



A Neutrosophic Micro Vague Correlation Measure: Application to Multi-Criteria Decision Making Problems

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Abstract: Statistical methods called correlation measures are employed to express the degree of association or relationship between two variables. The correlation coefficient which may be computed in a variety of ways is the most often used kind of correlation measure in many disciplines including biology, psychology, economics, and finance. Knowing the correlations between variables is essential for analysis and decision-making. This paper aims to commence a novel type of correlation measure called Neutrosophic Micro vague Correlation Measure and Neutrosophic Micro Vague Weighted Correlation Measure. Further demonstrates the implementation of the Neutrosophic Micro Vague correlation measure in the MCDM Problem. By adopting a more inclusive approach to Neutrosophic micro-vague correlation analysis, researchers and practitioners can gain a more comprehensive understanding of the intricate relationships within their datasets.

Keywords: Neutrosophic Vague Sets (NVS); Neutrosophic Micro Vague sets; Neutrosophic Micro Vague Correlation Measure; Neutrosophic Micro Vague Weighted Correlation Measure.

1. Introduction

L.A. Zadeh [1] designed a fuzzy set in 1965 and it remains one of several well-known ideas in modern the mathematical discipline. Fuzzy topology had been initially suggested in 1968 by C.L. Chang [2]. The Intuitionistic Fuzzy Set which has been extensively utilized in numerous mathematical fields was originated by Atanassov [3]. In 1993, Gau and Buehrer [4] have extended fuzzy sets into Vague sets. Unreliable and ambiguous data was not handled well by the IFS. Consequently, Smarandache [5, 6] formed the Neutrosophic sets (in short NS), in which all of the components correspond to either the real or not conventional standard range $]0,1+[$ and has an absolute subscription amount an unpredictability subscription measure, and an untruth subscription amount. Yager [7] presented a fresh concept of Pythagorean fuzzy sets with a constraint that the total of the squares representing true and untrue subscriptions is either lower or equal to 1. By combining NSs with VSs, the notion of NVS was originated in 2015 by Shawkat Alkhazaleh [8]. Later in 2023, Vargees Vahini T and Trinita Pricilla M investigated PNV generalised β closed sets [9].

M. Lellis Thivagar [10] initially released Nano topology in 2013. It has no more than 5 Nano open sets and at least 3 Nano open sets which includes the universal set and the empty set. Later in 2019, S. Chandrasekar [11] designed Micro Topology employing the simple extension notion on this. Vargees Vahini T and Trinita Pricilla M [12] constructed the latest topological space known as Micro Vague Topological Space by integrating Micro and Vague topological spaces in 2023.

I. M. Hanafy [13, 15] proposes a correlation between NSs and Data in Probability Spaces. Broumi. S and Smarandache. F [14] established the Correlation of INS in 2013. Broumi et al. [16] devised the correlation gauge for NRS and used it towards the problem of medical prognosis. In 2017, Surapati Pramanik et al. [17] determined the association coefficient comparing two rough NS and used in an

MCDM. To solve MCDM, Surapati Pramanik et al. [18] devised Correlation Coefficient Measures of Bipolar Neutrosophic Sets. The correlation measure was given in 2019 by R. Jansi et al. [19] for PNS with T and F depending on NS. In 2020, the correlation for SVNS was implemented by Shchur Iryna et al. [20]. For intuitionistic fuzzy sets, Jyoti Bajaj and Satish Kumar [21] have implemented the correlation coefficient.

Neutrosophic Linguistic valued Hypersoft Set with Application in Medical Diagnosis and Treatment has been studied by Muhammad Saqlain, Poom Kumam, and Wiyada Kumam [23]. Neutrosophic Fixed Point Theorems and Cone Metric Spaces [24], (Φ, Ψ) -Weak Contractions in Neutrosophic Cone Metric Spaces via Fixed Point Theorems [25] were introduced by Smarandache, Florentin and Wadei F. Al-Omeri. Using the Max Product of Complement in Neutrosophic Graphs [26] Al-Omeri et al. identified Internet streaming services. They also introduced Neutrosophic G^* - Closed Sets in Neutrosophic Topological Spaces [29], studied Neutrosophic Graphs with Application on Earthquake Response Centers in Japan [28], and discussed The Property (P) and New Fixed-Point Results on Ordered Metric Spaces in Neutrosophic Theory [27].

This study presents the Neutrosophic Micro Vague Correlation Measure and Neutrosophic Micro Vague Weighted Correlation Measure and their application in MCDM.

2. Preliminaries

Definition 2.1. [8]: A \mathcal{NVS} φ on Ω is written as $\varphi = \{(\nu; \hat{\beta}(\nu), \hat{\delta}(\nu), \hat{\eta}(\nu)) | \nu \in \Omega\}$. Given that.

$$\hat{\beta}(\nu) = [\beta^-, \beta^+], \hat{\delta}(\nu) = [\delta^-, \delta^+], \hat{\eta}(\nu) = [\eta^-, \eta^+]$$

Where,

- i). $\beta^+ = 1 - \eta^-$
- ii). $\eta^+ = 1 - \beta^-$ and
- iii). $0 \leq (\beta^-)^2 + (\delta^-)^2 + (\eta^-)^2 \leq 2^+$.

Definition 2.2. [12]: Let us consider a Nano Vague Topological Space $(\mathcal{U}, \xi_{\mathfrak{R}}(\mathcal{Z}))$. Let $\eta_{\mathfrak{R}}(\mathcal{Z}) = \{\kappa \cup (\omega \cap \eta) : \kappa, \omega \in \xi_{\mathfrak{R}}(\mathcal{Z}) \text{ and } \eta \notin \xi_{\mathfrak{R}}(\mathcal{Z})\}$. Then, $\eta_{\mathfrak{R}}(\mathcal{Z})$ is termed the Micro Vague Topology (\mathcal{MVT} in short) of $\eta_{\mathfrak{R}}(\mathcal{Z})$ by η . We refer to the trio $(\mathcal{U}, \xi_{\mathfrak{R}}(\mathcal{Z}), \eta_{\mathfrak{R}}(\mathcal{Z}))$ as the Micro Vague Topological Space ($\mathcal{MVT\mathcal{S}}$ in short). Micro Vague open sets (\mathcal{MVOS} in short) are made up of the elements of $\eta_{\mathfrak{R}}(\mathcal{Z})$, while Micro Vague closed sets (\mathcal{MVCS} in short) are the complement of \mathcal{MVOS} .

Definition 2.3. [22]: Consider a Neutrosophic Vague Nano Topological Space $(\mathcal{U}, \tau_{\mathfrak{R}}(\mathcal{S}))$. Assume $\eta_{\mathfrak{R}}(\mathcal{S}) = \{\chi \cup (\xi \cap \eta) : \chi, \xi \in \tau_{\mathfrak{R}}(\mathcal{S}) \text{ and } \eta \notin \tau_{\mathfrak{R}}(\mathcal{S})\}$. If $\eta_{\mathfrak{R}}(\mathcal{S})$ Meets the following axioms, then it is referred to as the Neutrosophic Micro Vague Topology of $\tau_{\mathfrak{R}}(\mathcal{S})$ By η .

- i). $0_{\mathcal{NMV}}, 1_{\mathcal{NMV}} \in \eta_{\mathfrak{R}}(\mathcal{S})$.
- ii). Any subcollection of $\eta_{\mathfrak{R}}(\mathcal{S})$ have its components unioned into $\eta_{\mathfrak{R}}(\mathcal{S})$.
- iii). Any finite sub-collection of $\eta_{\mathfrak{R}}(\mathcal{S})$ has its intersection points inside $\eta_{\mathfrak{R}}(\mathcal{S})$.

As of right now, $\eta_{\mathfrak{R}}(\mathcal{S})$ is known as the Neutrosophic Micro Vague Topology (*Shortly NMVT*). The trio $(\mathcal{U}, \tau_{\mathfrak{R}}(\mathcal{S}), \eta_{\mathfrak{R}}(\mathcal{S}))$ is called the Neutrosophic Micro Vague Topological Space (*Shortly NMVTS*).

3. Neutrosophic Micro Vague Correlation Measure

Definition 3.1. Let $(\Psi, \Delta_Y(\chi), \Gamma_Y(\chi))$ be \mathcal{NMVTS} . Let $\mathfrak{K} = \{< \mathcal{G}_i, \mathcal{F}_{\mathfrak{K}}(\mathcal{G}_i), \mathcal{J}_{\mathfrak{K}}(\mathcal{G}_i), \mathcal{F}_{\mathfrak{K}}(\mathcal{G}_i) >\}$ and $\mathfrak{R} = \{< \mathcal{G}_i, \mathcal{F}_{\mathfrak{R}}(\mathcal{G}_i), \mathcal{J}_{\mathfrak{R}}(\mathcal{G}_i), \mathcal{F}_{\mathfrak{R}}(\mathcal{G}_i) >\}$ be two \mathcal{NMV} sets in $(\Psi, \Delta_Y(\chi), \Gamma_Y(\chi))$. Then the \mathcal{NMV} Correlation Measure (*shortly, NMVCM*) between \mathfrak{K} and \mathfrak{R} is given as follows:

$$\hat{Z}(\mathfrak{N}, \mathfrak{R}) = \frac{\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R})}{\sqrt{\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N}) \cdot \tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{R})}} \rightarrow (1)$$

Where,

$$\begin{aligned} \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R}) &= \sum_{s=1}^n [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_s) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_s) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_s)] \\ \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N}) &= \sum_{s=1}^n [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_s) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_s) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_s)] \\ \tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{R}) &= \sum_{s=1}^n [\hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_s) + \hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_s) + \hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_s)] \end{aligned}$$

Proposition 3.2. The \mathcal{NMVCM} between \mathfrak{N} and \mathfrak{R} fulfills the subsequent criteria.

- i). $0 \leq \hat{Z}(\mathfrak{N}, \mathfrak{R}) \leq 1$.
- ii). $\hat{Z}(\mathfrak{N}, \mathfrak{R}) = 1$ iff $\mathfrak{N} = \mathfrak{R}$.
- iii). $\hat{Z}(\mathfrak{N}, \mathfrak{R}) = \hat{Z}(\mathfrak{R}, \mathfrak{N})$.

Proof: 1. $0 \leq \hat{Z}(\mathfrak{N}, \mathfrak{R}) \leq 1$

As $\hat{\mathcal{T}}, \hat{\mathcal{I}}$ and $\hat{\mathcal{F}}$ of \mathcal{NMVs} ranges from 0 to 1, $\hat{Z}(\mathfrak{N}, \mathfrak{R})$ also lies between 0 and 1.

$$\begin{aligned} \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R}) &= \sum_{s=1}^n [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_s) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_s) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_s)] \\ &= [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_1) \cdot \hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_1) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_1) \cdot \hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_1) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_1) \cdot \hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_1)] \\ &\quad + [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_2) \cdot \hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_2) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_2) \cdot \hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_2) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_2) \cdot \hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_2)] + \dots \\ &\quad + [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_n) \cdot \hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_n) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_n) \cdot \hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_n) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_n) \cdot \hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_n)] \end{aligned}$$

Using Cauchy Schwarz inequality,

$$\begin{aligned} \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R})^2 &\leq \left\{ \left[(\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_1))^2 + (\hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_1))^2 + (\hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_1))^2 \right] + \left[(\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_2))^2 + (\hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_2))^2 + (\hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_2))^2 \right] + \dots \right. \\ &\quad \left. + \left[(\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_n))^2 + (\hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_n))^2 + (\hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_n))^2 \right] \right\} \\ &\quad \times \left\{ \left[(\hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_1))^2 + (\hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_1))^2 + (\hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_1))^2 \right] + \left[(\hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_2))^2 + (\hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_2))^2 + (\hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_2))^2 \right] + \dots \right. \\ &\quad \left. + \left[(\hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_n))^2 + (\hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_n))^2 + (\hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_n))^2 \right] \right\} \\ &\leq \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N}) \cdot \tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{R}) \end{aligned}$$

Hence, $\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R})^2 \leq \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N}) \cdot \tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{R})$. Thus, $\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R}) \leq 1$. Therefore, $0 \leq \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R}) \leq 1$.

2. Let $\mathfrak{N} = \mathfrak{R}$. (i.e) $\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_i) = \hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_i)$, $\hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_i) = \hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_i)$ and $\hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_i) = \hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_i)$. $\therefore \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N}) = \tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{R})$

$$\begin{aligned} \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R}) &= \sum_{s=1}^n [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{R}}(\mathcal{G}_s) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{R}}(\mathcal{G}_s) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{R}}(\mathcal{G}_s)] \\ &= \sum_{s=1}^n [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{N}}(\mathcal{G}_s) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{N}}(\mathcal{G}_s) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{N}}(\mathcal{G}_s)] \\ &= \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N}). \end{aligned}$$

$$\text{Therefore, } \hat{Z}(\mathfrak{N}, \mathfrak{R}) = \frac{\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R})}{\sqrt{\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N}) \cdot \tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{R})}} = \frac{\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N})}{\sqrt{\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N}) \cdot \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N})}} = 1.$$

$$\text{This is possible only if } \frac{\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R})}{\sqrt{\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N}) \cdot \tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{R})}} = 1.$$

Hence, $\hat{Z}(\mathfrak{N}, \mathfrak{R}) = 1$ iff $\mathfrak{N} = \mathfrak{R}$.

$$\begin{aligned} 3. \quad \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R}) &= \sum_{s=1}^n [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathfrak{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{R}}(\mathfrak{G}_s) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathfrak{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{R}}(\mathfrak{G}_s) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathfrak{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{R}}(\mathfrak{G}_s)] \\ &= \sum_{s=1}^n [\hat{\mathcal{T}}_{\mathfrak{R}}(\mathfrak{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{N}}(\mathfrak{G}_s) + \hat{\mathcal{I}}_{\mathfrak{R}}(\mathfrak{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{N}}(\mathfrak{G}_s) + \hat{\mathcal{F}}_{\mathfrak{R}}(\mathfrak{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{N}}(\mathfrak{G}_s)] \\ &= \tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{N}) \end{aligned}$$

$$\text{Therefore, } \hat{Z}(\mathfrak{N}, \mathfrak{R}) = \frac{\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{R})}{\sqrt{\tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N}) \cdot \tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{R})}} = \frac{\tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{N})}{\sqrt{\tilde{\mathfrak{C}}(\mathfrak{R}, \mathfrak{R}) \cdot \tilde{\mathfrak{C}}(\mathfrak{N}, \mathfrak{N})}} = \hat{Z}(\mathfrak{R}, \mathfrak{N})$$

Hence the proof.

Definition 3.3. Since distinct sets may have applied different weights in numerous real-life instances, for the elements $x_s \in \mathfrak{X}$, the weight q_s should be considered where $s = 1, 2, \dots, p$. We cultivate a Neutrosophic Micro Vague Weighted Correlation Measure (*NMVWCM for short*) for the *NMV* sets \mathfrak{N} and \mathfrak{R} .

Let $q = \{q_1, q_2, \dots, q_p\}^j$ represents the weight vector of $\mathfrak{G}_s, \{s = 1, 2, \dots, p\}$ with $q_s \geq 0$ and $\sum_{s=1}^p q_s = 1$. Here, a *NMVWCM* is constructed by extending the *NMVC*M.

$$\hat{Z}_q(\mathfrak{N}, \mathfrak{R}) = \frac{\tilde{\mathfrak{C}}_q(\mathfrak{N}, \mathfrak{R})}{\sqrt{\tilde{\mathfrak{C}}_q(\mathfrak{N}, \mathfrak{N}) \cdot \tilde{\mathfrak{C}}_q(\mathfrak{R}, \mathfrak{R})}} \rightarrow (2)$$

Where,

$$\begin{aligned} \tilde{\mathfrak{C}}_q(\mathfrak{N}, \mathfrak{R}) &= \sum_{s=1}^p q_s [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathfrak{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{R}}(\mathfrak{G}_s) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathfrak{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{R}}(\mathfrak{G}_s) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathfrak{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{R}}(\mathfrak{G}_s)] \\ \tilde{\mathfrak{C}}_q(\mathfrak{N}, \mathfrak{N}) &= \sum_{s=1}^p q_s [\hat{\mathcal{T}}_{\mathfrak{N}}(\mathfrak{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{N}}(\mathfrak{G}_s) + \hat{\mathcal{I}}_{\mathfrak{N}}(\mathfrak{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{N}}(\mathfrak{G}_s) + \hat{\mathcal{F}}_{\mathfrak{N}}(\mathfrak{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{N}}(\mathfrak{G}_s)] \\ \tilde{\mathfrak{C}}_q(\mathfrak{R}, \mathfrak{R}) &= \sum_{s=1}^p q_s [\hat{\mathcal{T}}_{\mathfrak{R}}(\mathfrak{G}_s) \cdot \hat{\mathcal{T}}_{\mathfrak{R}}(\mathfrak{G}_s) + \hat{\mathcal{I}}_{\mathfrak{R}}(\mathfrak{G}_s) \cdot \hat{\mathcal{I}}_{\mathfrak{R}}(\mathfrak{G}_s) + \hat{\mathcal{F}}_{\mathfrak{R}}(\mathfrak{G}_s) \cdot \hat{\mathcal{F}}_{\mathfrak{R}}(\mathfrak{G}_s)] \end{aligned}$$

4. Application - Neutrosophic Micro Vague Correlation Measure in MCDM

The essential ideas behind *NMVC*M as well as their mathematical underpinnings and real-world applications will be discussed in this section. Here it is demonstrated how these measurements might improve our capacity to find correlations in confusing and unclear data through case studies and illustrated examples.

4 persons say $\check{P}_1, \check{P}_2, \check{P}_3$ and \check{P}_4 Would like to purchase an automobile based on the characteristics of fuel economy, safety features, maintenance cost, performance, and warranty. They have a total of five distinct automobile brands say $\check{V}_1, \check{V}_2, \check{V}_3, \check{V}_4$ and \check{V}_5 . Based on the features they

require in their vehicle they want to determine which car matches their needs best. Using *NMVCC*, the solution is obtained as follows in Tables 1 and 2:

Table 1. The relations between persons and features.

Persons / Features	Fuel Economy	Safety Features	Maintenance Cost	Performance	Warranty
\check{P}_1	< [0.1, 0.4], [0.3, 0.5], [0.6, 0.9] >	< [0.3, 0.6], [0.2, 0.2], [0.4, 0.7] >	< [0.4, 0.5], [0.0, 0.2], [0.5, 0.6] >	< [0.1, 0.7], [0.1, 0.8], [0.3, 0.9] >	< [0.4, 0.5], [0.3, 0.7], [0.5, 0.6] >
\check{P}_2	< [0.0, 0.5], [0.2, 0.5], [0.5, 1.0] >	< [0.3, 0.4], [0.3, 0.9], [0.6, 0.7] >	< [0.9, 0.9], [0.6, 0.7], [0.1, 0.1] >	< [0.3, 0.6], [0.2, 0.6], [0.4, 0.7] >	< [0.5, 0.5], [0.6, 0.7], [0.5, 0.5] >
\check{P}_3	< [0.4, 0.5], [0.2, 0.3], [0.5, 0.6] >	< [0.7, 0.8], [0.1, 0.6], [0.2, 0.3] >	< [0.5, 0.9], [0.3, 0.7], [0.1, 0.5] >	< [0.3, 0.6], [0.2, 0.5], [0.4, 0.7] >	< [0.3, 0.6], [0.3, 0.9], [0.4, 0.7] >
\check{P}_4	< [0.3, 0.9], [0.4, 0.4], [0.1, 0.7] >	< [0.8, 0.8], [0.1, 0.9], [0.2, 0.2] >	< [0.0, 0.1], [0.3, 0.7], [0.9, 1.0] >	< [0.5, 0.9], [0.4, 0.9], [0.1, 0.5] >	< [0.3, 0.6], [0.1, 0.2], [0.4, 0.7] >

Table 2. The relationship between attributes and automobiles.

Features / Automobiles	\check{V}_1	\check{V}_2	\check{V}_3	\check{V}_4	\check{V}_5
Fuel Economy	< [0.0, 0.4], [0.8, 0.9], [0.6, 1.0] >	< [0.3, 0.3], [0.0, 0.5], [0.7, 0.7] >	< [0.1, 0.2], [0.5, 0.6], [0.8, 0.9] >	< [0.2, 0.7], [0.2, 0.8], [0.3, 0.8] >	< [0.6, 0.9], [0.0, 0.1], [0.1, 0.4] >
Safety Features	< [0.4, 0.7], [0.9, 0.9], [0.3, 0.6] >	< [0.3, 0.5], [0.2, 0.8], [0.5, 0.7] >	< [0.5, 0.6], [0.1, 0.7], [0.4, 0.5] >	< [0.4, 0.8], [0.2, 0.3], [0.2, 0.6] >	< [0.6, 0.7], [0.3, 0.9], [0.3, 0.4] >
Maintenance Cost	< [0.0, 0.8], [0.1, 0.5], [0.2, 1.0] >	< [0.4, 0.6], [0.5, 0.8], [0.4, 0.6] >	< [0.2, 0.9], [0.4, 0.4], [0.1, 0.8] >	< [0.3, 0.4], [0.1, 0.5], [0.6, 0.7] >	< [0.7, 0.8], [0.1, 0.2], [0.2, 0.3] >
Performance	< [0.8, 0.8], [0.8, 0.9], [0.2, 0.2] >	< [0.5, 0.7], [0.4, 0.8], [0.3, 0.5] >	< [0.1, 0.4], [0.1, 0.6], [0.6, 0.9] >	< [0.7, 0.8], [0.5, 0.6], [0.2, 0.3] >	< [0.5, 0.9], [0.2, 0.6], [0.1, 0.5] >
Warranty	< [0.6, 0.9], [0.1, 0.7], [0.1, 0.4] >	< [0.3, 0.8], [0.0, 0.6], [0.2, 0.7] >	< [0.6, 0.7], [0.1, 0.6], [0.3, 0.4] >	< [0.1, 0.8], [0.2, 0.2], [0.2, 0.9] >	< [0.2, 0.5], [0.5, 0.8], [0.5, 0.8] >

The following Table 3 provides the *NMVCM* values between persons and automobiles.

Table 3. *NMVCM*.

Persons / Automobiles	\check{V}_1	\check{V}_2	\check{V}_3	\check{V}_4	\check{V}_5
\check{P}_1	0.822481	0.883675	0.909356	0.865635	0.820215
\check{P}_2	0.793594	0.899957	0.881937	0.78449	0.854698
\check{P}_3	0.814676	0.912279	0.910692	0.844582	0.915496
\check{P}_4	0.821317	0.849844	0.770644	0.903916	0.806466

The comparison of the above Table 3 is given in the following Figure 1.

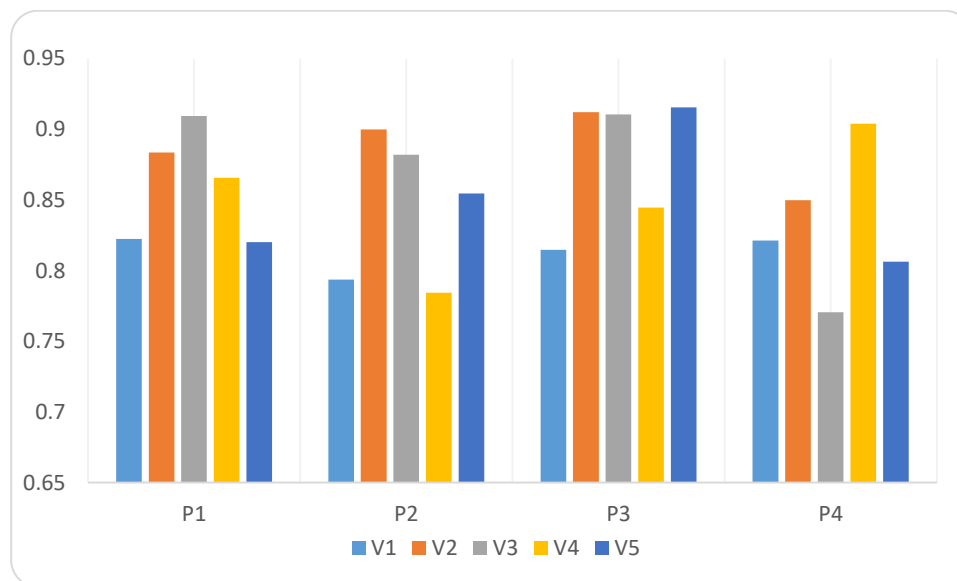


Figure 1. The comparison analyze.

From Table 3 and Figure 1, the $NMVMC$ results that the automobile. \check{V}_3 has a higher chance of matching the features of the person \check{P}_1 Needs. Similarly, \check{V}_2 matches the needs of \check{P}_2 , \check{V}_5 matches the needs of \check{P}_3 and \check{V}_4 matches the needs of \check{P}_4 .

5. Conclusions

Correlation measures are crucial in discovering how different criteria are wired collectively and for gauging how strongly the variables are related to one another. The Neutrosophic Micro Vague correlation measure and Neutrosophic Micro Vague Weighted correlation measure are discussed in this study. Furthermore, certain fundamental characteristics of the $NMVMC$ were demonstrated. In addition, the proposed $NMVMC$ approach is applied to the MCDM, and the solution is obtained. In research and data analysis, $NMVMC$ analysis can be employed to investigate the association between parameters. It will be possible to explore the unknown environment, improve data analysis techniques, and gain a better understanding of the inherent vagueness that exists in real-world events by applying $NMVMC$.

Declarations

Ethics Approval and Consent to Participate

The results/data/figures in this manuscript have not been published elsewhere, nor are they under consideration by another publisher. All the material is owned by the authors, and/or no permissions are required.

Consent for Publication

This article does not contain any studies with human participants or animals performed by any of the authors.

Availability of Data and Materials

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Competing Interests

The authors declare no competing interests in the research.

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Author Contribution

All authors contributed equally to this research.

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