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*"***Neutrosophic Systems with Applications***"* has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

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Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e., notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjointed two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of classical statistics.

What distinguishes neutrosophic from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Compactness and Neutrosophic Topological Space via Grills

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Abstract: The aim of this paper is to introduce the concept of various types of compactness in neutrosophic topological space via grills. We shall generalize neutrosophic C - compact space and neutrosophic G - compact space and introduce $C(G)$ - compact space in neutrosophic topological space with respect to grills. We shall call it as neutrosophic C - compact with respect to grills and term it as neutrosophic $C(G)$ - compact space. We shall also investigate some of its basic properties and characterization theorems. We shall also study the neutrosophic quasi - H - closed space with respect to a grill.

Keywords: Neutrosophic Space; Grill; Neutrosophic G – compact; Neutrosophic C – compact; Neutrosophic quasi – H - closed.

1. Introduction

The notion of a grill was initiated by Choquet [1]. Subsequently it turned out to be a very convenient tool for various topological and neutrosophic topological investigations. From the literature it finds that in many situations, grills are more effective than certain similar concepts like nets and filters. According to Choquet, a grill G on a topological space X is a non - null collection of nonempty subsets of X satisfying two conditions: (i) $A \in G$ and $A \subseteq B \subseteq X \Rightarrow B \in G$ and (ii) A, $B \subseteq X$ and $A \cup B \in G \Rightarrow A \in G$ or $B \in G$.

Zadeh [2] introduced the notion of fuzzy set. As it was not sufficient to control uncertainty, Atanasov [3] introduced the notion of intuitionistic fuzzy set with membership and non -membership values. Thereafter, Smarandache [4] considered the elements with membership, non - membership and indeterministic values and introduced the notion of neutrosophic set in order to overcome all sorts of difficulty to handle all types of problems under uncertainty. The notion of neutrosophic topological space was first introduced by Salama and Alblowi [5], followed by Salama and Alblowi [6]. Alimohammady and Roohi [7] introduced fuzzy minimal structure and fuzzy minimal vector spaces. Alimohammady and Roohi [8] introduced the notion of compactness in fuzzy minimal spaces. Pal et al. [9] introduced the notion of grill in neutrosophic topological space. Pal and Dhar [10] introduced the notion of compactness in neutrosophic minimal space. Roy and Mukherjee [11] introduced the notion of compactness in topological space. Gupta and Gaur [12] introduced the notion of C - compactness in topological space by grills.Besides them, many researchers [13, 14, 15, 16, 17,18, 19] contributed compactness in neutrosophic topological space. Following their works we would introduce and study C - cocompactness via grills in neutrosophic topological space. We would also introduce neutrosophic quasi - H - closed space with respect to a grill.

2. Preliminaries

In this section, we recall some basic concepts and results which are relevant for this article.

Definition 2.1. [11] Let G be a grill on a topological space (X, τ). A cover $\{U_\alpha : \alpha \in \Lambda\}$ of X is said to be a G - cover if there exists a finite subset Λ_0 of Λ such that $X - \cup_{\alpha \in \Lambda_0} U\alpha \notin G$.

Definition 2.2. [4] Let X be an universal set. A neutrosophic set A in X is a set contains triplet having truthness, falseness and indeterminacy membership values that can be characterized independently, denoted by T_A, F_A, I_A in [0,1]. The neutrosophic set is denoted as follows:

$A = \{(x, T_A(x), F_A(x), I_A(x)): x \in X$, and $T_A(x), F_A(x), I_A(x) \in [0,1]\}$.

There is no restriction on the sum of $T_A(x)$, $F_A(x)$ and $I_A(x)$, so $0 \leq T_A(x) + F_A(x) + I_A(x) \leq 3$.

Definition 2.3. [5] Let X be a non - empty set and T be the collection of neutrosophic subsets of X. Then T is said to be a neutrosophic topology (in short NT) on X if the following properties hold:

(i) $0_N, 1_N \in T$.

(ii) $U_1, U_2 \in T \Rightarrow U_1 \cap U_2 \in T$.

(iii) $\cup_{i\in\Delta} u_i \in T$, for every $\{u_i: i \in \Delta\} \subseteq T$.

Then (X, T) is called a neutrosophic topological space (in short NTS) over X. The members of T are called neutrosophic open sets (in short NOS). A neutrosophic set D is calledneutrosophic closed set (in short NCS) if and only if D^c is a neutrosophic open set.

Definition 2.4. [9] Let X be a set and $P(X)$ denotes the power set of X. A family M of neutrosophic subsets of X where $M \subset P(X)$ is said to be a minimal structure on X if 0_N and 1_N belong to M. By (X, M), we denote the nutrosophic minimal space.

We consider the elements of M as neutrosophic m - open subset of X. The complement of neutrosophic m - open set A is called a neutrosophic m-closed set.

Definition 2.5. [9] Let X be a set and $P(X)$ denotes the power set of X. A sub – collection of neutrosophic sets G (not containing 0_N) of $P(X)$ is called a grill on X if G satisfies the following conditions:

(i) $A \in G$ and $A \subseteq B$ implies $B \in G$.

(ii) $A, B \subseteq X$ *and* $A \cup B \in G$ implies that $A \in G$ or $B \in G$.

Remark 2.6. [9] Since $0_N \notin G$, so G is not a minimal structure on X. A minimal structure with a grill is called as a grill minimal space, denoted by (X, M, G).

3. Neutrosophic C - Compactness with Respect to a Grill

In this section, our main focus is to propose the concept of neutrosophic C - compactness with respect to a grill and to investigate various properties of this notion.

Definition 3.1. A neutrosophicspace (X, T) is said to be neutrosophic C - compact if for each neutrsophic closed set A and each neutrosophic T - open covering U of A, there exists a finite subfamily $\{U_1, U_2, U_3 \dots \dots, U_n\}$ such that $A \subset \bigcup_{i=1}^{n} Cl(U_i)$.

Definition 3.2. Let (X, T) be a neutrosophic topological space and G be a grill on X. (X, T) is said to be neutrosophic C - compact with respect to grill or just $NC(G)$ if for every neutrosophic T -open covering of U of A, there exists a finite subfamily $\{U_1, U_2, U_3, \dots, U_n\}$ of U such that $A - \sum_{i=1}^{n} cl(U_i) \notin G$.

Every neutrosophic C - compact space (X, T) is NC(G) - compact for any grill G on X.

Theorem 3.3. For a neutrosophic topological space (X, T), the following statements are true:

(a) (X, T) is neutrosophic C - compact.

(b) (X, T) is neutrosophic $C(0_N)$ - compact.

(c) (X, T) is neutrosophic $C(1_N)$ - compact.

Proof. Obvious.

Theorem 3.4. If a space is neutrosophic G - compact then it is neutrosophic $C(G)$ - compact.

Proof. Let X be a neutrosophic G - compact space, A a neutrosophic closed subset of X and $\{V_\alpha\}_{\alpha\in\Lambda}$an open cover of A. Then $(X-A)\cup_{\alpha\in\Lambda}(V_\alpha)$ is a neutrosophic open cover of X. Since X is neutrosophic G - compact, therefore there exists finite$\Lambda_0\subseteq\Lambda$such that $X-\{(X-A)\cup_{\alpha\in\Lambda_0}(V_\alpha)\}\notin G$. This implies$A-\{\cup_{\alpha\in\Lambda_0}(V_\alpha)\}\notin G$. Since$V_\alpha\subset Cl(V_\alpha)$, therefore $A-\cup_{\alpha\in\Lambda_0}cl(V_\alpha)\notin G$, implies that X is neutrosophic $C(G)$ - compact.

Theorem 3.5. Let(X, T) be a neutrosophic space and G be a grill on X. Then the following are equivalent:

(a) (X,T) is neutrosophic $C(G)$ - compact.

(b) For each neutrosophic closed subset A of X and each family of neutrosophic closed subsets of X such that $\cap\{T\cap A: T\in F\}=0_N$, there is a finite subfamily $\{T_1,T_2,T_3,\dots\dots..,T_n\}$ such that $\cap_{i=1}^{n}(int(T_i))\cap A\in G$.

(c) For each neutrosophic closed set A and each family F of neutrosophic closed subsets of Xsuch that $\{int(T)\cap A: T\in F\}$ (G) FIP, one has $\cap\{T\cap A: T\in F\}\neq 0_N$.

(d) For each neutrosophic closed set A and each neutrosophic regular open cover U of A, there exists a finite sub collection $\{U_1,U_2,\ U_3\dots\dots,U_n\}$ such that $A-\cup_{i=1}^{n}Cl(U_i)\notin G$.

(e) For each neutrosophic closed set$A$ and each family F of neutrosophic regular closed sets such that $\cap\{T\cap A : T\in F\}=0_N$, there is a finite subfamily$\{T_1,T_2,T_3,\dots\dots..,T_n\}$ such that $\cap_{i=1}^{n}(int(T_i))\cap A\notin G$.

(f) For each neutrosophic closed set A and each family F of neutrosophic regular closed sets such that $\{int(T)\cap A :\ T\in F\}$ has grill neutrosophic finite intersection property, one has $\cap\{T\cap A : T\in F\}\neq 0_N$.

(g) For each neutrosophic closed set A, each neutrosophic open cover U of $X-A$ and each neutrosophic open neighbourhood U of A, there exists a finite subfamily $\{U_1,U_2,\ U_3\dots\dots,U_n\}$ of U such that $X-(U\cup(\cup_{i=1}^{n}Cl(U_i)))\notin G$.

(h) For each neutrosophic closed set A and each neutrosophic open filter base Bon X such that $\{B\cap A :\ B\in B\}\subset G$, one has $\cap\{Cl(B): B\in B\}\cap A\neq 0_N$.

Proof. (a)⇒ (b). Let (X,T) be neutrosophic $C(G)$ - compact, A a neutrosophic closed subset and F family of neutrosophic closed subsets with $\cap\{T\cap A: T\in F\}=0_N$. Then $\{X-T\ :\ T\in F\}$ is a neutrosophic open cover of A and hence admits a finite subfamily $\{X\backslash T_i : i=1,2,\dots\dots.n\}$ such that $A-\cup_{i=1}^{n}Cl(X-T_i)\notin G$ is easily seen to be $\{\cap_{i=1}^{n}(int(T_i))\cap A\}$.

(b) ⇒(c). It is obvious.

(c) ⇒(a). Let A be a neutrosophic closed subset. Let U be a neutrosophic open cover of A with the property that for no finite subfamily $\{U_1,U_2,\ U_3,\dots\dots,U_n\}$ of$U$, one has $A-\cup_{i=1}^{n}Cl(U_i)\notin G$. Then $\{X-U_i\ :\ U_i\in U, i=1,2,\dots\dots..,n\}$ is a family of closed sets. Since $\cap_{i=1}^{n}\{(X\backslash Cl(U_i))\}\cap A=\cap_{i=1}^{n}\{A-Cl(A_i)\}=A-\cup_{i=1}^{n}Cl(U_i)$, the family $\{int(X\backslash U_i)\cap A : U_i\in U, i=1,2,\dots\dots..n\}$ has neutrosophic finite intersection property with a grill G. By the hypothesis $\cap\{(X-U_i)\cap A :\ U_i\in U\}\neq 0_N\Rightarrow\cap\{A-U_i\ :\ U_i\in U, i=1,2,\dots\dots..,n\}\neq 0_N\Rightarrow A-\cup\{U_i : U_i\in U\}\neq 0_N\Rightarrow U$ is not a cover of A, a contradiction.

(d) ⇒(a). Let A be a neutrosophic closed subset of X and U be a neutrosophic open cover of A. Then $\{int(Cl(U_i)) : U_i\in U\}$ is a neutrosophic regular open cover of A. Let $\{int(Cl(U_i)), i=$

$1, 2, \ldots\ldots, n\}$ be a finite subfamily such that $A - \bigcup_{i=1}^{n} Cl\left(int\left(Cl(U_i)\right)\right) \notin G$. Since U_i is neutrosophic open and for each open set U_i, we have, $Cl\left(int\left(Cl(U_i)\right)\right) = Cl(U_i)$. We have $A - \bigcup_{i=1}^{n} Cl(U_i) \notin G$. Hence X is neutrosophic $C(G)$ - compact.

(a) ⇒(d). It is obvious.

(d) ⇒(e) ⇒(f) ⇒(d) are same as **(a)⇒(b)⇒(c)⇒(a)** respectively.

(a) ⇒(g). Let A be a neutrosophic closed set, Va neutrosophic open neighborhood of A and U an open cover of $X - A$. Since $X - V \subset X - A$, U is a neutrosophic open cover of $X - V$. Let $\{U_1, U_2, U_3, \ldots\ldots, U_n\}$ be a finite collection of$U$, such that $(X - V) - \bigcup_{i=1}^{n} Cl(U_i) \notin G$. Since $(X - V) - \bigcup_{i=1}^{n} Cl(U_i) = X - (V \cup (\bigcup_{i=1}^{n} Cl(U_i)))$.This shows $X - (V \cup (\bigcup_{i=1}^{n} Cl(U_i))) \notin G$.

(g) ⇒(a). Let Abe a neutrosophic closed X and U a neutrosophic open covering of A. If H denotes the union of members of U, then $T = X - H$ is neutrosophic closed set and $X - A$ is a neutrosophic open neighborhood of T. Also U is a neutrosophic open cover of $X - T$. By hypothesis, there is a finite sub - collection $\{U_1, U_2, U_3 \ldots\ldots, U_n\}$ of U, such that $X - ((X - A) \cup (\bigcup_{i=1}^{n} Cl(U_i))) \notin G$. However, this set not in $G$is nothing but $A - \bigcup_{i=1}^{n} Cl(U_i)$.

(a) ⇒(h). Suppose$A$ is a neutrosophic closed set and B is neutrosophic open filter base on X with $\{D \cap A : D \in B\} \subset G$. Suppose, if possible, $\cap \{Cl(D) : D \in B\} \cap A = 0_N$. Then $\{X - Cl(D) : D \in B\}$ is a neutrosophic open cover of A. By hypothesis, there exists a finite subfamily $\{X - Cl(D_i) : i = 1, 2, \ldots\ldots, n\}$ such that $A - \bigcup_{i=1}^{n} Cl(X - Cl(D_i)) \notin G$. However, this set is $A \cap (\bigcap_{i=1}^{n} int(Cl(D_i)))$ and $A \cap (\bigcap_{i=1}^{n}(D_i)$ is a subset of it. Therefore, $A \cap (\bigcap_{i=1}^{n}(D_i) \notin G$. Since B is a filter base, we have a $D \in B$ such that $D \subset \bigcap_{i=1}^{n} D_i$. But then $A \cap D \notin G$ which contradicts the fact that $\{D \cap A : D \in B\} \subset G$.

(h) ⇒(a). Suppose that (X, T) is not neutrosophic $C(G)$ - compact. Then there exists a neutrosophic closed subset A of X and a neutrosophic open cover U of A such that for any finite subfamily $\{U_1, U_2, U_3 \ldots\ldots, U_n\}$ of U, we have $A - \bigcup_{i=1}^{n} Cl(A_i) \in G$. We may assume that U is neutrosophic closed under finite unions. Then the family $B = \{X - Cl(U_i) : U_i \in U, i = 1, 2, \ldots\ldots, n\}$ is a neutrosophic open filter base on X such that $\{D \cap A : D \in B\} \subset G$. So, by the hypothesis, $\cap \{Cl\left(X - Cl(U_i)\right) : U_i \in U\} \cap A \neq 0_N$. Let x be a neutrosophic point in the intersection. Then$x \in A$ and $x \in Cl\left(X - Cl(U)\right) \subset X - U$ for each $U_i \in V$. But this contradicts the fact that U is a cover of A. Hence, (X, T) is a neutrosophic $C(G)$ - compact.

Definition 3.6. A neutrosophic filter base B is said to be neutrosophic G adherent if for every neutrosophic neighborhood N of the adherent set of B, there exists an element$D \in B$ such that $(X - N) \cap D \notin G$.

Theorem 3.7. A space (X, T) is neutrosophic $C(G)$ - compact if and only if every neutrosophic open filter base on G is G - adherent convergent.

Proof. Let (X, T) be neutrosophic $C(G)$ - compact and let B be an open filter base on G with A as its adherent set. Let G be an open neighborhoodof A. Then $A = \cap \{Cl(D) : D \in B\}$, $A \subset G$ and $X -$ G is neutrosophic closed. Now $\{X - cl(D) : D \in B\}$ is a neutrosophic open cover of $X - G$ and so by the hypothesis, it admits a finite subfamily $\{X - cl(D_i) : i = 1, 2, \ldots\ldots, n\}$ such that $(X - G) - \bigcup_{i=1}^{n} cl(X - cl(D_i)) \notin G$. But this implies $(X - G) \cap (\bigcap_{i=1}^{n} int(cl((D_i))) \notin G$. However, $D_i \subset int(cl(D_i))$ implies

$(X-G)\cap(\cap_{i=1}^{n}(D_i)) \notin G$. Since B is a filter base and $D_i \in B$, there is a $E \in B$ such that $E \subset \cap_{i=1}^{n}(B_i)$. But $(X-G)\cap E \notin G$ is required.

Conversely, let (X, T) be not neutrosophic $C(G)$ - compact, and A be a neutrosophic closed set and U be aneutrosophic open cover of A such that for no finite subfamily$\{U_1, U_2, U_3 \dots\dots, U_n\}$ of U, one has $A - \cup_{i=1}^{n} cl(U_i) \in G$. Without loss of generality, we may assume that U is closed for finite unions. Therefore, $B = \{X - cl(U_i) : U_i \in U\}$ becomes a neutrosophic filter base on G. If x is a neutrosophic adherent point of B, that is, if $x \in \{cl(X - cl(U_i)) : U_i \in U\} = X - \cup\{int(cl(U_i)) : U_i \in U\}$, then $x \notin A$, because U is a neutrosophic open cover of A and for $U_i \in U$, $U_i \subset int(cl(U_i))$. Therefore, the neutrosophic adherent set of B is contained in $X - A$, which is a neutrosophic open set. By hypothesis, there exists an element $D \in B$ such that $(X-(X-A)) \cap D \notin$ G, that is, $A \cap D \notin G$, that is $A \cap (X - cl(U_i) \notin G$ for some $U \in V$. This however contradicts our assumption. This completes the proof.

4. Neutrosophic Quasi - H - Closed with Respect to a Grill

In this section, our aim is to introduce the concept of neutrosophic quasi - H - closed and study various properties of this notionwith respect to a grill.

Definition 4.1. A neutrosophic topological space (X, T) is said to be neutrosophic quasi - H-closed or simply NQHC, if for every open cover U of X, there exists a finite subfamily $\{U_1, U_2, U_3, \dots, U_n\}$ such that $X = \sum_{i=1}^{n} cl(U_i)$.

Definition 4.2. Let (X, T) be a neutrosophic topological space and G be a grill on X. X is neautrosophic quasi - H - closed with respect to G or just NQHC(G) if for every open cover U of X, there exists a finite subfamily $\{U_1, U_2, U_3, \dots, U_n\}$ of U such that$X - \sum_{i=1}^{n} cl(U_i) \notin G$.

Definition 4.3. A grill G of subsets of a neutrosophic topological space (X, T) is said to be co non - dense if the complement of each of its members is non - dense.

Definition 4.4. Let (X, T) be a neutrosophic topological space. A family F of subsets of X is said to have the finite intersection property with respect to a grill G on X or just FIP(G) if the intersection of finite subfamily of F is a member of G.

Theorem 4.5. For a neutrosophic topological space (X, T) and a grill G on X, the following are equivalent:

(a) (X, T) is NQHC(G).

(b) For each family F of closed sets having empty intersection, there is a finite subfamily $\{F_1, F_2, F_3, \dots, F_n\}$ such that $\cap_{i=1}^{n} int(F_i) \notin G$.

(c) For each family K of neutrosophic closed sets such that $\{int(F) : F \in K\}$ has FIP(G), one has $\cap\{F : F \in K\} \neq 0_N$.

(d) Every neutrosophic regular open cover has a proximate G cover.

(e) For each family F of non empty neutrosophic regular closed sets having empty intersection, there is a finite subfamily $\{F_1, F_2, F_3, \dots, F_n\}$ such that $\cap_{i=1}^{n} int(F_i) \notin G$.

(f) For each collection K of non emptyneutrosophicregular closed sets such that $\{int(F) : F \in K\}$ has FIP(G), one has $\cap\{F : F \in K\} \neq 0_N$.

(g) For each neutrosophicopen filter base Con G, $\cap\{cl(B) : B \in C\} \neq 0_N$.

(h) Every neutrosophicopen ultra filter on G converges.

Proof. Obvious.

5. Conclusion

In this article, we have defined neutrosophic C - compactness with respect to a grill. We have investigated some properties of this newly defined compactness. Some characterization theorems have also been established. We have also defined neutrosophic quasi - H - closed with respect to a grill. Some characterization theorems on this newly concept have been investigated.It is expected that the work done will help in further investigation of the compactness in neutrosophic topological space.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Towards a Responsive Resilient Supply Chain based on Industry 5.0: A Case Study in Healthcare Systems

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Abstract: Executives and academics have presented the idea of Industry 5.0, which is an attempt to build upon the previous iteration, Industry 4.0, by including many important tenets such as human-centricity, resilience, and sustainability. Because of the significance of this idea, the current study provides a decision-making framework to examine a responsive supply chain dubbed responsive supply chain 5.0 for healthcare systems. This framework takes into consideration the aspects of Industry 5.0. In order to do this, at the onset, the most important connected factors and strategies are determined by consulting the relevant experts and body of published research. This problem has been considered a multi-criteria decision-making (MCDM) problem. Initially, the DEMATEL method was used to assess and prioritize the main criteria and their sub-indicators. Secondly, the CRADIS method was used to evaluate and rank the strategies used to make the supply chain more responsive. The results indicate that the collaboration and sharing of information strategy is the most appropriate strategy if applied.

Keywords: Supply chain 5.0; Responsiveness; Industry 4.0; Industry 5.0; MCDM; DEMATEL method; CRADIS method.

1. Introduction

In today's world, when there has been such a dramatic increase in the level of competitiveness in the market environment, supply chain management has developed into an essential component of each and every company [1]. In recent years, there has been a growing trend among scholars toward the issue of the supply chain, which can be traced back to the aforementioned argument. As a result of the fact that consumers are an essential component of any supply chain, meeting the customers' demands is often seen as one of the most important objectives of supply networks [2]. When seen from this angle, the idea of responsiveness stands out as one of the most essential criteria in the supply chain. The capability of a supply chain to satisfy the demand of consumers within a certain time frame is referred to as its responsiveness.

The substantial improvements that have been made in information technology and the digital industry over the course of the last decade have resulted in significant changes to the business settings that are associated with the so-called "Industry 4.0." Artificial intelligence and information technology have been the focus of efforts under the Industry 4.0 initiative, which aims to boost overall industrial productivity [3]. However, despite its many positive attributes, Industry 4.0 is only a techno-economic vision in nature that has centred its attention on the part that technology plays in enhancing the operational effectiveness of businesses. Therefore, as a result of the excessive focus that Industry 4.0 places on digitalization and technology powered by artificial intelligence, several vitally important concepts, such as sustainability and the role that people play in the sector, have been neglected. Researchers have recently presented Industry 5.0, which aims to complete and expand the characteristics of Industry 4.0. This is because of the concerns that were highlighted, as well as the vulnerabilities of today's industry, which were aggravated during the COVID-19 outbreak [4]. In this manner, the foundation for Industry 5.0 has been laid, and it is founded on the following three primary dimensions: (i) sustainability; (ii) resilience; and (iii) human-centricity. In general, the issue changes into a sustainable one when the social, environmental, and financial factors are all examined at the same time. On the other hand, the capacity of a supply chain to lessen the effects of prospective interruptions or the chance of such disruptions occurring, as well as to shorten the amount of time required to resume and restore operations, is what is meant by the term "resilience." In conclusion, human-centricity refers to the practice of taking into account the role that people play in both society and business, as well as giving precedence to the requirements of humans.

As a result of the COVID-19 epidemic, it has become clear how important health systems are to issues of public health, social cohesion, faith in governments, and economic development. Therefore, doing research into healthcare systems has the potential to enhance the circumstances of the aforementioned factors, particularly during current challenging times [5]. According to the studies that have been conducted, the primary focus in the healthcare supply chains that existed prior to the adoption of Industry 5.0 was on the implementation of contemporary technologies (such as the Internet of Things, Big Data, and Blockchain) in order to cut down on the amount of time spent on operations, cut down on the amount of money spent on operations, and increase the effectiveness of healthcare systems.

However, after the implementation of Industry 5.0, in addition to taking into account the beneficial role that technologies play, it will be necessary to include a number of other essential ideas, including human-centricity, sustainability, and resilience. As a result of this, those in charge of running healthcare systems should make use of newly developed technology in order to make their organizations more resilient and sustainable. For instance, the use of 3D printing in the production of goods such as medical gadgets may significantly cut down on waste and the harm it does to the environment [6]. The use of information-sharing systems may also lead to an improvement in the system's openness and visibility, which both contribute to an increase in the system's resilience. After the implementation of Industry 5.0, it is essential for managers of healthcare facilities to take into account a number of crucial factors, including the part that people play in both society and operations.

On the other hand, when it comes to responsiveness, the policies outlined in Industry 5.0 may be of great assistance. In this respect, for example, the power of a system to deal with disturbances or recover after disruptions significantly rises when resilience techniques are employed inside the system. As a result, this system does not go into maintenance mode following disturbances and is able to continue providing the services that users demand. This ensures that it keeps its responsiveness.

Concerning the aforementioned considerations, the present study explores the qualities of the responsive supply chain according to the dimensions of Industry 5.0 utilizing the multi-criteria decision-making (MCDM) methodologies [7]. This investigation was prompted by real-world case studies. In addition, since the significance of the medical devices business as one of the essential components of healthcare systems has been brought into the spotlight in a significant way in the aftermath of the coronavirus epidemic, this paper takes that industry into consideration as a case study. The following is a list of the most important goals that this research work aims to achieve:

- Looking at the primary components of a responsive supply chain in the age of Industry 5.0.
- Determining the primary approaches that may be used in order to get towards a responsive supply chain 5.0.

2. Criteria and Alternatives

In this section, the main criteria and their sub-indicators related to improving the response aspect of the supply chain are presented. Also, four strategies used as alternatives are presented in the study. Figure 1 presents the main objective of the study, evaluation criteria, and four solution strategies used in the study. It is worth noting that the standards of the sustainability, the resiliency, and human-centricity are basic elements in Industry 5.0.

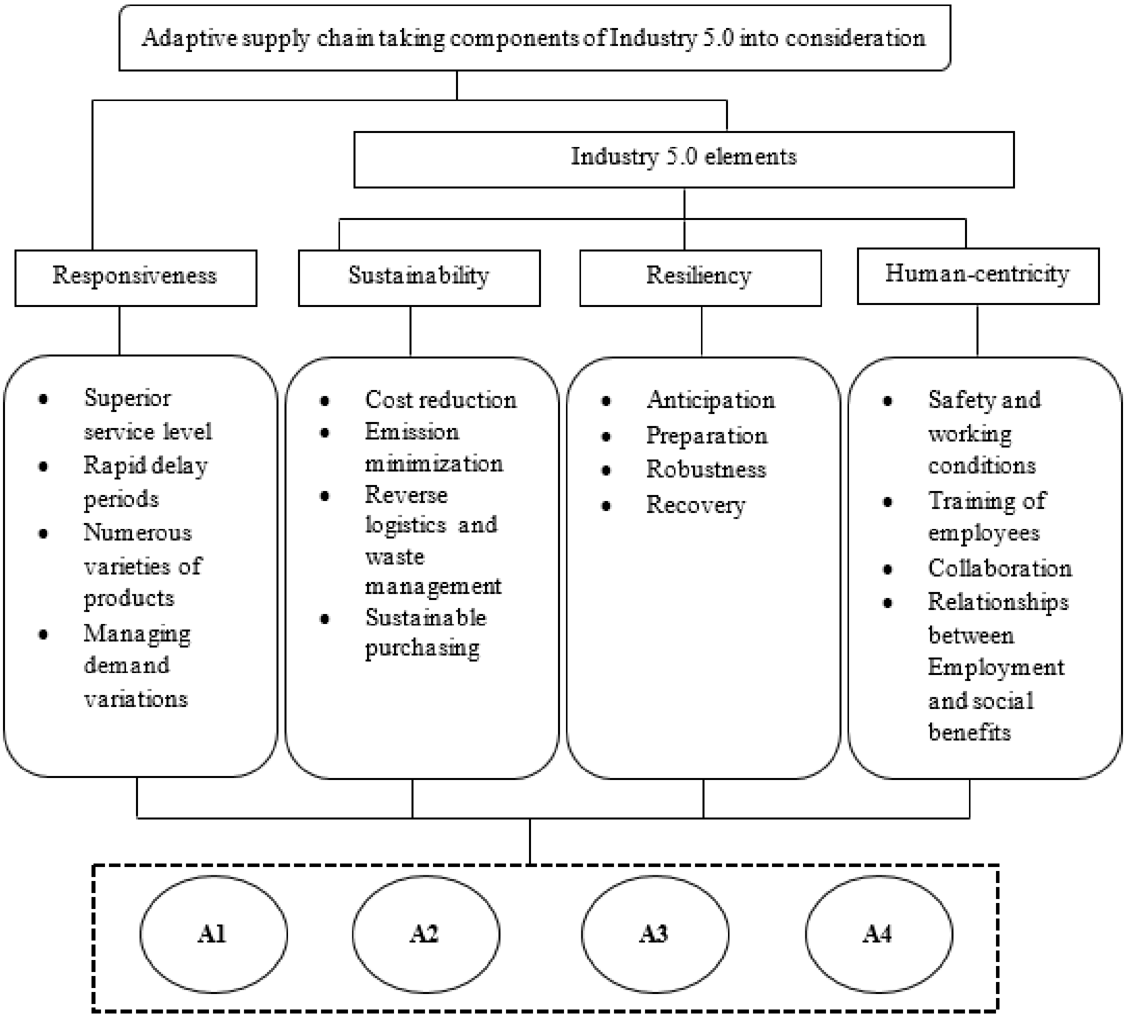

Figure 1. Elements of the research problem.

2.1 Criteria

2.1.1 Responsiveness criterion C_1

In general, the amount of responsiveness that a supply chain has may be characterized by how well it is able to satisfy the criteria that are posed by the customers. In this regard, the responsiveness criterion includes several sub-indicators, namely: superior service level (C_{1_1}), rapid delay periods (C_{1_2}), numerous varieties of products (C_{1_3}), and managing demand variations (C_{1_4}).

2.1.2 Sustainability criterion C_2

The economic, environmental, and social factors are all included in the sustainability element. In this regard, the sustainability criterion includes several sub-indicators, namely: cost reduction (C_{2_1}), emission minimization (C_{2_2}), reverse logistics and waste management (C_{2_3}), and sustainable purchasing (C_{2_4}).

2.1.3 Resiliency criterion C_3

According to the available research, supply chain resilience refers to an organization's capacity to rebound from interruptions and continue meeting the needs of its consumers. In this regard, the resiliency criterion includes several sub-indicators, namely: anticipation (C_{3_1}), preparation (C_{3_2}), robustness (C_{3_3}), and recovery (C_{3_4}).

2.1.4 Human-centricity criterion C_4

Researchers have regarded the human-centricity component as one of the primary pillars of Industry 5.0 in order to get rid of the flaw that was discussed with regard to Industry 4.0. This is because Industry 4.0 has placed an excessive emphasis on the role that technology plays in industries while ignoring the part that people play in such sectors. In this regard, the human-centricity criterion includes several sub-indicators, namely: safety and working conditions (C_{4_1}), training of employee (C_{4_2}), collaboration (C_{4_3}), and relationships between employment and social benefits (C_{4_4}).

2.2 Alternatives

In this part, the four strategies used in this research work to improve the responsiveness aspect of the supply chain are listed. The four strategies are using cutting-edge technology (A_1), collaboration and sharing of information (A_2), intelligent warehousing (A_3), and postponement (A_4).

2.2.1 Using cutting-edge technology

The use of more sophisticated technologies, which are characterized by greater adaptability and dependability, has the potential to boost the responsiveness of the supply chain.

2.2.2 Collaboration and sharing of information

It is possible to boost the responsiveness of the supply chain by using tactics that include cooperation and the exchange of information.

2.2.3 Intelligent warehousing

A structure that makes use of computers and other types of machinery and is used for the storage of finished goods and raw materials.

2.2.4 Postponement

A delay in the operations of assembly and distribution is the cause of the postponement. This delay will continue until there is sufficient information available regarding the client order.

3. Methodology

In this section, the proposed DEMATEL-CRADIS methodology is presented to evaluate several strategies to make the supply chain more responsive. The DEMATEL refers to the Decision Making Trial and Evaluation Laboratory Method. Also, the CRADIS method is a combination of several methods are the Additive Ratio Assessment (ARAS), the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), and the Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS).

Step 1: A set of alternatives are identified to be used in the evaluation process. The set strategies = $(A_1, A_2, \dots, A_m)$ having i = 1, 2... m alternatives, is measured by n decision criteria and indicators of C_j = $(C_1, C_2, \dots, C_n)$, with j = 1, 2...n. Let $w = (w_1, w_2, \dots, w_n)$ be the vector set utilized for defining the criteria and indicators weights, $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$.

Step 2: A set of linguistic terms and their corresponding triangular neutrosophic numbers (TNNs) are defined to help participants prioritize the main criteria and their sub-indicators as provided in Table 1.

Table 1. Linguistic terms and their equivalent TNNs for evaluating criteria and alternatives.

Linguistic terms	Abbreviations	TNNs
Fully Low Value	FLV	⟨(0.1, 0.2, 0.3); 0.4, 0.1, 0.3⟩
Very Low Value	VLV	⟨(0.2, 0.3, 0.4); 0.5, 0.1, 0.3⟩
Low Value	LOV	⟨(0.3, 0.4, 0.5); 0.6, 0.2, 0.1⟩
Modest Low Value	MLV	⟨(0.4, 0.5, 0.6); 0.7, 0.3, 0.2⟩
Roughly Value	ROV	⟨(0.5, 0.6, 0.7); 0.8, 0.3, 0.3⟩
Modest High Value	MHV	⟨(0.6, 0.7, 0.8); 0.9, 0.4, 0.4⟩
High Value	HVV	⟨(0.7, 0.8, 0.9); 1.0, 0.3, 0.5⟩
Very High Value	VHV	⟨(0.8, 0.9, 1.0); 1.0, 0.2, 0.3⟩
Fully High Value	FHV	⟨(0.9, 1.0, 1.0); 1.0, 0.2, 0.2⟩

Step 3: Create a pairwise comparison matrix amongst the main criteria and itself by all experts to clarify their preferences for these criteria.

Step 4: Convert TNNs to real values by applying the score function according to Eq. (1).

$$S(\tilde{x}_{ij}) = \frac{1}{8}(l + m + u) \times (2 + \alpha_{\tilde{x}} - \theta_{\tilde{x}} - \beta_{\tilde{x}}) \quad (1)$$

Step 5: Calculating the generalized direct relation matrix (g) for all criteria by using Eqs. (2-3).

$$Q = \frac{1}{\mathrm{Max}_{1 \le i \le n} \sum_{j=1}^{n} x_{ij}} \quad (2)$$

$$g = Q \times P \quad (3)$$

Step 6: Calculating the total relation matrix (T) for all criteria by using Eq. (4).

$$T = g \times (I - g)^{-1} \quad (4)$$

Where I is the identity matrix.

Step 7: Calculating the sum of rows and columns expressed as R and C, respectively, in Eq. (5) and Eq. (6). Then, the horizontal axis vector R+C is calculated, and the vertical axis vector R–C.

$$R = \left[\sum_{i=1}^{n} t_{ij}\right]_{1\times n} \quad (5)$$

$$C = \left[\sum_{j=1}^{n} t_{ij}\right]_{n\times 1} \quad (6)$$

Step 8: Attaining the weights of the main criteria $C_1, C_2, \dots, C_n$ based on the R and C that obtaining from expert opinions according to Eq. (7).

$$w = \frac{R+C}{\sum_{i=1}^{n} R+C} \quad (7)$$

Step 9: Constructing the assessment decision matrix by all experts between the determined sub-indicators and the available alternatives using the linguistic terms as presented in Table 1. Then, convert TNNs to real values by applying the score function according to Eq. (1).

Step 10: Calculating the normalized decision matrix for the benefit indicators according to Eq. (8), and for cost indicators according Eq. (9).

$$\text{normalized}_{\text{ij}} = \frac{y_{\text{ij}}}{y_{\text{jmax}}} \quad (8)$$

$$\text{normalized}_{\text{ij}} = \frac{y_{\text{jmin}}}{y_{\text{ij}}} \quad (9)$$

Step 11: Computing the weighted evaluation decision matrix by multiplying the value of the normalized decision matrix by the corresponding weights according to Eq. (10).

$$v_{\text{ij}} = \text{normalized}_{\text{ij}} \times w_{\text{j}} \quad (10)$$

Step 12: Determining the ideal and anti-ideal solution by using Eq. (11) and Eq. (12), respectively.

$$t_{\text{i}} = \max v_{\text{ij}} \quad (11)$$

$$t_{\text{ai}} = \min v_{\text{ij}} \quad (12)$$

Step 13: Computing of deviations from ideal and anti-ideal solutions, respectively according to Eq. (13) and Eq. (14).

$$d^{+} = t_{\text{i}} - v_{\text{ij}} \quad (13)$$

$$d^{-} = v_{\text{ij}} - t_{\text{ai}} \quad (14)$$

Step 14: Determining the degrees to which specific alternatives deviate from ideal and anti-ideal solutions and then computing those degrees according to Eq. (15) and Eq. (16).

$$s_i^{+} = \sum_{j=1}^{n} d^{+} \quad (15)$$

$$s_i^{-} = \sum_{j=1}^{n} d^{-} \quad (16)$$

Step 15: Computing of the utility function for each alternative in relation to the deviations from the optimal alternatives according to Eq. (17) and Eq. (18).

$$K_i^{+} = \frac{s_0^{+}}{s_i^{+}} \quad (17)$$

$$K_i^{-} = \frac{s_i^{-}}{s_0^{-}} \quad (18)$$

Step 16: Calculating the final order by looking for the average deviation of the alternatives from the degree of utility according to Eq. (19). Then, rank the alternatives, the best alternative is the one that has the greatest valueQ_i.

$$Q_i = \frac{K_i^{+} + K_i^{-}}{2} \quad (19)$$

4. Application

4.1 Case study

During the last two years, the epidemic of coronavirus has caused a significant amount of disturbance all across the globe. This illness has been responsible for a number of deaths as well as significant economic losses. The significance of medical gadgets has been brought into sharper focus as a direct result of the epidemic that was discussed. As a result, a business operating in the field of medical equipment and situated in Egypt has been chosen. This company manufactures a wide range of medical equipment, including the blood bank refrigerator, the vaccine refrigerator, the oxygen concentrator device, and a variety of other similar products. The current epidemic has presented this business with a number of significant hurdles. A significant rise in the number of people needing medical equipment has made it difficult for this organization to satisfy the needs of its clientele, which brings us to our first point. In this respect, the notion of responsiveness may be of assistance to the administrators of this organization in their efforts to address the aforementioned problem. To acquire a competitive edge, the management of this firm are interested, on the other hand, in applying the aspects of Industry 5.0 inside their organization.

4.2 Application of the suggested approach

In this part, the steps of the proposed methodology DEMATEL-CRADIS are applied. The proposed approach is applied under a neutrosophic environment.

Step 1: A pairwise comparison matrix was created amongst the main criteria and itself by all experts to clarify their preferences for these criteria using linguistic terms as presented in Table 2. Then, TNNs were converted to real values by applying the score function according to Eq. (1).

Step 2: The generalized direct relation matrix was computed for all main criteria by using Eqs. (2-3), as presented in Table 3.

Step 3: The total relation matrix was determined for all main criteria by using Eq. (4), as presented in Table 4.

Step 4: The weights of the main criteria were obtained according to Eq. (7), as presented in Table 4 and shown in Figure 2.

Table 2. Evaluation of main criteria using linguistic terms by all experts.

$Expert_s$	C_1	C_2	C_3	C_4
C_1	*	MLV	LOV	FHV
C_2	VLV	*	MHV	FLV
C_3	VHV	MHV	*	ROV
C_4	ROV	VHV	VHV	*

Table 3. Generalized relation matrix of main criteria by all experts.

$Expert_s$	C_1	C_2	C_3	C_4
C_1	0.1866	0.1530	0.1306	0.3507
C_2	0.0896	0.1866	0.2052	0.0560
C_3	0.3134	0.2052	0.1866	0.1866
C_4	0.1866	0.3134	0.3134	0.1866

Table 4. Total relation matrix of main criteria by all experts.

	C_1	C_2	C_3	C_4	R_i	C_i	$R_i + C_i$	$R_i - C_i$	Identity	Weight
C_1	1.0428	1.1193	1.0744	1.2043	4.441	4.033	8.473	0.408	Cause	0.252
C_2	0.6105	0.7487	0.7536	0.5564	2.669	4.449	7.118	-1.779	Effect	0.212
C_3	1.2095	1.1912	1.1463	1.0957	4.643	4.338	8.981	0.305	Cause	0.267
C_4	1.1698	1.3895	1.3638	1.1422	5.065	3.999	9.064	1.067	Cause	0.269

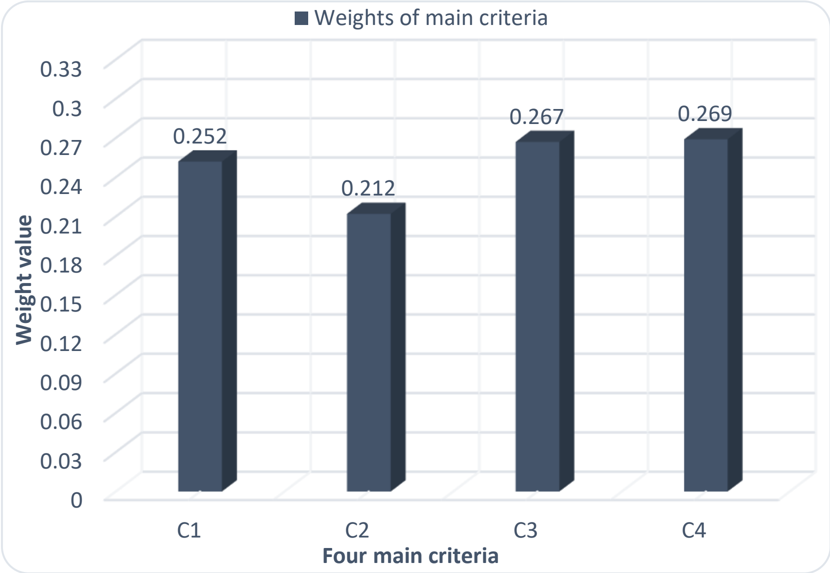

Figure 2. Final weights of main criteria.

Step 5: A pairwise comparison matrix was created amongst the responsiveness criterion's sub-indicators and itself by all experts to clarify their preferences for these indicators using linguistic terms as presented in Table 5. Then, TNNs were converted to real values by applying the score function according to Eq. (1).

Step 6: The generalized direct relation matrix was computed for responsiveness criterion's sub-indicators by using Eqs. (2-3), as presented in Table 6.

Step 7: The total relation matrix was determined for responsiveness criterion's sub-indicators by using Eq. (4), as presented in Table 7.

Step 8: The weights of the responsiveness criterion's sub-indicators were obtained according to Eq. (7), as presented in Table 7 and shown in Figure 3.

Table 5. Evaluation of responsiveness criterion's sub-indicators using linguistic terms.

Expert_s	C_{1_1}	C_{1_2}	C_{1_3}	C_{1_4}
C_{1_1}	$*$	MLV	VHV	MHV
C_{1_2}	MLV	$*$	FHV	HVV
C_{1_3}	VHV	MHV	$*$	VHV
C_{1_4}	ROV	HVV	VHV	$*$

Table 6. Generalized relation matrix of responsiveness criterion's sub-indicators.

Expert_s	C_{1_1}	C_{1_2}	C_{1_3}	C_{1_4}
C_{1_1}	0.1832	0.1502	0.3077	0.2015
C_{1_2}	0.1502	0.1832	0.3443	0.2418
C_{1_3}	0.3077	0.2015	0.1832	0.3077
C_{1_4}	0.1832	0.2418	0.3077	0.1832

Table 7. Total relation matrix of responsiveness criterion's sub-indicators.

	C_{1_1}	C_{1_2}	C_{1_3}	C_{1_4}	R_i	C_i	$R_i + C_i$	$R_i - C_i$	Identity	Weight
C_{1_1}	2.5991	2.3621	3.4368	2.8813	11.279	11.305	22.584	-0.026	Effect	0.230
C_{1_2}	2.7994	2.6116	3.7752	3.1814	12.368	10.395	22.762	1.973	Cause	0.232
C_{1_3}	3.1024	2.7766	3.8491	3.4135	13.142	14.775	27.917	-1.634	Effect	0.285
C_{1_4}	2.8041	2.6444	3.7145	3.0976	12.261	12.574	24.835	-0.313	Effect	0.253

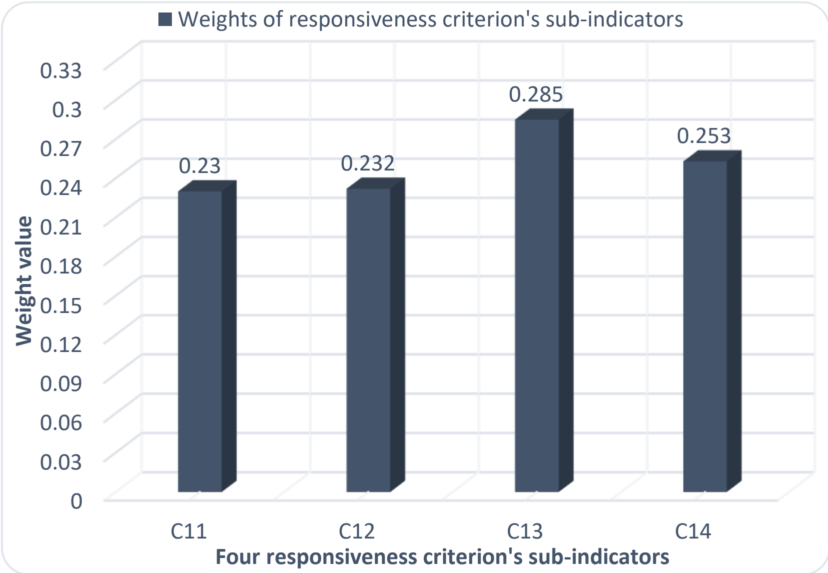

Figure 3. Final weights of responsiveness criterion's sub-indicators.

Step 9: A pairwise comparison matrix was created amongst the sustainability criterion's sub-indicators and itself by all experts to clarify their preferences for these indicators using linguistic terms as presented in Table 8. Then, TNNs were converted to real values by applying the score function according to Eq. (1).

Step 10: The generalized direct relation matrix was computed for sustainability criterion's sub-indicators by using Eqs. (2-3), as presented in Table 9.

Step 11: The total relation matrix was determined for sustainability criterion's sub-indicators by using Eq. (4), as presented in Table 10.

Step 12: The weights of the sustainability criterion's sub-indicators were obtained according to Eq. (7), as presented in Table 10 and shown in Figure 4.

Table 8. Evaluation of sustainability criterion's sub-indicators using linguistic terms.

Expert_s	C_{2_1}	C_{2_2}	C_{2_3}	C_{2_4}
C_{2_1}	*	FHV	HVV	MHV
C_{2_2}	FLV	*	FLV	HVV
C_{2_3}	FLV	LOV	*	VHV
C_{2_4}	ROV	LOV	FHV	*

Table 9. Generalized relation matrix of sustainability criterion's sub-indicators.

$Expert_s$	C_{2_1}	C_{2_2}	C_{2_3}	C_{2_4}
C_{2_1}	0.1887	0.3547	0.2491	0.2075
C_{2_2}	0.0566	0.1887	0.0566	0.2491
C_{2_3}	0.0566	0.1321	0.1887	0.3170
C_{2_4}	0.1887	0.1321	0.3547	0.1887

Table 10. Total relation matrix of sustainability criterion's sub-indicators.

	C_{2_1}	C_{2_2}	C_{2_3}	C_{2_4}	R_i	C_i	$R_i + C_i$	$R_i - C_i$	Identity	Weight
C_{2_1}	0.6667	1.1055	1.1138	1.2009	4.087	2.042	6.129	2.045	Cause	0.235
C_{2_2}	0.3367	0.6015	0.5640	0.7981	2.300	3.161	5.461	-0.861	Effect	0.210
C_{2_3}	0.4148	0.6514	0.8562	1.0313	2.954	3.696	6.650	-0.743	Effect	0.255
C_{2_4}	0.6238	0.8026	1.1624	1.0927	3.681	4.123	7.804	-0.442	Effect	0.300

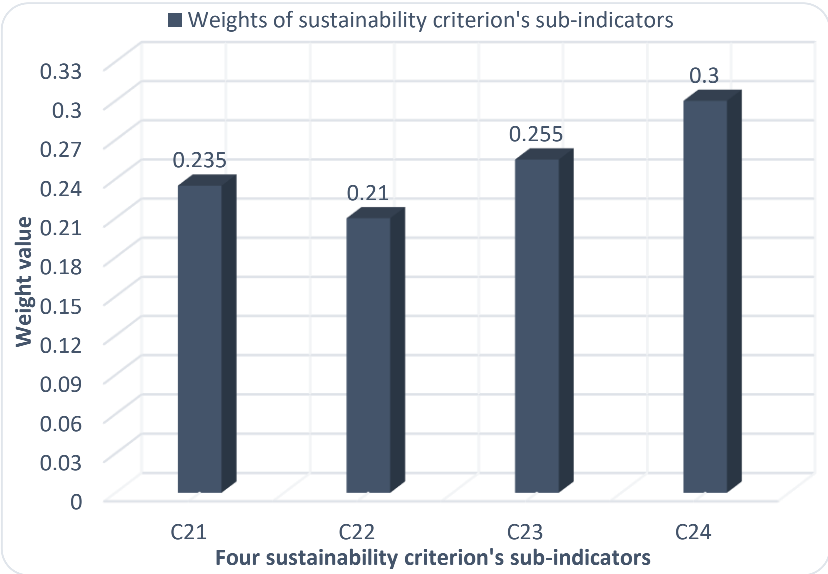

Figure 4. Final weights of sustainability criterion's sub-indicators

Step 13: A pairwise comparison matrix was created amongst the resiliency criterion's sub-indicators and itself by all experts to clarify their preferences for these indicators using linguistic terms as presented in Table 11. Then, TNNs were converted to real values by applying the score function according to Eq. (1).

Step 14: The generalized direct relation matrix was computed for resiliency criterion's sub-indicators by using Eqs. (2-3), as presented in Table 12.

Step 15: The total relation matrix was determined for resiliency criterion's sub-indicators by using Eq. (4), as presented in Table 13.

Step 16: The weights of the resiliency criterion's sub-indicators were obtained according to Eq. (7), as presented in Table 13 and shown in Figure 5.

Table 11. Evaluation of resiliency criterion's sub-indicators using linguistic terms.

Expert_s	C_{3_1}	C_{3_2}	C_{3_3}	C_{3_4}
C_{3_1}	*	LOV	HVV	MHV
C_{3_2}	LOV	*	FLV	LOV
C_{3_3}	FLV	LOV	*	VHV
C_{3_4}	ROV	LOV	FHV	*

Table 12. Generalized relation matrix of resiliency criterion's sub-indicators.

Expert_s	C_{3_1}	C_{3_2}	C_{3_3}	C_{3_4}
C_{3_1}	0.2183	0.1528	0.2882	0.2402
C_{3_2}	0.1528	0.2183	0.0655	0.1528
C_{3_3}	0.0655	0.1528	0.2183	0.3668
C_{3_4}	0.2183	0.1528	0.4105	0.2183

Table 13. Total relation matrix of resiliency criterion's sub-indicators.

	C_{3_1}	C_{3_2}	C_{3_3}	C_{3_4}	R_i	C_i	$R_i + C_i$	$R_i - C_i$	Identity	Weight
C_{3_1}	1.1071	1.0828	1.7501	1.6804	5.620	3.887	9.508	1.733	Cause	0.233
C_{3_2}	0.7176	0.8051	0.9515	1.0199	3.494	4.064	7.558	-0.570	Effect	0.185
C_{3_3}	0.8744	0.9969	1.5595	1.6647	5.096	6.280	11.376	-1.185	Effect	0.279
C_{3_4}	1.1881	1.1789	2.0190	1.8224	6.208	6.187	12.396	0.021	Cause	0.304

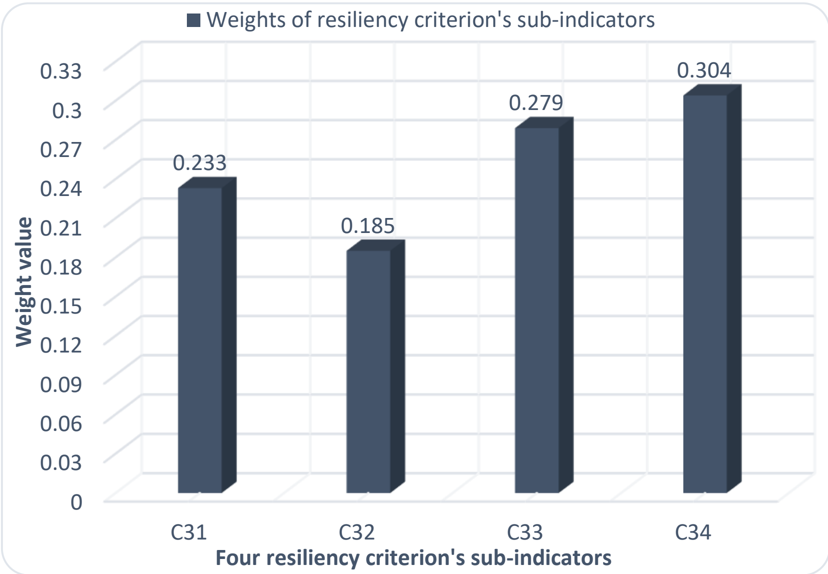

Figure 5. Final weights of resiliency criterion's sub-indicators.

Step 17: A pairwise comparison matrix was created amongst the human-centricity criterion's sub-indicators and itself by all experts to clarify their preferences for these indicators using linguistic terms as presented in Table 14. Then, TNNs were converted to real values by applying the score function according to Eq. (1).

Step 18: The generalized direct relation matrix was computed for human-centricity criterion's sub-indicators by using Eqs. (2-3), as presented in Table 15.

Step 19: The total relation matrix was determined for human-centricity criterion's sub-indicators by using Eq. (4), as presented in Table 16.

Step 20: The weights of the human-centricity criterion's sub-indicators were obtained according to Eq. (7), as presented in Table 16 and shown in Figure 6.

Step 21: The global weights of the sub-indicators are calculated by multiplying the weights of the main criteria by the weights of the local criteria for the sub-indicators, as in Figure 7.

Table 14. Evaluation of human-centricity criterion's sub-indicators using linguistic terms.

Expert_s	C_{4_1}	C_{4_2}	C_{4_3}	C_{4_4}
C_{4_1}	*	LOV	HVV	MHV
C_{4_2}	HVV	*	FLV	LOV
C_{4_3}	FLV	VHV	*	LOV
C_{4_4}	HVV	VHV	FLV	*

Table 15. Generalized relation matrix of human-centricity criterion's sub-indicators.

Expert_s	C_{4_1}	C_{4_2}	C_{4_3}	C_{4_4}
C_{4_1}	0.2326	0.1628	0.3070	0.2558
C_{4_2}	0.3070	0.2326	0.0698	0.1628
C_{4_3}	0.0698	0.3907	0.2326	0.1628
C_{4_4}	0.3070	0.3907	0.0698	0.2326

Table 16. Total relation matrix of human-centricity criterion's sub-indicators.

	C_{4_1}	C_{4_2}	C_{4_3}	C_{4_4}	R_i	C_i	$R_i + C_i$	$R_i - C_i$	Identity	Weight
C_{4_1}	2.3472	2.6551	1.7670	2.0537	8.823	8.848	17.671	-0.025	Effect	0.271
C_{4_2}	2.0530	2.2124	1.2617	1.6334	7.161	10.289	17.450	-3.128	Effect	0.267
C_{4_3}	1.8917	2.4971	1.4392	1.6777	7.506	6.039	13.544	1.467	Cause	0.207
C_{4_4}	2.5560	2.9245	1.5709	2.1086	9.160	7.473	16.633	1.687	Cause	0.255

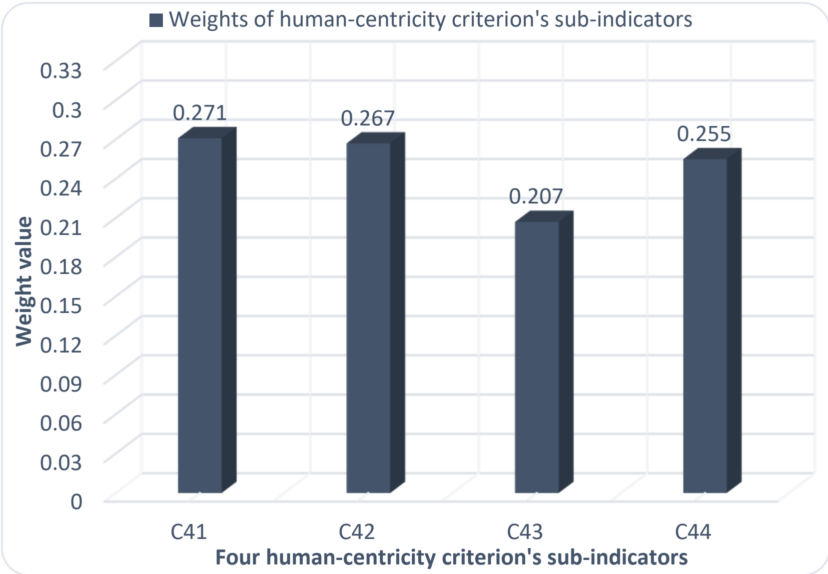

Figure 6. Final weights of human-centricity criterion's sub-indicators.

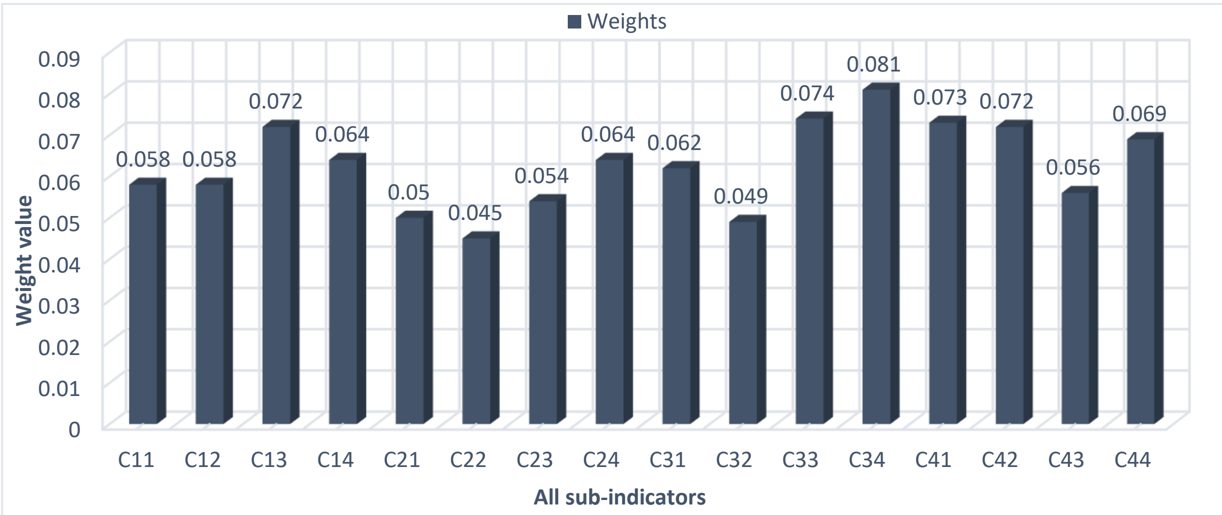

Figure 7. Final global weights of all sub-indicators.

Step 22: The assessment decision matrix was constructed by all experts between the determined sub-indicators and the available alternatives using the linguistic terms provided in Table 1, as presented in Table 17. Then, the TNNs were converted to real values by applying the score function according to Eq. (1).

Step 23: The normalized decision matrix was constructed for the benefit indicators according to Eq. (8), and for cost indicators according Eq. (9), as presented in Table 18.

Step 24: The weighted evaluation decision matrix was computed by multiplying the value of the normalized decision matrix by the corresponding weights according to Eq. (10), as exhibited in Table 19.Then, the ideal and anti-ideal solution were determined by using Eq. (11) and Eq. (12), respectively.

Step 25: The degrees to which specific alternatives deviate from ideal and anti-ideal solutions were determined and then computing those degrees according to Eq. (15) and Eq. (16), as presented in Table 20.

Step 26: The final order by looking for the average deviation of the alternatives were determined from the degree of utility according to Eq. (19), as presented in Table 21 and shown in Figure 8.

Table 17. Evaluation matrix of four strategies regarding all sub-indicators using linguistic terms.

Experts	C_{1_1}	C_{1_2}	C_{1_3}	C_{1_4}	C_{2_1}	C_{2_2}	C_{2_3}	C_{2_4}
A_1	HVV	FHV	MHV	VHV	ROV	HVV	FLV	VLV
A_2	FHV	VHV	VHV	FHV	HVV	VHV	FHV	FHV
A_3	VHV	FLV	MLV	FLV	MHV	MLV	VLV	VLV
A_4	MHV	MHV	ROV	VHV	FHV	MHV	VHV	HVV
Experts	C_{3_1}	C_{3_2}	C_{3_3}	C_{3_4}	C_{4_1}	C_{4_2}	C_{4_3}	C_{4_4}
A_1	MLV	MLV	MHV	FLV	VLV	MHV	ROV	HVV
A_2	FHV	VHV	HVV	FHV	VHV	VHV	FHV	FHV
A_3	VLV	LOV	ROV	LOV	HVV	MHV	HVV	VHV
A_4	FHV	ROV	VLV	HVV	FHV	MLV	LOV	MHV

Table 18. Normalized matrix of four strategies regarding all sub-indicators.

Experts	C_{1_1}	C_{1_2}	C_{1_3}	C_{1_4}	C_{2_1}	C_{2_2}	C_{2_3}	C_{2_4}
A_1	0.835	1.000	0.653	0.178	1.000	0.782	0.159	0.251
A_2	0.585	0.895	1.000	0.159	0.750	1.000	1.000	1.000
A_3	0.653	0.159	0.489	1.000	0.898	0.489	0.251	0.251
A_4	1.000	0.585	0.587	0.178	0.525	0.653	0.895	0.700
Experts	C_{3_1}	C_{3_2}	C_{3_3}	C_{3_4}	C_{4_1}	C_{4_2}	C_{4_3}	C_{4_4}
A_1	0.438	0.836	0.429	1.000	0.251	0.653	0.525	0.835
A_2	1.000	0.409	0.358	0.159	0.895	1.000	1.000	0.585
A_3	0.251	1.000	0.477	0.435	0.700	0.653	0.700	0.653
A_4	1.000	0.697	1.000	0.227	1.000	0.489	0.366	1.000

Table 19. Weighted normalized matrix of four strategies regarding all sub-indicators.

Experts	C_{1_1}	C_{1_2}	C_{1_3}	C_{1_4}	C_{2_1}	C_{2_2}	C_{2_3}	C_{2_4}
A_1	0.048	0.058	0.047	0.011	0.050	0.035	0.009	0.016
A_2	0.034	0.052	0.072	0.010	0.037	0.045	0.054	0.064
A_3	0.038	0.009	0.035	0.064	0.045	0.022	0.014	0.016
A_4	0.058	0.034	0.042	0.011	0.026	0.029	0.048	0.045
t_i	0.058	0.058	0.072	0.064	0.050	0.045	0.054	0.064
t_{ai}	0.034	0.009	0.035	0.010	0.026	0.022	0.009	0.016
Experts	C_{3_1}	C_{3_2}	C_{3_3}	C_{3_4}	C_{4_1}	C_{4_2}	C_{4_3}	C_{4_4}
A_1	0.027	0.041	0.032	0.081	0.018	0.047	0.029	0.057
A_2	0.062	0.020	0.027	0.013	0.065	0.072	0.056	0.040
A_3	0.016	0.049	0.036	0.035	0.051	0.047	0.039	0.045
A_4	0.062	0.034	0.074	0.018	0.073	0.035	0.020	0.069

t_i	0.062	0.049	0.074	0.081	0.073	0.072	0.056	0.069
t_{ai}	0.016	0.020	0.027	0.013	0.018	0.035	0.020	0.040

Table 20. The grades of the deviation of individual alternatives from ideal and anti-ideal solutions.

Experts	C_{1_1}	C_{1_2}	C_{1_3}	C_{1_4}	C_{2_1}	C_{2_2}	C_{2_3}	C_{2_4}
A_1	0.015	0.000	0.025	0.001	0.024	0.010	0.045	0.048
A_2	0.000	0.006	0.000	0.000	0.011	0.000	0.000	0.000
A_3	0.004	0.049	0.037	0.054	0.019	0.023	0.041	0.048
A_4	0.024	0.024	0.030	0.001	0.000	0.015	0.006	0.019
Experts	C_{3_1}	C_{3_2}	C_{3_3}	C_{3_4}	C_{4_1}	C_{4_2}	C_{4_3}	C_{4_4}
A_1	0.035	0.021	0.005	0.068	0.055	0.025	0.026	0.017
A_2	0.000	0.000	0.000	0.000	0.008	0.000	0.000	0.000
A_3	0.047	0.029	0.009	0.022	0.022	0.025	0.017	0.005
A_4	0.000	0.014	0.048	0.006	0.000	0.037	0.035	0.028

Table 21. Final ranking of the four strategies.

Strategies	S^+_i	S^-_i	K^+_i	K^-_i	Q_i	Rank
A_1	0.269	0.151	0.458	0.712	0.585	2
A_2	0.014	0.011	0.476	0.738	0.607	1
A_3	0.307	0.141	0.347	0.538	0.443	4
A_4	0.166	0.121	0.428	0.653	0.540	3

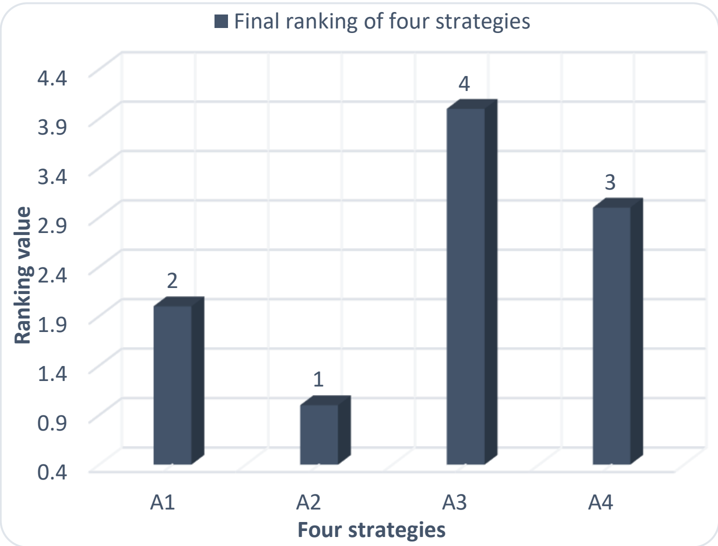

Figure 8. Final ranking of four strategies using CRADIS method.

4.3 Discussion

In this part, the results obtained from the application of the proposed methodology DEMATEL-CRADIS under the neutrosophic environment are discussed.

Initially, the four main criteria were evaluated and prioritized using the DEMATEL method. The results indicate that the human-centricity criterion is the most influential criterion with a weight of 0.269, followed by the resiliency criterion, while the sustainability criterion is the least influential with a weight of 0.212.

Also, the responsiveness criterion's sub-indicators were evaluated and prioritized using the DEMATEL method. The results indicate that the numerous varieties of products indicator is the most influential criterion with a weight of 0.285, followed by the managing demand variations indicator, while the superior service level indicator is the least influential with a weight of 0.230.

5. Conclusion

In this research, the responsive supply chain was investigated based on the Industry 5.0 dimensions, which were given the moniker responsive supply chain 5.0. Both the indicators and the alternatives have been determined. After that, the necessary data were collected, the weights of the indicators were calculated, and the alternatives were prioritized while taking into consideration a case study of the healthcare systems. In further research, several aspects of Industry 5.0, such as global supply chains and agile supply chains, might be the subject of investigation. In this respect, researchers have the opportunity to add aspects of the global supply chain, such as the capability of the supply chain to link to other supply networks located all over the world. On the other hand, there are aspects of an agile supply chain that may be modified, such as lead time flexibility and dependability. The use of MCDM in conjunction with artificial intelligence as a methodology for researching the research subject is yet another potential path for future research. Finally, researchers are able to create a supply chain network based on the pillars proposed by Industry 5.0 using the mathematical programming methods that they have proposed.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Rank and Analysis Several Solutions of Healthcare Waste to Achieve Cost Effectiveness and Sustainability Using Neutrosophic MCDM Model

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Abstract: Managing healthcare waste (HCWTT) from healthcare facilities is difficult. It's high up on the list of health concerns. This growth in HCWTT has been especially visible in recent years, as the quantity of medical services available has increased. Because of the potential danger, this garbage poses to people and the planet, it must be properly disposed of. Because of ineffective waste management practices, inadequate financial resources, and a lack of adequate facilities, HCWT administration is especially crucial in developing nations. Reducing HCWT via appropriate treatment is important for the area's financial and environmental health. In order to solve single-valued neutrosophic (SVN) group decision-making issues with missing weight details, this research creates a unique multi-criterion decision-making (MCDM) approach. First, it's important to remember that various decision-makers (DMs) have varying levels of expertise. To get over the problem that the standards cannot compensate for each other, an improved version of ELECTRE is studied. The ELECTRE method is used under neutrosophic environment to rank solutions of HCWT.

Keywords: Sustainability, Healthcare Waste Management, MCDM, Neutrosophic Set.

1. Introduction

The leadership of healthcare waste (HCWT) is an important aspect of public health since it entails concern for the environment. Medical and laboratory facilities often produce hazardous waste, or HCWT. The disposal of medical waste has evolved into a difficult and intricate issue. Healthcare facilities produce HCWTs, which have the potential to harm humans and their surroundings. Managing HCWT waste may be done in a number of ways. To properly handle this waste and lessen its impact on human health and the contamination of the surroundings, the right procedure and proper equipment for HCWT handling must be chosen[1], [2].

The gathering of waste from hospitals, choosing the delivery mode and routes to purifying facilities, the determination of treatment technological advances, and the choice of a disposal area are only few of the many steps that make up HCWT administration. Difficulties in HCWT administration include reducing waste and increasing recycling rates, limiting the release of harmful gases from incinerators, and developing new methods of burning. Since there are several HCWT solutions available, converting HCWT to municipal waste is only one option for handling the same. To this end, a variety of mathematical tools and techniques were used to determine the best approach to HCWT administration[3], [4].

Various multicriteria methods for the HCWT problem of management may be found in previously completed research, such as the fuzzy approach and a fuzzy set, interval fuzzy logic, and intuitionistic hesitant fuzzy sets[5], [6].

Smarandache first introduced the concept of a neutrosophic set (NS) with three possible memberships (truth, indeterminacy, and falsehood). When representing information that is ambiguous, partial, or inconsistent, NSs are preferred over FSs and IFSs because of their versatility and usefulness[7]–[9]. This paper ranks and analysis of HCWT solution by applying ELECTRE method under neutrosophic environment. ELECTRE is a multi-criteria decision-making approach from the family of outranking methods; it involves building an over-classification connection that takes into account the decision-makers created choices in light of the assessed criteria and the choices that are presented[10], [11].

2. Healthcare Waste Risks

When we talk about healthcare waste, we're referring to everything that is thrown away when providing medical treatment in a hospital, clinic, or private home. There are many different ways to categorize HCWT, however, the most common methods separate it into risky and nonhazardous components that account for 75–90% and 10–25%, respectively. The non-hazardous portion of healthcare waste (HCWT), referred to as general HCWT, is composed mostly of paper, plastic, glass, and food scraps and jars and is comparable to municipal solid trash[12], [13].

Risky waste, in contrast to non-hazardous trash, may pose a variety of chemical and physical dangers to the natural world and human health. Type, origin, and potential hazards during collection, transportation, storage, and disposal all contribute to the different groups into which hazardous household waste (HCWT) falls. Sharps, infectious trash, dated chemicals, medications, anatomical/pathological waste, and radioactive material are all part of this category of garbage. In particular, the expense of getting rid of hazardous trash is multiplied by 10 compared to that of regular garbage. Therefore, accurately determining the kinds and amounts of HCWT generated when measuring HCWT production rates is very important in appropriate and safe HCWT administration[14]–[16].

3. Healthcare Waste Sustainability

As among the most rapidly expanding industries worldwide, the healthcare business is also one of the most wasteful since it offers so many products and services to prevent and cure illness. Healthcare waste (HCWT) has the potential to significantly impact local ecology and public wellness. In addition, the worldwide supply of HCWT increases at a rate of 2% to 3% each year in line with the rise of the overall population index and the expansion of healthcare infrastructure. China has the fastest-growing HCWT market, with a projected volume of 2.496 million tons in 2023. As an important ecological problem, HCWTs need careful oversight and the implementation of appropriate treatment procedures prior to disposal[17]–[19].

HCWT administration is crucial for ensuring safeguards for the environment and economical sustainability since it provides the means to correctly classify, collect, transport, process, and discard of HCWT. However, there are several obstacles in the way of effective implementation of HCWT management strategies, including inadequate funding from hospital management, inexperienced personnel dealing with infectious materials, and antiquated technology and procedures for disposing of HCWT. For example, just 58% of institutions examined from 24 countries throughout the globe had sufficient mechanisms for coping with the secure removal of HCWT, based to an evaluation published by the globe Health Organization.

The past ten years have seen an explosion in study of HCWT in all its forms. HCWT administration difficulties throughout the COVID-19 pandemic have all received considerable attention in previous research. Waste reduction and the implementation of programs to prepare for recycling, composting, and restoration according to the circular economy (CE) model should be explored in the healthcare business to conserve both ecological and monetary assets without compromising the industry's top objective of providing high-quality care to patients.

Despite the fact that HCWT is a severe concern to human and environmental health owing to its infectious and dangerous properties, there is a paucity of information in the literature about how a CE model may be used to deal with HCWT. However, the COVID-19 pandemic's breakout has added another layer of complexity to the already difficult task of disposing of HCWT in an ecologically sound manner, given the prevalence of highly transmissible waste generated by both patients and healthcare personnel. In addition, more study is necessary because of the haze that surrounds a comprehensive structure of HCWT research topics and developments towards a CE transformation and ecological sustainability[20], [21].

3.1 Plastic Waste

How much plastics have improved our lives or how much they may complicate them in the future may be the most divisive topic of conversation as we enter the 21st century. Because of its cheap cost, adaptability, and resilience, plastic is one of the most important materials used in the packaging industry. Although mass manufacture of plastics began roughly 60 years ago, it has recently increased so much that 8.3 billion metric tons have been produced, the majority of which are in disposable goods that end up as rubbish. Plastic is just a long-chain polymer molecule, chemically synthesized from the repeating structural components of a single monomer. Polyethene is the molecular term for the plastic used to make things such as grocery bags, foil, and various types of toys. Up to 20,000 separate ethane molecules are linked to make each polyethylene chain under extreme conditions of heat and pressure. The enormous number of chains of polymers in such plastic makes it challenging to break down in the environment. The vast majority of polymers used in consumer goods today have their origins in fossil fuels.

It's no surprise that polymer substances have had a significant effect on our way of life, and it's not easy to dispose of plastics in an environmentally responsible manner. Most typical plastic items employ polymers that are not biodegradable and have a degradation time in moist soil of more than a century. Only 16% of plastic trash gets recycled into new plastics, despite the fact that more than 380 million tons of polymers are generated annually across the globe. To aid with the recycling procedure and various forms of recycling, the Society of the Plastics Industry (SPI) has allocated a recycling code (ranging from 1 to 7). It is crucial to create a system for managing plastic trash that has a good effect on the environment via recycling, reusing, and appropriate disposal[18], [19].

3.2 Medical Waste

However, biodegradable artificial polymers are extensively used in the healthcare and bio-sector due to their important qualities to manage the function of efficacy and rate of biodegradability. Drug administration systems, implants for surgery, spectacles, sutures, tissue manipulation, and many more medical device applications make extensive use of artificial biodegradable polymers. There are two primary sources of medical waste: hospitals and other institutional settings (such as pathology labs) and individuals (who throw away their own personal supplies of medicine).

Cytotoxic waste (waste containing materials with cytotoxic effects) make up 15% of all medical wastes, as reported by the World Health Most of these waste products are damaging to the ecosystem and the medical field because they include poisonous components and hazardous compounds. Infectious diseases may spread to both medical patients and healthcare workers if medical waste is not disposed of properly. In addition, the improper disposal of healthcare wastes is harming landfills, water, and the surrounding environment, all of which pose potential indirect health concerns. It's crucial to sort medical wastes by category before properly disposing of them, taking all necessary precautions along the way.

Various kinds of healthcare waste are often stored in containers with corresponding tiers and color codes. IV bottles, IV sets, infection dressing, aprons, and gloves are all placed in red plastic bags or containers before being autoclaved or microwaved. Solid hazardous or pathogenic items, including cotton buds, dressing substances, and anatomic or bodily tissues, should be placed in yellow boxes or plastic bags before being incinerated, plasma paralyzed or deep buried in a landfill. Ampoules, syringes, glass, scalp veins, needles, and blades that have been contaminated are thrown away in either a blue container or a white (or transparent) container, depending on the kind of contamination. The contents of the blue box are usually sterilized in a sterilizer, microwave, or hydrolase before being recycled. A sealed lead bucket marked "radioactive" is used for disposing of radioactive materials. Wrappers, food ingredients, and papers are thrown out in a black container or bag[20], [21].

3.3 Electronic Waste

E-waste, or electronic garbage, is an increasing problem in addition to the more common non-biodegradable plastic. Humanity's propensity to generate e-waste is expanding as a result of the widespread use of digital technologies, which are transforming everything from our daily routines to our sports and our health care. There has been a 21% increase in e-waste output over the last five years, and most of it is not routinely collected or reused, as reported by the United Nations Global E-waste Report 2020. The analysis estimates that in 2019, Asia produced 24.9 Mt of e-waste, the Americas produced 13.1 Mt, and Europe produced 12 Mt; Africa and Oceania produced 2.9 Mt and 0.7 Mt, respectively. Unfortunately, much of the electronic garbage produced in the developed world ends up in the landfills of nations in Africa, Asia, and Latin America. The poisonous chemicals and dangerous elements included in electronic trash, such as mercury, are harmful to human health and the environment. The World Health Organization reports that millions of individuals, mainly children and women, have their health put at risk due to the informal processing of discarded electronic trash. It is not good for children and women to gather metals and precious items from electronics dumps, which is what most underdeveloped nations do. This is a major problem[22].

4. Healthcare Waste and COVID-19

The recent spread of the pandemic COVID-19 has caused widespread alarm. Social isolation, lockdowns, border closures, protective clothing, aprons, and face shields are only some of the preventative measures that have been used in every nation throughout the world. The number of infected individuals and mortality rates continues to rise every day despite the efforts of international and local governments in most nations. By October 15, 2021, it is expected that 240 million people will have been affected and 4.90 million will have lost their lives due to the disease. Personal protective equipment (PPE) including masks, gloves, and face shields are used to prevent the spread of disease, but their usage results in an alarming amount of hospital and medical waste. Low-income nations are particularly vulnerable to the dangers posed by medical waste. Infections spread by improperly discarded medical supplies kill at least 5.2 million people per year, including 4 million children. The amount of medical waste produced as a result of this epidemic has been documented in a number of studies[23], [24].

Medical waste in China amounts to around 469 tons per day, as reported by Peng et al. A total of 12,740 tons of medical waste were produced in only 60 days after the first incidence was discovered in Indonesia. Due to their potential as a communication channel, great care must be taken while disposing of infectious wastes. Many nations' environments and populations are at risk because of the waste generated by this epidemic. Used protective equipment (PPE) plastic contamination has received widespread attention and will continue to add to the accumulation of microplastic Many poor and rising nations lack adequate regulations for the disposal of such materials, which might lead to the eventual spread of this virus.

This virus may live on pavement for up to 9 days, as shown by Kampf et al., which raises worry in many nations due to the potential for general waste contamination in the absence of waste disposal programs. In addition, there is a higher risk of transmission since recycling employees in numerous nations gather items without wearing sufficient PPE and then reuse these substances. Inadequate disposal of this garbage will increase the risk of disease transmission both during and after the pandemic. As a result, managing biomedical waste is crucial, particularly in emerging and low-income nations, to stop the spread of this epidemic[25], [26].

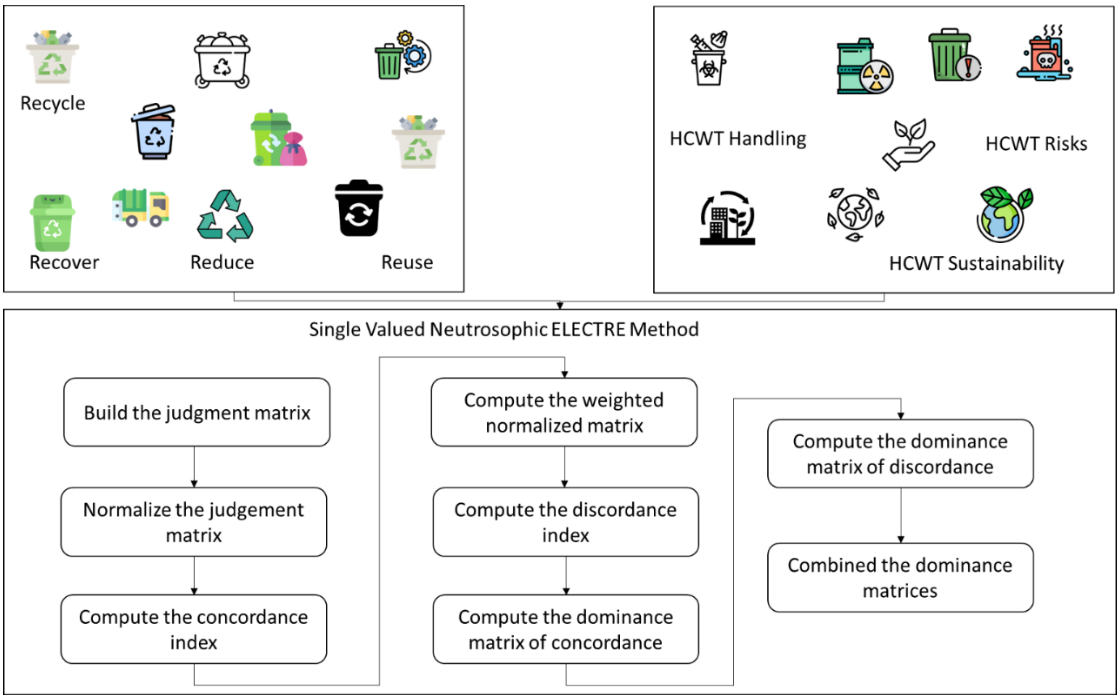

Figure 1. The combined between HCWT criteria and proposed method.

5. MCDM Methodology

The Roy-proposed ELECTRE technique is the most widely used multi-criteria analysis approach within the group of outranking techniques. Contradictory choice factors and their relative relevance may be used to build ordering connections among options, allowing for the definition of harmony and disagreement indices. As a result, the approach aids in identifying the group of solutions that are most preferable, even if they are not ideal. Similarly, to other European School MCDM approaches, the ELECTRE approach does not offer a ranking of options but rather groups them together based on outranking indices. The best alternatives from the perspective of all the criteria analyzed simultaneously are those in the kernel set, which is the set that examines other options not over-classified by any other. The number of viable options may be narrowed down in this manner. One way to categorize the other sets is by the options that are being passed up. As a result, the user's preferences may be taken into account by increasing the size of the pool of viable options by establishing the maximum amount of outranking connections[27]–[29]. Figure 1 shows the framework of the proposed method. The HCWT suitability, handling and risks are entered as an input of criteria to the proposed method, also the recycle, reuse, recover and reduce are criteria of HCWT. Also, the figure show the steps of the proposed method.

5.1) Build the judgment matrix.

5.2) Normalize the judgement matrix

$$T_{ij} = \frac{x_{ij}}{\sqrt{\sum_i x_{ij}^2}} \quad (1)$$

5.3) Compute the concordance index.

The next step of the procedure involves contrasting each set of options using a different set of criteria.

$$d_{mm} = \sum_{j|\, T_{ij}>T_{nj}} w_j + 0.5 \sum_{j|\, T_{ij}=T_{nj}} w_j \quad (2)$$

5.4) Compute the weighted normalized matrix.

$$WT_{ij} = T_{ij} * w_j \tag{3}$$

5.5) Compute the discordance index

$$S_{mn} = \frac{j| \max\limits_{T_{mj}<T_{nj}} |WT_{mj} - WT_{nj}|}{\max\limits_{j} |WT_{mj} - WT_{nj}|} \tag{4}$$

5.6) Compute the dominance matrix of concordance

$$O_{mn} = \begin{cases} 1, if\ d_{mn} > c \\ 0, if\ d_{mn} \leq c \end{cases} \tag{5}$$

5.7) Compute the dominance matrix of discordance

$$P_{mn} = \begin{cases} 1, if\ S_{mn} < d \\ 0, if\ S_{mn} \geq c \end{cases} \tag{6}$$

5.8) Combined the dominance matrices

$$L_{mn} = o_{mn}.P_{mn} \tag{7}$$

6. Results

The efficient leadership of HCWT is a critical and challenging issue for all hospitals and clinics. Multi-criteria decision-making is necessary to address this issue. The single valued neutrosophic numbers are used in the context of HCWT leadership in this research. This section provides the application of the proposed neutrosophic method under single valued neutrosophic set. This study gathered 15 criteria as a feature of HCWT as shown in Figure 2 and 10 solutions.

Table 1. Initial matrix by single valued neutrosophic numbers

	HCW_1	HCW_2	HCW_3	HCW_4	HCW_5	HCW_6	HCW_7	HCW_8	HCW_9	HCW_{10}	HCW_{11}	HCW_{12}	HCW_{13}	HCW_{14}	HCW_{15}
HCA_1	<0.2, 0.8, 0.7>	<0.65, 0.35, 0.3>	<0.3, 0.7, 0.6>	<0.2, 0.8, 0.7>	<0.3, 0.7, 0.6>	<0.2, 0.8, 0.7>	<0.8, 0.2, 0.15>	<0.2, 0.8, 0.7>	<0.9, 0.1, 0.05>	<0.3, 0.7, 0.6>	<0.2, 0.8, 0.7>	<0.3, 0.7, 0.6>	<0.3, 0.7, 0.6>	<0.2, 0.8, 0.7>	<0.9, 0.1, 0.05>
HCA_2	<0.9, 0.1, 0.05>	<0.5, 0.5, 0.45>	<0.9, 0.1, 0.05>	<0.65, 0.35, 0.3>	<0.8, 0.2, 0.15>	<0.9, 0.1, 0.05>	<0.65, 0.35, 0.3>	<0.9, 0.1, 0.05>	<0.65, 0.35, 0.3>	<0.3, 0.7, 0.6>	<0.2, 0.8, 0.7>	<0.3, 0.7, 0.6>	<0.9, 0.1, 0.05>	<0.65, 0.35, 0.3>	<0.2, 0.8, 0.7>
HCA_3	<0.2, 0.8, 0.7>	<0.65, 0.35, 0.3>	<0.8, 0.2, 0.15>	<0.5, 0.5, 0.45>	<0.8, 0.2, 0.15>	<0.2, 0.8, 0.7>	<0.65, 0.35, 0.3>	<0.3, 0.7, 0.6>	<0.65, 0.35, 0.3>	<0.8, 0.2, 0.15>	<0.8, 0.2, 0.15>	<0.5, 0.5, 0.45>	<0.8, 0.2, 0.15>	<0.3, 0.7, 0.6>	<0.3, 0.7, 0.6>
HCA_4	<0.8, 0.2, 0.15>	<0.3, 0.7, 0.6>	<0.9, 0.1, 0.05>	<0.5, 0.5, 0.45>	<0.9, 0.1, 0.05>	<0.5, 0.5, 0.45>	<0.65, 0.35, 0.3>	<0.9, 0.1, 0.05>	<0.5, 0.5, 0.45>	<0.9, 0.1, 0.05>	<0.5, 0.5, 0.45>	<0.9, 0.1, 0.05>	<0.65, 0.35, 0.3>	<0.65, 0.35, 0.3>	<0.2, 0.8, 0.7>

HCA_5	<0.2, 0.8, 0.7>	<0.65, 0.35, 0.3>	<0.5, 0.5, 0.45>	<0.2, 0.8, 0.7>	<0.5, 0.5, 0.45>	<0.3, 0.7, 0.6>	<0.5, 0.5, 0.45>	<0.2, 0.8, 0.7>	<0.65, 0.35, 0.3>	<0.3, 0.7, 0.6>	<0.8, 0.2, 0.15>	<0.5, 0.5, 0.45>	<0.3, 0.7, 0.6>	<0.65, 0.35, 0.3>	<0.3, 0.7, 0.6>
HCA_6	<0.9, 0.1, 0.05>	<0.5, 0.5, 0.45>	<0.3, 0.7, 0.6>	<0.3, 0.7, 0.6>	<0.65, 0.35, 0.3>	<0.65, 0.35, 0.3>	<0.65, 0.35, 0.3>	<0.3, 0.7, 0.6>	<0.65, 0.35, 0.3>	<0.65, 0.35, 0.3>	<0.2, 0.8, 0.7>	<0.5, 0.5, 0.45>	<0.65, 0.35, 0.3>	<0.8, 0.2, 0.15>	<0.2, 0.8, 0.7>
HCA_7	<0.2, 0.8, 0.7>	<0.65, 0.35, 0.3>	<0.65, 0.35, 0.3>	<0.2, 0.8, 0.7>	<0.65, 0.35, 0.3>	<0.2, 0.8, 0.7>	<0.65, 0.35, 0.3>	<0.2, 0.8, 0.7>	<0.65, 0.35, 0.3>	<0.3, 0.7, 0.6>	<0.8, 0.2, 0.15>	<0.5, 0.5, 0.45>	<0.3, 0.7, 0.6>	<0.3, 0.7, 0.6>	<0.2, 0.8, 0.7>
HCA_8	<0.8, 0.2, 0.15>	<0.9, 0.1, 0.05>	<0.65, 0.35, 0.3>	<0.65, 0.35, 0.3>	<0.8, 0.2, 0.15>	<0.5, 0.5, 0.45>	<0.8, 0.2, 0.15>	<0.5, 0.5, 0.45>	<0.8, 0.2, 0.15>	<0.8, 0.2, 0.15>	<0.2, 0.8, 0.7>	<0.65, 0.35, 0.3>	<0.3, 0.7, 0.6>	<0.3, 0.7, 0.6>	<0.3, 0.7, 0.6>
HCA_9	<0.8, 0.2, 0.15>	<0.3, 0.7, 0.6>	<0.8, 0.2, 0.15>	<0.2, 0.8, 0.7>	<0.5, 0.5, 0.45>	<0.2, 0.8, 0.7>	<0.8, 0.2, 0.15>	<0.2, 0.8, 0.7>	<0.8, 0.2, 0.15>	<0.8, 0.2, 0.15>	<0.3, 0.7, 0.6>	<0.8, 0.2, 0.15>	<0.8, 0.2, 0.15>	<0.2, 0.8, 0.7>	<0.8, 0.2, 0.15>
HCA_{10}	<0.2, 0.8, 0.7>	<0.3, 0.7, 0.6>	<0.9, 0.1, 0.05>	<0.3, 0.7, 0.6>	<0.2, 0.8, 0.7>	<0.9, 0.1, 0.05>	<0.3, 0.7, 0.6>	<0.2, 0.8, 0.7>	<0.9, 0.1, 0.05>	<0.3, 0.7, 0.6>	<0.2, 0.8, 0.7>	<0.9, 0.1, 0.05>	<0.2, 0.8, 0.7>	<0.9, 0.1, 0.05>	<0.2, 0.8, 0.7>

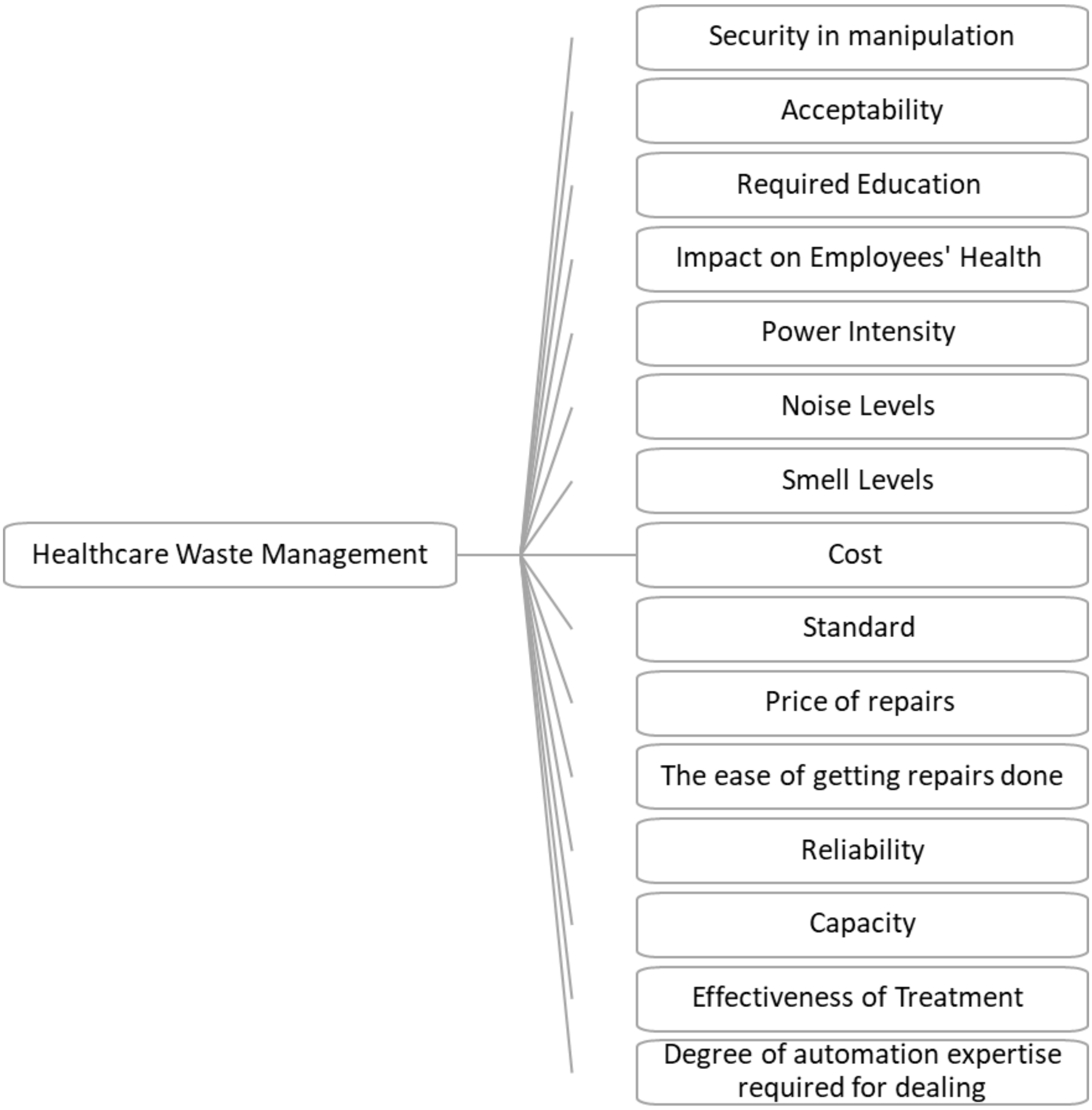

Figure 2. The healthcare waste criteria.

Then apply the steps of the proposed method using single valued neutrosophic numbers as shown in Table 1. Then let experts to evaluate the criteria and alternatives. Then apply the proposed method. Then normalize the dataset to ensure all data in the same range as shown in Table 2 using Eq. (1). Then compute the weights of criteria, and multiply it by the normalization decision matrix as shown in Table 3 using Eq. (3).

Table 2. Normalization decision matrix

	HCW_1	HCW_2	HCW_3	HCW_4	HCW_5	HCW_6	HCW_7	HCW_8	HCW_9	HCW_{10}	HCW_{11}	HCW_{12}	HCW_{13}	HCW_{14}	HCW_{15}
HCA_1	0.042813	0.118694	0.048309	0.058824	0.05291	0.048611	0.123116	0.055777	0.125285	0.05848	0.052239	0.055096	0.061162	0.044872	0.235043

HCA_2	0.168196	0.091988	0.13285	0.168067	0.12963	0.190972	0.100503	0.219124	0.091116	0.05848	0.052239	0.055096	0.168196	0.128205	0.059829
HCA_3	0.042813	0.118694	0.118357	0.130252	0.12963	0.048611	0.100503	0.079681	0.091116	0.143275	0.182836	0.085399	0.149847	0.064103	0.08547
HCA_4	0.149847	0.059347	0.13285	0.130252	0.145503	0.107639	0.100503	0.219124	0.070615	0.160819	0.115672	0.151515	0.122324	0.128205	0.059829
HCA_5	0.042813	0.118694	0.074879	0.058824	0.082011	0.069444	0.077889	0.055777	0.091116	0.05848	0.182836	0.085399	0.061162	0.128205	0.08547
HCA_6	0.168196	0.091988	0.048309	0.084034	0.10582	0.138889	0.100503	0.079681	0.091116	0.116959	0.052239	0.085399	0.122324	0.157051	0.059829
HCA_7	0.042813	0.118694	0.096618	0.058824	0.10582	0.048611	0.100503	0.055777	0.091116	0.05848	0.182836	0.085399	0.061162	0.064103	0.059829
HCA_8	0.149847	0.163205	0.096618	0.168067	0.12963	0.107639	0.123116	0.123506	0.111617	0.143275	0.052239	0.110193	0.061162	0.064103	0.08547
HCA_9	0.149847	0.059347	0.118357	0.058824	0.082011	0.048611	0.123116	0.055777	0.111617	0.143275	0.074627	0.134986	0.149847	0.044872	0.209402
HCA_{10}	0.042813	0.059347	0.13285	0.084034	0.037037	0.190972	0.050251	0.055777	0.125285	0.05848	0.052239	0.151515	0.042813	0.176282	0.059829

Table 3. weighted normalization decision matrix

	HCW_1	HCW_2	HCW_3	HCW_4	HCW_5	HCW_6	HCW_7	HCW_8	HCW_9	HCW_{10}	HCW_{11}	HCW_{12}	HCW_{13}	HCW_{14}	HCW_{15}
HCA_1	0.763936	1.583661	0.576983	1.024913	0.664066	0.867386	1.413443	0.928153	1.37442	0.973126	1.004749	1.149265	0.730492	0.898042	3.911217
HCA_2	3.001179	1.227337	1.586703	2.928324	1.626962	3.407589	1.153831	3.646314	0.999578	0.973126	1.004749	1.149265	2.008854	2.565835	0.995582
HCA_3	0.763936	1.583661	1.413608	2.269451	1.626962	0.867386	1.153831	1.325932	0.999578	2.384158	3.516621	1.78136	1.789706	1.282918	1.422261
HCA_4	2.673778	0.79183	1.586703	2.269451	1.826182	1.920641	1.153831	3.646314	0.774673	2.676096	2.224801	3.160478	1.460985	2.565835	0.995582
HCA_5	0.763936	1.583661	0.894324	1.024913	1.029302	1.239123	0.894219	0.928153	0.999578	0.973126	3.516621	1.78136	0.730492	2.565835	1.422261
HCA_6	3.001179	1.227337	0.576983	1.464162	1.328132	2.478246	1.153831	1.325932	0.999578	1.946251	1.004749	1.78136	1.460985	3.143148	0.995582
HCA_7	0.763936	1.583661	1.153966	1.024913	1.328132	0.867386	1.153831	0.928153	0.999578	0.973126	3.516621	1.78136	0.730492	1.282918	0.995582
HCA_8	2.673778	2.177534	1.153966	2.928324	1.626962	1.920641	1.413443	2.055195	1.224483	2.384158	1.004749	2.298529	0.730492	1.282918	1.422261

HC A_9	2.67 3778	0.79 183	1.41 3608	1.02 4913	1.02 9302	0.86 7386	1.41 3443	0.92 8153	1.22 4483	2.38 4158	1.43 5356	2.81 5699	1.78 9706	0.89 8042	3.48 4539
HC A_{10}	0.76 3936	0.79 183	1.58 6703	1.46 4162	0.46 4846	3.40 7589	0.57 6916	0.92 8153	1.37 442	0.97 3126	1.00 4749	3.16 0478	0.51 1345	3.52 8023	0.99 5582

Then compute the concordance index using Eq. (2). Then compute the discordance index using Eq. (4). Then compute the dominance matrix of concordance using Eq. (5). Then compute the dominance matrix of discordance using Eq. (6). Then compute the dominance matrices using Eq. (7) to obtain the final score as shown in Figure 3.

The second solution is the best followed by third, fourth, and tenth solutions. The seventh solution is the worst and least importance.

Figure 3. Values of combined the dominance matrices.

7. Conclusion

An essential part of the healthcare infrastructure is the problem of how to best manage healthcare workers. Hazardous healthcare waste (HCWT) is produced by healthcare facilities and may be harmful to humans and ecosystems. The risks associated with HCWT, however, must be mitigated to the greatest degree feasible by appropriate treatment. This research addresses the issue of acquiring a new HCWT sterilization plant to convert the same into utility trash, one of several potential approaches to dealing with HCWT. Choosing a new building is an example of a typical decision-making issue that may be solved using MCDM techniques. This paper applied the ELECTRE method under single valued neutrosophic set. The single valued neutrosophic set is used to overcome the uncertain data. Then the ELECTRE method is applied into single valued neutrosophic numbers. This study used 15 criteria and 10 solutions to give best solution in HCWT. The main results show the second solution is the best followed by third, fourth, and tenth solutions. The seventh solution is the worst and least importance.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Separation Axioms in Neutrosophic Topological Spaces

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Abstract: In this article, we first establish some results based on single-valued neutrosophic sets. Next, we define a subspace topology in a neutrosophic topological space and investigate some properties. We then define the neutrosophic T_0, T_1, T_2-spaces and study their various properties offering adequate examples.

Keywords: Neutrosophic subspace; Neutrosophic T_0-space; Neutrosophic T_1-space; Neutrosophic T_2 -space; Neutrosophic Compact set; Neutrosophic continuous function; Neutrosophic homeomorphism.

1. Introduction

Zadeh [36] uncovered the concept of a fuzzy set in 1965, and Atanassov [1] introduced the intuitionistic fuzzy set, a generalized version of a fuzzy set, in 1986. After a decade, Florentin Smarandache [26-28] developed and studied a new branch of philosophy called "Neutrosophy". Smarandache [28] demonstrated that a neutrosophic set is a generalization of an intuitionistic fuzzy set. Just like an intuitionistic fuzzy set, a neutrosophic set assigns degrees of membership and non-membership to its elements. However, it incorporates an additional measure called the degree of indeterminacy to determine the level of membership. In a neutrosophic set, all three neutrosophic components are independent of one another, which is an important characteristic of the neutrosophic set.

After Smarandache had brought the thought of neutrosophy, it was studied and taken ahead by many researchers [6, 30, 34, 35]. Due to its flexibility and effectiveness, neutrosophy is attracting researchers from various fields around the world, and it has proven to be useful not only in the development of science and technology but also in other areas. For example, Abdel-Basset et al. [3, 4] studied the applications of neutrosophic theory in several scientific fields, while Pramanik and Roy [23] analyzed the conflict between India and Pakistan over Jammu-Kashmir using neutrosophic game theory. Furthermore, researchers have applied neutrosophic theory to medical diagnosis [5, 15], decision-making problems [13, 22], image processing [16], and many other fields.

In 2002, Smarandache [27] introduced the concept of neutrosophic topology on the non-standard interval, and Lupiáñez [18-20] subsequently investigated many properties of neutrosophic topological spaces. In 2012, Salama & Alblowi[29] revealed the idea of neutrosophic topological space as an extension of intuitionistic fuzzy topological space developed by D.Coker [10] in 1997. Salama et al.[32] later introduced the concept of neutrosophic continuous functions. In 2016, Karatas and Kuru[17] redefined single-valued neutrosophic set operations and examined important properties associated with neutrosophic topological spaces. Subsequently, various notions related to neutrosophic topological spaces were developed by numerous researchers [2, 11, 12, 14, 24, 25, 30, 31, 33]. For instance, Al-Nafee et al. [8] utilized neutrosophic crisp points to construct separation axioms in neutrosophic crisp topological spaces and examined the relationships between them. In 2020, Ahu and Ferhat [7] introduced the concept of neutrosophic pre-separation axioms in neutrosophic soft topological spaces and explored the connections among these separation axioms. Additionally, A. Mehmood et al. [21] developed and studied the neutrosophic soft p-separation axioms in neutrosophic soft topological spaces, while V. Amarendra Babu and J. Aswini [9] investigated separation axioms in supra neutrosophic crisp topological spaces in 2021.

The primary objective of this article is to define and explore the separation axioms in neutrosophic topological spaces. Prior to that, we shall first investigate some properties of single-valued neutrosophic sets. Additionally, we shall define the subspace topology (relative topology) in a neutrosophic topological space and examine a few properties.

The article is organized by conferring some basic notions in section 2. In section 3, we establish some results in connection with single-valued neutrosophic sets. We then define neutrosophic subspace with example and investigate some properties. In section 4, we define neutrosophic T_0, T_1, T_2-spaces and study various properties. In section 5, we confer a conclusion.

2. Preliminaries

2.1. Definition: [26] Let X be the universe of discourse. A neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where the functions $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are real standard or non-standard subsets of $]^-0, 1^+[$, i.e., $\mathcal{T}_A : X \to \,]^-0, 1^+[$, $\mathcal{I}_A : X \to \,]^-0, 1^+[$, $\mathcal{F}_A : X \to \,]^-0, 1^+[$ and $^-0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3^+$.

The neutrosophic set A is characterized by the truth-membership function $\mathcal{T}_A$, indeterminacy-membership function $\mathcal{I}_A$, falsehood-membership function $\mathcal{F}_A$.

2.2. Definition: [35] Let X be the universe of discourse. A single-valued neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are functions from X to $[0,1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.

The set of all single-valued neutrosophic sets over X is denoted by $\mathcal{N}(X)$.

Throughout this article, a neutrosophic set (NS, for short) will mean a single-valued neutrosophic set.

2.3. Definition: [17] Let $A, B \in \mathcal{N}(X)$. Then

(i) (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.

(ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

(iii) (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x)\rangle : x \in X\}$.

(iv) (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x)\rangle : x \in X\}$.

(v) (Complement): The complement of the NS A, denoted by A^c, is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x)\rangle : x \in X\}$.

(vi) (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

(vii) (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

2.4. Definition: [29] Let $\{A_i : i \in \Delta\} \subseteq \mathcal{N}(X)$, where Δ is an index set. Then

(i) $\cup_{i\in\Delta} A_i = \{\langle x, \vee_{i\in\Delta} \mathcal{T}_{A_i}(x), \wedge_{i\in\Delta} \mathcal{I}_{A_i}(x), \wedge_{i\in\Delta} \mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

(ii) $\cap_{i\in\Delta} A_i = \{\langle x, \wedge_{i\in\Delta} \mathcal{T}_{A_i}(x), \vee_{i\in\Delta} \mathcal{I}_{A_i}(x), \vee_{i\in\Delta} \mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

2.5. Definition:[17] Let $\tau \subseteq \mathcal{N}(X)$. Then τ is called a neutrosophic topology on X if

(i) $\widetilde{\emptyset}$ and $\tilde{X}$ belong to τ.

(ii) Arbitrary union of neutrosophic sets in τ is in τ.

(iii) Intersection of any two neutrosophic sets in τ is in τ.

If τ is a neutrosophic topology on X then the pair (X, τ) is called a neutrosophic topological space (NTS, for short) over X. The members of τ are called neutrosophic τ-open sets (neutrosophic open sets or open sets, for short) in X. If for an NS A, $A^c \in \tau$ then A is said to be a neutrosophic τ-closed set (neutrosophic closed set or closed set, for short) in X.

2.6. Definition: [24] Let $\mathcal{N}(X)$ be the set of all neutrosophic sets over X. An NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic point (NP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = \alpha, \mathcal{I}_P(y) = \beta, \mathcal{F}_P(y) = \gamma$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$. A neutrosophic point $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ will be denoted by $x_{\alpha,\beta,\gamma}$. For the NP $x_{\alpha,\beta,\gamma}$, x will be called its support. The complement of the NP $x_{\alpha,\beta,\gamma}$ will be denoted by $(x_{\alpha,\beta,\gamma})^c$. An NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic crisp point (NCP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = 1, \mathcal{I}_P(y) = 0, \mathcal{F}_P(y) = 0$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$.

2.7. Definition: [32] Let X and Y be two non-empty sets and $f : X \to Y$ be a function. Also let $A \in \mathcal{N}(X)$ and $B \in \mathcal{N}(Y)$. Then

(i) Image of A under f is defined by $f(A) = \{\langle y, f(\mathcal{T}_A)(y), f(\mathcal{I}_A)(y), (1 - f(1 - \mathcal{F}_A))(y)\rangle : y \in Y\}$,

where $f(\mathcal{T}_A)(y) = \begin{cases} \sup\{\mathcal{T}_A(x) : x \in f^{-1}(y)\} & if \;\; f^{-1}(y) \neq \emptyset \\ 0 & if \;\; f^{-1}(y) = \emptyset \end{cases}$

$$f(\mathcal{I}_A)(y) = \begin{cases} \inf\{\mathcal{I}_A(x) : x \in f^{-1}(y)\} & if \;\; f^{-1}(y) \neq \emptyset \\ 1 & if \;\; f^{-1}(y) = \emptyset \end{cases}$$

$$(1 - f(1 - \mathcal{F}_A))(y) = \begin{cases} \inf\{\mathcal{F}_A(x) : x \in f^{-1}(y)\} & if \;\; f^{-1}(y) \neq \emptyset \\ 1 & if \;\; f^{-1}(y) = \emptyset \end{cases}$$

(ii) Pre-image of B under f is defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mathcal{T}_B)(x), f^{-1}(\mathcal{I}_B)(x), f^{-1}(\mathcal{F}_B)(x)\rangle : x \in X\}$

2.8. Theorem: [32] Let $f : X \to Y$ be a function. Also let $A, A_i \in \mathcal{N}(X), i \in I$ and $B, B_j \in \mathcal{N}(Y), j \in J$. Then the following hold.

(i) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$, $B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.

(ii) $A \subseteq f^{-1}(f(A))$ and if f is injective then $A = f^{-1}(f(A))$.

(iii) $f^{-1}(f(B)) \subseteq B$ and if f is surjective then $f^{-1}(f(B)) = B$.

(iv) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$ and $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$.

(v) $f(\cup A_i) = \cup f(A_i)$, $f(\cap A_i) \subseteq \cap f(A_i)$ and if f is injective then $f(\cap A_i) = \cap f(A_i)$.

(vi) $f^{-1}(\widetilde{\emptyset}_Y) = \widetilde{\emptyset}_X$, $f^{-1}(\tilde{Y}) = \tilde{X}$.

(vii) $f(\widetilde{\emptyset}_X) = \widetilde{\emptyset}_Y$, $f(\tilde{X}) = \tilde{Y}$ if f is surjective.

2.9. Definition: [33] Let f be a function from an NTS (X, τ) to another NTS (Y, σ). Then

(i) f is called a neutrosophic continuous function if $f^{-1}(G) \in \tau$ for all $G \in \sigma$ then

(ii) f is called a neutrosophic open function if $f(G) \in \sigma$ for all $G \in \tau$.

(iii) f is called a neutrosophic closed function if $f(G)$ is a neutrosophic closed set in Y for every neutrosophic closed set G in X.

(iv) f is called a neutrosophic homeomorphism if the following three conditions hold:

a. f is a bijective function.

b. f is a neutrosophic continuous function.

c. f^{-1} is a neutrosophic continuous function.

2.10. Definition:[24] Let (X, τ) be a neutrosophic topological space. An NS $A \in \mathcal{N}(X)$ is called a neutrosophic neighbourhood or simply neighbourhood (nhbd for short) of an NP $x_{\alpha,\beta,\gamma}$ iff there exists an NS $B \in \tau$ such that $x_{\alpha,\beta,\gamma} \in B \subseteq A$.

2.11. Definition: [14] Let (X, τ) be a neutrosophic topological space. A subcollection $\mathcal{B}$ of τ is called a base for τ iff for each $A \in \tau$, there exists a subcollection $\{A_i : i \in \Delta\} \subseteq \mathcal{B}$ such that $A = \cup \{A_i : i \in \Delta\}$, where Δ is an index set.

2.12. Definition:[14] Let (X, τ) be a neutrosophic topological space and $A \in \mathcal{N}(X)$. A collection $C = \{G_\lambda : \lambda \in \Delta\}$ of neutrosophic open sets of X is called a neutrosophic open cover (NOC, in short) of A if $A \subseteq \cup_{\lambda \in \Delta} G_\lambda$. We then say C covers A. In particular, C is said to be a NOC of X iff $\tilde{X} = \cup_{\lambda \in \Delta} G_\lambda$.

Let C be a NOC of the NS A and $C' \subseteq C$. Then C' is called a neutrosophic open subcover (NOSC, in short) of C if C' covers A.

2.13. Definition:[14] An NS A in an NTS (X, τ) is said to be neutrosophic compact set iff every NOC of A has a finite NOSC. In particular, the space X is said to be neutrosophic compact space iff every NOC of X has a finite NOSC.

3. Neutrosophic Subspaces

In this section we try to establish some results related to single-valued neutrosophic sets. After that, we define neutrosophic subspace with example and then investigate some properties.

3.1. Definition: Let X, Y be two crisp sets such that $Y \neq \emptyset$ and $Y \subseteq X$. We define $\tilde{Y} = \{\langle x, \alpha, \beta, \gamma \rangle : x \in X\}$, where $\alpha = 1, \beta = 0, \gamma = 0$ if $x \in Y$ and $\alpha = 0, \beta = 1, \gamma = 1$ if $x \in X \backslash Y$. The set of all single-valued neutrosophic sets over Y will be denoted by $\mathcal{N}(Y)$.

3.2. Definition: Let X, Y be two crisp sets such that $Y \neq \emptyset$ and $Y \subseteq X$. Then for an NS $A \in \mathcal{N}(X)$, we define $A|_Y = \{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x) \rangle : x \in X\}$, where $\mathcal{T}_{A|_Y}(x) = \mathcal{T}_A(x)$, $\mathcal{I}_{A|_Y}(x) = \mathcal{I}_A(x)$, $\mathcal{F}_{A|_Y}(x) = \mathcal{F}_A(x)$ if $x \in Y$ and $\mathcal{T}_{A|_Y}(x) = 0$, $\mathcal{I}_{A|_Y}(x) = 1$, $\mathcal{F}_{A|_Y}(x) = 1$ if $x \in X \backslash Y$.

3.3. Remark: From the definitions 3.1 and 3.2, it is clear that

1. $A|_Y \in \mathcal{N}(Y)$ for every $A \in \mathcal{N}(X)$.
2. Every NS A over Y can be considered as an NS over X by taking $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X \backslash Y$.
3. $\tilde{X}|_Y = \tilde{Y}$ and $\tilde{\emptyset}|_Y = \tilde{\emptyset}$.

3.4. Proposition: Let X, Y, Z be three sets such that $\emptyset \neq Z \subseteq Y \subseteq X$. Let $A \in \mathcal{N}(X)$ and $\{A_\lambda: \lambda \in \Delta\} \subseteq \mathcal{N}(X)$, where Δ is an index set. Then

(i) $(\cup_{\lambda\in\Delta} A_\lambda)|_Y = \cup_{\lambda\in\Delta} (A_\lambda|_Y)$.

(ii) $(\cap_{\lambda\in\Delta} A_\lambda)|_Y = \cap_{\lambda\in\Delta} (A_\lambda|_Y)$.

(iii) $A^c|_Y = (A|_Y)^c$.

(iv) $(A|_Y)|_Z = A|_Z$.

Proofs:

(i)
$$\begin{aligned}(\cup_{\lambda\in\Delta} A_\lambda)|_Y &= \{\langle x, \mathcal{T}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in X\} \\
&= \{\langle x, \mathcal{T}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in Y\} \cup \\
&\qquad \{\langle x, \mathcal{T}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in X\backslash Y\} \\
&= \{\langle x, \mathcal{T}_{\cup_{\lambda\in\Delta}A_\lambda}(x), \mathcal{I}_{\cup_{\lambda\in\Delta}A_\lambda}(x), \mathcal{F}_{\cup_{\lambda\in\Delta}A_\lambda}(x)\rangle : x \in Y\} \cup \\
&\qquad \{\langle x, 0,1,1\rangle : x \in X\backslash Y\} \\
&= \{\langle x, \vee_{\lambda\in\Delta} \mathcal{T}_{A_\lambda}(x), \wedge_{\lambda\in\Delta} \mathcal{I}_{A_\lambda}(x), \wedge_{\lambda\in\Delta} \mathcal{F}_{A_\lambda}(x)\rangle : x \in Y\} \\
&= \{\langle x, \vee_{\lambda\in\Delta} \mathcal{T}_{A_\lambda|_Y}(x), \wedge_{\lambda\in\Delta} \mathcal{I}_{A_\lambda|_Y}(x), \wedge_{\lambda\in\Delta} \mathcal{F}_{A_\lambda|_Y}(x)\rangle : x \in Y\} \\
&= \cup_{\lambda\in\Delta} [\{\langle x, \mathcal{T}_{A_\lambda|_Y}(x), \mathcal{I}_{A_\lambda|_Y}(x), \mathcal{F}_{A_\lambda|_Y}(x)\rangle : x \in Y\} \cup \{\langle x, 0,1,1\rangle : x \in X\backslash Y\}] \\
&= \cup_{\lambda\in\Delta} (A_\lambda|_Y)\end{aligned}$$

(ii)
$$\begin{aligned}(\cap_{\lambda\in\Delta} A_\lambda)|_Y &= \{\langle x, \mathcal{T}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in X\} \\
&= \{\langle x, \mathcal{T}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in Y\} \cup \\
&\qquad \{\langle x, \mathcal{T}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in X\backslash Y\} \\
&= \{\langle x, \mathcal{T}_{\cap_{\lambda\in\Delta}A_\lambda}(x), \mathcal{I}_{\cap_{\lambda\in\Delta}A_\lambda}(x), \mathcal{F}_{\cap_{\lambda\in\Delta}A_\lambda}(x)\rangle : x \in Y\} \cup \\
&\qquad \{\langle x, 0,1,1\rangle : x \in X\backslash Y\} \\
&= \{\langle x, \wedge_{\lambda\in\Delta} \mathcal{T}_{A_\lambda}(x), \vee_{\lambda\in\Delta} \mathcal{I}_{A_\lambda}(x), \vee_{\lambda\in\Delta} \mathcal{F}_{A_\lambda}(x)\rangle : x \in Y\} \\
&= \{\langle x, \wedge_{\lambda\in\Delta} \mathcal{T}_{A_\lambda|_Y}(x), \vee_{\lambda\in\Delta} \mathcal{I}_{A_\lambda|_Y}(x), \vee_{\lambda\in\Delta} \mathcal{F}_{A_\lambda|_Y}(x)\rangle : x \in Y\} \\
&= \cap_{\lambda\in\Delta} [\{\langle x, \mathcal{T}_{A_\lambda|_Y}(x), \mathcal{I}_{A_\lambda|_Y}(x), \mathcal{F}_{A_\lambda|_Y}(x)\rangle : x \in Y\} \cup \{\langle x, 0,1,1\rangle : x \in X\backslash Y\}] \\
&= \cap_{\lambda\in\Delta} (A_\lambda|_Y)\end{aligned}$$

(iii)
$$\begin{aligned}A^c|_Y &= \{\langle x, \mathcal{T}_{A^c|_Y}(x), \mathcal{I}_{A^c|_Y}(x), \mathcal{F}_{A^c|_Y}(x)\rangle : x \in X\} \\
&= \{\langle x, \mathcal{T}_{A^c}(x), \mathcal{I}_{A^c}(x), \mathcal{F}_{A^c}(x)\rangle : x \in Y\} \cup \{\langle x, 0,1,1\rangle : x \in X\backslash Y\} \\
&= \{\langle x, \mathcal{T}_{A^c}(x), \mathcal{I}_{A^c}(x), \mathcal{F}_{A^c}(x)\rangle : x \in Y\} \\
&= \{\langle x, \mathcal{T}_{A}(x), \mathcal{I}_{A}(x), \mathcal{F}_{A}(x)\rangle : x \in Y\}^c \\
&= \{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x)\rangle : x \in Y\}^c \\
&= (\{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x)\rangle : x \in Y\} \cup \{\langle x, 0,1,1\rangle : x \in Y\})^c \\
&= \{\langle x, \mathcal{T}_{(A|_Y)^c}(x), \mathcal{I}_{(A|_Y)^c}(x), \mathcal{F}_{(A|_Y)^c}(x)\rangle : x \in X\} \\
&= (A|_Y)^c\end{aligned}$$

(iv)
$$\begin{aligned}(A|_Y)|_Z &= \{\langle x, \mathcal{T}_{(A|_Y)|_Z}(x), \mathcal{I}_{(A|_Y)|_Z}(x), \mathcal{F}_{(A|_Y)|_Z}(x)\rangle : x \in X\} \\
&= \{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x)\rangle : x \in Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Z\} \\
&= \{\langle x, \mathcal{T}_{A}(x), \mathcal{I}_{A}(x), \mathcal{F}_{A}(x)\rangle : x \in Y \cap Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Y \cap Z\}\end{aligned}$$

$= \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Z\}$
$= \{\langle x, \mathcal{T}_{A|_Z}(x), \mathcal{I}_{A|_Z}(x), \mathcal{F}_{A|_Z}(x)\rangle : x \in Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Z\}$
$= \{\langle x, \mathcal{T}_{A|_Z}(x), \mathcal{I}_{A|_Z}(x), \mathcal{F}_{A|_Z}(x)\rangle : x \in X\}$
$= A|_Z$

***3.5. Proposition*:** Let Y, Z be two non-empty subsets of X and let $A \in \mathcal{N}(X)$. Then $A|_{(Y\cap Z)} = (A|_Y) \cap (A|_Z)$.

Proof:

$$
\begin{aligned}
A|_{(Y\cap Z)} &= \{\langle x, \mathcal{T}_{A|_{(Y\cap Z)}}(x), \mathcal{I}_{A|_{(Y\cap Z)}}(x), \mathcal{F}_{A|_{(Y\cap Z)}}(x)\rangle : x \in X\} \\
&= \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in Y\cap Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Y\cap Z\} \\
&= \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in Y\cap Z\} \\
&= \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in Y\} \cap \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in Z\} \\
&= [\{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x)\rangle : x \in Y\} \cup \{\langle x, 0,1,1\rangle : x \notin Y\}] \cap \\
&\quad [\{\langle x, \mathcal{T}_{A|_Z}(x), \mathcal{I}_{A|_Z}(x), \mathcal{F}_{A|_Z}(x)\rangle : x \in Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Z\}] \\
&= \{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x)\rangle : x \in X\} \cap \{\langle x, \mathcal{T}_{A|_Z}(x), \mathcal{I}_{A|_Z}(x), \mathcal{F}_{A|_Z}(x)\rangle : x \in X\} \\
&= (A|_Y) \cap (A|_Z)
\end{aligned}
$$

3.6. Proposition: Let (X, τ) be an NTS. Let $\emptyset \neq Y \subseteq X$ and $\tau|_Y = \{G|_Y : G \in \tau\}$. Then $(Y, \tau|_Y)$ is an NTS.

Proof:

1. $\tilde{X}, \tilde{\emptyset} \in \tau \Rightarrow \tilde{X}|_Y, \tilde{\emptyset}|_Y \in \tau|_Y$. As $\tilde{Y} = \tilde{X}|_Y$ and $\tilde{\emptyset} = \tilde{\emptyset}|_Y$, so $\tilde{Y}, \tilde{\emptyset} \in \tau|_Y$.
2. Let $\{G_i : i \in \Delta\} \subseteq \tau|_Y$ Then for each $i \in \Delta$, $G_i = G_i'|_Y$ for some $G_i' \in \tau$. Now $\cup_{i\in\Delta} G_i = \cup_{i\in\Delta} (G_i'|_Y) == (\cup_{i\in\Delta} G_i')|_Y \in \tau|_Y$ [$\because \cup_{i\in\Delta} G_i' \in \tau$ and by 3.4(i)].
3. Let $G, H \in \tau|_Y$. Then $G = G'|_Y$ and $H = H'|_Y$ for some $G', H' \in \tau$. Now $G \cap H = (G'|_Y) \cap (H'|_Y) = (G' \cap H')|_Y \in \tau|_Y$ [$\because G' \cap H' \in \tau$ and by 3.4(ii)]

   Hence $(Y, \tau|_Y)$ is an NTS.

3.7. Definition: Let (X, τ) be an NTS. Let $\emptyset \neq Y \subseteq X$ and $\tau|_Y = \{G|_Y : G \in \tau\}$. Then $(Y, \tau|_Y)$ [by 3.6] is an NTS. The topology $\tau|_Y$ is called the neutrosophic relative topology of τ on Y or the neutrosophic subspace topology of Y and the NTS $(Y, \tau|_Y)$ is called a neutrosophic subspace (or a subspace, for short) of the NTS (X, τ).

Members of $\tau|_Y$ are called $\tau|_Y$-open sets in Y. An NS $A \in \mathcal{N}(Y)$ such that $A^c \in \tau|_Y$ is called a $\tau|_Y$-closed set in Y.

$(Y, \tau|_Y)$ is called a neutrosophic open subspace or neutrosophic closed subspace of (X, τ) according as $\tilde{Y} \in \tau$ or $\tilde{Y} \in \tau^c$.

3.8. Example: Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B, A\cap B, A\cup B\}$, where $A = \{\langle a, 0.5,0.4,0.2\rangle, \langle b, 0.6,0.3,0.5\rangle\}$ and $B = \{\langle a, 0.3,0.4,0.6\rangle, \langle b, 0.4,0.7,0.3\rangle\}$. Clearly (X, τ) is an NTS. Let $Y = \{a\}$. Then $\tilde{X}|_Y = \{\langle a, 1,0,0\rangle, \langle b, 0,1,1\rangle\} = \tilde{Y}$, $\tilde{\emptyset}|_Y = \{\langle a, 0,1,1\rangle, \langle b, 0,1,1\rangle\} = \tilde{\emptyset}$, $A|_Y =$

$\{\langle a, 0.5,0.4,0.2\rangle, \langle b, 0,1,1\rangle\}$, $B|_Y = \{\langle a, 0.3,0.4,0.6\rangle, \langle b, 0,1,1\rangle\}$, $(A \cap B)|_Y = \{\langle a, 0.3,0.4,0.6\rangle, \langle b, 0,1,1\rangle\}$, $(A \cup B)|_Y = \{\langle a, 0.5,0.4,0.2\rangle, \langle b, 0,1,1\rangle\}$.

Clearly $\tau|_Y = \{\widetilde{\emptyset}, \widetilde{Y}, A|_Y, B|_Y, (A \cap B)|_Y, (A \cup B)|_Y\}$ is a neutrosophic subspace topology of Y, i.e., $(Y, \tau|_Y)$ is a neutrosophic subspace of (X, τ).

3.9. Proposition: Let (Y, σ) be a subspace of an NTS (X, τ) and (Z, μ) be a subspace of (Y, σ). Then (Z, μ) is a subspace of (X, τ).

Proof: Since $Z \subseteq Y \subseteq X$, so $Z \subseteq X$. We need to show that $\tau|_Z = \mu$. Let $G \in \mu$. Since (Z, μ) is a subspace of (Y, σ), so there exists $H \in \sigma$ such that $G = H|_Z$. Again since (Y, σ) is a subspace of (X, τ), so there exists $K \in \tau$ such that $H = K|_Y$. Then $G = H|_Z = (K|_Y)|_Z = K|_Z$ [by 3.4(iv)]. Since $K|_Z \in \tau|_Z$, so $G \in \tau|_Z$. Therefore $\mu \subseteq \tau|_Z$. Next suppose that $U \in \tau|_Z$. Then there exists $V \in \tau$ such that $U = V|_Z$. Since (Y, σ) is a subspace of (X, τ), so $V|_Y \in \sigma$. Again since (Z, μ) is a subspace of (Y, σ), so $(V|_Y)|_Z \in \mu \Rightarrow V|_Z \in \mu \Rightarrow U \in \mu$. Therefore $\tau|_Z \subseteq \mu$. Hence $\tau|_Z = \mu$, i.e., (Z, μ) is a subspace of (X, τ).

3.10. Proposition: Let Y and Z be two subspaces of an NTS (X, τ). If $Y \subseteq Z$ then Y is a subspace of Z.

Proof: Let (Y, σ) and (Z, μ) be the subspaces of the NTS (X, τ). Then $\tau|_Y = \sigma$ and $\tau|_Z = \mu$. Now $\mu|_Y = \{A|_Y : A \in \mu\} = \{(B|_Z)|_Y : B \in \tau$ and $B|_Z = A \in \mu\} = \{B|_Y : B \in \tau\} = \tau|_Y = \sigma$. Since $\mu|_Y = \sigma$, so Y is a subspace of Z.

3.11. Proposition: Let $(Y, \tau|_Y)$ be a subspace of an NTS (X, τ) and $A \in \mathcal{N}(Y)$. Then A is $\tau|_Y$-closed iff $A = F|_Y$ for some τ-closed set F in X.

Proof:A is $\tau|_Y$-closed in $Y \Leftrightarrow A^c$ is $\tau|_Y$-open in $Y \Leftrightarrow A^c = G|_Y$ for some $G \in \tau \Leftrightarrow A = (G|_Y)^c \Leftrightarrow A = G^c|_Y$ [3.4(iii)] $\Leftrightarrow A = F|_Y$, where $F = G^c$ is a τ-closed set in X.

3.12. Remark: From 3.11, it is easy to conclude that if $(Y, \tau|_Y)$ is a subspace of an NTS (X, τ) then $(\tau|_Y)^c = \tau^c|_Y$.

3.13. Proposition: Let $(Y, \tau|_Y)$ be a subspace of an NTS (X, τ) and let $\mathcal{B}$ be a base for τ. Then $\mathcal{B}|_Y = \{B|_Y : B \in \mathcal{B}\}$ is a base for $\tau|_Y$.

Proof: Let H be a $\tau|_Y$-open set in Y. Also let $x_{\alpha,\beta,\gamma} \in H$ be an arbitrary NP. Then there exists a τ-open set G such that $H = G|_Y$. Since $\mathcal{B}$ is a base for τ, so there exists a $B \in \mathcal{B}$ such that $x_{\alpha,\beta,\gamma} \in B \subseteq G$. Therefore $x_{\alpha,\beta,\gamma} \in B|_Y \subseteq G|_Y = H$ as $x_{\alpha,\beta,\gamma} \in \mathcal{N}(Y)$. Thus for any $x_{\alpha,\beta,\gamma} \in H$, there exists a member $B|_Y$ of $\mathcal{B}|_Y$ such that $x_{\alpha,\beta,\gamma} \in B|_Y \subseteq H$. Therefore $H = \cup \{B|_Y : B|_Y \in \mathcal{B}|_Y$ and $B|_Y \subseteq H\}$. Hence $\mathcal{B}|_Y$ is a base for $\tau|_Y$.

4. Neutrosophic Separation Axioms

Here we study the separation axioms in neutrosophic topological spaces. But, before that, we put forward two definitions.

4.1. Definition: A property of an NTS (X, τ) is said to be hereditary if whenever the space X has that property, then so does every subspace of it.

***4.2. Definition*:** A property of an NTS (X,τ) is said to be a topological property or topological invariant if each space homeomorphic to X has that property whenever the space X has that property. In other words, a property of an NTS is said to be a topological property iff it is preserved under homeomorphism.

***4.3. Definition*:** An NTS (X,τ) is called a neutrosophic T_0-space or (NT_0-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

***4.4. Example*:** Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly (X,τ) is an NTS and it is a NT_0-space.

***4.5. Example*:** Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \tilde{X}\}$. Clearly (X,τ) is an NTS but it is not a NT_0-space.

***4.6. Proposition*:** Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is finer than τ. If (X,τ) is a NT_0-space then (X,τ^*) is also a NT_0-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is a NT_0-space, so there exists a $G \in \tau$ such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a $H \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since τ^* is finer than τ, so $G, H \in \tau \Rightarrow G, H \in \tau^*$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $G \in \tau^*$ such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a $H \in \tau^*$ such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X,τ^*) is also a NT_0-space.

***4.7. Proposition*:** Let (X,τ) be a NT_0-space. Then every neutrosophic subspace of X is a NT_0-space and hence the property is hereditary.

Proof: Let $(Y,\tau|_Y)$ be a neutrosophic subspace of (X,τ), where $\tau|_Y = \{G|_Y : G \in \tau\}$. We want to show $(Y,\tau|_Y)$ is a NT_0-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X,τ) is a NT_0-space, so there exists a τ-open NS U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists a τ-open NS V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Then $(x_{\alpha,\beta,\gamma} \in U|_Y,\ y_{\alpha',\beta',\gamma'} \notin U|_Y)$ or $(x_{\alpha,\beta,\gamma} \notin V|_Y,\ y_{\alpha',\beta',\gamma'} \in V|_Y)$. Also $U|_Y, V|_Y \in \tau|_Y$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x \neq y$, there exists a $\tau|_Y$-open NS $U|_Y$ such that $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \notin U|_Y$ or there exists a $\tau|_Y$-open NS $V|_Y$ such that $x_{\alpha,\beta,\gamma} \notin V|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ Therefore $(Y,\tau|_Y)$ is a NT_0-space and hence the property is hereditary.

***4.8. Proposition*:** Let (X,τ) be an NTS. Then X is a NT_0-space iff for any two distinct neutrosophic crisp points $x_{1,0,0}$ and $y_{1,0,0}$ in X, $(x_{1,0,0})\hat{q}[cl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[cl(x_{1,0,0})]$.

Proof: Necessary part: Suppose that both $(x_{1,0,0})\hat{q}[cl(y_{1,0,0})]$ and $(y_{1,0,0})\hat{q}[cl(x_{1,0,0})]$ are false. Then $(x_{1,0,0})q[cl(y_{1,0,0})]$ and $(y_{1,0,0})q[cl(x_{1,0,0})]$ are true. Now $(x_{1,0,0})q[cl(y_{1,0,0})] \Rightarrow x_{1,0,0} \notin [cl(y_{1,0,0})]^c \Rightarrow x_{1,0,0} \notin [\cap\{G : G$ is a τ-closed NS and $y_{1,0,0} \in G\}]^c \Rightarrow x_{1,0,0} \notin \cup\{G^c : G^c$ is a τ-open NS and $y_{1,0,0} \notin G^c\} \Rightarrow x_{1,0,0} \notin G^c$ for all τ-open NSs G^c such that $y_{1,0,0} \notin G^c$. This ensures that if H is a τ-open NS such that $y_{1,0,0} \in H$ then $x_{1,0,0} \in H$. Similarly $(y_{1,0,0})q[cl(x_{1,0,0})]$ implies that if K is a τ-open NS such that $x_{1,0,0} \in K$ then $y_{1,0,0} \in K$. Thus every τ-open NS containing one of $x_{1,0,0}$ and $y_{1,0,0}$ must contain the other. But this is a contradiction to our assumption that X is a NT_0-space. Therefore $(x_{1,0,0})\hat{q}[cl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[cl(x_{1,0,0})]$.

Converse part: $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ be any two NPs in X such that $x \neq y$. Now by hypothesis, $(x_{1,0,0})\hat{q}[cl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[cl(x_{1,0,0})]$. If $(x_{1,0,0})\hat{q}[cl(y_{1,0,0})]$ then $x_{1,0,0} \in [cl(y_{1,0,0})]^c$, which gives $x_{\alpha,\beta,\gamma} \in [cl(y_{1,0,0})]^c$. Obviously $y_{p,q,r} \notin [cl(y_{1,0,0})]^c$. Since $cl(y_{1,0,0})$ is a τ-closed NS, so $[cl(y_{1,0,0})]^c$ is a τ-open NS. Thus there exists a τ-open NS $[cl(y_{1,0,0})]^c$ in X such that $x_{\alpha,\beta,\gamma} \in [cl(y_{1,0,0})]^c$ but $y_{p,q,r} \notin [cl(y_{1,0,0})]^c$. Similarly if $(y_{1,0,0})\hat{q}[cl(x_{1,0,0})]$ then there exists a τ-open NS $[cl(x_{1,0,0})]^c$ in X such that $x_{\alpha,\beta,\gamma} \notin [cl(x_{1,0,0})]^c$ but $y_{p,q,r} \in [cl(x_{1,0,0})]^c$. Therefore (X,τ) is a NT_0-space.
Hence proved.

4.9. Proposition: Let f be a one-one neutrosophic continuous function from an NTS (X,τ) to the NTS (Y,σ). If (Y,σ) is NT_0 then (X,τ) is also a NT_0-space.
Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_0, so there exists a σ-open NS G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-open NS H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Again, since f is neutrosophic continuous, so $f^{-1}(G)$ is a τ-open NS. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-open NS such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-open NS $f^{-1}(G)$ such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ or there exists a τ-open NS $f^{-1}(H)$ such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is a NT_0-space. Hence proved.

4.10. Proposition: The property of being NT_0-space is preserved under a bijective neutrosophic open function.
Proof: Let (X,τ) and (Y,σ) be two NTSs. Also let (X,τ) be a NT_0-space and $f: X \rightarrow Y$ be a bijective neutrosophic open function. We show that (Y,σ) ia a NT_0-space. Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ be two NPs in Y such that $y^1 \neq y^2$. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_0, so there exists a τ-open NS G such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-open NS H such that $x^1_{\alpha,\beta,\gamma} \notin H$, $x^2_{\alpha',\beta',\gamma'} \in H$. Suppose G exists such that $x^1_{\alpha,\beta,\gamma} \in G$ and $x^2_{\alpha',\beta',\gamma'} \notin G$. Since f is a neutrosophic open function, so $f(G)$ is a σ-open NS such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly if H exists such that $x^1_{\alpha,\beta,\gamma} \notin H$ and $x^2_{\alpha',\beta',\gamma'} \in H$ then $f(H)$ is a σ-open NS such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \notin f(H)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there exists a σ-open NS $f(G)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \notin f(G)$ or there exists a σ-open NS $f(H)$ such that $y^1_{p,q,r} \notin f(H)$, $y^2_{p',q',r'} \in f(H)$. Therefore (Y,σ) is a NT_0-space. Hence proved.

4.11. Proposition: The property of being NT_0-space is a topological property.

Proof: Let (X,τ) and (Y,σ) be two NTSs. Also let (X,τ) be a NT_0-space and $f: X \to Y$ be a neutrosophic homeomorphism. Since f is a neutrosophic homeomorphism, so f is a bijective neutrosophic open function. Therefore by the proposition 4.10, (Y,σ) is a NT_0-space. Hence proved.

***4.12. Definition*:** An NTS (X,τ) is called a neutrosophic T_1-space (NT_1-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

***4.13. Example*:** Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \widetilde{X}, A, B\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly (X,τ) is an NTS and it is a NT_1-space.

***4.14. Example*:** Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \widetilde{X}\}$. Clearly (X,τ) is an NTS but it is not a NT_1-space.

***4.15. Proposition*:** Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is finer than τ. If (X,τ) is a NT_1-space then (X,τ^*) is also a NT_1-space.
Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is a NT_1-space, so there exists a $G \in \tau$ such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a $H \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since τ^* is finer than τ, so $G, H \in \tau \Rightarrow G, H \in \tau^*$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exists a $G \in \tau^*$ such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a $H \in \tau^*$ such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X,τ^*) is a NT_1-space.

***4.16. Proposition*:** Let (X,τ) be an NTS. If (X,τ) is a NT_1-space then it is a NT_0-space.
Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since X is NT_1-space, so there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Hence (X,τ) is a NT_0-space.

***4.17. Remark*:** Converse of the proposition 4.16 is not true. We establish it by the following counter example.
Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \widetilde{X}, A\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$. Clearly (X,τ) is a NT_0-space but not a NT_1-space.

***4.18. Proposition*:** Let (X,τ) be a NT_1-space. Then every neutrosophic subspace of X is a NT_1-space and hence the property is hereditary.
Proof: Let $(Y,\tau|_Y)$ be a neutrosophic subspace of (X,τ), where $\tau|_Y = \{G|_Y: G \in \tau\}$. We want to show $(Y,\tau|_Y)$ is a NT_1-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X,τ) is NT_1-space, so there exists a τ-open NS U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a τ-open NS V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Then $(x_{\alpha,\beta,\gamma} \in U|_Y, y_{\alpha',\beta',\gamma'} \notin U|_Y)$ and $(x_{\alpha,\beta,\gamma} \notin V|_Y, y_{\alpha',\beta',\gamma'} \in V|_Y)$. Also $U|_Y, V|_Y \in \tau|_Y$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x \neq y$, there exists a $\tau|_Y$-open NS $U|_Y$ such that $x_{\alpha,\beta,\gamma} \in U|_Y$,

$y_{\alpha',\beta',\gamma'} \notin U|_Y$ and there exists a $\tau|_Y$-open NS $V|_Y$ such that $x_{\alpha,\beta,\gamma} \notin V|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ Therefore $(Y, \tau|_Y)$ is a NT_1-space and hence the property is hereditary.

4.19. Proposition: Let (X, τ) be an NTS. Then every NCP in X is a τ-closed NS iff X is a NT_1-space.
Proof: Necessary part: Let $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ be two NPs in X such that $x \neq y$. Since $x \neq y$, so $x_{\alpha,\beta,\gamma} \in (y_{1,0,0})^c$. By hypothesis, $y_{1,0,0}$ is a τ-closed NS. Therefore $(y_{1,0,0})^c$ is a τ-open NS. Thus there exists a τ-open NS $(y_{1,0,0})^c$ such that $x_{\alpha,\beta,\gamma} \in (y_{1,0,0})^c$ but $y_{p,q,r} \notin (y_{1,0,0})^c$. Similarly $(x_{1,0,0})^c$ is a τ-open NS such that $y_{p,q,r} \in (x_{1,0,0})^c$ but $x_{\alpha,\beta,\gamma} \notin (x_{1,0,0})^c$. Therefore X is a NT_1-space.
Sufficient part: Let $x_{1,0,0}$ be an NCP in X. Also let $y_{p,q,r}$ be an NP in X such that $x \neq y$. Then $y_{p,q,r} \in (x_{1,0,0})^c$. Let us consider an NP $x_{\alpha,\beta,\gamma}$ with support x. Since X is a NT_1-space, so for $y_{p,q,r}$ and $x_{\alpha,\beta,\gamma}$, there exists a τ-open NS G such that $y_{p,q,r} \in G$ and $x_{\alpha,\beta,\gamma} \notin G$. Since for all α, β, γ with $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$, one such G exists, therefore we must have a τ-open NS H such that $y_{p,q,r} \in H$ and $x_{1,0,0} \cap H = \widetilde{\emptyset}$, i.e., $y_{p,q,r} \in H \subseteq (x_{1,0,0})^c$. Therefore $(x_{1,0,0})^c$ is a τ-open NS and hence $x_{1,0,0}$ is a τ-closed NS.
Hence proved.

4.20. Proposition: Let f be a one-one neutrosophic continuous function from an NTS (X, τ) to the NTS (Y, σ). If (Y, σ) is NT_1 then (X, τ) is also a NT_1-space.
Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_1, so there exists a σ-open NS G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-open NS H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is neutrosophic continuous, so $f^{-1}(G)$ and $f^{-1}(H)$ are τ-open NSs. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-open NS $f^{-1}(G)$ such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ and there exists a τ-open NS $f^{-1}(H)$ such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X, τ) is a NT_1-space. Hence proved.

4.21. Proposition: The property of being NT_1-space is preserved under a bijective neutrosophic open function.
Proof: Let (X, τ) and (Y, σ) be two NTSs. Also let (X, τ) be a NT_1-space and $f: X \to Y$ be a bijective neutrosophic open function. We show that (Y, σ) ia a NT_1-space. Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, be two NPs in Y. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_1, so there exists a τ-open NS G such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-open NS H such that $x^1_{\alpha,\beta,\gamma} \notin H$, $x^2_{\alpha',\beta',\gamma'} \in H$. Since f is a neutrosophic open function, so $f(G)$ is a σ-open NS such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly $f(H)$ is a σ-open NS such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \notin f(H)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there

exists a σ-open NS $f(G)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \notin f(G)$ and there exists a σ-open NS $f(H)$ such that $y^1_{p,q,r} \notin f(H)$, $y^2_{p',q',r'} \in f(H)$. Therefore (Y,σ) is a NT_1-space. Hence proved.

***4.22. Proposition*:** The property of being NT_1-space is a topological property.

Proof: Let (X,τ) and (Y,σ) be two NTSs. Also let (X,τ) be a NT_1-space and $f: X \to Y$ be a neutrosophic homeomorphism. Since f is a neutrosophic homeomorphism, so f is a bijective neutrosophic open function. Therefore by the proposition 4.21, (Y,σ) is a NT_1-space. Hence proved.

***4.23. Proposition*:** Let (X,τ) be an NTS. Then X is NT_1 iff the intersection of all the neutrosophic neighbourhoods of an arbitrary NP of X is an NP.

Proof: Necessary part: Let $x_{\alpha,\beta,\gamma}$ be an arbitrary NP in X and N be the intersection of all the neutrosophic neighbourhoods of $x_{\alpha,\beta,\gamma}$. Also let $y_{p,q,r}$ be any NP in X such that $x \neq y$. Since X is NT_1, so there exists a neutrosophic neighbourhood G of $x_{\alpha,\beta,\gamma}$ such that $y_{p,q,r} \notin G$ and consequently $y_{p,q,r} \notin N$. Since $y_{p,q,r}$ is arbitrary, so $N = x_{\alpha,\beta,\gamma}$.

Sufficient part: Let $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ be any two NPs in X such that $x \neq y$. By hypothesis, the intersection of all the neutrosophic neighbourhoods of $x_{\alpha,\beta,\gamma}$ is $x_{\alpha,\beta,\gamma}$. So, there must exist a neutrosophic neighbourhood of $x_{\alpha,\beta,\gamma}$ which does not contain $y_{p,q,r}$. Similarly, there must exist a neutrosophic neighbourhood of $y_{p,q,r}$ which does not contain $x_{\alpha,\beta,\gamma}$. Therefore X is a NT_1-space. Hence proved.

***4.24 Definition*:** An NTS (X,τ) is called a neutrosophic T_2-space or neutrosophic Hausdorff space (NT_2-space or N-Hausdorff space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exist $U, V \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \widetilde{\emptyset}$.

***4.25. Example*:** Let $X = \{a, b\}$ and $\tau = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a, 1,0,0\rangle, \langle b, 0,1,1\rangle\}$ and $B = \{\langle a, 0,1,1\rangle, \langle b, 1,0,0\rangle\}$. Clearly (X,τ) is an NTS and it is a NT_2-space.

***4.26. Example*:** Let $X = \{a, b\}$ and $\tau = \{\widetilde{\emptyset}, \tilde{X}\}$. Clearly (X,τ) is an NTS but it is not a NT_2-space.

***4.27. Proposition*:** Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is finer than τ. If (X,τ) is a NT_2-space then (X,τ^*) is also a NT_2-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is a NT_2-space, so there exist $G, H \in \tau$ such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \widetilde{\emptyset}$. Since τ^* is finer than τ, so $G, H \in \tau \Rightarrow G, H \in \tau^*$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exist $G, H \in \tau^*$ such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \widetilde{\emptyset}$.. Hence (X,τ^*) is a NT_2-space.

***4.28. Proposition*:** Let (X,τ) be an NTS. If (X,τ) is a NT_2-space then it is a NT_1-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x \neq y$. Since (X,τ) is a NT_2-space, so there exist τ-open NSs H and K such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \in K$ and $H \cap K = \widetilde{\emptyset}$. Since $x_{\alpha,\beta,\gamma} \in H$ and $H \cap K = \widetilde{\emptyset}$, so $x_{\alpha,\beta,\gamma} \notin K$. Similarly, $y_{\alpha',\beta',\gamma'} \notin H$. Thus there exists a $H \in \tau$ such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \notin H$ and there exists a $K \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin K$, $y_{\alpha',\beta',\gamma'} \in K$. Hence (X,τ) is a NT_1-space.

***4.29. Lemma*:** The co-finite NTS $(\mathbb{N},\tau)$ is not a NT_2-space, where $\mathbb{N}$ is the set of all natural numbers.

Proof: Let $\widetilde{\mathbb{N}} = \{\langle x, 1,0,0\rangle : x \in \mathbb{N}\}$ and $\widetilde{\emptyset} = \{\langle x, 0,1,1\rangle : x \in \mathbb{N}\}$. Given that τ is a co-finite topology on $\mathbb{N}$, so τ is the set containing $\widetilde{\emptyset}$ and all those neutrosophic sets over $\mathbb{N}$ whose complements are finite. We show that the co-finite NTS $(\mathbb{N}, \tau)$ is not a NT_2-space.
Suppose, on the contrary, that $(\mathbb{N}, \tau)$ is a NT_2-space. Then for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in $\mathbb{N}$ such that $x \neq y$, there exist τ-open NSs G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \widetilde{\emptyset}$. Now $G \cap H = \widetilde{\emptyset} \Rightarrow (G \cap H)^c = (\widetilde{\emptyset})^c \Rightarrow G^c \cup H^c = \widetilde{\mathbb{N}}$, which is not possible as $\widetilde{\mathbb{N}}$ is an infinite neutrosophic set and $G^c \cup H^c$ is a finite neutrosophic set being the union of two finite neutrosophic sets G^c and H^c.
Therefore the co-finite NTS $(\mathbb{N}, \tau)$ is a not a NT_2-space.

***4.30. Remark*:** Converse of the proposition 4.28 is not true. We establish it by the following counter example.
We consider the co-finite NTS $(\mathbb{N}, \tau)$, where $\mathbb{N}$ is the set of all natural numbers. In the lemma 4.29, we have shown that $(\mathbb{N}, \tau)$ is not a NT_2-space.
We now show that $(\mathbb{N}, \tau)$ is a NT_1-space. Let $\widetilde{\mathbb{N}} = \{\langle x, 1,0,0\rangle : x \in \mathbb{N}\}$ and $\widetilde{\emptyset} = \{\langle x, 0,1,1\rangle : x \in \mathbb{N}\}$. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in $\mathbb{N}$ such that $x \neq y$. Now $(\widetilde{\mathbb{N}} \backslash x_{1,0,0})^c = x_{1,0,0}$, a finite NS. Therefore $\widetilde{\mathbb{N}} \backslash x_{1,0,0}$ is a τ-open NS. Obviously $y_{\alpha',\beta',\gamma'} \in \widetilde{\mathbb{N}} \backslash x_{1,0,0}$ but $x_{\alpha,\beta,\gamma} \notin \widetilde{\mathbb{N}} \backslash x_{1,0,0}$. Similarly $\widetilde{\mathbb{N}} \backslash y_{1,0,0}$ is a τ-open NS such that $x_{\alpha,\beta,\gamma} \in \widetilde{\mathbb{N}} \backslash y_{1,0,0}$ but $y_{\alpha',\beta',\gamma'} \notin \widetilde{\mathbb{N}} \backslash y_{1,0,0}$. Therefore $(\mathbb{N}, \tau)$ is a NT_1-space.
Thus the co-finite NTS $(\mathbb{N}, \tau)$ is a NT_1-space but not a NT_2-space.

***4.31. Proposition*:** Let (X, τ) be a NT_2-space. Then every neutrosophic subspace of X is a NT_2-space and hence the property is hereditary.
Proof: Let $(Y, \tau|_Y)$ be a neutrosophic subspace of (X, τ), where $\tau|_Y = \{G|_Y : G \in \tau\}$. We want to show $(Y, \tau|_Y)$ is a NT_2-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X, τ) is NT_2-space, so there exist τ-open NSs U, V such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \widetilde{\emptyset}$. Then $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ and $(U|_Y) \cap (V|_Y) = (U \cap V)|_Y = \widetilde{\emptyset}|_Y = \widetilde{\emptyset}$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x \neq y$, there exist $\tau|_Y$-open NSs $U|_Y$, $V|_Y$ such that $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ and $(U|_Y) \cap (V|_Y) = \widetilde{\emptyset}$. Therefore $(Y, \tau|_Y)$ is a NT_2-space and hence the property is hereditary.

4.32. Proposition: If f is a one-one neutrosophic continuous function from an NTS (X, τ) to a neutrosophic Hausdorff space (Y, σ) then (X, τ) is also a neutrosophic Hausdorff space.
Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since (Y, σ) is neutrosophic Hausdorff, so there exist σ-open NSs H_1, H_2 such that $y^1_{p,q,r} \in H_1$, $y^2_{p',q',r'} \in H_2$ and $H_1 \cap H_2 = \widetilde{\emptyset}$. Since f is neutrosophic continuous, so $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are τ-open NSs. Now $f^{-1}(H_1) \cap f^{-1}(H_2) = f^{-1}(H_1 \cap H_2) = f^{-1}(\widetilde{\emptyset}) = \widetilde{\emptyset}$. Also $y^1_{p,q,r} \in H_1 \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(H_1) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exist τ-

open NSs $f^{-1}(H_1)$, $f^{-1}(H_2)$ such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$ and $f^{-1}(H_1) \cap f^{-1}(H_2) = \tilde{\emptyset}$. Therefore (X,τ) is a neutrosophic Hausdorff space. Hence proved.

***4.33. Proposition*:** The property of being NT_2-space is preserved under a bijective neutrosophic open function.

Proof: Let (X,τ) and (Y,σ) be two NTSs. Also let (X,τ) be a NT_2-space and $f: X \to Y$ be a bijective neutrosophic open function. We show that (Y,σ) ia a NT_2-space. Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, be two NPs in Y. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_2, so there exist τ-open NSs G, H such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Since f is a neutrosophic open function, so $f(G), f(H)$ are σ-open NSs such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$, $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Again since f is bijective, so $f(G) \cap f(H) = f(G \cap H) = f(\tilde{\emptyset}) = \tilde{\emptyset}$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there exist σ-open NSs $f(G), f(H)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \in f(H)$ and $f(G) \cap f(H) = \tilde{\emptyset}$. Therefore (Y,σ) is a NT_2-space. Hence proved.

***4.34. Proposition*:** The property of being NT_2-space is a topological property.

Proof: Let (X,τ) and (Y,σ) be two NTSs. Also let (X,τ) be a NT_2-space and $f: X \to Y$ be a neutrosophic homeomorphism. Since f is a neutrosophic homeomorphism, so f is a bijective neutrosophic open function. Therefore by the proposition 4.33, (Y,σ) is a NT_2-space. Hence proved.

***4.35. Proposition*:** Let A be a neutrosophic compact subset of a neutrosophic Hausdorff space (X,τ) such that $A \cap A^c = \tilde{\emptyset}$. Then A is a neutrosophic closed set.

Proof: We want to show that A is τ-closed, i.e. A^c is τ-open. Let $x_{\alpha,\beta,\gamma}$ be an NP in A^c. Since X is neutrosophic Hausdorff, so for any NP $y^i_{p,q,r} \in A$ (Obviously $x \neq y$ as $A \cap A^c = \tilde{\emptyset}$), there exist τ-open NSs $G_i(x_{\alpha,\beta,\gamma})$, $H_i(y^i_{p,q,r})$ such that $x_{\alpha,\beta,\gamma} \in G_i(x_{\alpha,\beta,\gamma})$, $y^i_{p,q,r} \in H_i(y^i_{p,q,r})$ and $G_i(x_{\alpha,\beta,\gamma}) \cap H_i(y^i_{p,q,r}) = \tilde{\emptyset}$ for each $i \in \Delta$, where Δ is an index set. Clearly $\{H_i(y^i_{p,q,r}): i \in \Delta\}$ is a NOC of A. Since A is neutrosophic compact, so A has a finite NOSC, i.e., $A \subseteq \bigcup_{k=1}^{n} H_{i_k}(y^{i_k}_{p,q,r})$. Let $G_{i_k}(x_{\alpha,\beta,\gamma})$ be the neutrosophic open sets corresponding to the neutrosophic open sets $H_{i_k}(y^{i_k}_{p,q,r})$, $k = 1,2,3,\cdots,n$. Let $M = \bigcap_{k=1}^{n} G_{i_k}(x_{\alpha,\beta,\gamma})$ and $N = \bigcup_{k=1}^{n} H_{i_k}(y^{i_k}_{p,q,r})$. Obviously M is a τ-open set. We claim that $M \cap N = \tilde{\emptyset}$. Let $z_{\alpha',\beta',\gamma'}$ be an arbitrary NP in N. Then $z_{\alpha',\beta',\gamma'} \in H_{i_k}(y^{i_k}_{p,q,r})$ for some $k, 1 \le k \le n \Rightarrow z_{\alpha',\beta',\gamma'} \notin G_{i_k}(x_{\alpha,\beta,\gamma})$ for some $k, 1 \le k \le n \Rightarrow z_{\alpha',\beta',\gamma'} \notin M$. Again if $u_{r,s,t} \in M$ be an arbitrary NP then $u_{r,s,t} \in G_{i_k}(x_{\alpha,\beta,\gamma})$ for all $k, 1 \le k \le n \Rightarrow u_{r,s,t} \notin H_{i_k}(y^{i_k}_{p,q,r})$ for all $k, 1 \le k \le n \Rightarrow u_{r,s,t} \notin N$. Thus $M \cap N = \tilde{\emptyset}$. Since $A \subseteq N$ and since $M \cap N = \tilde{\emptyset}$, so $A \cap M = \tilde{\emptyset}$ and therefore $M \subseteq A^c$. Since M is a τ-open set and since $x_{\alpha,\beta,\gamma} \in M$, so M is a τ-neighbourhood of $x_{\alpha,\beta,\gamma}$. Since $M \subseteq A^c$, so A^c is also a τ-neighbourhood of $x_{\alpha,\beta,\gamma}$. Since $x_{\alpha,\beta,\gamma}$ is an arbitrary NP in A^c, so A^c is a τ-neighbourhood of each of its NPs. Therefore A^c is τ-open, i.e., A is a τ-closed NS. Hence proved.

***4.36. Proposition*:** Let (X,τ) be a neutrosophic Hausdorff space. If $x_{\alpha,\beta,\gamma}$ is an NP in X and A is a neutrosophic compact subset of X such that $x_{\alpha,\beta,\gamma} \cap A = \tilde{\emptyset}$ then $x_{\alpha,\beta,\gamma}$ and A can be separated by two disjoint neutrosophic open sets.

Proof: Since $x_{\alpha,\beta,\gamma} \cap A = \widetilde{\emptyset}$, so $x_{\alpha,\beta,\gamma} \in A^c$. Since X is neutrosophic Hausdorff, so for any NP $y^i_{p,q,r} \in A$, $x \neq y$, there exist τ-open NSs $G_i(x_{\alpha,\beta,\gamma})$, $H_i(y^i_{p,q,r})$ such that $x_{\alpha,\beta,\gamma} \in G_i(x_{\alpha,\beta,\gamma})$, $y^i_{p,q,r} \in H_i(y^i_{p,q,r})$ and $G_i(x_{\alpha,\beta,\gamma}) \cap H_i(y^i_{p,q,r}) = \widetilde{\emptyset}$ for each $i \in \Delta$, where Δ is an index set. Clearly $\{H_i(y^i_{p,q,r}) : i \in \Delta\}$ is a NOC of A. Since A is neutrosophic compact, so A has a finite NOSC, i.e., $A \subseteq \bigcup_{k=1}^{n} H_{i_k}(y^{i_k}_{p,q,r})$. Let $G_{i_k}(x_{\alpha,\beta,\gamma})$ be the τ-open NSs corresponding to the τ-open NSs $H_{i_k}(y^{i_k}_{p,q,r})$, $k = 1,2,3,\cdots,n$. Let $M = \bigcap_{k=1}^{n} G_{i_k}(x_{\alpha,\beta,\gamma})$ and $N = \bigcup_{k=1}^{n} H_{i_k}(y^{i_k}_{p,q,r})$. Obviously M and N are neutrosophic open sets such that $x_{\alpha,\beta,\gamma} \in M$ and $A \subseteq N$. We claim that $M \cap N = \widetilde{\emptyset}$. Let $z_{\alpha',\beta',\gamma'}$ be an arbitrary NP in N. Then $z_{\alpha',\beta',\gamma'} \in H_{i_k}(y^{i_k}_{p,q,r})$ for some $k, 1 \leq k \leq n \Rightarrow z_{\alpha',\beta',\gamma'} \notin G_{i_k}(x_{\alpha,\beta,\gamma})$ for some $k, 1 \leq k \leq n \Rightarrow z_{\alpha',\beta',\gamma'} \notin M$. Again if $u_{r,s,t} \in M$ be an arbitrary NP then $u_{r,s,t} \in G_{i_k}(x_{\alpha,\beta,\gamma})$ for all $k, 1 \leq k \leq n \Rightarrow u_{r,s,t} \notin H_{i_k}(y^{i_k}_{p,q,r})$ for all $k, 1 \leq k \leq n \Rightarrow u_{r,s,t} \notin N$. Therefore $M \cap N = \widetilde{\emptyset}$. Hence proved.

***4.37. Proposition*:** Let A be a neutrosophic compact subset of a neutrosophic Hausdorff space (X, τ). If $x_{\alpha,\beta,\gamma}$ is an NP in X such that $x_{\alpha,\beta,\gamma} \cap A = \widetilde{\emptyset}$ then there exists a neutrosophic open set G such that $x_{\alpha,\beta,\gamma} \in G \subseteq A^c$.

Proof: Immediately from 4.36.

***4.38. Proposition*:** Let A and B be disjoint neutrosophic compact subsets of a neutrosophic Hausdorff space (X, τ). Then there exist disjoint neutrosophic open sets G and H such that $A \subseteq G$ and $B \subseteq H$.

Proof: Let $x_{\alpha,\beta,\gamma} \in A$. Then $x_{\alpha,\beta,\gamma} \notin B$ as $A \cap B = \widetilde{\emptyset}$. Since X is neutrosophic Hausdorff, so for any $y_{\alpha',\beta',\gamma'} \in B$, there exist disjoint τ-open NSs $G(y_{\alpha',\beta',\gamma'})$ and $H(y_{\alpha',\beta',\gamma'})$ such that $x_{\alpha,\beta,\gamma} \in G(y_{\alpha',\beta',\gamma'})$ and $y_{\alpha',\beta',\gamma'} \in H(y_{\alpha',\beta',\gamma'})$. The collection $\{H(y_{\alpha',\beta',\gamma'}) : y_{\alpha',\beta',\gamma'} \in B\}$ is evidently a NOC of B. Since B is neutrosophic compact, so there exist finitely many NPs $y^i_{p,q,r}, i = 1,2,3,\ldots,n$ of B such that $B \subseteq \bigcup_{i=1}^{n} H(y^i_{p,q,r})$. Let $H(x_{\alpha,\beta,\gamma}) = \bigcup_{i=1}^{n} H(y^i_{p,q,r})$ and $G(x_{\alpha,\beta,\gamma}) = \bigcap_{i=1}^{n} G(y^i_{p,q,r})$, where $G(y^i_{p,q,r})$ are the τ-open NSs corresponding to the τ-open NSs $H(y^i_{p,q,r})$. Then clearly $H(x_{\alpha,\beta,\gamma})$ and $G(x_{\alpha,\beta,\gamma})$ are τ-open NSs such that $x_{\alpha,\beta,\gamma} \in G(x_{\alpha,\beta,\gamma}), B \subseteq H(x_{\alpha,\beta,\gamma})$ and $G(x_{\alpha,\beta,\gamma}) \cap H(x_{\alpha,\beta,\gamma}) = \widetilde{\emptyset}$. Now suppose that $x_{\alpha,\beta,\gamma}$ is an arbitrary NP in A. We construct $G(x_{\alpha,\beta,\gamma})$ and $H(x_{\alpha,\beta,\gamma})$ as above. Evidently $\{G(x_{\alpha,\beta,\gamma}) : x_{\alpha,\beta,\gamma} \in A\}$ is a NOC of A. Since A is neutrosophic compact, so there exist finitely many NPs $x^j_{r,s,t}, j = 1,2,3,\ldots,m$ of A such that $A \subseteq \bigcup_{j=1}^{m} G(x^j_{r,s,t})$. Let $G = \bigcup_{j=1}^{m} G(x^j_{r,s,t})$ and $H = \bigcap_{j=1}^{m} H(x^j_{r,s,t})$, where $H(x^j_{r,s,t})$ are the τ-open NSs corresponding to the τ-open NSs $G(x^j_{r,s,t})$. Clearly G and H are neutrosophic open sets such that $A \subseteq G$, $B \subseteq H$ and $G \cap H = \widetilde{\emptyset}$. Hence proved.

5. Conclusion

In this article, our primary objective was to explore the separation axioms in neutrosophic topological spaces. Just like in the study of topological spaces in classical, fuzzy or other settings, the significance of subspace topology and subspaces can not be overlooked, as many properties of topological spaces are interconnected with subspaces. Therefore, in section 3, we have introduced the concept of neutrosophic subspace, and investigated a few properties of it. Before delving into neutrosophic subspaces, we have laid the groundwork by establishing some results based on single-valued neutrosophic sets which have played a crucial role in the study of neutrosophic subspaces.

Moving forward in section 4, we have defined neutrosophic T_0, T_1 and T_2-spaces in relation to neutrosophic topological spaces and examined various properties associated with these separation axioms. Our future research will aim to explore other notions associated with neutrosophic topological spaces. We hope that the findings presented in this article will prove beneficial to the research community and contribute to the advancement of various aspects of neutrosophic topology.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Neutrosophic CRITIC MCDM Methodology for Ranking Factors and Needs of Customers in Product's Target Demographic in Virtual Reality Metaverse

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Abstract: Affective design has come to place a premium on customer-centric creativity, which ultimately results in the creation of a product tailored to the requirements of a certain demographic. Designing a fresh and original item that appeals to clients remains challenging, despite the abundance of literature on customer-centric creativity and impact product design. This is so because optimizing technological and aesthetical design aspects and variables for a group of consumers is highly complicated, and for the same reasons that it is hard to know a customer's choice, enable the product's functioning, etc. One of the most important parts of creating cutting-edge technological items is the technical product requirement. In reality, we can generally glean insights about a product's design by looking at the hidden desires of a comparable set of potential buyers. In this study, we provide a unique combined structure for evaluating the factors and needs of customers in product design in virtual reality (VR) metaverse by merging the CRiteria Importance Through Intercriteria Correlation (CRITIC) approach with single-valued neutrosophic sets (SVNSs). The characteristic weights in this approach are calculated using the CRITIC technique. The CRITIC method computes the rank of needs. The neutrosophic set is used to overcome the uncertain data. The application of this study shows thermal dissipation, airtight construction, weight, and control mechanisms are among the most crucial considerations.

Keywords: Metaverse; Virtual Reality; Product Design; Neutrosophic CRITIC MCDM.

1. Introduction

Even though we describe it perfectly now, the phrase "Metaverse" was originally used in 1992 by Neal Stephenson, a student at Boston University who is particularly interested in computers, in his book Snow Crash. The narrative also makes contemporary references to avatars and virtual reality goggles. There's more to life than merely chatting with friends online, and the metaverse promises to provide. Pre-metaverse platforms are a kind of promising idea. Sandbox, for instance, provides blockchain-based video games, virtual worlds, and e-commerce. Non-fungible tokens (NFTs) exist in their own universe as well. It allows users to build their own virtual environments. The 3D scanning functionality of the Metahero project shines through, allowing users to create their own avatar in 16K resolution and convert it to NFT [1], [2].

In addition to providing a fresh take on the Internet, the Metaverse is quickly becoming a lucrative market for a wide range of industries. The number of potential financial gains resulting from this is straightforward to estimate. Major investments are being made by some of the world's most prestigious IT firms in the hope that the metaverse will one day become mainstream. Companies like Meta, Microsoft, and Epic Games are good examples. To make the metaverse a reality, we need advancements in areas such as wearable gadgets, internet connection methods, etc. Research on open standards are being conducted, and a standards forum for the metaverse is currently being developed. Recent years have seen significant development in the realms of both virtual reality (VR) and augmented reality (AR). However, when using data flow from senses with different technologies and sophisticated methods, safety and confidentiality problems emerge [3]–[5].

The growth of the theory of single-valued neutrosophic sets (SVNSs) inspired us to investigate its potential use in factors and needs of customers of product design assessment in VR and metaverse associated with imprecise expertise, the inaccurate human mind, and unreliable data. In order to evaluate the factors and needs of customers of product design selection based on several criteria, this research creates a framework using the CRiteria Importance Through Intercriteria Correlation (CRITIC) method in an SVNSs setting for the first time. This is because the CRITIC method's calculation of objective attribute weights is more in line with common sense when applied to MCDM systems [6]–[8].

1.1 Factors and needs of customer in product design in VR Metaverse

Recent decades have seen a rise in the importance of customer-centric innovation as a method for facilitating the production of cutting-edge manufactured goods. Innovation and product design that resonates with the intended market are propelled by a customer-centric approach. Commercial product design creates an item that not only prioritizes its functionality, dependability, manufacturability, and novelty, but also places an emphasis on the outlook, form, shape, and look that piques consumers' attention and influences their purchase decisions [9].

Marketing and selling a product that is tailored to the needs of a certain demographic requires an innovative approach that puts the consumer at the center. Given that product, perspectives are often the central problem influencing the idea development of a product, item enclosure design is crucial in the focused on client's design approach. There have been many academic articles written on customer-centric creativity and effective design for goods, but it is still difficult to create anything really novel that will find an audience among your target consumers. Knowing what a consumer wants, implementing that want into a product, etc., is difficult [10].

It is challenging to simultaneously maximize the technical and aesthetic qualities of a product's design. Immersive devices (e.g., virtual reality/mixed reality headsets), AI structures, mobile phones, Internet of Things (IoT) goods, etc. all rely heavily on the design of their respective product enclosures. The product enclosures safeguard the product's essential electromechanical parts, allowing it to function as intended per the design's requirements and specifications. Creating original tech products relies heavily on adhering to technical design specifications [11], [12].

The main contributions of this study are organized as:

- The first study applied the MCDM model to rank factors and needs of customer in product design on VR metaverse.
- This is the first study applied the neutrosophic set with the VR metaverse in product design.
- The CRITIC method is used to compute and rank the factors and needs of customer in product design in VR metaverse.
- The weights of factors are computed based on the standard deviation and correlation coefficient between factors.

2. Metaverse

In recent years, corporations have been focusing on joining the Metaverse, a permanent and decentralized online environment. The use of HMDs is crucial to the development of the Metaverse. The Metaverse will likely consist of mixed work surroundings, virtual NFT exchanges, shared and distributed immersive experiences, and immersive simulations of performances as the outcome of the combination of HMDs alongside additional immersive and novel technologies like blockchain, no fungible tokens (NFTs), artificial intelligence (AI), and Web 3.0. There are numerous potential benefits and drawbacks of exploring the immersive Metaverse and its effects on people, organizations, and society. Researchers in the field of IS will be able to learn more about these novel occurrences and determine which methods work best for conducting studies in the immersive, physically-boundless Metaverse [13], [14].

Virtual reality (VR) apps, for instance, aim to trick users into thinking they are physically experiencing the virtual environment in which they are interacting. In other words, not only does the simulation of the actual world blur the border between physical and virtual reality, but the blending of human and technical endeavors has also contributed to this phenomenon. Because of this, HMDs enable new socio-material practices in the workplace and in daily life by extending the capacities of both humans and technology. The overlapping of real and virtual identities, increased cybersecurity concerns, security, legal and moral difficulties, and VR-related technostress are all examples of potential unintended consequences that might arise from a more porous boundary among the real and virtual worlds [15], [16].

Since the Metaverse is so immersive, it provides IS researchers with unprecedented possibilities for trying out novel strategies (such as the socio-material method), techniques, and theoretical concepts. Anonymity, trust, user behavior and misbehavior, and the organizational effects of using immersive Metaverses are all areas where this framework can be fruitfully applied to further research [17].

3. Virtual Reality

The term "virtual reality" (VR) is used to describe computer-generated environments that are designed to look and feel like our everyday physical world. Virtual reality (VR) as an idea has been around for quite some time. Recent advances in consumer virtual reality head-mounted displays (HMDs) have led to a surge in the technology's popularity. Due to its superior technological features, including as high-quality picture rendering and a variety of accessories that enable unrestricted movement in the virtual world, the HMDs provide the most realistic VR experiences to date. These technology advancements pave the way for novel user and business scenarios. The $2.3 billion invested in VR startups in 2016 will grow to a $6.1 billion industry by 2020 as a result of the growing demand for head-mounted display (HMD)-based virtual reality (VR) innovation. Big Tech is shifting its attention back to virtual reality (VR) for both retail and enterprise use, and the industry is projected to grow to $20.9 billion by 2025. The virtual reality market is so promising that it is now included among the most important technological trends that governments may pursue [18], [19].

Many academic disciplines, from computer science to the health sciences, have taken an interest in virtual reality. Despite recommendations for greater study in this area, the majority of VR-related articles published in mainstream IS journals have dealt with non-immersive VR (i.e. desktop-based VR) topics such as virtual worlds and 3D objects. Journals in fields including medicine, retail, travel, HCMI, and educational technology are where you'll find the bulk of the known and expanding literature on immersive VR [20], [21].

Virtual reality (VR) is distinguished by its immersive qualities. The ability of virtual reality (VR) gear and software to fool the senses into thinking they are in a different environment is what we mean when we talk about immersion. The favorable impact that immersion has on the user's sense of presence in a virtual environment makes it crucial. Awareness, on the other hand, is a subjective feeling of being fully immersed in a certain place or situation. Users' actions in virtual reality become increasingly consistent with those in the real world as their sense of presence increases, erasing the barriers among the two worlds. Therefore, being immersed in a virtual environment is not just VR's ultimate objective, but also its distinguishing characteristic [22], [23].

4. Single Valued Neutrosophic Sets and MCDM

To cope with the ensuing ambiguity in a variety of fields, the notion of fuzzy sets (FSs) has been widely used. Pattern recognition, making choices, computational imaging, and other fields have all benefited from the various generalizations of FSs that have been described and applied over the last several decades [24]. However, FSs and their expansions are only capable of dealing with partial and inaccurate data, and not the indeterminate and unreliable information that arises in practical MCDM issues [25], [26]. Smarandache devised the neutrosophic set (NS) notion to sidestep this difficulty. Standalone and residing in are the truth, the indeterminacy, and the falsity. The NS is the generalization of FSs as stated by Smarandache. As a result, it has found widespread use in a variety of contexts [27], [28]. Figure 1 shows the SVNSs with the CRIRIC method. The proposed method is starting by using VR and metaverse to rank the customer needs and factors in product design.

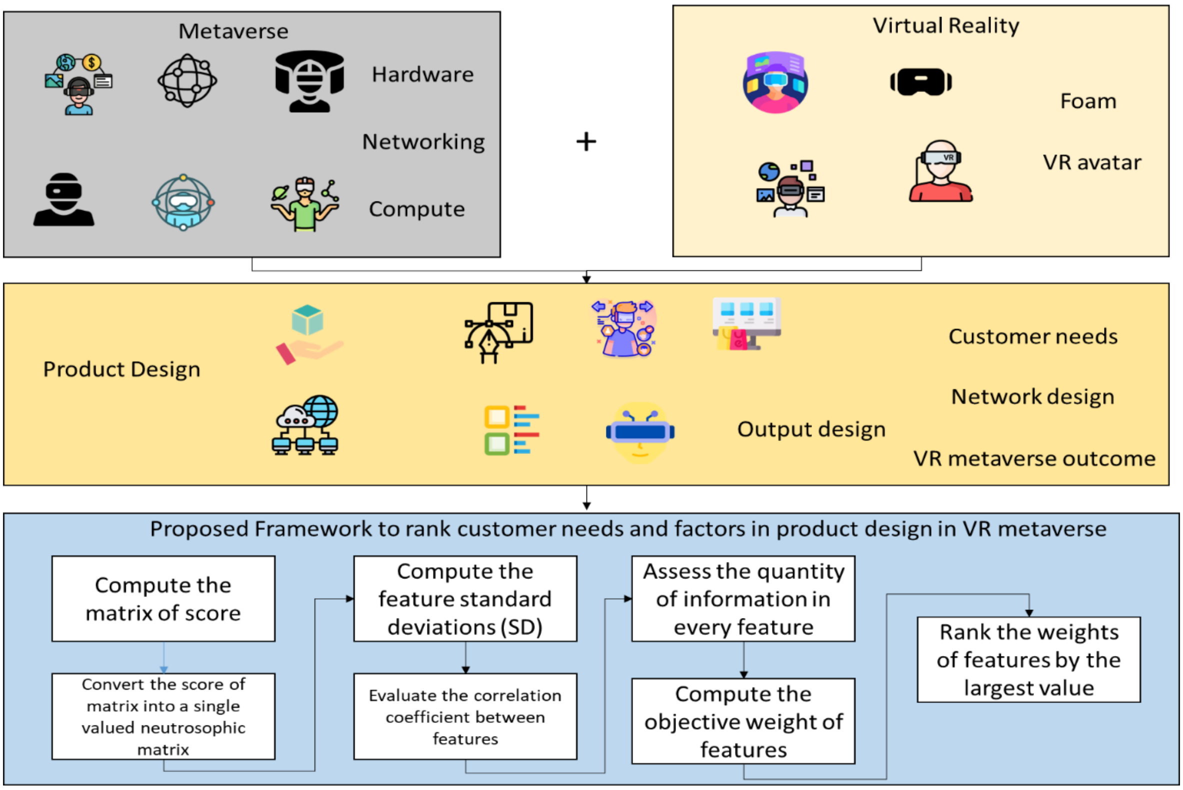

Figure 1. The VR metaverse in product design with the neutrosophic CRITIC method.

5. Metaverse and Authentication

Some people dismiss the enhanced virtual reality technology known as the Metaverse as pure fantasy. However, verses are virtual worlds, and the metaverse is the overarching word for the future Internet. Video games also include digital environments, but the metaverse differs in important ways. In order to draw in a diverse audience, Metaverse should provide engaging content.

The greatest possible user experience requires that all user actions occur simultaneously and in real time. The token economy is the key to ensuring the system's long-term viability. The security of the network, token economy, and decentralized identities all depend on decentralization. Wallets will be used to store and access the digital assets across platforms. The need to safeguard user histories and digital assets will grow. We'll need stronger protections in metaverse, notably during the authentication process[29], [30].

Users and programs must go through authentication before being granted access to a protected area. After successfully authenticating, the user should have access to all resources; nevertheless, security and privacy must be guaranteed. Security refers to the safeguarding of people and their belongings against any and all threats to life and limb. When information, transaction data, or communication belonging to the parties engaged in a transaction is shielded from third parties, we say that the parties have privacy. As our lives grow increasingly intertwined with technological systems, protecting the confidentiality of private information becomes more challenging. Some methods that may be used to verify a user's identity in the metaverse include full-body scanning, face recognition software, DNA identification, and retinal recognition. Because of the sensitive nature of this information, protecting the privacy and security of biometric data is more important than ever before[31], [32].

6. The CRITIC Method

The CRITIC method evaluates the relative importance of factors in MCDM problems based on hard data. The objective weights do this by combining a measure of correlation coefficient (CRC) between qualities with an evaluation of the intensity contrast standard deviation (SD) between features[33], [34]. The weights are $e = (e_1, e_2, \ldots\ldots e_n)^T, and\ \sum_{j=1}^{n} e_j = 1\,, e_j \in [0,1]$. The steps involved in configuring single-valued neutrosophic sets are outlined below.

(i) Compute the matrix of score

$$S(b_{ij}) = \frac{3+T_{ij}-2I_{ij}-F_{ij}}{4} \quad (1)$$

(ii) Convert the score of matrix into a single valued neutrosophic matrix.

$$\tilde{B}_{ij} = \begin{cases} \frac{B_{ij}-\min_j B_{ij}}{\max_j B_{ij}-\min_j B_{ij}} & for\ positive\ criteria \\ \frac{\max_j B_{ij}-B_{ij}}{\max_j B_{ij}-\min_j B_{ij}} & for\ cost\ criteria \end{cases} \quad (2)$$

(iii) $$SD_j = \sqrt{\frac{\sum_{i=1}^{m}\left(\tilde{B}_{ij}-\sum_{i=1}^{m}\frac{\tilde{B}_{ij}}{m}\right)^2}{m}} \quad (3)$$

(iv) Evaluate the correlation coefficient between features.

$$q_{ij} = \frac{\sum_{i=1}^{m}\left(\tilde{B}_{ij}-\sum_{j=1}^{m}\frac{\tilde{B}_j}{m}\right)\left(\tilde{B}_{ij}-\sum_{i=1}^{m}\frac{\tilde{B}_i}{m}\right)}{\sqrt{\sum_{i=1}^{m}\left(\tilde{B}_{ij}-\sum_{j=1}^{m}\frac{\tilde{B}_j}{m}\right)^2\left(\tilde{B}_{ij}-\sum_{i=1}^{m}\frac{\tilde{B}_i}{m}\right)^2}} \quad (4)$$

(v) Assess the quantity of information in every feature.

$$P_j = SD_j \sum_{t=1}^{n} (1 - q_{jt}) \quad (5)$$

(vi) Compute the objective weight of features.

$$e_j = \frac{P_j}{\sum_{j=1}^{n} P_j} \quad (6)$$

(vii) Rank the weights of features by the largest value.

7. Application

As the metaverse industry evolves rapidly, so are the technical standards for the gear used in it. The proposed hybrid approach is useful for evaluating the design of any technology item since it takes into account both technological and aesthetic factors. To show how the suggested hybrid MCDM technique may be utilized to rank the factors and needs of customers in designing products in the VR metaverse.

It is important to give each element and sub-factor enough consideration when ranking the factors and needs of the customer in product design in the VR metaverse. The variables and sub-factor weights were determined by polling an expert panel. Stakeholders were polled via the use of questionnaires.

Brainstorming sessions or in-depth interviews with prospective customers, key consumers, and technical teams are often used to assess consumer requirements. Next, the requirements need to be weighted based on how important they are. To achieve this objective, it is helpful to ask participants to compare various design elements in a survey by having them rank them on a scale of importance using SVNSs CRITIC. The technical design aspects that are most significant to VR head-mounted systems are identified via market analysis of technical requirements.

The SVNSs CRITIC is used to compute the weights of factors and needs of customers in product design in the VR metaverse. We collected four main factors and 14 sub-factors in the needs of customers to compute the weight as shown in Figure 2.

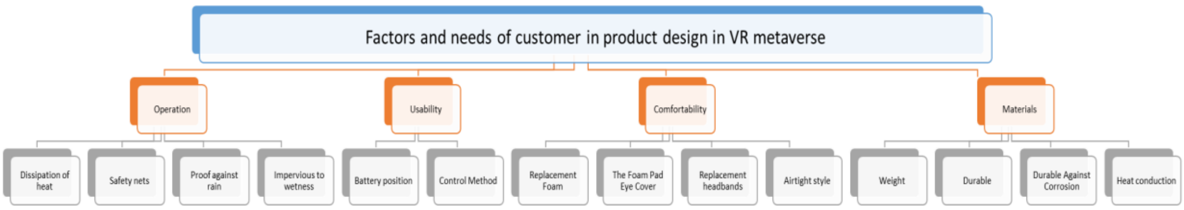

Figure 2. The needs and factors of customer in product design in VR metaverse.

The SVNSs CRITIC method is applied in the data, which collected from survey and previous studies. We collect experts have expertise in product design and VR metaverse to assess the factors and needs of customer in product design in VR metaverse. The experts build the score matrix by using single valued neutrosophic numbers by using Eq. (1). Then compute the normalization score matrix by using Eq. (2). Then compute the standard deviation by using Eq. (3). Then compute the correlation between features by using Eq. (4) as shown in Table 1. Then assess the information by all features by using Eq. (5) as shown in Table 2. Then compute the objective weight by using Eq. (6) as shown in Figure 3. The rank the weights of features. From Figure 3, thermal dissipation, airtight construction, weight, and control mechanisms are among the most crucial considerations.

Table 1. Correlation coefficient of all features.

	VRM_1	VRM_2	VRM_3	VRM_4	VRM_5	VRM_6	VRM_7	VRM_8	VRM_9	VRM_{10}	VRM_{11}	VRM_{12}	VRM_{13}	VRM_{14}
VRM_1	1	0.066872	0.122363	-0.00619	-0.72244	0.315618	-0.09081	0.151651	0.310365	0.251944	0.320882	0.643458	0.290807	0.331133
VRM_2	0.066872	1	-0.58167	0.013252	-0.08454	-0.32151	-0.20769	0.040565	0.536038	-0.28328	0.258597	-0.55979	0.360556	0.596401
VRM_3	0.122363	-0.58167	1	-0.76654	0.205529	0.637387	-0.61338	-0.38597	-0.06301	0.240776	-0.48929	0.623583	-0.55511	-0.04538
VRM_4	-0.00619	0.013252	-0.76654	1	-0.30576	-0.47856	0.768982	0.701272	-0.38212	-0.02174	0.638936	-0.25582	0.684383	-0.44129
VRM_5	-0.72244	-0.08454	0.205529	-0.30576	1	-0.40293	-0.24295	-0.15749	0.139099	-0.53968	-0.11869	-0.60432	-0.43863	0.001495
VRM_6	0.315618	-0.32151	0.637387	-0.47856	-0.40293	1	-0.32274	-0.43155	-0.15148	0.863602	-0.61363	0.746693	-0.27668	0.058412
VRM_7	-0.09081	-0.20769	-0.61338	0.768982	-0.24295	-0.32274	1	0.121191	-0.1641	0.116103	0.13875	-0.14608	0.094001	-0.2111
VRM_8	0.151651	0.040565	-0.38597	0.701272	-0.15749	-0.43155	0.121191	1	-0.44436	-0.2482	0.895879	-0.12301	0.892859	-0.54417
VRM_9	0.310365	0.536038	-0.06301	-0.38212	0.139099	-0.15148	-0.1641	-0.44436	1	-0.26232	-0.01759	-0.24655	-0.31766	0.96073
VRM_{10}	0.251944	-0.28328	0.240776	-0.02174	-0.53968	0.863602	0.116103	-0.2482	-0.26232	1	-0.4526	0.57702	-0.05055	-0.04129
VRM_{11}	0.320882	0.258597	-0.48929	0.638936	-0.11869	-0.61363	0.13875	0.895879	-0.01759	-0.4526	1	-0.24777	0.814418	-0.16837
VRM_{12}	0.643458	-0.55979	0.623583	-0.25582	-0.60432	0.746693	-0.14608	-0.12301	-0.24655	0.57702	-0.24777	1	-0.11328	-0.16477
VRM_{13}	0.290807	0.360556	-0.55511	0.684383	-0.43863	-0.27668	0.094001	0.892859	-0.31766	-0.05055	0.814418	-0.11328	1	-0.31543
VRM_{14}	0.331133	0.596401	-0.04538	-0.44129	0.001495	0.058412	-0.2111	-0.54417	0.96073	-0.04129	-0.16837	-0.16477	-0.31543	1

Table 2. The evaluation of information of every feature.

	VRM_1	VRM_2	VRM_3	VRM_4	VRM_5	VRM_6	VRM_7	VRM_8	VRM_9	VRM_{10}	VRM_{11}	VRM_{12}	VRM_{13}	VRM_{14}
VRM_1	0	0.933128	0.877637	1.006193	1.722438	0.684382	1.090812	0.848349	0.689635	0.748056	0.679118	0.356542	0.709193	0.668867
VRM_2	0.933128	0	1.58167	0.986748	1.084544	1.321506	1.207689	0.959435	0.463962	1.283275	0.741403	1.559792	0.639444	0.403599
VRM_3	0.877637	1.58167	0	1.766544	0.794471	0.362613	1.613379	1.385973	1.063014	0.759224	1.489289	0.376417	1.555109	1.045381
VRM_4	1.006193	0.986748	1.766544	0	1.305759	1.478565	0.231018	0.298728	1.382121	1.021737	0.361064	1.25582	0.315617	1.44129
VRM_5	1.722438	1.084544	0.794471	1.305759	0	1.40293	1.242946	1.15749	0.860901	1.539678	1.118695	1.604316	1.438634	0.998505
VRM_6	0.684382	1.321506	0.362613	1.478565	1.40293	0	1.32274	1.431549	1.151483	0.136398	1.613633	0.253307	1.276678	0.941588
VRM_7	1.090812	1.207689	1.613379	0.231018	1.242946	1.32274	0	0.878809	1.164098	0.883897	0.86125	1.146084	0.905999	1.211096
VRM_8	0.848349	0.959435	1.385973	0.298728	1.15749	1.431549	0.878809	0	1.444365	1.248201	0.104121	1.123011	0.107141	1.544165
VRM_9	0.689635	0.463962	1.063014	1.382121	0.860901	1.151483	1.164098	1.444365	0	1.262317	1.017591	1.246552	1.317659	0.03927
VRM_{10}	0.748056	1.283275	0.759224	1.021737	1.539678	0.136398	0.883897	1.248201	1.262317	0	1.4526	0.42298	1.050545	1.041289
VRM_{11}	0.679118	0.741403	1.489289	0.361064	1.118695	1.613633	0.86125	0.104121	1.017591	1.4526	0	1.247769	0.185582	1.168373
VRM_{12}	0.356542	1.559792	0.376417	1.25582	1.604316	0.253307	1.146084	1.123011	1.246552	0.42298	1.247769	0	1.113285	1.164774
VRM_{13}	0.709193	0.639444	1.555109	0.315617	1.438634	1.276678	0.905999	0.107141	1.317659	1.050545	0.185582	1.113285	0	1.315434
VRM_{14}	0.668867	0.403599	1.045381	1.44129	0.998505	0.941588	1.211096	1.544165	0.03927	1.041289	1.168373	1.164774	1.315434	0

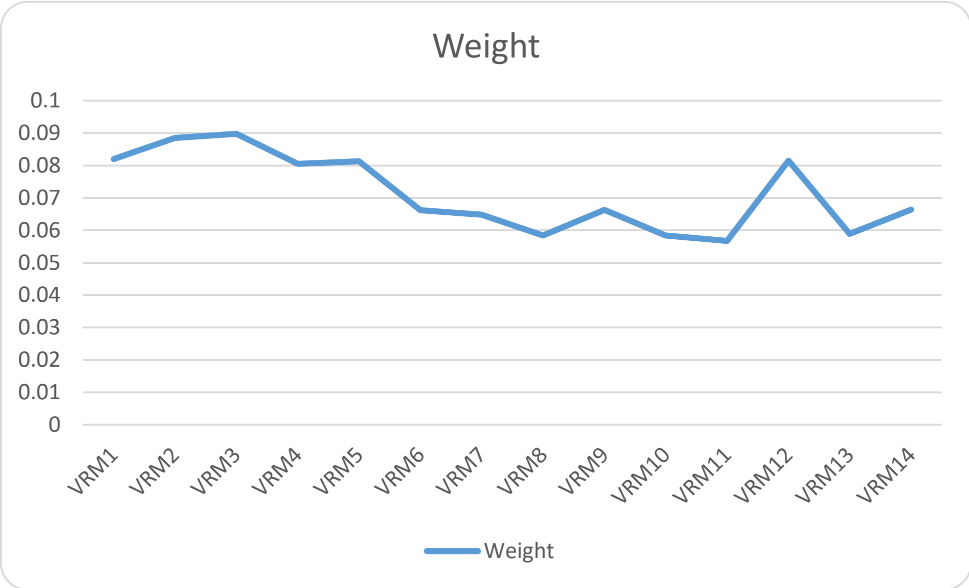

Figure 3. The objective weight of all features.

8. Conclusion

As technology firms shift their focus to immersive VR as a strategic chance, the VR sector is quickly developing increasingly powerful and adaptable HMDs and VR accessories. The VR metaverse has a big attention in product design, so many firms seek to obtain the needs of customers to improve their product design. So this paper used the MCDM method to rank factors and needs of customers in product design in the VR metaverse. The purpose of this research is to propose a novel decision-making approach to evaluating and ranking factors and needs of customers in product design in the VR metaverse in an SVNSs setting. First, a novel approach was presented that used the CRITIC technique while also making use of SVNSs. The CRITIC approach was used to calculate the relative importance of each criterion in this technique. The needs of customers are ranked based on the CRITIC method. The neutrosophic set is used to overcome incomplete and vague data. Thermal dissipation, airtight construction, weight, and control mechanisms are among the most crucial considerations.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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