} \left\{ \boxed{\max\limits_{\sigma_n \in \hat{A}, \rho \in \hat{G}}} \begin{pmatrix} \left[\alpha_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_3), \alpha_1{}^{+}{}_{(\sigma_n,\rho_1)}(\xi_3)\right], \\ \left[\beta_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_3), \beta_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_3)\right], \\ \left[\alpha_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_3), \alpha_2{}^{+}{}_{(\sigma_n,\rho_2)}(\xi_3)\right], \\ \left[\beta_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_3), \beta_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_3)\right], \\ \eta_{1(\sigma_n,\rho_1)}(\xi_3), \eta_{2(\sigma_n,\rho_2)}(\xi_3) \end{pmatrix} \right\} \dots\dots\dots, \\ \min\limits_{\xi \in \hat{U}} \left\{ \boxed{\max\limits_{\sigma_n \in \hat{A}, \rho \in \hat{G}}} \begin{pmatrix} \left[\alpha_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_m), \alpha_1{}^{+}{}_{(\sigma_n,\rho_1)}(\xi_m)\right], \\ \left[\beta_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_m), \beta_1{}^{-}{}_{(\sigma_n,\rho_1)}(\xi_m)\right], \\ \left[\alpha_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_m), \alpha_2{}^{+}{}_{(\sigma_n,\rho_2)}(\xi_m)\right], \\ \left[\beta_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_m), \beta_2{}^{-}{}_{(\sigma_n,\rho_2)}(\xi_m)\right], \\ \eta_{1(\sigma_n,\rho_1)}(\xi_m), \eta_{2(\sigma_n,\rho_2)}(\xi_m) \end{pmatrix} \right\} \end{array} \quad (4.11)$$

And so on. Where

$$\left(\left[\alpha_1{}^{-}{}_{\sigma}^{\rho}(\xi), \alpha_1{}^{+}{}_{\sigma}^{\rho}(\xi)\right], \left[\beta_1{}^{-}{}_{\sigma}^{\rho}(\xi), \beta_1{}^{+}{}_{\sigma}^{\rho}(\xi)\right]\right) = \frac{1}{3}\begin{pmatrix} \alpha_1{}^{-}{}_{\sigma}^{\rho}(\xi) + \alpha_1{}^{+}{}_{\sigma}^{\rho}(\xi) - \\ \alpha_1{}^{-}{}_{\sigma}^{\rho}(\xi)\alpha_1{}^{+}{}_{\sigma}^{\rho}(\xi) + \\ \beta_1{}^{-}{}_{\sigma}^{\rho}(\xi) + \beta_1{}^{+}{}_{\sigma}^{\rho}(\xi) - \\ \beta_1{}^{-}{}_{\sigma}^{\rho}(\xi)\beta_1{}^{+}{}_{\sigma}^{\rho}(\xi) \end{pmatrix}, \quad (4.12)$$

$$\left(\left[\alpha_2{}^{-}{}_{\sigma}^{\rho}(\xi), \alpha_2{}^{+}{}_{\sigma}^{\rho}(\xi)\right], \left[\beta_2{}^{-}{}_{\sigma}^{\rho}(\xi), \beta_2{}^{+}{}_{\sigma}^{\rho}(\xi)\right]\right) = \frac{1}{3}\begin{pmatrix} \alpha_2{}^{-}{}_{\sigma}^{\rho}(\xi) + \alpha_2{}^{+}{}_{\sigma}^{\rho}(\xi) - \\ \alpha_2{}^{-}{}_{\sigma}^{\rho}(\xi)\alpha_2{}^{+}{}_{\sigma}^{\rho}(\xi) + \\ \beta_2{}^{-}{}_{\sigma}^{\rho}(\xi) + \beta_2{}^{+}{}_{\sigma}^{\rho}(\xi) - \\ \beta_2{}^{-}{}_{\sigma}^{\rho}(\xi)\beta_2{}^{+}{}_{\sigma}^{\rho}(\xi) \end{pmatrix}, \quad (4.13)$$

Then the similarity measures between the GIVIFSESs $£_{\eta_1}$ and Γ_{η_2} is defined as,

$$\widetilde{SM}\left(£_{\eta_1}, \Gamma_{\eta_2}\right) = \frac{1}{n}\Sigma S_i\left(£_{\eta_1}, \Gamma_{\eta_2}\right) \qquad (4.14)$$

Theorem 4.3 If $\widetilde{SM}\left(£_{\eta 1}, \Gamma_{\eta_2}\right)$ be the similarity measures between two GIVIFSESs $£_{\eta_1}$ and Γ_{η_2} Then.

i. $\widetilde{SM}\left(£_{\eta_1}, \Gamma_{\eta_2}\right) = \widetilde{SM}\left(\Gamma_{\eta_2}, £_{\eta_1}\right)$,
ii. $0 \leq \widetilde{SM}\left(£_{\eta_1}, \Gamma_{\eta_2}\right) \leq 1$,
iii. $\widetilde{SM}\left(£_{\eta_1}, \Gamma_{\eta_2}\right) = 1 \Leftrightarrow £_{\eta_1} = \Gamma_{\eta_2}$.

4.3 Type-III Similarity Measures for Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert Sets

Definition 4.4 Let $\xi = \{\xi_x : x = 1,2,3,\ldots\ldots,m\}$ be the finite universal set of elements ξ_x, $\hat{A} = \{\sigma_{\hat{y}} : \hat{y} = 1,2,3,\ldots\ldots,n\}$ be the set of attributes and $\hat{G} = \{\rho_{\check{z}} : \check{z} = 1.2.3,\ldots\ldots,t\}$ be the set of experts (or decision makers). Let $£_{\eta_1}$ and Γ_{η_2} be two GIVIFSESs over ξ. Let $\widehat{SM}(£_{\eta_1},\Gamma_{\eta_2})$ specify the similarity measures between GIVIFSESs $£_{\eta_1}$ and Γ_{η_2}. We must first find similarities between their $\rho_{\check{z}}$th approximations to see similar measurements $£_{\eta_1}$ and Γ_{η_2}. Let $S_i(£_{\eta_1},\Gamma_{\eta_2})$ indicate the similarity measure between the two $\rho_{\check{z}}$th approximation of

$$£_{\eta_1}{}_{\sigma}^{\rho} = \left(\left\{< \xi, \left[\alpha_1{}^{-}{}_{\sigma}^{\rho}(\xi), \alpha_1{}^{+}{}_{\sigma}^{\rho}(\xi)\right], \left[\beta_1{}^{-}{}_{\sigma}^{\rho}(\xi), \beta_1{}^{+}{}_{\sigma}^{\rho}(\xi)\right] >\right\}, \eta_1{}_{\sigma}^{\rho}(\xi)\right),$$

$$\Gamma_{\eta_2}{}_{\sigma}^{\rho} = \left(\left\{< \xi, \left[\alpha_2{}^{-}{}_{\sigma}^{\rho}(\xi), \alpha_2{}^{+}{}_{\sigma}^{\rho}(\xi)\right], \left[\beta_2{}^{-}{}_{\sigma}^{\rho}(\xi), \beta_2{}^{+}{}_{\sigma}^{\rho}(\xi)\right] >\right\}, \eta_2{}_{\sigma}^{\rho}(\xi)\right),$$

Then $S_i(£_{\eta_1},\Gamma_{\eta_2})$ is defined as follow:

$S_i(£_{\eta_1},\Gamma_{\eta_2}) =$

$$\left[\sum_{\check{z}=1}^{t}\sum_{\hat{y}=1}^{n}\sum_{x=1}^{m}\left(\frac{\left|\left[\alpha_1{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x),\alpha_1{}^{+}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x)\right],\left[\beta_1{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x),\beta_1{}^{+}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x)\right]-\eta_1{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x)\right| \wedge \left|\left[\alpha_2{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x),\alpha_2{}^{+}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x)\right],\left[\beta_2{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x),\beta_2{}^{+}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x)\right]-\eta_2{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x)\right|}{\left|\left[\alpha_1{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x),\alpha_1{}^{+}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x)\right],\left[\beta_1{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x),\beta_1{}^{+}{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x)\right]-\eta_1{}_{(\sigma_{\hat{y}},\rho_{\check{z}})}(\xi_x)\right| \vee \left|\left[\alpha_2{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x),\alpha_2{}^{+}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x)\right],\left[\beta_2{}^{-}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x),\beta_2{}^{+}{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x)\right]-\eta_2{}_{(\sigma_{\hat{y}},\rho_{\check{z}+1})}(\xi_x)\right|}\right)\right] \quad (4.15)$$

Where i=1, 2, 3,......., l and

$$\left(\left[\alpha_1{}^{-}{}_{\sigma}^{\rho}(\xi), \alpha_1{}^{+}{}_{\sigma}^{\rho}(\xi)\right], \left[\beta_1{}^{-}{}_{\sigma}^{\rho}(\xi), \beta_1{}^{+}{}_{\sigma}^{\rho}(\xi)\right]\right) = \frac{1}{2}\begin{pmatrix}\alpha_1{}^{-}{}_{\sigma}^{\rho}(\xi) + \alpha_1{}^{+}{}_{\sigma}^{\rho}(\xi)\\ +\\ \beta_1{}^{-}{}_{\sigma}^{\rho}(\xi) + \beta_1{}^{+}{}_{\sigma}^{\rho}(\xi)\end{pmatrix}, \quad (4.16)$$

$$\left(\left[\alpha_2{}^{-}{}_{\sigma}^{\rho}(\xi), \alpha_2{}^{+}{}_{\sigma}^{\rho}(\xi)\right], \left[\beta_2{}^{-}{}_{\sigma}^{\rho}(\xi), \beta_2{}^{+}{}_{\sigma}^{\rho}(\xi)\right]\right) = \frac{1}{2}\begin{pmatrix}\alpha_2{}^{-}{}_{\sigma}^{\rho}(\xi) + \alpha_2{}^{+}{}_{\sigma}^{\rho}(\xi)\\ +\\ \beta_2{}^{-}{}_{\sigma}^{\rho}(\xi) + \beta_2{}^{+}{}_{\sigma}^{\rho}(\xi)\end{pmatrix}, \quad (4.17)$$

then the similarity measures between the GIVIFSESs $£_{\eta_1}$ *and* Γ_{η_2} is described as,

$$\widehat{SM}(£_{\eta_1},\Gamma_{\eta_2}) = \frac{1}{n}\Sigma S_i(£_{\eta_1},\Gamma_{\eta_2}) \qquad (4.18)$$

Theorem 4.4 If $\widehat{SM}(£_{\eta_1},\Gamma_{\eta_2})$ be the similarity measures between two GIVIFSESs $£_{\eta_1}$ and Γ_{η_2} then.

i. $\widehat{SM}(£_{\eta_1},\Gamma_{\eta_2}) = \widehat{SM}(\Gamma_{\eta_2},£_{\eta_1})$
ii. $0 \le \widehat{SM}(£_{\eta_1},\Gamma_{\eta_2}) \le 1$
iii. $\widehat{SM}(£_{\eta_1},\Gamma_{\eta_2}) = 1 \Leftrightarrow £_{\eta_1} = \Gamma_{\eta_2}$

5. Application in Decision-Making Problems

In this section, we will apply the proposed similarity measure for the decision-making problem in the healthcare field. The step-by-step algorithm is defined below.

5.1 Algorithm for Decision-making based on the Proposed Model

Here, we introduce an algorithm depending on the similarity measures of two GIVIFSESs to evaluate the possibility that the sick person having specific symptoms is suffering from typhoid fever. For this purpose, we first established a model of GIVIFSESs by using the opinion of a specialist over the criteria of the symptoms for a sick person and another GIVIFSESs using a different opinion of the next specialist using the same criteria for sickness. Then we calculate the similarity measures of these two GIVIFSESs. We also suppose that if the similarity measures between these two GIVIFSESs is greater than or equal to 0.5 then we decide that the person is possibly affected by typhoid fever.

The algorithm of the similarity measures for GIVIFSESs is as follows:
Step 1: Construct a model GIVIFSES over the universe ξ based on the opinion of a specialist for a sick person.
Step 2**:** Construct GIVIFSESs over the universe ξ based on another specialist opinion for sickness.
Step 3**:** Calculate Type-I, Type-II, and Type-III similarity measures between the two GIVIFSESs.
Step 4**:** Estimate the result by using similarity measures, where if the similarity measures between these two GIVIFSESs is greater than or equal to 0.5 then the person is possibly affected by typhoid fever, and if the similarity measures between these two GIVIFSESs is less than 0.5 then the person is not possibly affected by typhoid fever.

5.2 Mathematical Applications of Proposed Model

Let $\xi = \{\xi_1, \xi_2\}$ be the universal set, where ξ holds only two elements which are ξ_1=typhoid fever and ξ_2= Not typhoid fever. Here the set of attributes σ is the set of certain visible symptoms of typhoid fever. Let $\hat{A} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$, where σ_1 =body temperature, σ_2 =diarrhea, σ_3 =weakness, σ_4 =muscle aches and $\hat{G} = \{\rho_1, \rho_2\}$Be the set of specialists.

Step 1: Construct a model for GIVIFSES $£_{\eta_1}$ over the universe ξ based on the opinion of a specialist ρ_1 for a sick person as shown in Table 3:

Table 3. Opinion of specialist $\boldsymbol{\rho_1}$.

$£_{\eta_1}$	$(\boldsymbol{\sigma_1}, \boldsymbol{\rho_1}), \eta^{\rho}_{\sigma}(\xi)$	$(\boldsymbol{\sigma_2}, \boldsymbol{\rho_1}), \eta^{\rho}_{\sigma}(\xi)$	$(\boldsymbol{\sigma_3}, \boldsymbol{\rho_1}), \eta^{\rho}_{\sigma}(\xi)$	$(\boldsymbol{\sigma_4}, \boldsymbol{\rho_1}), \eta^{\rho}_{\sigma}(\xi)$
$\boldsymbol{\xi_1}$	$\begin{pmatrix}[0.1,0.3],[0.2,0.4],\\0.1\end{pmatrix}$	$\begin{pmatrix}[0.1,0.2],[0.3,0.4],\\0.2\end{pmatrix}$	$\begin{pmatrix}[0.3,0.4],[0.4,0.5],\\0.4\end{pmatrix}$	$\begin{pmatrix}[0.4,0.6],[0.1,0.5],\\0.6\end{pmatrix}$
$\boldsymbol{\xi_2}$	$\begin{pmatrix}[0.3,0.4],[0.3,0.5],\\0.5\end{pmatrix}$	$\begin{pmatrix}[0.2,0.3],[0.3,0.4],\\0.4\end{pmatrix}$	$\begin{pmatrix}[0.7,0.8],[0.1,0.2],\\0.3\end{pmatrix}$	$\begin{pmatrix}[0.3,0.7],[0.2,0.4],\\0.7\end{pmatrix}$

Step 2: Construct a model for GIVIFSES Γ_{η_2}over the universe ξ for typhoid fever based on another specialist ρ_2 opinion for a sickness as shown in Table 4:

Table 4. Opinion of specialist $\boldsymbol{\rho_2}$.

Γ_{η_2}	$(\boldsymbol{\sigma_1}, \boldsymbol{\rho_2}), \eta^{\rho}_{\sigma}(\xi)$	$(\boldsymbol{\sigma_2}, \boldsymbol{\rho_2}), \eta^{\rho}_{\sigma}(\xi)$	$(\boldsymbol{\sigma_3}, \boldsymbol{\rho_2}), \eta^{\rho}_{\sigma}(\xi)$	$(\boldsymbol{\sigma_4}, \boldsymbol{\rho_2}), \eta^{\rho}_{\sigma}(\xi)$
$\boldsymbol{\xi_1}$	$\begin{pmatrix}[0.4,0.5],[0.1,0.2],\\0.4\end{pmatrix}$	$\begin{pmatrix}[0.3,0.4],[0.2,0.3],\\0.7\end{pmatrix}$	$\begin{pmatrix}[0.4,0.5],[0.3,0.4],\\0.0\end{pmatrix}$	$\begin{pmatrix}[0.4,0.7],[0.1,0.3],\\0.5\end{pmatrix}$
$\boldsymbol{\xi_2}$	$\begin{pmatrix}[0.6,0.7],[0.2,0.4],\\0.32\end{pmatrix}$	$\begin{pmatrix}[0.4,0.5],[0.1,0.4],\\0.9\end{pmatrix}$	$\begin{pmatrix}[0.0,0.3],[0.1,0.5],\\0.8\end{pmatrix}$	$\begin{pmatrix}[0.6,0.7],[0.1,0.4],\\0.1\end{pmatrix}$

Step 3: Calculate Type-I, Type-II, and Type-III similarity measures between the two GIVIFSESs $£_{\eta_1}$ and Γ_{η_2} Respectively, we have.

Type-I similarity measure for Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert sets:
Now we use equations 4.5 and 4.6 respectively, we get.

$$£_{\eta_1} = \begin{Bmatrix} ((\sigma_1,\rho_1),\eta_1(\xi)) & ((\sigma_2,\rho_1),\eta_1(\xi)) & ((\sigma_3,\rho_1),\eta_1(\xi)) & ((\sigma_4,\rho_1),\eta_1(\xi)) \\ \xi_1 = (0.29,0.1) & (0.28,0.2) & (0.42,0.4) & (0.43,0.6) \\ \xi_2 = (0.41,0.5) & (0.34,0.4) & (0.40,0.3) & (0.43,0.7) \end{Bmatrix},$$

$$\Gamma_{\eta_2} = \begin{Bmatrix} ((\sigma_1,\rho_2),\eta_1(\xi)) & ((\sigma_2,\rho_2),\eta_1(\xi)) & ((\sigma_3,\rho_2),\eta_1(\xi)) & ((\sigma_4,\rho_2),\eta_1(\xi)) \\ \xi_1 = (0.32,0.4) & (0.34,0.7) & (0.42,0.0) & (0.39,0.5) \\ \xi_2 = (0.46,0.32) & (0.38,0.9) & (0.28,0.8) & (0.44,0.1) \end{Bmatrix},$$

Then by using Eq. (4.1) when i=1

$$S_1(£_{\eta_1},\Gamma_{\eta_2}) = max_{\xi_1}\begin{Bmatrix} min \\ \rho_1,\rho_2(0.29, 0.1, 0.32, 0.4) \\ \sigma_1 \end{Bmatrix}, max_{\xi_2}\begin{Bmatrix} min \\ \rho_1,\rho_2(0.41, 0.5, 0.46, 0.32) \\ \sigma_1 \end{Bmatrix}, max\{(0.1, 0.32)\}, 0.32$$

Similarly, by using Eq. (4.1) when i=2

$$S_2(£_{\eta_1},\Gamma_{\eta_2}) = max_{\xi_1}\begin{Bmatrix} min \\ \rho_1,\rho_2(0.28, 0.2, 0.34, 0.7) \\ \sigma_2 \end{Bmatrix}, max_{\xi_2}\begin{Bmatrix} min \\ \rho_1,\rho_2(0.34, 0.4, 0.38, 0.9) \\ \sigma_2 \end{Bmatrix}, max\{(0.2, 0.34)\}, 0.34$$

Similarly, by using Eq. (4.1) when i=3

$$S_3(£_{\eta_1},\Gamma_{\eta_2}) = max_{\xi_1}\begin{Bmatrix} min \\ \rho_1,\rho_2(0.42, 0.4, 0.42, 0.0) \\ \sigma_3 \end{Bmatrix}, max_{\xi_2}\begin{Bmatrix} min \\ \rho_1,\rho_2(0.40, 0.3, 0.28, 0.8) \\ \sigma_3 \end{Bmatrix}, max\{(0.0, 0.28)\}, 0.28$$

Similarly, by using Eq. (4.1) when i=4

$$S_4(£_{\eta_1},\Gamma_{\eta_2}) = max_{\xi_1}\begin{Bmatrix} min \\ \rho_1,\rho_2(0.43, 0.6, 0.39, 0.5) \\ \sigma_4 \end{Bmatrix}, max_{\xi_2}\begin{Bmatrix} min \\ \rho_1,\rho_2(0.43, 0.7, 0.44, 0.1) \\ \sigma_4 \end{Bmatrix}, max\{(0.39, 0.1)\}, 0.39$$

Now by using Eq. (4.7) the similarity measure of Type-I between the GIVIFSESs $£_{\eta_1}$ and Γ_{η_2} is,

$$SM(£_{\eta_1},\Gamma_{\eta_2}) = \frac{1}{4}(0.32 + 0.34 + 0.28 + 0.39)$$

$$= 0.33$$

Type-II similarity measure for Generalized Interval Valued Intuitionistic Fuzzy Soft Expert sets: Now by using Eq. (4.8) we have, when i = 1

$$S_1(£_{\eta_1}, \Gamma_{\eta_2}) = min_{\xi_1}\left\{\begin{matrix} max \\ \rho_1, \rho_2(0.29, 0.1, 0.32, 0.4) \\ \sigma_1 \end{matrix}\right\}, \quad min_{\xi_2}\left\{\begin{matrix} max \\ \rho_1, \rho_2(0.41, 0.5, 0.46, 0.32) \\ \sigma_1 \end{matrix}\right\}, \quad min\{(0.4, 0.5)\}, \quad 0.4$$

Similarly, by using Eq. (4.8) when i=2

$$S_2(£_{\eta_1}, \Gamma_{\eta_2}) = min_{\xi_1}\left\{\begin{matrix} max \\ \rho_1, \rho_2(0.28, 0.2, 0.34, 0.7) \\ \sigma_2 \end{matrix}\right\}, \quad min_{\xi_2}\left\{\begin{matrix} max \\ \rho_1, \rho_2(0.34, 0.4, 0.38, 0.9) \\ \sigma_2 \end{matrix}\right\}, \quad min\{(0.7, 0.9)\}, \quad 0.7$$

Similarly, by using Eq. (4.8) when i=3

$$S_3(£_{\eta_1}, \Gamma_{\eta_2}) = min_{\xi_1}\left\{\begin{matrix} max \\ \rho_1, \rho_2(0.42, 0.4, 0.42, 0.0) \\ \sigma_3 \end{matrix}\right\}, \quad min_{\xi_2}\left\{\begin{matrix} max \\ \rho_1, \rho_2(0.40, 0.3, 0.28, 0.8) \\ \sigma_3 \end{matrix}\right\}, \quad min\{(0.42, 0.8)\}, \quad 0.42$$

Similarly, by using Eq. (4.8) when i=4

$$S_4(£_{\eta_1}, \Gamma_{\eta_2}) = min_{\xi_1}\left\{\begin{matrix} max \\ \rho_1, \rho_2(0.43, 0.6, 0.39, 0.5) \\ \sigma_4 \end{matrix}\right\}, \quad min_{\xi_2}\left\{\begin{matrix} max \\ \rho_1, \rho_2(0.43, 0.7, 0.44, 0.1) \\ \sigma_4 \end{matrix}\right\}, \quad min\{(0.6, 0.7)\}, \quad 0.6$$

Now by using Eq. (4.14) the similarity measure of Type-II between the GIVIFSESs $£_{\eta_1}$ and Γ_{η_2} is,

$$\widetilde{SM}\,(£_{\eta_1}, \Gamma_{\eta_2}) = \frac{1}{4}(0.4 + 0.7 + 0.42 + 0.6)$$

$$= 0.53$$

Type-III similarity measure for Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert sets:

Now using Eq. (4.16) and Eq. (4.17)

$$£_{\eta 1} = \left\{\begin{matrix} ((\sigma_1, \rho_1) - \eta_1(\xi)) & ((\sigma_2, \rho_1) - \eta_1(\xi)) & ((\sigma_3, \rho_1) - \eta_1(\xi)) & ((\sigma_4, \rho_1) - \eta_1(\xi)) \\ \xi_1 = |0.5 - 0.1| = 0.4 & |0.5 - 0.2| = 0.3 & |0.8 - 0.4| = 0.4 & |0.8 - 0.6| = 0.2 \\ \xi_2 = |0.75 - 0.5| = 0.25 & |0.6 - 0.4| = 0.2 & |0.9 - 0.3| = 0.6 & |0.8 - 0.7| = 0.1 \end{matrix}\right\},$$

$$\Gamma_{\eta_2} = \left\{\begin{matrix} ((\sigma_1, \rho_2) - \eta_2(\xi)) & ((\sigma_2, \rho_2) - \eta_2(\xi)) & ((\sigma_3, \rho_2) - \eta_2(\xi)) & ((\sigma_4, \rho_2) - \eta_2(\xi)) \\ \xi_1 = |0.6 - 0.4| = 0.2 & |0.6 - 0.7| = 0.1 & |0.8,0.0| = 0.8 & |1.75,0.5| = 0.25 \\ \xi_2 = |0.95 - 0.32| = 0.63 & |0.7 - 0.9| = 0.2 & |0.45 - 0.8| = 0.35 & |0.9,0.1| = 0.8 \end{matrix}\right\},$$

Now by using Eq. (4.15), we have, when i= 1

$$S_1(£_{\eta_1}, \Gamma_{\eta_2}) = \frac{0.2 + 0.25}{0.4 + 0.63} = 0.43,$$

Now by using Eq. (4.15), we have, when i= 2

$$S_2(£_{\eta_1}, \Gamma_{\eta_2}) = \frac{0.1 + 0.2}{0.3 + 0.2} = 0.6,$$

Now by using Eq. (4.15), we have, when i= 3

$$S_3(£_{\eta_1}, \Gamma_{\eta_2}) = \frac{0.4 + 0.35}{0.8 + 0.6} = 0.53,$$

Now by using Eq. (4.15), we have, when i= 4

$$S_4(£_{\eta_1}, \Gamma_{\eta_2}) = \frac{0.2 + 0.1}{0.25 + 0.8} = 0.28,$$

Therefore, by using Eq. (4.18) the similarity measure between GIVIFSESs $£_{\eta_1}$ and Γ_{η_2} is specified by.

$$\widehat{SM}(£_{\eta_1}, \Gamma_{\eta_2}) = \frac{1}{4}(0.43 + 0.6 + 0.53 + 0.28),$$

$$= 0.46$$

Step 4: Now we can easily see that from Type-I and Type-III there are less than 0.5 so we can say the person is not possibly affected by typhoid fever. But on another hand the result from Type-II similarity measures is $\widehat{SM}(£_{\eta_1}, \Gamma_{\eta_2}) = 0.53 > 0.5$ then we conclude that the person is possibly affected by typhoid fever.

6. Conclusion

In conclusion, this paper has introduced a novel framework of Generalized Interval-Valued Intuitionistic Fuzzy Soft Expert Sets (GIVIFSESs) and a suite of similarity measures—Type-I, Type-II, and Type-III—demonstrating their practical application in the medical diagnosis domain. The flexibility of GIVIFSESs in handling uncertainty and imprecision, along with the ability to quantify resemblance between medical cases, enhances decision-making accuracy in diagnostic processes. The empirical validation confirms the potential of these measures to support more informed and reliable medical decisions.

For future research, several promising directions can be explored. First, the extension of the proposed similarity measures to other domains, such as risk assessment, financial forecasting, and environmental monitoring, could further validate the adaptability of GIVIFSESs in handling complex, uncertain data. Second, integrating these similarity measures with machine learning algorithms may enhance automated decision-support systems, combining the strengths of data-driven models with expert knowledge. Additionally, more sophisticated types of similarity measures could be developed to capture higher-order relationships between variables. Lastly, conducting large-scale real-world experiments across different medical fields will further solidify the practical benefits of GIVIFSESs in improving diagnostic outcomes in the given fuzzy data [30-33].

Declarations

Ethics Approval and Consent to Participate

The results/data/figures in this manuscript have not been published elsewhere, nor are they under consideration by another publisher. All the material is owned by the authors, and/or no permissions are required.

Consent for Publication

This article does not contain any studies with human participants or animals performed by any of the authors.

Availability of Data and Materials

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Competing Interests

The authors declare no competing interests in the research.

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