



# On Neutrosophic $p\delta s$ -irresolute Functions in Neutrosophic Topological Spaces

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**Abstract:** In general topology the notion of  $p\delta s$ -irresolute and  $\alpha\delta s$ -irresolute functions were introduced by Beceren and Noiri. In the present paper, these concepts of  $p\delta s$ -irresolute (briefly,  $Np\delta s$ -irresolute) and  $\alpha\delta s$ -irresolute (briefly,  $N\alpha\delta s$ -irresolute) functions are explored for the first time in neutrosophic topological spaces (NTS) as generalized version. We proved that every  $N\alpha\delta s$ -irresolute function is  $Np\delta s$ -irresolute function but not conversely. Some characterizations, counter examples, and fundamental features are also presented. By neutrosophic pre-open, neutrosophic  $\delta$ -open, and neutrosophic  $\delta$ -semi-open sets, some new fundamental properties of such functions are provided. Furthermore, under  $Np\delta s$ -irresolute functions, the behavior of neutrosophic semi-connected, neutrosophic pre-connected, neutrosophic pre- $T_2$ , neutrosophic  $\delta$ -semi- $T_2$ , neutrosophic pre-compact, and neutrosophic  $s$ -closed spaces are investigated.

**Keywords:** Neutrosophic Set; Neutrosophic  $\delta$ -cluster Point; Neutrosophic  $\delta$ -open Set; Neutrosophic- $\delta$ -semi Open Set; Neutrosophic pre-open Set; Neutrosophic  $\alpha$ -open Set; Neutrosophic  $p\delta s$ -irresolute Function; Neutrosophic  $\alpha\delta s$ -irresolute function, Neutrosophic  $\delta$ -semi  $T_2$  space, Neutrosophic Pre-connected Space; Neutrosophic Pre-compact Space.

## 1. Introduction and Literature Review

Based on classical data general topology is intended to find applications in various fields such as robotics, network design, data analysis, material science, physics, and biology. It helps us understand shapes, connectivity, and spatial relationships in complex systems. Two key mappings for describing topological properties in topological spaces are continuity and irresolute functions. The concept of irresoluteness has constituted the basis of many precious researches in general topology. The results of these studies have been reflected in numerous applications, and they have played important roles in many facets of real life. But as technology has advanced and the industry has undergone a revolution, people's requirements have also evolved, and as a result, general topology has lagged in practical applications. Besides, these findings' effects on actual practices have diminished. The phenomena of making decisions accurately is nowadays the most complicated task in the real world due to the presence of parameters of uncertainty and imprecise. Furthermore, in several other fields, including economics, engineering, the environment, social science, medical science, etc., classical approaches are inadequate for addressing several real-world issues. Thus, it is now necessary for scientists to re-examine some of the basic problems in mathematics and discover new kinds of spaces and functions, according to the new theories proposed (see [16-22]). In these topological spaces, irresolute functions have taken up a significant amount of characteristics.

Zadeh [9] proposed the novel concept of fuzzy set (FS) in handling vague, imprecise, and uncertain types of data in 1965. Later, the paper Chang [1] paved the way for the subsequent development of numerous fuzzy topological concepts. As a generalization of a fuzzy set, Atanassov [8] created an intuitionistic fuzzy set (IFS) in 1986. His theory thereafter became widely acknowledged as an essential resource in the fields of mathematics, engineering [15], and medicine [18]. Coker [3]

defined intuitionistic fuzzy topological spaces using the concept of intuitionistic fuzzy sets in 1997 and studied some engrossing results. In 1995, the idea of the neutrosophic set (in short, NS) was proposed by Smarandache [11] as a generalization of the intuitionistic fuzzy set (IFS). Neutrosophic logic [11], neutrosophic vector space [29], neutrosophic topological space [12], neutrosophic group theory [30], neutrosophic ring theory [31], and dombi neutrosophic graph [32] are only a few of the many academics who have contributed to the field of neutrosophic theory. The idea of a neutrosophic set and its wide range of applications are being developed by numerous academicians. The attributes of neutrosophic logic and applications were developed by El-Hamed E.A. et al [2]. Salama and Al-Blawi [12] initiated the thought of neutrosophic topological spaces (shortly, NTS) utilizing a neutrosophic set. The notion of the univalent neutrosophic number (SVN-number) has been introduced in a different mode by investigating its structural aspects by Bera et al. [23]. They have constructed an assignment issue model in the neutrosophic context, along with the solution methodology. A Data Envelopment Analysis (DEA) model has been introduced by Yang et al. [24] with neutrosophic sets. The healthcare system has adopted this new concept. As a result, they have produced beneficial real-world outcomes. Triangular neutrosophic numbers (TNNs) are used as the information on decision-making units in Edalatpanah's attempt to create a novel DEA model in [25]. The use of neutrosophic logic in the confirmatory data analysis of the life satisfaction measure was first presented by Duran et al. [26]. The idea of an algorithm-based neutrosophic soft matrix has been applied by Dhar [27] to address some of the issues with disease diagnosis brought on by the emergence of different symptoms in patients.

The idea of quadripartitioned neutrosophic Pythagorean sets has been applied to Lie algebras by Radha and Stanis Arul Mary [28]. Arokiarani I. et al. [5] coined the concept of neutrosophic semi-open sets in NTS. Subsequently, several authors intended and introduced various types of open sets in NTS. Weaker and stronger forms of open and closed sets play a vital role in neutrosophic topology because the significance of NS is rapidly rising in many areas of applications and they are the present research area of many researchers over the entire Globe. The concept of neutrosophic pre-open sets was conceived by Arokiarani I. [5], as a weaker version of  $\alpha$ -open set in NTS. Kannan and Chandrasekar [34] studied the concept of neutrosophic pre- $\alpha$ , semi- $\alpha$ , pre- $\beta$  irresolute open and closed mappings, and Priya and Chandrasekar [33] proposed neutrosophic  $\alpha$ gs-continuity and irresolute maps. It has been observed that neutrosophic  $\alpha$ -open sets and neutrosophic  $\delta$ -semi-open sets introduced by Vadivel A. et al. [7] play a vital role in our present research.

### 1.1 Research Gaps and Contribution of the Paper

During the literature survey, we observe that

- i). Neutrosophic topological spaces and topological properties have almost been established. However, very little study has been addressed on the preservation aspects of these properties under irresolute functions.
- ii). Only some works have been done on continuous and irresolute functions with the help of neutrosophic semi-open, regular open, pre-open,  $\alpha$ -open,  $\beta$ -open sets.
- iii). We notice that only a few papers used  $N\delta$ -open and  $N\delta$ -semi-open sets to describe continuity. This motivated us to introduce the novel idea of  $p\delta$ s-irresolute and  $\alpha\delta$ s-irresolute functions in a neutrosophic environment.

In the present work we contribute the following notions to fill the gaps:

- i). We define the idea of  $Np\delta$ s-irresolute and  $N\alpha\delta$ s-irresolute functions in neutrosophic topological spaces. Further, we prove that every neutrosophic  $\alpha\delta$ s-irresolute function is  $Np\delta$ s-irresolute function. But with a counter-example, we show the converse is false.
- ii). Moreover, some characterizations, counterexamples and basic properties of  $Np\delta$ s-irresolute function are investigated.

- iii). Finally, the behaviors of neutrosophic semi-connected, neutrosophic pre-connected, neutrosophic pre- $T_2$ , neutrosophic  $\delta$ -semi  $T_2$ , neutrosophic pre-compact, neutrosophic s-closed spaces are examined in the light of  $Np\delta s$ -irresolute function.

### 1.2 Roadmap of the Paper

This work is organized into the following five sections: we presented the introduction and literature review in the first section. Section 2 focuses on common definitions and preliminaries. Section 3 describes the concept of  $p\delta s$ -irresolute and  $\alpha\delta s$ -irresolute functions in neutrosophic topological spaces. We proved that every neutrosophic  $\alpha\delta s$ -irresolute function is  $p\delta s$ -irresolute function but not conversely. Some characterizations, counterexamples, and basic properties are also investigated. In Section 4, the behavior of neutrosophic semi-connected, neutrosophic pre-connected, neutrosophic pre- $T_2$ , neutrosophic  $\delta$ -semi  $T_2$ , neutrosophic pre-compact, neutrosophic s-closed spaces are examined in the light of neutrosophic  $p\delta s$ -irresolute function. Finally, a concluding remark on our work and future research direction has been presented in Section 5.

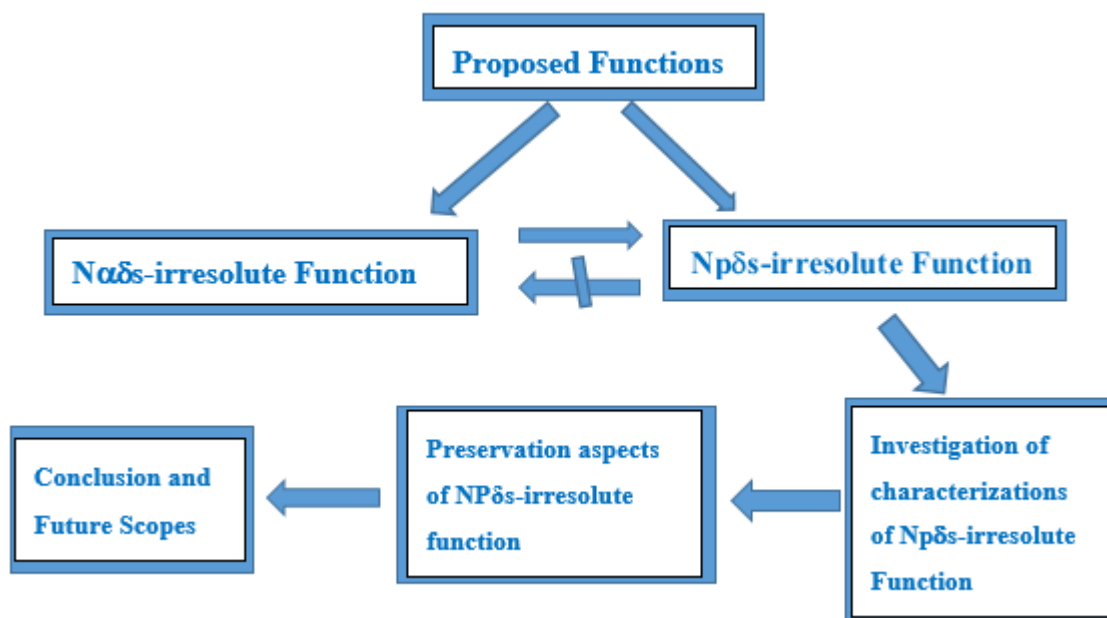


Figure 1. Flow chart of the proposed work.

## 2. Mathematical Preliminaries

This section consists of some common notations and definitions which have been involved in the course of the paper. Throughout the paper, the function  $\psi:(X,\tau_X)\rightarrow (Y,\nu_Y)$  will represent a single-valued neutrosophic function from a neutrosophic topological space (in short, NTS)  $(X,\tau_X)$  into another neutrosophic topological space  $(Y,\nu_Y)$  and ordered pairs  $(X,\tau_X)$  and  $(Y,\nu_Y)$  stand neutrosophic topological spaces  $X$  and  $Y$  respectively. For any neutrosophic subset  $W \subset X$ ,  $NInt(W)$ ,  $NSInt(W)$ ,  $NCI(W)$  and  $NSCI(W)$  represent the neutrosophic interior of  $W$ , neutrosophic semi interior of  $W$ , neutrosophic closure of  $W$  and neutrosophic semi-closure of  $W$  respectively.

**Definition 2.1.** [9] Let  $U$  be a universe of discourse. Then the fuzzy set on  $U$  is described as  $F = \{x, \mu(x) : x \in X\}$  where,  $\mu(x) \in [0,1]$ , denotes the degree of membership of  $x \in X$ .

**Definition 2.2.** [4] Let  $U$  be an initial universe and  $E$  be a set of parameters. Let  $P(U)$  denote the power set of  $U$  and  $A \subset E$ . Then the pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$  such that  $F(e) \in P(U)$ .

**Definition 2.3.** [13] Let  $U$  be an initial universe and  $E$  be a set of parameters. Let  $F(U)$  denote the set of all fuzzy sets of  $U$ . Then the fuzzy soft set  $F_A$  can be represented by the set of ordered pairs  $F_A = \{(x, \gamma_A(x)): x \in E, \gamma_A(x) \in F(U)\}$ .

**Definition 2.4.** [8] Let  $U$  be a universe of discourse. Then the intuitionistic fuzzy set on  $U$  is described as  $F = \{(x, \mu(x), \nu(x)): x \in X, \text{ where, } \mu(x), \nu(x) \in [0,1], \text{ indicating the degrees of membership and non-membership respectively such that } 0 \leq \mu(x) + \nu(x) \leq 1 \text{ and the hesitant/ indeterminate part } = 1 - \mu(x) - \nu(x)\}$ .

**Definition 2.5.** [11] Let  $X$  be a universe of discourse. Then the neutrosophic set is defined by  $N = \{(x, \tau(x), \lambda(x), \eta(x)), x \in X\}$ , where,  $\tau(x), \lambda(x), \eta(x) \in [0,1]$ , indicating the degrees of truth, indeterminacy and falsehood respectively that satisfy  $0 \leq \inf(\tau) + \inf(\lambda) + \inf(\eta) \leq \sup(\tau) + \sup(\lambda) + \sup(\eta) \leq 3$ .

**Definition 2.6.** [15] A Neutrosophic point (N-point)  $S_{(t,i,f)}$  of  $NTS(X, \tau)$  is a neutrosophic set defined by

$$S_{(t,i,f)}(x) = \begin{cases} (t, i, f), & \text{if } x = s \\ (0, 1, 1), & \text{if } x \neq s \end{cases}$$

where,  $t, i, f \in [0,1]$  such that  $0 < t+i+f \leq 3$ . In this context, 'S' is referred to as the support of the N-point and  $(t, i, f)$  is the value of the N-point.

A N-point  $S_{(t,i,f)} \in N = \{(x, \tau(x), \lambda(x), \eta(x)), x \in X\}$  iff  $t \leq \tau(s), i \geq \lambda(s)$  and  $f \geq \eta(s)$ .

**Definition 2.7.** [7] If  $W$  is a neutrosophic subset of a  $NTS(X, \tau)$ . Then a N-point  $S_{(t,i,f)}$  of  $X$  is said to be  $N\delta$ -cluster point of  $W$  iff for every regular open  $q$ -nbd  $G$  of  $S_{(t,i,f)}$  is  $q$ -coincident with  $W$ . The collection of all possible  $N\delta$ -cluster points of  $W$  is called the  $N\delta$ -closure of  $W$  which we denote by  $NCl_\delta(W)$  and  $q$ -neighbourhood and  $q$ -coincident is defined in [2].

**Definition 2.8.** [7] If  $W$  is a neutrosophic subset of a  $NTS(X, X)$ . Then  $W$  is said to be  $N\delta$ -closed if  $NCl_\delta(W) = W$  and the complement of  $N\delta$ -closed set will be called  $N\delta$ -open set in  $X$ . The set of all  $N\delta$ -open (respectively,  $N\delta$ -closed) sets of  $X$  is denoted by  $N\delta O(X)$  (respectively,  $N\delta C(X)$ ).

**Definition 2.9.** [7] A neutrosophic subset  $W$  of a  $NTS(X, \tau)$  is called  $N\delta$ -semi-open if there exists a  $N\delta$ -open set  $G$  of  $X$  such that  $G \subseteq W \subseteq Ncl(G)$ . The complement of a  $N\delta$ -semi-open set is called  $N\delta$ -semi-closed set in  $X$  and a neutrosophic subset  $V$  is said to be  $N\delta$ -semi-closed set in  $X$  if there exists a  $N\delta$ -closed set  $F$  of  $X$  such that  $Nint(F) \subseteq V \subseteq F$ . We denote the collection of all possible  $N\delta$ -semi-open (resp.  $N\delta$ -semi-closed) subsets of  $X$  by  $N\delta SO(X)$  (resp.  $N\delta SC(X)$ ).

**Definition 2.10.** [5] Any neutrosophic subset  $W$  of a  $NTS(X, X)$  is called

- a) Neutrosophic semi-open [briefly, NSO] set if  $W \subseteq Ncl(Nint(W))$ .
- b) Neutrosophic pre-open [briefly, NPO] set if  $W \subseteq Nint(Ncl(W))$ .
- c) Neutrosophic regular open [briefly, NRO] set if  $W = Nint(Ncl(W))$ .
- d) Neutrosophic  $\alpha$ -open [briefly,  $N\alpha O$ ] set if  $W \subseteq Nint(Ncl(Nint(W)))$ .

The complement of NSO (resp. NPO, NRO,  $N\alpha O$ ) set is called a neutrosophic semi [resp. Pre, regular,  $\alpha$ ] closed [shortly, NSC (resp. NPC, NRC,  $N\alpha C$ )] set in  $X$ . The family of all possible NSO [resp. NPO, NRO,  $N\alpha O$ , NPC, NRC,  $N\alpha C$ ] sets of  $X$  are denoted by  $NSO(X)$  [resp.  $NPO(X), NRO(X), N\alpha O(X), NPC(X), NRC(X), N\alpha C(X)$ ].

### 3. Characterizations of $Np\delta s$ -irresolute Function and Basic Properties

In this research work, the notion of  $Np\delta s$ -irresolute functions is initiated and studied in neutrosophic topological spaces as a generalized version of neutrosophic  $\alpha\delta s$ -irresolute functions. Some characterizations, counterexamples, and fundamental features are also investigated.

**Definition 3.1.** A neutrosophic function  $\psi:(X,\tau_X)\rightarrow (Y,\nu_Y)$  is called

- a) Neutrosophic  $\rho\delta$ -irresolute (in short,  $N\rho\delta$ -irresolute) function if for each  $N\delta$ -semi open set  $W$  of  $Y$ ,  $\psi^{-1}(W)$  is neutrosophic pre-open (shortly, NP-open) set in  $X$ .
- b) Neutrosophic  $\alpha\delta$ -irresolute (briefly,  $N\alpha\delta$ -irresolute) function if for each  $N\delta$ -semi open set  $W$  of  $Y$ ,  $\psi^{-1}(W)$  is neutrosophic  $\alpha$ -open (in short,  $N\alpha$ -open) set in  $X$ .

**Theorem 3.2.** Every  $N\alpha\delta$ -irresolute function  $\psi: (X,\tau_X)\rightarrow (Y,\nu_Y)$  is  $N\rho\delta$ -irresolute function.

**Proof.** Let  $H$  be an arbitrary neutrosophic  $\delta$ -semi open set in  $Y$ . As  $\psi$  is  $N\alpha\delta$ -irresolute,  $\psi^{-1}(H)\in N\alpha O(X)$ . Being  $N\alpha$ -open set is NP-open set,  $\psi^{-1}(H)\in PO(X)$  which makes  $\psi$  a  $N\rho\delta$ -irresolute function.

**Remark 3.3.** From the above it is clear that:  $N\alpha\delta$ -irresoluteness  $\Rightarrow N\rho\delta$ -irresoluteness but the reverse implication is not true which can be seen from the following example:

**Example 3.4.** Let us consider two neutrosophic topological spaces  $(X,\tau_X)$  and  $(Y,\nu_Y)$ , where,  $X=\{x,y,z\}$  and  $Y=\{a,b,c\}$  with two neutrosophic topologies  $\tau_X$  and  $\nu_Y$  defined by  $\tau_X=\{0_X,1_X, W_1, W_2\}$   $\nu_Y=\{0_Y,1_Y, V_1\}$ , where,  $V_1=\{\langle a, 0.25, 0.55, 0.85 \rangle, \langle b, 0.15, 0.55, 0.75 \rangle, \langle c, 0.25, 0.55, 0.65 \rangle\}$ ,  $W_1=\{\langle x, 0.25, 0.55, 0.85 \rangle, \langle y, 0.35, 0.55, 0.75 \rangle, \langle z, 0.4, 0.5, 0.6 \rangle\}$ ,  $W_2=\{\langle x, 0.15, 0.55, 0.95 \rangle, \langle y, 0.15, 0.55, 0.95 \rangle, \langle z, 0.45, 0.55, 0.65 \rangle\}$ ,  $W_3=\{\langle x, 0.15, 0.55, 0.95 \rangle, \langle y, 0.15, 0.55, 0.95 \rangle, \langle z, 0.45, 0.55, 0.65 \rangle\}$ . If  $\psi:(X,\tau_X)\rightarrow (Y,\nu_Y)$  is a neutrosophic function defined by  $\psi(x)=a$ ,  $\psi(y)=b$ ,  $\psi(z)=c$ , then  $\psi$  is  $N\rho\delta$ -irresolute function but it is not  $N\alpha\delta$ -irresolute function since  $\psi^{-1}(V_1)$  is neutrosophic pre-open but not neutrosophic  $\alpha$ -open set of  $(X,\tau_X)$ .

**Theorem 3.5.** If  $\psi:(X,\tau_X)\rightarrow (Y,\nu_Y)$  is a function between two topological spaces, then the following are equivalent:

- i).  $\psi$  is  $N\rho\delta$ -irresolute function.
- ii). For each  $u\in X$  and for each  $N\delta$ -semi open set  $B$  of  $Y$  containing  $\psi(u)$ ,  $\exists$  a neutrosophic pre-open set  $A$  of  $X$  with  $u\in A$  such that  $\psi(A) \subset B$ .
- iii).  $\psi^{-1}(B)\subset NCl(NInt(NCl(\psi^{-1}(B)))) \cup NInt(NCl(\psi^{-1}(B)))$ , for every  $N\delta$ -semi open subset  $B$  of  $Y$ .
- iv).  $\psi^{-1}(F)$  is neutrosophic pre-closed in  $X$ , for every  $N\delta$ -semi closed subset  $F$  of  $Y$ .
- v).  $NInt(NCl(NInt(\psi^{-1}(NSCl(B)))) \cap NCl(NInt(\psi^{-1}(NSCl(B)))) \subset \psi^{-1}(NSCl(B))$ , for every neutrosophic subset  $B$  of  $Y$ .
- vi).  $\psi[NInt(NCl(NInt(A))) \cap NCl(NInt(A))] \subset NSCl(\psi(A))$ , for every neutrosophic subset  $A$  of  $X$ .

**Proof.**

- (i)  $\rightarrow$ (ii): Let  $u\in X$  and  $B$  be arbitrary  $N\delta$ -semi open set of  $Y$  such that  $\psi(u)\in B$ . Then  $\psi^{-1}(B)$  is NP-open in  $X$  containing  $u$ . Let  $A=\psi^{-1}(B)$ . So, using (i),  $A$  is an NP-open subset of  $X$  including  $u$  and  $\psi(A) \subset B$ .
- (ii)  $\rightarrow$ (iii): Let  $B$  be arbitrary  $N\delta$ -semi open set in  $Y$  and  $u\in\psi^{-1}(B)$ . From (ii),  $\exists$  an NP-open set  $A$  of  $M$  including  $u$  satisfying  $\psi(A) \subset B$ . So, we have  $u\in A\subset NCl(NInt(NCl(A))) \cup NInt(NCl(A)) \subset NCl(NInt(NCl(\psi^{-1}(B)))) \cup NInt(NCl(\psi^{-1}(B)))$ . Hence,  $\psi^{-1}(B)\subset NCl(NInt(NCl(\psi^{-1}(B)))) \cup NInt(NCl(\psi^{-1}(B)))$ .
- (iii)  $\rightarrow$ (iv): Suppose that  $F \subset Y$  and it is  $N\delta$ -semi closed subset in  $Y$ . Let  $D=Y-F$ . Then  $D$  is  $N\delta$ -semi open in  $Y$ . using (iii),  $\psi^{-1}(D)\subset NCl(NInt(NCl(\psi^{-1}(D)))) \cup NInt(NCl(\psi^{-1}(D)))$ . Thus,  $\psi^{-1}(F)=\psi^{-1}(Y-D)=X-\psi^{-1}(D)\supset X-[NCl(NInt(NCl(\psi^{-1}(D)))) \cup NInt(NCl(\psi^{-1}(D)))] = NInt(NCl(NInt(\psi^{-1}(D)))) \cap NCl(NInt(\psi^{-1}(D)))$ . Thus,  $\psi^{-1}(F)$  is NP-closed set in  $X$ .
- (iv)  $\rightarrow$ (v): Suppose that  $B \subset Y$ . As  $NSCl(B)$  is  $N\delta$ -semi closed in  $Y$ , by (iv),  $\psi^{-1}(NSCl(B))$  is NP-closed in  $X$  from which it follows that,  $NInt(NCl(NInt(\psi^{-1}(NSCl(B)))) \cap NCl(NInt(\psi^{-1}(NSCl(B)))) \subset \psi^{-1}(NSCl(B))$ .

- (v)  $\rightarrow$  (vi): Suppose that  $A \subset X$ . By virtue of (v),  $NInt(NCl(NInt(A))) \cap NCl(NInt_s(A)) \subset NInt(NCl(NInt(\psi^{-1}(\psi(A)))) \cap NCl(NInt_s(\psi^{-1}(\psi(A)))) \subset \psi^{-1}(NSCl_s(\psi(A)))$  and so,  $\psi[NInt(NCl(NInt(A))) \cap NCl(NInt_s(A))] \subset NSCl_s(\psi(A))$ .
- (vi)  $\rightarrow$  (i): Let  $D$  be arbitrary  $N\delta$ -semi open set in  $Y$ . Since  $\psi^{-1}(Y-D) = X - \psi^{-1}(D)$  is a neutrosophic subset of  $X$ . using (6), we have,  $\psi[NInt(NCl(NInt(\psi^{-1}(Y-D)))) \cap NCl(NInt_s(\psi^{-1}(Y-D)))] \subset NSCl_s(\psi(\psi^{-1}(Y-D))) \subset NSCl_s(Y-D) = Y - NSInt_s(D) = Y - D$  and hence,  $X - [NCl(NInt(NCl(\psi^{-1}(D)))) \cap NInt(NCl_s(\psi^{-1}(D)))] = NInt(NCl(NInt(X - \psi^{-1}(D)))) \cap NCl(NInt_s(X - \psi^{-1}(D))) = NInt(NCl(NInt(\psi^{-1}(Y-D)))) \cap NCl(NInt_s(\psi^{-1}(Y-D))) \subset \psi^{-1}(Y-D) = X - \psi^{-1}(D)$ . Thus,  $\psi^{-1}(D) \subset NCl(NInt(NCl(\psi^{-1}(D)))) \cup NInt(NCl_s(\psi^{-1}(D)))$ . This implies that  $\psi^{-1}(D)$  is NP-open set in  $X$ . Thus,  $\psi$  is NP $\delta$ s-irresolute function.

**Lemma 3.6.** [2]: Let  $W_1$  and  $W_2$  be neutrosophic subsets of a NTS  $(X, \tau_X)$ . If  $W_1$  is NSO set in  $(X, \tau_X)$  and  $W_2$  is NPO set in  $(X, \tau_X)$ , then  $W_1 \cap W_2$  is a NPO set in  $(W_1, \tau_{W_1})$ .

**Theorem 3.7.** If the neutrosophic function  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is NP $\delta$ s-irresolute function and  $D \subset X$  is NS-open in  $X$ , then the restriction  $\psi|_D : D \rightarrow Y$  is necessarily NP $\delta$ s-irresolute function on  $D$ .

**Proof:** Suppose that  $G$  is an arbitrary  $N\delta$ -semi open subset of  $N$ . As  $\psi$  is NP $\delta$ s-irresolute,  $\psi^{-1}(G)$  is NP-open in  $X$ .  $D$  being NS-open set in  $X$ , so in the light of Lemma 3.6,  $\psi|_D^{-1}(G) = D \cap \psi^{-1}(G)$  is NP-open set in  $D$  and thus,  $\psi|_D^{-1}$  is NP $\delta$ s-irresolute function on  $D$ .

**Lemma 3.8.** Suppose  $W_1$  and  $W_2$  are two neutrosophic subsets of a NTS  $(X, \tau_X)$  such that  $W_1 \subset W_2 \subset X$ . If  $W_1 \in NPO(W_2)$  and  $W_2 \in N\delta O(X)$ , then  $W_1 \in NPO(X)$ .

**Proof.** Obvious and hence omitted.

**Theorem 3.9.** Let  $\{A_\mu; \mu \in \Lambda\}$  be a pre-open cover of a topological space  $(X, \tau_X)$ . Then  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is the NP $\delta$ s-irresolute function if for each  $\mu \in \Lambda$ , the neutrosophic restriction function.  $\psi|_{A_\mu} : A_\mu \rightarrow Y$  is NP $\delta$ s-irresolute function.

**Proof.** Let  $G \subset N$  be arbitrary  $N\delta$ -semi open. By hypothesis,  $\psi|_{A_\mu}$  is NP $\delta$ s-irresolute function. So,  $(\psi|_{A_\mu})^{-1}(G)$  is NP-open in  $A_\mu$ . Since for each  $\mu \in \Lambda$ ,  $A_\mu$  is NP-open set in  $M$ , Lemma 3.8, ensures that  $(\psi|_{A_\mu})^{-1}(G)$  will be NP-open in  $M$ . Now  $\psi^{-1}(G) = M \cap \psi^{-1}(G) = \cup\{A_\mu \cap \psi^{-1}(G) : \mu \in \Lambda\} = \cup\{(\psi|_{A_\mu})^{-1}(G) : \mu \in \Lambda\}$ . Because the union of the arbitrary number of NP-open sets is necessarily NP-open,  $\psi^{-1}(G)$  will be NP-open set in  $M$ . Thus  $\psi$  becomes NP $\delta$ s-irresolute function.

**Theorem 3.10.** If  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is NP $\delta$ s-irresolute function and  $\phi: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  is  $N\delta$ -irresolute function. Then  $\phi \circ \psi: (X, \tau_X) \rightarrow (Z, \tau_Z)$  is NP $\delta$ s-irresolute function.

**Proof.** Suppose that  $W \subset Z$  is  $N\delta$ -semi open in  $Z$ . Now  $\phi$  being  $N\delta$ -irresolute function,  $\phi^{-1}(W)$  is  $N\delta$ -semi open subset of  $Y$ . By given hypothesis  $\psi$  is NP $\delta$ s-irresolute function, therefore,  $(\phi \circ \psi)^{-1}(W) = \psi^{-1}(\phi^{-1}(W))$  is NP-open in  $X$ . Thus, the composition  $\phi \circ \psi$  is NP $\delta$ s-irresolute function.

**Theorem 3.11.** If  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is NP-irresolute function and  $\phi: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  is NP $\delta$ s-irresolute function. Then  $\phi \circ \psi: (X, \tau_X) \rightarrow (Z, \tau_Z)$  is NP $\delta$ s-irresolute function.

**Proof.** Suppose that  $W \subset Z$  is  $N\delta$ -semi open in  $Z$ . Now  $\phi$  being NP $\delta$ s-irresolute function,  $\phi^{-1}(W)$  is NP-open subset in  $Y$ . By condition  $\psi$  is NP-irresolute function, therefore,  $(\phi \circ \psi)^{-1}(W) = \psi^{-1}(\phi^{-1}(W))$  is NP-open in  $X$ . Thus, the composition  $\phi \circ \psi$  is NP $\delta$ s-irresolute function.

**Theorem 3.12.** If  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is NP $\delta$ s-irresolute function and  $\phi: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  is  $N\delta$ -semi continuous function. Then  $\phi \circ \psi: (X, \tau_X) \rightarrow (Z, \tau_Z)$  is NP-continuous function.

**Proof.** Suppose that  $W \subset Z$  is arbitrary neutrosophic open in  $Z$ . Now  $\phi$  being  $N\delta$ -semi continuous function,  $\phi^{-1}(W)$  is  $N\delta$ -semi open set in  $Y$ . By given condition  $\psi$  is  $NP\delta s$ -irresolute function, therefore,  $(\phi \circ \psi)^{-1}(W) = \psi^{-1}(\phi^{-1}(W))$  is pre-open in  $X$ . Thus, the composition  $\phi \circ \psi$  is  $NP$ -continuous function.

**Lemma 3.13.** Let  $\{X_r: r \in \Lambda\}$  be an arbitrary collection of NTS and  $U_{r_i}$  be a non-empty neutrosophic subset of  $X_{r_i}$  for each  $i=1,2,3,\dots,n$ . Then  $M = \prod_{r \neq r_i} X_r \times \prod_{i=1}^n U_{r_i}$  will be a non-empty neutrosophic  $\delta$ semi-open (respectably, neutrosophic pre-open) subset of  $\prod X_r \Leftrightarrow U_{r_i}$  is neutrosophic  $\delta$ semi-open (respectably, neutrosophic pre-open) in  $X_{r_i}$ , for all  $i=1,2,3,\dots,n$ .

**Proof.** Obvious and hence omitted.

**Theorem 3.14.** Let  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be a mapping and the graph  $g: X \rightarrow X \times Y$  of  $\psi$ , defined by  $g(u) = (u, \psi(u))$  for each  $u \in X$ , be  $p\delta s$ -irresolute, then  $\psi$  is  $p\delta s$ -irresolute function.

**Proof.** Suppose that  $u \in X$  is an arbitrary member. Let  $W$  be  $N\delta s$ -open subset of  $Y$  with  $\psi(u) \in W$ . With the Lemma 3.13,  $X \times W$  will be  $NP\delta s$ -open subset of  $X \times Y$  which contains  $g(u)$ . By assumption  $g$  is  $NP\delta s$ -irresolute. So,  $\exists$  an  $NP$ -open subset  $F$  of  $X$  which contains  $u$  such that  $g(u) \in X \times W$  which implies that  $\psi(F) \subset W$ . Consequently,  $\psi$  is  $NP\delta s$ -irresolute function.

**Theorem 3.15.** Let the product function  $\psi: \prod_{r \in \Delta} M_r \rightarrow \prod_{r \in \Delta} N_r$  is  $NP\delta s$ -irresolute function from the product space  $\prod_{r \in \Delta} M_r$  into another product space  $\prod_{r \in \Delta} N_r$ , then for each  $r \in \Delta$ ,  $\psi_r: M_r \rightarrow N_r$  is  $NP\delta s$ -irresolute.

**Proof.** Let  $r_0 \in \Delta$  be any fixed index and  $G_{r_0}$  be an arbitrary neutrosophic  $\delta$ -semi-open subset of  $N_{r_0}$  which includes  $\psi(t)$ . So, by virtue of 3.13,  $\prod_{r \in \Delta} N_r \times G_{r_0}$  is neutrosophic  $\delta$ -semi open set of  $\prod_{r \in \Delta} N_r$  where,  $r_0 \neq r \in \Delta$ . As  $\psi$  is  $NP\delta s$ -irresolute function,  $\psi^{-1}(\prod_{r \in \Delta} N_r \times G_{r_0}) = \prod_{r \in \Delta} M_r \times \psi_{r_0}^{-1}(G_{r_0})$  is  $NP$ -open subset in  $\prod_{r \in \Delta} M_r$ . Hence, by Lemma 3.13,  $\psi_{r_0}^{-1}(G_{r_0})$  is  $NP$ -open in  $M_{r_0}$ . Thus, for each  $r \in \Delta$ ,  $\psi_r$  is  $NP\delta s$ -irresolute function.

#### 4. Preservation Properties and Applications of $NP\delta s$ -irresolute Function

In this section, the behavior of neutrosophic semi-connected, neutrosophic pre-connected, neutrosophic pre- $T_2$ , neutrosophic  $\delta$ -semi- $T_2$ , neutrosophic pre-compact, neutrosophic closed spaces are examined in the light of  $NP\delta s$ -irresolute function. Also, the preservation aspects are investigated.

**Definition 4.1.** The neutrosophic topological space  $(X, \tau_X)$  is called neutrosophic semi-connected (shortly,  $NS$ -connected) if  $X$  cannot be decomposed as the union of two nonempty neutrosophic semi-open sets which are disjoint.

**Definition 4.2.** The neutrosophic topological space  $(X, \tau_X)$  is called neutrosophic pre-connected (shortly,  $NP$ -connected) if  $X$  cannot be represented as the union of two nonempty neutrosophic pre-open sets which are disjoint.

**Lemma 4.3.** Let  $(X, \tau_X)$  be a neutrosophic topological space and  $W \subset X$ . Then  $W$  is neutrosophic  $\delta$ -semi-open if  $W$  is a neutrosophic semi-clo-open set.

**Lemma 4.4.** Let  $(X, \tau_X)$  be a neutrosophic topological space and  $W \subset X$ . Then  $NSCI(W)$  is neutrosophic semi-cloopen if  $W$  is a neutrosophic semi-open set.

**Lemma 4.5.** Let  $W$  be a neutrosophic subset of a neutrosophic topological space  $(X, \tau_X)$ . Then  $NSCI(W)$  is a neutrosophic  $\delta$ -semi open set if  $W$  is a neutrosophic semi-open set in  $X$ .

**Proof.** Since  $W$  is neutrosophic semi-open, by virtue of 4.4,  $NSCI(W)$  is semi-cloopen. By Lemma 4.3, every neutrosophic semi-clo-open set is neutrosophic  $\delta$ -semi open. Thus  $NSCI(W)$  is neutrosophic  $\delta$ -semi open in  $X$ .

**Theorem 4.6.** Let  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be NP $\delta$ s-irresolute and surjection function. Then  $(Y, \tau_Y)$  is NS-connected if  $(X, \tau_X)$  is NP-connected space.

**Proof.** If possible let  $Y$  be not NS-connected. So, there are two non-empty NS-open sets  $G$  and  $H$  such that  $Y = G \cup H$  and  $G \cap H = \emptyset$ . by 4.4,  $NSCI(G)$  will be NS-open in  $Y$  such that  $NSCI(G) \cap H = \emptyset$ . Then  $NSCI(G) \cap NSCI(H) = \emptyset$ . By means of 4.5,  $NSCI(G)$  and  $NSCI(H)$  will be N $\delta$ S-open sets and  $N = NSCI(G) \cup NSCI(H)$  from which it follows that  $X = \psi^{-1}(NSCI(G)) \cup \psi^{-1}(NSCI(H))$  and  $\psi^{-1}(NSCI(G)) \cap \psi^{-1}(NSCI(H)) = \emptyset$ . Because of NP $\delta$ s-irresolute function  $\psi$ ,  $\psi^{-1}(NSCI(G))$  and  $\psi^{-1}(NSCI(H))$  must be non-empty NS-open sets. Thus  $(X, \tau_X)$  is not NP-connected, a contradiction to the hypothesis. Hence,  $Y$  is NS-connected.

**Definition 4.7.** A NTS  $(X, \tau_X)$  will be called

- a) Neutrosophic pre- $T_0$  (shortly, NP- $T_0$ ) if for every distinct neutrosophic points  $S_{(t,i,f)}$  and  $P_{(t,i,f)}$  in  $X$ ,  $\exists$  a non-empty NP-open set  $W$  such that either  $S_{(t,i,f)} \in W, P_{(t,i,f)} \notin W$  or  $P_{(t,i,f)} \in W, S_{(t,i,f)} \notin W$ .
- b) Neutrosophic pre- $T_1$  (shortly, NP- $T_1$ ) if for every distinct neutrosophic points  $S_{(t,i,f)}$  and  $P_{(t,i,f)}$  in  $M$ ,  $\exists$  two non-empty NP-open sets  $W_1$  and  $W_2$  containing  $s$  and  $P_{(t,i,f)}$  respectably, such that  $P_{(t,i,f)} \notin W_1$  and  $S_{(t,i,f)} \notin W_2$ .
- c) Neutrosophic pre- $T_2$  (shortly, NP- $T_2$ ) if for every distinct neutrosophic points  $S_{(t,i,f)}$  and  $P_{(t,i,f)}$  in  $M$ ,  $\exists$  two non-empty NP-open sets  $W_1$  and  $W_2$  in  $M$  which are disjoint in such a way that  $S_{(t,i,f)} \in W_1$  and  $P_{(t,i,f)} \in W_2$ .

**Definition 4.8.** A NTS  $(X, \tau_X)$  will be called neutrosophic  $\delta$ -semi- $T_2$  (shortly, N $\delta$ S- $T_2$ ) if for every pair of distinct neutrosophic points  $S_{(t,i,f)}$  and  $P_{(t,i,f)}$  in  $X$ ,  $\exists$  two non-empty N $\delta$ S-open sets  $W_1$  and  $W_2$  in  $X$  which are disjoint in such a way that  $S_{(t,i,f)} \in W_1$  and  $P_{(t,i,f)} \in W_2$ .

**Theorem 4.9.** Let  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be NP $\delta$ s-irresolute and surjection function. If  $(Y, \tau_Y)$  is N $\delta$ -semi- $T_i$  Space, then  $(X, \tau_X)$  is NP- $T_i$  Space, for each  $i=0,1,2$ .

**Proof.** We prove the theorem for  $i=2$ . Suppose  $s$  and  $t$  are any two distinct pairs of neutrosophic points of  $M$ . Then  $\psi(s) \neq \psi(t)$ . Since  $Y$  is N $\delta$ -semi- $T_2$  space. So,  $\exists$  non-empty disjoint N $\delta$ -semi open sets  $W_1$  and  $W_2$  of  $Y$  such that  $\psi(s) \in W_1$  and  $\psi(t) \in W_2$  respectably. Since  $\psi$  is NP $\delta$ s-irresolute,  $\exists$  two NP-open sets  $U_1$  and  $U_2$  of  $X$  containing respectably  $s$  and  $t$  that satisfy  $\psi(U_1) \subset W_1, \psi(U_2) \subset W_2. \Rightarrow U_1 \subset \psi^{-1}(W_1)$  and  $U_2 \subset \psi^{-1}(W_2) \Rightarrow U_1 \cap U_2 \subset \psi^{-1}(W_1) \cap \psi^{-1}(W_2) = \psi^{-1}(W_1 \cap W_2) = \psi^{-1}(\emptyset) = \emptyset$ . So,  $U_1 \cap U_2 = \emptyset$ . Consequently,  $(X, \tau_X)$  is NP- $T_2$  space.

**Proposition 4.10.** Suppose that  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be NP $\delta$ s-irresolute function. If  $(Y, \tau_Y)$  is neutrosophic  $\delta$ -semi- $T_2$  space, then the set  $G = \{(s,t): \psi(s) = \psi(t), s \in X, t \in X\}$  is NP-closed in the product space  $X \times X$ .

**Proof.** Suppose that  $(s,t) \notin G$ . So,  $\psi(s) \neq \psi(t)$ . Since  $(Y, \tau_Y)$  is neutrosophic  $\delta$ -semi- $T_2$  space. So,  $\exists$  two non-empty disjoint neutrosophic  $\delta$ -semi open sets  $W_1$  and  $W_2$  in  $Y$  such that  $\psi(s) \in W_1$  and  $\psi(t) \in W_2$  respectably. As  $\psi$  is NP $\delta$ s-irresolute,  $\exists$  two NP-open sets  $U_1$  and  $U_2$  in  $X$  with  $s \in U_1$  and  $t \in U_2$  that satisfy  $\psi(U_1) \subset W_1$  and  $\psi(U_2) \subset W_2$ . Let  $H = U_1 \times U_2$ , then  $H$  is NP-open set in  $X \times X$  such that  $(s,t) \in H$  and  $G \cap H = \emptyset \Rightarrow NPCI(G) \subset G$ . Hence  $G$  is an NP-closed set of  $X \times X$ .

**Definition 4.11:** Let  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be any neutrosophic function, then the graph  $G$  of  $\psi$  denoted by  $G(\psi) = \{(z, \psi(z)): z \in X\}$  will be called NP $\delta$ s-closed if for each  $(z,t) \in (X \times Y) - G(\psi)$ , there is a  $W_1 \in NPO(X)$  with  $z \in W_1$  and  $W_2 \in N\delta SO(Y)$  that contains  $t$  in such a way that  $(W_1 \times W_2) \cap G(\psi) = \emptyset$ .



**Proposition 4.12.** If  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is NP $\delta$ s-irresolute function and  $(Y, \tau_Y)$  is neutrosophic  $\delta$ -semi  $T_2$ -space, then the graph of  $\psi$  i.e.  $G(\psi)$  is NP $\delta$ s-closed in  $X \times Y$ .

**Proof.** Suppose that  $(s, t) \in (X \times X) - G(\psi)$ . So,  $\psi(s) \neq t$ . Now  $Y$  being neutrosophic  $\delta$ -semi  $T_2$ -space, there are neutrosophic  $\delta$ -semi open sets  $W_1$  and  $W_2$  of  $Y$  with  $\psi(s) \in W_1$  and  $t \in W_2$ , respectively. As  $\psi$  is NP $\delta$ s-irresolute, there is a NP-open subset  $H_1$  in  $X$  with  $s \in H_1$  that satisfies  $\psi(H_1) \subset W_1$ . So,  $\psi(H_1) \cap W_2 = \emptyset$  and consequently  $(H_1 \times W_1) \cap G(\psi) = \emptyset$ . Hence,  $G(\psi)$  is NP $\delta$ s-closed in  $X \times Y$ .

**Theorem 4.13.** If  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is NP $\delta$ s-irresolute and injection function. Then  $X$  is NP- $T_2$ -space, if the graph  $G(\psi) = \{(z, \psi(z)) : z \in M\}$  of  $\psi$  is NP $\delta$ s-closed.

**Proof.** Suppose  $z$  and  $t$  are arbitrary elements of  $X$ . So,  $\psi(z) \neq \psi(t)$ . So,  $(z, \psi(t)) \in (X \times Y) - G(\psi)$ . Now  $G(\psi)$  being NP $\delta$ s-closed, there is a NP-open set  $W_1$  of  $X$  with  $z \in W_1$  and a N $\delta$ semi open set  $W_2$  of  $Y$  with  $\psi(z) \in W_2$  satisfying  $\psi(W_1) \cap W_2 = \emptyset$ . As  $\psi$  is NP $\delta$ s-irresolute, there is a NP-open subset  $H_1$  in  $X$  with  $t \in H_1$  that satisfies  $\psi(H_1) \subset W_2$ . So,  $\psi(W_1 \cap H_1) \subset \psi(W_1) \cap \psi(H_1) = \emptyset$  which shows  $W_1 \cap H_1 = \emptyset$  and so,  $X$  is NP- $T_2$ -space.

**Definition 4.14.** The neutrosophic subset  $W$  of a NTS  $(X, \tau_X)$  is said to be neutrosophic pre-compact (shortly, NP-compact) relative to  $(X, \tau_X)$  if, for every neutrosophic pre-open cover  $\{V_r : r \in \Lambda, \text{ an indexing set}\}$  of  $W$ , there exists a finite subset  $\Delta$  of  $\Lambda$  such that  $W \subset \bigcup_{r \in \Delta} V_r$ . The neutrosophic topological space  $(X, \tau_X)$  is NP-compact if  $X$  is NP-compact relative to  $(X, \tau_X)$ .

**Definition 4.15.** The neutrosophic subset  $W$  of a NTS  $(X, \tau_X)$  is said to be neutrosophic semi-compact (shortly, NS-compact) relative to  $(X, \tau_X)$  if for every neutrosophic semi-open cover  $\{U_r : r \in \Lambda, \text{ an indexing set}\}$  of  $W$ , there exists a finite subset  $\Delta$  of  $\Lambda$  such that  $W \subset \bigcup_{r \in \Delta} U_r$ . The neutrosophic topological space  $(X, \tau_X)$  is NS-compact if  $X$  is NS-compact relative to  $(X, \tau_X)$ .

**Theorem 4.16.** If  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is NP $\delta$ s-irresolute and  $W$  is NP-compact subset concerning  $X$ . Then  $\psi(W)$  is neutrosophic semi-closed for  $Y$ .

**Proof.** Consider the neutrosophic semi-open cover  $\{V_r : r \in \Lambda, \text{ an indexing set}\}$  of  $\psi(W)$  where,  $V_r$  is NS-open in  $Y$  for each  $r \in \Lambda$ . Under 4.5,  $NSCI(V_r)$  will be N $\delta$ -semi open set in  $N$ . Being  $\psi$  an NP $\delta$ s-irresolute function,  $\psi^{-1}(NSCI(V_r))$  becomes NP-open in  $X$ , for each  $r \in \Lambda$ . Since  $\psi(W) \subset \bigcup_{r \in \Lambda} V_r \subset \bigcup_{r \in \Lambda} NSCI(V_r)$ . Then,  $W \subset \psi^{-1}(\psi(W)) \subset \psi^{-1}(\bigcup_{r \in \Lambda} NSCI(V_r))$  which implies that  $W \subset \bigcup_{r \in \Lambda} \psi^{-1}(NSCI(V_r))$ . By hypothesis  $W$  is an NP-compact for  $X$ , so  $W$  has a finite NP-open sub-cover i.e. there exists a finite subset  $\Delta$  of  $\Lambda$  satisfying  $W \subset \bigcup_{r \in \Delta} \psi^{-1}(NSCI(V_r)) \Rightarrow \psi(W) \subset \bigcup_{r \in \Delta} NSCI(V_r)$ . So,  $\psi(W)$  is semi-closed relative to  $Y$ .

**Theorem 4.17.** If  $\psi: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is NP $\delta$ s-irresolute and surjection function. Then  $Y$  will be neutrosophic semi-closed if  $X$  is NP-compact.

**Proof.** Consider the neutrosophic semi-open cover  $\{V_r : r \in \Lambda, \text{ an indexing set}\}$  of  $Y = \psi(X)$  where  $V_r$  is neutrosophic semi-open in  $Y$  for each  $r$ . By 4.5,  $NSCI(V_r)$  will be N $\delta$ -semi open set in  $Y$ . Being  $\psi$  an NP $\delta$ s-irresolute and surjection function, so for each  $r \in \Lambda$ ,  $\psi^{-1}(NSCI(V_r))$  is NP-open in  $X$ . As  $Y = \psi(X) \subset \bigcup_{r \in \Lambda} V_r \subset \bigcup_{r \in \Lambda} NSCI(V_r)$ . Then  $X = \psi^{-1}(\psi(X)) \subset \psi^{-1}(\bigcup_{r \in \Lambda} NSCI(V_r))$ . This implies that  $M \subset \bigcup_{r \in \Lambda} \psi^{-1}(NSCI(V_r))$ . Being  $X$  an NP-compact, it has a finite NP-open sub cover i.e. there exists a finite subset  $\Delta$  of  $\Lambda$  satisfying  $X \subset \bigcup_{r \in \Delta} \psi^{-1}(NSCI(V_r))$  which gives  $Y = \psi(X) \subset \bigcup_{r \in \Delta} NSCI(V_r)$ . So,  $Y$  is neutrosophic semi-closed.

## 5. Conclusion

Because of the presence of indeterminacy in neutrosophic set theory and the corresponding membership functions being crucial, it has many real-life applications. In this paper, we describe the concept of N $\alpha$  $\delta$ s-irresolute and NP $\delta$ s-irresolute functions in neutrosophic topological spaces and prove that every N $\alpha$  $\delta$ s-irresolute function is NP $\delta$ s-irresolute function but not conversely. Some

characterizations, counterexamples, and basic properties are also investigated. Finally, the behavior of neutrosophic semi-connected, neutrosophic pre-connected, neutrosophic pre-T<sub>2</sub>, neutrosophic  $\delta$ -semi-T<sub>2</sub>, neutrosophic pre-compact, and neutrosophic s-closed spaces are examined in the light of NP $\delta$ s-irresolute function. Irresoluteness and continuity feature distinguished positions in the characterization of topological spaces. As a result, this concept, in the future, can be a vital tool in describing various topological structures. Moreover, several neutrosophic irresolute functions such as Nb $\delta$ s-irresolute, N $\beta$  $\delta$ s-irresolute, N $\beta^*$  $\delta$ s-irresolute, Ne $^*$  $\delta$ s-irresolute functions can be explored as generalized functions in the light of neutrosophic b-open, neutrosophic  $\beta$ -open, neutrosophic  $\beta^*$ -open and neutrosophic e $^*$ -open sets in NTS and the concepts can be generalized in neutrosophic bitopological spaces.

### Declarations

#### Ethics Approval and Consent to Participate

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#### Consent for Publication

This article does not contain any studies with human participants or animals performed by any of the authors.

#### Availability of Data and Materials

The data that support the findings of this study are available from the corresponding author upon reasonable request.

#### Competing Interests

The authors declare no competing interests in the research.

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