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Multi-Criteria Decision-Making Approach Based on Correlation Coefficient for Multi-Polar Interval-Valued Neutrosophic Soft Set

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Abstract: The correlation coefficient between two factors is crucial in statistical computation, indicating the extent and evolution of the appropriate link. The precision of applicability evaluations frequently relies on the thoroughness and caliber of data obtained from a certain dataset. Statistical research sometimes entails data marked by intrinsic trade-offs and uncertainty. This study seeks to present m-polar interval-valued neutrosophic soft sets (mPIVNSSs) through the integration of m-polar fuzzy sets with interval-valued neutrosophic soft sets. The suggested mPIVNSS structure is a significantly generalized version of m-polar neutrosophic soft sets and serves as a substantial extension of interval-valued neutrosophic soft sets. In this scenario, we delineate the correlation coefficient (CC) and the weighted correlation coefficient (WCC), together with their pertinent features, specifically designed for mPIVNSSs. A multi-criteria decision-making (MCDM) technique has been established based on the proposed correlation measures. To demonstrate the efficacy and relevance of the MCDM method, we present a detailed mathematical illustration. The study emphasizes the utility, impact, and flexibility of the created method through comparison analysis with traditional methodologies, illustrating its efficacy in resolving complicated decision-making situations.

Keywords: Multipolar Interval-valued Neutrosophic Set; Multipolar Interval-valued Neutrosophic Soft Set; MCDM; Correlation Coefficient; Weighted Correlation Coefficient.

1. Introduction

Multi-criteria decision-making (MCDM) is a systematic approach employed to choose the optimal alternative from a range of accessible possibilities, evaluated against various criteria or qualities. Traditionally, it is presumed that the information affecting the decision, encompassing criteria, and their associated weights, is articulated as precise numerical values. In practical situations, decision-making frequently transpires in contexts marked by uncertainty and ambiguity, when goals and limitations are not distinctly articulated. To mitigate such uncertainties, Zadeh [1] invented fuzzy sets (FS), an effective instrument for managing ambiguity and imprecision in decision-making. Although useful, fuzzy sets can fail to adequately address specific complex scenarios. To address these constraints, Turksen [2] introduced the notion of interval-valued fuzzy sets (IVFS), which enhance fuzzy sets by permitting membership degrees to be expressed as intervals instead of singular values. Nonetheless, some circumstances necessitate the consideration of non-membership values of components, a need that cannot be sufficiently met by either FS or IVFS. To address this issue, Atanassov [3] proposed the notion of intuitionistic fuzzy sets (IFS), which include both membership and non-membership values, facilitating a more thorough depiction of uncertainty. Notwithstanding its benefits, the IFS architecture remains constrained in addressing data incompatibilities and inaccuracies. Yager [4] extended the IFS to Pythagorean fuzzy sets with some fundamental operations and developed a decision model to solve MCDM complications. Zhang et al. [5-10] used the different fuzzy models in different fields of life such as supply chain management, emergency model robust emergency strategy, etc. Sarwar and Li [11] used the fuzzy fixed point results to ordinary fuzzy differential equations with their applications.

Smarandache [12] introduced the notion of neutrosophic sets (NS) to rectify these deficiencies, aimed at managing inconsistent, partial, and inaccurate data. Approximately concurrently, Molodtsov [13] introduced soft sets (SS), a multifaceted mathematical instrument intended to mitigate uncertainty in diverse decision-making scenarios. Maji [14] developed the neutrosophic soft set (NSS) by integrating the concepts of soft sets (SS) and neutrosophy (NS), thereby providing a more comprehensive framework for addressing uncertainty in decision-making. Karaaslan [15] subsequently enhanced the NSS framework by introducing its probabilistic variant and proposing a decision-making methodology designed to address intricate MCDM issues within the NSS framework. This development underscores the ongoing advancement of decision-making approaches to tackle the increasing complexity and ambiguity of real-world issues. Broumi [16] presented the most comprehensive variant of neutrosophic soft sets (NSS), integrating fundamental operations and formulating a decision-making (DM) approach to tackle intricate issues. Wang et al. [17] enhanced the research on single-valued neutrosophic sets (SVNSs) by proposing a correlation coefficient (CC) for these sets and formulating a decision-making approach grounded in their established correlation measure. Correlation analysis is essential in these investigations as it evaluates the interrelationships between two variable quantities. Yu [18] advanced this research by introducing a correlation coefficient for fuzzy numbers, facilitating the assessment of links between them. Gerstenkorn and Mafiko [19] advanced this domain by proposing a method for correlating intuitionistic fuzzy sets (IFS), which they designated as characteristic coefficients.

Garg and Arora [20] significantly advanced the field by formulating the correlation coefficient (CC) and weighted correlation coefficient (WCC) for intuitionistic fuzzy soft sets (IFSS). They also presented an innovative Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) methodology, employing their CC and WCC to efficiently address multi-attribute decision-making (MADM) challenges. Zulqarnain et al. [21] developed the TOPSIS technique using the CC in the interval-valued intuitionistic fuzzy soft structure and used their established method to solve decision-making problems. Zulqarnain et al. [22] examined the soft set framework and employed the TOPSIS approach for decision-making. They also developed several aggregation operators and the TOPSIS technique in Pythagorean fuzzy soft structure to solve decision-making complexities [23-28]. Several researchers used different fuzzy models for decision-making in diverse fields of life [29-34].

Saeed et al. [35] similarly introduced the concept of m-polar neutrosophic soft sets (mPNSS) and their fundamental operations. They formulated a distance-based similarity metric and illustrated its application in data mining and clinical diagnostics. Deli [36] introduced the interval-valued neutrosophic soft sets with their basic operations and properties. Chen et al. [37] established the m-polar fuzzy sets which is the generalized version of bi-polar fuzzy sets. Furthermore, multiple researchers [38-51] have thoroughly investigated diverse extensions of fuzzy sets, enhancing their applicability and utility in decision-making. These initiatives have resulted in the creation of various decision-making models customized for distinct configurations of fuzzy sets. The developments encompass modifications and expansions, including intuitionistic fuzzy sets, interval-valued fuzzy sets, neutrosophic sets, single-valued neutrosophic sets, and multipolar fuzzy sets, among others. Each expansion is meticulously crafted to tackle distinct issues arising from ambiguity, vagueness, and imprecision in real-world decision-making contexts. The researchers have developed novel approaches to improve the usefulness of fuzzy sets, encompassing the development of aggregation operators, similarity measures, correlation coefficients, and weighted correlation coefficients. Additionally, they have suggested other methodologies, including TOPSIS, multi-attribute decision-making (MADM) methods, and MCDM frameworks. These models and methodologies have been utilized across diverse domains, including engineering, management, healthcare, and environmental sciences, illustrating the versatility and applicability of fuzzy set extensions in addressing intricate decision-making challenges. Hashmi [52] Presented the m-polar neutrosophic set (mPNS) and its corresponding topological structure by integrating m-polar fuzzy sets and neutrosophic sets, examining their features, operations, and scoring functions through illustrated examples. Furthermore, it introduces similar metrics and formulates three algorithms for multi-criteria decision-making (MCDM) in medical diagnosis and clustering analysis, showcasing their benefits, adaptability, and superiority compared to current methodologies.

Experts acknowledge that contemporary natural systems and processes are progressively transitioning towards multi-polarization, a phenomenon characterized by the interplay of several elements or components that enhance a system's complexity. Multi-polarized information is essential for the advancement of fields including medicine, research, engineering, and the arts. In neurobiology, multipolar neurons in the brain process and integrate extensive information from other neurons, illustrating the intrinsic significance of multipolarity in complex systems. This study aims to enhance and unify research in this field by systematically building an exhaustive structure. We illustrate that diverse hybrid configurations of fuzzy sets (FS) can be converted into specialized m-polar interval-valued neutrosophic soft sets (mPIVNSS) under suitable conditions, highlighting the versatility and applicability of these sophisticated structures. This research introduces the novel concept of m-polar neutrosophic soft sets (mPNSS), advocating for their effectiveness, adaptability, superiority, and unique advantages in addressing complicated decision-making situations.

The framework established in this study exemplifies a highly adaptable approach, proficient at synthesizing and applying knowledge across many real-world contexts. The present study establishes a basis for expanding these concepts to further hybrid structures and decision-making methodologies, hence facilitating applications in several domains like healthcare, environmental management, and artificial intelligence.

The paper is structured to ensure a logical progression and thorough comprehension of the presented principles as follows: Section 2 presents essential principles requisite for the investigation. The document encompasses a comprehensive analysis of neutrosophic sets (NS), soft sets (SS), neutrosophic soft sets (NSS), multipolar neutrosophic sets (mPNS), and interval-valued neutrosophic soft sets (IVNSS). These principles establish a theoretical foundation for the later development of the suggested framework. A new framework, the m-polar interval-valued neutrosophic soft set (mPIVNSS), is presented in Section 3 by combining the m-polar fuzzy set (mPFS) with interval-valued neutrosophic soft set (IVNSS). This section delineates and expounds upon the correlation coefficient (CC) and weighted correlation coefficient (WCC) for mPIVNSS, encompassing their mathematical formulations and theoretical implications. Section 4 delineates a decision-making (DM) methodology employing the established CC and WCC. It emphasizes the advantageous characteristics of the suggested measures and illustrates their implementation in decision-making. A thorough numerical example is provided to validate the practicality and effectiveness of the methodology, demonstrating the real-world applicability of the suggested method. A comparative analysis is conducted in Section 5 to assess the progressive method in relation to established techniques. This section highlights the superiority, practicality, and adaptability of the proposed framework, illustrating its benefits above traditional methods. The discourse emphasizes the superiority of the created method in managing intricate decision-making situations with enhanced efficiency and adaptability.

2. Materials and Method (Preliminaries)

In the following section, we will present some necessary definitions, which help us build the current article's structure.

Definition 2.1. [52] Let $\mathcal{U}$ be the universal set and $\wp_{\mathfrak{R}}$ is said to multipolar neutrosophic set if

$\wp_{\Re} = \{(u, u_{\alpha}(u), v_{\alpha}(u), w_{\alpha}(u)): u \in \mathcal{U}, \alpha = 1,2,3,\ldots,m\}$, where $u_{\alpha}(u)$, $v_{\alpha}(u)$, and $w_{\alpha}(u)$ represents the truthiness, indeterminacy, and falsity, respectively, $u_{\alpha}(u), v_{\alpha}(u), w_{\alpha}(u) \subseteq [0,1]$ and $0 \leq u_{\alpha}(u) + v_{\alpha}(u) + w_{\alpha}(u) \leq 3$, for all $\alpha = 1, 2, 3, \ldots, m$; and $u \in \mathcal{U}$.

Definition 2.2. [13] Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called a soft set over $\mathcal{U}$, and its mapping is given as

$$\mathcal{F}: \mathcal{A} \to \mathcal{P}(\mathcal{U})$$

It is also defined as:

$$(\mathcal{F}, \mathcal{A}) = \{\mathcal{F}(e) \in \mathcal{P}(\mathcal{U}): e \in \mathcal{E}, \mathcal{F}(e) = \emptyset \ if \ e \neq \mathcal{A}\}$$

Definition 2.3. [14] Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the set of NSs over $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called an NSS over $\mathcal{U}$ and its mapping is given as

$$\mathcal{F}: \mathcal{A} \to \mathcal{P}(\mathcal{U})$$

Definition 2.4. [35] Let $\mathcal{U}$ be a universe of discourse and $\mathcal{E}$ be a set of attributes, an m-polar neutrosophic soft set (mPNSS) $\wp_{\Re}$ over $\mathcal{U}$ defined as

$\wp_{\Re} = \{(e, \{(u, u_{\alpha}(u), v_{\alpha}(u), w_{\alpha}(u)): u \in \mathcal{U}, \alpha = 1,2,3,\ldots,m\}): e \in \mathcal{E}\}$,

where $u_{\alpha}(u)$, $v_{\alpha}(u)$, and $w_{\alpha}(u)$ represent the truthiness, indeterminacy, and falsity respectively, $u_{\alpha}(u), v_{\alpha}(u), w_{\alpha}(u) \subseteq [0,1]$ and $0 \leq u_{\alpha}(u) + v_{\alpha}(u) + w_{\alpha}(u) \leq 3$, for all $\alpha = 1, 2, 3, \ldots, m$; $e \in \mathcal{E}$ and $u \in \mathcal{U}$. Simply an m-polar neutrosophic number (mPNSN) can be expressed as $\wp = \{\langle u_{\alpha}, v_{\alpha}, w_{\alpha}\rangle\}$, where $0 \leq u_{\alpha} + v_{\alpha} + w_{\alpha} \leq 3$ and $\alpha = 1, 2, 3 \ldots, m$.

Definition 2.5. [36] Let $\mathcal{U}$ be a universe of discourse and $\mathcal{E}$ be a set of attributes, an IVNSS $\wp_{\Re}$ over $\mathcal{U}$ defined as

$\wp_{\Re} = \{(e, \{(u, u_{\Re}(u), v_{\Re}(u), w_{\Re}(u)): u \in \mathcal{U}, \alpha = 1,2,3,\ldots,m\}): e \in \mathcal{E}\}$,

where $u_{\Re}(u) = [u_{\Re}^{\ell}(u), u_{\Re}^{u}(u)]$, $v_{\Re}(u) = [v_{\Re}^{\ell}(u), v_{\Re}^{u}(u)]$, $w_{\Re}(u) = [w_{\Re}^{\ell}(u), w_{\Re}^{u}(u)]$, represents the interval truthiness, indeterminacy, and falsity respectively, $u_{\Re}(u), v_{\Re}(u), w_{\Re}(u) \subseteq [0,1]$ and $0 \leq u_{\Re}^{u}(u) + v_{\Re}^{u}(u) + w_{\Re}^{u}(u) \leq 3$, for each $e \in \mathcal{E}$ and $u \in \mathcal{U}$.

3. Correlation Coefficient for Multi-polar Interval Valued Neutrosophic Soft Set

The m-polar fuzzy set (mPFS), presented by Chen et al. [37], was developed to manage ambiguous and imprecise data encompassing multi-polar information. Neutrosophic sets (NS) encompass truth, falsehood, and indeterminacy for a singular criterion but are constrained in their capacity to manage multi-criteria, multi-source, and multi-polar information integration. Saeed et al. [35] addressed this restriction by introducing the m-polar neutrosophic soft set (mPNSS), which integrates m-polar neutrosophic sets with the soft set (SS) framework. Deli [36] progressed this field by creating the interval-valued neutrosophic soft set (IVNSS), which integrates the principles of interval-valued neutrosophic sets IVNS soft sets. This study builds on existing foundations to establish the core principles of mPNSS and extends them to the m-polar interval-valued neutrosophic soft set (mPIVNSS), introducing diverse operations and properties that augment its applicability and versatility in complex decision-making contexts.

Definition 3.1 Let $\mathcal{U}$ be a universe of discourse and $\mathcal{E}$ be a set of attributes, an mPIVNSS $\wp_{\Re}$ over $\mathcal{U}$ defined as

$\wp_{\Re} = \{(e, \{(u, u_{\alpha}(u), v_{\alpha}(u), w_{\alpha}(u)): u \in \mathcal{U}, \alpha = 1,2,3,\ldots,m\}): e \in \mathcal{E}\}$, where $\mathcal{u}_\alpha(u) = [\mathcal{u}_\alpha^{\ell}(u), \mathcal{u}_\alpha^{\mathrm{u}}(u)]$, $\mathcal{v}_\alpha(u) = [\mathcal{v}_\alpha^{\ell}(u), \mathcal{v}_\alpha^{\mathrm{u}}(u)]$, $\mathcal{w}_\alpha(u) = [\mathcal{w}_\alpha^{\ell}(u), \mathcal{w}_\alpha^{\mathrm{u}}(u)]$, represent the interval truthiness, indeterminacy, and falsity respectively, $\mathcal{u}_\alpha(u), \mathcal{v}_\alpha(u), \mathcal{w}_\alpha(u) \subseteq [0,1]$ and $0 \leq \mathcal{u}_\alpha^{\mathrm{u}}(u) + \mathcal{v}_\alpha^{\mathrm{u}}(u) + \mathcal{w}_\alpha^{\mathrm{u}}(u) \leq 3$ for all $\alpha = 1, 2, 3, \ldots, m$; $e \in \mathcal{E}$ and $u \in \mathcal{U}$. Simply an m-polar interval-valued neutrosophic soft number (mPIVNSN) can be expressed as $\wp = \{[\mathcal{u}_\alpha^{\ell}(u), \mathcal{u}_\alpha^{\mathrm{u}}(u)], [\mathcal{v}_\alpha^{\ell}(u), \mathcal{v}_\alpha^{\mathrm{u}}(u)], [\mathcal{w}_\alpha^{\ell}(u), \mathcal{w}_\alpha^{\mathrm{u}}(u)]\}$, where $0 \leq \mathcal{u}_\alpha^{\mathrm{u}}(u) + \mathcal{v}_\alpha^{\mathrm{u}}(u) + \mathcal{w}_\alpha^{\mathrm{u}}(u) \leq 3$ and $\alpha = 1, 2, 3, \ldots, m$.

Definition 3.2 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, $\wp_{\mathfrak{R}}$ is called an m-polar interval-valued neutrosophic soft subset of $\wp_{\mathcal{L}}$. If

$\mathcal{u}_\alpha^{\ell\mathfrak{R}}(u) \leq \mathcal{u}_\alpha^{\ell\mathcal{L}}(u)$, $\mathcal{u}_\alpha^{\mathrm{u}\mathfrak{R}}(u) \leq \mathcal{u}_\alpha^{\mathrm{u}\mathcal{L}}(u)$

$\mathcal{v}_\alpha^{\ell\mathfrak{R}}(u) \geq \mathcal{v}_\alpha^{\ell\mathcal{L}}(u)$, $\mathcal{v}_\alpha^{\mathrm{u}\mathfrak{R}}(u) \geq \mathcal{v}_\alpha^{\mathrm{u}\mathcal{L}}(u)$

$\mathcal{w}_\alpha^{\ell\mathfrak{R}}(u) \geq \mathcal{w}_\alpha^{\ell\mathcal{L}}(u)$, $\mathcal{w}_\alpha^{\mathrm{u}\mathfrak{R}}(u) \geq \mathcal{w}_\alpha^{\mathrm{u}\mathcal{L}}(u)$

for all $\alpha = 1, 2, 3, \cdots, m$; $e \in \mathcal{E}$ and $u \in \mathcal{U}$.

Definition 3.3 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, $\wp_{\mathfrak{R}} = \wp_{\mathcal{L}}$, if

$\mathcal{u}_\alpha^{\ell\mathfrak{R}}(u) \leq \mathcal{u}_\alpha^{\ell\mathcal{L}}(u)$, $\mathcal{u}_\alpha^{\ell\mathcal{L}}(u) \leq \mathcal{u}_\alpha^{\ell\mathfrak{R}}(u)$ and $\mathcal{u}_\alpha^{\mathrm{u}\mathfrak{R}}(u) \leq \mathcal{u}_\alpha^{\mathrm{u}\mathcal{L}}(u)$, $\mathcal{u}_\alpha^{\mathrm{u}\mathcal{L}}(u) \leq \mathcal{u}_\alpha^{\mathrm{u}\mathfrak{R}}(u)$

$\mathcal{v}_\alpha^{\ell\mathfrak{R}}(u) \geq \mathcal{v}_\alpha^{\ell\mathcal{L}}(u)$, $\mathcal{v}_\alpha^{\ell\mathcal{L}}(u) \geq \mathcal{v}_\alpha^{\ell\mathfrak{R}}(u)$ and $\mathcal{v}_\alpha^{\mathrm{u}\mathfrak{R}}(u) \geq \mathcal{v}_\alpha^{\mathrm{u}\mathcal{L}}(u)$, $\mathcal{v}_\alpha^{\mathrm{u}\mathcal{L}}(u) \geq \mathcal{v}_\alpha^{\mathrm{u}\mathfrak{R}}(u)$

$\mathcal{w}_\alpha^{\ell\mathfrak{R}}(u) \geq \mathcal{w}_\alpha^{\ell\mathcal{L}}(u)$, $\mathcal{w}_\alpha^{\ell\mathcal{L}}(u) \geq \mathcal{w}_\alpha^{\ell\mathfrak{R}}(u)$ and $\mathcal{w}_\alpha^{\mathrm{u}\mathfrak{R}}(u) \geq \mathcal{w}_\alpha^{\mathrm{u}\mathcal{L}}(u)$, $\mathcal{w}_\alpha^{\mathrm{u}\mathcal{L}}(u) \geq \mathcal{w}_\alpha^{\mathrm{u}\mathfrak{R}}(u)$

for all $\alpha = 1, 2, 3 \ldots, m$; $e \in \mathcal{E}$ and $u \in \mathcal{U}$.

Definition 3.4 Let

$$\wp_{\mathfrak{R}} = \left\{\left(e, \left\{\left(u, \left[\mathcal{u}_\alpha^{\ell\mathfrak{R}}(u_j), \mathcal{u}_\alpha^{\mathrm{u}\mathfrak{R}}(u_j)\right], \left[\mathcal{v}_\alpha^{\ell\mathfrak{R}}(u_j), \mathcal{v}_\alpha^{\mathrm{u}\mathfrak{R}}(u_j)\right], \left[\mathcal{w}_\alpha^{\ell\mathfrak{R}}(u_j), \mathcal{w}_\alpha^{\mathrm{u}\mathfrak{R}}(u_j)\right]\right) : u_j \in \mathcal{U}, \alpha = 1,2,3,\ldots,m\right\}\right) : e \in \mathcal{E}\right\}$$

and

$$\wp_{\mathcal{L}} = \left\{\left(e, \left\{\left(u, \left[\mathcal{u}_\alpha^{\ell\mathcal{L}}(u_j), \mathcal{u}_\alpha^{\mathrm{u}\mathcal{L}}(u_j)\right], \left[\mathcal{v}_\alpha^{\ell\mathcal{L}}(u_j), \mathcal{v}_\alpha^{\mathrm{u}\mathcal{L}}(u_j)\right], \left[\mathcal{w}_\alpha^{\ell\mathcal{L}}(u_j), \mathcal{w}_\alpha^{\mathrm{u}\mathcal{L}}(u_j)\right]\right) : u_j \in \mathcal{U}, \alpha = 1,2,3,\ldots,m\right\}\right) : e \in \mathcal{E}\right\}$$

be two mPIVNSSs over the universe of discourse $\mathcal{U}$. Then, informational neutrosophic energies for mPIVNSS can be presented as

$$\varsigma_{mPIVNSS}(\wp_{\mathfrak{R}}) = \sum_{\alpha=1}^{m}\sum_{j=1}^{n}\left(\left(\mathcal{u}_\alpha^{\ell\mathfrak{R}}(u_j)\right)^2 + \left(\mathcal{u}_\alpha^{\mathrm{u}\mathfrak{R}}(u_j)\right)^2 + \left(\mathcal{v}_\alpha^{\ell\mathfrak{R}}(u_j)\right)^2 + \left(\mathcal{v}_\alpha^{\mathrm{u}\mathfrak{R}}(u_j)\right)^2 + \left(\mathcal{w}_\alpha^{\ell\mathfrak{R}}(u_j)\right)^2 + \left(\mathcal{w}_\alpha^{\mathrm{u}\mathfrak{R}}(u_j)\right)^2\right) \tag{3.1}$$

$$\varsigma_{mPIVNSS}(\wp_{\mathcal{L}}) = \sum_{\alpha=1}^{m}\sum_{j=1}^{n}\left(\left(\mathcal{u}_\alpha^{\ell\mathcal{L}}(u_j)\right)^2 + \left(\mathcal{u}_\alpha^{\mathrm{u}\mathcal{L}}(u_j)\right)^2 + \left(\mathcal{v}_\alpha^{\ell\mathcal{L}}(u_j)\right)^2 + \left(\mathcal{v}_\alpha^{\mathrm{u}\mathcal{L}}(u_j)\right)^2 + \left(\mathcal{w}_\alpha^{\ell\mathcal{L}}(u_j)\right)^2 + \left(\mathcal{w}_\alpha^{\mathrm{u}\mathcal{L}}(u_j)\right)^2\right) \tag{3.2}$$

Definition 3.5 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs. Then, the correlation between them is defined as

$$\mathcal{C}_{mPIVNSS}(\wp_{\mathfrak{R}}, \wp_{\mathcal{L}}) = \sum_{\alpha=1}^{m}\sum_{j=1}^{n}\left(\mathcal{u}_\alpha^{\ell\mathfrak{R}}(u_j) * \mathcal{u}_\alpha^{\ell\mathcal{L}}(u_j) + \mathcal{u}_\alpha^{\mathrm{u}\mathfrak{R}}(u_j) * \mathcal{u}_\alpha^{\mathrm{u}\mathcal{L}}(u_j) + \mathcal{v}_\alpha^{\ell\mathfrak{R}}(u_j) * \mathcal{v}_\alpha^{\ell\mathcal{L}}(u_j) + \mathcal{v}_\alpha^{\mathrm{u}\mathfrak{R}}(u_j) * \mathcal{v}_\alpha^{\mathrm{u}\mathcal{L}}(u_j) + \mathcal{w}_\alpha^{\ell\mathfrak{R}}(u_j) * \mathcal{w}_\alpha^{\ell\mathcal{L}}(u_j) + \mathcal{w}_\alpha^{\mathrm{u}\mathfrak{R}}(u_j) * \mathcal{w}_\alpha^{\mathrm{u}\mathcal{L}}(u_j)\right). \tag{3.3}$$

Theorem 3.1 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs and $\mathcal{C}_{mPIVNSS}(\wp_{\mathfrak{R}}, \wp_{\mathcal{L}})$ represents the correlation between them. Then, the subsequent possessions hold.

$\mathcal{C}_{mPIVNSS}(\wp_{\mathfrak{R}}, \wp_{\mathfrak{R}}) = \varsigma_{mPIVNSS}(\wp_{\mathfrak{R}})$

$\mathcal{C}_{mPIVNSS}(\wp_{\mathcal{L}}, \wp_{\mathcal{L}}) = \varsigma_{mPIVNSS}(\wp_{\mathcal{L}})$

Proof: The proof is trivial.

Definition 3.6 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, their CC is given as $\delta_{mPIVNSS}(\wp_{\mathfrak{R}}, \wp_{\mathcal{L}})$ and is stated as follows:

$$\delta_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = \frac{\mathcal{C}_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}})}{\sqrt{\varsigma_{mPIVNSS}(\wp_{\Re}) * \varsigma_{mPIVNSS}(\wp_{\mathcal{L}})}} \tag{3.4}$$

$$\delta_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = \frac{\sum_{\alpha=1}^{m}\sum_{j=1}^{n}\begin{pmatrix} u_{\alpha}^{\ell\Re}(u_j)*u_{\alpha}^{\ell\mathcal{L}}(u_j)+ u_{\alpha}^{u\Re}(u_j)*u_{\alpha}^{u\mathcal{L}}(u_j)+v_{\alpha}^{\ell\Re}(u_j)*v_{\alpha}^{\ell\mathcal{L}}(u_j)+ \\ v_{\alpha}^{u\Re}(u_j)*v_{\alpha}^{u\mathcal{L}}(u_j)+w_{\alpha}^{\ell\Re}(u_j)*w_{\alpha}^{\ell\mathcal{L}}(u_j)+ w_{\alpha}^{u\Re}(u_j)*w_{\alpha}^{u\mathcal{L}}(u_j)\end{pmatrix}}{\begin{pmatrix} \sqrt{\sum_{\alpha=1}^{m}\sum_{j=1}^{n}\left(\left(u_{\alpha}^{\ell\Re}(u_j)\right)^2+\left(u_{\alpha}^{u\Re}(u_j)\right)^2+\left(v_{\alpha}^{\ell\Re}(u_j)\right)^2+\left(v_{\alpha}^{u\Re}(u_j)\right)^2+\left(w_{\alpha}^{\ell\Re}(u_j)\right)^2+\left(w_{\alpha}^{u\Re}(u_j)\right)^2\right)} \\ \sqrt{\sum_{\alpha=1}^{m}\sum_{j=1}^{n}\left(\left(u_{\alpha}^{\ell\mathcal{L}}(u_j)\right)^2+\left(u_{\alpha}^{u\mathcal{L}}(u_j)\right)^2+\left(v_{\alpha}^{\ell\mathcal{L}}(u_j)\right)^2+\left(v_{\alpha}^{u\mathcal{L}}(u_j)\right)^2+\left(w_{\alpha}^{\ell\mathcal{L}}(u_j)\right)^2+\left(w_{\alpha}^{u\mathcal{L}}(u_j)\right)^2\right)}\end{pmatrix}} \tag{3.5}$$

Theorem 3.2 Let $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, the following properties are satisfied:

$0 \leq \delta_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) \leq 1$

$\delta_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = \delta_{mPIVNSS}(\wp_{\mathcal{L}}, \wp_{\Re})$

If $\wp_{\Re} = \wp_{\mathcal{L}}$, that is $\forall\ j,\ \alpha,\ u_{\alpha}^{\ell\Re}(u_j) = u_{\alpha}^{\ell\mathcal{L}}(u_j),\ u_{\alpha}^{u\Re}(u_j) = u_{\alpha}^{u\mathcal{L}}(u_j),\ v_{\alpha}^{\ell\Re}(u_j) = v_{\alpha}^{\ell\mathcal{L}}(u_j),\ v_{\alpha}^{u\Re}(u_j) = v_{\alpha}^{u\mathcal{L}}(u_j),\ w_{\alpha}^{\ell\Re}(u_j) = w_{\alpha}^{\ell\mathcal{L}}(u_j),\ w_{\alpha}^{u\Re}(u_j) = w_{\alpha}^{u\mathcal{L}}(u_j)$, then $\delta_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = 1$.

Proof 1 The proof is obvious.

Definition 3.7 Let $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, their CC has also been given as $\delta^{1}_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}})$ and is expressed as follows:

$$\delta^{1}_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = \frac{\mathcal{C}_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}})}{max\{\varsigma_{mPIVNSS}(\wp_{\Re}), \varsigma_{mPIVNSS}(\wp_{\mathcal{L}})\}} \tag{3.6}$$

$$\delta^{1}_{IVIFSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = \frac{\sum_{\alpha=1}^{m}\sum_{j=1}^{n}\begin{pmatrix} u_{\alpha}^{\ell\Re}(u_j)*u_{\alpha}^{\ell\mathcal{L}}(u_j)+ u_{\alpha}^{u\Re}(u_j)*u_{\alpha}^{u\mathcal{L}}(u_j)+v_{\alpha}^{\ell\Re}(u_j)*v_{\alpha}^{\ell\mathcal{L}}(u_j)+ \\ v_{\alpha}^{u\Re}(u_j)*v_{\alpha}^{u\mathcal{L}}(u_j)+w_{\alpha}^{\ell\Re}(u_j)*w_{\alpha}^{\ell\mathcal{L}}(u_j)+ w_{\alpha}^{u\Re}(u_j)*w_{\alpha}^{u\mathcal{L}}(u_j)\end{pmatrix}}{max\begin{Bmatrix} \sum_{\alpha=1}^{m}\sum_{j=1}^{n}\left(\left(u_{\alpha}^{\ell\Re}(u_j)\right)^2+\left(u_{\alpha}^{u\Re}(u_j)\right)^2+\left(v_{\alpha}^{\ell\Re}(u_j)\right)^2+\left(v_{\alpha}^{u\Re}(u_j)\right)^2+\left(w_{\alpha}^{\ell\Re}(u_j)\right)^2+\left(w_{\alpha}^{u\Re}(u_j)\right)^2\right), \\ \sum_{\alpha=1}^{m}\sum_{j=1}^{n}\left(\left(u_{\alpha}^{\ell\mathcal{L}}(u_j)\right)^2+\left(u_{\alpha}^{u\mathcal{L}}(u_j)\right)^2+\left(v_{\alpha}^{\ell\mathcal{L}}(u_j)\right)^2+\left(v_{\alpha}^{u\mathcal{L}}(u_j)\right)^2+\left(w_{\alpha}^{\ell\mathcal{L}}(u_j)\right)^2+\left(w_{\alpha}^{u\mathcal{L}}(u_j)\right)^2\right)\end{Bmatrix}} \tag{3.7}$$

Theorem 3.3 Let $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, the subsequent possessions are fulfilled:

$0 \leq \delta^{1}_{IVIFSS}(\wp_{\Re}, \wp_{\mathcal{L}}) \leq 1$

$\delta^{1}_{IVIFSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = \delta^{1}_{IVIFSS}(\wp_{\mathcal{L}}, \wp_{\Re})$

If $\wp_{\Re} = \wp_{\mathcal{L}}$, that is $\forall\ j,\ \alpha,\ u_{\alpha}^{\ell\Re}(u_j) = u_{\alpha}^{\ell\mathcal{L}}(u_j),\ u_{\alpha}^{u\Re}(u_j) = u_{\alpha}^{u\mathcal{L}}(u_j),\ v_{\alpha}^{\ell\Re}(u_j) = v_{\alpha}^{\ell\mathcal{L}}(u_j),\ v_{\alpha}^{u\Re}(u_j) = v_{\alpha}^{u\mathcal{L}}(u_j),\ w_{\alpha}^{\ell\Re}(u_j) = w_{\alpha}^{\ell\mathcal{L}}(u_j),\ w_{\alpha}^{u\Re}(u_j) = w_{\alpha}^{u\mathcal{L}}(u_j)$, then $\delta^{1}_{IVIFSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = 1$.

Proof The proof is obvious.

Determining the weights of interval-valued neutrosophic soft sets (IVNSS) is essential for real-world use. The analytical results may change dramatically when a decision-maker allocates different weights to each piece within the universe of discourse. Consequently, it is imperative to thoroughly assess the ramifications of these weight assignments prior to reaching a choice. Let $\dot{\omega} = \{\dot{\omega}_1, \dot{\omega}_2, \dot{\omega}_3, \ldots, \dot{\omega}_m\}$ denote the weight vector for experts, where $\dot{\omega}_k > 0$, $\sum_{k=1}^{m}\dot{\omega}_k = 1$. Let $\gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_n\}$ represent the weight vector for parameters, where $\gamma_i > 0$, $\sum_{i=1}^{n}\gamma_i = 1$. The concept of the weighted correlation coefficient (WCC) is elaborated upon based on Definitions 3.6 and 3.7 in the subsequent sections.

Definition 3.8 Let $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ are two mPIVNSS over $\mathcal{U}$. Then, their WCC is given as $\delta_{WmPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}})$ and expressed as follows:

$$\delta_{WmPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = \frac{\mathcal{C}_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}})}{\sqrt{\varsigma_{mPIVNSS}(\wp_{\Re}) * \varsigma_{mPIVNSS}(\wp_{\mathcal{L}})}} \tag{3.8}$$

$$\delta_{WmPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = \frac{\sum_{\alpha,k=1}^{m} \dot{\omega}_k \left(\sum_{i,j=1}^{n} \gamma_i \left(\begin{array}{c} u_{\alpha}^{\ell\Re}(u_j) * u_{\alpha}^{\ell\mathcal{L}}(u_j) + u_{\alpha}^{\mathrm{u}\Re}(u_j) * u_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) + v_{\alpha}^{\ell\Re}(u_j) * v_{\alpha}^{\ell\mathcal{L}}(u_j) + \\ v_{\alpha}^{\mathrm{u}\Re}(u_j) * v_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) + w_{\alpha}^{\ell\Re}(u_j) * w_{\alpha}^{\ell\mathcal{L}}(u_j) + w_{\alpha}^{\mathrm{u}\Re}(u_j) * w_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) \end{array} \right) \right)}{\left(\begin{array}{c} \sqrt{\sum_{\alpha,k=1}^{m} \dot{\omega}_k \left(\sum_{i,j=1}^{n} \gamma_i \left(\left(u_{\alpha}^{\ell\Re}(u_j) \right)^2 + \left(u_{\alpha}^{\mathrm{u}\Re}(u_j) \right)^2 + \left(v_{\alpha}^{\ell\Re}(u_j) \right)^2 + \left(v_{\alpha}^{\mathrm{u}\Re}(u_j) \right)^2 + \left(w_{\alpha}^{\ell\Re}(u_j) \right)^2 + \left(w_{\alpha}^{\mathrm{u}\Re}(u_j) \right)^2 \right) \right)} \\ \sqrt{\sum_{\alpha,k=1}^{m} \dot{\omega}_k \left(\sum_{i,j=1}^{n} \gamma_i \left(\left(u_{\alpha}^{\ell\mathcal{L}}(u_j) \right)^2 + \left(u_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) \right)^2 + \left(v_{\alpha}^{\ell\mathcal{L}}(u_j) \right)^2 + \left(v_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) \right)^2 + \left(w_{\alpha}^{\ell\mathcal{L}}(u_j) \right)^2 + \left(w_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) \right)^2 \right) \right)} \end{array} \right)}$$

Definition 3.9 Let $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, their WCC has also given as $\delta_{WmPIVNSS}^{1}(\wp_{\Re}, \wp_{\mathcal{L}})$ is defined as follows:

$$\delta_{WmPIVNSS}^{1}(\wp_{\Re}, \wp_{\mathcal{L}}) = \frac{\mathcal{C}_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}})}{max\{\varsigma_{mPIVNSS}(\wp_{\Re}), \varsigma_{mPIVNSS}(\wp_{\mathcal{L}})\}} \tag{3.9}$$

$$\delta_{WmPIVNSS}^{1}(\wp_{\Re}, \wp_{\mathcal{L}}) = \frac{\sum_{\alpha,k=1}^{m} \dot{\omega}_k \left(\sum_{i,j=1}^{n} \gamma_i \left(\begin{array}{c} u_{\alpha}^{\ell\Re}(u_j) * u_{\alpha}^{\ell\mathcal{L}}(u_j) + u_{\alpha}^{\mathrm{u}\Re}(u_j) * u_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) + v_{\alpha}^{\ell\Re}(u_j) * v_{\alpha}^{\ell\mathcal{L}}(u_j) + \\ v_{\alpha}^{\mathrm{u}\Re}(u_j) * v_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) + w_{\alpha}^{\ell\Re}(u_j) * w_{\alpha}^{\ell\mathcal{L}}(u_j) + w_{\alpha}^{\mathrm{u}\Re}(u_j) * w_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) \end{array} \right) \right)}{max \left\{ \begin{array}{c} \sum_{\alpha,k=1}^{m} \dot{\omega}_k \left(\sum_{i,j=1}^{n} \gamma_i \left(\left(u_{\alpha}^{\ell\Re}(u_j) \right)^2 + \left(u_{\alpha}^{\mathrm{u}\Re}(u_j) \right)^2 + \left(v_{\alpha}^{\ell\Re}(u_j) \right)^2 + \left(v_{\alpha}^{\mathrm{u}\Re}(u_j) \right)^2 + \left(w_{\alpha}^{\ell\Re}(u_j) \right)^2 + \left(w_{\alpha}^{\mathrm{u}\Re}(u_j) \right)^2 \right) \right), \\ \sum_{\alpha,k=1}^{m} \dot{\omega}_k \left(\sum_{i,j=1}^{n} \gamma_i \left(\left(u_{\alpha}^{\ell\mathcal{L}}(u_j) \right)^2 + \left(u_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) \right)^2 + \left(v_{\alpha}^{\ell\mathcal{L}}(u_j) \right)^2 + \left(v_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) \right)^2 + \left(w_{\alpha}^{\ell\mathcal{L}}(u_j) \right)^2 + \left(w_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) \right)^2 \right) \right) \end{array} \right\}} \tag{3.10}$$

If we consider $\dot{\omega} = \{\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}\}$ and $\gamma = \{\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\}$, then $\delta_{WmPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}})$ and $\delta_{WmPIVNSS}^{1}(\wp_{\Re}, \wp_{\mathcal{L}})$ are reduced to $\delta_{mPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}})$ and $\delta_{mPIVNSS}^{1}(\wp_{\Re}, \wp_{\mathcal{L}})$ respectively defined in definitions 3.6 and 3.7.

Theorem 3.4 Let $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, WCC between them satisfies the succeeding properties

$0 \leq \delta_{WmPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) \leq 1$

$\delta_{WmPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = \delta_{WmPIVNSS}(\wp_{\mathcal{L}}, \wp_{\Re})$

If $\wp_{\Re} = \wp_{\mathcal{L}}$, that is $\forall\ j,\ \alpha,\ u_{\alpha}^{\ell\Re}(u_j) = u_{\alpha}^{\ell\mathcal{L}}(u_j),\ u_{\alpha}^{\mathrm{u}\Re}(u_j) = u_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j),\ v_{\alpha}^{\ell\Re}(u_j) = v_{\alpha}^{\ell\mathcal{L}}(u_j),\ v_{\alpha}^{\mathrm{u}\Re}(u_j) = v_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j),\ w_{\alpha}^{\ell\Re}(u_j) = w_{\alpha}^{\ell\mathcal{L}}(u_j),\ w_{\alpha}^{\mathrm{u}\Re}(u_j) = w_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j)$, then $\delta_{WmPIVNSS}(\wp_{\Re}, \wp_{\mathcal{L}}) = 1$.

Proof The proof is obvious.

4. Decision-making Approach Based on CC of mPIVNSS

Take a collection of s alternatives represented as $\beta = \{\beta^1, \beta^2, \beta^3, \ldots, \beta^s\}$, assessed by a panel of experts $\mathcal{U} = \{u_1, u_2, u_3, \ldots, u_n\}$ with proper weights $\Omega = (\Omega_1, \Omega_1, \ldots, \Omega_n)^T$, where $\Omega_i > 0$, $\sum_{i=1}^{n} \Omega_i = 1$. Furthermore, let $\mathcal{E} = \{e_1, e_2, \ldots, e_m\}$ Denote a collection of characteristics, accompanied by a weight vector $\gamma = (\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_m)^T$ such as $\gamma_j > 0$, $\sum_{j=1}^{m} \gamma_j = 1$. The group of specialists $\{u_i\colon\ i = 1, 2\ldots, n\}$ assesses the alternatives $\{\beta^{(z)}\colon\ z = 1, 2, \ldots, s\}$ based on the specified parameters $\{e_j\colon\ j = 1, 2, \ldots, m\}$. The assessments are articulated as m-polar interval-valued neutrosophic soft numbers (mPIVNSNs), represented as $\mathcal{L}_{ij}^{(z)} = \left(u_{\alpha_{ij}}^{(z)}, v_{\alpha_{ij}}^{(z)}, w_{\alpha_{ij}}^{(z)} \right)$, where $u_{\alpha_{ij}}^{(z)} = \left[u_{\alpha_{ij}}^{\ell}(u), u_{\alpha_{ij}}^{\mathrm{u}}(u) \right]$, $v_{\alpha_{ij}}^{(z)} = \left[v_{\alpha_{ij}}^{\ell}(u), v_{\alpha_{ij}}^{\mathrm{u}}(u) \right]$, and $w_{\alpha_{ij}}^{(z)} = \left[w_{\alpha_{ij}}^{\ell}(u), w_{\alpha_{ij}}^{\mathrm{u}}(u) \right]$, here $0 \leq u_{\alpha}^{\ell}(u), u_{\alpha}^{\mathrm{u}}(u)$, $v_{\alpha}^{\ell}(u), v_{\alpha}^{\mathrm{u}}(u)$, $w_{\alpha}^{\ell}(u), w_{\alpha}^{\mathrm{u}}(u) \leq 1$ and $0 \leq u_{\alpha_{ij}}^{\mathrm{u}}(u) + v_{\alpha_{ij}}^{\mathrm{u}}(u) + w_{\alpha_{ij}}^{\mathrm{u}}(u) \leq 3$. So $\mathcal{L}_{ij}^{(z)} = \left(\left[u_{\alpha_{ij}}^{\ell}(u), u_{\alpha_{ij}}^{\mathrm{u}}(u) \right], \left[v_{\alpha_{ij}}^{\ell}(u), v_{\alpha_{ij}}^{\mathrm{u}}(u) \right], \left[w_{\alpha_{ij}}^{\ell}(u), w_{\alpha_{ij}}^{\mathrm{u}}(u) \right] \right)$ for all $i,\ j$.

4.1. Algorithm for the Correlation Coefficient of mPIVNSS

Step 1. Select the set of attributes.

Step 2. According to the expert's opinion, develop the decision matrix for each alternative in the form of mPIVNSNs.

Step 3. Compute the informational neutrosophic energies for mPIVNSSs.

Step 4. By using the following formula, compute the correlation.

$$\mathcal{C}_{mPIVNSS}(\wp_{\mathfrak{R}},\wp_{\mathcal{L}}) = \sum_{\alpha=1}^{m}\sum_{j=1}^{n}\Big(u_{\alpha}^{\ell\mathfrak{R}}(u_j) * u_{\alpha}^{\ell\mathcal{L}}(u_j) + u_{\alpha}^{\mathrm{u}\mathfrak{R}}(u_j) * u_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) + v_{\alpha}^{\ell\mathfrak{R}}(u_j) * v_{\alpha}^{\ell\mathcal{L}}(u_j) + v_{\alpha}^{\mathrm{u}\mathfrak{R}}(u_j) * v_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j) + w_{\alpha}^{\ell\mathfrak{R}}(u_j) * w_{\alpha}^{\ell\mathcal{L}}(u_j) + w_{\alpha}^{\mathrm{u}\mathfrak{R}}(u_j) * w_{\alpha}^{\mathrm{u}\mathcal{L}}(u_j)\Big)$$

Step 5. By using the following formula, compute the CC between two mPIVNSSs

$$\delta_{mPIVNSS}(\wp_{\mathfrak{R}},\wp_{\mathcal{L}}) = \frac{\mathcal{C}_{mPIVNSS}(\wp_{\mathfrak{R}},\wp_{\mathcal{L}})}{\sqrt{\varsigma_{mPIVNSS}(\wp_{\mathfrak{R}}) * \varsigma_{mPIVNSS}(\wp_{\mathcal{L}})}}$$

Step 6. Analyze the results.

The flowchart of the proposed model is given in the following Figure 1.

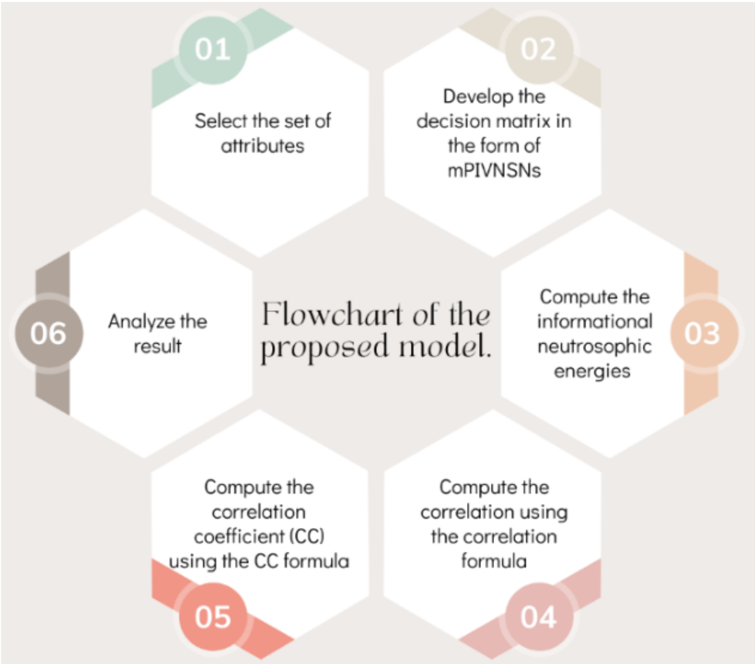

Figure 1. Flowchart of the proposed model.

4.2. Numerical Example

A university aims to appoint an associate professor and has shortlisted four candidates following an initial evaluation: $\{\beta^{(1)}, \beta^{(2)}, \beta^{(3)}, \beta^{(4)}\}$. To facilitate a thorough assessment, the institution's president has convened a panel of three experts, $\{u_1, u_2, u_3\}$, with designated weights of $(0.25, 0.30, 0.45)^T$, indicating the significance of each expert's contribution. The panel delineates three principal criteria for evaluating the candidates: e_1 = experience, e_2 = publications, and e_3 = Research quality with weights $(0.35, 0.25, 0.40)^T$. Each expert assesses the candidates based on these criteria, articulating their preferences as m-polar interval-valued neutrosophic soft numbers (mPIVNSNs). The evaluations are formulated as $\mathcal{L}_{ij}^{(z)} = \left(u_{\alpha_{ij}}^{(z)}, v_{\alpha_{ij}}^{(z)}, w_{\alpha_{ij}}^{(z)}\right)$, where $u_{\alpha_{ij}}^{(z)} = \left[u_{\alpha_{ij}}^{\ell}(u), u_{\alpha_{ij}}^{\mathrm{u}}(u)\right]$, $v_{\alpha_{ij}}^{(z)} = \left[v_{\alpha_{ij}}^{\ell}(u), v_{\alpha_{ij}}^{\mathrm{u}}(u)\right]$, and $w_{\alpha_{ij}}^{(z)} = \left[w_{\alpha_{ij}}^{\ell}(u), w_{\alpha_{ij}}^{\mathrm{u}}(u)\right]$, here $0 \leq u_{\alpha}^{\ell}(u), u_{\alpha}^{\mathrm{u}}(u), v_{\alpha}^{\ell}(u), v_{\alpha}^{\mathrm{u}}(u)$, $w_{\alpha}^{\ell}(u), w_{\alpha}^{\mathrm{u}}(u) \leq 1$ and $0 \leq u_{\alpha_{ij}}^{\mathrm{u}}(u) + v_{\alpha_{ij}}^{\mathrm{u}}(u) + w_{\alpha_{ij}}^{\mathrm{u}}(u) \leq 3$. So $\mathcal{L}_{ij}^{(z)} = \left(\left[u_{\alpha_{ij}}^{\ell}(u), u_{\alpha_{ij}}^{\mathrm{u}}(u)\right], \left[v_{\alpha_{ij}}^{\ell}(u), v_{\alpha_{ij}}^{\mathrm{u}}(u)\right], \left[w_{\alpha_{ij}}^{\ell}(u), w_{\alpha_{ij}}^{\mathrm{u}}(u)\right]\right)$ for all i, j.

Upon completion of the evaluations, the weighted correlation coefficient (WCC) is calculated for each candidate to assess the correlation between their evaluations and the institution's established ideal criteria. The candidates are rated according to these results, and the one with the highest correlation is chosen for the position. This methodical methodology guarantees an equitable, transparent, and systematic assessment procedure, considering many criteria and the diverse viewpoints of the expert panel.

4.3. Application of Proposed Approach

Analyze a collection of options, $\{\beta^{(1)}, \beta^{(2)}, \beta^{(3)}, \beta^{(4)}\}$, denoting the candidates selected for the interview procedure. Let $\mathcal{E}$ = $\{e_1$ = experience, e_2 = publications, e_3= research quality$\}$ represents the set of criteria for evaluating candidates for the position of associate professor. Let $\mathfrak{R}$ and $\mathcal{L} \subseteq \mathcal{E}$, then the specifications provided by the university administration, we formulate the 3-PIVNSS $\wp_{\mathfrak{R}}(e)$ as follows given in Table 1:

Table 1. Construction of 3-PIVNSS of alternatives according to management requirements.

$\wp_{\mathfrak{R}}(e)$	e_1	e_2	e_3
u_1	([.3,.5],[.2,.4],[.2,.6]), ([.2,.3],[.5,.7],[.1,.3]), ([.5,.6],[.1,.3],[.4,.6])	([.2,.4],[.3,.5],[.3,.6]), ([.2,.3],[.2,.4],[.4,.5]), ([.4,.6],[.1,.3],[.2,.4])	([.6,.7],[.2,.3],[.3,.4]), ([.4,.5],[.5,.8],[.1,.2]), ([.1,.2],[.5,.8],[.2,.4])
u_2	([.5,.7],[.1,.2],[.4,.6]), ([.2,.4],[.3,.4],[.2,.5]) ([.6,.8],[.1,.2],[.3,.5])	([.5,.6],[.2,.3],[.3,.4]), ([.4,.6],[.4,.5],[.3,.5]), ([.3,.5],[.4,.5],[.1,.3])	([.5,.7],[.1,.2],[.5,.6]), ([.2,.4],[.5,.6],[.4,.6]), ([.2,.4],[.3,.4],[.2,.5])
u_3	([.4 .6],[.2,.3],[.1,.4]), ([.2,.5],[.2,.3],[.1,.6]), ([.3,.4],[.2,.5],[.5,.7])	([.3,.5],[.4,.5],[.1,.3]), ([.2,.4],[.7,.8],[.1,.2]), ([.1,.2],[.7,.8],[.2,.3])	([.2,.3],[.5,.7],[.1,.3]), ([.3,.4],[.2,.5],[.5,.7]), ([.2,.4],[.3,.5],[.3,.6])

Construct the 3-PIVNSS $\wp_{\mathcal{L}}^{(t)}(e)$ for each alternative according to experts, where t = 1, 2, 3, 4.

Table 2. Evaluation report for alternative $\beta^{(1)}$.

$\wp_{\mathcal{L}}^{(1)}(e)$	e_1	e_2	e_3
u_1	([.2,.4],[.4,.5],[.3,.4]), ([.6,.7],[.1,.2],[.2,.3]), ([.3,.4],[.4,.5],[.2,.4])	([.3,.4],[.4,.5],[.2,.5]), ([.3,.6],[.2,.3],[.1,.2]), ([.4,.6],[.2,.3],[.4,.5])	([.2,.4],[.4,.6],[.1,.2]), ([.1,.3],[.6,.7],[.2,.3]), ([.4,.5],[.2,.5],[.2,.3])
u_2	([.5,.7],[.1,.2],[.2,.4]), ([.7,.8],[.1,.2],[.2,.4]) ([.1,.3],[.1,.5],[.2,.5])	([.1,.4],[.2,.4],[.1,.2]), ([.2,.5],[.2,.4],[.3,.5]), ([.3,.5],[.2,.4],[.4,.6])	([.5,.7],[.1,.2],[.5,.6]), ([.3,.5],[.3,.4],[.6,.7]), ([.2,.4],[.3,.4],[.2,.5])
u_3	([.4,.5],[.2,.5],[.1,.2]), ([.4,.7],[.1,.2],[.1,.2]), ([.3,.4],[.2,.5],[.5,.7])	([.6,.8],[.1,.2],[.1,.5]), ([.2,.4],[.7,.8],[.1,.2]), ([.5,.7],[.1,.2],[.2,.4])	([.5,.6],[.2,.3],[.4,.5]), ([.3,.4],[.4,.5],[.2,.4]), ([.2,.4],[.3,.5],[.3,.6])

Table 3. Evaluation report for alternative $\beta^{(2)}$.

$\wp_{\mathcal{L}}^{(2)}(e)$	e_1	e_2	e_3
u_1	([.2,.4],[.4,.6],[.4,.5]), ([.2,.3],[.4,.6],[.3,.5]), ([.1,.2],[.6,.8],[.2,.5])	([.4,.5],[.2,.5],[.1,.2]), ([.2,.3],[.4,.6],[.3,.5]), ([.1,.2],[.6,.8],[.2,.5])	([.7,.8],[.1,.2],[.2,.3]), ([.1,.3],[.6,.7],[.2,.5]), ([.4,.5],[.2,.5],[.1,.2])

u_2	([.5, .7], [.1, .2], [.2, .4]), ([.1, .3], [.6, .7], [.2, .6]) ([.1, .4], [.2, .5], [.4, .6])	([.1, .4], [.2, .4], [.1, .2]), ([.1, .2], [.2, .5], [.4, .6]), ([.1, .4], [.2, .5], [.4, .6])	([.1, .4], [.2, .5], [.4, .6]), ([.3, .4], [.2, .6], [.4, .6]), ([. 2, .4], [. 3, .4], [. 2, .5])
u_3	([.4, .5], [.2, .5], [.1, .2]), ([.1, .2], [.2, .5], [.4, .6]), ([.3, .5], [.3, .5], [.6, .7])	([.3, .5], [.3, .5], [.6, .7]), ([.1, .2], [.2, .5], [.4, .6]), ([.5, .7], [.1, .2], [.2, .4])	([.2, .4], [.4, .5], [.6, .8]), ([.3, .5], [.3, .5], [.6, .7]), ([.1, .2], [.2, .5], [.4, .6])

Table 4. Evaluation report for alternative $\beta^{(3)}$.

$\wp_{\mathcal{L}}^{(3)}(e)$	e_1	e_2	e_3
u_1	([.6, .7], [.1, .2], [.3, .5]), ([.6, .8], [.1, .2], [.2, .3]), ([.6, .7], [.3, .5], [.1, .2])	([.7, .8], [.1, .2], [.2, .5]), ([.6, .7], [.1, .2], [.1, .2]), ([.5, .8], [.1, .2], [.2, .4])	([.1, .3], [.6, .7], [.2, .5]), ([.7, .8], [.1, .2], [.2, .3]), ([.5, .7], [.3, .4], [.2, .3])
u_2	([.5, .7], [.2, .5], [.2, .3]), ([.7, .8], [.3, .5], [.1, .3]) ([.4, .7], [.2, .3], [.3, .7])	([.5, .6], [.3, .4], [.1, .2]), ([.1, .2], [.2, .5], [.4, .6]), ([.4, .6], [.2, .3], [.1, .2])	([.1, .4], [.2, .5], [.4, .6]), ([.4, .6], [.2, .3], [.1, .2]]), ([. 2, .4], [. 3, .4], [. 2, .5])
u_3	([.4, .6], [.2, .3], [.1, .2]), ([.1, .2], [.2, .5], [.4, .6]), ([.6, .8], [.1, .2], [.1, .3])	([.3, .5], [.3, .5], [.6, .7]), ([.6, .8], [.1, .2], [.1, .2]), ([.7, .8], [.1, .2], [.2, .4])	([.6, .8], [.3, .4], [.1, .2]), ([.5, .7], [.1, .2], [.4, .5]), ([.1, .2], [.2, .5], [.4, .6])

Table 5. Evaluation report for alternative $\beta^{(4)}$.

$\wp_{\mathcal{L}}^{(4)}(e)$	e_1	e_2	e_3
u_1	([.3, .5], [.2, .4], [.1, .2]), ([.3, .6], [.1, .2], [.4, .7]), ([.4, .7], [.3, .4], [.2, .3])	([.7, .8], [.2, .4], [.3, .5]), ([.5, .7], [.3, .4], [.2, .4]), ([.4, .6], [.2, .5], [.3, .4])	([.2, .3], [.5, .7], [.2, .4]), ([.5, .7], [.2, .4], [.3, .5]), ([.4, .5], [.5, .7], [.2, .4])
u_2	([.4, .7], [.3, .5], [.2, .4]), ([.5, .8], [.3, .6], [.2, .3]) ([.4, .6], [.2, .3], [.3, .5])	([.5, .8], [.3, .4], [.2, .3]), ([.2, .4], [.2, .3], [.4, .5]), ([.3, .5], [.2, .3], [.3, .5])	([.2, .4], [.2, .3], [.3, .6]), ([.4, .6], [.2, .3], [.1, .2]]), ([. 2, .4], [. 3, .4], [. 2, .5])
u_3	([.3, .5], [.3, .5], [.1, .2]), ([.1, .2], [.2, .5], [.4, .6]), ([.5, .7], [.2, .4], [.1, .3])	([.3, .5], [.4, .6], [.6, .7]), ([.5, .7], [.1, .2], [.4, .5]), ([.3, .5], [.2, .5], [.1, .3])	([.4, .6], [.3, .5], [.1, .2]), ([.6, .7], [.1, .2], [.3, .5]), ([.2, .5], [.2, .3], [.4, .6])

4.4. Solution by using the Developed Approach

Compute the CC among $\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(1)}(e))$, $\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(2)}(e))$, $\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(3)}(e))$, $\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(4)}(e))$ utilizing equation 3.5 from Table 1-5, such as

$$\mathcal{S}^{1}_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(1)}(e)) = \left\{\frac{(.3)(.2)+(.5)(.4)+(.2)(.4)+(.4)(.5)+(.2)(.3)+(.6)(.4)}{\sqrt{(.3)^2+(.5)^2+(.2)^2+(.4)^2+(.2)^2+(.6)^2}\sqrt{(.2)^2+(.4)^2+(.4)^2+(.5)^2+(.3)^2+(.4)^2}} + \frac{(.2)(.6)+(.3)(.7)+(.5)(.1)+(.7)(.2)+(.1)(.2)+(.3)(.3)}{\sqrt{(.2)^2+(.3)^2+(.5)^2+(.7)^2+(.1)^2+(.3)^2}\sqrt{(.6)^2+(.7)^2+(.1)^2+(.2)^2+(.2)^2+(.3)^2}} + \cdots + \frac{(.2)(.2)+(.4)(.5)+(.3)(.2)+(.5)(.3)+(.3)(.4)+(.6)(.6)}{\sqrt{(.2)^2+(.4)^2+(.3)^2+(.5)^2+(.3)^2+(.6)^2}\sqrt{(.2)^2+(.5)^2+(.2)^2+(.3)^2+(.4)^2+(.6)^2}}\right\} = \left(\frac{24.28}{27.7036}\right) = 0.87642.$$

Similarly, we can find the CC between $\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(2)}(e))$, $\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(3)}(e))$, $\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(4)}(e))$ given as

$\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(2)}(e)) = \left(\frac{25.04}{28.6727}\right) = 0.87330$, $\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(3)}(e)) = \left(\frac{23.73}{29.4968}\right) = 0.80449$, and $\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(4)}(e)) = \left(\frac{24.58}{28.7433}\right) = 0.85516$. This shows that $\delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(1)}(e)) > \delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(2)}(e)) > \delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(4)}(e)) > \delta_{mPIVNSS}(\wp_{\Re}(e), \wp_{\mathcal{L}}^{(3)}(e))$. The obtained results show that $\beta^{(1)}$ is the most suitable alternative. So, we can say that the $\beta^{(1)}$ is the best alternative. The alternatives ranking is given as $\beta^{(1)} > \beta^{(2)} > \beta^{(4)} > \beta^{(3)}$. The graphical representation of the results obtained is given in the following Figure 2.

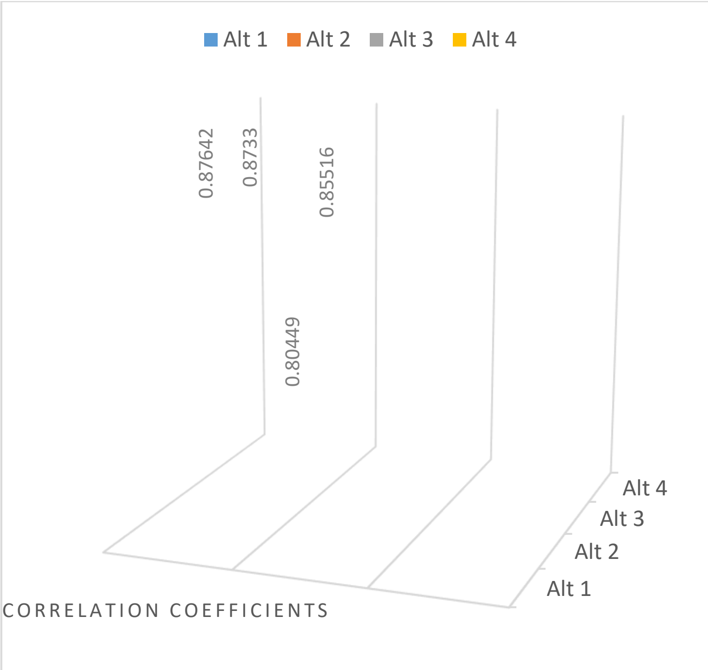

Figure 2. Correlation values of each alternative.

5. Discussion and Comparative Analysis:

The subsequent part will examine the efficacy, simplicity, adaptability, and benefits of the proposed strategy. A succinct comparison study is presented to assess our methodology with established techniques.

5.1. Advantages, Superiority, and Flexibility of Proposed Approach

The paper presents a novel decision-making technique based on the correlation coefficient utilizing m-polar interval-valued neutrosophic soft sets. This methodology has shown more effectiveness in resolving multi-criteria decision-making (MCDM) issues than current techniques, yielding more precise and dependable outcomes. Our comprehensive scientific study and comparison analysis reveal that the results of the proposed method are more extensive and generalizable than those of traditional approaches. The proposed strategy, in contrast to conventional DM methods, integrates supplementary data to adeptly address uncertainties and imprecisions. The hybrid building of various fuzzy set (FS) architectures is a specific instance of mPIVNSS under certain clearly defined parameters, hence augmenting its adaptability. The mPIVNSS framework enhances the representation of objects by the integration of supplementary information, rendering it a potent and effective instrument for managing ambiguous and indeterminate data in decision-making. Thus, our proposed method is more resilient and adaptive, exceeding the unique hybrid structures of FS in flexibility, appropriateness, and dependability for addressing complicated MCDM situations. This constitutes a substantial achievement in the field of decision-making under ambiguity.

Table 6. Comparison of existing methodologies versus the proposed methodology.

	Set	Truthiness	Indeterminacy	Falsity	Multi-polarity	Loss of information
Chen et al. [37]	mPFS	✓	×	×	✓	×
Zhang et al. [56]	IFS	✓	×	✓	×	✓
Talebi et al. [53]	mPIVIFS	✓	×	✓	✓	✓
Naeem et al. [54]	MPyFS	✓	×	✓	✓	×
Zhang et al. [55]	INSs	✓	✓	✓	×	×
Proposed approach	mPIVNSS	✓	✓	✓	✓	×

The subject of this article is a modern challenge: why must we develop new algorithms specifically suited to the present framework? The proposed methodology demonstrates considerable advantages compared to current approaches. Conventional frameworks, including intuitionistic fuzzy sets (IFS), picture fuzzy sets, fuzzy sets (FS), hesitant fuzzy sets, neutrosophic sets (NS), and other variations of fuzzy sets, are frequently limited by their hybrid structures and do not adequately encompass the comprehensive range of information necessary to tackle complex scenarios. These constraints hinder its application in situations requiring thorough decision-making. The created m-polar interval-valued neutrosophic soft set (mPIVNSS) model provides a superior framework for multi-criteria decision-making (MCDM). It encompasses the aspects of truthiness, indeterminacy, and falsity, offering a more comprehensive depiction of the choice space. The comparison study in Table 6 unequivocally illustrates the superiority of our technique relative to existing methodologies. We contend that the mPIVNSS framework exhibits superior efficacy compared to other models, such as intuitionistic, neutrosophic, hesitant, image, and ambiguous fuzzy sets, managing uncertainty and producing precise outcomes. Furthermore, the recently developed similarity measure for mPIVNSS demonstrates superior effectiveness and reliability compared to the similarity measures already utilized in MCDM approaches. The findings underscore the remarkable efficacy and relevance of the mPIVNSS paradigm in tackling modern decision-making difficulties.

5.2. Discussion

Chen et al. [37] originally presented the notion of multi-polar information in fuzzy sets, focusing on membership values, but this framework falls short in situations with both indeterminacy and falsehood. Zhang et al. [56] advanced intuitionistic fuzzy sets (IFS) to incorporate membership and non-membership values, yet they do not fully address multi-polar information or intrinsic uncertainties. Likewise, Naeem et al. [54] proposed m-polar fuzzy sets (mPyFS), which still struggle with accurately handling the indeterminacy associated with alternatives. Saqlain et al. [57] recent studies, in contrast, represent substantial progress, integrating truth, indeterminacy, and falsity to handle multi-polar and complex data effectively. Their application of linguistic hypersoft sets for rural health services and multi-polar interval-valued neutrosophic hypersoft sets enhances decision-making under uncertainty in fields like hydrogen technology evaluation and medical diagnostics [58-59]. Zhang et al. [55] also addressed truth, indeterminacy, and falsity grades, but their approach does not adequately handle multi-dimensional data in complex MCDM contexts. The approach defined by [60-61] overcomes these limitations by employing advanced aggregation operators and distance measures for wastewater treatment and water quality evaluation, providing specialists with a more thorough and reliable framework for MCDM applications [62-63]. This innovative approach supports complex decision-making scenarios with greater accuracy and reliability, marking a significant advancement in the field.

6. Conclusions

In this paper, we present a novel hybrid structure, the m-polar interval-valued neutrosophic soft set, by integrating two distinct structures: the m-polar fuzzy set and the interval-valued neutrosophic soft set. In this structure, we developed the CC and WCC, collectively with their relevant features, specifically designed for mPIVNSS. Additionally, we developed a novel algorithm employing these metrics to effectively tackle MCDM challenges. A thorough comparison analysis was performed to validate the proposed approach. The analytical results indicate the method's superior dependability, pragmatics, and efficacy, offering decision-makers a reliable and realistic instrument for complicated decisions. Our findings indicate that the established MCDM technique is exceptionally pertinent and appropriate for contemporary decision-making challenges. This methodology can be applied to additional fields in future endeavors, such as clinical diagnostics, mathematical programming, and cluster analysis, thereby enhancing its relevance and influence.

Declarations

Ethics Approval and Consent to Participate

The results/data/figures in this manuscript have not been published elsewhere, nor are they under consideration by another publisher. All the material is owned by the authors, and/or no permissions are required.

Consent for Publication

This article does not contain any studies with human participants or animals performed by any of the authors.

Availability of Data and Materials

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Competing Interests

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