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Solving N-players Continuous Differential Games under Neutrosophic Environment

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Abstract: Uncertainty plays a crucial role in decision-making problems, particularly in game theory. Various forms of uncertainty have been explored in the literature, including fuzzy, soft, rough, and interval-based approaches. Game theory has been extensively studied under these uncertainty models, with researchers addressing vagueness, and imprecision from multiple perspectives. More recently, neutrosophic sets have emerged as an alternative framework for handling uncertainty. Neutrosophic numbers effectively incorporate indeterminacy in decision-making by considering factors such as intuition, assumptions, judgment, behavior, evaluation, and preferences of decision-makers. This paper presents a novel approach to solving a new class of n-player continuous differential games within a neutrosophic framework. In the proposed methodology, the neutrosophic n-player continuous games are redefined into two separate crisp problems: the lower problem and the upper problem. The study further establishes the necessary and sufficient conditions for determining equilibrium strategies in neutrosophic continuous differential games. To demonstrate its effectiveness and practical applicability, the proposed method is validated through a numerical example.

Keywords: Nash Equilibrium Solutions; N-players Games; Differential Games; Neutrosophic Numbers; Sufficient and Necessary Conditions; Game Theory.

1. Introduction

Game theory is a mathematical framework used to model decision-making situations involving two or more participants, where the participants may either compete or cooperate. Each participant strives to outmaneuver the others in the game. Game theory provides a wide array of effective and efficient tools for formulating and solving multi-player problems, where players adopt various strategies to interact with rational decision-makers. This theory is extensively applied across numerous domains, including economics, management, social policy, and both international and national governance [1, 2]. Traditional game theory assumes that all game-related data is precisely known to the players. However, in many real-world scenarios, players may be unable to assess certain data accurately. In such cases, the uncertainty arises from the inaccuracy of information and the vague understanding of the situation by the players. In response, numerous researchers have contributed to the development of techniques aimed at identifying equilibrium strategies in such uncertain or imprecise games.

Fuzzy sets [3] are limited by their inability to effectively represent imprecise and inconsistent information, as they rely solely on the truth membership function. Intuitionistic fuzzy sets [4] extend this concept by incorporating both truth and falsity membership functions, offering an enhancement over traditional fuzzy sets. However, they still fall short of fully capturing the complexities of human-like decision-making processes. The neutrosophic set addresses these limitations by introducing indeterminacy as an independent and essential component. This innovative approach provides a new framework to accommodate inconsistency, incompleteness, and uncertainty. In 1999, Smarandache

[5] introduced the concept of neutrosophic sets, rooted in the theory of neutrosophy, specifically to manage inconsistent, partial, and indeterminate information where indeterminacy plays a significant role. Additionally, the concept of the neutrosophic number was proposed, incorporating indeterminacy as a fundamental element, and its essential properties were systematically defined and explained.

In recent decades, numerous studies have focused on matrix games under uncertainty. For instance, Jishu et al. [6] explored linguistic Pythagorean hesitant fuzzy matrix games and their application in multi-criteria decision-making. Karmakar et al. [7] proposed bi-matrix games within a dense fuzzy environment and applied this model to natural disaster management. Naqvi et al. [8] examined solutions for matrix games involving linguistic interval-valued intuitionistic fuzzy sets. Dong et al. [9] developed a type-2 interval-valued intuitionistic fuzzy matrix game and applied it to the energy vehicle industry. Seikh et al. [10] introduced a non-linear mathematical approach to solving matrix games with picture fuzzy payoffs, particularly for addressing cyber-terrorism attacks. Karmakar et al. [11] formulated a nonlinear programming approach for solving interval-valued intuitionistic hesitant noncooperative fuzzy matrix games. Seikh et al. [12] applied an interval neutrosophic matrix game-based method to address cybersecurity issues. Ahuja et al. [13] employed the Mehar approach to solve hesitant fuzzy linear programming problems. Kirti et al. [14] solved matrix games with payoffs of single-valued trapezoidal neutrosophic numbers. Li et al. [15] introduced a bilinear programming approach for solving bi-matrix games with intuitionistic fuzzy number payoffs. Figueroa et al. [16] analyzed group matrix games involving interval-valued fuzzy numbers. Jana et al. [17] presented dual hesitant fuzzy matrix games based on similarity measures. Singh et al. [18] discussed matrix games with 2-tuple linguistic information. Jana et al. [19] proposed solutions for matrix games with payoffs represented by generalized trapezoidal fuzzy numbers. Zhou et al. [20] constructed a new matrix game model with payoffs involving generalized dempster shafer structures. Roy et al. [21] proposed a rough set algorithm for bi-matrix games. Seikh et al. [22] addressed matrix games with hesitant fuzzy payoffs. Das et al. [23] developed a fuzzy-based genetic algorithm for solving entropy bi-matrix goal games. Han et al. [24] described a novel matrix game with payoffs based on the maxitive belief structure. Roy et al. [25] studied bi-matrix games with bi-fuzzy parameters. Mula et al. [26] applied a bi-rough programming technique to solve bi-matrix games within bi-rough environments. Roy et al. [27] examined Stackelberg games involving type-2 fuzzy variables. Hamiden [28] proposed a method for solving two-person zero-sum matrix games within a neutrosophic environment. Bhaumik et al. [29] considered a hesitant interval-valued intuitionistic fuzzy-linguistic term set approach in the Prisoners' Dilemma matrix game. Ammar et al. [30] solved bi-matrix games in tourism planning management using the rough interval approach. Brikaa et al. [31] developed a fuzzy multi-objective programming algorithm to solve fuzzy rough constrained matrix games. Bhaumik et al. [32] introduced a multi-objective linguistic-neutrosophic matrix game and its applications to tourism management. Brikaa et al. [33] applied a resolving indeterminacy method to solve multi-criteria matrix games with intuitionistic fuzzy goals. Smarandache et al. [34] investigated neutrosophic fixed point theorems within the context of cone metric spaces. Jafari et al. [35] introduced (Φ, Ψ)-weak contractions in neutrosophic cone metric spaces through fixed point theorems. Additionally, Khalil et al. [36] proposed the degree of (L, M)-fuzzy semi-precontinuous and (L, M)-fuzzy semi-preirresolute functions. In another contribution, Al-Omeri [37] introduced mixed b-fuzzy topological spaces and explored their properties. Furthermore, Kaviyarasu et al. [38] analyzed internet streaming services using the max-product of complements in neutrosophic graphs. Al-Omeri [39] discussed Property (P) and presented new fixed point results on ordered metric spaces within the framework of neutrosophic theory. Additionally, Al-Omeri [40] studied neutrosophic g-closed sets in neutrosophic topological spaces. Kaviyarasu et al, [41] examined the application of neutrosophic graphs in the earthquake response center in Japan.

In decision-making problems such as n-player continuous differential games, the exact determination of available data and system parameters is often infeasible. This uncertainty arises due to several factors, including insufficient input information, multiple and inconsistent data sources, fluctuations in parameter values, data noise, inaccurate statistical analysis, and subjective judgmental uncertainty. These challenges necessitate the development of robust mathematical frameworks to effectively model and analyze decision-making processes under such conditions of imprecision and variability. This paper introduces a novel approach for solving neutrosophic n-players' continuous differential games. The proposed methodology determines Nash equilibrium solutions, and the corresponding state trajectories as neutrosophic numbers in the form $\chi + \vartheta\, I$, where χ and ϑ are real numbers, and I represent indeterminacy. The neutrosophic set is applied in many decision-making problems to express the vagueness and achieved great success in realizing the day-to-day problems. To the best of our knowledge, there are no previous studies that solve the neutrosophic players continuous differential games. The main contributions and novelties of this study can be summarized as follows:

- A novel class of neutrosophic n-players continuous differential games is introduced in this study.
- The neutrosophic n-players continuous differential games problems are converted into two distinct crisp n-players continuous differential games problems.
- A new and effective methodology is developed to determine the Nash equilibrium solutions, along with the corresponding state trajectories for players.
- The Nash equilibrium solutions and the corresponding state trajectories for each player are obtained in neutrosophic form, which is desirable.
- A numerical example is presented to verify the proposed approach and to present its effectiveness and practicality.

The outlay of the paper is organized as follows: In Section 2; basic concepts and results related to interval numbers and neutrosophic numbers are recalled. In Section 3, classical n-players continuous differential game is discussed. In Section 4, n-players' continuous differential games under a neutrosophic environment are formulated and an approach for solving continuous differential games is proposed. In Section 5, a numerical example is given to illustrate the efficiency of the solution approach. Finally, some concluding remarks are reported in Section 6.

2. Preliminaries

In this section, certain fundamental definitions of interval numbers [42] and neutrosophic numbers [43] are presented.

2.1 Interval Numbers

An interval number on the real line $\mathcal{R}$ is defined as $M = [M^L, M^U] = \{m : M^L \leq m \leq M^U, m \in \mathcal{R}\}$, where M^U and M^L Expressed the right and left limit of the interval number M on $\mathcal{R}$.

Definition 2.1 Suppose $\rho(M)$ and $\sigma(M)$ be the width and midpoint of an interval $M = [M^L, M^U]$, respectively. Then,

i). $\rho(M) = \frac{1}{2}(M^L - M^U)$,

ii). $\sigma(M) = \frac{1}{2}(M^L + M^U)$.

Definition 2.2 The scalar multiplication on M is given as follows:

i). $a\,M = [a\,M^L, a\,M^U], a \geq 0$,

ii). $a\,M = [a\,M^U, a\,M^L], a \leq 0$.

Definition 2.3 The absolute value of M is given as follows:

i). $|M| = [M^L, M^U], M^L \geq 0$,

ii). $|M| = [0, \max(-M^L, M^U)], M^L < 0 < M^U$,

iii). $|M| = [-M^U, -M^L], M^U \leq 0$.

2.2 Neutrosophic Numbers

Definition 2.4 The mathematical expression of neutrosophic number is defined as $P^N = [\chi + \vartheta\, I]$, where ϑ, χ are real numbers where $\vartheta\, I$ is the indeterminate part and χ is the determinate component, and $I \in [I^L, I^U]$ represents indeterminacy, then $P^N = [\chi + \vartheta\, I] = [\chi + \vartheta\, I^L, +\vartheta\, I^U] = [P^L, P^U]$

Definition 2.5 Suppose that $P^N = [\chi_1 + \vartheta_1 I_1] = [\chi_1 + \vartheta_1 {I_1}^L, \chi_1 + \vartheta_1 {I_1}^U] = [P^L, P^U], Q^N = [\chi_2 + \vartheta_2 I_2] = [\chi_2 + \vartheta_2 {I_2}^L, \chi_2 + \vartheta_2 {I_2}^U] = [Q^L, Q^U]$ are two neutrosophic numbers where $I_1 \in [{I_1}^L, {I_1}^U], I_2 \in [{I_2}^L, {I_2}^U]$. We have:

i). $[P^L, P^U] + [Q^L, Q^U] = [P^L + Q^L, P^U + Q^U]$,

ii). $[P^L, P^U] - [Q^L, Q^U] = [P^L - Q^U, P^U - Q^L]$,

iii). $[P^L, P^U].[Q^L, Q^U] = [\,min(P^L.Q^L, P^L.Q^U, P^U.Q^L, P^U.Q^U)\,, max(P^L.Q^L, P^L.Q^U, P^U.Q^L, P^U.Q^U)]$,

iv). $\frac{[P^L, P^U]}{[Q^L, Q^U]} = min\left(\frac{P^L}{Q^L}, \frac{P^L}{Q^U}, \frac{P^U}{Q^L}, \frac{P^U}{Q^U}\right), max\left(\frac{P^L}{Q^L}, \frac{P^L}{Q^U}, \frac{P^U}{Q^L}, \frac{P^U}{Q^U}\right)$.

3. The Classical N-players Continuous Differential Games

The classical n-players continuous differential games can be expressed as follows:

$$E_i(\omega_1(t), \omega_2(t), \dots, \omega_n(t)\,) = \xi_i\left(z(t_f)\right) + \int_{t_0}^{t_f} P_i(z(t), \omega_1(t), \omega_2(t), \dots, \omega_n(t), t)\, dt,$$

Subject to

$$\dot{z}(t) = g(z(t), \omega_1(t), \omega_2(t), \dots, \omega_n(t), t),$$

$$z(t_0) = z_0, t \in [t_0, t_f], i = 1,2,3, \dots n\,.$$

where $z(t)$ is the trajectory state at time $t \in [t_0, t_f]$, E_i , $i = 1,2,3, \dots n$ is the cost function for players, $P_i(z(t), \omega_1(t), \omega_2(t), \dots, \omega_n(t), t)$ is the running payoff, $\xi_i\left(z(t_f)\right)$ is the terminal payoff, ω_i, $i = 1,2,3, \dots n$ denotes the control for players, the game starts at $t = t_0$ and ends when $t = t_f$ and $z(t_0)$ is the initial state known by all players.

Definition 3.1. The *n*-tuple $({\omega_1}^*, {\omega_2}^* \dots, {\omega_k}^*)$ is a Nash equilibrium strategy for *n*-players continuous differential games if,

$$E_1({\omega_1}^*, {\omega_2}^* \dots, {\omega_n}^*) \leq E_1(\omega_1, {\omega_2}^*, \dots, {\omega_n}^*)$$
$$\vdots$$
$$E_n({\omega_1}^*, {\omega_2}^* \dots, {\omega_n}^*) \leq E_n({\omega_1}^*, {\omega_2}^*, \dots, \omega_n).$$

Definition 3.2. The situation $(\omega_1, \omega_2, \dots, \omega_n)$ is named player's admissible situation if for any other strategy ${\omega_1}'$ $E_i(\omega_1, \omega_2, \dots, \omega_i, \dots, \omega_n) \leq E_i(\omega_1, \omega_2, \dots, {\omega_i}', \dots, \omega_n)$.

4. N-players Neutrosophic Continuous Differential Games Models and Solution Algorithm

The n-players' continuous differential games under a neutrosophic environment can be formulated as follows:

$$\min_{{\omega_i}^N} {E_i}^N ({\omega_1}^N(t), {\omega_2}^N(t), \dots, {\omega_n}^N(t)\,) = {\xi_i}^N\left(z^N(t_f)\right) + \int_{t_0}^{t_f} {P_i}^N(z^N(t), {\omega_1}^N(t), {\omega_2}^N(t), \dots, {\omega_n}^N(t), t)\, dt, \quad (1)$$

Subject to

$$\dot{z}^N(t) = g^N(z^N(t), {\omega_1}^N(t), {\omega_2}^N(t), \dots, {\omega_n}^N(t), t),$$

$$z^N(t_0) = {z_0}^N, t \in [t_0, t_f], i = 1,2,3, \dots n\,.$$

Problem (1) can be represented as:

$$\begin{aligned} &\min_{\omega_i^N} \left[E_i^L(\omega_1^L(t), \omega_2^L(t), \dots, \omega_n^L(t)), E_i^U(\omega_1^U(t), \omega_2^U(t), \dots, \omega_n^U(t))\right] \\ &= \left[\xi_i^L\left(z^L(t_f)\right), \xi_i^U\left(z^U(t_f)\right)\right] \\ &+ \int_{t_0}^{t_f} \left[P_i^L(z^L(t), \omega_1^L(t), \omega_2^L(t), \dots, \omega_n^L(t), t), P_i^U(z^U(t), \omega_1^U(t), \omega_2^U(t), \dots, \omega_n^U(t), t)\right] dt, \end{aligned} \tag{2}$$

Subject to

$$\begin{aligned} &[\dot{z}^L(t), \dot{z}^U(t)] \\ &= [g^L(z^L(t), \omega_1^L(t), \omega_2^L(t), \dots, \omega_n^L(t), t),\ g^U(z^U(t), \omega_1^U(t), \omega_2^U(t), \dots, \omega_n^U(t), t)], \end{aligned}$$

$[z^L(t_0), z^U(t_0)] = [z_0^L, z_0^U], t \in [t_0, t_f], i = 1,2,3, \dots n.$

Currently, problem (2) can be reformulated into two distinct models, as outlined below:

The lower problem:

$$\begin{aligned} &\min_{\omega_i^L} E_i^L(\omega_1^L(t), \omega_2^L(t), \dots, \omega_n^L(t)) \\ &\qquad = \xi_i^L\left(z^L(t_f)\right) + \int_{t_0}^{t_f} P_i^L(z^L(t), \omega_1^L(t), \omega_2^L(t), \dots, \omega_n^L(t), t)\, dt, \end{aligned} \tag{3}$$

Subject to

$$\dot{z}^L(t) = g^L(z^L(t), \omega_1^L(t), \omega_2^L(t), \dots, \omega_n^L(t), t),$$

$z^L(t_0) = z_0^L, t \in [t_0, t_f], i = 1,2,3, \dots n.$

The upper problem:

$$\begin{aligned} &\min_{\omega_i^U} E_i^U(\omega_1^U(t), \omega_2^U(t), \dots, \omega_n^U(t)) \\ &\qquad = \xi_i^U\left(z^U(t_f)\right) + \int_{t_0}^{t_f} P_i^U(z^U(t), \omega_1^U(t), \omega_2^U(t), \dots, \omega_n^U(t), t)\, dt, \end{aligned} \tag{4}$$

Subject to

$$\dot{z}^U(t) = g^U(z^U(t), \omega_1^U(t), \omega_2^U(t), \dots, \omega_n^U(t), t),$$

$z^U(t_0) = z_0^U, t \in [t_0, t_f], i = 1,2,3, \dots n.$

Hence, the neutrosophic Nash equilibrium strategy $\omega^N = [\omega^{\boldsymbol{L}}, \omega^{\boldsymbol{U}}]$ for the neutrosophic n-players continuous differential game can be obtained by solving the two formulated subproblems: the lower game model, represented by Eq. (3), and the upper game model, represented by Eq. (4).

5. Sufficient and Necessary Conditions

This section addresses the necessary and sufficient conditions for the Nash equilibrium solution in n-player continuous differential games within a neutrosophic environment.

5.1 Necessary Condition

Theorem 5.1. Suppose $P_i^L(z^L(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), t)$ and $g^L(z^L(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), t)$ be continuous differentiable functions. If $\left(\omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^{*L}(t) \dots, \omega_n^{*L}(t)\right)$ represents an open-loop Nash equilibrium with the corresponding state trajectory $z^{*L}(t), t \in [t_0, t_f]$, for the lower game problem (3), then there exist n-costate vector $\mathcal{M}_i^L(t): [t_0, t_f] \to \mathcal{R}^n$ and n-Hamiltonian functions $H_i^L,\ i = 1,2,3, \dots n$ are given by

$$H_i^L\big(z^L(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), \mathcal{M}_i^L(t), t\big) = P_i^L(z^L(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), t) + \left(\mathcal{M}_i^L(t)\right)^T g^L(z^L(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), t), \tag{5}$$

subject to

$$\dot{z}^{*L}(t) = g^L(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^{*L}(t) \dots, \omega_n^{*L}(t), t), \ z^{*L}(t_0) = z_0, \tag{6}$$

$$\dot{\mathcal{M}}_i^L(t)^T = -\frac{\partial H_i^L\big(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^{*L}(t) \dots, \omega_n^{*L}(t), \mathcal{M}_i^L(t), t\big)}{\partial z^L}, \tag{7}$$

$$\mathcal{M}_i^L(t_f) = \frac{\partial \xi_i^L\left(z^L(t_f)\right)}{\partial z^L(t_f)}, \tag{8}$$

$$\frac{\partial H_i^L\big(z^L(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), \mathcal{M}_i^L(t), t\big)}{\partial \omega_i^L} = 0, \tag{9}$$

$$\frac{\partial^2 H_i^L\big(z^L(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), \mathcal{M}_i^L(t), t\big)}{\partial (\omega_i^L)^2} \geq 0, \tag{10}$$

Proof Since the two functions $P_i^L(z^L(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), t)$ and $g^L(z^L(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), t)$ are continuous differentiable functions, then there exists a solution $\mathcal{F}_i^L$ of the following differential equation

$$\dot{\mathcal{M}}_i^L(t_f) = -\mathcal{M}_i^L(t_f) \frac{\partial g^L(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^{*L}(t) \dots, \omega_n^{*L}(t), t)}{\partial z^L} - \frac{\partial P_i^L(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^{*L}(t) \dots, \omega_n^{*L}(t), t)}{\partial z^L}, \tag{11}$$

subject to

$$\mathcal{M}_i^L(\mathrm{t_f}) = \frac{\partial \xi_i^L\left(z^L(t_f)\right)}{\partial z^L(t_f)}. \tag{12}$$

The adjoint equation corresponding to Eq. (11) is expressed as

$$\mathcal{M}_i^L(t)\, \delta\dot{z}^L(t) = \left[\mathcal{M}_i^L(t) \frac{\partial g^L(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^{*L}(t) \dots, \omega_n^{*L}(t), t)}{\partial z^L} + \frac{\partial P_i^L(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^{*L}(t) \dots, \omega_n^{*L}(t), t)}{\partial z^L}\right] \delta z^L(t) + H_i^L\big(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^{*L}(t) \dots, \omega_n^{*L}(t), \mathcal{M}_i^L(t), t\big) - H_i^L\big(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), \mathcal{M}_i^L(t), t\big). \tag{13}$$

The solution of Eq. (13) is $\delta z^L(t)$ with initial condition $\delta z^L(t_0) = 0$. Based on theorem (10.1) in [44], we have

$$\frac{d\left[\mathcal{M}_i^L(t)\, \delta\dot{z}^L(t)\right]}{dt} = H_i^L\big(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^{*L}(t) \dots, \omega_n^{*L}(t), \mathcal{M}_i^L(t), t\big) - H_i^L\big(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), \mathcal{M}_i^L(t), t\big). \tag{14}$$

By integrating, we have

$$\mathcal{M}_i^L(t_f)\, \delta z^L(t_f) = \int_{t_0}^{t_f} [H_i^L\big(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^{*L}(t) \dots, \omega_n^{*L}(t), \mathcal{M}_i^L(t), t\big) - H_i^L\big(z^{*L}(t), \omega_1^{*L}(t), \omega_2^{*L}(t), \dots, \omega_i^L(t) \dots, \omega_n^{*L}(t), \mathcal{M}_i^L(t), t\big)]dt. \tag{15}$$

Based on theorem (11.1) in [44], we have

$$\delta E_i{}^L(u^*,\omega_i) = \int_{t_0}^{t_f} [H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{*L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big) - H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big)]dt. \tag{16}$$

Where u^* represent the composite optimal control for the remaining players.

From Eq. (16), we have

$$\int_{t_0}^{t_f} \mathcal{M}^L(t)\; g^L{}_u(t,z,u)\,\omega(t)dt = \int_{t_0}^{t_f} [H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{*L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big) - H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big)]dt. \tag{17}$$

Based on theorem (11.2) in [44], we obtain

$$\mathcal{M}^L(t)\, g^L{}_u(t,z,u)\,\omega(t) \le 0. \tag{18}$$

From Eq. (17) and Eq. (18), we have

$$H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{*L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big) - H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big) \le 0, \tag{19}$$

Then we have

$$H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{*L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big) \le H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big). \tag{20}$$

In a similar manner, the necessary condition for the upper game problem, as described by Eq. (4), can be established.

5.2 *Sufficient Condition*

Theorem 5.2. Suppose $H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big)$ be twice differentiable on $\mathcal{R}^n$, then the n-dimensional vector $\big(\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{*L}(t)\dots,\omega_n{}^{*L}(t)\big)$ is an open-loop Nash equilibrium if an n-dimensional co-state vector $\mathcal{F}_i{}^L(t)$ Exists and satisfies the conditions (6-10).

Proof Since

$$H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big) = H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{*L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big) + \frac{\partial H_i{}^L}{\partial \omega_i{}^L}\;\delta\omega_i{}^{*L} + \frac{1}{2!}\frac{\partial^2 H_i{}^L}{\partial(\omega_i{}^L)^2}\;\delta^2\|\omega_i{}^{*L}\|^2. \tag{21}$$

From Eq. (6) and Eq. (10), we have

$$H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{*L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big) \le H_i{}^L\big(z^{*L}(t),\omega_1{}^{*L}(t),\omega_2{}^{*L}(t),\dots,\omega_i{}^{L}(t)\dots,\omega_n{}^{*L}(t),\mathcal{M}_i{}^L(t),t\big). \tag{22}$$

6. Numerical Example

A numerical experiment is now conducted to demonstrate the effectiveness of the proposed algorithm. The neutrosophic cost function for each player is defined as follows:

$$\min_{\omega_1{}^N} E_1{}^N\big(\omega_1{}^N(t),\omega_2{}^N(t)\big) = c^N\, z_1{}^N\big(t_f\big) + \int_{t_0}^{t_f} (c^N\,\omega_1{}^N(t) - c^N)^2\,dt,$$

$$\min_{\omega_2{}^N} E_2{}^N\left(\omega_1{}^N(t), \omega_2{}^N(t)\right) = d^N z_2{}^N(t_f) + \int_{t_0}^{t_f} (d^N \omega_2{}^N(t) - d^N)^2\, dt,$$

With the state trajectories:

$$\dot{z_1}{}^N(t) = c^N \omega_1{}^N(t) + c^N \omega_2{}^N(t), \quad \dot{z_2}{}^N(t) = -d^N \omega_1{}^N(t) - d^N \omega_2{}^N(t).$$

where $c^N = [3 + 5I]$ and $d^N = [2 + 4I]$ are neutrosophic numbers with $I \in [0, 1]$.

6.1 The Solution Procedure

This model can be decomposed into two submodels: the lower game problem and the upper game problem, as outlined below:

Lower problem

$$\min_{\omega_1{}^L} E_1{}^L\left(\omega_1{}^L(t), \omega_2{}^L(t)\right) = 3\, z_1{}^L(t_f) + \int_{t_0}^{t_f} (3\, \omega_1{}^L(t) - 3)^2\, dt,$$

$$\min_{\omega_2{}^L} E_2{}^L\left(\omega_1{}^L(t), \omega_2{}^L(t)\right) = 2\, z_2{}^L(t_f) + \int_{t_0}^{t_f} (2\, \omega_2{}^L(t) - 2)^2\, dt,$$

subject to

$$\left(\dot{z_1}(t)\right)^L = 3\, \omega_1{}^L(t) + 3\, \omega_2{}^L(t),$$

$$\left(\dot{z_2}(t)\right)^L = -3\, \omega_1{}^L(t) - 3\, \omega_2{}^L(t).$$

The Hamiltonian functions are defined as follows:

$$H_1{}^L = (3\, \omega_1{}^L(t) - 3)^2 + \mathcal{M}_1{}^L\, [3\, \omega_1{}^L(t) + 3\, \omega_2{}^L(t)] + \mathcal{M}_2{}^L[-3\, \omega_1{}^L(t) - 3\, \omega_2{}^L(t)],$$

$$H_2{}^L = (2\, \omega_2{}^L(t) - 2)^2 + \mathcal{M}_1{}^L\, [3\, \omega_1{}^L(t) + 3\, \omega_2{}^L(t)] + \mathcal{M}_2{}^L[-3\, \omega_1{}^L(t) - 3\, \omega_2{}^L(t)].$$

From Eq. (7) and Eq. (8), we have $\mathcal{M}_1{}^L(t) = c_1$ and $\mathcal{M}_1{}^L(t) = 3$,
Likewise, we obtain $\mathcal{M}_2{}^L(t) = c_2$ and $\mathcal{M}_2{}^L(t) = 2$.
Then according to Eq. (9) and the values of $\mathcal{M}_1{}^L(t)$ and $\mathcal{M}_2{}^L(t)$, we obtain

$$\omega_1{}^L = 0.833,\ \ \omega_2{}^L = 0.625,\ z_1{}^L = 4.375\,(t_f - t_0) \text{ and } z_2{}^L = -4.375(t_f - t_0).$$

Upper problem

$$\min_{\omega_1{}^U} E_1{}^U\left(\omega_1{}^U(t), \omega_2{}^U(t)\right) = 8\, z_1{}^U(t_f) + \int_{t_0}^{t_f} (8\, \omega_1{}^U(t) - 8)^2\, dt,$$

$$\min_{\omega_2{}^U} E_2{}^U\left(\omega_1{}^U(t), \omega_2{}^U(t)\right) = 6\, z_2{}^U(t_f) + \int_{t_0}^{t_f} (6\, \omega_2{}^U(t) - 6)^2\, dt,$$

subject to

$$\left(\dot{z_1}(t)\right)^U = 8\, \omega_1{}^U(t) + 8\, \omega_2{}^U(t),$$

$$\left(\dot{z_2}(t)\right)^U = -8\, \omega_1{}^U(t) - 8\, \omega_2{}^U(t).$$

The Hamiltonian functions are

$$H_1{}^U = (8\,\omega_1{}^U(t) - 8)^2 + \mathcal{M}_1{}^U\,[8\;\omega_1{}^U(t) + 8\,\omega_2{}^U(t)] + \mathcal{M}_2{}^U[-8\,\omega_1{}^U(t) - 8\,\omega_2{}^U(t)],$$

$$H_2{}^U = (6\,\omega_2{}^U(t) - 6)^2 + \mathcal{M}_1{}^U\,[8\;\omega_1{}^U(t) + 8\,\omega_2{}^U(t)] + \mathcal{M}_2{}^U[-8\,\omega_1{}^U(t) - 8\,\omega_2{}^U(t)].$$

From Eq. (7) and Eq. (8), we have $\mathcal{M}_1{}^U(t)$= c_1 and $\mathcal{M}_1{}^U(t) = 8$,
Likewise, we obtain $\mathcal{M}_2{}^U(t)$= c_2 and $\mathcal{M}_2{}^U(t) = 6$.
Then according to Eq. (9) and the values of $\mathcal{M}_1{}^U(t)$ and $\mathcal{M}_2{}^U(t)$, we have

$$\omega_1{}^U = 0.875,\;\; \omega_2{}^U = 0.778,\; z_1{}^U = 13.222\,(t_f - t_0) \text{ and } z_2{}^U = -13.222\,(t_f - t_0).$$

The neutrosophic Nash equilibrium solutions, along with the corresponding state trajectories for both players, are provided as follows:
$\omega_1{}^N = [\omega_1{}^L,\, \omega_1{}^U] = [0.833, 0.875]$,
$\omega_2{}^N = [\,\omega_2{}^L,\, \omega_2{}^U] = [0.625, 0.778]$,
$z_1{}^N = [\,z_1{}^L,\, z_1{}^U] = [4.375, 13.222](t_f - t_0)$,
$z_2{}^N = [\,z_2{}^L,\, z_2{}^U] = [-13.222, -4.375\,](t_f - t_0)$.

6.2 Comparison with other Existing Methods

The proposed methodology addresses the problem of continuous differential games with parameters expressed as neutrosophic numbers in the form $\chi + \vartheta\, I$, where χ and ϑ are real numbers, and represent indeterminacy. There is no existing method in the literature that solves continuous differential games with neutrosophic numbers in this specific form. Therefore, this work constitutes the first approach within the neutrosophic numerical framework.

Several research studies have explored continuous differential games under different parameter environments. Brikaa et al. [45] examined fuzzy rough continuous differential games. Megahed et al. [46] analyzed min-max zero-sum two-person continuous differential games with fuzzy controls. Ammar et al. [47] developed two-person zero-sum rough interval continuous differential games, while Megahed et al. [48] proposed a min-max zero-sum two-person fuzzy continuous differential game. Brikaa et al. [49] employed a rough-set approach to investigate non-cooperative continuous differential games. Abo Elnaga et al. [50] considered Nash min-max hybrid continuous static games under fuzzy conditions, and Khalifa et al. [51] introduced an interactive approach for solving fuzzy cooperative continuous static games. El-Sobky et al. [52] presented a trust-region-based active-set interior-point algorithm for fuzzy continuous static games. Additionally, Abo Elnaga et al. [53] studied Nash equilibrium solutions for fuzzy rough continuous static games, and Megahed et al. [54] explored Nash equilibrium solutions in rough differential games.

Despite the extensive body of work on continuous differential games in various uncertain environments, no prior research has addressed the specific case of neutrosophic numbers in the form $\chi + \vartheta\, I$. Consequently, the parameterization and methodological environment of this study are distinct from those of existing research, making direct comparisons with previously developed methods infeasible.

7. Conclusion

In decision-making models, the data provided by the decision maker is frequently incomplete or imprecise due to factors such as insufficient information, time constraints, limitations in data processing capabilities, and the bounded attention of the decision maker. As a result, incorporating value and ambiguity indices into neutrosophic decision-making models is of significant importance for both practical applications and scientific research. This study introduces a novel solution algorithm for n-players continuous differential games characterized by neutrosophic controls and neutrosophic state trajectories. To address the upper and lower bounds of the neutrosophic set, two crisp formulations of n-players continuous differential games are developed. The necessary and sufficient conditions for the proposed model are rigorously established. Finally, a numerical example is examined to show the applicability and embodiment of the proposed method.

The limitation of the proposed approach is that it does not find the solution to the continuous differential game problem directly, as it considers the construction of two distinct problems: the lower problem and the upper problem.

Future research in neutrosophic numbers holds significant potential for further enhancing decision-making models. While the current study demonstrates the application of neutrosophic numbers in addressing antagonistic decision-making problems, such as those found in finance, business, military strategy, and economics, additional exploration is needed. In particular, the utilization of various forms of neutrosophic numbers, such as triangular, trapezoidal, intuitionistic, convex, and L-R neutrosophic numbers, could provide more flexible and precise representations of uncertainty, imprecision, and indeterminacy in complex systems.

Furthermore, future work could extend the proposed algorithm to tackle a variety of game-theoretic models, such as neutrosophic non-cooperative games, neutrosophic bi-matrix games, neutrosophic coalition games, and multi-criteria decision-making problems. The application of neutrosophic numbers in these contexts could open new avenues for solving real-world problems with inherent uncertainty, providing valuable insights across multiple domains.

Declarations

Ethics Approval and Consent to Participate

The results/data/figures in this manuscript have not been published elsewhere, nor are they under consideration by another publisher. All the material is owned by the authors, and/or no permissions are required.

Consent for Publication

This article does not contain any studies with human participants or animals performed by any of the authors.

Availability of Data and Materials

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Competing Interests

The authors declare no competing interests in the research.

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Author Contribution

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