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*"***Neutrosophic Systems with Applications***"* has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e., notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjointed two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (*T*), a degree of indeterminacy (*I*), and a degree of falsity (*F*), where *T, I, F* are standard or non-standard subsets of *]-0, 1+[*.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of classical statistics.

What distinguishes neutrosophic from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Contents

Sudeep Dey, Gautam Chandra Ray, **Properties of Redefined Neutrosophic Composite Relation**..1

Nabil M. AbdelAziz, Safa Al-Saeed, **Mitigating Landslide Hazards in Qena Governorate of Egypt: A GIS-based Neutrosophic PAPRIKA Approach**...13

D. Sasikala, B.Divya, **A Newfangled Interpretation on Fermatean Neutrosophic DOMBI Fuzzy Graphs**..36

N. Raksha Ben, G. Hari Siva Annam, G.Helen Rajapushpam, **A Perspective Note on μ_N σ Baire's Space** 54

Uma. G, Nandhitha. S, **An Investigative Study on Quick Switching System using Fuzzy and Neutrosophic Poisson Distribution**..61

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Properties of Redefined Neutrosophic Composite Relation

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Abstract: The notion of single-valued neutrosophic composite relation was redefined by S.Dey and G.C.Ray in the year 2022. In this article, we investigate some basic properties of the redefined neutrosophic composite relation.

Keywords: Neutrosophic set; Single-valued neutrosophic set; Single-valued neutrosophic relation; Redefined neutrosophic composite relation.

1. Introduction

The concept of neutrosophic set was introduced by Smarandache [13,14] in the 1990's. Afterwards many researchers [7,8,11,12,15] studied and developed it. Since its inception, the neutrosophic set has garnered significant interest from researchers worldwide due to its flexibility and effectiveness. It has proven to be not only valuable in the advancement of science and technology but also applicable in various other fields. For instance, works[1,2,6,18,19] on medical diagnosis, decision-making problems, image processing, social issues etc. had also been done in a neutrosophic environment.

In 2010, Wang et al.[16] further developed the notion of a single-valued neutrosophic set. Salma et.al. [9,10] added the thinking of neutrosophic relation and studied some of its properties. Building upon these concepts, Yang et al.[17] in 2016 introduced single-valued neutrosophic relation and investigated some properties. Taking the concept forward, Kim et al.[5] generalized the notion of a of single-valued neutrosophic relation from a set X to a set Y. The authors also introduced the composition of two neutrosophic relations and throughly examined various properties associated with it.

More recently, in 2022, S.Dey and G.C.Ray [3] introduced a novel definition for the neutrosophic composite relation of two single-valued neutrosophic relations. In this article, we aim to explore and investigate some properties related to the redefined neutrosophic composite relation.

2. Preliminaries

In this section we confer some basic concepts which will be helpful in the later sections.

2.1. Definition: [13] Let X be the universe of discourse. A neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where the functions $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are real standard or non-standard subsets of $]^-0, 1^+[$, i.e., $\mathcal{T}_A : X \to]^-0, 1^+[, \mathcal{I}_A : X \to]^-0, 1^+[, \mathcal{F}_A : X \to]^-0, 1^+[$ and $^-0 \le \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \le 3^+$.

The neutrosophic set A is characterized by the truth-membership function $\mathcal{T}_A$, indeterminacy-membership function $\mathcal{I}_A$, falsehood-membership function $\mathcal{F}_A$.

2.2. Definition:[16] Let X be the universe of discourse. A single-valued neutrosophic set (SVNS, for short) A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are functions from X to $[0,1]$ and $0 \le \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \le 3$.

The functions $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ denote respectively the degrees of truth-membership, indeterminacy-membership, falsehood-membership of the element $x \in X$ in A.
The set of all single-valued neutrosophic sets over X is denoted by $\mathcal{N}(X)$.
2.3. Definition:**[4]** Let $A, B \in \mathcal{N}(X)$. Then

i. (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.

ii. (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

iii. (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x) \rangle : x \in X\}$.

iv. (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x) \rangle : x \in X\}$.

v. (Complement): The complement of the NS A, denoted by A^c, is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x) \rangle : x \in X\}$

vi. (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

vii. (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\widetilde{\emptyset}$.

2.4. Definition: **[4]** Let $A, B \in \mathcal{N}(X)$ and $\{A_i : i \in \Delta\} \subseteq \mathcal{N}(X)$, Δ is an index set. Then the following hold.

i. $A \cup A = A$ and $A \cap A = A$

ii. $A \cup B = B \cup A$ and $A \cap B = B \cap A$

iii. $A \cup \widetilde{\emptyset} = A$ and $A \cup \tilde{X} = \tilde{X}$

iv. $A \cap \widetilde{\emptyset} = \widetilde{\emptyset}$ and $A \cap \tilde{X} = A$

v. $A \cap (B \cap C) = (A \cap B) \cap C$ and $A \cup (B \cup C) = (A \cup B) \cup C$

vi. $(A^c)^c = A$

vii. $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$

viii. $(\cup_{i \in \Delta} A_i)^c = \cap_{i \in \Delta} A_i^c$ and $(\cap_{i \in \Delta} A_i)^c = \cup_{i \in \Delta} A_i^c$

ix. $B \cup (\cap_{i \in \Delta} A_i) = \cap_{i \in \Delta} (B \cup A_i)$

x. $B \cap (\cup_{i \in \Delta} A_i) = \cup_{i \in \Delta} (B \cap A_i)$

2.5. Definition: **[5]** Let X, Y, Z be three ordinary sets. Then R is called a single-valued neutrosophic relation (SVNR, for short) from X to Y if it is a SVNS in $X \times Y$ having the form $R = \{\langle (x, y), \mathcal{T}_R(x, y), \mathcal{I}_R(x, y), \mathcal{F}_R(x, y) \rangle : (x, y) \in X \times Y\}$, where $\mathcal{T}_R : X \times Y \to [0,1], \mathcal{I}_R : X \times Y \to [0,1], \mathcal{F}_R : X \times Y \to [0,1]$ denote respectively the truth-membership function, indeterminacy-membership function, falsity-membership function.
In particular, a SVNR from from X to X is called a SVNR in X.
The empty SVNR and the whole SVNR in X, denoted by $\widetilde{\emptyset}_N$ and $\tilde{X}_N$ respectively, are defined as $\widetilde{\emptyset}_N = \{\langle (x, y), 0,1,1 \rangle : (x, y) \in X \times X\}$ and $\tilde{X}_N = \{\langle (x, y), 1,0,0 \rangle : (x, y) \in X \times X\}$.

The set of all SVNRs from X to Y is denoted by $SVNR(X \times Y)$ and the set of all SVNRs in X is denoted by $SVNR(X)$.

2.6. Definition: [5] Let $R \in SVNR(X \times Y)$. Then

i. The inverse of R, denoted by R^{-1}, is a SVNR from Y to X defined as $R^{-1}(y,x) = R(x,y)$ for each $(y,x) \in Y \times X$.

ii. The complement of R, denoted by R^c, is a SVNR from X to Y defined as $\mathcal{T}_R^c(x,y) = \mathcal{F}_R(x,y), \mathcal{I}_R^c(x,y) = 1 - \mathcal{I}_R(x,y), \mathcal{F}_R^c(x,y) = \mathcal{T}_R(x,y)$ for each $(x,y) \in X \times Y$.

2.7. Definition: **[5]** Let $R, S \in SVNR(X \times Y)$. Then

i. R is said to be contained in S, denoted by $R \subseteq S$, if $\mathcal{T}_R(x,y) \leq \mathcal{T}_S(x,y), \mathcal{I}_R(x,y) \geq \mathcal{I}_S(x,y)$, $\mathcal{F}_R(x,y) \geq \mathcal{I}_S(x,y)$ for each $(x,y) \in X \times Y$.

ii. R is said to be equal to S, denoted by $R = S$, if $R \subseteq S$ and $S \subseteq R$.

iii. The intersection of R and S, denoted by $R \cap S$, is defined as $R \cap S = \{\langle (x,y), \mathcal{T}_R(x,y) \wedge \mathcal{T}_S(x,y), \mathcal{I}_R(x,y) \vee \mathcal{I}_S(x,y), \mathcal{F}_R(x,y) \vee \mathcal{F}_S(x,y) \rangle : (x,y) \in X \times Y\}$.

iv. The union of R and S, denoted by $R \cup S$, is defined as $R \cup S = \{\langle (x,y), \mathcal{T}_R(x,y) \vee \mathcal{T}_S(x,y), \mathcal{I}_R(x,y) \wedge \mathcal{I}_S(x,y), \mathcal{F}_R(x,y) \wedge \mathcal{F}_S(x,y) \rangle : (x,y) \in X \times Y\}$.

2.8. Definition: **[5]** Let X, Y, Z be three ordinary sets. Also let $R \in SVNR(X \times Y)$ and $S \in SVNR(Y \times Z)$. Then the composition(max-min-max composition) of R and S, denoted by $S \circ R$, is a SVNR from X to Z defined as

$$S \circ R = \{\langle (x,z), \mathcal{T}_{S\circ R}(x,z), \mathcal{I}_{S\circ R}(x,z), \mathcal{F}_{S\circ R}(x,z) \rangle : (x,z) \in X \times Z\},$$

where

$$\mathcal{T}_{S\circ R}(x,z) = \vee_{y\in Y} (\mathcal{T}_R(x,y) \wedge \mathcal{T}_S(y,z)),$$
$$\mathcal{I}_{S\circ R}(x,z) = \wedge_{y\in Y} (\mathcal{I}_R(x,y) \vee \mathcal{I}_S(y,z)),$$
$$\mathcal{F}_{S\circ R}(x,z) = \wedge_{y\in Y} (\mathcal{F}_R(x,y) \vee \mathcal{F}_S(y,z)).$$

2.9. Definition:**[5]**

i. The single-valued neutrosophic identity relation in X, denoted by I_X, is defined as : for each $(x,y) \in X \times X$, $\mathcal{T}_{I_X}(x,y) = 1, \mathcal{I}_{I_X}(x,y) = 0, \mathcal{F}_{I_X}(x,y) = 0$ if $x = y$ and $\mathcal{T}_{I_X}(x,y) = 0, \mathcal{I}_{I_X}(x,y) = 1, \mathcal{F}_{I_X}(x,y) = 1$ if $x \neq y$.

ii. A SVNR R in X is said to be reflexive if for each $x \in X$, $\mathcal{T}_R(x,x) = 1, \mathcal{I}_R(x,x) = 0, \mathcal{F}_R(x,x) = 0$.

iii. A SVNR R in X is said to be symmetric if for each $(x,y) \in X \times X$, $\mathcal{T}_R(x,y) = \mathcal{T}_R(y,x), \mathcal{I}_R(x,y) = \mathcal{I}_R(y,x), \mathcal{F}_R(x,y) = \mathcal{F}_R(y,x)$.

iv. A SVNR R in X is said to be transitive if $R \circ R \subseteq R$, i.e., $R^2 \subseteq R$.

2.10. Proposition:**[5]** Let X be an ordinary set and $R \in SVNR(X)$. Then R is symmetric iff $R^{-1} = R$.

***2.11. Definition:*[3]** Let X, Y, Z be three ordinary sets. Also let $R \in SVNR(X \times Y)$ and $S \in SVNR(Y \times Z)$. Then the redefined neutrosophic composite relation of the SVNRs R and S, denoted by $S \circ R$, is a SVNR from X to Z defined as

$$S \circ R = \{\langle (x,z), \mathcal{T}_{S\circ R}(x,z), \mathcal{I}_{S\circ R}(x,z), \mathcal{F}_{S\circ R}(x,z)\rangle : (x,z) \in X \times Z\},$$

where

$$\mathcal{T}_{S\circ R}(x,z) = \vee_{y\in Y} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_S(y,z)}{2},$$

$$\mathcal{I}_{S\circ R}(x,z) = \wedge_{y\in Y} \frac{\mathcal{I}_R(x,y)+\mathcal{I}_S(y,z)}{2},$$

$$\mathcal{F}_{S\circ R}(x,z) = \wedge_{y\in Y} \frac{\mathcal{F}_R(x,y)+\mathcal{F}_S(y,z)}{2}.$$

***2.12 Example*:** Let $X = \{a, b\}, Y = \{p, q\}, Z = \{u, v\}$. Also let $R \in SVNR(X \times Y)$ and $S \in SVNR(Y \times Z)$ be given by the Table-1, Table-2.

Table-1

R	p	q
a	(0.6, 0.1, 0.2)	(0.1, 0.2, 0.7)
b	(0.5, 0.6, 0.7)	(0.3, 0.2, 0.1)

Table-2

S	u	v
p	(0.5, 0.3, 0.2)	(0.6, 0.4, 0.3)
q	(0.9, 0.1, 0.2)	(0.2, 0.5, 0.4)

Then by using the definition 2.11, we have

$$\mathcal{T}_{S\circ R}(a,u) = \vee_{y\in Y} \frac{\mathcal{T}_R(a,y)+\mathcal{T}_S(y,u)}{2} = \vee \{\frac{0.6+0.5}{2}, \frac{0.1+0.9}{2}\} = 0.55.$$

$$\mathcal{I}_{S\circ R}(a,u) = \wedge_{y\in Y} \frac{\mathcal{I}_R(a,y)+\mathcal{I}_S(y,u)}{2} = \wedge\{\frac{0.1+0.3}{2}, \frac{0.2+0.1}{2}\} = 0.15.$$

$$\mathcal{F}_{S\circ R}(a,u) = \wedge_{y\in Y} \frac{\mathcal{F}_R(a,y)+\mathcal{F}_S(y,u)}{2} = \wedge \{\frac{0.2+0.2}{2}, \frac{0.7+0.2}{2}\} = 0.20.$$

Similarly proceeding for the pairs $(a, v), (b, u), (b, v)$, we get the redefined neutrosophic composite relation $S \circ R \in SVNR(X \times Z)$ as shown in the following Table-3.

Table-3

$S \circ R$	u	v
a	(0.55, 0.15, 0.20)	(0.60, 0.25, 0.25)
b	(0.60, 0.15, 0.15)	(0.55, 0.35, 0.25)

Main Results: In this section we study the properties of redefined neutrosophic composite relation.

***3.1. Proposition*:** Let X, Y, Z be three ordinary sets. Also let $R, S \in SVNR(X \times Y)$ and $P \in SVNR(Y \times Z)$. Then

i. $P \circ (R \cup S) = (P \circ R) \cup (P \circ S)$.

ii. $R \subseteq S \Rightarrow P \circ R \subseteq P \circ S$.

iii. $(P \circ R)^{-1} = R^{-1} \circ P^{-1}$.

Proof:

i. Clearly $P \circ (R \cup S), (P \circ R) \cup (P \circ S) \in SVNR(X \times Z)$. Let $(x, z) \in X \times Z$. Then

$$\mathcal{T}_{P\circ(R\cup S)}(x,z) = \bigvee_{y\in Y} \frac{\mathcal{T}_{R\cup S}(x,y)+\mathcal{T}_P(y,z)}{2}$$

$$= \bigvee_{y\in Y} \frac{(\mathcal{T}_R(x,y)\vee\mathcal{T}_S(x,y))+\mathcal{T}_P(y,z)}{2}$$

$$= \bigvee_{y\in Y} [\frac{\mathcal{T}_R(x,y)+\mathcal{T}_P(y,z)}{2} \vee \frac{\mathcal{T}_S(x,y)+\mathcal{T}_P(y,z)}{2}]$$

$$= [\bigvee_{y\in Y} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_P(y,z)}{2}] \vee [\bigvee_{y\in Y} \frac{\mathcal{T}_S(x,y)+\mathcal{T}_P(y,z)}{2}]$$

$$= \mathcal{T}_{P\circ R}(x,z) \vee \mathcal{T}_{P\circ S}(x,z)$$
$$= \mathcal{T}_{(P\circ R)\cup(P\circ S)}(x,z)$$

Similarly we can show that $\mathcal{I}_{P\circ(R\cup S)}(x,z) = \mathcal{I}_{(P\circ R)\cup(P\circ S)}(x,z)$ and $\mathcal{F}_{P\circ(R\cup S)}(x,z) = \mathcal{F}_{(P\circ R)\cup(P\circ S)}(x,z)$. Therefore $P \circ (R \cup S) = (P \circ R) \cup (P \circ S)$.

ii. Clearly $P \circ R, P \circ S \in SVNR(X \times Z)$. Let $(x, z) \in X \times Z$. Then

$$\mathcal{T}_{P\circ R}(x,z) = \bigvee_{y\in Y} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_P(y,z)}{2} \leq \bigvee_{y\in Y} \frac{\mathcal{T}_S(x,y)+\mathcal{T}_P(y,z)}{2} [\because R \subseteq S] = \mathcal{T}_{P\circ S}(x,z).$$

Therefore, $\mathcal{T}_{P\circ R}(x,z) \leq \mathcal{T}_{P\circ S}(x,z)$.
Similarly we can show that $\mathcal{I}_{P\circ R}(x,z) \geq \mathcal{I}_{P\circ S}(x,z)$ and $\mathcal{F}_{P\circ R}(x,w) \geq \mathcal{F}_{P\circ S}(x,z)$.
Hence $P \circ R \subseteq P \circ S$.

iii. Clearly $P \circ R \in SVNR(X \times Z)$ and $(P \circ R)^{-1}, R^{-1} \circ P^{-1} \in SVNR(Z \times X)$. Let $(z, x) \in Z \times X$. Then

$$\mathcal{T}_{(P\circ R)^{-1}}(z,x) = \mathcal{T}_{P\circ R}(x,z)$$
$$= \bigvee_{y\in Y} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_P(y,z)}{2}$$

$$= \bigvee_{y\in Y} \frac{\mathcal{T}_{R^{-1}}(y,x)+\mathcal{T}_{P^{-1}}(z,y)}{2}$$

$$= \mathcal{T}_{R^{-1}\circ P^{-1}}(z,x)$$

Similarly we can show that $\mathcal{I}_{(P\circ R)^{-1}}(z,x) = \mathcal{I}_{R^{-1}\circ P^{-1}}(z,x)$ and $\mathcal{F}_{(P\circ R)^{-1}}(z,x) = \mathcal{F}_{R^{-1}\circ P^{-1}}(z,x)$.
Therefore $(P \circ R)^{-1} = R^{-1} \circ P^{-1}$.

3.2. Remark: Redefined neutrosophic composite relation is not commutative. We shall establish by the following counter example.
Let $X = \{a, b\}, Y = \{p, q\}, Z = \{u, v\}$. Also let $R \in SVNR(X \times Y), P \in SVNR(Y \times Z)$. Obviously $P \circ R \in SVNR(X \times Z)$ and $R \circ P \in SVNR(Y \times Y)$. Therefore $P \circ R \neq R \circ P$.

3.3. Remark: Redefined neutrosophic composite relation is not associative. We shall establish by the following counter example.
Let $X = \{a, b\}, Y = \{p, q\}, Z = \{u, v\}, W = \{x, y\}$. Also let $R \in SVNR(X \times Y), P \in SVNR(Y \times Z)$ and $Q \in SVNR(Z \times W)$ be given by the following Table-4, Table-5, Table-6.

Table-4

R	p	q
a	(.6,.1,.2)	(.1,.2,.7)
b	(.5,.6,.7)	(.3,.2,.1)

Table-5

P	u	v
p	(.5,.3,.2)	(.6,.4,.3)
q	(.9,.1,.2)	(.3,.2,.1)

Table-6

Q	x	y
u	(.5,.4,.2)	(.5,.3,.1)
v	(.8,.2,.1)	(.3,.6,.4)

Then by using the definition 2.11, we find the redefined neutrosophic composite relations $P \circ R \in SVNR(X \times Z), Q \circ P \in SVNR(Y \times W), Q \circ (P \circ R) \in SVNR(X \times W), (Q \circ P) \circ R \in SVNR(X \times W)$ as shown in the following Table-7, Table-8, Table-9, Table-10.

Table-7

$P \circ R$	u	v
a	(.55,.15,.20)	(.60,.25,.25)
b	(.60,.15,.15)	(.55,.35,.25)

Table-8

$Q \circ P$	x	y
p	(.70,.20,.15)	(.50,.40,.25)
q	(.70,.25,.15)	(.70,.30,.20)

Table-9

$Q \circ (P \circ R)$	x	y
a	(.475,.225,.225)	(.475,.225,.225)
b	(.475,.225,.225)	(.475,.225,.225)

Table-10

$(Q \circ P) \circ R$	x	y
a	(.65,.15,.175)	(.55,.25,.225)
b	(.60,.225,.125)	(.50,.25,.15)

We see that $\mathcal{T}_{Q\circ(P\circ R)}(a,x) = 0.475$ and $\mathcal{T}_{(Q\circ P)\circ R}(a,x) = 0.65$. Since $\mathcal{T}_{Q\circ(P\circ R)}(a,x) \neq \mathcal{T}_{(Q\circ P)\circ R}(a,x)$, so $Q \circ (P \circ R) \neq (Q \circ P) \circ R$.

***3.4. Remark*:** Redefined neutrosophic composite relation is not distributive over intersection. We shall establish by the following counter example.
Let $X = \{a, b\}, Y = \{p, q\}, Z = \{u, v\}, W = \{x, y\}$. Also let $R, S \in SVNR(X \times Y), P \in SVNR(Y \times Z)$ be given by the Table-11, Table-12, Table-13.

Table-11

R	p	q
a	(.6,.1,.2)	(.1,.2,.7)
b	(.5,.6,.7)	(.3,.2,.1)

Table-12

S	p	q
a	(.8,.7,.3)	(.2,.0,.7)
b	(.7,.2,.3)	(.5,.6,.4)

Table-13

P	u	v
p	(.5,.3,.2)	(.6,.4,.3)
q	(.9,.1,.2)	(.3,.2,.1)

Then by using the definition 4(`)@, we find the SVNRs $R \cap S \in SVNR(X \times Y), P \circ (R \cap S) \in SVNR(X \times Z), P \circ R \in SVNR(X \times Z), P \circ S \in SVNR(X \times Z)$ and $(P \circ R) \cap (P \circ S) \in SVNR(X \times Y)$ as shown in the following Table-14, Table-15, Table-16, Table-17, Table-18.

Table-14

$P \circ R$	u	v
a	(.55,.15,.20)	(.60,.25,.25)
b	(.60,.15,.15)	(.55,.35,.25)

Table-15

$P \circ S$	u	v
a	(.65,.05,.25)	(.70,.25,.30)
b	(.70,.25,.25)	(.65,.30,.30)

Table-16

$R \cap S$	p	q
a	(.6,.7,.3)	(.1,.2,.7)
b	(.5,.6,.7)	(.3,.6,.4)

Table-17

$P \circ (R \cap S)$	u	v
a	(.55,.15,.25)	(.60,.35,.30)
b	(.60,.35,.30)	(.55,.50,.40)

Table-18

$(P \circ R) \cap (P \circ S)$	u	v
a	(.55,.05,.20)	(.60,.25,.25)
b	(.60,.15,.15)	(.55,.30,.25)

From the Table-17 and Table-18, it is easy to see that

$$\mathcal{I}_{P\circ(R\cap S)}(a,u) = .15 \quad \text{and} \quad \mathcal{I}_{(P\circ R)\cap(P\circ S)}(a,u) = .05.$$

Therefore $P \circ (R \cap S) \neq (P \circ R) \cap (P \circ S)$.

3.5. Proposition: Let X be an ordinary set and $R, S \in SVNR(X \times X)$. If R, S are reflexive then $S \circ R$ is reflexive.

Proof: For any two elements $x, y \in X$, we have

$$\mathcal{T}_{S\circ R}(x,x) = \bigvee_{y\in X} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_S(y,x)}{2}$$

$$= [\bigvee_{y\neq x} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_S(y,x)}{2}] \vee [\frac{\mathcal{T}_R(x,x)+\mathcal{T}_S(x,x)}{2}]$$

$$= [\bigvee_{y\neq x} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_S(y,x)}{2}] \vee [\frac{1+1}{2}] (\because R \text{ and } S \text{ are reflexive})$$

$$= [\bigvee_{y\neq x} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_S(y,x)}{2}] \vee 1 = 1$$

Again

$$\mathcal{I}_{S\circ R}(x,x) = \bigwedge_{y\in X} \frac{\mathcal{I}_R(x,y)+\mathcal{I}_S(y,x)}{2}$$

$$= [\bigwedge_{y\neq x} \frac{\mathcal{X}_R(x,y)+\mathcal{I}_S(y,x)}{2}]\wedge[\frac{\mathcal{I}_R(x,x)+\mathcal{I}_S(x,x)}{2}]$$

$$= [\bigwedge_{y\neq x} \frac{\mathcal{I}_R(x,y)+\mathcal{I}_S(y,x)}{2}]\wedge [\frac{0+0}{2}](\because R \text{ and } S \text{ are reflexive})$$

$$= [\bigwedge_{y\neq x} \frac{\mathcal{I}_R(x,y)+\mathcal{I}_S(y,x)}{2}]\wedge 0$$

$$= 0$$

Similarly we can show that $\mathcal{F}_{S\circ R}(x,x) = 0$.

Therefore, $S \circ R$ is reflexive.

3.6. Remark: Let X be an ordinary set and $R, P \in SVNR(X)$. If R, P are symmetric then $P \circ R$ may not be symmetric. We shall establish it by a counter example.

Let $X = \{a, b\}$. Also let $R, P \in SVNR(X)$ be given by the Table-19 and Table-20.

Table-19

R	a	b
a	(0.6, 0.1, 0.2)	(0.5, 0.6, 0.7)
b	(0.5, 0.6, 0.7)	(0.3, 0.2, 0.1)

Table-20

P	a	b
a	(0.5, 0.3, 0.2)	(0.6, 0.4, 0.3)
b	(0.6, 0.4, 0.3)	(0.2, 0.5, 0.4)

Then $\mathcal{T}_{P\circ R}(a,b) = \vee_{y\in X} \frac{\mathcal{T}_R(a,y)+\mathcal{T}_S(y,b)}{2} = \vee \ \{\frac{0.6+0.6}{2}, \frac{0.5+0.2}{2}\} = 0.6$

and $\mathcal{T}_{P\circ R}(b,a) = \vee_{y\in X} \frac{\mathcal{T}_R(b,y)+\mathcal{T}_S(y,a)}{2} = \vee \ \{\frac{0.5+0.5}{2}, \frac{0.3+0.6}{2}\} = 0.5$.

We can see that $\mathcal{T}_{P\circ R}(a,b) = 0.6 \neq 0.5 = \mathcal{T}_{P\circ R}(b,a)$. Therefore $P \circ R$ is not symmetric.

***3.7. Proposition*:** Let X be an ordinary set and $R, S \in SVNR(X \times X)$ are symmetric. Then $S \circ R$ is symmetric iff $S \circ R = R \circ S$.

Proof: Since R and S are symmtric, so $R^{-1} = R$ and $S^{-1} = S$ [by 2.10]. First suppose that $S \circ R$ is symmetric. Then $S \circ R = (S \circ R)^{-1} = R^{-1} \circ S^{-1} = R \circ S$. Conversely suppose that $S \circ R = R \circ S$. Then $(S \circ R)^{-1} = R^{-1} \circ S^{-1} = R \circ S$,i.e., $S \circ R$ is symmetric.

3.8. Proposition: Let X be an ordinary set and $R \in SVNR(X \times X)$ be transitive. Then $R \circ R$ is transitive.

Proof: Since R is transitive, so $R \circ R \subseteq R$, i.e., $R^2 \subseteq R$. Now

$$\mathcal{T}_{R^2 \circ R^2}(x,z) = \bigvee_{y \in X} \frac{\mathcal{T}_{R^2}(x,y) + \mathcal{T}_{R^2}(y,z)}{2}$$

$$\leq \bigvee_{y \in X} \frac{\mathcal{T}_R(x,y) + \mathcal{T}_R(y,z)}{2}$$

$$= \mathcal{T}_{R \circ R}(x,z)$$

$$= \mathcal{T}_{R^2}(x,z)$$

Similarly we can show that $\mathcal{I}_{R^2 \circ R^2}(x,z) \geq \mathcal{I}_{R^2}(x,z)$ and $\mathcal{F}_{R^2 \circ R^2}(x,z) \geq \mathcal{F}_{R^2}(x,z)$. Therefore $R^2 \circ R^2 \subseteq R^2$. Hence R^2, i.e., $R \circ R$ is transitive.

***3.9. Proposition*:** Let X be an ordinary set. If $R \in SVNR(X)$ is transitive R^{-1} is also transitive.

Proof: Since R is transitive, so $R \circ R \subseteq R$. Now

$$\mathcal{T}_{R^{-1} \circ R^{-1}}(x,z) = \bigvee_{y \in X} \frac{\mathcal{T}_{R^{-1}}(x,y) + \mathcal{T}_{R^{-1}}(y,z)}{2}$$

$$= \bigvee_{y \in X} \frac{\mathcal{T}_R(y,x) + \mathcal{T}_R(z,y)}{2}$$

$$= \mathcal{T}_{R \circ R}(z,x)$$

$$\leq \mathcal{T}_R(z,x)$$

$$= \mathcal{T}_{R^{-1}}(x,z)$$

Similarly we can show that $\mathcal{I}_{R^{-1} \circ R^{-1}}(x,z) \geq \mathcal{I}_{R^{-1}}(x,z)$ and $\mathcal{F}_{R^{-1} \circ R^{-1}}(x,z) \geq \mathcal{F}_{R^{-1}}(x,z)$. Therefore $R^{-1} \circ R^{-1} \subseteq R^{-1}$ and so, R^{-1} is transitive.

3.10. Remark: Let X be an ordinary set and $R, S \in SVNR(X)$. If R, S are transitive then $R \cup S$ and $R \cap S$ may not be transitive. We shall establish it by a counter example. Let $X = \{a, b\}$. Also let $R, S \in SVNR(X)$ be given by the Table-21 and Table-22.

Table-21

R	a	b
a	(0.8, 0.5, 0.4)	(0.6, 0.4, 0.5)
b	(0.7, 0.6, 0.2)	(0.7, 0.6, 0.3)

Table-22

S	a	b
a	(0.7, 0.4, 0.2)	(0.4, 0.6, 0.4)
b	(0.5, 0.4, 0.3)	(0.5, 0.4, 0.4)

Clearly R and S are transitive.

Then the relations $R \cup S$ and $R \cap S$ are as given in Table-23 and Table-24.

Table-23

$R \cup S$	a	b
a	(0.8, 0.4, 0.2)	(0.6, 0.4, 0.4)
b	(0.7, 0.4, 0.2)	(0.7, 0.4, 0.3)

Table-24

$R \cap S$	a	b
a	(0.7, 0.5, 0.4)	(0.4, 0.6, 0.5)
b	(0.5, 0.6, 0.3)	(0.5, 0.6, 0.4)

Now,

$$\mathcal{T}_{(R\cup S)\circ(R\cup S)}(a,b) = \vee_{y\in X} \frac{\mathcal{T}_R(a,y)+\mathcal{T}_S(y,b)}{2} = \vee \left\{\frac{0.8+0.6}{2}, \frac{0.6+0.7}{2}\right\} = 0.7$$

$$\text{and } \mathcal{T}_{(R\cap S)\circ(R\cap S)}(a,b) = \vee_{y\in X} \frac{\mathcal{T}_R(a,y)+\mathcal{T}_S(y,b)}{2} = \vee \left\{\frac{0.5+0.7}{2}, \frac{0.5+0.5}{2}\right\} = 0.6.$$

We can see that $\mathcal{T}_{(R\cup S)\circ(R\cup S)}(a,b) = 0.7 > 0.6 = \mathcal{T}_{R\cup S}(a,b)$, i.e. $(R\cup S)\circ(R\cup S) \nsubseteq R\cup S$. Therefore $R\cup S$ is not transitive.

We can also see that $\mathcal{T}_{(R\cap S)\circ(R\cap S)}(b,a) = 0.6 > 0.5 = \mathcal{T}_{R\cap S}(b,a)$, i.e. $(R\cap S)\circ(R\cap S) \nsubseteq R\cap S$. Therefore $R\cap S$ is not transitive.

3. Conclusion

In this article, we have investigated various properties in connection with redefined neutrosophic composite relation. Our investigations into the neutrosophic composite relation provide valuable insights and pave the way for further advancements in the field of neutrosophic algebra. We anticipate that the findings presented in this study will serve as a significant resource for researchers and scholars, enabling them to build upon our work and contribute to the ongoing development and exploration of neutrosophic algebra.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Mitigating Landslide Hazards in Qena Governorate of Egypt: A GIS-based Neutrosophic PAPRIKA Approach

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Abstract: This paper presents a novel approach to landslide susceptibility assessment in the Qena Governorate, Egypt, integrating the neutrosophic Multi-Criteria Decision-Making (MCDM) method, the Potentially All Pairwise RanKings of all possible Alternatives (PAPRIKA), and the ArcGIS weighted overlay technique. The research focuses on the quantification and prioritization of eight criteria: slope, aspect, proximity to road, soil type, proximity to river, land cover, elevation, and Lithology. These factors are evaluated under the uncertainty and indeterminacy of the neutrosophic environment by employing the PAPRIKA method. The results of the analysis are visualized and interpreted using ArcGIS weighted overlay, offering spatially explicit insights into the landslide-prone areas. This study's outcomes could significantly contribute to the overall understanding of landslide hazards in Qena, promoting better hazard management and mitigation strategies. The results of the study demonstrated varying levels of landslide susceptibility within the study area: 2% of the area was identified as having Very High Susceptibility, 17% presented High Susceptibility, 28% had Moderate Susceptibility, 44% indicated Low Susceptibility, 8% showed Very Low Susceptibility, and 1% with Practically No Susceptibility. These findings can aid local authorities and policy-makers in prioritizing areas for mitigation efforts based on their susceptibility to landslides. The study also incorporates a sensitivity analysis, exploring ten different scenarios to ensure the robustness and reliability of the results. In the first scenario, we adhere to our initial criteria weights to represent the current situation. In the second scenario, all criteria are accorded equal significance to check the model's steadfastness when no one criterion outweighs another. Scenarios three to ten each elevate the weightage of one criterion, allowing for a comprehensive understanding of each individual criterion's influence on the decision-making process. This systematic alteration helps pinpoint the salient features driving landslides and aids in fortifying our mitigation strategies.

Keywords: Neutrosophic Set; Geographic Information System; PAPRIKA; Multi-Criteria Decision-Making; Qena.

1. Introduction

Landslides represent a significant natural hazard that have the potential to cause substantial socio-economic harm and loss of life, particularly in areas characterized by challenging terrain and environmental conditions [1]. Notably, the Qena Governorate of Egypt is one such area that has been recurrently plagued by such events. As it encompasses a mix of various topographic and environmental features, this region has witnessed a rise in landslide incidents over the past decades [2]. The aftermath of these landslides often leads to severe economic implications, including the destruction of infrastructure, loss of arable land, and disruption of communication routes, thereby causing profound socioeconomic setbacks for the local inhabitants [3]. Moreover, there's a profound impact on the life of the local population, often leading to forced displacement, immediate danger to personal safety, and lasting trauma. As such, mitigating the frequency and intensity of these hazards becomes an issue of utmost importance. Furthermore, the complexities involved due to the various intrinsic and extrinsic factors contributing to landslide occurrences implicate the need for a comprehensive, integrated multi-criteria assessment method. This would not only enable predicting areas with high susceptibility but also aid in drawing effective mitigation strategies, consequently contributing toward a safer and economically stable Qena region.

Traditional solutions to predicting and managing landslide hazards generally revolve around a blend of geotechnical measures, engineering solutions, and land-use planning strategies [4, 5]. Some of these approaches include mechanical stabilization measures, such as building retaining walls and terraces, redirecting landslip flow paths, and slope degrading. Land-use strategies revolve around preventing infrastructural development in landslide-prone areas [6]. While these solutions can be effective, they often come with high implementation costs and feasibility issues, particularly in regions with complex socio-economic and environmental conditions. Additionally, conventional methodologies that aim to predict landslide-prone zones often rely on simple statistical models or deterministic methods. These methods use geological, topographic, and meteorological data to estimate landslide susceptibility. However, they often fail to adequately account for uncertain or indeterminate information and the inherent subjectivity in human decision-making, thereby reducing the overall accuracy of susceptibility maps. Furthermore, there is a limitation in their ability to quantify landslide hazards in a highly efficient, refined manner due to the lack of comprehensive integration of various environmental factors contributing to landslides. Hence, there is a pressing need for an approach that overcomes these shortcomings and enhances the understanding and prediction of landslide hazards.

To address the limitations of existing approaches, the focus of this study is to adopt an innovative approach involving the application of the neutrosophic Multi-Criteria Decision-Making (MCDM) method. This method allows for a more nuanced interpretation of landslide hazards by effectively capturing, representing, and processing uncertain, incomplete, and indeterminate information inherent in the decision-making process. The advantage of using the neutrosophic MCDM method lies in its ability to handle uncertainties and ambiguities, which is a common challenge in the environmental sciences. Besides, the integration of this method with the "Potentially All Pairwise RanKings of all Possible Alternatives" (PAPRIKA) method allows for a simplified, user-friendly, and easily understandable decision-making process. The proposed solution also integrates the use of the ArcGIS weighted overlay, aiding in the unification and management of diverse geographic data for hazard mapping, thus refining the analysis of landslide susceptibility zones. Consequently, it provides improved strategies for hazard mitigation and land-use planning, thereby assisting efforts in controlling landslide hazards in the Qena Governorate. In summary, the proposed neutrosophic MCDM approach using neutrosophic PAPRIKA and ArcGIS weighted overlay offers a more comprehensive, effective, and economically viable solution for landslide risk prediction and mitigation in Qena Governorate, transcending the limitations inherent in traditional methodologies.

1.1 Study Aims and Objectives

The specific aims and objectives of this study are as follows:

- To develop a comprehensive evaluation model for landslide susceptibility using the neutrosophic MCDM method integrated with PAPRIKA and ArcGIS weighted overlay. This model is intended to overcome the limitations of existing approaches, providing not only a more accurate understanding of landslide hazards but also more effective strategies for mitigation.
- To evaluate this model's effectiveness using eight predetermined evaluation criteria. These criteria include geological, hydrological, and environmental factors known to influence landslip occurrence such as slope gradient, slope aspect, elevation, lithology, land cover, distance from

roads, distance from rivers, and soil type. These were specifically chosen because they represent the key factors that contribute to landslide susceptibility.

- To test the models against actual observed landslide events in the Qena Governorate to verify their predictive capability and accuracy.
- To ultimately inform better land-use planning strategies and mitigation measures that can enhance community resilience against landslide hazards in the Qena Governorate of Egypt.
- To conduct sensitivity analysis that serves as a vital step to ascertain how different values of inputs in a given quantitative model influence the outputs. This step enables us to robustly evaluate landslide susceptibility under various scenarios, thereby contributing to our overarching objective of devising effective, scenario-specific mitigation strategies.

In the end, the objective is to provide a practical, cost-effective, and technologically innovative solution for mitigating landslide hazards. This approach has the potential to improve current prediction methods and inform land-use planning strategies, thus managing and reducing the risk of landslides in the Qena Governorate, as well as other regions facing similar hazards.

1.2 Contributions of this Study

This study carries several significant contributions:

- It proposes a novel, integrated approach to landslide hazard mitigation, combining the neutrosophic MCDM method with PAPRIKA and ArcGIS weighted overlay. This multidisciplinary approach brings together insights from environmental science, geographic information science, and decision theory to provide a more comprehensive solution to landslide hazard management.
- By employing the neutrosophic MCDM method, the study contributes to a more nuanced understanding of the complex, uncertain, and often ambiguous nature of landslide hazards. This method presents a way to navigate these uncertainties, thus enhancing the accuracy of landslide susceptibility modelling.
- This work introduces a practical utilization of the PAPRIKA method in environmental science, demonstrating its value in simplifying complex multi-criteria decision-making processes.
- It offers an advance in geospatial analysis for landslide hazard by showcasing the utility of ArcGIS weighted overlay in combining diverse geospatial data for hazard mapping.
- Notably, by testing the model against actual observed landslide events in Qena Governorate, it provides tangible evidence of the model's effectiveness, contributing to future research and practice in landslide hazard mitigation.
- The study ultimately encourages better informed, more sustainable land-use planning and policy decisions, contributing to enhanced community resilience and safety in landslide-prone areas.

1.3 Structure of the Study

This study is methodically organized in the following manner:

- Introduction: This section provides the background and context of the study, presenting the research problem and the proposed solution using a Geographic Information System (GIS)-based neutrosophic MCDM Approach.
- Literature Review: It summarizes the relevant previous research in the field, highlighting gaps that this study aims to fill.
- PAPRIKA: This section provides a basic pseudocode representation of the traditional PAPRIKA method to demonstrate its logic and operation.
- Neutrosophic PAPRIKA: Presents a thorough discussion about the neutrosophic PAPRIKA approach and its innovative application for landslide mitigation.
- Methodology: It outlines the step-by-step flowchart followed in this research, detailing the creation of the evaluation model using the neutrosophic MCDM method, PAPRIKA, and ArcGIS weighted overlay.

- Case Study Application and Discussion: This section applies the methodology to an actual case study of landslide hazard in Qena Governorate of Egypt, accompanied by a thoughtful discussion of the results.
- Sensitivity Analysis: An analysis is conducted to establish the robustness of the model, investigating how slight variations in input parameters affect the results.
- Conclusions: Renders a summary of the study's main findings, the implications for the field, and suggestions for future research.
- References: Cites all sources and previous works referenced throughout the study.

This structured approach ensures a comprehensive presentation of the research undertaken and facilitates a coherent presentation of its findings.

2. Literature Review

Landslides are geological phenomena characterized by the mass movement of rock, debris, or earth down slopes due to gravitational forces [7]. According to the International Association of Engineering Geology and the Environment (IAEG), landslides can be classified along two dimensions: their mechanism of movement and the type of material involved [8]. The three fundamental mechanisms include fall, topple, and slide, with composite movements often observed. The type of material can range from rock to soil and in between. This classification helps experts predict potential landslide events and devise appropriate responses [9].

The understanding of landslides has significantly evolved over time. In the early stages, landslides were perceived as random natural disasters that pose uncontrollable risks [10]. With advancements in geotechnical and geological understanding in the mid-20th century, researchers started to delve into the mechanics and triggers of landslides. Significant works by researchers like Terzaghi (1950), who introduced the concept of effective stress and its impact on soil shear strength [11], and Skempton (1977), who developed the concept of pore pressure and its pivotal role in landslide occurrence, have laid the foundation for our modern understanding of landslides [12].

Landslides generally occur due to a combination of natural and anthropogenic factors [13]. Natural factors include geomorphology (for example, the shape and structure of land), geology (the type of rock or soil, its condition, and water content), climate (specifically, prevailing weather patterns and incidents of heavy rainfall), and seismic activities (such as earthquakes and volcanic eruptions) [14]. Anthropogenic, or man-made factors, tend to exacerbate the vulnerability of areas to landslides [15]. These factors include deforestation, which reduces the water-holding capacity of the soil, and over-exploitation of land resources. Uncontrolled urban development and unregulated mining activities can also destabilize slopes and amplify the risk of landslides [16, 17]. In many instances, it is a combination of these factors that eventually trigger a landslide, which makes their prediction and prevention a complex task. These complexities further underline the need for an advanced and sophisticated approach like the neutrosophic MCDM for landslide hazard mitigation. The field of landslide research is rich with numerous notable studies. A seminal work in this regard is by Aleotti and Chowdhury (1999), who employed Fuzzy Logic to evaluate the uncertainties in the spatial prediction of landslides [18]. This study underscored the importance of accounting for uncertainty in landslide research and further paved the way for the application of advanced decision-making frameworks in this domain. Another pioneering study by Guzzetti et al. (1999) utilized GIS to map landslide susceptibility across Italy and demonstrated the powerful potential of GIS in landslide research [19]. Similarly, a more recent study conducted by Wang et al. (2020) used machine learning algorithms in GIS for landslide susceptibility modeling, revealing a new frontier in the application of artificial intelligence in landslide research [20]. Hungr et al. (2014) comprehensively reviewed and synthesized various quantitative risk assessment tools for landslide hazards, providing a reference point for future methodological development [21]. These studies are instrumental in augmenting the existing knowledge and developing more sophisticated landslide mitigation strategies, including the application of the neutrosophic MCDM approach.

Traditional methods of landslide susceptibility mapping have included deterministic and statistical models. While deterministic models focus on a specified set of physical variables (e.g., slope stability or shear strength), statistical models apply past landslide data to identify and analyze trends. For instance, Guzzetti et al. (1999) employed a traditional deterministic method to produce a map of landslide susceptibility for Italy, incorporating variables such as slope angle and climatology [22]. The resultant map was used in planning developments and managing natural hazards. However, these traditional models have limitations. They often rely heavily on specific physical parameters making it difficult to account for variations in local environmental conditions and human activity. They are also predicated on the availability of accurate historical landslide data, a criterion that is not always met, especially in regions where landslides are less frequent. More worryingly, traditional models are usually unable to effectively project future landslide events, making it challenging to predict when and where landslides might occur next. Their failure to accommodate uncertainty further hampers their effectiveness. Given these concerns with traditional models, it is critical that we explore improved methods for landslide susceptibility mapping. Advanced methodologies, like the one this paper presents, could potentially overcome these limitations and provide a more accurate, comprehensive, predictive tool for landslide risk assessment and mitigation.

GIS have revolutionized the field of environmental studies, offering a robust tool for spatial data analysis and representation. Literature reflects numerous applications of GIS in this field, particularly in hazard or susceptibility mapping. A clear example is illuminated by Carrara et al. (1991), who applied GIS techniques for landslide susceptibility mapping in Italy [23]. Their work set a precedent for the integration of GIS in environmental hazard management. Landslide hazard mapping, along with other forms of environmental susceptibility mapping, would benefit significantly from these GIS capabilities. Yet another compelling application is found in the work of Casagli et al. (2023). They used RS techniques to predict landslide susceptibility – a feat unachievable without GIS technology. However, while the utility of GIS in hazard mapping is well-documented, its scope is seldom fully realized due to inherent complexities in environmental systems. Also, the existing GIS-based models often fall short on adequately handling uncertainties in spatial data and employ static modeling approaches. As such, more advanced, dynamic, and inclusive GIS-based models, like the one proposed in this paper, are required for a comprehensive understanding and prediction of environmental susceptibilities. The neutrosophic MCDM approach could be instrumental in addressing these existing gaps.

MCDM involves navigating complex decision-making scenarios involving multiple, often competing criteria. The neutrosophic set theory, introduced by Smarandache (1999), extends fuzzy sets and introduces the concept of indeterminacy, providing a more flexible and inclusive model for decision-making [24]. In neutrosophic MCDM, decisions can be evaluated based on truth-membership (T), indeterminacy-membership (I), and falsehood-membership (F) functions, allowing for the accommodation of vagueness, ambiguity, and uncertainty in parameters. This approach can prove beneficial when exact data isn't available or when decisions have to be made considering various conflicting criteria. An application of neutrosophic MCDM can be seen in the study by Biswas et al. (2019), where it was used for socioeconomic development ranking [25]. Their work highlighted the robustness and flexibility of the neutrosophic MCDM, particularly when dealing with unstructured and indeterminate information. However, its application in environmental science, specifically in landslide susceptibility mapping, is relatively unexplored. Given its potential to address uncertainties and incorporate multiple criteria in an inclusive and flexible manner, the neutrosophic MCDM holds promise in enhancing the precision and applicability of landslide susceptibility mapping. The use of neutrosophic MCDM, in conjunction with GIS techniques, as proposed in this paper, represents a novel approach to landslide hazard mitigation.

The PAPRIKA Method: The acronym PAPRIKA stands for Potentially All Pairwise RanKings of all possible Alternatives, a multi-criteria decision-making method developed by Hiroshi and Eichler (2008) [26]. The method is known for its simplicity and intuitive appeal. It is used to derive a ratio scale on criteria by comparing pairs of alternatives that differ in at most two factors. Belton and Stewart (2012) emphasized that the PAPRIKA method's advantage lies in its ability to handle complex decisions involving multiple criteria in a simple, hierarchic, and comprehensible way [27]. PAPRIKA's successful applications are numerous, but its integration with neutrosophic principles is still uncommon in literature. The fusion of PAPRIKA's simplicity with the inherent ability of neutrosophic MCDM to handle uncertainty and fuzziness could indeed provide a powerful tool for landslide susceptibility mapping. However, further research is needed to establish this approach's efficacy and reliability.

Egypt's topography, geology, and climate make it prone to various natural hazards, including landslides [28]. According to data from the Egyptian Geological Survey and Mining Authority (EGSMA), the country has experienced several significant landslide events, particularly in mountainous and hilly regions. Qena is one of the 27 governorates of Egypt, located in the south, known for its unique geographical characteristics. The area is characterized by its steep slopes, loose soil composition, and high rainfall during certain seasons, which all contribute to its landslide susceptibility. Moreover, rapid urbanization and human activities, especially in the form of unregulated construction on steep hillsides, have further aggravated the situation. There is a substantial body of literature on landslides in Egypt. However, a comprehensive review indicates that there's a relative dearth of literature on the application of modern decision-making models like MCDM, and especially, neutrosophic-based approaches in assessing landslide susceptibility in Qena. Therefore, the proposed approach of applying GIS-based neutrosophic MCDM using the PAPRIKA method for landslide mitigation in Qena represents a valuable contribution to the field.

While extensive research has been conducted on mitigating landslide hazards, particularly in areas like Qena Governorate with unique geographical attributes, a gap remains in applying more contemporary decision-making models in this context. Notably, most studies dominated by traditional and statistical likelihood approaches have limited abilities to handle complex uncertainties inherent in environmental and geographical data. Furthermore, despite the recognized advantages and potential of the PAPRIKA method in multi-criteria decision-making, its integration with the neutrosophic principles in the context of landslide mitigation is still uncommon. This is evidenced by the limited examples in the existing literature. Therefore, the current study intends to bridge this gap by proposing an innovative GIS-based neutrosophic MCDM approach leveraging the PAPRIKA method's benefits. This new approach is expected to offer a more systematic and robust tool to assess landslide susceptibility, thereby enhancing the current mitigation strategies in Qena Governorate, Egypt.

3. PAPRIKA

Here is a simple pseudocode that outlines the PAPRIKA method:

```
// Input: Decision criteria and alternatives.
ALGORITHM PAPRIKA_Method(decisionCriteria, alternatives)
BEGIN
// function, evaluatePair(), to conduct pairwise comparisons
FUNCTION evaluatePair(alternative1, alternative2)
BEGIN
    ASK the decision-maker to compare alternative1 and alternative2
        STORE the preference by the decision-maker
        RETURN preference
END FUNCTION
```

```
// function, rankPairs(), to sort comparisons.
   FUNCTION rankPairs(pairs)
BEGIN
    FOR each pair in pairs
        CALL evaluatePair() for each pair
        STORE results of each evaluation with its corresponding pair
    SORT pairs based on results with most preferred at the top and least preferred at the bottom
    RETURN most and least-preferred pairs
END FUNCTION
//function, interpolatePoints(), to establish additional ranking points.
   FUNCTION interpolatePoints()
BEGIN
    RETRIEVE most and least preferred pairs using rankPairs()
    FOR each pair between the most and least preferred pairs
        CALCULATE interpolation
    STORE interpolated points in points set
    RETURN points set
END FUNCTION
//function, sortFurther(), to rank remaining comparison pairs.
FUNCTION sortFurther(remainingPairs)
BEGIN
    RETRIEVE reference points by using interpolatePoints()
    FOR each pair in remainingPairs
        COMPARE pair using evaluatePair() with reference points
        RANK pair based on comparison with reference points
            RETURN ranked remaining pairs
END FUNCTION
//function, calcWeights(), to assign weights to criteria.
FUNCTION calcWeights(criteria)
BEGIN
    DECLARE a list weights
    FOR each criterion in criteria
        CALCULATE the difference between the two extreme points of that criterion
        STORE this difference to weights
    RETURN weights
END FUNCTION
// function, rankAlternatives(), to score and rank alternatives.
FUNCTION rankAlternatives(alternatives, criteria)
BEGIN
    RETRIEVE weights by calling calcWeights(criteria)
    FOR each alternative in alternatives
        CALCULATE score of alternative by using weights
        STORE the score of each alternative
    RANK alternatives based on their scores
    RETURN ranked alternatives
END FUNCTION
//Final rankings of alternatives for decision making.
OUTPUT final rankings of alternatives for decision making.
END PAPRIKA_Method
```

4. Neutrosophic PAPRIKA

The PAPRIKA method under a neutrosophic environment follows the same basic steps as classic PAPRIKA, but they are adapted to suit the indeterminacy of neutrosophic decision-making.

Step 1. Identify a panel of experts most suitable for the problem domain.

Step 2. Expert consultation.

Define the criteria suitable for the problem using expert consultations. Each criterion represents a different aspect of the problem you're trying to solve.

Step 3. Assign neutrosophic values for criterion evaluations

The expert assigns a neutrosophic set to each criterion for evaluation. Using the scale shown in Table 1 demonstrates a neutrosophic set that includes three parts - truth, indeterminacy, and falsity.

Table 1. Linguistic variables for determining likelihood of criteria and decision maker's confidence.

Linguistic terms for likelihood	(L, M, U)	Decision maker's confidence degree (DM)
Absolutely unlikely	⟨(0,0,0)⟩	Absolutely unsure (0,1,1)
Presumably unlikely	⟨(0,0,2)⟩	Unsure (0.25 ,0.75, 0.75)
Somewhat likely	⟨(1,2,4)⟩	Somewhat confident (0.45 ,0.60,0.60)
Moderately likely	⟨(2,4,6)⟩	Moderately confident (0.50 ,0.50,0.50)
Likely	⟨(4,6,8)⟩	Confident (0.75 ,0.20,0.20)
Very likely	⟨(6,8,10)⟩	Very confident (0.85 ,0.15,0.15)
Absolutely likely	⟨(8,10,12)⟩	Absolutely confident (1.00 ,0.00, 0.0)

Step 4. Compute average to aggregate experts opinions.

Let j represents the criterion index, k is the expert index, n is the number of experts, and $AggID[j]$ is the aggregated importance degree for criteria$j$.

Compute$SumID[j]$, shown in Eq. (1) which is the sum of importance degree for criterion j across all experts:

$SumID[j] = \sum(ID\_kj)$ for all k in Experts (1)

Then, compute the aggregated score for each criterion, marked as$AggID[j]$ in Eq. (2):

$AggID[j] = SumID[j] / n$ for all j in criteria (2)

Step 5. In order to rank main criteria the score function shown in Eq. (3) is used.

Let $B\tilde{}1 = (B1, B2, B3)$; $\eta B\tilde{}1, \theta B\tilde{}1, \alpha B\tilde{}1$ be a triangular neutrosophic number then the score function equals

$SB\tilde{}1 = 1/2 * (B1 + 2B2 + B3) * 2 + \eta B\tilde{}1 - \theta B\tilde{}1 - \alpha B\tilde{}1$ (3)

Step 6. Calculate criteria weights using the traditional PAPRIKA method.

5. Methodology

The methodology of the study began with the identification of the problem, specifically, landslide susceptibility in the Qena Governorate. This fed into an extensive data collection and analysis phase, which incorporated landslide inventory data and relevant environmental and topographical factors. These criteria were then evaluated through a neutrosophic MCDM process, ranking the varying factors according to their contribution to landslide risks. The resulting composite was integrated through the Weighted Overlay Analysis in ArcGIS, combining the data layers with assigned weights. Following this, the neutrosophic PAPRIKA method was deployed to prioritize the areas that needed urgent attention. The culmination of these procedures was visually represented in a Landslide Susceptibility Map. This map was then used to devise suitable mitigation strategies, making the findings of the study practical and implementable. Throughout the methodology, the researched also conducted a sensitivity analysis involving ten separate scenarios to test the robustness and reliability of the model. The study flowchart shown in Figure 1.

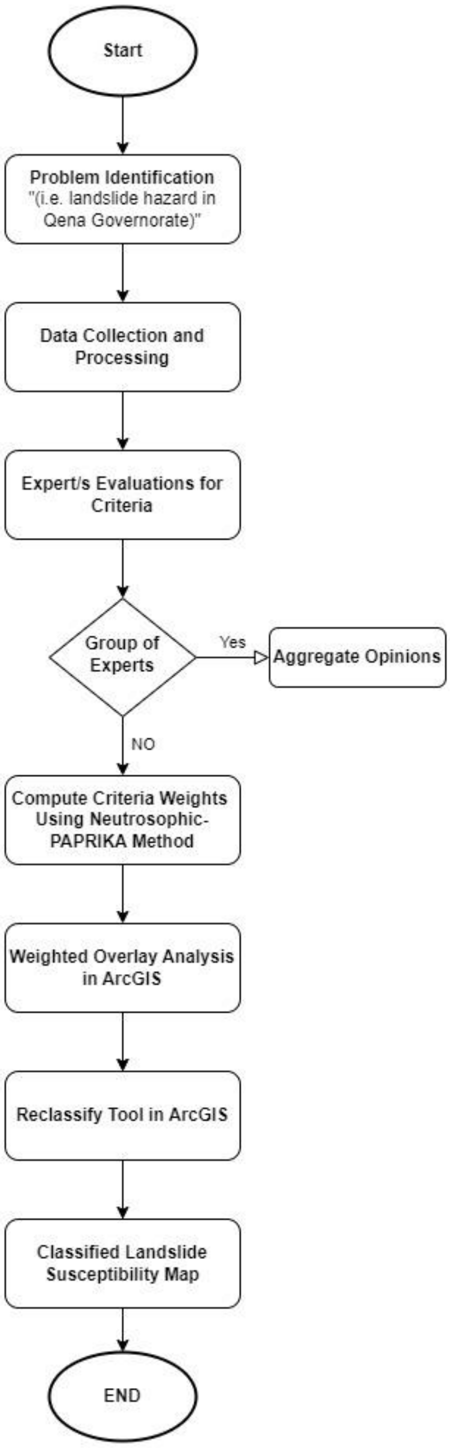


Figure 1. Study Flowchart.

5.1 Study Area

Qena Governorate is located in the southern part of Egypt, and it covers an area of about 10798 km² as shown in Figure 2. It is geographically positioned between latitudes 25.5° and 26.6° N and longitudes 32.2° and 33.2° E [29]. Qena is characterized by its rugged and hilly terrain, interrupted by desert plains [30]. The region is significantly influenced by the course of the Nile River, which cuts across it and contributes to its varied topography [31]. Enhanced by its positioning between the Nile and the Eastern Desert, Qena experiences a desert climate with hot summers and moderate winters. The geographic attributes of Qena, along with the complex geology and presence of vulnerable populations and infrastructure, pose significant landslide risks [32]. These systemic factors, therefore, justify Qena as an ideal field for the proposed research on mitigating landslide disasters. It offers a distinctive chance to explore how GIS-based neutrosophic MCDM methods can be employed to enhance existing landslide mitigation strategies.

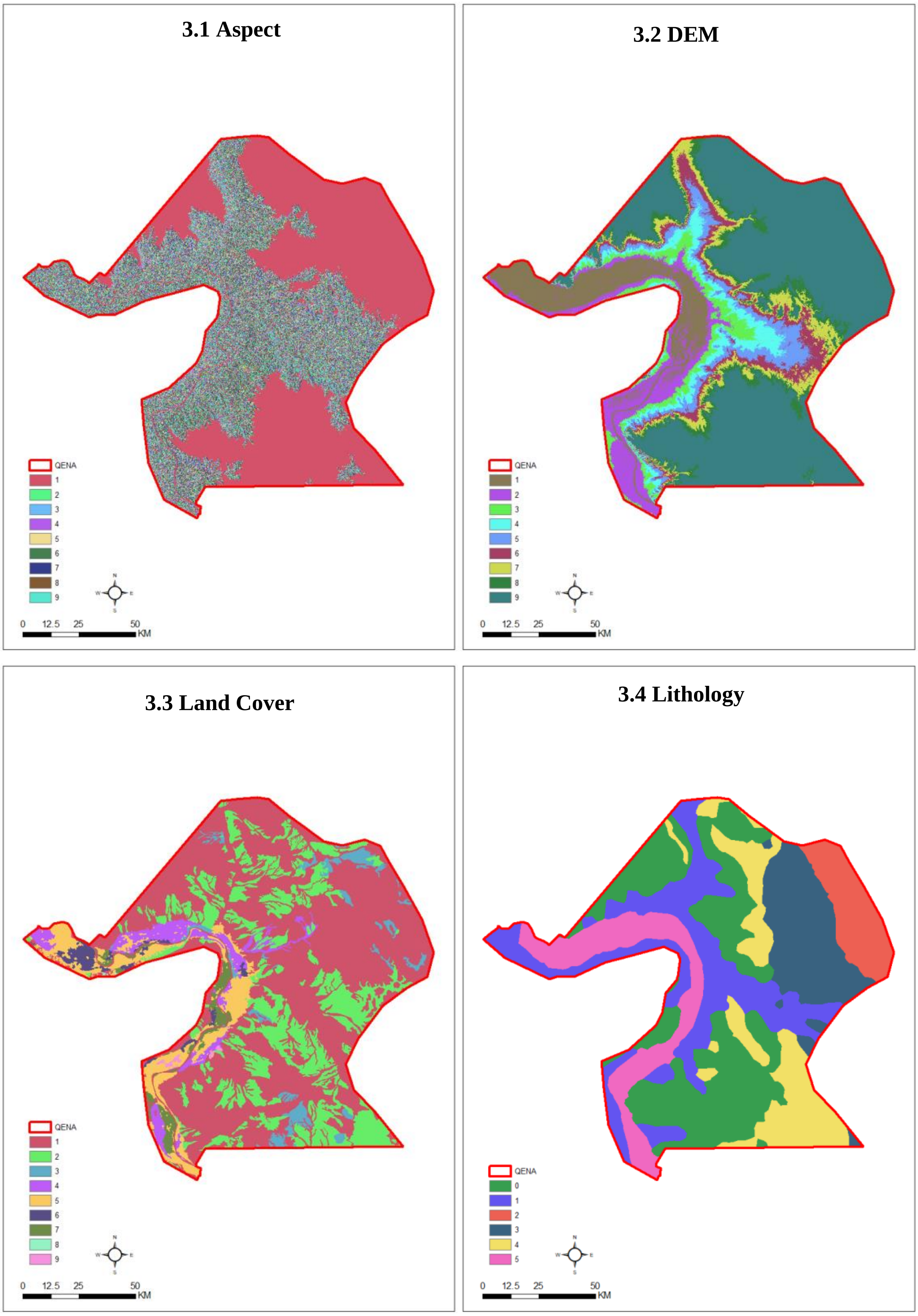
Figure 2. Study Area.

5.2 Data Collection and Processing

This study encompasses eight distinct criteria integral to understanding landslide triggers and susceptibility. These criteria, along with their detailed descriptions and sources, are extensively laid out in Table 2. Further the classified spatial representation of these criteria, illuminating their specific geographic distribution, is visualized in Figure 3. The hierarchical structure of the neutrosophic Preference Ranking Interactive Multi-criteria Analytic (n-PAPRIKA) problem, instrumental in our analysis, can be visualized in Figure 4. This Figure delineates the interconnections and dependencies among the various factors considered in the n-PAPRIKA problem, consequently aiding in a more comprehensive understanding of the landslide mitigation approach we have adopted.

Table 2. Study criteria, description, and source.

Criteria	Relevance to Study	Source
Slope (C1)	Steeper slopes are typically more prone to landslides.	Slope tool ArcGIS
DEM (C2)	Higher elevations may have increased landslide susceptibility.	USGS Earth Explorer website
Soil Type (C3)	Certain soils (e.g., sandy or clay-rich soils) can be particularly susceptible to landslides.	FAO Soil Portal website
Proximity to River (C4)	Areas close to rivers can be more susceptible due to erosion and increased water saturation	The Egyptian National Authority for Remote Sensing and Space Sciences (NARSS)
Land Cover (C5)	Vegetation can stabilize soils and reduce landslide risks.	United Nations Land Cover, Egypt (Africover, FAO)
Aspect (C6)	The direction a slope faces can influence its moisture levels and the freeze-thaw cycle, both factors in landslide risk.	Aspect tool ArcGIS
Proximity to Road (C7)	Construction and maintenance of roads can destabilize slopes and increase landslide risk.	The Egyptian National Authority for Remote Sensing and Space Sciences (NARSS)
Lithology (C8)	The type of bedrock can greatly impact landslide susceptibility.	USGS World Geologic Maps


3.1 Aspect
3.2 DEM
3.3 Land Cover
3.4 Lithology
QENA
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2
3
4
5
6
7
8
9
0
0 12.5 25 50
KM

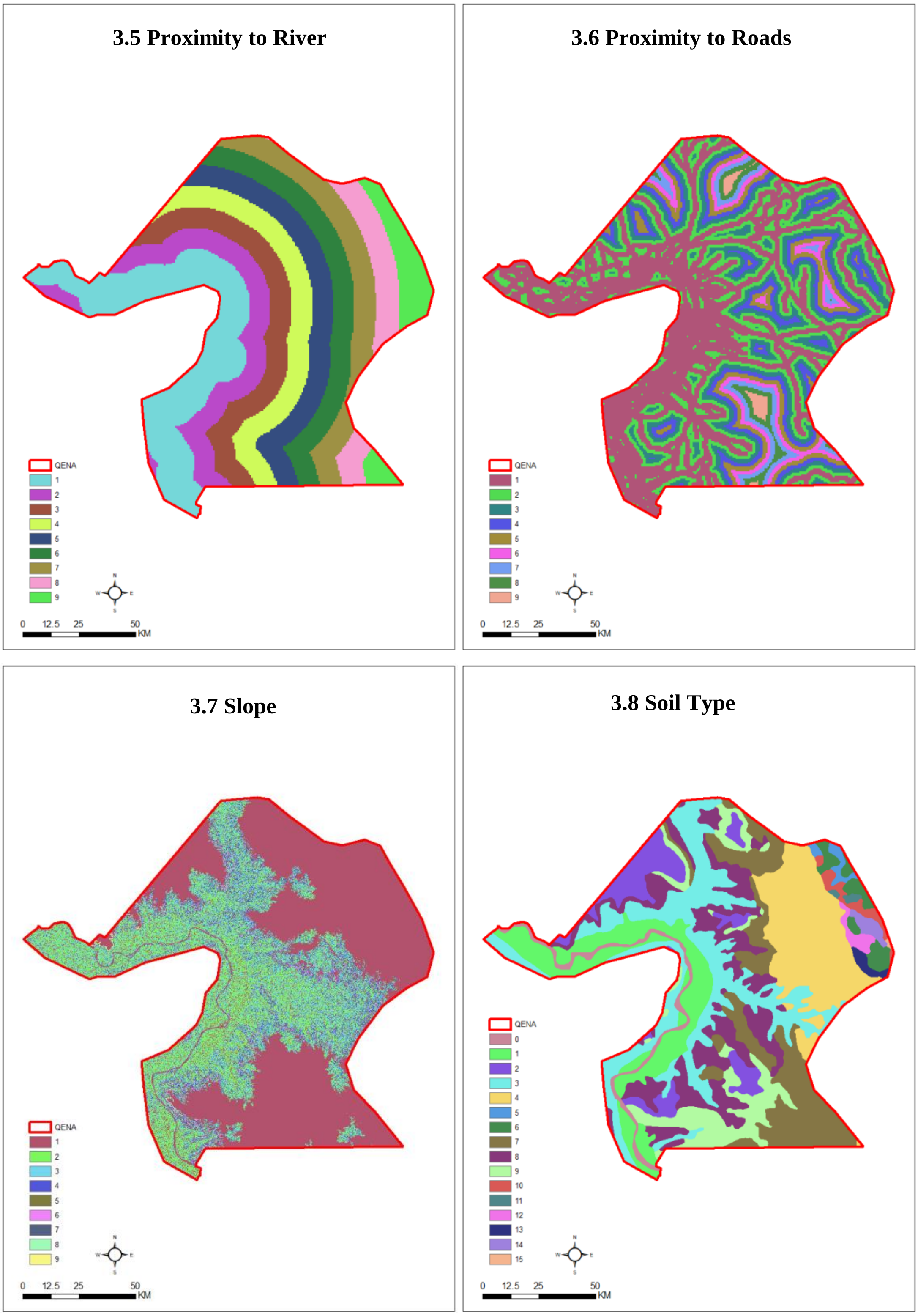


Figure 3. Classified Spatial Maps of the criteria.

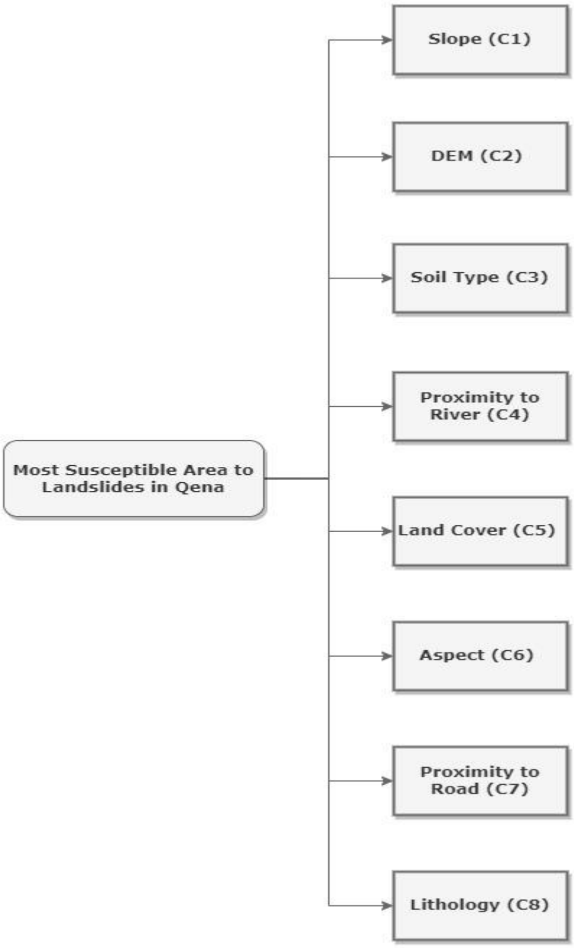


Figure 4. Hierarchy of the N-PAPRIKA MCDM problem.

5.3 Panel of Experts

The panel of experts for examining the mitigation of landslide hazards in Qena Governorate using a GIS-based neutrosophic MCDM approach includes the following:

E1. Geologist: Geologist is vital as they understand the processes causing landslides and can provide valuable data about the geological properties of local areas.

E2. Civil Engineer (Geotechnical Specialist): Provide insights on infrastructure vulnerability and the effectiveness of proposed landslide mitigation strategies.

E3. Emergency Manager /Disaster Risk Reduction Expert: Professional that can identify vulnerable populations and infrastructure in Qena, and propose measures to increase resistance against landslides.

E4. Urban Planner or Local Government Official: Provide details about local zoning codes and land development plans for future prevention of landslide-prone areas.

This multidisciplinary panel would aid in creating comprehensive, evidence-based solution for landslide mitigation in the Qena Governorate.

6. Case Study: Results and Discussion

1. After selecting the group of experts and the criteria appropriate for the study. The expert's panel begins to evaluate the main criteria using the linguistic variables shown in Table 1. The evaluations of the expert panel are available in Table 3.

2. Since group expert opinions are being assessing the criteria aggregation of the opinions is necessary as shown in Table 2.

Table 3. Experts evaluations and ranks of the main criteria.

Criteria	Aggregated Experts Opinions	Score Function	Criteria Rank
Slope (C1)	⟨(24,32,40), (3.45,0.5,0.5)⟩	11.87	1
DEM (C2)	⟨(16,24,32), (2.75,1.2,1.2)⟩	4.7	4
Soil Type (C3)	⟨(22,30,38), (3.35,0.55,0.55)⟩	10.63	2
Proximity to River (C4)	⟨(12,20,28), (2.5,1.4,1.4)⟩	2.83	5
Land Cover (C5)	⟨(10,18,26), (2.25,1.7,1.7)⟩	1.275	6
Aspect (C6)	⟨(9,16,24), (2.2,1.8,1.8)⟩	0.81	7
Proximity to Road (C7)	⟨(8,14,22), (2.15,1.9,1.9)⟩	0.42	8
Lithology (C8)	⟨(20,28,36), (3.1,0.85,0.85)⟩	7.93	3

3. After opinion aggregation, score function shown in Eq. (3) is used to compute criteria ranks.

Applying the PAPRIKA method for criteria (C1 to C8) based on their ranks:

- Calculate Minimum-to-Preference Point Distances (MPP): The distance is the difference between the least preferred level and the ranking. Here, since the ranks themselves act as the preference point, the MPP distances are:

$C1 = 8$

$C2 = 4$

$C3 = 6$

$C4 = 4$

$C5 = 3$

$C6 = 2$

$C7 = 1$

$C8 = 5$

- Normalize the Distances: Divide each MPP distance by the sum of all MPP distances to calculate the weights for each criterion. Sum all distances (C1 to C8) as shown in Eq. (4):

$$Sum = 4 + 1 + 3 + 8 + 6 + 4 + 5 + 2 = 33 \quad (4)$$

The weight of each criterion is calculated as follows:

$W(C1) = 8/33 \approx 0.242$

$W(C2) = 4/33 \approx 0.121$

$W(C3) = 6/33 \approx 0.182$

$W(C4) = 4/33 \approx 0.121$

$W(C5) = 3/33 \approx 0.09$

$W(C6) = 2/33 \approx 0.06$

$W(C7) = 1/33 \approx 0.03$

$W(C8) = 5/33 \approx 0.152$

These weights reflect the relative importance of each criterion as informed by the decision-maker's ranking and are shown in Figure 5.

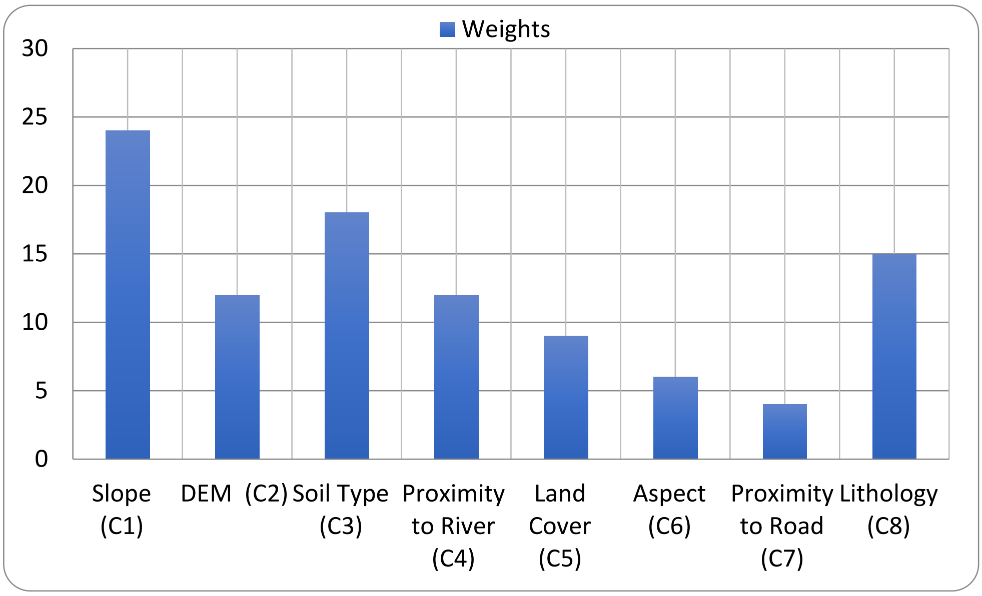


Figure 5. Weights of the main criteria.

The weighted average Tool in ArcGIS plays a pivotal role in our study by enabling us to allocate appropriate weights to each criterion, reflecting its relative significance in landslide occurrence. The integration of Criteria Weights and geographic features through this approach facilitates a nuanced computation of areas predisposed to landslides a process captured in Figure 6. These visual insights thus underscore the spatial heterogeneity of landslide vulnerability across the Qena Governorate. The classified susceptibility resulting map classified into the following classes:

- Very High Susceptibility: Areas with steep slopes, certain soil types - clay-rich or sandy soils, high elevations, near rivers, with certain types of bedrock, high levels of human activities like road construction, and sparse vegetation.
- High Susceptibility: Areas with moderately steep slopes, near rivers, higher elevations, certain risky soil types, and lesser vegetation.
- Moderate Susceptibility: Areas with moderate slopes, some distance from rivers, middle range elevations, mixed soil types, and light to moderate vegetation.
- Low Susceptibility: Areas with gentle slopes, further from rivers, lower elevations, stable soil types, and substantial vegetation.
- Very Low Susceptibility: Areas with very gentle slopes, far from rivers, at the lowest elevations, with stable types of soil, and dense vegetation.
- Practically No Susceptibility: Areas with flat terrain, far from rivers, at the lowest elevations, with highly stable soil types, dense vegetation, and little to no human activity.

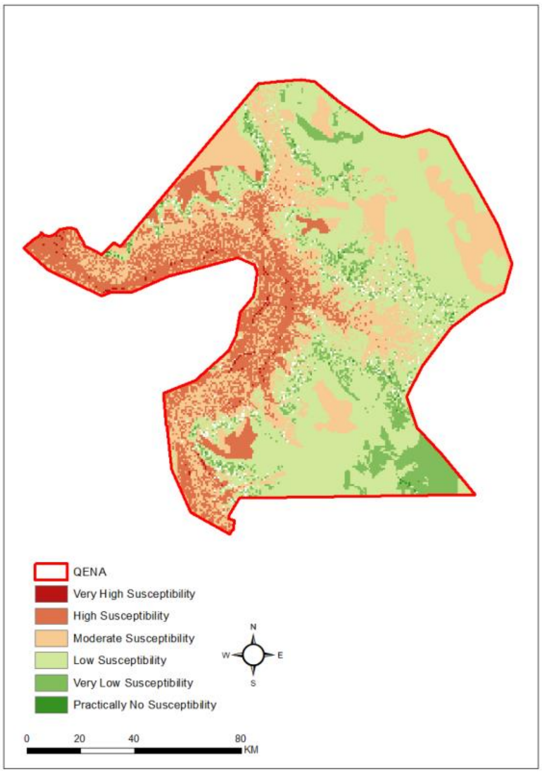


Figure 6. Classified Susceptibility Map.

7. Sensitivity Analysis

Sensitivity analysis holds significant importance to this study because it provides insightful details regarding the robustness of our model and the impact of individual parameters on landslides susceptibility. By analyzing how variations in input parameters influence the model outcome, it enables us to discern critical parameters and more accurately predict potential landslide-prone areas. The current study scenarios are as follows:

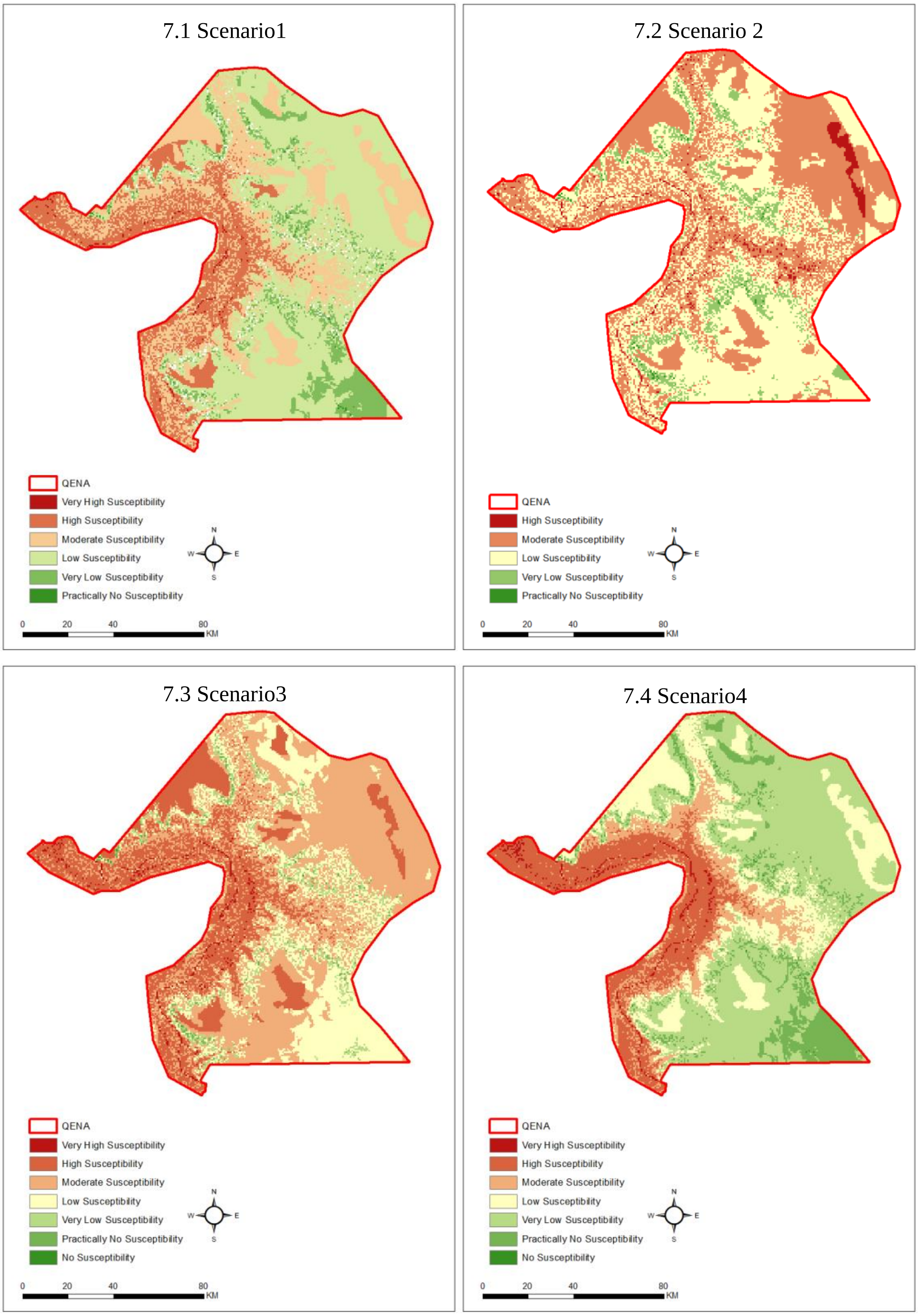
- Scenario 1: maintain the obtained criteria weights. This is the base scenario, reflecting the existing situation without any modifications.
- Scenario 2: Assign equal weights to each criterion. This scenario checks the robustness of the model when all criteria have equal significance.
- Scenarios 3-10: Increase the weight of each criterion (C1 to C8 consecutively) by 20% individually. These scenarios help understand the influence of each criterion on the overall decision-making process.

The detailed scenarios can be shown in Table 4. Classified sensitivity analysis maps shown in Figure 7.

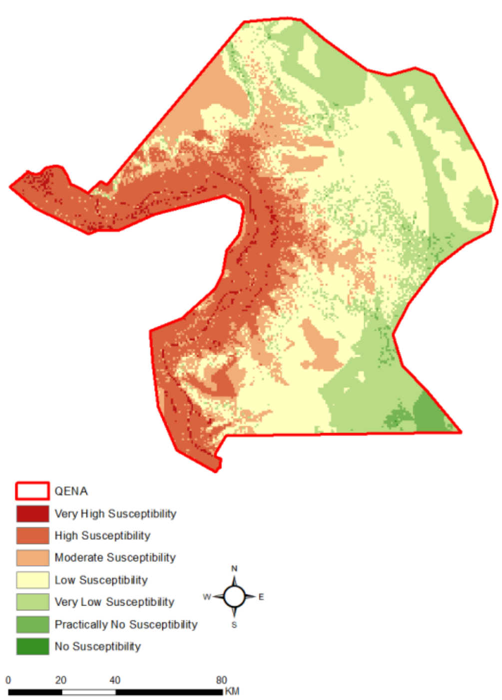
Table 4. Sensitivity analysis scenarios.

Scenario #	Description
S1	Original obtained criteria weights
S2	Equal criteria weights
S3	Slope Criteria (C1) + 20%
S4	DEM Criteria (C2) + 20%
S5	Soil Type Criteria (C3) + 20%
S6	Proximity to River Criteria (C4) + 20%

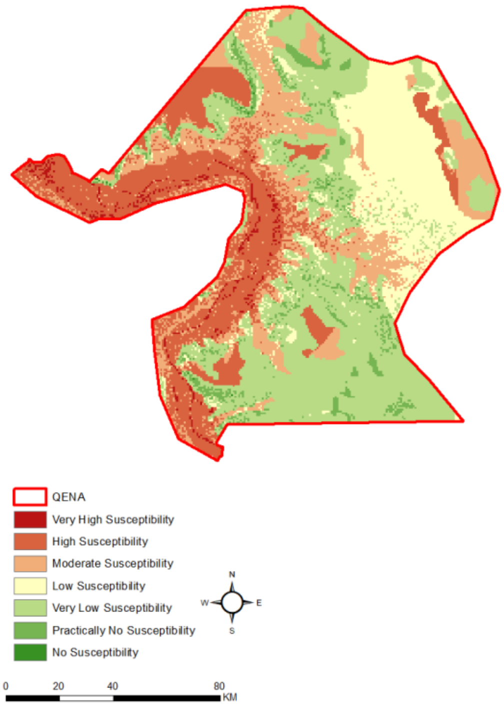
S7	Land Cover Criteria (C5) + 20%
S8	Aspect Criteria (C6) + 20%
S9	Proximity to Road Criteria (C7) + 20%
S10	Lithology Criteria (C8) + 20%



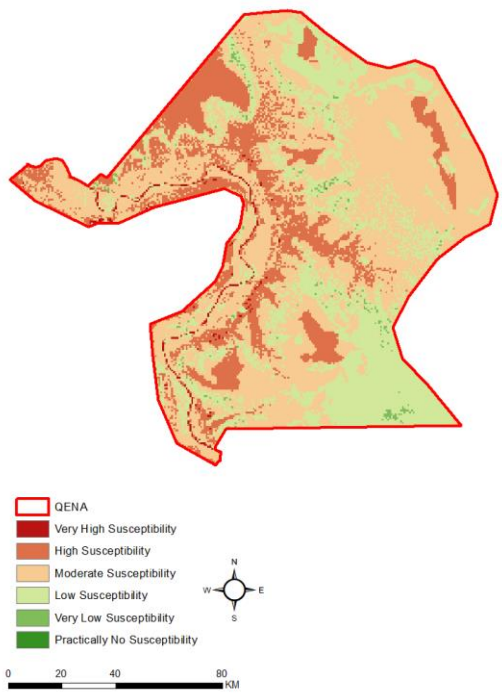
7.5 Scenario 5



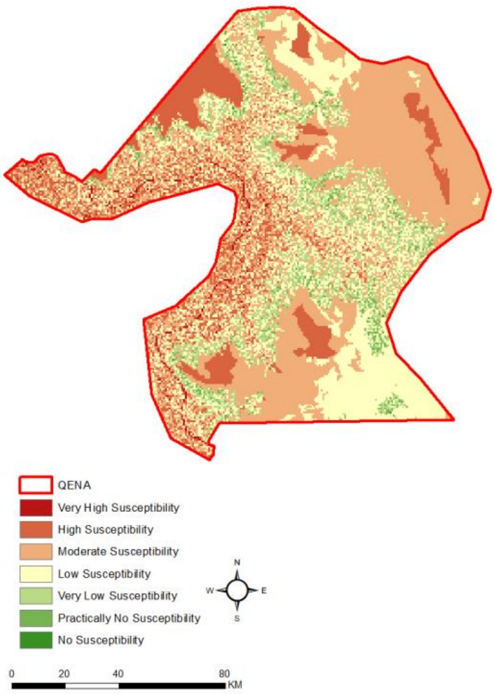
7.6 Scenario 6



7.7 Scenario 7



7.8 Scenario 8



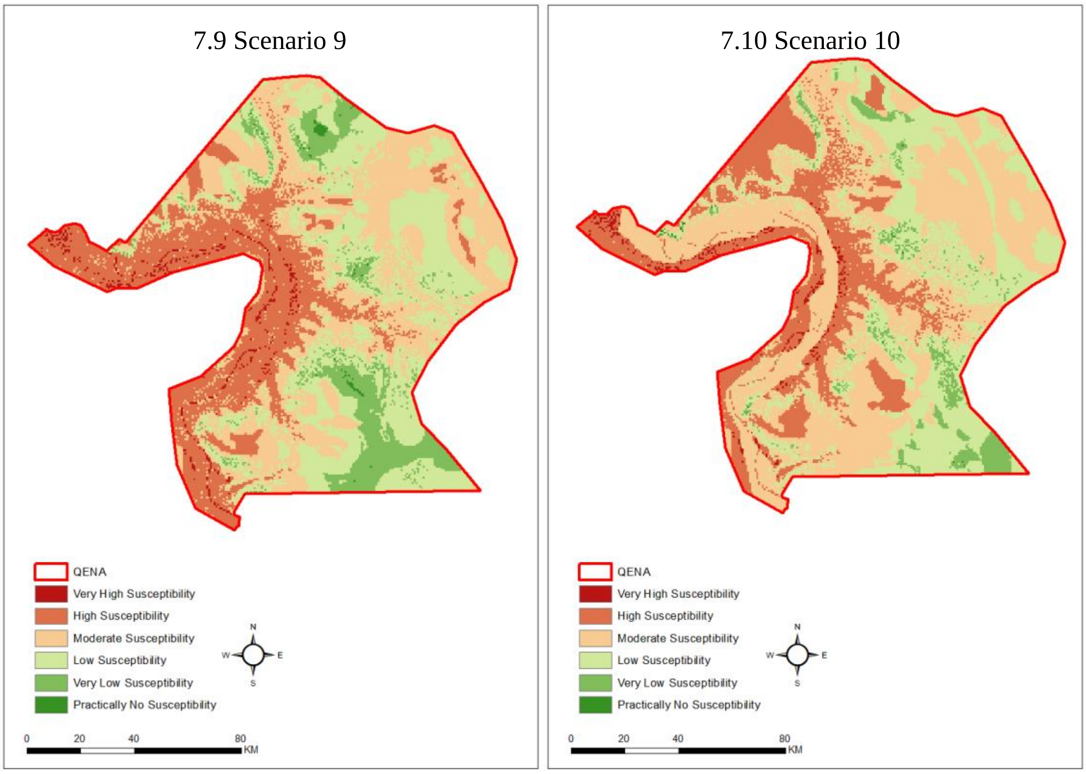


Figure 7. Classified sensitivity snalysis Maps.

The sensitivity analysis results in Figure 8 demonstrate that there are significant variations in the susceptibility categories across the different scenarios. Scenario 2, for instance, reveals a larger percentage of areas with High Susceptibility (43%) and Moderate Susceptibility (48%) than most others, implying more vulnerability to landslides under the conditions of that scenario. Conversely, Very Low Susceptibility is absent, which may indicate extreme risk conditions. In contrast, Scenarios 4 and 5 depict a comparatively higher percentage of areas with Very Low Susceptibility (42% and 34% respectively), suggesting less vulnerability to landslides under those conditions. Overall, most of the scenarios indicated Moderate to High Susceptibility, highlighting the need for prioritized and bespoke mitigation strategies. Also, the variation in susceptibility across scenarios underscores the importance of considering multiple future scenarios in vulnerability assessment for more robust mitigation planning.

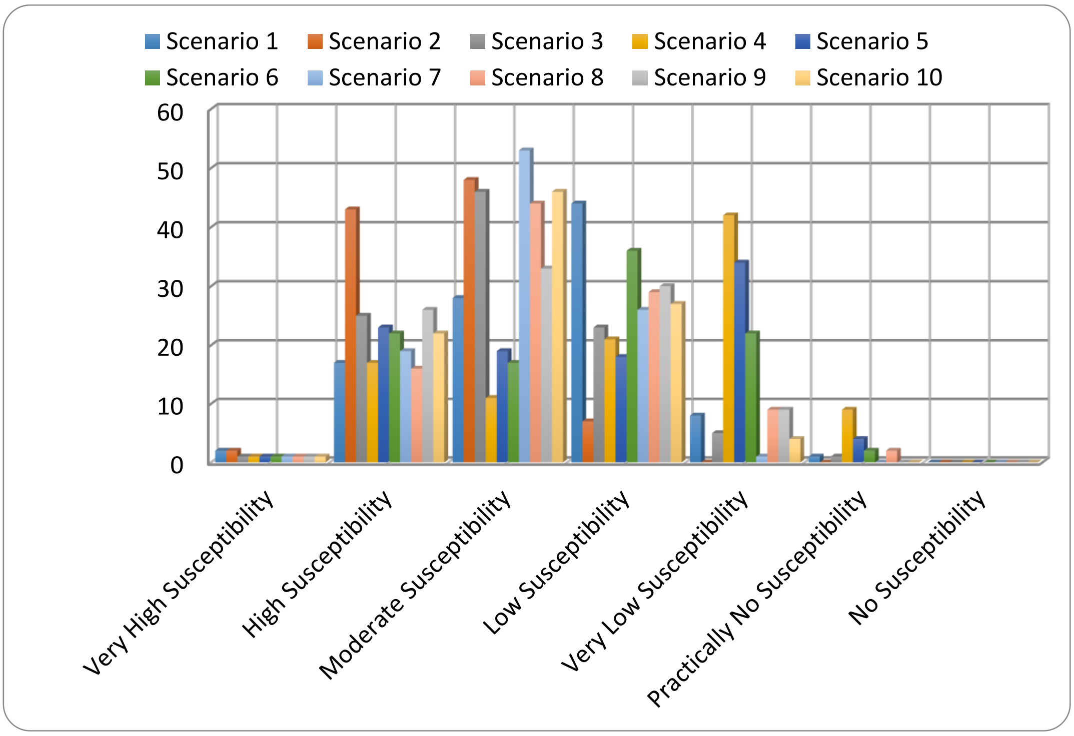


Figure 8. Susceptibility classes for sensitivity analysis.

8. Conclusion

In conclusion, this study puts forth a robust method for mitigating landslide hazards in the Qena Governorate using a GIS-based neutrosophic MCDM approach, employing the neutrosophic paprika and ArcGIS weighted overlay. Through the rigorous application of sensitivity analysis and careful consideration of multiple criteria, we identify potential landslide-prone areas in the region. This research serves as a substantial contribution to the field, providing a framework for similar studies focused on landslide hazard mitigation using GIS technologies and neutrosophic decision-making. Regarding future research directions, it would be beneficial to validate and refine this model with new and updated datasets as they become available. Future studies might also consider additional criteria or alternative decision-making methods to cater to the complex and dynamic nature of landslides. A greater focus on integrating local community inputs and incorporating socio-economic considerations could also offer further depth and applicability to these models.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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A Newfangled Interpretation on Fermatean Neutrosophic Dombi Fuzzy Graphs

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Abstract: Neutrosophic Dombi fuzzy graph is an advancement of the Dombi fuzzy graph and intuitionistic Dombi fuzzy graph. In this paper, we have initiated a new concept of the Fermatean neutrosophic Dombi fuzzy graph. Further, we identified a few products of the direct, cartesian, composition of Fermatean neutrosophic Dombi fuzzy graphs. Also, we examined the related proposition with suitable illustrations with graphs.

Keywords: Neutrosophic Graphs; Neutrosophic Fuzzy Graph; Neutrosophic Fuzzy Edge Graph; Neutrosophic Dombi Fuzzy Graph; Fermatean Neutrosophic Dombi Fuzzy Graph.

1. Introduction

In 1965, Zadeh [17] developed the idea of a fuzzy set and the term degree of membership to address imprecision. Using the degree of non-membership in the fuzzy set idea as an independent variable, Antanassov [5] proposed intuitionistic fuzzy sets (IFS) in 1983. Florentin Samarandache created a neutrosophic set with a degree of indeterminacy in 2005 [15] by utilizing the concept of intuitionistic fuzzy sets. The neutrosophic sets are defined by truth, indeterminacy, and false membership function.

In many fields, including geometry, number theory, topology, optimization, and computer science, graph theory is utilized to solve combinatorial problems. A graph is made up of nodes and arcs. Fuzzy logic is an extension of classical logic, where each and every item has a different grade of membership. The concept of fuzzy graphs was first introduced and explained by Kaufmann in 1975 [10]. IFS relationships and intuitionistic fuzzy graphs were discussed in 2006 by Shannon andAtanassov [14]. Neutrosophic graphs were first developed by Ghoei and Pal[8], and they are used to model a variety of real-world problems.

The Dombi operator with relevant parameter was inaugurated by Dombi [7] in 1982, and the concept of the Dombi fuzzy graph was developed by Ashraf et al. (2018) [4]. The Dombi operator is crucial in simulating and resolving numerous problems encountered in everyday life. In order to take use of this advantage, Mijanur Rahman Seikh and Utpal Mandal (2021) [11] applied Dombi operations to intuitionistic fuzzy graphs and created a multiple attribute group decision-making problem. The neutrosophic Dombi graph was invented and refined by Tejindarsingh lakhwani, Karthick [16].In addition to proposing Pythagorean neutrosophic fuzzy graphs using the Dombi operator, Ajay et al.[1] presented an innovative concept of a Pythagorean neutrosophic fuzzy graph. In this research, we introduced a new emergent notion of the Fermatean neutrosophic Dombi fuzzy graph using the Dombi operator. the primary consideration, In Section 2, we developed and remembered the fundamental concepts and notions used in this part. In Section 3, we described the new concepts of Fermatean neutrosophic Dombi fuzzy graphs and establish their proposition with relatable illustrations and graphs.

2. Preliminaries

2.1. Definition: Let ϑ_D be a non-empty set. A fuzzy set A^{μ} in $\vartheta_D$is distinguished by its membership function $\alpha_{\mu_D}(\vartheta_D)$:→[0,1] and $\mu_D(n)$ is interpreted as the degree of member of elementn^{μ} in a fuzzy set N^{μ} , for each $n^{\mu} \in \vartheta_D$ .It is clear that N^{μ} is determined by the set of tuples of N^{μ} = {(n^{μ},$\alpha_{\mu_D}(\vartheta_D), n^{\mu} \in \vartheta_D$}.

2.2. Definition: An intuitionistic fuzzy set (briefly IFS) N^{μ} is an object of having the form A^{μ}={<n^{μ^*},$\alpha_{\mu_D}(\vartheta_D), \beta_{\mu_D}(\vartheta_D)$> : $n^{\mu} \in R_{N^{\mu}}$} where the functions $\alpha_{\mu_D}(\vartheta_D)$:→[0,1] and $\beta_{\mu_D}(\vartheta_D)$:→[0,1] denote the degree of membership and the degree non-membership of each element $n^{\mu} \in \vartheta_D$to the set G^* respectively, and 0≤ $\alpha_{\mu_D}(\vartheta_D)$+$\beta_{\mu_D}(\vartheta_D) \leq 1$ for each $n^{\mu} \in \vartheta_D$. Denote by IFS($R_{N^{\mu}}$), The set of all intuitionistic fuzzy sets in ϑ_D. An intuitionistic fuzzy set G^* in ϑ_D is simply denoted by N^{μ}= <n^{μ}, $\alpha_{\mu_D}(\vartheta_D), \beta_{\mu_D}(\vartheta_D)$> instead of denoting A^{μ}= {(n^{μ},$\alpha_{\mu_D}(\vartheta_D), \beta_{\mu_D}(\vartheta_D)$) : $n^{\mu} \in \vartheta_D$}.

2.3. Definition Let ϑ_D be a non-empty set. A Neutrosophic set (NS) A^{μ} in $\vartheta_D$is characterized by a truth-membership function α_{μ_D} , an indeterminacy-membership function β_{μ_D} , and a falsity - membership functionγ_{μ_D} is $\alpha_{\mu_D}(\vartheta_D), \beta_{\mu_D}(\vartheta_D), \gamma_{\mu_D}(\vartheta_D)$ are real or non-standard subsets of] $0^-, 1^+$[on ϑ_D.

i.e.)$\alpha_{\mu_D}(\vartheta_D): \vartheta_D \to\ ]0^-, 1^+[$

$\beta_{\mu_D}(\vartheta_D):\ \vartheta_D \to\ ]0^-, 1^+[$

$\gamma_{\mu_D}(\vartheta_D): \vartheta_D \to\ ]0^-, 1^+[$

2.4. Definition: A fuzzy graph of the graph D^*= (φ_D, ς_D) is a pair of D =(μ_D , ν_D), Where $\mu_D \to$ [0,1] is a fuzzy set on φ_D and ν_D :$\varphi_D X \varphi_D \to$ [0,1] is a fuzzy relation on φ_D such that $\nu_D\ (n,t) \leq$ μ_D (n)∧ μ_D (t),∀(n,t)∈ $\varphi_D X \varphi_D$.

2.5.Definition: A binary function Ŧ :[0,1]X[0,1] → [0,1] is known as triangular norm (t-norm) if for all n,t,s ∈ [0,1], it satisfied the following conditions

(1) (Neutral property or boundary condition) *Ŧ(n,1) = n*

(2) (commutativity) *Ŧ(n,t) = Ŧ(t,n)*

(3) (associativity) *Ŧ(n,(t,s)) = Ŧ((n,t),s)*

(4) (monotonicity) *Ŧ(n,t) ≤Ŧ(s,d) if* $n \leq s$ *and* $t \leq d$

2.6.Definition: A binary function $\mathcal{M}$: [0,1]X[0,1] → [0,1] is known as triangular conorm (t-conorm) if and only if there exists a t-norm *Ŧ* for all *Ŧ(n,t)* ∈ [0,1]X[0,1]

$\mathcal{M}$(n,t)=1-Ŧ(1-n,1-t)

Preferred options for t-norms are:

- The minimum operator $\mathcal{M}$ *(n,t)=MIN(n,t)*
- The product operator $\mathcal{P}$*(n,t)=nt*
- The dombi's t-norm,$\frac{1}{1+([(\frac{1-n}{n})]^{\delta}+[(\frac{1-t}{t})]^{\delta})^{\frac{1}{\delta}}}$, δ >0

Preferred options for related t-conorms are:

- The maximum operator $\mathcal{M}^*$*(n,t)=MAX (n,t)*

- The Probabilistic sum $\mathrm{P}^*(n,t) = n+t-nt$
- The Dombi's t-conorm $\frac{1}{1+([(\frac{1-n}{n})]^{-\delta}+[(\frac{1-t}{t})]^{-\delta})^{-\frac{1}{\delta}}}$, $\delta > 0$

One more pair of $\mathcal{F}$-operator is $\mathcal{F}(n,t)=\frac{nt}{n+t-nt}$

$\mathcal{P}(n,t)=\frac{n+t-2nt}{1-nt}$,which is obtained by substituting $\delta = 1$ in dombi's t-norm and t-conorm. Also

$\mathcal{P}(n,t) \leq \frac{nt}{n+t-nt} \leq \mathcal{M}(n,t)$ and $\mathcal{M}^*(n,t) \leq \frac{n+t-2nt}{1-nt} \leq \mathrm{P}^*(n,t)$.

2.7. Definition: A dombi fuzzy graph with a countable set φ_D as the elementary set is a pair $D=(\mu_D, \nu_D)$,where $\mu_D \to [0,1]$ is a symmetric fuzzy on φ_D such that $\varsigma_D(nt) \leq \frac{\varphi_D(n)\varphi_D(t)}{\varphi_D(n)+\varphi_D(t)-\varphi_D(n)\varphi_D(t)}$, $\forall\, n,t \in \varphi_D$.

2.8. Definition: Let $D^* = (\varphi_D, \varsigma_D)$ be a crisp undirected graph contain no self loop and parallel edges. Let $\mu_D = (\alpha_{\mu_D}, \beta_{\mu_D}, \gamma_{\mu_D})$ such that $\alpha_{\mu_D} : \vartheta_D \to [0,1]$, $\beta_{\mu_D} : \vartheta_D \to [0,1]$, $\gamma_{\mu_D} : \vartheta_D \to [0,1]$ $\alpha_{\nu_D} : \vartheta_D \to [0,1]$, $\beta_{\nu_D} : \vartheta_D \to [0,1]$, $\gamma_{\nu_D} : \vartheta_D \to [0,1]$ .Here $\alpha_{\mu_D}, \alpha_{\nu_D} \to$ The membership function, $\beta_{\mu_D}, \beta_{\nu_D} \to$ The indeterminacy function, $\gamma_{\mu_D}, \gamma_{\nu_D} \to$ The falsity function in the neutrosophic dombi fuzzy graph, $\varsigma_F \subset \vartheta_D X \vartheta_D$ Then the neutrosophic dombi fuzzy graph, $D = (\varphi_D, \mu_D, \nu_D)$

$$\alpha_{\varsigma_D}(nt) \leq \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)} , \forall\, nt \in \varsigma_D$$

$$\beta_{\varsigma_D}(nt) \leq \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)} , \forall\, nt \in \varsigma_D$$

$$\gamma_{\varsigma_D}(nt) \leq \frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)} , \forall\, nt \in \varsigma_D$$

3. Fermatean Neutrosophic Dombi Fuzzy Graph

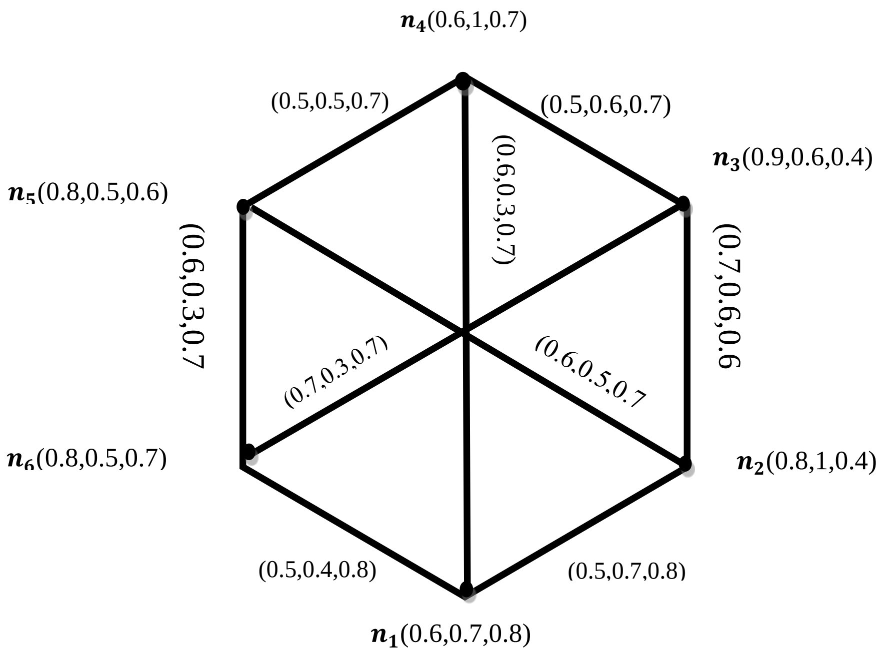
3.1 Definition: A fermatean neutrosophic dombi fuzzy graph is defined [its indicated by *FNDFG*] with a finite elementary set ϑ_D of its order pair $d=(\varphi_D, \varsigma_D)$ where $\varphi_D : \vartheta_D \to [0,1]$.Here we consider, $\mu_D = (\alpha_{\varphi_D}, \beta_{\mu_D}, \gamma_{\nu_D})$ such that $\alpha_{\varphi_D} : \vartheta_D \to [0,1]$, $\beta_{\mu_D} : \vartheta_D \to [0,1]$, $\gamma_{\nu_D} : \vartheta_D \to [0,1]$, $\alpha_{\mu_D} : \vartheta_D \to [0,1]$, $\beta_{\mu_D} : \vartheta_D \to [0,1]$, $\gamma_{\nu_D} : \vartheta_D \to [0,1]$ .Here $\alpha_{\mu_D}, \alpha_{\nu_D} \to$ The membership function, $\beta_{\mu_D}, \beta_{\nu_D} \to$ The indeterminacy function, $\gamma_{\mu_D}, \gamma_{\nu_D} \to$ The falsity function in the Fermatean neutrosophic dombi fuzzy graph, $\varsigma_F \subset \vartheta_D X \vartheta_D$ Then the fermatean neutrosophic dombi fuzzy graph, $d = (\varphi_D, \mu_D, \nu_D)$

$$\alpha_{\varsigma_D}(nt) \leq \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)} , \forall\, nt \in \varsigma_D$$

$$\beta_{\varsigma_D}(nt) \leq \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)} , \forall\, nt \in \varsigma_D$$

$\gamma_{\varsigma_D}(nt) \leq \frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}$, $\forall\, nt \in \varsigma_D$ And $0 \leq {\alpha_{\nu_D}}^3(nt) + {\beta_{\nu_D}}^3(nt) + {\gamma_{\nu_D}}^3(nt) \leq 2$; $0 \leq {\alpha_{\nu_D}}^3(nt) + {\gamma_{\nu_D}}^3(nt) \leq 1$; $0 \leq {\beta_{\nu_D}}^3(nt) \leq 1$.

3.1 Example: Define a graph D=(φ_D, ς_D). Here φ_D ={ $n_1, n_2, n_3, n_4, n_5, n_6$ } and ς_D={$n_1n_2, n_1n_6, n_2n_3, n_2n_5, n_3n_4, n_3n_6, n_4n_5, n_5n_6$}.Let $\emptyset_D$ and δ_D be fermatean neutrosophic dombi fuzzy graph vertex set and fermatean neutrosophic dombi fuzzy edge set specified on φ_D, ς_D respectively.



Fermatean Neutrosophic Dombi Fuzzy graphs

$$\emptyset_D = \begin{bmatrix} \frac{n_1}{0.6}, \frac{n_2}{0.8}, \frac{n_3}{0.9}, \frac{n_4}{0.6}, \frac{n_5}{0.8}, \frac{n_6}{0.8} \\ \frac{n_1}{0.7}, \frac{n_2}{1}, \frac{n_3}{0.6}, \frac{n_4}{1}, \frac{n_5}{0.5}, \frac{n_6}{0.7} \\ \frac{n_1}{0.8}, \frac{n_2}{0.4}, \frac{n_3}{0.4}, \frac{n_4}{0.7}, \frac{n_5}{0.6}, \frac{n_6}{0.7} \end{bmatrix} \quad \varsigma_D = \begin{bmatrix} \frac{n_1n_2}{0.52}, \frac{n_1n_6}{0.52}, \frac{n_2n_3}{0.73}, \frac{n_2n_5}{0.66}, \frac{n_3n_4}{0.56}, \frac{n_3n_6}{0.73}, \frac{n_4n_5}{0.52}, \frac{n_5n_6}{0.66} \\ \frac{n_1n_2}{0.7}, \frac{n_1n_6}{0.41}, \frac{n_2n_3}{0.6}, \frac{n_2n_5}{0.5}, \frac{n_3n_4}{0.6}, \frac{n_3n_6}{0.37}, \frac{n_4n_5}{0.5}, \frac{n_5n_6}{0.33} \\ \frac{n_1n_2}{0.82}, \frac{n_1n_6}{0.86}, \frac{n_2n_3}{0.62}, \frac{n_2n_5}{0.71}, \frac{n_3n_4}{0.75}, \frac{n_3n_6}{0.75}, \frac{n_4n_5}{0.79}, \frac{n_5n_6}{0.79} \end{bmatrix}$$

Definition 3.2: Consider, ζ_D ={nt,$\emptyset_{\zeta_D}(nt)$, $\mu_{\zeta_D}(nt)$,$\nu_{\zeta_D}(nt)$, $nt \in \varsigma_D$} be a fermatean neutrosophic dombi fuzzy edge set in D:

- The order of O_D is established by $\rho(O_D)$ =[$\sum_{n\in\varphi} \emptyset_\varphi$,$\sum_{n\in\varphi} \mu_\varphi$,$\sum_{n\in\varphi} \nu_\varphi$] from illustration 3.1, order of D, $\rho(O_D)$ = [3.4,4.3,2.9]
- The size of O_{s_D} is symbolized by $\rho_S(O_D)$=[$\sum_{n\in\varphi} \emptyset_\varsigma(nt)$, $\sum_{n\in\varphi} \mu_\varsigma(nt)$, $\sum_{n\in\varphi} \nu_\varsigma(nt)$] from illustration 3.1, order of D,$\rho_S(O_D)$ = [5.33,4.71,6.95]
- The degree of vertex n∈ φ_D is denoted by $ɖ_{D_N}$its described by
  $ɖ_{D_N}$=[$ɖ_{\emptyset_\varphi}(n)$, $ɖ_{\mu_\varphi}(n)$, $ɖ_{\nu_\varphi}(n)$],where

  $ɖ_{\emptyset_\varphi}(n)$=$\sum_{n,t\neq n\in\varphi} \emptyset_\varsigma(nt)$ =$\sum_{n,t\neq n\in\varphi} \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}$,

  $$\sum_{n,t\neq n\in\varphi} \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)},$$

$$\sum_{n,t\neq n\in\varphi}\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}.$$

From illustration 3.1, $ȡ_{D_N}=\begin{bmatrix}\frac{n_1}{1.45},\frac{n_2}{1.91},\frac{n_3}{2.023},\frac{n_4}{1.513},\frac{n_5}{1.84},\frac{n_6}{1.91}\\ \frac{n_1}{1.81},\frac{n_2}{1.8},\frac{n_3}{1.575},\frac{n_4}{1.8},\frac{n_5}{1.33},\frac{n_6}{1.16}\\ \frac{n_1}{2.54},\frac{n_2}{2.155},\frac{n_3}{2.125},\frac{n_4}{2.4},\frac{n_5}{2.29},\frac{n_6}{2.4}\end{bmatrix}$

- The Total degree of vertex n $\in \varphi_D$ is specified by T[$ȡ_{D_N}$]its explained by T[$ȡ_{D_N}$]=[$ȡ_{\emptyset_\varphi}(n), ȡ_{\mu_\varphi}(n), ȡ_{\nu_\varphi}(n)$],where
- $[Tȡ]_{\emptyset_\varphi}(n)=\sum_{n,t\neq n\in\varphi}\emptyset_\varsigma(nt)=\sum_{n,t\neq n\in\varphi}\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}+\emptyset_{\varphi_D}(n)$,

$$\sum_{n,t\neq n\in\varphi}\emptyset_\varsigma(nt) = \sum_{n,t\neq n\in\varphi}\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}+\emptyset_{\varphi\mu_D}(n),$$

$$\sum_{n,t\neq n\in\varphi}\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}+\emptyset_{\nu_D}(n)$$

from illustration 3.1, $[Tȡ]_{D_N}=\begin{bmatrix}\frac{n_1}{2.05},\frac{n_2}{2.71},\frac{n_3}{2.723},\frac{n_4}{2.413},\frac{n_5}{2.64},\frac{n_6}{2.8}\\ \frac{n_1}{2.51},\frac{n_2}{2.8},\frac{n_3}{2.375},\frac{n_4}{2.3},\frac{n_5}{1.83},\frac{n_6}{2.99}\\ \frac{n_1}{2.94},\frac{n_2}{2.65},\frac{n_3}{2.725},\frac{n_4}{2.8},\frac{n_5}{2.11},\frac{n_6}{3.1}\end{bmatrix}$

Definition 3.3: Let $d = (\emptyset_D,\mu_{D,}\nu_D)$ and $F = (\emptyset_F,\mu_{F,}\nu_F)$ of the graphs $D^*=(\varphi_D,\varsigma_D)$ and $F^*=(\varphi_F,\varsigma_F)$ respectively and its established by the union of two fermatean neutrosophic dombi fuzzy graphare $D\cup F=(\emptyset_D\cup\emptyset_F,\mu_D\cup\mu_F,\nu_D\cup\nu_F)$.
$\emptyset_D\cup\emptyset_F=[\alpha_{\emptyset_D\cup\emptyset_F},\beta_{\emptyset_D\cup\emptyset_F},\gamma_{\emptyset_D\cup\emptyset_F}]$
$\mu_D\cup\mu_F=[\alpha_{\mu_D\cup\mu_F},\beta_{\mu_D\cup\mu_F},\gamma_{\mu_D\cup\mu_F}]$
$\nu_D\cup\nu_F=[\alpha_{\nu_D\cup\nu_F},\beta_{\nu_D\cup\nu_F},\gamma_{\nu_D\cup\nu_F}]$ such that

$$[\alpha_{\mu_D\cup\mu_F}](\chi)=\begin{cases}[\alpha_{\mu_D}](\chi), \text{if } \chi\in\varphi_D-\varphi_F\\ [\alpha_{\mu_F}](\chi), \text{if } \chi\in\varphi_F-\varphi_D\\ \frac{\alpha_{\mu_D}(\chi)+\alpha_{\mu_D}(\chi)-2\alpha_{\mu_D}(\chi)\alpha_{\mu_D}(\chi)}{1-\alpha_{\mu_D}(\chi)\alpha_{\mu_D}(\chi)}, \text{if } \chi\in\varphi_D\cap\varphi_F\end{cases}$$

$$[\beta_{\mu_D\cup\mu_F}](\chi)=\begin{cases}[\beta_{\mu_D}](\chi), \text{if } \chi\in\varphi_D-\varphi_F\\ [\beta_{\mu_F}](\chi), \text{if } \chi\in\varphi_F-\varphi_D\\ \frac{\beta_{\mu_D}(\chi)+\beta_{\mu_D}(\chi)-2\beta_{\mu_D}(\chi)\beta_{\mu_D}(\chi)}{1-\beta_{\mu_D}(\chi)\beta_{\mu_D}(\chi)}, \text{if } \chi\in\varphi_D\cap\varphi_F\end{cases}$$

$$[\gamma_{\mu_D\cup\mu_F}](\chi)=\begin{cases}[\gamma_{\mu_D}](\chi), \text{if } \chi\in\varphi_D-\varphi_F\\ [\gamma_{\mu_F}](\chi), \text{if } \chi\in\varphi_F-\varphi_D\\ \frac{\gamma_{\mu_D}(\chi)\gamma_{\mu_D}(\chi)}{\gamma_{\mu_D}(\chi)+\gamma_{\mu_D}(\chi)-\gamma_{\mu_D}(\chi)\gamma_{\mu_D}(\chi)}, \text{if } \chi\in\varphi_D\cap\varphi_F\end{cases}$$

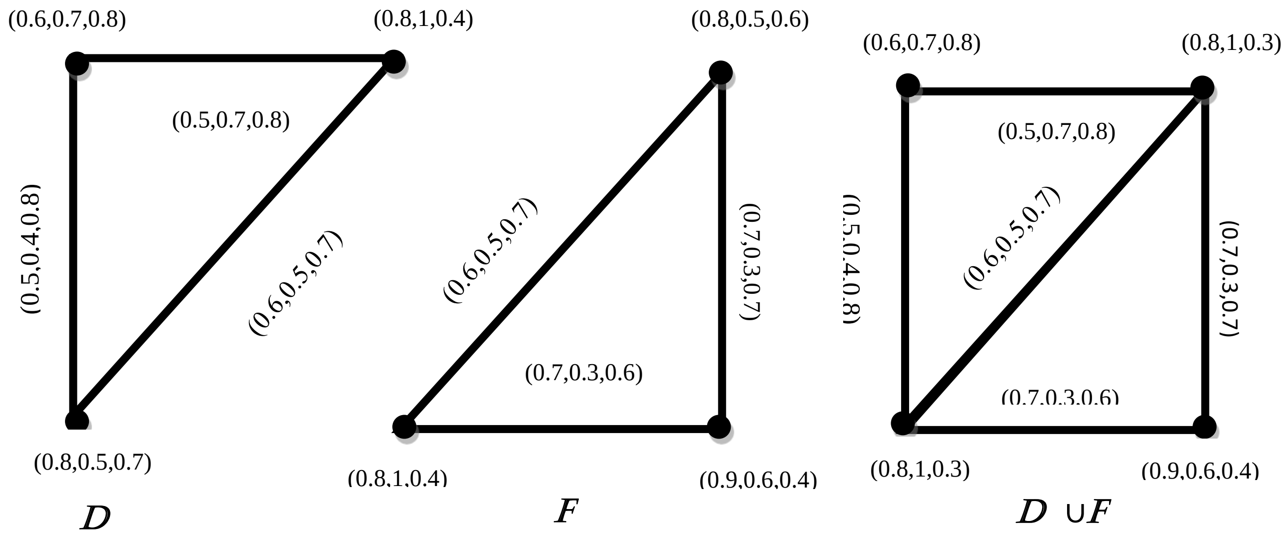
$$[\alpha_{\nu_D\cup\nu_F}](\text{nt})=\begin{cases}[\alpha_{\mu_D}](\text{nt}), \text{if nt}\in\varsigma_D-\varsigma_F\\ [\alpha_{\mu_F}](\text{nt}), \text{if nt}\in\varsigma_F-\varsigma_D\\ \frac{\alpha_{\nu_D}(nt)+\alpha_{\nu_D}(nt)-2\alpha_{\nu_D}(nt)\alpha_{\nu_D}(nt)}{1-\alpha_{\nu_D}(nt)\alpha_{\nu_D}(nt)}, \text{if nt}\in\varsigma_D\cap\varsigma_F\end{cases}$$

$$[\beta_{\nu_D \cup \nu_F}](nt) = \begin{cases} [\beta_{\mu_D}](nt), \text{if } nt \in \varsigma_D - \varsigma_F \\ [\beta_{\mu_F}](nt), \text{if } nt \in \varsigma_F - \varsigma_D \\ \frac{\beta_{\nu_D}(nt)+\beta_{\nu_D}(nt)-2\beta_{\nu_D}(nt)\beta_{\nu_D}(nt)}{1-\beta_{\nu_D}(nt)\beta_{\nu_D}(nt)}, \text{if } nt \in \varsigma_D \cap \varsigma_F \end{cases}$$

$$[\gamma_{\nu_D \cup \nu_F}](nt) = \begin{cases} [\gamma_{\mu_D}](nt), \text{if } nt \in \varsigma_D - \varsigma_F \\ [\gamma_{\mu_F}](nt), \text{if } nt \in \varsigma_F - \varsigma_D \\ \frac{\gamma_{\nu_D}(nt)\gamma_{\nu_D}(nt)}{\gamma_{\nu_D}(nt)+\gamma_{\nu_D}(nt)-\gamma_{\nu_D}(nt)\gamma_{\nu_D}(nt)}, \text{if } nt \in \varsigma_D \cap \varsigma_F \end{cases}$$

Where $0 \leq \alpha_{\nu_D}{}^3(nt)+\beta_{\nu_D}{}^3(nt)+\gamma_{\nu_D}{}^3(nt) \leq 2$

Example 3.3: Let us consider $D = (\emptyset_D, \mu_D, \nu_D)$ and $\boldsymbol{F} = (\emptyset_F, \mu_F, \nu_F)$ $D^* = (\varphi_D, \varsigma_D)$ and $F^* = (\varphi_F, \varsigma_F)$ separately. Here φ_D = { a,b,c}, ς_F ={ab,bc,ca}, φ_F={b,c,d} ,ς_F={bc,cd,da}.Then the union of two neutrosophic dombi fuzzy graphs,$\varphi_D \cup \varphi_F$ is



Definition 3.4: Let $D = (\emptyset_D, \mu_D, \nu_D)$ and $\boldsymbol{F} = (\emptyset_F, \mu_F, \nu_F)$ of the graphs $D^* = (\varphi_D, \varsigma_D)$ and $F^* = (\varphi_F, \varsigma_F)$ respectively and its established by the intersection of two fermatean neutrosophic dombi fuzzy graph $d \cap F$ *is*

$d \cap F = (\emptyset_D \cap \emptyset_F, \mu_D \cap \mu_F, \nu_D \cap \nu_F)$.Here,

$\emptyset_D \cap \emptyset_F = [\alpha_{\emptyset_D \cap \emptyset_F}, \beta_{\emptyset_D \cap \emptyset_F}, \gamma_{\emptyset_D \cap \emptyset_F}]$

$\mu_D \cap \mu_F = [\alpha_{\mu_D \cap \mu_F}, \beta_{\mu_D \cap \mu_F}, \gamma_{\mu_D \cap \mu_F}]$

$\nu_D \cap \nu_F = [\alpha_{\nu_D \cap \nu_F}, \beta_{\nu_D \cap \nu_F}, \gamma_{\nu_D \cap \nu_F}]$ such that

- $[\alpha_{\mu_D \cap \mu_F}](\chi) = \frac{\alpha_{\mu_D}(\chi)+\alpha_{\mu_D}(\chi)-2\alpha_{\mu_D}(\chi)\alpha_{\mu_D}(\chi)}{1-\alpha_{\mu_D}(\chi)\alpha_{\mu_D}(\chi)}$, if $\chi \in \varphi_D \cap \varphi_F$

$$[\beta_{\mu_D \cap \mu_F}](\chi) = \frac{\beta_{\mu_D}(\chi)+\beta_{\mu_D}(\chi)-2\beta_{\mu_D}(\chi)\beta_{\mu_D}(\chi)}{1-\beta_{\mu_D}(\chi)\beta_{\mu_D}(\chi)}, \text{if } \chi \in \varphi_D \cap \varphi_F$$

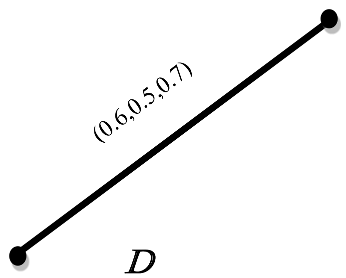
$$[\gamma_{\mu_D \cap \mu_F}](\chi) = \frac{\gamma_{\mu_D}(\chi)\gamma_{\mu_D}(\chi)}{\gamma_{\mu_D}(\chi)+\gamma_{\mu_D}(\chi)-\gamma_{\mu_D}(\chi)\gamma_{\mu_D}(\chi)}, \text{if } \chi \in \varphi_D \cap \varphi_F$$

- $[\alpha_{\nu_D \cap \nu_F}](\text{nt}) = \frac{\alpha_{\nu_D}(nt)+\alpha_{\nu_D}(nt)-2\alpha_{\nu_D}(nt)\alpha_{\nu_D}(nt)}{1-\alpha_{\nu_D}(nt)\alpha_{\nu_D}(nt)}$, if nt $\in \varsigma_D \cap \varsigma_F$

$[\beta_{\nu_D \cap \nu_F}](\text{nt}) = \frac{\beta_{\nu_D}(nt)+\beta_{\nu_D}(nt)-2\beta_{\nu_D}(nt)\beta_{\nu_D}(nt)}{1-\beta_{\nu_D}(nt)\beta_{\nu_D}(nt)}$, if nt $\in \varsigma_D \cap \varsigma_F$

$[\gamma_{\nu_D \cap \nu_F}](\text{nt}) = \frac{\gamma_{\nu_D}(nt)\gamma_{\nu_D}(nt)}{\gamma_{\nu_D}(nt)+\gamma_{\nu_D}(nt)-\gamma_{\nu_D}(nt)\gamma_{\nu_D}(nt)}$, if nt $\in \varsigma_D \cap \varsigma_F$

Example 3.4: Let us consider the two neutrosophic dombi fuzzy graphs $D = (\emptyset_D, \mu_{D,}\nu_D)$ and F=$(\emptyset_F, \mu_{F,}\nu_F)$D*= (φ_D, ς_D) and F^*=(φ_F, ς_F) separately. Here φ_D = { a,b,c}, ς_F ={ab,bc,ca}, φ_F={b,c,d} ,ς_F={bc,cd,da}.Then the intersection of $\varphi_D \cap \varphi_F$ is



Definition 3.5:[Direct product] Let $\emptyset_i$ be a fermatean neutrosophic fuzzy subset of φ_D and ς_i be a neutrosophic fuzzy subset $\varsigma_D$of ς_i=1,2,3.The direct product of fermatean neutrosophic dombi fuzzy graph. $d = (\emptyset_D, \mu_{D,}\nu_D)$ and $F = (\emptyset_F, \mu_{F,}\nu_F)$ D*= (φ_D, ς_D) and F^*=(φ_F, ς_F) separately is established by dXF is

$d\, XF = (\emptyset_D X \emptyset_F, \mu_D X \mu_F, \nu_D X \nu_F)$.

(i.e) $\emptyset_D X \emptyset_F$=$[\alpha_{\emptyset_D X \emptyset_F}, \beta_{\emptyset_D X \emptyset_F}, \gamma_{\emptyset_D X \emptyset_F}]$

$\mu_D X \mu_F$=$[\alpha_{\mu_D X \mu_F}, \beta_{\mu_D X \mu_F}, \gamma_{\mu_D X \mu_F}]$

$\nu_D X \nu_F$=$[\alpha_{\nu_D X \nu_F}, \beta_{\nu_D X \nu_F}, \gamma_{\nu_D X \nu_F}]$ such that

$\xi_{D\ XF}$ $=\{(n,n_2),(n,t_2)$: $x \in \varphi_D$,$n_2t_2 \in \varsigma_F\} \cup \{(n_1,z),(n_2,z)\ :\ n_1n_2 \in \varsigma_D, z \in \varphi_F\}$ such that,

for all $(n_1,n_2) \in \varphi_D$xφ_F ,

- $\alpha_{\emptyset_D\ X\emptyset_F}\ (n_1,n_2) = \frac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_F}(n_2)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}$

$\beta_{\mu_D X \mu_F}(n_1,n_2) = \frac{\beta_{\mu_D}(n_1)\beta_{\mu_F}(n_2)}{\beta_{\mu_D}(n_1)+\beta_{\mu_F}(n_2)-\beta_{\mu_D}(\chi)\beta_{\mu_F}(n_2)}$

$\gamma_{\nu_D\ X\nu_F}(n_1,n_2) = \frac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_F}(n_2)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)}$

$$(\varsigma_D \text{x} \varsigma_F)(n_1,n_2)(t_1,t_2) = \frac{\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}{\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)+\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)-\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)},$$

$$\frac{\varsigma_{\beta_{\mu_D}}(n_1t_1)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}{\varsigma_{\beta_{\mu_D}}(n_1t_1)+\varsigma_{\beta_{\mu_F}}(n_2t_2)-\varsigma_{\beta_{\mu_D}}(n_1t_1)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)},$$

$$\frac{\varsigma_{\gamma_{\nu_D}}(n_1t_1)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)-\varsigma_{\gamma_{\nu_D}}(n_1t_1)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)}{1-\varsigma_{\gamma_{\nu_D}}(n_1t_1)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)}.$$

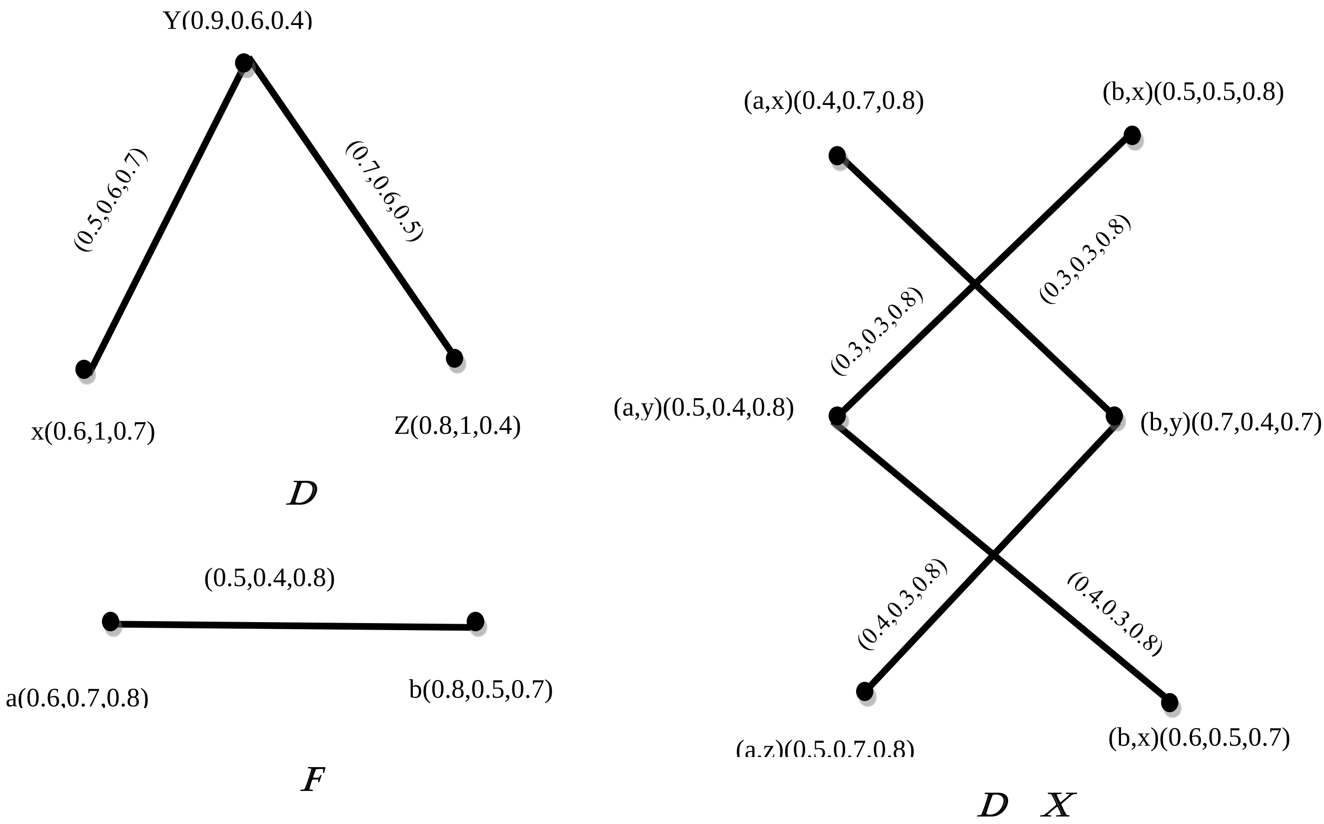
For all $n_1t_1 \in \varsigma_D, n_2t_2 \in \varsigma_F$

Example 3.5: Consider two neutrosophic dombi fuzzy graphs $d = (\emptyset_D, \mu_{D,} \nu_D)$ and $F = (\emptyset_F, \mu_{F,} \nu_F)$.Here φ_D={a,b},φ_F={x,y,z},$\varsigma_D = \{ab\}$, ς_F={xy,yz} where

$\varphi_D = \left\{\frac{a}{(0.6,0.7,0.8)}, \frac{b}{(0.8,0.5,0.7)}\right\}$,$\varphi_F$={$\frac{x}{(0.6,1,0.7)}$,$\frac{y}{(0.9,0.6,0.4)}$,$\frac{z}{(0.8,1,0.4)}$},$\varsigma_D = \left\{\frac{ab}{(0.5,0.4,0.8)}\right\}$,

ς_F={$\frac{xy}{(0.5,0.6,0.7)}$,$\frac{yz}{(0.7,0.6,0.5)}$ }. Then we have

$(\varphi_D \text{x} \varphi_F)$(a,b) =(0.5,0.4,0.8), $(\varphi_D \text{x} \varphi_F)$(x,y)=(0.5,0.6,0.7) , $(\varphi_D \text{x} \varphi_F)$(y,z)=(0.7,0.6,0.5),
$(\varphi_D \text{x} \varphi_F)$(a,x)=(0.4,0.7,0.8), $(\varphi_D \text{x} \varphi_F)$(a,y)=(0.5,0.4,0.8), $(\varphi_D \text{x} \varphi_F)$(a,z)=(0.5,0.7,0.8)
$(\varphi_D \text{x} \varphi_F)$(b,x)=(0.5,0.5,0.8), $(\varphi_D \text{x} \varphi_F)$(b,y)=(0.7,0.4,0.7), $(\varphi_D \text{x} \varphi_F)$(b,z)=(0.6,0.5,0.7)
$(\varsigma_D \text{x} \varsigma_F)(a,x)(b,y)$=(0.4,0.3,0.8),$(\varsigma_D \text{x} \varsigma_F)(a,y)(b,z) = (0.3,0.3,0.8)$,
$(\varsigma_D \text{x} \varsigma_F)(b,x)(a,y)$=(0.4,0.3,0.8), $(\varsigma_D \text{x} \varsigma_F)(b,y)(b,z)$=(0.3,0.3,0.8).



Theorem 3.1: The direct product of two fermatean neutrosophic dombi fuzzy graph is a fermatean neutrosophic dombi fuzzy graph.

Proof:

Let d and F be the fermatean neutrosophic dombi fuzzy graphs $d = (\emptyset_D, \mu_{D,} \nu_D)$ and $F = (\emptyset_F, \mu_{F,} \nu_F)$ respectively .

Consider,for all $n_1t_1 \in \varsigma_D, n_2t_2 \in \varsigma_F$ such that

$(\varsigma_D \text{x} \varsigma_F)(n_1, n_2)(t_1, t_2)$

The direct product d XF= T[$\varsigma_D(n_1 t_1)$,$\varsigma_F(n_2 t_2)$]

$$\leq \text{T}\begin{bmatrix} \frac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(n_2)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_D}(n_2)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(n_2)}, \\ \frac{\beta_{\mu_D}(n_1)\beta_{\mu_D}(n_2)}{\beta_{\mu_D}(n_1)+\beta_{\mu_D}(n_2)-\beta_{\mu_D}(n_2)\beta_{\mu_D}(n_2)}, \\ \frac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_D}(n_2)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(n_2)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(n_2)} \end{bmatrix}$$

$(\varsigma_D \text{x} \varsigma_F)(n_1, n_2)(t_1, t_2)$

$$\leq \begin{cases} \frac{(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1 t_2)}{(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1 n_2)+(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1 t_2)-(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1 t_2)}, \\ \frac{(\varphi_D \text{x} \varphi_F)\beta_{\mu_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1 t_2)}{(\varphi_D \text{x} \varphi_F \beta_{\mu_D}(n_1 n_2)+(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1 t_2)-(\varphi_D \text{x} \varphi_F)\beta_{\mu_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1 t_2)}, \\ \frac{(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1 n_2)(+\varphi_D \text{x} \varphi_F)\gamma_{\nu_F}(t_1 t_2)-2(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1 t_2)}{1-(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1 t_2)} \end{cases}$$

Corollary 3.2: The product of two fermatean neutrosophic dombi fuzzy graph is the fermatean neutrosophic dombi fuzzy graph

From the illustration 3.5

$(\varphi_D \text{x} \varphi_F)$(a,x)=(0.4,0.7,0.8), $(\varphi_D \text{x} \varphi_F)$(b,y)=(0.7,0.4,0.7),$(\varsigma_D \text{x} \varsigma_F)(a, y)(b, z) = (0.3,0.3,0.8)$

$(\varsigma_D \text{x} \varsigma_F)(a, y)(b, z) = (0.3,0.3,0.8)$

$$= \begin{cases} \frac{(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1 t_2)}{(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1 n_2)+(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1 t_2)-(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1 t_2)}, \\ \frac{(\varphi_D \text{x} \varphi_F)\beta_{\mu_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1 t_2)}{(\varphi_D \text{x} \varphi_F \beta_{\mu_D}(n_1 n_2)+(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1 t_2)-(\varphi_D \text{x} \varphi_F)\beta_{\mu_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1 t_2)}, \\ \frac{(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1 n_2)(+\varphi_D \text{x} \varphi_F)\gamma_{\nu_F}(t_1 t_2)-2(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1 t_2)}{1-(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1 n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1 t_2)} \end{cases}$$

=(0.34,0.34,0.86)

Therefore the product of two fermatean neutrosophic dombi fuzzy graph is fermatean neutrosophic dombi fuzzy graph.

Definition 3.6:[Cartesian product] Let $\emptyset_i$ be a fermatean neutrosophic fuzzy subset of φ_D and ς_i be a neutrosophic fuzzy subset $\varsigma_D$of ς_i=1,2,3.The cartesian product of fermatean neutrosophic dombi fuzzy graph.$\boldsymbol{d}$ = $(\emptyset_D, \mu_{D,} \nu_D)$ and $\boldsymbol{F}$ = $(\emptyset_F, \mu_{F,} \nu_F)$ D*= (φ_D, ς_D) and F^*=(φ_F, ς_F) separately is established by $d \, \square \, F$ is defined as

$d \, \square \, F$= $(\emptyset_D \square \emptyset_F, \mu_D \square \mu_F, \nu_D \square \nu_F)$.

(i.e)$\emptyset_D \square \emptyset_F$=$[\alpha_{\emptyset_D \square \emptyset_F}, \beta_{\emptyset_D \square \emptyset_F}, \gamma_{\emptyset_D \square \emptyset_F}]$

$\mu_D \square \mu_F$=$[\alpha_{\mu_D \square \mu_F}, \beta_{\mu_D \square \mu_F}, \gamma_{\mu_D \square \mu_F}]$

$\nu_D \square \nu_F$=$[\alpha_{\nu_D \square \nu_F}, \beta_{\nu_D \square \nu_F}, \gamma_{\nu_D \square \nu_F}]$

$\xi_{D \,\square F} = \{(n, n_2), (n, t_2):\ x \in \varphi_D, n_2 t_2 \in \varsigma_F\} \cup \{(n_1, z), (n_2, z)\ :\ n_1 n_2 \in \varsigma_D, z \in \varphi_F\} \cup$

$\{(n_1, n_2)(t_1, t_2): n_1 n_2 \in \varsigma_D, , n_2 \neq t_2\}$

For all $(n_1, n_2) \in \varphi_D \text{x} \varphi_F$ such that,

$$i)\ \varphi_D \square \varphi_F(n_1,n_2) = \begin{cases} \frac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_F}(n_2)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}, \\ \frac{\beta_{\mu_D}(n_1)\beta_{\mu_F}(n_2)}{\beta_{\mu_D}(n_1)+\beta_{\mu_F}(n_2)-\beta_{\mu_D}(n_1)\beta_{\mu_F}(n_2)}, \\ \frac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_F}(n_2)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)} \end{cases}$$

$$ii)\ (\varsigma_D \square \varsigma_F)\ (\mathrm{n}, n_2)(t,t_2) = \begin{cases} \frac{\varphi_{\alpha_{\emptyset_D}}(n)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}{\varphi_{\alpha_{\emptyset_D}}(n)+\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)-\varphi_{\alpha_{\emptyset_D}}(n)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}, \\ \frac{\varphi_{\beta_{\mu_D}}(n)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}{\varphi_{\beta_{\mu_D}}(n)+\varsigma_{\beta_{\mu_F}}(n_2t_2)-\varphi_{\beta_{\mu_D}}(n)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}, \\ \frac{\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)-\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)}{1-\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)} \end{cases}$$

For all $n \in \varphi_D$ and $n_2t_2 \in \varsigma_F$

$$(\varsigma_D \square \varsigma_F)\ (n_1,n_2)(t_1,t_2) = \begin{cases} \frac{\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}{\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)+\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)-\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}, \\ \frac{\varsigma_{\beta_{\mu_D}}(n_1t_1)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}{\varsigma_{\beta_{\mu_D}}(n_1t_1)+\varsigma_{\beta_{\mu_F}}(n_2t_2)-\varsigma_{\beta_{\mu_D}}(n_1t_1)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}, \\ \frac{\varsigma_{\gamma_{\nu_D}}(n_1t_1)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)-\varsigma_{\gamma_{\nu_D}}(n_1t_1)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)}{1-\varsigma_{\gamma_{\nu_D}}(n_1t_1)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)} \end{cases}$$

For all $n_1t_1 \in \varsigma_D, n_2t_2 \in \varsigma_F$

Example 3.5: Consider two Fermatean neutrosophic dombi fuzzy graphs $d = (\emptyset_D, \mu_{D,}\nu_D)$ and $F = (\emptyset_F, \mu_{F,}\nu_F)$.Here φ_D={x,y,z},φ_F={a,b},ς_D = {xy, yz }, ς_F= $\{ab\}$where

$$\varphi_D = \{\frac{x}{(0.7,0.6,0.5)}, \frac{y}{(0.8,0.7,0.5)}, \frac{z}{(0.9,1,0.6)}\}, \varphi_F = \left\{\frac{a}{(0.7,0.8,0.5)}, \frac{b}{(0.8,0.7,0.6)}\right\},\ \varsigma_F = \left\{\frac{ab}{(0.6,0.5,0.7)}\right\}$$

$\varsigma_D = \left\{\frac{xy}{(0.6,0.5,0.7)}, \frac{yz}{(0.7,0.7,0.7)}\right\}$. Then we have

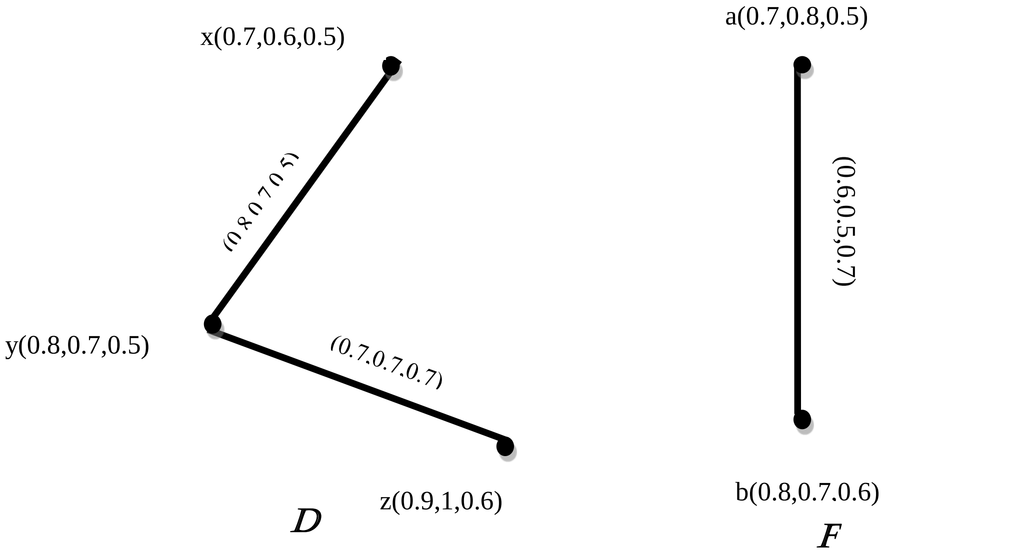
$\varphi_D \square \varphi_F$(x,y) = (0.6,0.5,0.7), $\varphi_D \square \varphi_F(y,z) = (0.7,0.7,0.7)$, $\varphi_D \square \varphi_F(a,b) = ((0.6,0.5,0.7)$,
$\varphi_D \square \varphi_F(x,a) = (0.5,0.5,0.7)$, $\varphi_D \square \varphi_F(x,b) = (0.6,0.5,0.7)$, $\varphi_D \square \varphi_F(y,a) = (0.6,0.6,0.7)$,
$\varphi_D \square \varphi_F(y,b) = (0.6,0.5,0.7)$, $\varphi_D \square \varphi_F(z,a) = (0.6,0.8,0.7)$, $\varphi_D \square \varphi_F(z,b) = (0.7,0.7,0.8)$,

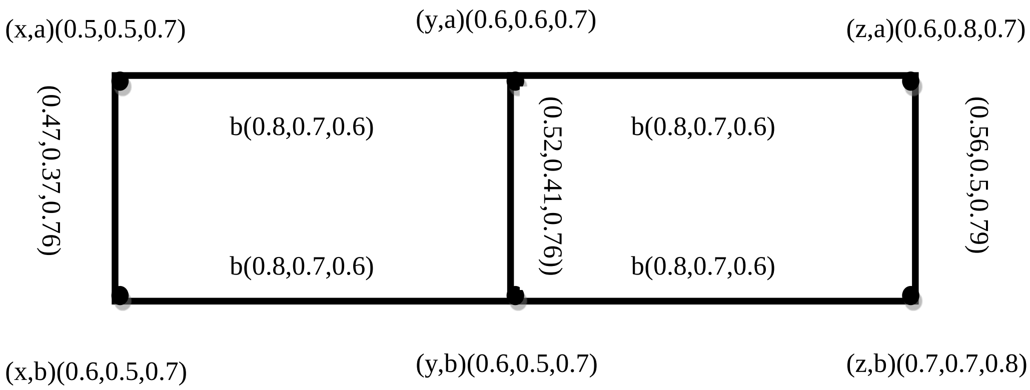
$(\varsigma_D \square \varsigma_F)$(x,a)(x,b) = (0.47,0.37,0.76),$(\varsigma_D \square \varsigma_F)$(y,a)(y,b) = (0.52,0.41,0.76)

$(\varsigma_D \square \varsigma_F)(z,a)(z,b) = (0.56,0.5,0.79)$,$(\varsigma_D \square \varsigma_F)$ (x,a)(y,a)=(0.47,0.44,0.76),

$(\varsigma_D \square \varsigma_F)$(y,a)(z,a)=(0.53,0.6,0.76),$(\varsigma_D \square \varsigma_F)$(x,b)(y,b)=(0.52,0.41,0.79)

$(\varsigma_D \square \varsigma_F)$(y,b)(z,b)=(0.6,0.53,0.79).





$D\square F$

Corollary 3.3: The Cartesian product of two fermatean neutrosophic dombi fuzzy graph is not necessiarily to a fermatean neutrosophic dombi fuzzy graph.

From the graph $d \square F$,

$(\varsigma_D \square \varsigma_F)$(y,a)(y,b)=(0.52,0.41,0.76)

$(\varsigma_D \square \varsigma_F)$(y,a)(y,b)

$$==\begin{cases} \frac{(\varphi_D\square\varphi_F)\alpha_{\emptyset_D}(y,a)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(y,b)}{(\varphi_D\square\varphi_F)\alpha_{\emptyset_D}(y,a)+(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(y,b)-(\varphi_D\square\varphi_F)\alpha_{\emptyset_D}(y,a)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(y,b)}, \\ \frac{(\varphi_D\square\varphi_F)\beta_{\mu_D}(y,a)(\varphi_D\square\varphi_F)\beta_{\mu_F}(y,b)}{(\varphi_D\square\varphi_F)\beta_{\mu_D}(y,a)+(\varphi_D\square\varphi_F)\beta_{\mu_F}(y,b)-(\varphi_D\square\varphi_F)\beta_{\mu_D}(y,a)(\varphi_D\square\varphi_F)\beta_{\mu_F}(y,b)}, \\ \frac{(\varphi_D\square\varphi_F)\gamma_{\nu_D}(y,a)+(\varphi_D\square\varphi_F)\gamma_{\nu_F}(y,b)-2(\varphi_D\square\varphi_F)\gamma_{\nu_D}(y,a)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(y,b)}{1-(\varphi_D\square\varphi_F)\gamma_{\nu_D}(y,a)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(y,b)} \end{cases}$$

=(0.42,0.375,0.72)

$(\varsigma_D \square \varsigma_F)$(y,a)(y,b)=(0.52,0.41,0.76)$\nleq$ (0.42,0.375,0.72)

From the illustration, Cartesian of two fermatean neutrosophic dombi fuzzy graph is not neutrosophic dombi fuzzy graph.

Definition 3.7: [Fermatean neutrosophic dombi fuzzy edge graph]

$D^*$is defined the fermatean neutrosophic dombi fuzzy edge graph if a neutrosophic fuzzy fuzzy by a truth-membership function, an indeterminacy-membership function, and a falsity - membership function is attached from (0,1) to each edge of the fermatean neutrosophic dombi fuzzy edge graph $D^*$of a graph d and each vertex φ_D is crisply in d.

Theorem 3.2: The cartesian product of two fermatean neutrosophic dombi fuzzy edge graph is a fermatean neutrosophic dombi fuzzy graph

Proof:

Let d and F be the fermatean neutrosophic dombi fuzzy edge graphs $d = (\emptyset_D, \mu_{D,}\nu_D)$ and $F = (\emptyset_F, \mu_{F,}\nu_F)$ $D^* = (\varphi_D, \varsigma_D)$ and $F^*=(\varphi_F, \varsigma_F)$ separately Then the Cartesian product $d\square F$

Consider,

i)For all $n \in \varsigma_D, n_2t_2 \in \varsigma_F$ such that

$(\varsigma_D\square\varsigma_F)(n, n_2)(n, t_2)$= T[$\varphi_D(n)$,$\varsigma_F(n_2t_2)$]=T[1,$\varsigma_F(n_2t_2)$]

$(\varsigma_D\square\varsigma_F)(n, n_2)(n, t_2)$

$$\leq T\begin{bmatrix} \frac{\alpha_{\emptyset_D}\varphi(n)\alpha_{\emptyset_F}\varsigma(n_2t_2)}{\alpha_{\emptyset_D}\varphi(n)+\alpha_{\emptyset_F}\varsigma(n_2t_2)-\alpha_{\emptyset_D}\varphi(n)\alpha_{\emptyset_F}\varsigma(n_2t_2)}, \\ \frac{\beta_{\mu_D}\varphi(n)\beta_{\mu_F}\varsigma(n_2t_2)}{\beta_{\mu_D}\varphi(n)+\beta_{\mu_F}\varsigma(n_2t_2)-\beta_{\mu_D}\varphi(n)\beta_{\mu_F}\varsigma(n_2t_2)}, \\ \frac{\gamma_{\nu_D}\varphi(n)+\gamma_{\nu_F}\varsigma(n_2t_2)-2\gamma_{\nu_D}\varphi(n)\gamma_{\nu_F}\varsigma(n_2t_2)}{\gamma_{\nu_D}\varphi(n)\gamma_{\nu_F}\varsigma(n_2t_2)} \end{bmatrix}$$

$$\varsigma_F(n_2t_2)=\begin{cases} \frac{\alpha_{\emptyset_D}(n_2)\alpha_{\emptyset_F}(t_2)}{\alpha_{\emptyset_D}(n_2)+\alpha_{\emptyset_F}(t_2)-\alpha_{\emptyset_D}(n_2)\alpha_{\emptyset_F}(t_2)}, \\ \frac{\beta_{\mu_D}(n_2)\beta_{\mu_F}(t_2)}{\beta_{\mu_D}(n_2)+\beta_{\mu_F}(t_2)-\beta_{\mu_D}(n_2)\beta_{\mu_F}(t_2)}, \\ \frac{\gamma_{\nu_D}(n_2)+\gamma_{\nu_F}(t_2)-2\gamma_{\nu_D}(n_2)\gamma_{\nu_F}(t_2)}{\gamma_{\nu_D}(n_2)\gamma_{\nu_F}(t_2)} \end{cases}$$

$$=\begin{cases} \frac{(\varphi_D\square\varphi_F)\alpha_{\emptyset_D}(n,n_2)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(n,t_2)}{(\varphi_D\square\varphi_F)\alpha_{\emptyset_D}(n,n_2)+(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(n,t_2)-(\varphi_D\square\varphi_F)\alpha_{\emptyset_D}(n,n_2)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(n,t_2)}, \\ \frac{(\varphi_D\square\varphi_F)\beta_{\mu_D}(n,n_2)(\varphi_D\square\varphi_F)\beta_{\mu_F}(n,t_2)}{(\varphi_D\square\varphi_F)\beta_{\mu_D}(n,n_2)+(\varphi_D\square\varphi_F)\beta_{\mu_F}(n,t_2)-(\varphi_D\square\varphi_F)\beta_{\mu_D}(n,n_2)(\varphi_D\square\varphi_F)\beta_{\mu_F}(n,t_2)}, \\ \frac{(\varphi_D\square\varphi_F)\gamma_{\nu_D}(n,n_2)+(\varphi_D\square\varphi_F)\gamma_{\nu_F}(n,t_2)-2(\varphi_D\square\varphi_F)\gamma_{\nu_D}(n,n_2)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(n,t_2)}{1-(\varphi_D\square\varphi_F)\gamma_{\nu_D}(n,n_2)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(n,t_2)} \end{cases}$$

------------------------(1)

For all $n \in \varsigma_D, n_2t_2 \in \varsigma_F$

i) Now consider $n_1t_1 \in \varsigma_D$,z $\in \varphi_F$

$(\varsigma_D\square\varsigma_F)$ $(n_1, z)(t_1, z)$ = T[$\varsigma_D(n_1t_1)$,φ_F(z)]

$$\varsigma_D(n_1t_1) \leq \begin{cases} \frac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(t_1)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_D}(t_1)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(t_1)}, \\ \frac{\beta_{\mu_D}(n_1)\beta_{\mu_D}(t_1)}{\beta_{\mu_D}(n_1)+\beta_{\mu_D}(t_1)-\beta_{\mu_D}(n_1)\beta_{\mu_D}(t_1)}, \\ \frac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_D}(t_1)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(t_1)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(t_1)} \end{cases}$$

$$= \begin{cases} \frac{(\varphi_D \square \varphi_F)\alpha_{\emptyset_D}(n_1,z)(\varphi_D \square \varphi_F)\alpha_{\emptyset_F}(t_1,z)}{(\varphi_D \square \varphi_F)\alpha_{\emptyset_D}(n_1,z)+(\varphi_D \square \varphi_F)\alpha_{\emptyset_F}(t_1,z)-(\varphi_D \square \varphi_F)\alpha_{\emptyset_D}(n_1,z)(\varphi_D \square \varphi_F)\alpha_{\emptyset_F}(t_1,z)}, \\ \frac{(\varphi_D \square \varphi_F)\beta_{\mu_D}(n_1,z)(\varphi_D \square \varphi_F)\beta_{\mu_F}(t_1,z)}{(\varphi_D \square \varphi_F)\beta_{\mu_D}(n_1,z)+(\varphi_D \square \varphi_F)\beta_{\mu_F}(t_1,z)-(\varphi_D \square \varphi_F)\beta_{\mu_D}(n_1,z)(\varphi_D \square \varphi_F)\beta_{\mu_F}(t_1,z)}, \\ \frac{(\varphi_D \square \varphi_F)\gamma_{\nu_D}(n_1,z)+(\varphi_D \square \varphi_F)\gamma_{\nu_F}(t_1,z)-2(\varphi_D \square \varphi_F)\gamma_{\nu_D}(n_1,z)(\varphi_D \square \varphi_F)\alpha_{\emptyset_F}(t_1,z)}{1-(\varphi_D \square \varphi_F)\gamma_{\nu_D}(n_1,z)(\varphi_D \square \varphi_F)\alpha_{\emptyset_F}(t_1,z)} \end{cases}$$

---------------------------(2)

For all $n_1t_1 \in \varsigma_D$ and z $\in \varphi_F$

From (1) & (2)

Every Cartesian product of two fermateanneutrosophicdombi fuzzy edge graph is a fermateanneutrsophicdombi fuzzy edge graph.

Definition 3.8: [Composition or Lexicographic product]: Let $\emptyset_i$ be a fermatean neutrosophic fuzzy subset of φ_D and ς_i be a fermateean neutrosophic fuzzy subset of ς_i=1,2,3.The composition of fermatean neutrosophic dombi fuzzy graphs ***d***= $(\emptyset_D, \mu_{D,}\nu_D)$ and ***F*** = $(\emptyset_F, \mu_{F,}\nu_F)$ D*= (φ_D, ς_D) and F^*=(φ_F, ς_F) separately is established by $d \circ F$and it is defined as

$d \circ F$= $(\emptyset_D \circ \emptyset_F, \mu_D \circ \mu_F, \nu_D \circ \nu_F)$.Here,

$\emptyset_D \circ \emptyset_F$ =$[\alpha_{\emptyset_D \circ \emptyset_F}, \beta_{\emptyset_D \circ \emptyset_F}, \gamma_{\emptyset_D \circ \emptyset_F}]$

$\mu_D \circ \mu_F$=$[\alpha_{\mu_D \circ \mu_F}, \beta_{\mu_D \circ \mu_F}, \gamma_{\mu_D \circ \mu_F}]$

$\nu_D \circ \nu_F$ =$[\alpha_{\nu_D \circ \nu_F}, \beta_{\nu_D \circ \nu_F}, \gamma_{\nu_D \circ \nu_F}]$

$\xi_{\ D \circ F}$ = $\{(n, n_2), (n, t_2)\colon\ x \in \varphi_D, n_1n_2 \in \varsigma_F\} \cup \{(n_1, z), (n_2, z)\ :\ n_1n_2 \in \varsigma_D, z \in \varphi_F\} \cup$

$\{\ (n_1, n_2)(t_1, t_2)\colon n_1n_2 \in \varsigma_D, , n_2 \neq t_2\}$

$$i)(\varphi_D \circ \varphi_F)(n_1, n_2)\ = \begin{cases} \frac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_F}(n_2)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}, \\ \frac{\beta_{\mu_D}(n_1)\beta_{\mu_F}(n_2)}{\beta_{\mu_D}(n_1)+\beta_{\mu_F}(n_2)-\beta_{\mu_D}(n_1)\beta_{\mu_F}(n_2)} \\ \frac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_F}(n_2)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)} \end{cases}$$

For all $(n_1, n_2) \in \varphi_D \circ \varphi_F$ such that,

$ii)(\varsigma_D \circ \varsigma_F)$ $(\mathrm{n}, n_2)(t, t_2)$

$$= \begin{cases} \frac{\varphi_{\alpha_{\emptyset_D}}(n)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}{\varphi_{\alpha_{\emptyset_D}}(n)+\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)-\varphi_{\alpha_{\emptyset_D}}(n)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}, \\ \frac{\varphi_{\beta_{\mu_D}}(n)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}{\varphi_{\beta_{\mu_D}}(n)+\varsigma_{\beta_{\mu_F}}(n_2t_2)-\varphi_{\beta_{\mu_D}}(n)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}, \\ \frac{\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)-\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)}{1-\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)} \end{cases}$$

For all $n \in \varphi_D$ and $n_2t_2 \in \varsigma_F$

$$iii)(\varsigma_D \circ \varsigma_F)\ (n_1, z)(t_1, z) = \begin{cases} \frac{\alpha_{\emptyset_F}(z)\alpha_{\emptyset_D}(n_1t_1)}{\alpha_{\emptyset_F}(z)+\alpha_{\emptyset_D}(n_1t_1)-\alpha_{\emptyset_F}(z)\alpha_{\emptyset_D}(n_1t_1)}, \\ \frac{\beta_{\mu_F}(z)\beta_{\mu_D}(n_1t_1)}{\beta_{\mu_F}(z)+\beta_{\mu_D}(n_1t_1)-\beta_{\mu_F}(z)\beta_{\mu_D}(n_1t_1)}, \\ \frac{\gamma_{\nu_F}(z)+\gamma_{\nu_D}(n_1t_1)-2\gamma_{\nu_F}(z)\gamma_{\nu_D}(n_1t_1)}{\gamma_{\nu_F}(z)\gamma_{\nu_D}(n_1t_1)} \end{cases}$$

ii) $(\varsigma_D \circ \varsigma_F)\ (n_1, n_2)(t_1, t_2)$

$$= \begin{cases} \frac{\alpha_{\emptyset_F}(n_2)\alpha_{\emptyset_F}(t_2)\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)}{\alpha_{\emptyset_F}(n_2)\alpha_{\emptyset_F}(t_2)+\alpha_{\emptyset_F}(t_2)\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)+\alpha_{\emptyset_F}(n_2)\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)\alpha_{\emptyset_F}(n_2)\alpha_{\emptyset_F}(t_2)\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)}, \\ \frac{\beta_{\mu_F}(n_2)\beta_{\mu_F}(t_2)\varsigma_{\beta_{\mu_D}}(n_1t_1)}{\beta_{\mu_F}(n_2)\beta_{\mu_F}(t_2)+\beta_{\mu_F}(t_2)\varsigma_{\beta_{\mu_D}}(n_1t_1)+\beta_{\mu_F}(n_2)\varsigma_{\beta_{\mu_D}}(n_1t_1)-2\beta_{\mu_F}(n_2)\beta_{\mu_F}(t_2)\varsigma_{\beta_{\mu_D}}(n_1t_1)}, \\ \frac{\gamma_{\nu_F}(n_2)\gamma_{\nu_F}(t_2)+\gamma_{\nu_F}(t_2)\varsigma_{\gamma_{\nu_D}}(n_1t_1)+\gamma_{\nu_F}(n_2)\varsigma_{\gamma_{\nu_D}}(n_1t_1)-2\gamma_{\nu_F}(n_2)\gamma_{\nu_F}(t_2)\varsigma_{\gamma_{\nu_D}}(n_1t_1)}{1-\gamma_{\nu_F}(n_2)\gamma_{\nu_F}(t_2)\varsigma_{\gamma_{\nu_D}}(n_1t_1)} \end{cases}$$

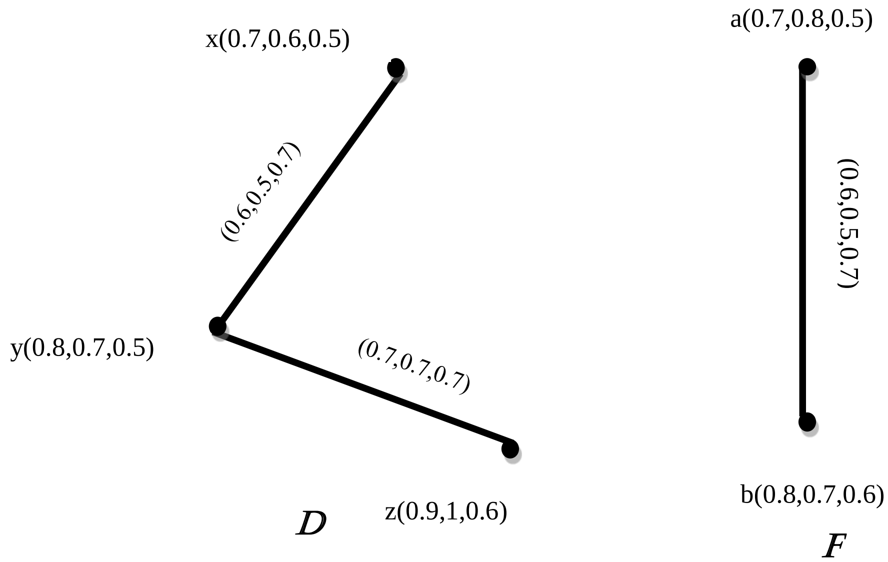
For all $n_1t_1 \in \varsigma_D, n_2 \neq t_2$

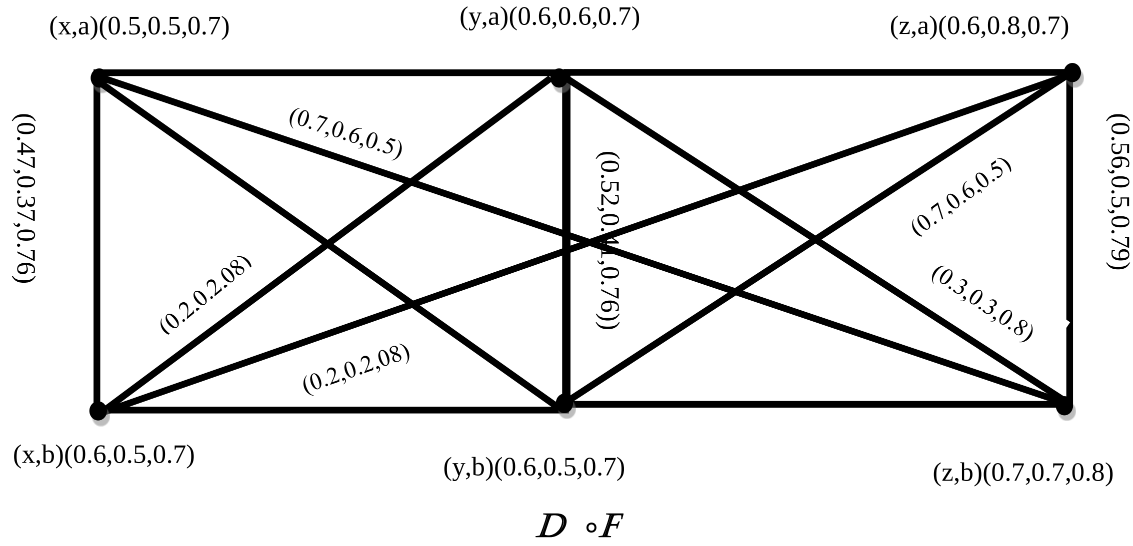
Example 3.6: Consider two Fermatean neutrosophicdombi fuzzy graphs ***d***= $(\emptyset_D, \mu_{D,}\nu_D)$ and ***F***= $(\emptyset_F, \mu_{F,}\nu_F)$.Here φ_D={x,y,z},φ_F={a,b},$\varsigma_D = \{xy, yz\}$, $\varsigma_F = \{ab\}$where

$\varphi_D = \{\frac{x}{(0.7,0.6,0.5)}, \frac{y}{(0.8,0.7,0.5)}, \frac{z}{(0.9,1,0.6)}\}$,$\varphi_F=\left\{\frac{a}{(0.7,0.8,0.5)}, \frac{b}{(0.8,0.7,0.6)}\right\}$,$\varsigma_F=\left\{\frac{ab}{(0.6,0.5,0.7)}\right\}$,

$\varsigma_D = \{\frac{xy}{(0.6,0.5,0.7)}, \frac{yz}{(0.7,0.7,0.7)}\}$, .Then we have

$\varphi_D \circ \varphi_F$(x,y) = (0.6,0.5,0.7), $\varphi_D \circ \varphi_F(y,z) = (0.7,0.7,0.7)$, $\varphi_D \circ \varphi_F(a,b) = (0.6,0.5,0.7)$,
$\varphi_D \circ \varphi_F(x,a) = (0.5,0.5,0.7)$, $\varphi_D \circ \varphi_F(x,b) = (0.6,0.5,0.7)$, $\varphi_D \circ \varphi_F(y,a) = (0.6,0.6,0.7)$,
$\varphi_D \circ \varphi_F(y,b) = (0.6,0.5,0.7)$, $\varphi_D \circ \varphi_F(z,a) = (0.6,0.8,0.7)$, $\varphi_D \circ \varphi_F(z,b) = (0.7,0.7,0.8)$,
$(\varsigma_D \circ \varsigma_F)$(x,a) (x,b) = (0.47,0.37,0.76), $(\varsigma_D \circ \varsigma_F)$(y,a)(y,b) = (0.52,0.41,0.76),
$(\varsigma_D \circ \varsigma_F)(z,a)(z,b) = (0.56,0.5,0.79)$, $(\varsigma_D \circ \varsigma_F)$ (x,a)(y,a) = (0.47,0.44,0.76),
$(\varsigma_D \circ \varsigma_F)$(y,a)(z,a)=(0.53,0.6,0.76), $(\varsigma_D \circ \varsigma_F)$(x,b)(y,b)=(0.52,0.41,0.79),
$(\varsigma_D \circ \varsigma_F)$(y,b)(z,b) = (0.6,0.53,0.79), $(\varsigma_D \circ \varsigma_F)$(x,a)(y,b) = (0.29,0.27,0.82),
$(\varsigma_D \circ \varsigma_F)$(y,a)(z,b)=(0.32,0.32,0.82),$(\varsigma_D \circ \varsigma_F)$(x,b)(z,a)=(0.29,0.29,0.82).





Corollary 3.4: The composition of two fermatean neutrosophic dombi fuzzy graph is not necessarily to a fermatean neutrosophic dombi fuzzy graph. $(\varsigma_D \circ \varsigma_F)$(y,a)(z,a)=(0.53,0.6,0.76)

Edge product of (y,a) and (z,a) is

$$=\left[\frac{0.6X0.6}{0.6+0.6-(0.6X0.6)}, \frac{0.6X0.8}{0.6+0.8-(0.6X0.8)}, \frac{0.7X0.7}{0.7X0.7-(0.7X0.7)}\right]=(0.42,0.52,0.82)$$

(0.53,0.6,0.76)$\nleq$(0.42,0.52,0.82)

Theorem 3.4: The Composition of two fermatean neutrosophic dombi fuzzy edge graph is a fermatean neutrosophic dombi fuzzy graph.

Proof:

Let d and F be the fermatean neutrosophic dombi fuzzy edge graphs $d = (\emptyset_D, \mu_{D,} \nu_D)$ and $F = (\emptyset_F, \mu_{F,} \nu_F)$ D*= (φ_D, ς_D) and $F^*= (\varphi_F, \varsigma_F)$ separately Then the Composition product $d \circ F$

Consider, For all $n \in \varsigma_D, n_2 t_2 \in \varsigma_F$ such that

$(\varsigma_D \circ \varsigma_F)(n, n_2)(n, t_2)$= T[$\varphi_D(n)$,$\varsigma_F(n_2 t_2)$]=T[1,$\varsigma_F(n_2 t_2)$]

$(\varsigma_D \circ \varsigma_F)(n, n_2)(n, t_2)$

$$\leq T\left[\frac{\alpha_{\emptyset_D}\varphi(n)\alpha_{\emptyset_F}\varsigma(n_2t_2)}{\alpha_{\emptyset_D}\varphi(n)+\alpha_{\emptyset_F}\varsigma(n_2t_2)-\alpha_{\emptyset_D}\varphi(n)\alpha_{\emptyset_F}\varsigma(n_2t_2)}, \frac{\beta_{\mu_D}\varphi(n)\beta_{\mu_F}\varsigma(n_2t_2)}{\beta_{\mu_D}\varphi(n)+\beta_{\mu_F}\varsigma(n_2t_2)-\beta_{\mu_D}\varphi(n)\beta_{\mu_F}\varsigma(n_2t_2)}, \frac{\gamma_{\nu_D}\varphi(n)+\gamma_{\nu_F}\varsigma(n_2t_2)-2\gamma_{\nu_D}\varphi(n)\gamma_{\nu_F}\varsigma(n_2t_2)}{\gamma_{\nu_D}\varphi(n)\gamma_{\nu_F}\varsigma(n_2t_2)}\right]$$

$$\varsigma_F(n_2t_2)=\begin{cases}\frac{\alpha_{\emptyset_D}(n_2)\alpha_{\emptyset_F}(t_2)}{\alpha_{\emptyset_D}(n_2)+\alpha_{\emptyset_F}(t_2)-\alpha_{\emptyset_D}(n_2)\alpha_{\emptyset_F}(t_2)}, \\ \frac{\beta_{\mu_D}(n_2)\beta_{\mu_F}(t_2)}{\beta_{\mu_D}(n_2)+\beta_{\mu_F}(t_2)-\beta_{\mu_D}(n_2)\beta_{\mu_F}(t_2)}, \\ \frac{\gamma_{\nu_D}(n_2)+\gamma_{\nu_F}(t_2)-2\gamma_{\nu_D}(n_2)\gamma_{\nu_F}(t_2)}{\gamma_{\nu_D}(n_2)\gamma_{\nu_F}(t_2)}\end{cases}$$

$$=\begin{cases}\frac{(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(n,t_2)}{(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n,n_2)+(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(n,t_2)-(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(n,t_2)}, \\ \frac{(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n,n_2)(\varphi_D\circ\varphi_F)\beta_{\mu_F}(n,t_2)}{(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n,n_2)+(\varphi_D\circ\varphi_F)\beta_{\mu_F}(n,t_2)-(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n,n_2)(\varphi_D\circ\varphi_F)\beta_{\mu_F}(n,t_2)}, \\ \frac{(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n,n_2)+(\varphi_D\circ\varphi_F)\gamma_{\nu_F}(n,t_2)-2(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(n,t_2)}{1-(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(n,t_2)}\end{cases}$$

------------------------(3)

For all $n \in \varsigma_D, n_2 t_2 \in \varsigma_F$

i) Now consider $n_1 t_1 \in \varsigma_D$,z $\in \varphi_F$

$(\varsigma_D \circ \varsigma_F)$ $(n_1, z)(t_1, z)$ = T[$\varsigma_D(n_1 t_1)$,φ_F(z)]

$$\varsigma_D(n_1t_1) \leq \begin{cases}\frac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(t_1)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_D}(t_1)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(t_1)}, \\ \frac{\beta_{\mu_D}(n_1)\beta_{\mu_D}(t_1)}{\beta_{\mu_D}(n_1)+\beta_{\mu_D}(t_1)-\beta_{\mu_D}(n_1)\beta_{\mu_D}(t_1)}, \\ \frac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_D}(t_1)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(t_1)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(t_1)}\end{cases}$$

$$=\begin{cases}\frac{(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n_1,z)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,z)}{(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n_1,z)+(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,z)-(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n_1,z)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,z)}, \\ \frac{(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n_1,z)(\varphi_D\circ\varphi_F)\beta_{\mu_F}(t_1,z)}{(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n_1,z)+(\varphi_D\circ\varphi_F)\beta_{\mu_F}(t_1,z)-(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n_1,z)(\varphi_D\circ\varphi_F)\beta_{\mu_F}(t_1,z)}, \\ \frac{(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n_1,z)+(\varphi_D\circ\varphi_F)\gamma_{\nu_F}(t_1,z)-2(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n_1,z)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,z)}{1-(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n_1,z)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,z)}\end{cases}$$

------------(4)

For all $n_1 t_1 \in \varsigma_D$ and z $\in \varphi_F$

Now consider $n_1 t_1 \in \varsigma_D, n_2 = t_2$.

$(\varsigma_D \circ \varsigma_F)$ $[(n_1, n_2)(t_1, t_2)]$ =T[T($\varphi_F(n_2)$, $\varphi_F(t_2)$,$\varsigma_D(n_1 t_1)$)]

=T[(T(1,1),$\varsigma_{\mu_D}(n_1 t_1)$),(T(0,0),$\varsigma_{\nu_F}(n_1 t_1)$)

= T[(T(1,0),T(1,0)),(T($\varsigma_{\mu_D}(n_1 t_1)$,$\varsigma_{\nu_F}(n_1 t_1)$)

$$\leq\begin{cases}\dfrac{(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n_1,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,t_2)}{(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n_1,n_2)+(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,t_2)-(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n_1,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,t_2)},\\ \dfrac{(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n_1,n_2)(\varphi_D\circ\varphi_F)\beta_{\mu_F}(t_1,t_2)}{(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n_1,n_2)+(\varphi_D\circ\varphi_F)\beta_{\mu_F}(t_1,t_2)-(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n_1,n_2)(\varphi_D\circ\varphi_F)\beta_{\mu_F}(t_1,t_2)},\\ \dfrac{(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n_1,n_2)+(\varphi_D\circ\varphi_F)\gamma_{\nu_F}(t_1,t_2)-2(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n_1,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,t_2)}{1-(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n_1,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,t_2)}\end{cases} \qquad \text{--------------(5)}$$

From (3), (4) & (5)

Every Composition of two fermatean neutrosophic dombi fuzzy edge graph is a fermatean neutrsophic dombi fuzzy edge graph.

4. Conclusion

Dombi neutrosophic fuzzy graph is more and more interesting by researches. There are many theoretical and applied results on neutrosophic fuzzy graphs that are built and developed. The concept of neutrosophic graphs can be used in many applications like decision-making problem, transportation, and computer networks. In this paper, we have introduced Fermatean neutrosophic Dombi fuzzy graphs and then we presented and studied some of its properties. Also, we investigated the relationship between the other existing Dombi neutrosophic fuzzy graphs. This shall be extended in the future to investigate the operations of the strong,semi-strong, complement of Fermatean neutrosophic Dombi fuzzy graphs with few real-life applications with relatable illustrations.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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A Perspective Note on $\mu_N\ \sigma$ Baire's Space

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Abstract: This paper presents an introduction to many novel types of sets, including μ_N strongly dense sets, μ_N strongly nowhere dense sets, μ_N strongly first category sets, and μ_N strongly nowhere residual sets. The features of these sets are briefly elucidated. In addition, by the use of these techniques, we have successfully obtained the highly Baire space μ_N, and it is imperative to elucidate its inherent features.

Keywords: μ_N Strongly Dense; μ_N Strongly Nowhere Dense; μ_N Strongly First Category Sets.

1. Introduction

The idea of fuzziness introduced by Zadeh has had a significant influence on several disciplines within the realm of mathematics. The concepts introduced by C.L. Chang [1] were subsequently integrated, resulting in the development of fuzzy topological spaces. This fusion of ideas included the principles of fuzziness inside the framework of topological spaces, establishing the foundation for the theory of fuzzy topological spaces. The discovery of intuitionistic fuzzy sets was attributed to K.T. Attanasov [2], who, in collaboration with Stoeva [3], further extended this study by introducing a generalization known as intuitionistic L-fuzzy sets. Smarandache [7] focused his research on the concept of indeterminacy and introduced the concept of neutrosophic sets. Subsequently, the neutrosophic topological spaces were introduced by A.A. Salama and Albowi [13] by the use of neutrosophic sets. The authors [12] developed a novel concept called μ_N TS, which involves the construction of Generalised topological spaces via the use of neutrosophic sets. This approach was inspired by previous studies in the field. The notion of Baire space in μ_NTS was introduced by the authors, and in this study, we further explore the robust properties of μ_N Baire space.

2. Necessities

Definition 2.1 [4-10, 14]: Let X be a non-empty fixed set. A Neutrosophic set [NS for short] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function , the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 2.2.[14] Every intuitionistic fuzzy set A is a non empty set in X is obviously on Neutrosophic sets having the form $A = \{\langle \mu_A(x), 1 - \mu_A(x) + \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.Since our main purpose is to construct the tools for developing Neutrosophic Set and Neutrosophic topology , we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

0_N may be defined as follows

$(0_1) 0_N = \{\langle x, 0,1,1\rangle : x \epsilon X\}$

1_N may be defined as follows

$(1_1) 1_N = \{\langle x, 1,0,0\rangle : x \epsilon X\}$

Definition 2.3.[14] Let $A = \{\langle \mu_A, \sigma_A, \gamma_A \rangle\}$ be a NS on X , then the complement of the set A [C(A) for short] may be defined

(C_1) $C(A) = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

Definition 2.4.[14] Let X be a non-empty set and neutrosophic sets A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$.

$A \subseteq B$ may be defined as :

$(A \subseteq B) \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x), \gamma_A(x) \geq \gamma_{\boldsymbol{B}}(x) \; \forall x \in X$

Proposition 2.5. [14] For any neutrosophic set A, the following conditions holds:

$0_N \subseteq$ A,

$A \subseteq 1_N$

Definition 2.6. [14] Let X be a non empty set and $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$

$B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$ are NSs. Then $A \cap B$ may be defined as :

(I_1) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle$

$A \cup B$ may be defined as :

(I_1) $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle$

Definition 2.7[12]. A μ_N topology on a non - empty set X is a family of neutrosophic subsets in X $satisfying\ the\ following\ axioms$:

$(\mu_{N_1}) 0_N \in \mu_N$

$(\mu_{N_2}) Union\ of\ any\ number\ of\ \mu_N\ open\ sets\ is\ \mu_N\ open.$

Remark 2.8.[12] The elements of μ_N are μ_N open sets and their complement is called μ_N closed sets.

Definition 2.9.[12]The $\mu_N -$ Closure of A is the intersection of all μ_N closed sets containing A.

Definition 2.10.[12]The $\mu_N -$ Interior of A is the union of all μ_N open sets contained in A.

Definition 2.11.[13]. : A neutrosophic set A in μ_N TS (X, μ_N) is called μ_N dense set if there exists no μ_N closed set B in (X, μ_N) such that $A \subset B \subset 1_N$

Definition 2.12.[13]. The μ_N Topological spaces is said to be μ_N Baire's Space if $\mu_N\ Int(\bigcup_{i=1}^{\infty} G_i) = 0_N$ where $G_i's$ are μ_N nowhere dense set in (X, μ_N).

Theorem 2.13.[13]: Let (X, μ_N) be a μ_NTS. Then the following are equivalent.

(i) (X, μ_N) is μ_NBaire's Space.

(ii) $\mu_N\ Int(A) = 0_N$, for all μ_N first category set in (X, μ_N).

(iii) $\mu_N\ Cl(A) = 1_N$, μ_N Residual set in (X, μ_N).

3. $\mu_N\ \sigma$ *Nowhere Dense sets*

Definition 3.1: A neutrosophic set A in X is called $\mu_N\ \sigma$ rare set if A is a $\mu_N\ F_\sigma$ set such that μ_NInt(A)= 0_N.

Definition 3.2 :A neutrosophic set A in X is called $\mu_N\ \sigma$ nowhere dense set if A is a $\mu_N\ F_\sigma$ set such that μ_NInt(μ_N Cl A)= 0_N.

Remark 3.3 :If A is a $\mu_N\ F_\sigma$ set and μ_N Nowhere dense set in X then A is $\mu_N\ \sigma$ rare set.

Example 3.4: Let $X = \{a\}$ define neutrosophic sets $0_N = \{\langle 0,1,1\rangle\}$, $A = \{\langle 0.1,0.4,0.6\rangle\}, B = \{\langle 0.2,0.3,0.5\rangle\}, C = \{\langle 0.6,0.6,0.1\rangle\}, 1_N = \{\langle 1,0,0\rangle\}$ and we define a μ_N TS $\mu_N = \{0_N, A, C\}$.Here $\overline{A}$ and $\overline{B}$ are $\mu_N\ \sigma$ rare sets.

Theorem 3.5: A neutrosophic set A in X is μ_N σ rare set iff $\overline{A}$ is μ_N dense and μ_N G_δ set.

Proof: Let A be μ_N σ rare set in X.Then A is μ_N F_σ set such that $\mu_N Int(A)= 0_N$ which implies us that $\mu_N Cl(\overline{A})= 1_N$ and $\overline{A} = \overline{\bigcup_{i=1}^{\infty} A_i} = \bigcap_{i=1}^{\infty} A_i$ where A's are μ_N open sets.Therefore $\overline{A}$ is μ_N dense and μ_N G_δ set in (X, μ_N).Conversely,assume that $\overline{A}$ is μ_N dense and μ_N G_δ set in (X, μ_N). Then $\overline{A} = \bigcap_{i=1}^{\infty} A_i \Rightarrow A = \bigcup_{i=1}^{\infty} A_i$,where A_i's are μ_N closed sets .From this we retrieve that A in (X, μ_N) is μ_N F_σ and also $\mu_N Cl(\overline{A})= 1_N$ that implies $\mu_N Int(A)= 0_N$.From this we say that A is μ_N σ rare set.

Corollary 3.6: A neutrosophic set A in X is μ_N σ rare set iff μ_N Ext $(\overline{A}) = 0_N$ and $\overline{A}$ is μ_N G_δ set.

Proof: Let A be a μ_N σ rare set in (X, μ_N). Then A is μ_N F_σ set such that μ_N Ext$(A) = 0_N$. Now μ_N Ext $(\overline{A}) = \mu_N$ Ext$(A) = 0_N$ and $\overline{A} = \overline{\bigcup_{i=1}^{\infty} A_i} = \bigcap_{i=1}^{\infty} \overline{A_i}$ where $\overline{A_i} \in \mu_N$ open sets in (X, μ_N). Therefore, μ_N Ext $(\overline{A}) = 0_N$ and $\overline{A}$ is a μ_N G_δ set. Conversely, assume that μ_N Ext $(\overline{A}) = 0_N$ and $\overline{A}$ is a μ_N G_δ set in (X, μ_N). Then $\overline{A} = \bigcap_{i=1}^{\infty} \overline{A_i} \Rightarrow A = \bigcup_{i=1}^{\infty} A_i$ where A_i's is μ_N closed sets in $(X, \mu_N) \Rightarrow$ A in X is μ_N F_σ set. Also μ_N Int $(A) = \mu_N$ Int$(\overline{\overline{A}}) = \mu_N$ Ext $(\overline{A}) = 0_N$. Therefore, A is μ_N σ Rare set.

Theorem 3.7: If a neutrosophic set in X is μ_N σ rare set then μ_N border is a subset of μ_N frontier.

Proof: Suppose A in X is μ_N σ rare set then A is a μ_N F_σ set and μ_N Int $(A) = 0_N$ that implies A $=\bigcup_{i=1}^{\infty} A_i$ where A_i's are μ_N closed sets in (X, μ_N).Now μ_N Br(A)=A $-$ μ_N Int (A) = A and μ_N Fr(A)= $\mu_N Cl(A) - \mu_N$ Int (A) $= \mu_N$ Cl A. Henceforth μ_N border is a subset of μ_N frontier.

Theorem 3.8: If a neutrosophic set A in X is μ_N σ rare set then A is μ_N strongly first category set.

Proof: Suppose A is μ_N σ rare set then A is a μ_N F_σ set and μ_N Int $(A) = 0_N$ that implies us that A $= \bigcup_{i=1}^{\infty} A_i$ where A_i's are μ_N closed sets in (X, μ_N) and μ_N Int $(A) = 0_N$. We know that $\bigcup_{i=1}^{\infty} \mu_N$ Int $(A) \subseteq \mu_N$ Int$(\bigcup_{i=1}^{\infty} A_i) = \mu_N$ Int $(A) = 0_N \Rightarrow \mu_N$ Int $(A) = 0_N$, where A_i's is μ_N closed sets. We have if A is μ_N closed set with μ_N Int $(A) = 0_N$, then A is a μ_N strongly nowhere dense sets. By using this we get A_i's are μ_N strongly nowhere dense sets and hence A $=\bigcup_{i=1}^{\infty} A_i$ where A_i's are μ_N strongly nowhere dense sets. Therefore then A is a μ_N strongly first category set.

Remark 3.9: The converse of the above theorem not true. Let $X = \{a\}$ define neutrosophic sets $0_N = \{\langle 0,1,1\rangle\}, A = \{\langle 0.1,0.4,0.6\rangle\}, B = \{\langle 0.2,0.3,0.5\rangle\}, C = \{\langle 0.6,0.6,0.1\rangle\}, 1_N = \{\langle 1,0,0\rangle\}$ and we define a μ_N TS $\mu_N = \{0_N, A, C\}$.Here $\overline{A}$ and $\overline{B}$ are μ_N σ rare sets,{A,B,C,D, $0_N, \overline{A}, \overline{B}, \overline{C}$ and $\overline{D}$ are μ_N strongly first category set.We can analyse that $\overline{C}$ and $\overline{D}$ are μ_N strongly first category sets but not μ_N σ rare sets.

Theorem 3.10: Every μ_N σ Nowhere dense set is μ_N σ rare set.

Proof: Let $A \subseteq X$ be a μ_N σ nowhere dense set. Then A is μ_N F_σ set and μ_N nowhere dense set. Using theorem 2.3, A is μ_N F_σ set and μ_N Int $(A) = 0_N$. Hence A is a μ_N σ rare set.

Corollary 3.11: A neutrosophic set A in X is μ_N σ rare set and μ_N closed set then A is μ_N σ nowhere dense set.

Proof: Given that A in X is μ_N σ rare set and μ_N closed set. Then A is μ_N F_σ set with μ_N Int $(A) = 0_N$ and μ_N Cl$(A) = A$, we know Let $A \subseteq X$. If μ_N closed set with μ_N Int $(A) = 0_N$. Then A is μ_N nowhere dense set in μ_NTS.

Remark 3.12: Every μ_N σ nowhere dense set is μ_N nowhere dense set but the reverse is not valid.

Example: Let $X = \{a\}$ define neutrosophic sets $0_N = \{\langle 0,1,1\rangle\}$, $A = \{\langle 0.1,0.4,0.6\rangle\}, B = \{\langle 0.2,0.3,0.5\rangle\}, C = \{\langle 0.6,0.6,0.1\rangle\}$, $D = \{\langle 0.5,0.7,0.2\rangle\} 1_N = \{\langle 1,0,0\rangle\}$ and we define a μ_N TS $\mu_N = \{0_N, A, B\}$.Here the μ_N nowhere dense sets { $C, D, 0_N, \overline{A}, \overline{B}$ } and the μ_N nowhere dense sets {

$\overline{A}, \overline{B}$}.From this we conclude that Every μ_N nowhere dense set need not be $\mu_N\sigma$ nowhere dense set.

Theorem 3.13: If a neutrosophic set A in (X, μ_N) is $\mu_N\,\sigma$ nowhere dense set then A is μ_N strongly first category set.

Proof: We have "Every $\mu_N\,\sigma$ nowhere dense set is $\mu_N\,\sigma$ rare set." And "If A in (X, μ_N) is $\mu_N\,\sigma$ rare set then A is μ_N strongly first category set." Using these theorem's, we get A is μ_N strongly first category set.

Theorem 3.14: If a neutrosophic set A in X is $\mu_N\,\sigma$ nowhere dense set then $\overline{A}$ is μ_N dense set and $\mu_N G_\delta$ set in (X, μ_N).

Proof: Using Corollary 3.6 and theorem 3.5, $\mu_N\,\text{Ext}\,(\overline{A}) = 0_N$ and $\overline{A}$ is $\mu_N G_\delta$ set.

Theorem 3.15: If a neutrosophic set in (X, μ_N) is $\mu_N\,\sigma$ nowhere dense set then μ_N border is a subset of μ_N Frontier.

Theorem 3.16: (i) Every subset of a $\mu_N\,\sigma$ rare set is $\mu_N\,\sigma$ rare set.

(ii) Every subset of a $\mu_N\,\sigma$ nowhere dense set is $\mu_N\,\sigma$ nowhere dense set.

Definition 3.17: A neutrosophic set A is said to be $\mu_N\,\sigma$- category I set, if $A = \bigcup_{i=1}^{\infty} A_i$ where A_i's are $\mu_N\,\sigma$ rare set. Remaining sets are called $\mu_N\,\sigma$ category II sets. The complement set of $\mu_N\,\sigma$ first category sets are named as $\mu_N\,\sigma$ complement set.

Theorem 3.18: Every subset of $\mu_N\,\sigma$ category I set is $\mu_N\,\sigma$ category I set.

Theorem 3.19: If A is μ_N dense and $\mu_N G_\delta$ set then $\overline{A}$ is $\mu_N\,\sigma$ category I set.

Theorem 3.20: If A is $\mu_N\,\sigma$ category I set in X then $A \subseteq \eta$ where η is a non-void $\mu_N\,F_\sigma$ set in X.

Theorem 3.21: If A is $\mu_N\,\sigma$ complement set in X then there exists a $\mu_N G_\delta$ set B such that $A \subseteq B$.

Proof: Let A be $\mu_N\,\sigma$ complement set in X. Then $\overline{A}$ is $\mu_N\,\sigma$ first category set by using theorem 3.20, we have there is a non-void $\mu_N\,F_\sigma$ set B in X such that $\overline{A} \subseteq B$. Hence $\overline{A} \subseteq B$ and $\overline{B}$ is a $\mu_N G_\delta$ set. Take $A = \overline{B}$. Therefore we have $A \subseteq B$.

Theorem 3.22:

(i) Every $\mu_N\,\sigma$ category I set is a $\mu_N\,F_\sigma$ set.

(ii) Every $\mu_N\,\sigma$ complement set is a $\mu_N G_\delta$ set.

Definition 3.23: A neutrosophic set A is said to be $\mu_N\,\sigma$-first category in μ_NTS if $A = \bigcup_{i=1}^{\infty} A_i$ where A_i's are $\mu_N\,\sigma$ nowhere dense sets. Remaining sets are $\mu_N\,\sigma$ second category set. The complement of $\mu_N\,\sigma$ first category set is named as $\mu_N\,\sigma$ residual set.

Theorem 3.24: Every subset of $\mu_N\,\sigma$ first category set is $\mu_N\,\sigma$ first category set.

Theorem 3.25: If A is μ_N dense and $\mu_N G_\delta$ set then $\overline{A}$ is $\mu_N\,\sigma$ first category set.

Theorem 3.26: If A is $\mu_N\,\sigma$ first category set in X then $A \subseteq B$ where A is a non-void $\mu_N\,F_\sigma$ set in X.

Theorem 3.27: If A is $\mu_N\,\sigma$ residual set in X then there exists a $\mu_N G_\delta$ set B such that $A \subseteq B$.

Proof: Let A be a $\mu_N\,\sigma$ residual set in X. Then $\overline{A}$ is $\mu_N\,\sigma$ first category set by using theorem 3.24, we have there is a non-void $\mu_N\,F_\sigma$ set B in X such that $\overline{A} \subseteq B$. Hence $\overline{A} \subseteq B$ and $\overline{B}$ is a $\mu_N G_\delta$ set. Take $A = \overline{B}$. Therefore we have $A \subseteq B$.

Theorem 3.28:

(i) Every $\mu_N\,\sigma$ first category set is a $\mu_N\,F_\sigma$ set.

(ii) Every $\mu_N\,\sigma$ residual set is a $\mu_N G_\delta$ set.

4. $\mu_N\ B_\sigma$ Space and $\mu_N\ \sigma$ Baire Space

Definition 4.1: If $\mu_N\ \mathrm{Int}(\bigcup_{i=1}^{\infty} A_i) = 0_N$ where A_i's are $\mu_N\ \sigma$ rare set then X is a $\mu_N\ B_\sigma$ Space.

Definition 4.2: If $\mu_N\ \mathrm{Int}(\bigcup_{i=1}^{\infty} A_i) = 0_N$ where A_i's are $\mu_N\ \sigma$ nowhere dense set then X is a $\mu_N\ B_\sigma$ Baire space.

Theorem 4.3: If $\mu_N\ \mathrm{Cl}(\bigcap_{i=1}^{\infty} \delta_i) = 1_N$ where δ_i's are μ_N dense set and $\mu_N G_\delta$ set then (X, μ_N) is a $\mu_N\ B_\sigma$ Baire space.

Proof: Given that $\mu_N\ \mathrm{Cl}(\bigcap_{i=1}^{\infty} \delta_i) = 1_N$ which gives that $\overline{\mu_N\ \mathrm{Cl}(\bigcap_{i=1}^{\infty} \delta_i)} = 0_N \Rightarrow \mu_N\ \mathrm{Int}(\bigcup_{i=1}^{\infty} \delta_i) = 0_N$. Take $\mathbf{B}_i = \overline{\delta_i}$. Then $\mu_N\ \mathrm{Int}(\bigcup_{i=1}^{\infty} B_i) = \zeta$. Now δ_i's are μ_N dense set and $\mu_N G_\delta$ set in X. Then by theorem 3.8 $\overline{\delta_i}$ is a $\mu_N\ \sigma$ rare set and hence $\mu_N\ \mathrm{Int}(\bigcup_{i=1}^{\infty} B_i) = 0_N$ where B_i's are $\mu_N\ \sigma$ rare set. Therefore (X, μ_N) is a $\mu_N\ B_\sigma$ Baire space.

Theorem 4.4: Let (X, μ_N) be μ_NTS. Then the following are equivalent.

(i) (X, μ_N) is a $\mu_N\ B_\sigma$ Baire space.

(ii) $\mu_N\ \mathrm{Int}(\delta_i) = 0_N$ for every $\mu_N\ \sigma$ first category set in X.

(iii) $\mu_N\ \mathrm{Cl}(\delta_i) = 1_N$ for every $\mu_N\ \sigma$ residual set in X.

Proof: (i)⇒(ii) Let δ_i be μ_N first category set in X. Then $\delta_i = \bigcup_{i=1}^{\infty} \delta_i$ where δ_i's are $\mu_N\ \sigma$ rare set and $\mu_N\ \mathrm{Int}(\delta_i) = \mu_N\ \mathrm{Int}(\bigcup_{i=1}^{\infty} \delta_i)$ since (X, μ_N) is a $\mu_N\ B_\sigma$ space. $\mu_N\ \mathrm{Int}(\delta_i) = 0_N$.

(ii)⇒(iii) Let δ_i be $\mu_N\ \sigma$ complement set in X. Then $\overline{\delta_i}$ is a $\mu_N\ \sigma$ first category set in X. From (ii), $\mu_N\ \mathrm{Int}(\delta_i) = 0_N \Rightarrow \overline{\mu_N\ \mathrm{Cl}(\delta_i)} = 0_N$. Hence $\mu_N\ \mathrm{Cl}(\delta_i) = 1_N$.

(iii)⇒(i) Let δ_i be $\mu_N\ \sigma$ first category set in X. Then $\delta = \bigcup_{i=1}^{\infty} \delta_i$ where δ_i's are $\mu_N\ \sigma$ rare set. We have if δ is $\mu_N\ \sigma$ first category set in X then $\overline{\delta}$ is $\mu_N\ \sigma$ residual set. By (iii) we get $\mu_N\ \mathrm{Cl}(\overline{\delta}) = 1_N$ which gives $\overline{\mu_N\ \mathrm{Int}(\delta)} = 0_N$. Therefore $\mu_N\ \mathrm{Int}(\delta) = 0_N$ and hence $\mu_N\ \mathrm{Int}(\bigcup_{i=1}^{\infty} \delta_i) = 0_N$ where δ_i's are $\mu_N\ \sigma$ rare set. Hence (X, μ_N) is a $\mu_N\ B_\sigma$ Baire space.

Theorem 4.5: If $\mu_N\ \mathrm{Int}(A) = 0_N$ for each $\mu_N\ F_\sigma$ set A in X then X is a $\mu_N\ B_\sigma$ Baire space.

Proof: Let A be a $\mu_N\ \sigma$ first category set in X. Then $A \subseteq B$ where A is a non-void $\mu_N\ F_\sigma$ set in $X \Rightarrow \mu_N\ \mathrm{Int}(A) \subseteq \mu_N\ \mathrm{Int}(B) = 0_N$ and hence $\mu_N\ \mathrm{Int}(A) = 0_N$ for each μ_N first category set A in X. By theorem 4.4 X is a $\mu_N\ B_\sigma$ Baire space.

Theorem 4.6: If $\mu_N\ \mathrm{Cl}(A) = 1_N$ for each $\mu_N G_\delta$ set A in X then X is a $\mu_N\ B_\sigma$ Baire space.

Proof: Let A be a $\mu_N\ \sigma$ first category set in X. Then $A \subseteq B$ where A is a non-empty $\mu_N\ F_\sigma$ set in X. Since B is a $\mu_N\ F_\sigma$ set, $\overline{A}$ is $\mu_N\ G_\delta$ set and then $\mu_N\ \mathrm{Cl}(\overline{B}) = 1_N \Rightarrow \mu_N\ \mathrm{Int}(A) = 0_N$. Now $A \subseteq B \Rightarrow \mu_N\ \mathrm{Int}(A) \subseteq \mu_N\ \mathrm{Int}(B) = 0_N$. Hence $\mu_N\ \mathrm{Int}(A) = 0_N$. By theorem 4.4, X is a $\mu_N\ B_\sigma$ Baire space.

Theorem 4.7: If $\mu_N\ \mathrm{Int}(\bigcup_{i=1}^{\infty} A_i) = 0_N$, where A_i's are μ_N closed set and $\mu_N\ \sigma$ rare set in X, then (X, μ_N) is a $\mu_N\ B_\sigma$ Baire space.

Proof: Given that $\mu_N\ \mathrm{Int}(\bigcup_{i=1}^{\infty} A_i) = 0_N$, where A_i's are μ_N closed set and $\mu_N\ \sigma$ rare set. By corollary 3.11, A_i's are $\mu_N\ \sigma$ nowhere dense sets. Therefore $\mu_N\ \mathrm{Int}(\bigcup_{i=1}^{\infty} A_i) = 0_N$, where A_i's are $\mu_N\ \sigma$ nowhere dense set and hence (X, μ_N) is a $\mu_N\ B_\sigma$ Baire space.

Remark 4.8: Every $\mu_N\ B_\sigma$ Baire space is a μ_N Baire space if every $\mu_N\ \sigma$ rare set is μ_N closed.

Theorem 4.9: Every $\mu_N\ \sigma$ Baire space isμ_N Baire space.

Theorem 4.10: Let (X, μ_N) be μ_NTS. Then the following are equivalent.

(i) (X, μ_N) is a $\mu_N\ \sigma$ Baire space.

(ii) $\mu_N \operatorname{Int}(A) = 0_N$ for every $\mu_N\ \sigma$ first category set in X.

(iii) $\mu_N \operatorname{Cl}(A) = 1_N$ for every $\mu_N\ \sigma$ residual set in X.

Proof: (i) ⇒ (ii) Let A be $\mu_N\ \sigma$ first category set in X. Then $A = \bigcup_{i=1}^{\infty} A_i$ where A_i's are $\mu_N\ \sigma$ nowhere dense sets and $\mu_N \operatorname{Int}(A) = \mu_N \operatorname{Int}(\bigcup_{i=1}^{\infty} A_i)$ since (X, μ_N) is a $\mu_N\ \sigma$ Baire space. $\mu_N \operatorname{Int}(A) = 0_N$.

(ii)⇒(iii) Let A be $\mu_N\ \sigma$ residual set in X. Then $\overline{A}$ is a $\mu_N\ \sigma$ first category set in X. From (ii), $\mu_N \operatorname{Int}(\overline{A}) = 0_N \Rightarrow \overline{\mu_N \operatorname{Cl}(A)} = 0_N$. Hence $\mu_N \operatorname{Cl}(A) = 1_N$.

(iii)⇒(i) Let A be $\mu_N\ \sigma$ first category set in X. Then $\delta A = \bigcup_{i=1}^{\infty} A_i$ where A_i's are $\mu_N\ \sigma$ nowhere dense sets. We have, if A is $\mu_N\ \sigma$ first category set in X then $\overline{A}$ is $\mu_N\ \sigma$ residual set. By (iii) we get $\mu_N \operatorname{Cl}(\overline{A}) = 1_N$ which gives us that $\overline{\mu_N \operatorname{Int}(A)} = 1_N$. Therefore $\mu_N \operatorname{Int}(A) = 0_N$ and hence $\mu_N \operatorname{Int}(\bigcup_{i=1}^{\infty} A_i) = 0_N$ where A_i's are $\mu_N\ \sigma$ rare set. Hence (X, μ_N) is a $\mu_N\ \sigma$ Baire space.

Theorem 4.11: If $\mu_N \operatorname{Int}(A) = 0_N$, for each $\mu_N\ F_\sigma$ set A in X, then X is a $\mu_N\ \sigma$ Baire space.

Proof: Let A be $\mu_N\ \sigma$ first category set in X. Then $A \subseteq B$ where B is a non-empty $\mu_N\ F_\sigma$ set in X. $\Rightarrow \mu_N \operatorname{Int}(A) \subseteq \mu_N \operatorname{Int}(B) = 0_N$ and so $\mu_N \operatorname{Int}(A) = 0_N$ for each $\mu_N\ \sigma$ first category set. By theorem 4.10 ⇒ X is a $\mu_N\ \sigma$ Baire space.

Theorem 4.12: If $\mu_N \operatorname{Cl}(A) = 1_N$, for each $\mu_N G_\delta$ set A in X, then X is a $\mu_N\ \sigma$ Baire space.

Proof: Let A be a $\mu_N\ \sigma$ first category set in X. Then $A \subseteq B$ where B is a non-empty $\mu_N\ F_\sigma$ set in X. Since B is a $\mu_N\ F_\sigma$ set, $\overline{B}$ is $\mu_N\ G_\delta$ set and then $\mu_N \operatorname{Cl}(\overline{B}) = 1_N \Rightarrow \mu_N \operatorname{Int}(B) = 0_N$. Now $A \subseteq B \Rightarrow \mu_N \operatorname{Int}(A) \subseteq \mu_N \operatorname{Int}(B) = 0_N$. Hence $\mu_N \operatorname{Int}(A) = 0_N$. By theorem 4.10,X is a $\mu_N\ \sigma$ Baire space.

5. Conclusion

In this paper, we provide many new sorts of sets, including μ_N strongly dense sets, μ_N strongly nowhere dense sets, μ_N strongly first category sets, and μ_N strongly nowhere residual sets, as well as a short explanation of the characteristics that distinguish each of these sets. In addition to this, with their help, we were able to obtain the μ_N powerfully Baire space.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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An Investigative Study on Quick Switching System using Fuzzy and Neutrosophic Poisson Distribution

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Abstract: Stephens and Larson (1967) stated the sampling system as allotted grouping of two or three sampling plans and the rules for switching between the plans for sentencing the lots of manufactured products. Quick Switching System (QSS) by Romboski (1969) is a sampling system with reference to Single Sampling Plan (SSP) involves Normal and Tightened plans by adopting a switching rule. QSS provides quality protection and reduction in cost of inspection. The idea of Fuzzy Logic is adopted in system to handle the situations of fraction non-confirming or uncertainty or vagueness present in the parameters. The sampling plans based on Fuzzy Set Theory in literature is used for defuzzification of fuzzy numbers to the interval sets, able to solve problem with certain values for upper and lower limit of intervals. As one of expansion of Fuzzy Sets in this era, a new philosophy is stimulated as a substantial new theoretical development is Neutrosophy Sets (NSs) and used for rise in sensitiveness that makes flexibility in the plans and systems that can be applied in the manufacturing industries. This paper enhances the determination of QSS using Neutrosophic Poisson distribution and compared with the existing Fuzzy Poisson distribution through Operating Characteristic (OC) curves.

Keywords: QSS, OC, SSP, Fuzzy Set, Neutrosophic Set, Poisson distribution.

1. Introduction

Quick Switching System detailed in this article consists of two sampling plans - Normal and Tightened plans with a switching instantaneously between the two plans. Quick Switching System starts with Normal Plan for the good quality period of smaller sample size to reduce the cost of inspection and tightened plans is designed for a high level protection either by tightening the sample size or the acceptance number. Accordingly, QSS is branded into two means

i) Acceptance Number Tightening - **(n;** c_N , c_T**)**
ii) Sample Size Tightening - (n, k; c_0), k >1

Classical Acceptance sampling plans use certain mass quality metrics and quality specifications that may not be certain in some real-world applications due to including uncertainties in any form. Fuzzy Set (FS) theory defined by Zadeh (1965) and extended as an intuitionistic Fuzzy Set (IFS) by Attanasov (1986) applied in all the fields of domain especially in medical and manufacturing industries interms of cluster analysis, using similarity and distance measure approaches. However, FSs are not considered to be suitable to deal with indeterminate and inconsistent information which frequently exists in reality. As an advancement Samarandache (1996) introduced Neutroshophic

Logic and Neutroshopic Set (NS) theory which is the generalization of intuitionistic Fuzzy Set. NS is used to tackle uncertainty using the truth, indeterminacy and falsity membership grades which are considered as independent. The paper focused on comparing Quick Switching Systems under Fuzzy Logic and Neutrosophic Logic through Operating Characteristic (OC) curves of various acceptance numbers and fixed sample size in order to provide high level of protection.

2. Literature Review

QSS were originally proposed by Dodge (1967) and later investigated by Romboski (1969) and Govindaraju (1991). Based on Romboski's study, Devaraj Arumainayagam (1991) has studied Quick Switching System with reference to sampling plans like Single Sampling Plan, Double Sampling Plan for both acceptance number tightening and sample size tightening, Taylor (1992) designed QSS with Reduced and Tightened Sampling plans and evaluated with MILSTD 10E. Uma and Nandhinidevi (2018) studied the determination of QSS using both fuzzy Binomial distribution and fuzzy Poisson distribution with OC curves of various sample size and fixed acceptance number. Nandhinidevi and Uma (2018) analysed fuzzy logic importance on QSS by attributes using the Poisson distribution. Uma, Nandhinidevi and Manjula (2020) studied the impact of fuzzy logic on Quick Switching Single Double sampling plan with the acceptance number tightening criteria.

Aslam (2019) initiated to study both attribute and variable acceptance sampling plans and proposed a new attribute sampling plan by employing the Neutrosophic Interval method and the Neutrosophic Binomial distribution is utilized for computing the lot acceptance, rejection and indeterminate probabilities at various specified sample size and acceptance number parameters. Uma and Nandhitha (2022) reviewed the significance of Neutrosophic set on Acceptance Sampling plans (attribute and variable). Uma and Nandhitha (2023) have analyzed and evaluated the Quick Switching System using Neutrosophic Poisson Distribution with respective OC curve and necessary tables are constructed.

3. Quick Switching System

Dodge (1967) proposed a new sampling system consisting of pairs of normal and tightened plans. The application of the system is as follows

- Adopt a pair of sampling plans, a normal plan (N) and tightened plan (T), the plan T to be tightened OC curve wise than plan N.
- Use plan N for the first lot (optional): can start with plan T; the OC curve properties are the same; but first lot protection is greater if plan T is used.
- For each lot inspected; if the lot is accepted, use plan N for the next lot and if the lot is rejected, use plan T for the next lot'.

Due to instantaneous switching between normal and tightened plan, this system is referred as "Quick Switching System ". The OC function of QSS-1 is derived by Romboski (1969) as

$$P_a(p) = \frac{P_T}{(1-P_N+P_T)} \quad (1)$$

Conditions for Application

- The production is steady so that results on current and preceding lots are broadly indicative of a continuing process and submitted lots are expected to be essentially of the same quality.
- Lots are submitted substantially in the order of production.

- Inspection is by attributes with quality defined as fraction nonconforming.

Operating Procedure for QSS (n; c_N , c_T)

Step 1: From a lot, take a random sample of size 'n' at the normal level. Count the number of defectives 'd'

i) If d < c_N , accept the lot and repeat step 1

ii) If d > c_N, reject the lot and go to step 2.

Step 2: From the next lot, take a random sample of size n at the tightened level. Count the number of defectives 'd.

i) If d < c_T, accept the lot and use step 1

ii) If d > c_T, reject the lot and repeat step 2

Where c_N , c_T are the acceptance numbers in the Normal and Tightened Sampling plans

4. Preliminaries and Definitions

4.1 Fuzzy Set

Parameter 'p' (probability of a success in each experiment) of the crisp binomial distribution is known exactly, but sometimes we are not able to obtain exact some uncertainty in the value 'p' and is to be estimated from a random sample or from expert opinion. The crisp Poisson distribution has one parameter, which we also assume is not known exactly.

Definition 1: The fuzzy subset $\widetilde{N}$ of real line IR, with the membership function μ_N: IR→ $[0,1]$ is a fuzzy number if and only if (a) $\widetilde{N}$ is normal (b) $\widetilde{N}$ is fuzzy convex (c)μ_N is upper semi continuous (d) supp ($\widetilde{N}$) is bounded.

Definition 2: A triangular fuzzy number $\widetilde{N}$ is fuzzy number that membership function defined by three numbers $a_1 < a_2 < a_3$where the base of the triangle is the interval $[a_1, a_3]$ and vertex is at $x = a_2$.

Definition 3: The α - cut of a fuzzy number $\widetilde{N}$ is a non-fuzzy set defined as $N[\alpha] = \{x \in IR;\ \mu_N(x) \geq \alpha\}$. Hence $N[\alpha] = [N_\alpha^L, N_\alpha^U]$ where $N_\alpha^L = inf\{x \in IR;\ \mu_N(x) \geq \alpha\}$

$$N_\alpha^U = sup\{x \in IR;\ \mu_N(x) \geq \alpha\}$$

Definition 4: Due to the uncertainty in the l_i's values we substitute $\widetilde{l_\iota}$, a fuzzy number, for each l_i and assume that 0<$\widetilde{l_\iota} < 1$ all i. Then X together with the $\widetilde{l_\iota}$ value is a discrete fuzzy probability distribution. We write $\tilde{p}$ for fuzzy P and we have $\tilde{P}(\{x_i\})$ = $\widetilde{l_\iota}$

Let A = $\{x_1, x_2, \ldots x_l\}$ be subset of X. Then define:

$$\tilde{P}(A)[\alpha] = \frac{\sum_{i=1}^{l} l_i}{s} \quad (2)$$

For 0 < $\alpha < 1$, where stands for the statement "$l_i \in \ \widetilde{k_\iota}[\alpha], 1 < i < n, \sum_{i=1}^{l} l_i = 1$". This is our fuzzy arithmetic.

Definition 5: Let x be a random variable having the Poisson mass function. If P(x) stands for the probability that X=x, then

$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ (3)

For x=0,1,2, and **λ** >0.

Now substitute fuzzy number $\tilde{\lambda}$>0 for **λ** to produce the fuzzy Poisson probability mass function. Let P(x) to be the fuzzy probability that X=x. Then α −cut of this fuzzy number as

$\widetilde{P}$ (x)[a] =$\left\{\frac{e^{-\lambda}\lambda^x}{x!} \middle| \lambda \in \lambda[\alpha]\right\}$ (4)

For all $\alpha \in [0,1]$. Let X be a random variable having the fuzzy binomial distribution and $\tilde{P}$ in the definition 4 are small. That is all are p∈ $\tilde{p}$ sufficiently small. Then $\tilde{P}$[a,b] $[\alpha]$ using the fuzzy poisson approximation.

Then

$\widetilde{P}$ [a,b] [a] = $x = \sum_{x=a}^{b} \frac{e^{-\lambda}\lambda^x}{x!}$ (5)

4.2 Neutrosophic Sets

Type-1 fuzzy sets or classical fuzzy sets consider only the membership and only have $\mu(x) \in [0,1]$ in the membership function. Since non-membership is states as 1- $\mu(x)$, type-1 fuzzy sets are only usable in complete information case. Intuitionistic sets (ISs) have functions for both membership and non-membership. While $\mu(x) \in [0,1]$ is membership function and $\vartheta(x) \in [0,1]$ is non-membership function, the condition $0 \leq \mu(x) + \vartheta(x) \leq 1$ is satisfied (Atanassov, 2003). If the sum of membership and non-membership values is less than 1, it means incomplete information (Wang et al.,2005). NSs are the generalized form of ISs. It handles membership (truthiness), non-memership (falsity) and indeterminacy cases independent from each other. This independency makes possible to use inconsistent data in modelling (Smarandache, 2005). NSs can be formulated as in Eq. (5) (Wang et al., 2010):

(t, i, f) = (truthiness, indeterminacy, falsity)

$0 \leq t + i + f \leq 3, \quad t, i, f \in [0,1]$ (6)

Truthiness, indetermin6cy and falsity values can be real numbers or interval-valued numbers. If these are interval-valued numbers, the set is named as interval Neutrosophic set and it is represented with three intervals. Summation of the biggest upper limits of these three intervals must between o and 3 (Wang et al., 2005). Representation of interval NSs is shown in Eq. (6):

$$x = \langle [Tx_L Tx_U], [Fx_L Fx_U], [Ix_L Ix_U] \rangle$$

$$T_x, I_x, F_x \in [0,1]$$

$0 \leq sup\ T_x + sup\ F_x + sup\ I_x \leq 3$ (7)

5. Construction of QSS using Fuzzy Poisson Distribution

If the size of sample be large and 'p' is small then the random variable 'd' has a Poisson approximation distribution with $\lambda = np$.So, the probability for the number of defective items to be exactly equal to 'd' is

P(d) = $\frac{e^{-np}np^d}{d!}$ and the probability for acceptance of the lot (P_a) is:

$$P_a = P(d \leq c) = \sum_{d=0}^{c} \frac{e^{-np}np^d}{d!}$$

Suppose that we want to inspect a lot with the large size of 'N', such that the proportion of damaged items is not known precisely. So we represent this parameter with a fuzzy number $\tilde{p}$ as follows:

$$\tilde{p} = (a_1, a_2, a_3), p \in \tilde{p}[1], q \in \tilde{q}[1],$$

$$P + q = 1.$$

A single sampling plan with a fuzzy parameter if defined by the sample size 'n', and acceptance number 'c', and if the number of observation defective product is less than or equal to 'c', the lot will be acceptance. If 'N' is a large number, then the number of defective items in this sample (d) has a fuzzy Poisson distribution with parameter $\tilde{\lambda} = \widetilde{np}$. So the fuzzy probability for the number of defective items in a sample size that is exactly equal to 'd' is

$$\tilde{P}(d - defective)[a] = [P^L[a], P^U[a]]$$

$$P^L[a] = \min \left\{ \frac{e^{-np} np^d}{d!} \middle| \lambda \in \widetilde{np}[a] \right\}$$

$$P^U[a] = \max \left\{ \frac{e^{-np} np^d}{d!} \middle| \lambda \in \widetilde{np}[a] \right\}$$

and fuzzy acceptance probability is as follows:

$$\tilde{P}_a = \left\{ \sum_{d=0}^{c} \frac{e^{-np} np^d}{d!} \middle| \lambda \in \tilde{\lambda}[a] \right\} = [P^L[a], P^U[a]] \tag{8}$$

$$P^L[a] = min \left\{ \sum_{d=0}^{c} \frac{e^{-np} np^d}{d!} \middle| \lambda \in \tilde{\lambda}[a] \right\} \tag{9}$$

$$P^U[a] = max \left\{ \sum_{d=0}^{c} \frac{e^{-np} np^d}{d!} \middle| \lambda \in \tilde{\lambda}[a] \right\} \tag{10}$$

5.1 OC Band with Fuzzy Parameter

Operating characteristic curve is one of the important criteria in the sampling plan. By this curve, one could be determined the probability of acceptance or rejection of a lot having some specific defective items. The OC curve represents the performance of the acceptance sampling plans by plotting the probability of acceptance a lot versus its production quality, which is expressed by the proportion of nonconforming items in the lot. OC curve aids in selection of plans that are effective in reducing risk and indicates discriminating power of the plan.

The fuzzy probability of acceptance a lot in terms of fuzzy fraction of defective items would be as a band with upper and lower bounds. The uncertainty degree of a proportion parameter is one of the factors that bandwidth depends on that. The less uncertainty value results in less bandwidth, and if proportion parameter gets a crisp value., lower and upper bounds will become equal, which that OC curve is in classic state. Knowing the uncertainty degree of proportion parameter and variation of its position on horizontal axis, we have different fuzzy number ($\tilde{P}$) and hence we will have different proportion (p) which the OC bands are plotted in terms of it.

6. Construction of QSS using Neutrosophic Poisson distribution

In this section, an attribute sampling plan having certain plan parameters and neutrosophic defection status is offered based on Poisson distribution. Difference from classical acceptance sampling plans is considering the indeterminacy case as a defection status.

Formulation of single sampling plan based on Poisson distribution has two frequency values as defect frequency λ_F= n.P(F) and indeterminacy frequency λ_I= n. P(I). If the neutrosophic set A has inconsistency, the probability values should be normalized by dividing each of them with total probability to make

t + I + f = 1. This normalization is offered by Smarandache [14].

The acceptance probability of lot (P_a) is calculated as shown in Eq (11).

For normal single sampling plan, the Neutrosophic probability of acceptance is represented as 'P_{aN}', the rejection probability is represented as 'P_{rN}' and the indeterminacy probability is represented as 'P_{iN}'.

$$P_{aN} = \sum_{d=0}^{C_N} \frac{\lambda_F^d}{d!} \left[\sum_{i=o}^{min(I,n-d)} \frac{\lambda_I^i}{i!} e^{-(\lambda_I+\lambda_F)}\right] \tag{11}$$

$$P_{rN} = \sum_{d=c_{N+}1}^{C_N} \frac{\lambda_F^d}{d!} \left[\sum_{i=o}^{n-d} \frac{\lambda_I^i}{i!} e^{-(\lambda_I+\lambda_F)}\right] \tag{12}$$

$$P_{iN} = \sum_{i=I+1}^{n} \frac{\lambda_I^i}{i!} \left[\sum_{d=o}^{min(N,n-i)} \frac{\lambda_F^d}{d!} e^{-(\lambda_I+\lambda_F)}\right] \tag{13}$$

$\boldsymbol{P_{aN} + P_{rN} + P_{iN} = 1}$ ………….. total probability

Similarly, for tightened single sampling plan, the Neutrosophic probability of acceptance is represented as 'P_{aT}', the rejection probability is represented as 'P_{rT}' and the indeterminacy probability is represented as 'P_{iT}'

$$P_{aT} = \sum_{d=0}^{C_T} \frac{\lambda_F^d}{d!} \left[\sum_{i=o}^{min(I,n-d)} \frac{\lambda_I^i}{i!} e^{-(\lambda_I+\lambda_F)}\right] \tag{14}$$

$$P_{rT} = \sum_{d=c_{N+}1}^{C_N} \frac{\lambda_F^d}{d!} \left[\sum_{i=o}^{n-d} \frac{\lambda_I^i}{i!} e^{-(\lambda_I+\lambda_F)}\right] \tag{15}$$

$$P_{iT} = \sum_{i=I+1}^{n} \frac{\lambda_I^i}{i!} \left[\sum_{d=o}^{min(T,n-i)} \frac{\lambda_F^d}{d!} e^{-(\lambda_I+\lambda_F)}\right] \tag{16}$$

$\boldsymbol{P_{aT} + P_{rT} + P_{iT} = 1}$ ………….. total probability

Therefore, the probability of acceptance 'P_a'is calculated using,

$$P_a(p) = \frac{P_{(a)T}}{1 - P_{(a)N} + P_{(a)T}} \tag{17}$$

Where, $P_{(a)N}$ is the proportion of lots expected to be accepted using Neutrosophic Normal SSP.$P_{(a)T}$ is the proportion of lots accepted to be accepted using Neutrosophic Tightened SSP.

Illustration 1:

To illustrate the application capability of the proposed plans in real world, an example scenario can be such that:

A company purchases tyres from a supplier to use it as an accessory of its products. Most of the defects are considered as slight defect but some of the defects are not acceptable. The operator may become undecided in some cases, the major customers choose and inspect 40 items of the available product to buy them. If the number of non-conforming items in this sample equals three or two, the customer will buy all the products.

If the number of non-conforming increases, the customer will not buy them, Because of the proportion of defective products has explained linguistically, a fuzzy number P̂= (0,0.005,0.01) is considered. Therefore, the probability purchasing would be described in the following:

In the normal plan **n=40, C_N=3,** $\widehat{P}$= (0,0.005,0.01) and in the tightened plan **n=40, C_T=2,** $\widehat{P}$= (0,0.005,0.01) of acceptance of the system. Hence, Normal single sampling plan the

P_N = (1,0.9964,1) and tightened single sampling plan the P_T = [0.9920, 1] Then, the probability of acceptance of the QSSFP (40;3,2) is $\widehat{P}_a$ [0] = [0.9916, 1], that is it is executed that for every 100 lots in a manufacturing process, 99 to 100 lots will be accepted.

Table 1. Probability of acceptance for QSSFP (n=40, CN=3, CT=2)

Li	P	QSS$_F$P
0	[0,0.01]	[1,0.9916]
0.01	[0.01,0.02]	[0.9916,0.9446]
0.02	[0.02,0.03]	0.9446,0.8461]
0.03	[0.03,0.04]	[0.8461,0.7078]
0.04	[0.04,0.05]	[0.7078,0.5566]
0.05	[0.05,0.06]	0.5566,0.4175]
0.06	[0.06,0.07]	[0.4175,0.3034]
0.07	[0.07,0.08]	[0.3034,0.2163]
0.08	[0.08,0.09]	[0.2163,0.1527]
0.09	[0.09,0.10]	[0.1527,0.1073]
0.1	[0.1,0.11]	[0.1073,0.0752]

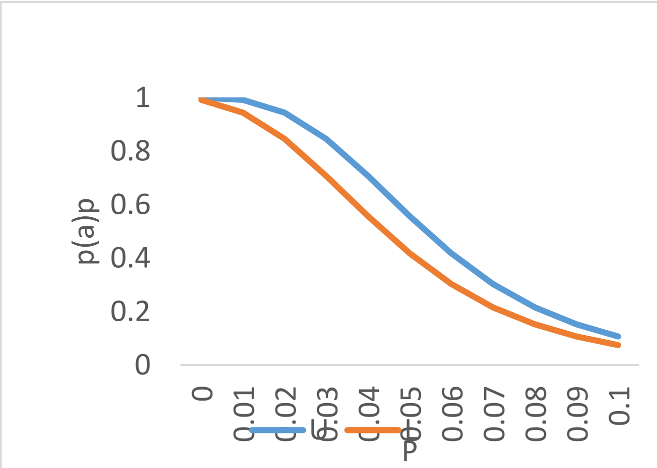

Figure 1. OC band for QSS using Fuzzy Poisson.

From the OC band, the systems are well defined since if the fraction of defective items is crisp, reduce to classical plans. The uncertainty degree of a proportion parameter is one of the factors that bandwidth depends on the band of the curve. The less uncertainty value results in less bandwidth, and greater uncertainty values results in wider bandwidth. From this it is suggested that, can adopt this system to predict the uncertainty level. Based on this system, one can achieve better outcome with minimum sampling cost and time.

Illustration 2:

To illustrate the application capability of the proposed plans in real world, an example scenario can be such that:

A company purchases tires from a supplier to use it as an accessory of its products. The tires can have several defects having different importance levels such as cap ply defects and skim stock defects. Some of the items can have multiple defects at the same time. Most of these defects are considered as slight defect but some are not acceptable. The operator may become undecided in some cases the items have multiple defects with multiple levels.

The agreement is made between the company and supplier depending on a quality level. The supplier declares an item non defectiveness probability, an item indeterminacy probability, and an item defectiveness probability for the incoming lots. The company controls the quality of the product by applying Quick Switching System based on Neutrosophic Poisson Distribution.

Step 1. From the lot, take a random sample of size 'n (40)' at the normal level and count the number of defective items that is 'd' and indeterminate items 'i'.

- a) If $d \le C_N$ (3) and $i \le I$ (2), accept the lot and repeat step 1 for the next lot.
- b) If $d > C_N$ (3), reject the lot and go to step 2.
- c) If $d \le C_N$ (3), $i > I$ (2), the lot is indeterminate.

Step 2. From the next lot, take a random sample of size 'n (40)' at the tightened level and count the number of defective items 'd' and number of indeterminate items 'i'.

- a) If $d \le C_T$ (2), $i \le I$ (2), accept the lot and repeat step 1 for the next lot.
- b) If $d > C_T$ (2), reject the lot and repeat step 2.
- c) If $d \le C_T$ (2), $i > I$ (2), the lot is indeterminate,

Table 2. Probability of acceptance for QSS_NP

N	n	c_N	c_T	I	P(S)	P(F)	P(I)	P_{aN}	P_{rN}	P_{iN}	P_{aT}	P_{rT}	P_{iT}	QSS (Pa)
600	30	3	2	2	0.95	0.05	0.05	0.779	0.057	0.164	0.6528	0.1902	0.154	0.747
600	30	2	1	2	0.95	0.04	0.02	0.862	0.118	0.02	0.4502	0.4146	0.178	0.7653
600	30	2	1	2	0.83	0.03	0.04	0.781	0.08	0.139	0.6788	0.2271	0.092	0.756
1200	30	3	2	2	0.95	0.05	0.05	0.779	0.057	0.164	0.6528	0.1902	0.154	0.747
1200	30	2	1	2	0.95	0.05	0.05	0.683	0.173	0.143	0.4502	0.797	0.106	0.5868
1200	40	3	2	2	0.95	0.05	0.05	0.458	0.113	0.488	0.4575	0.2492	0.207	0.4575

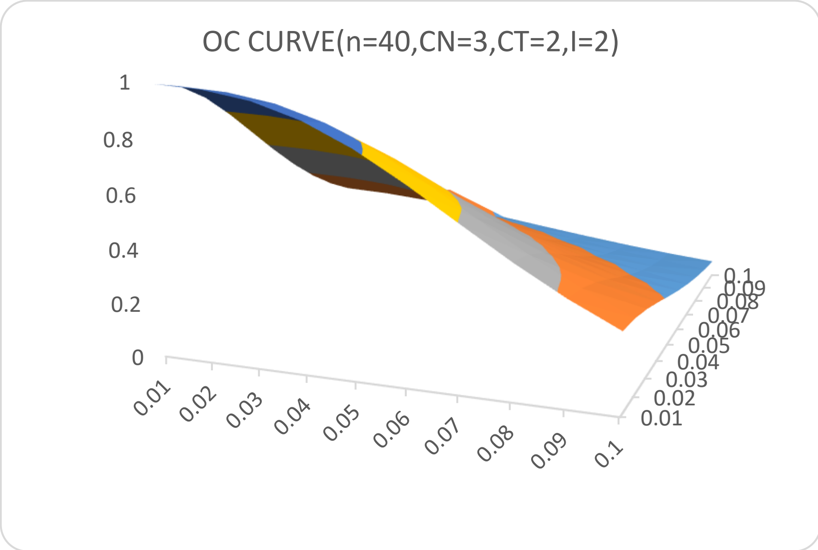

Figure 2. OC Surface for QSS using Neutrosophic Poisson distribution.

OC is formed as a surface depending on P(F), P(I) and P_a. The system has three possible outcomes: accept, reject and indeterminate. According to figure, rejection is dominant to indeterminate case. While C=I and P(F) =P(I), P_r is observed bigger than P_i.

7. Conclusion

In this article, Construction and designing of Quick Switching system QSS_{FP} and QSS_{NP} (n; C_N,C_T) with reference to Single sampling plan using Neutrosophic Poisson Distribution is studied and compared with Fuzzy Poisson Distribution for various Quality Characteristics. Both Fuzzy and Neutrosophic concepts are applied in uncertainty environment where NSs include indeterminacy term which is similar to human thinking. On comparison, it is concluded that QSS using Neutrosophic Poisson distribution gives high probability of acceptance and well suited for uncertainty environment than QSS Using Fuzzy Poisson distribution with the indeterminacy term. As a future study, these plans and systems can be extended for other distributions for various characteristic measures such as AQL, LQL, AOQL with NSs.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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