NEUTROSOPHIC SYSTEMS WITH APPLICATIONS

AN INTERNATIONAL JOURNAL ON INFORMATICS, DECISION SCIENCE, INTELLIGENT SYSTEMS APPLICATIONS

ISSN (ONLINE):2993-7159

ISSN (PRINT): 2993-7140

VOLUME 11

2023

NSWA
Neutrosophic Systems with Applications
An International Journal on Informatics, Decision Science, Intelligent Systems Applications
ISSN (online): 2993-7159
ISSN (print): 2993-7140

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An International Journal on Informatics, Decision Science, Intelligent Systems Applications

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*"***Neutrosophic Systems with Applications***"* has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e., notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjointed two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^{-}0, 1^{+}[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of classical statistics.

What distinguishes neutrosophic from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Neutrosophic Systems with Applications, Vol. 11, 2023
https://doi.org/10.61356/j.nswa.2023.83

Interval-Valued Fermatean Neutrosophic Shortest Path Problem via Score Function

Said Broumi [1,2,*], S. krishna Prabha [3] and Vakkas Uluçay [4]

[1] Laboratory of Information Processing, Faculty of Science Ben M'Sik, University of Hassan II, Casablanca, Morocco.
[2] Regional Center for the Professions of Education and Training (C.R.M.E.F), Casablanca- Settat, Morocco.
[3] Department of Mathematics, PSNA College of Engineering and Technology, Dindigul, Tamil Nadu, India; krishna_praba@psnacet.edu.in.
[4] Department of Mathematics, Faculty of Science, Kilis 7 Aralık University, Kilis, Turkey; vakkas.ulucay@kilis.edu.tr.

* Correspondence: s.broumi@flbenmsik.ma.

Abstract: Contemporary mathematical techniques have been crafted to address the uncertainty of numerous real-world settings, including Fermatean neutrosophic fuzzy set theory. Fermatean neutrosophic fuzzy set is an extension of combining Fermatean and neutrosophic sets. Fermatean neutrosophic set was developed to enable the analytical management of ambiguous data from relatively typical real-world decision-making scenarios. Decision-makers find it challenging to determine the degree of membership (MG) and non-membership (NG) with sharp values due to the insufficient data provided. Intervals MG and NG are suitable options in these circumstances. In this article, the shortest route issue is formulated using an interval set of values in a Fermatean neutrosophic setting. A de-neutrosophication technique utilizing a scoring function is then suggested. A mathematical version is also included to show the framework's usefulness and viability in more detail.

Keywords: Fermatean Neutrosophic Shortest Path Problem; Fermatean Neutrosophic Fuzzy Set; Shortest Path Problem; Network.

1. Introduction

For processing conceptual data, graph models are frequently used in a variety of domains, including operations science, networks, data analysis, pattern discovery, the field of finance, and visual design. In 1965, Zadeh [1] presented the Fuzzy Set (FS) as a magic solution to uncertainty and ambiguity. The FS theory is demonstrated in a variety of real-world problems in numerous practical applications. Atanassov [2] first presented the Intuitionistic Fuzzy Set (IFS) model in 1986. In IFS, membership and non-membership are used to characterize every item (totals are always capped at 1). Yager has introduced the Pythagorean fuzzy set (PFS) notion as a generalization of the intuitionistic fuzzy set (IFS) [3] to manage the complex imprecision and uncertainty in real-world decision-making difficulties.

By relaxing the requirement that the square root of the sum of the membership degree and non-membership degree must be greater than one, the Pythagorean fuzzy model varies from other fuzzy models. Neutrosophic sets, a concept first put forth by Smarandache [4] in 1995, can be used to overcome issues including insufficient, ambiguous, and inaccurate information. Senapati and Yager [5] introduce the idea of Fermatean fuzzy sets (FFS) with the restriction that the sum of the cubes expressing membership and non-membership degrees cannot be greater than one. The FFS is a useful method to accept ambiguity and vagueness since it increases the relative volume of membership and non-membership in fuzzy and PFSs. The Fermatean fuzzy TOPSIS approach with Dombi aggregation operators was presented by Aydemer et al. in 2020 [6]. Barraza et al. [7] in 2020 provide an application of Fermatean fuzzy matrices in the co-design of urban projects. Broumi et al. [8] proposed the concept of a complex Fermatean Neutrosophic graph and its use in decision-making in 2023.

The theory and applications of Fermatean Neutrosophic Graphs were provided in 2022 by Broumi et al. [9]. In 2022, Ganie [10] presented a method for multi-criteria decision-making based on the distance and knowledge measures of FFSs. Broumi also looks into bipolar single-valued neutrosophic graphs in [11], as well as the associated properties. In [12, 13], Sundareswaran et al. described and looked into the neutrosophic environment's susceptible features. A correlation metric for Pythagorean Neutrosophic Sets was presented in 2019 by Jansi et al. [14]. A New Decision-Making Approach Based on FFSs and WASPAS for Green Construction Supplier Evaluation was given by Keshavarz-Ghorabaee et al. [15] in 2020. The Fermatean fuzzy WASPAS method-based multi-criteria healthcare waste disposal location selection was proposed by Mishra. On interval neutrosophic sets, Broumi et al. [16] studied some operations on interval-valued Fermatean neutrosophic sets and their use in multicriteria decision-making. Broumi et al. [17] proposed a new intelligent algorithm for trapezoidal interval-valued neutrosophic network analysis. In 2016, Dey et al. [18] researched the fuzzy version of the shortest path problem (SPP). Using interval-valued triangular fuzzy arc weight, Ebrahimnejad et al. [19] suggested an optimization method for unraveling SPPs in 2020. In 2020, Singh [20] presented a fuzzy SPP from the perspective of a startup founder. SPP was first proposed by Jan A et al. [21] in 2022 using Pythagorean fuzzy components with interval values.

A new emergent concept of Fermatean neutrosophic was introduced by Antony and Jansi [22] in 2021 by fusing the concepts of Neutrosophic sets and FFSs. To determine the shortest path, the Fermatean neutrosophic graphs are examined in this work. Asim Bash et al. [23] provide a solution for neutrosophic Pythagorean fuzzy shortest path in a network. In 2023, Sasikala [24] presented her interpretation of Fermatean Neutrosophic Dombi Fuzzy Graphs. Mary et al. [25] provide a solution approach to the minimum spanning tree problem under the Fermatean fuzzy environment. Fermatean fuzzy hypergraph and some of its characteristics were proposed by Thamizhendhi [26] in 2021. By Vidhya [27] in 2022, an enhanced A search algorithm for the shortest path in a Pythagorean fuzzy environment with interval values. Broumi et al. [28] studied the concept of interval-valued Fermatean Neutrosophic graphs. Raut et al. [29] studied the problem of the shortest path on Fermatean Neutrosophic Networks. To the best of our knowledge, there is no study on interval Fermatean Neutrosophic Networks.

The organization of this paper is as follows: Section 1 covers the context and significant applications that provided inspiration for the proposed study. Section 2 provides a list of some fundamental definitions. A framework for the interval-valued Fermatean neutrosophic SPP is provided in Section 3, a quantified example is given in Section 4, and the study is summarized, possible future directions are discussed, and the benefits of the suggested work are emphasized in Section 5.

2. Preliminaries

In the following, some basic concepts and definitions of PFS, FFS, interval Fermatean neutrosophic sets, and interval valued Fermatean neutrosophic graph are reviewed from the literature.

Definition 2.1 [3] A PFS A on a universe of discourse X, is a structure having the form as

$$A_{PyFS}=\{\langle x, T_A(x), F_A(x)\rangle \mid x\in X\}$$

where $T_A(x): X\rightarrow[0,1]$ indicates the degree of membership and $F_A(x): X\rightarrow[0,1]$ indicates the degree of non-membership of every element $x\in X$ to the set A, respectively, with the constraints: $0\leq\left(T_A(x)\right)^2+\left(F_A(x)\right)^2\leq 1$.

Senpati et al. [5] suggested the idea of FFS considering more possible types of uncertainty. These are defined below,

Definition 2.2: [5] A FFS A on a universe of discourse X is a structure defined as,

$$A_{\text{FFS}}= \{\langle x, T_A(x), F_A(x)\rangle \mid x \in X\}$$

where $T_A(x): X \to [0,1]$ indicates the degree of membership, and $F_A(x): X \to [0,1]$ indicates the degree of non-membership of the element $x \in X$ to the set A, respectively, with the constraints:

$$0 \le \left(T_A(x)\right)^3 + \left(F_A(x)\right)^3 \le 1.$$

Definition 2.3: [16] An interval valued Fermatean neutrosophic number $A = \langle [T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U] \rangle$ is supposedly. Zero interval valued Fermatean neutrosophic number if and only if $T_A^L = 0$, $T_A^U = 0$, $I_A^L = 1$, $I_A^U = 1$, $F_A^L = 1$ and $F_A^U = 1$.

Definition 2.4: [16] An interval-valued Fermatean neutrosophic set (IVFNS) A on the universe of discourse X is of the structure: $A_{IVFNS} = \{\langle x, \tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)\rangle \mid x \in X\}$, where, $\tilde{T}_A(x) = [T_A^L(x), T_A^U(x)]$ $\tilde{I}_A(x) = [I_A^L(x), I_A^U(x)]$ and $\tilde{F}_A(x) = [F_A^L(x), F_A^U(x)]$ represents the truth-membership degree, indeterminacy-membership degree and falsity-membership degree, respectively. Consider the mapping $\tilde{T}_A(x): X \to D[0,1]$, $\tilde{I}_A(x): X \to D[0,1]$, $\tilde{F}_A(x): X \to D[0,1]$ and $0 \le \left(T_A^U(x)\right)^3 + \left(F_A^U(x)\right)^3 \le 1$ and $0 \le \left(I_A^U(x)\right)^3 \le 1$,

$$0 \le \left(T_A^U(x)\right)^3 + \left(I_A^U(x)\right)^3 + \left(F_A^U(x)\right)^3 \le 2, \forall x \in X.$$

Definition 2.5: [28] An Interval-Valued Fermatean Neutrosophic Graph (IVFNG) of a graph $G* =(V,E)$ we mean a pair $G =(A, B)$, where $A = \langle [\boldsymbol{T_A^L}, \boldsymbol{T_A^U}], [\boldsymbol{I_A^L}, \boldsymbol{I_A^U}], [\boldsymbol{F_A^L}, \boldsymbol{F_A^U}] \rangle$ is an interval-valued Fermatean neutrosophic set on V; and $B = \langle [\boldsymbol{T_B^L}, \boldsymbol{T_B^U}], [\boldsymbol{I_B^L}, \boldsymbol{I_B^U}], [\boldsymbol{F_B^L}, \boldsymbol{F_B^U}] \rangle$ is an interval valued Fermatean neutrosophic relation on E satisfying the following condition:

- $V = \{ v_1, v_2, \dots, v_n \}$, such that $T_A^L: V \to [0, 1]$, $T_A^U: V \to [0, 1]$, $I_A^L: V \to [0, 1]$, $I_A^U: V \to [0, 1]$ and $F_A^L: V \to [0, 1]$, $F_A^U: V \to [0, 1]$ denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element $y \in V$, respectively, and $0 \le (T_A(v_i))^3 + (I_A(v_i))^3 + (F_A(v_i))^3 \le 2$, *for all* $v_i \in V$, $(i = 1, 2, \dots, n)$ means $0 \le (T_A^U(v_i))^3 + (\boldsymbol{I}_A^U(v_i))^3 + (\boldsymbol{F}_A^U(v_i))^3 \le 2$, $\forall x \in X$.
- The functions $T_B^L : V{\times}V \to [0,1]$, $T_B^U : V{\times}V \to [0,1]$, $I_B^L : V{\times}V \to [0,1]$, $I_B^U : V{\times}V \to [0,1]$ and $F_B^L: V{\times}V \to [0,1]$, $F_B^U: V{\times}V \to [0,1]$ are such that $T_B^L(\{v_i, v_j\}) \le \min[T_A^L(vi), T_A^L(v_j)]$, $T_B^U(\{v_i, v_j\} \le \min[T_A^U(v_i), T_A^U(v_j)]$, $I_B^L(\{v_i, v_j\}) \ge \max[I_A^L(v_i), I_A^L(v_j)]$, $I_B^U(\{v_i, v_j\}) \ge \max[I_A^U(v_i), I_a^U(v_j)]$, $F_B^L(\{v_i, v_j\}) \ge \max[F_A^L(v_i), F_A^L(v_j)]$, $F_B^U(\{v_i, v_j\}) \ge \max[F_A^U(v_i), F_A^U(v_j)]$ denoting the degree of truth-membership, indeterminacy-membership and falsity membership of the edge $(v_i, v_j) \in E$ respectively, where $0 \le \left(T_B(v_i, v_j)\right)^3 + \left(\boldsymbol{I}_B(v_i, v_j)\right)^3 + \left(\boldsymbol{F}_B(v_i, v_j)\right)^3 \le 2$ for all $\{v_i, v_j\} \in E$ $(i,j = 1, 2, \dots, n)$ means $0 \le \left(T_B^U(v_i, v_j)\right)^3 + \left(\boldsymbol{I}_B^U(v_i, v_j)\right)^3 + \left(\boldsymbol{F}_B^U(v_i, v_j)\right)^3 \le 2$, $\forall x \in X$.

Definition 2.6: [16] Broumi et al. [29] defined the average possible membership degree of element x to interval valued Fermatean neutrosophic set A = $\langle [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)], \rangle$ as follows:

$$S_{Broumi}(x) = \frac{\left(T_A^L(x)\right)^3 + \left(T_A^U(x)\right)^3 + \left(I_A^L(x)\right)^3 + \left(I_A^U(x)\right)^3 + \left(F_A^L(x)\right)^3 + \left(F_A^U(x)\right)^3}{2}$$

Definition 2.7: [16] Let $\mathbb{A} = \langle[\mathbb{T}^L,\mathbb{T}^U],[I^L,I^U],[\mathbb{F}^L,\mathbb{F}^U]\rangle, \mathbb{A}_1 = [\mathbb{T}_1^L,\mathbb{T}_1^U],[I_1^L,I_1^U],[\mathbb{F}_1^L,\mathbb{F}_1^U]\ \rangle$ and $\mathbb{A}_2 = \langle[\mathbb{T}_2^L,\mathbb{T}_2^U],[I_2^L,I_2^U],[\mathbb{F}_2^L,\mathbb{F}_2^U]\rangle$ be three interval valued fermatean neutrosophic numbers and $\lambda > 0$. Then, the operations rules are described as follows;

- $\mathbb{A}_1 \oplus \mathbb{A}_2 = \langle\left[\sqrt[3]{\mathbb{T}_1^{L^3} + \mathbb{T}_2^{L^3} - \mathbb{T}_1^{L^3}\mathbb{T}_2^{L^3}}, \sqrt[3]{\mathbb{T}_1^{U^3} + \mathbb{T}_2^{U^3} - \mathbb{T}_1^{U^3}\mathbb{T}_2^{U^3}}\right], [I_1^L I_2^L, I_1^U I_2^U], [\mathbb{F}_1^L\mathbb{F}_2^L, \mathbb{F}_1^U\mathbb{F}_2^U]\rangle$

- $\mathbb{A}_1 \otimes \mathbb{A}_2 = \langle[\mathbb{T}_1^L\mathbb{T}_2^L, \mathbb{T}_1^U\mathbb{T}_2^U], \left[\sqrt[3]{I_1^{L^3} + I_2^{L^3} - I_1^{L^3}I_2^{L^3}}, \sqrt[3]{I_1^{U^3} + I_2^{U^3} - I_1^{U^3}I_2^{U^3}}\right],$
$\left[\sqrt[3]{\mathbb{F}_1^{L^3} + \mathbb{F}_2^{L^3} - \mathbb{F}_1^{L^3}\mathbb{F}_2^{L^3}}, \sqrt[3]{\mathbb{F}_1^{U^3} + \mathbb{F}_2^{U^3} - \mathbb{F}_1^{U^3}\mathbb{F}_2^{U^3}}\right]\rangle$

- $\lambda\,\mathbb{A} = \langle\left[\sqrt[3]{1-\left(1-\mathbb{T}^{L^3}\right)^{\lambda}}, \sqrt[3]{1-\left(1-\mathbb{T}^{U^3}\right)^{\lambda}}\right], \left[I^{L^\lambda}, I^{U^\lambda}\right], \left[\mathbb{F}^{L^\lambda}, \mathbb{F}^{U^\lambda}\right]\rangle$

- $\mathbb{A}^{\lambda} = \langle\left[\mathbb{T}^{L^\lambda}, \mathbb{T}^{U^\lambda}\right],$
$\left[\sqrt[3]{1-\left(1-I^{L^3}\right)^{\lambda}}, \sqrt[3]{1-\left(1-I^{U^3}\right)^{\lambda}}\right],$
$\left[\sqrt[3]{1-\left(1-\mathbb{F}^{L^3}\right)^{\lambda}}, \sqrt[3]{1-\left(1-\mathbb{F}^{U^3}\right)^{\lambda}}\right]\rangle$

3. Fermatean Neutrosophic Shortest Path Algorithm

One of the prominent graph theory puzzles is the shortest path problem. The shortest path problem has been extensively examined with respect to almost every fuzzy structure in fuzzy graph theory. The novelty of the suggested method is in its capacity to deal with problems arising in interval-valued Fermatean neutrosophic numbers. The algorithm we employed is relatively simple to use and yields results much faster than other methodologies. This strategy can be applied to any type of neutrosophic structure. Whether in the context of machine learning, shipping, computerized systems, labs or manufacturing facilities, etc., this algorithmic rule can be used to meet the demand for shortest path explanations.

A technique for figuring out the shortest path between each node and its predecessor is suggested in this portion of the article. In practical applications, this approach can be employed to determine the shortest path in a network.

Step 1: Prioritize v_1 and v_n as the destination's first and last nodes, respectively.

Step 2: Considering that node 1 is not isolated from itself by any distance, let $d_1 = \langle[0,0],[1,1],[1,1]\rangle$ Additionally, add the label $(\langle[0,0],[1,1],[1,1]\rangle\ ,\ -)$ to the first node.

Step 3: Find $d_j = \min\{d_i \oplus d_{ij}\}$. For j = 2, 3 …n. use the Score function for de-neutrosophication of IVFNS.

$$S_{Broumi}(x) = \frac{\left(T_A^L(x)\right)^3 + \left(T_A^U(x)\right)^3 + \left(I_A^L(x)\right)^3 + \left(I_A^U(x)\right)^3 + \left(F_A^L(x)\right)^3 + \left(F_A^U(x)\right)^3}{2},\ where\ score(A) \in [0,1].$$

Step 4: If a unique distance value is encountered at i = r. hence j is thus designated as $[d_j$, r].
If there is no unique match between the distance measurements.
It indicates that there are several IVFNS pathways leading from a node.
Use the score feature of IVFNS to find the shortest path out of multiple options.

Step 5: Let the destination node be labeled as $[d_n, k]$.where d_n is the shortest displacement between initial and final node.

Step 6: Therefore, we check the label of node k to get the IVFN shortest path from the first to the last node. Let it be. Next, we evaluate node l's label of node *l*, and so forth.
To obtain the initial node, repeat the steps above.
Step 7: Consequently, step 6 can be used to determine the IVFN shortest path.

4. Numerical Example

Presume a network of IVFNG shown in Figure 1. The shortest path is computed using the proposed technique in the approach shown below.

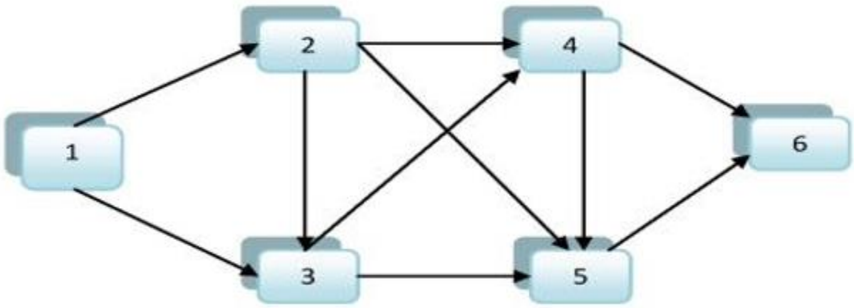

Figure 1. IVFN network.

In Table 1, IVFNG s is utilized to illustrate the path between each pair of nodes.

Table 1. Distance between the nodes in IVFN Network Edges.

Edges	Distance
(1, 2)	⟨[0.4, 0.6], [0.1, 0.3][0.2, 0.3]⟩
(1, 3)	⟨[0.2, 0.7], [0.1, 0.5], [0.1, 0.3]⟩
(2, 3)	⟨[0.1, 0.7], [0.2, 0.4], [0.3, 0.5]⟩
(2, 4)	⟨[0.4, 0.5], [0.7, 0.8], [0.1, 0.2]⟩
(2, 5)	⟨[0.5, 0.6], [0.5, 0.7], [0.3, 0.4]⟩
(3, 4)	⟨[0.6, 0.7], [0.4, 0.6], [0.3, 0.5]⟩
(3, 5)	⟨[0.6, 0.7], [0.3, 0.6], [0.2, 0.5]⟩
(4, 5)	⟨[0.4, 0.7], [0.5, 0.8], [0.1, 0.6]⟩
(4, 6)	⟨[0.3, 0.5], [0.3, 0.8], [0.1, 0.2]⟩
(5, 6)	⟨[0.5, 0.8], [0.5, 0.6], [0.2, 0.4]⟩

In Table 1, IVFNG s is utilized to illustrate the path between each pair of nodes.
Now, utilizing the methodology described, we determine the shortest path as specified:
The destination node being 6, n = 6.
If you mark the source node as (⟨[0,0],[1,1],[1,1]⟩ , −) (let's say node 1) and set d_1 = ⟨[0,0], [1,1], [1,1]⟩ to those coordinates, you can find d_j as follows.

Iteration 1: Since node 2 has only one predecessor, we set i = 1 and j = 2, which results in d_2 as

d_2= min $\{d_1 \oplus d_{12}\}$

$$= \min (\langle[0,0], [1,1], [1,1]\rangle \oplus \langle[0.4,0.6], [0.1,0.3][0.2,0.3]\rangle)$$
$$= \langle[0.4,0.6], [0.1,0.3], [0.2,0.3]\rangle$$

When i = 1, the minimum value is attained. Thus, vertex 2 is labeled as

$$\langle[0.4,0.6],[0.1,0.3],[0.2,0.3]\ \rangle,-1]$$

Iteration 2: Set i = 1, 2 and j = 3, since node 3's predecessors are 1 and 2.

$$d_3 = \min\{d_1 \oplus d_{13}, d_2 \oplus d_{23}\}$$

$$= \min\left\{\begin{matrix}\langle[0,0],[1,1],[1,1]\rangle \oplus \langle[0.2,0.7],[0.1,0.5],[0.1,0.3]\rangle, \\ \langle[0.4,0.6],[0.1,0.3][0.2,0.3]\rangle \oplus \langle[0.1,0.7],[0.2,0.4],[0.3,0.5]\rangle\end{matrix}\right\}$$

$$= \min\left\{\begin{matrix}\langle[0.2,0.7],[0.1,0.5],[0.1,0.3]\rangle, \\ \langle[0.0216,0.1616],[0.02,0.12],[0.06,0.15]\rangle\end{matrix}\right\}$$

Score function enables us to identify the absolute minimum:

$$S_{Broumi}(x) = \frac{\left(T_A^L(x)\right)^3+\left(T_A^U(x)\right)^3+\left(I_A^L(x)\right)^3+\left(I_A^U(x)\right)^3+\left(F_A^L(x)\right)^3+\left(F_A^U(x)\right)^3}{2}$$

S(⟨[0.2,0.7], [0.1,0.5], [0.1,0.3]⟩) = 0.2525, and
S(⟨[0.0216, 0.1616], [0.02, 0.12], [0.06, 0.15]⟩) = 0.048
So, the d_3 =⟨[0.1,0.7], [0.2,0.4], [0.3,0.5]⟩

When i = 2, the minimum value is attained. Thus, vertex 3 is labeled as [⟨[0.1,0.7], [0.2,0.4], [0.3,0.5]⟩, 2].

Iteration 3: Set i = 2, 3, and j = 4, since node 4's predecessors are 2 and 3.

$$d_4 = \min\{d_2 \oplus d_{24}, d_3 \oplus d_{34}\}$$

$$= \min\left\{\begin{matrix}\langle[0.4,0.6],[0.1,0.3],[0.2,0.3]\rangle \oplus \langle[0.4,0.5],[0.7,0.8],[0.1,0.2]\rangle\rangle, \\ \langle[0.1,0.7],[0.2,0.4],[0.3,0.5]\rangle \oplus \langle[0.6,0.7],[0.4,0.6],[0.3,0.5]\rangle\end{matrix}\right\}$$

$$= \min\left\{\begin{matrix}\langle[0.0413,0.1047],[0.1146,0.1751],[0.003,0.0116]\rangle, \\ \langle[0.0723,0.1895],[0.0238,0.887],[0.018,0.0781]\rangle\end{matrix}\right\}$$

Score function enables us to identify the absolute minimum:
S([0.0413 ,0.1047], [0.1146, 0.1751], [0.003,0.0116]⟩) = 0.004, and
S([0.0723,0.1895], [0.0238,0.887], [0.018, 0.0781]) = 0.0042
Hence d_4 = ⟨[0.0413 ,0.1047], [0.1146, 0.1751], [0.003,0.0116]⟩
When i = 2, the minimum value is attained. Thus, vertex 4 is labeled as [⟨[0.0413 , 0.1047], [0.1146, 0.1751], [0.003, 0.0116]⟩, 2].

Iteration 4: Set i = 2, 3, 4 and j = 5, since node 5's predecessors are 2, 3 and 4.

$$d_5 = \min\{d_2 \oplus d_{25}, d_3 \oplus d_{35}, d_4 \oplus d_{45}\}$$

$$= \min\left\{\begin{matrix}\langle[0.4,0.6],[0.1,0.3],[0.2,0.3]\rangle\rangle\rangle \oplus \langle[0.5,0.6],[0.5,0.7],[0.3,0.4]\rangle, \\ \langle[0.1,0.7],[0.2,0.4],[0.3,0.5]\rangle \oplus \langle[0.6,0.7],[0.3,0.6],[0.2,0.5]\rangle, \\ \langle[0.0413,0.1047],[0.1146,0.1751],[0.003,0.0116]\rangle \oplus \langle[0.4,0.7],[0.5,0.8],[0.1,0.6]\rangle\rangle\end{matrix}\right\}$$

$$= \min\left\{\begin{matrix}\langle[0.0603,0.1284],[0.042,0.1202],[0.012,0.0298]\rangle, \\ \langle[0.0723,0.1895],[0.0116,0.0887],[0.012,0.0781]\rangle, \\ \langle[0.0214,0.1146],[0.0421,0.1715],[0.0003,0.072]\rangle\end{matrix}\right\}$$

Score function enables us to identify the absolute minimum:
S(⟨[0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298]⟩) =0.0021
S(⟨[0.0723, 0.1895], [0.0116, 0.0887], [0.012, 0.0781]⟩) =0.0042, and
S(⟨[0.0214, 0.1146], [0.0421,0.1715], [0.0003, 0.072]⟩) =0.0035

So, the d_5 $\langle[0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298]\rangle$

When i = 2, the minimum value is attained. Thus, vertex 5 is labeled as

$\langle[0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298]\rangle$, 2].

Iteration 5: Set i = 4, 5 and j = 6, since node 6's predecessors are 4 and 5.

$$d_6 = \min\ \{d_4 \oplus d_{46}, d_5 \oplus d_{56}\}$$

$$= \min\begin{Bmatrix}\langle[0.0413\ ,0.1047], [0.1146, 0.1751], [0.003, 0.0116]\rangle \oplus \langle[0.3,0.5], [0.3,0.8], [0.1,0.2]\rangle, \\ \langle[0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298]\rangle \oplus \langle[0.5,0.8], [0.5,0.6], [0.2,0.4]\rangle\end{Bmatrix}$$

$$= \min\begin{Bmatrix}\langle[0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027]\rangle, \\ \langle[0.0417, 0.171], [0.0417, 0.0725], [0.0027, 0.0213]\rangle\end{Bmatrix}$$

Score function enables us to identify the absolute minimum:

$S(\langle[0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027]\rangle)$ =0.0026, and

$S(\langle[0.0417, 0.171], [0.0417, 0.0725], [0.0027, 0.0213]\rangle)$ = 0.0028

So, the $d_6 = \langle[0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027]\rangle$

When i = 4, the minimum value is attained.
Thus, vertex 6 is labeled as [$\langle[0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027]\rangle$, 4].
Since d_6 is the final destination. So, the shortest displacement is specified as proceeding from vertex one to six.

$$\langle[0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027]\rangle$$

The shortest way can be determined as follows:
Node 6 is labeled [$\langle[0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027]\rangle$, 4].
Node 5 is labeled as$\langle[0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298]\rangle$, 2].
Node 4 is labeled as [$\langle[0.0413\ , 0.1047], [0.1146,\ 0.1751], [0.003, 0.0116]\rangle$, 2].
Node 3 is labeled as as [$\langle[0.1,0.7], [0.2,0.4], [0.3,0.5]\rangle$, 2].
Consequently, the shortest route is 1→2→ 4→6 with the IVFN value of distance being $\langle[0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027]\rangle$
The shortest path is depicted in Figure 2 by the dotted line, and the paths of various nodes are shown in Table 2.

Table 2. Shortest Path of the above network.

Nodes No.(j)	d_i	Shortest path from 1st node to j^{th} node
2	$\langle[0.4,0.6], [0.1,0.3], [0.2,0.3]\rangle$	1 ⟶ 2
3	$\langle[0.1,0.7], [0.2,0.4], [0.3,0.5]\rangle$	1 ⟶ 3
4	$\langle[0.0413\ ,0.1047], [0.1146, 0.1751], [0.003,0.0116]\rangle$	1 ⟶ 2⟶ 4
5	$\langle[0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298]\rangle$	1 ⟶ 2⟶ 5
6	[$\langle[0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027]\rangle$, 4]	1 ⟶ 2 ⟶ 4⟶ 6

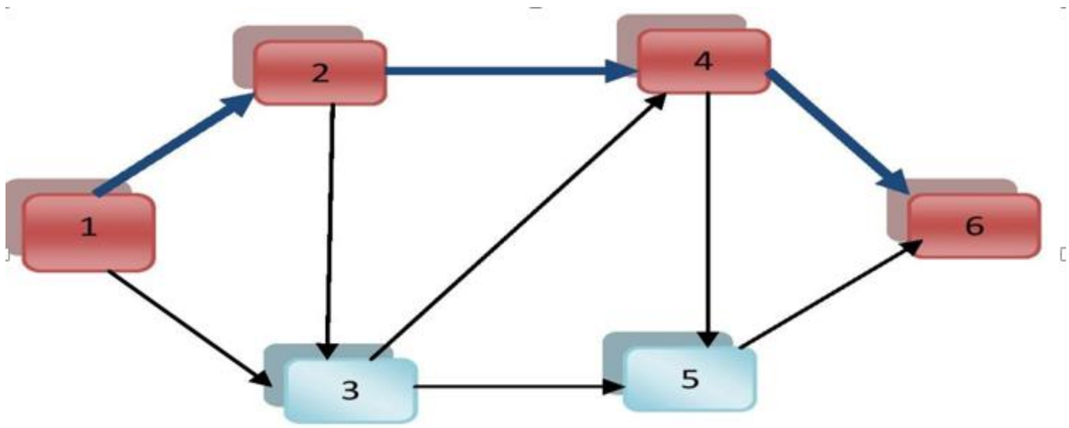

Figure 2. Shortest path IVFN network

5. Conclusions

The paper explores the idea of an Interval-Valued Fermatean Neutrosophic graph. The shortest path of an IVFNG has been determined via an algorithm. The suggested approach is employed to identify the network's shortest path across all possible paths in a numerical example. This research will be highly helpful to researchers who want to provide fresh approaches to the shortest path problem. New frameworks and algorithms will be created in the future to determine the best path for a specific network in various fixed contexts under various neutrosophic environments utilizing the findings of the present study.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors.

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Received: 30 May 2023, **Revised:** 19 Sep 2023,
Accepted: 05 Oct 2023, **Available online:** 13 Oct 2023.

Neutrosophic Systems with Applications, Vol. 11, 2023
https://doi.org/10.61356/j.nswa.2023.78

3-Dimensional Quartic Bézier Curve Approximation Model by Using Neutrosophic Approach

Siti Nur Idara Rosli [1] and **Mohammad Izat Emir Zulkifly** [2,*]

[1,2,*] Department of Mathematics, Faculty of Sciences, University of Technology Malaysia (UTM), Johor Bahru, Malaysia; Emails: sitinuridara99@gmail.com / idara-1999@graduate.utm.my; izatemir@utm.my.

* Correspondence: izatemir@utm.my.

Abstract: In a 3-dimensional data collection process, there exists noise data that cannot be included to visualize the process. Therefore, it is difficult to deal with since fuzzy set and intuitionistic fuzzy set theories did not consider the indeterminacy problem. However, using a neutrosophic approach with three memberships: truth, false, and indeterminacy membership function, the error data will be treated as uncertain data by using the indeterminacy degree. Thus, this study will visualize the 3-dimensional quartic Bézier curve model by using neutrosophic set theory. To construct the model, the neutrosophic quartic control point must first be introduced to approximate the neutrosophic quartic Bézier curve. Next, the Bernstein basis function as the methodology of this study will be blended with the neutrosophic fundamental notion. At the end of this paper, a numerical example of the 3-dimensional neutrosophic quartic Bézier curve will be visualized by using the approximation method as the finding of this study. Finally, this study provides significant contributions to making it easier for data collectors to visualize all data, which means that no data will be eliminated since the uncertainty data will also be used.

Keywords: Neutrosophic Set Theory; Bézier Geometric Modelling; Approximation Method; 3-Dimensional Quartic Modelling.

1. Introduction

Coping with insufficient 3-dimensional data collection is a significant challenge in a variety of areas, including finance, engineering, and research. Statistical inference, Bayesian analysis, fuzzy logic, and neural networks have all been created to deal with uncertainty. These solutions enable the representation of uncertainty in data and can improve the accuracy and dependability of data-driven decisions, then visualize it in 3-dimensional axes. The fuzzy set (FS) model is a theoretical paradigm for dealing with data imprecision and ambiguity. It was created in the 1960s by Lotfi Zadeh to address the inherent ambiguity and vagueness of natural language and human intellect [1]. As a result, FS just analyzes truth and false membership data and ignores the inconsistent data. In 1986, Krassimir Atanassov [2] developed intuitionistic fuzzy set (IFS) theory, which is a generalization of FS that includes true, false, and uncertain information. It is excellent for dealing with ambiguity. Since FS theory only considers whole membership data, the IFS notion is an alternative method for establishing FS when the amount of information recorded is insufficient to classify and process. However, when approaching an advanced problem with intuitive and fuzzy components, it is difficult to deal with, and it is rarely handled in the framework of spline modeling. [3], [4], [5], [6], [7], [8], [9], [10], and [11] contain studies involving fuzzy and intuitionistic fuzzy set theory and spline modeling. As a result, there is a gap in this study since the previous study's limitations include IFS and FS, which cannot deal with the more sophisticated problem. It is also the motivation of this study to use neutrosophic set theory and blend it with the Bernstein basis function to visualize it in 3-dimensional form.

The neutrosophic technique was devised by Florentin Smarandache [12] as a mathematical application of the concept of neutrality that works with uncertain data. The neutrosophic set (NS) idea is defined by membership degrees, non-membership, and indeterminacy. In this sense, an NS refers to the resolution and representation of problems that cover numerous domains. Since true, false, and indeterminate membership degrees are independent in NS theory, an element can have any value at the same time. This enables the modeling of more complex forms of uncertainty and indeterminacy, such as when a statement can be both true and false at the same time. Tas and Topal [13,14] have generated the Bézier curve and surface generally without focusing on the detail of the blending Bernstein function with NS theory, which was published in the top journal namely Neutrosophic Set System (NSS) journal. However, their research does not properly demonstrate the process of blending neutrosophic theory and Bernstein basis function, and the visualization does not clearly show the control points approximate the curve and surface. Meanwhile, Rosli and Zulkifly [15] discuss in detail the application of the B-spline curve interpolation. They also visualize a neutrosophic bicubic B-spline surface interpolation model for uncertainty data [16]. Therefore, this study will visualize the neutrosophic Bézier curve for the quartic version in 3-dimensional by showing the neutrosophic control point approximating the neutrosophic Bézier curve by using a numerical example. This research can contribute to helping data analysts model their data in spline form without wasting any noisy data. For example, in the real case of bathymetry data, there will be uncertainty due to the wave of a lake that the data was collected from by using an echo-sounder on a ship. When the ship's position changes due to a wave, there will be noise data that cannot be determined whether it is the result or not, which means it is true or false, so it will be treated as an indeterminacy degree in this study.

This paper will focus on the 3-dimensional neutrosophic quartic Bézier curve (NQBC) approximation model. The first section of this paper discusses the background of this research and some literature reviews. To visualize the NQBC model, the next section will focus on neutrosophic control point relation (NCPR) that needs to be introduced first by using some properties of the NS such as the fundamental notion NS, neutrosophic relation (NR), neutrosophic point (NP), and neutrosophic point relation (NPR). The third section discusses the blending Bernstein function with NCPR. The fourth section of this paper will visualize the application of the NQBC approximation model by using a numerical example and the algorithm of NQBC. At the end of this paper is the summarization of all sections of this study.

2. Preliminaries

This section discusses the NS, including the core concepts of NS, NR, and NP, and will define the NCP. Smarandache emphasizes that the intuitionistic set in fuzzy systems can handle limited data but not paraconsistent data [12]. "There are three memberships: a truth membership function, *T*, an indeterminacy membership function, *I*, and a falsity membership function, *F*, with the parameter 'indeterminacy' added by the NS specification" [12].

Definition 1:[12]Let Y be the main of conversation, with element in Y denoted as y . The neutrosophic set is an object in the form.

$$\hat{A}=\{\left\langle y:T_{\hat{A}(y)},I_{\hat{A}(y)},F_{\hat{A}(y)}\right\rangle|\ y\in Y\} \tag{1}$$

where the functions $T,I,F:Y\to]^{-}0,1^{+}[$ define, respectively, the degree of truth membership, the degree of indeterminacy, and the degree of false membership of the element $y\in Y$ to the set $\hat{A}$ with the condition;

$$0^{-} \le T_{\hat{A}}(y) + I_{\hat{A}}(y) + F_{\hat{A}}(y) \le 3^{+} \quad (2)$$

There is no limit to the amount of $T_{\hat{A}}(y), I_{\hat{A}}(y)$ and $F_{\hat{A}}(y)$

A value is chosen by NS from one of the real standard subsets or one of the non-standard subsets of $]^{-}0,1^{+}[$. The actual value of the interval $[0,1]$, on the other hand, $]^{-}0,1^{+}[$ will be utilized in technical applications since its utilization in real data, such as the resolution of scientific challenges, will be physically impossible. As a direct consequence of this, membership value utilization is increased.

$$\hat{A} = \{\langle y : T_{\hat{A}(y)}, I_{\hat{A}(y)}, F_{\hat{A}(y)} \rangle | \ y \in Y\} \text{ and } T_{\hat{A}}(y), I_{\hat{A}}(y), F_{\hat{A}}(y) \in [0,1] \quad (3)$$

There is no restriction on the sum of $T_{\hat{A}}(y), I_{\hat{A}}(y), F_{\hat{A}}(y)$ .Therefore,

$$0 \le T_{\hat{A}}(y) + I_{\hat{A}}(y) + F_{\hat{A}}(y) \le 3 \quad (4)$$

Definition 2: [13, 14] Let $\hat{B} = \{\langle y : T_{\hat{B}(y)}, I_{\hat{B}(y)}, F_{\hat{B}(y)} \rangle | \ y \in Y\}$ and $\hat{C} = \{\langle z : T_{\hat{C}(z)}, I_{\hat{C}(z)}, F_{\hat{C}(z)} \rangle | \ z \in Z\}$ be neutrosophic elements. Thus, $NR = \{\langle (y,z) : T_{(y,z)}, I_{(y,z)}, F_{(y,z)} \rangle | \ y \in \hat{B}, z \in \hat{C}\}$ is a neutrosophic relation (NR) on $\hat{B}$ and $\hat{C}$.

Definition 3: [13, 14] Neutrosophic set of $\hat{B}$ in space Y is neutrosophic point (NP) and $\hat{B} = \{\hat{B}_i\}$ where $i = 0,...,n$ is a set of NPs where there exists $T_{\hat{B}} : Y \to [0,1]$ as truth membership, $I_{\hat{B}} : Y \to [0,1]$ as indeterminacy membership, and $F_{\hat{B}} : \hat{Y} \to [0,1]$ as false membership with

$$T_{\hat{B}}(\hat{B}) = \begin{cases} 0 & \text{if } \hat{B}_i \notin \hat{B} \\ a \in (0,1) & \text{if } \hat{B}_i \hat{\in} \hat{B} \\ 1 & \text{if } \hat{B}_i \in \hat{B} \end{cases}$$

$$I_{\hat{B}}(\hat{B}) = \begin{cases} 0 & \text{if } \hat{B}_i \notin \hat{B} \\ b \in (0,1) & \text{if } \hat{B}_i \hat{\in} \hat{B} \\ 1 & \text{if } \hat{B}_i \in \hat{B} \end{cases} \quad (5)$$

$$F_{\hat{B}}(\hat{B}) = \begin{cases} 0 & \text{if } \hat{B}_i \notin \hat{B} \\ c \in (0,1) & \text{if } \hat{B}_i \hat{\in} \hat{B} \\ 1 & \text{if } \hat{B}_i \in \hat{B} \end{cases}$$

2.1 Neutrosophic Point Relation

The previous section's discussion of the NS notion, NP, and NR will be used to create the foundation for neutrosophic point relation (NPR). It is a group of Euclid eternal space points. Rosli and Zulkifly [15] describe NPR as follows:

Definition 4. [15] Let N, M be a grouping of elements in global area that are part of a set that is not null and $N, M, O \in \mathbf{R} \times \mathbf{R} \times \mathbf{R}$, then the term "NPR" refers to

$$\hat{R} = \left\{ \begin{matrix} \left\langle \left(n_i, m_j\right), T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j) \right\rangle \\ \mid T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j) \in I \end{matrix} \right\} \tag{6}$$

where $\left(n_i, m_j\right)$ is a set of ordered positions and $\left(n_i, m_j\right) \in N \times M$, while $T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j)$ are the truth membership, the indeterminacy membership, and the false membership that follows the condition of the neutrosophic set which is, respectively, $0 \le T_{\hat{N}}(z) + I_{\hat{N}}(z) + F_{\hat{N}}(z) \le 3$.

2.2 *Neutrosophic Control Point Relation*

In computer graphics and mathematical modelling, a control point (CP) is a single point or set of points that impact the shape or behaviour of a curve, surface, or another geometric object. Non-uniform rational B-splines modelling (NURBS), B-splines, and Bézier curves are all examples of techniques that use CPs. The location and properties of the CPs define the geometric object's qualities and deformation. Aside from manipulating data, the form, curvature, and other features of the curve or surface can be changed. The CPs in this work are a group of points used to define the contours of a neutrosophic Bézier curve. It is also critical in geometric modelling for the derivation and fabrication of smooth curves. In this part, the idea of NS and its properties are used to define NCP. The FS idea is utilized to define fuzzy control points based on research in [17, 18]. Therefore, the NCPR for NQBC was introduced based on the idea from Rosli and Zulkifly [15] as follows:

Definition 5: Let $\hat{R}$ be an NPR, then NCPR is viewed as a group of points $n+1$ that denotes a locations and coordinates and is used to describe the curve and is indicated by

$$\begin{aligned} \hat{P}_i^T &= \left\{ \hat{p}_0^T, \hat{p}_1^T, \ldots, \hat{p}_n^T \right\} \\ \hat{P}_i^I &= \left\{ \hat{p}_0^I, \hat{p}_1^I, \ldots, \hat{p}_n^I \right\} \\ \hat{P}_i^F &= \left\{ \hat{p}_0^F, \hat{p}_1^F, \ldots, \hat{p}_n^F \right\} \end{aligned} \tag{7}$$

where $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ are NCP for truth, false, and indeterminacy membership function and i is one less than n.

Since this study concentrates on **quartic case**, therefore, the $n = 4$ to create the NQBC. Thus, the NCPR is as follows:

$$\begin{aligned} \hat{P}_i^T &= \left\{ \hat{p}_0^T, \hat{p}_1^T, \hat{p}_2^T, \hat{p}_3^T, \hat{p}_4^T \right\} \\ \hat{P}_i^I &= \left\{ \hat{p}_0^I, \hat{p}_1^I, \hat{p}_2^I, \hat{p}_3^I, \hat{p}_4^I \right\} \\ \hat{P}_i^F &= \left\{ \hat{p}_0^F, \hat{p}_1^F, \hat{p}_2^F, \hat{p}_3^F, \hat{p}_4^F \right\} \end{aligned} \tag{8}$$

3. Approximation of Neutrosophic Quartic Bézier curve

Geometric simulation frequently employs Bézier curves, which are parameterized curves guided by a control polygon [19, 20]. The number of data points utilized to construct the curve corresponds to the degree of the polynomial [21]. The following definition illustrates a Bézier curve created by integrating the Bernstein polynomial or basis function using NCPR. NCPR and *Definition 1* are used to build the NQBC, which is then combined in a geometric model with the Bézier blending function. The NQBC model's properties are then discussed. The notion of the Bézier curve for approximation method comes from Piegl and Tiller [22] and is then blended with NCPR as follows:

Definition 3: Let $\hat{P}_i^T = \{\hat{p}_0^T, \hat{p}_1^T, \hat{p}_2^T, \hat{p}_3^T, \hat{p}_4^T\}, \hat{P}_i^I = \{\hat{p}_0^I, \hat{p}_1^I, \hat{p}_2^I, \hat{p}_3^I, \hat{p}_4^I\}$, and $\hat{P}_i^F = \{\hat{p}_0^F, \hat{p}_1^F, \hat{p}_2^F, \hat{p}_3^F, \hat{p}_4^F\}$ where $i = 0,1,2,3,4$ is NCPR. NQBC is defined as $BC(t)$ with the curve position vector depending on the value of the value t, then blending with J_i by Bézier curves and represented as follows:

$$BC(t)^T = \sum_{i=0}^{4} \hat{P}_i^T J_{4,i}(t)$$

$$BC(t)^I = \sum_{i=0}^{n} \hat{P}_i^I J_{4,i}(t) \tag{9}$$

$$BC(t)^F = \sum_{i=0}^{n} \hat{P}_i^F J_{4,i}(t)$$

where $0 \le t \le 1$ and the blending function is a Bézier or Bernstein basis, J_i:

$$J_{(4,i)}(t) = \binom{4}{i} t^i (1-t)^{4-i} \qquad (0)^0 \equiv 1 \tag{10}$$

with

$$\binom{4}{i} = \frac{4!}{i!(4-1)!} \qquad (0)^0 \equiv 1 \tag{11}$$

Based on the approach by Zaidi and Zulkifly [23], the NQBC equation can also be stated in matrix multiplication. By extending the analytic formulation of the curve into its Bernstein polynomial coefficients and then expressing these coefficients using the polynomial power basis [23], NQBC can be represented as a matrix, as illustrated roughly below:

$$BC(t) = [J][P] \tag{12}$$

where;

$$[J] = [J_{4,0}, J_{4,1}, J_{4,2}, J_{4,3}, J_{4,4}] \tag{13}$$

$$[P]^T = [P_0, P_1, P_2, P_3, P_4] \tag{14}$$

3.1. Properties of Neutrosophic Quartic Bézier curve

A Bézier curve is a specific case that is determined by a control polygon in the context of NURBS curves. Since the Bézier basis is the same as the Bernstein basis, certain properties of Bézier curves are easily recognized. As a result, the NQBC has the following fundamental characteristics from the idea of fundamental Bézier basis features by Piegl and Tiller [22].

- The NQBC's fundamental features are genuine.
- There are less control polygon points than the degree of the polynomial defining the curve segment.
- In most cases, the NQBC will conform to the outline of the control polygon.
- The NQBC's starting and ending positions also happen to be the start and end points of the control polygon.
- The initial and last polygon spans correspond in the direction of the tangent vectors at the ends of the NQBC.

- The NQBC is located inside the largest convex polygon specified by the vertices of the control polygon, also known as the convex hull of the control polygon.
- The NQBC displays the phenomenon of declining variance. This means that the curve does not sway more frequently than the control polygon does around any given straight line.
- Affine transformations have no effect on the NQBC.

4. Visualization of 3-Dimensional Neutrosophic Quartic Bézier curve

In this section, the 3-dimensional NQBC approximation model for truth, false, and indeterminacy will be visualized. **Table 1** shows the NCPR for each membership. All values of NCPR follow the condition of NS, which is $0 \leq T_{\hat{B}}(y) + I_{\hat{B}}(y) + F_{\hat{B}}(y) \leq 3$.

Table 1. NCPR with its degrees.

NCPR, $\hat{P}_i$	Truth Degree, $\hat{P}_i^T$	False Degree, $\hat{P}_i^F$	Indeterminacy Degree, $\hat{P}_i^I$
$\hat{P}_0 = (3,3)$	0.7	0.4	0.2
$\hat{P}_1 = (8,9)$	0.6	0.4	0.3
$\hat{P}_2 = (12,14)$	0.8	0.3	0.2
$\hat{P}_4 = (18,19)$	0.6	0.2	0.5
$\hat{P}_5 = (26,24)$	0.3	0.4	0.6

Figure 1, Figure 2, and Figure 3 show the 3-dimensional neutrosophic quartic Bézier curves for truth, false, and indeterminacy membership, respectively. The red dot denotes the neutrosophic control point relation, and the yellow dash line denotes the control polygon for NCPR. Figure 4 and Figure 5 visualize the 3-dimensional NQBC for all memberships on one axis and different views of NQBC.

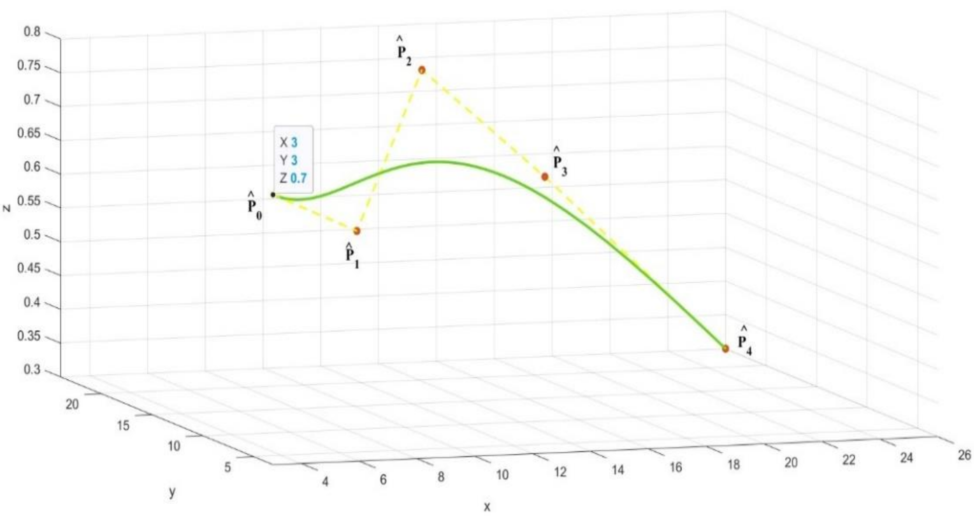

Figure 1. 3-dimensional NQBC for Truth Membership

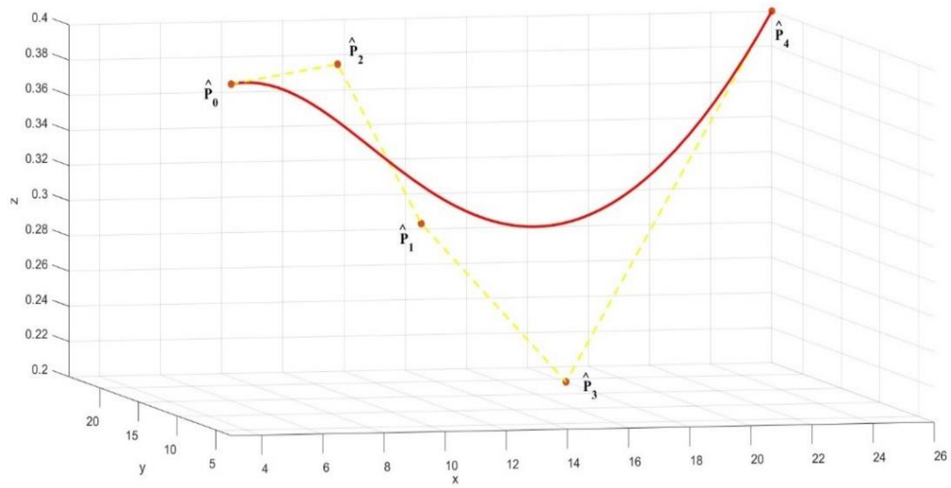

Figure 2. 3-dimensional NQBC for False Membership

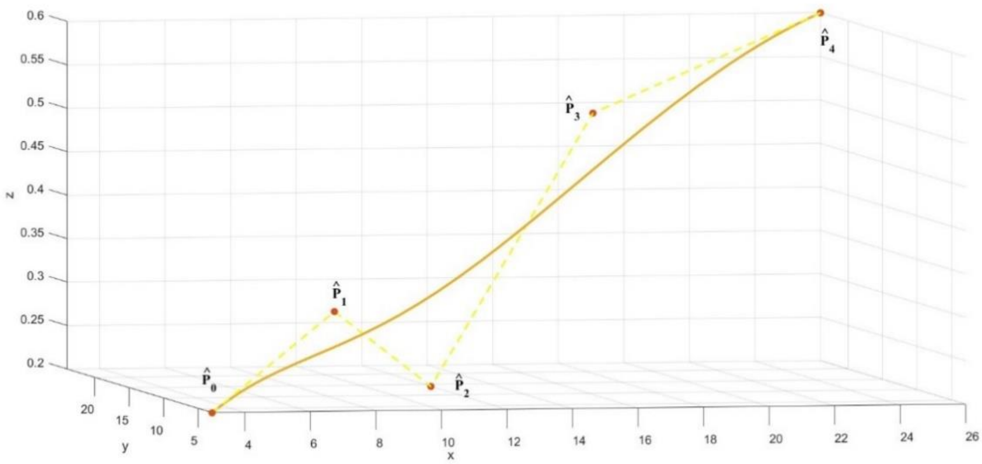

Figure 3. 3-dimensional NQBC for Indeterminacy Membership

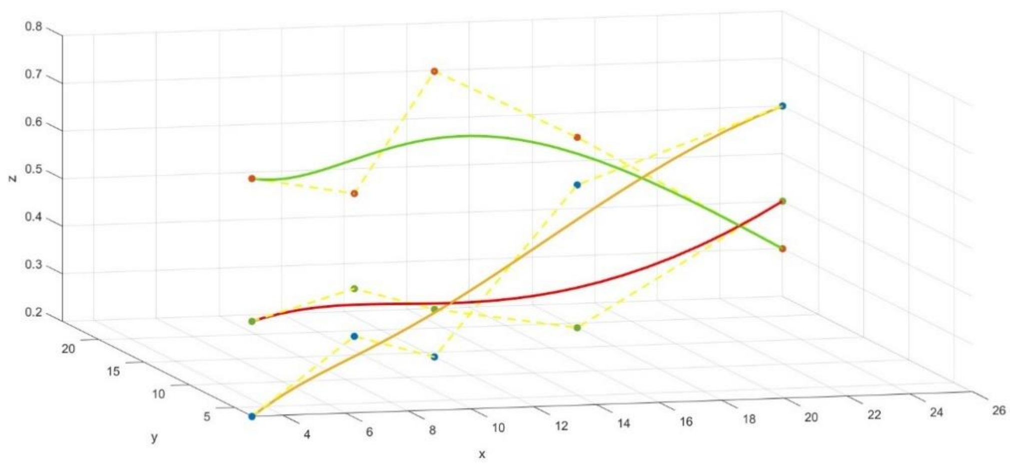

Figure 4. 3-dimensional NQBC for All Membership

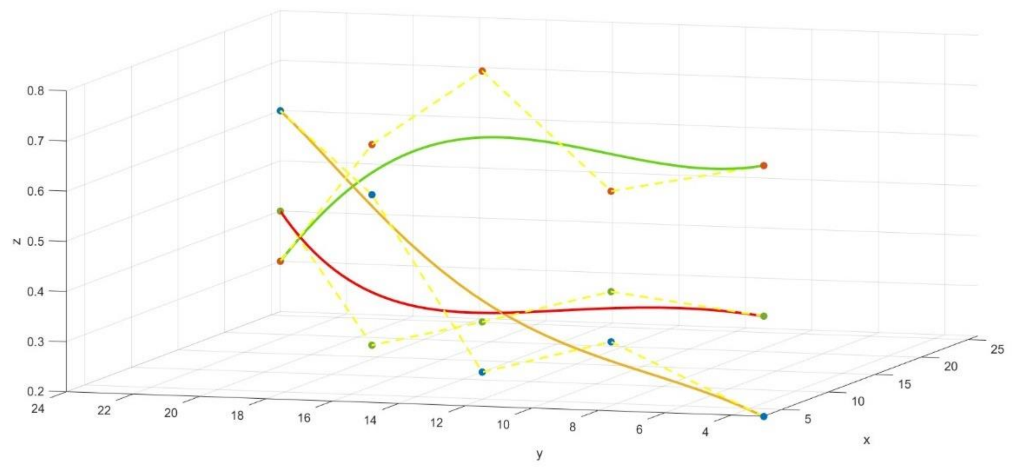

Figure 5. 3-dimensional NQBC in Different View

Based on the results for Figures 1 and 2, which are the truth and falsity membership 3-dimensional NQBC, it can be seen that the falsity membership NQBC is in the opposite direction to the truth membership, while the indeterminacy of NQBC in Figure 4 shows that it does not influence either of them but is clearly in the middle of the truth and falsity of the 3-dimensional NQBCs. This finding also demonstrates the characteristics of NS theory, wherein all memberships are considered to be independent and not influenced by one another. Nevertheless, in order for this study to be sensitive, it is necessary that all degrees comply to the condition of NS, which stands for $0 \leq T_{\hat{B}}(y) + I_{\hat{B}}(y) + F_{\hat{B}}(y) \leq 3$. According to Table 1, the variables for this study are membership values for truth, falsity, and indeterminacy. Figure 6 shows the algorithm for NQBC construction, the flowchart of this study, and an illustration of the procedure in matrix form:

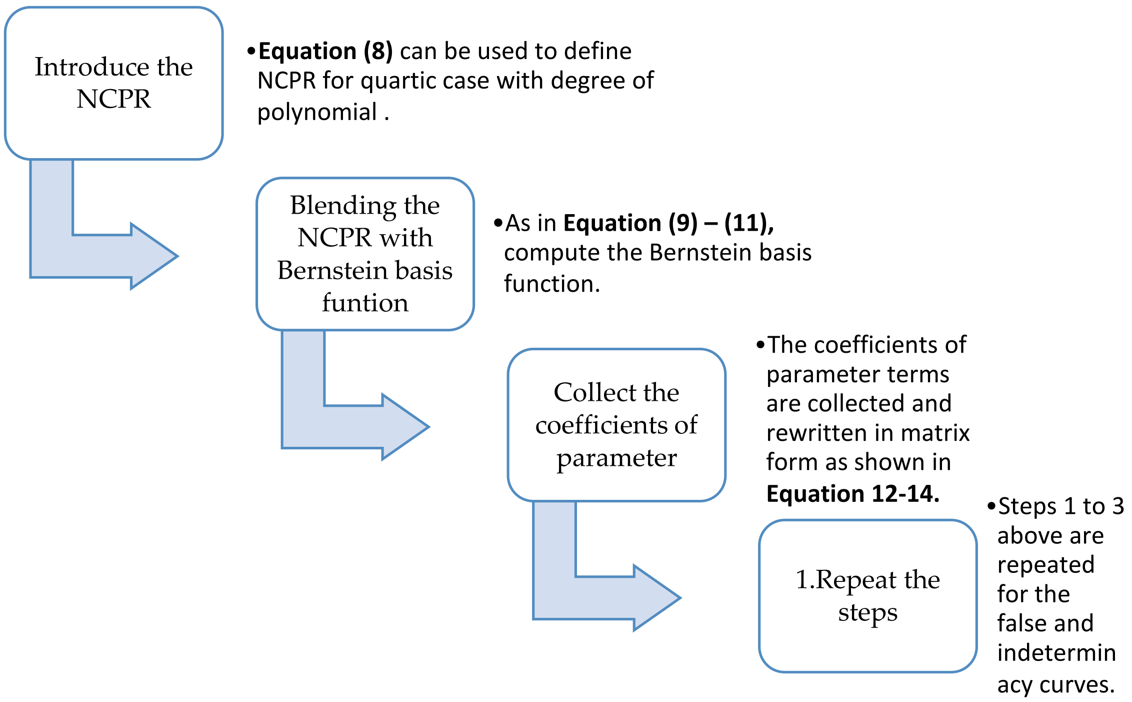

Figure 6. Flowchart of NQBC Construction

The novelties of this study are as follows:

- The NCPR for NQBC was introduced.
- The mathematical representation and properties for NQBC were analyzed and determined.
- The visualization of NQBC and the algorithm for constructing it were presented.

5. Conclusions

In this study, through NCPR, neutrosophic quartic Bézier curves approximation was introduced. Since it has a truth membership function, a false membership function, and an indeterminacy membership function, the NQBC model approximation is an excellent strategy for modeling data with neutrosophic properties. One of the key contributions of this study is that it has demonstrated that all data can be analyzed and processed using these functions. Besides that, the advantage of this model is its capacity to represent 3-dimensional neutrosophic data in the form of a Bézier curve, which is simple for data analysts to interpret and evaluate. Based on Figure 1–5, the neutrosophic data problem can be handled using the NQBC model. However, the limitation of this model is that it uses an approximation method, which simply approximates the curve by using the data, as compared to an interpolation method, which interpolates the curve using the given data. Therefore, this model can also be extended to tackle the neutrosophic data problem by using an interpolation approach on the surface that will be more accurate and precise. Besides that, future studies can also employ the quartic version by using B-spline and NURBS modeling.

Nomenclature:

Abbreviations and the variables

FS – Fuzzy Set

IFS – Intuitionistic Fuzzy Set

NS – Neutrosophic Set, $\hat{A}$

NP – Neutrosophic Point, $\hat{B}_i$

NR – Neutrosophic Relation, NR

NPR – Neutrosophic Point Relation, $\hat{R}$

NCP – Neutrosophic Control Point, $\hat{P}_i$

NCPR – Neutrosophic Control Points Relation, $\left\{\hat{p}_0^{T,I,F}, \hat{p}_1^{T,I,F}, ..., \hat{p}_n^{T,I,F}\right\}$

NQBC – Neutrosophic Quartic Bézier Curve, $BC(t)$

Variables

T - Truth Membership Degree

I - Indeterminacy Membership Degree

F - Falsity Membership Degree

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Received: 15 Jun 2023, **Revised:** 30 Sep 2023,

Accepted: 17 Oct 2023, **Available online:** 19 Oct 2023.

Neutrosophic Systems with Applications, Vol. 11, 2023
https://doi.org/10.61356/j.nswa.2023.90

ADM: Appraiser Decision Model for Empowering Industry 5.0-Driven Manufacturers toward Sustainability and Optimization: A Case Study

Gawaher Soliman Hussein [1], Abdel Nasser H. Zaied [2] and Mona Mohamed [3,*]

[1] Faculty of Computers and Informatics, Zagazig University, Zagazig 44519, Egypt; Gawahersoliman@zu.edu.eg.
[2] Misr International University, Information Systems Department, Egypt; Abdelnasser.riad@miuegypt.edu.eg.
[3] Higher Technological Institute, 10th of Ramadan City 44629, Egypt; mona.fouad@hti.edu.eg.

* Correspondence: mona.fouad@hti.edu.eg.

Abstract: Whilst it was thought that Industry 4.0 (I 4.0) would support sustainable growth, it overlooked or misinterpreted many current sustainability issues, which gave rise to the Industry 5.0 (I 5.0) agenda. Such a revolution facilitates sustainable development through its three dimensions. Therefore I 5.0 promotes more effective management of business environment as supply chain resources. Although artificial intelligence (AI) and big data analytics (BDA) are becoming more well-liked in the context of supply chains, research to this day is fragmented into research streams that are mostly determined by the publishing outlet. This study appraises the ability of these techniques in manufacturing enterprises toward sustainability based on a set of criteria. Hence, we identified the criteria which related to AI and BDA. The various techniques as entropy and weighted sum models of multi-criteria decision-making (MCDM) techniques are working under the authority of single values neutrosophic sets (SVNSs) to enhance and boost these techniques in uncertain situations. The constructed appraiser decision model (ADM) is applied to real enterprises to validate this model.

Keywords: Industry 5.0; Artificial Intelligence; Big Data Analytics; Sustainability; Single Values Neutrosophic Sets.

1. Introduction

Shocks from the outside world go beyond our prior experiences and have major repercussions, which might change the competitive environment in which firms compete. The pandemic caused by the COVID-19 virus has been described as a shock, and since it first appeared, it has been responsible for a considerable number of fatalities [1]. We have recently been witness to a variety of negative repercussions and company failures within the realm of commerce. Some examples of these include layoffs, firm closures, and bankruptcies. These effects were, to a significant degree, the result of the adoption of needed social distancing measures to reduce the transmission of the virus, which had a severe influence on the profitability and sustainability of various enterprises [2].

A significant aspect of this external shock is that there will be a significant increase in the amount of uncertainty that exists within the framework of operations and supply chains in particular. This has, in many instances, been noticed as a result of the widespread broadcast of fake news, which has resulted in additional disruption for companies and day-to-day life, to the extent that it has led to what has been referred to as an "infodemic," which is spreading via internet and mainstream media. This information epidemic has influenced consumer behavior, in which customers have resorted to panic purchasing and stockpiling of medical, cleaning, and non-perishable goods, driven by the worry that products may become unavailable. It should not come as a surprise that this sudden shift in consumer behavior has, in turn, resulted in disruptions in the supply chain. This is because companies are attempting to alter their supply chain and operations in order to deal with and foresee the shift in demand [3].

Because, ultimately, interruptions are perceived as possible hazards that need to be foreseen and managed against, risk management is generally considered as a lens through which such disruptions are viewed when viewed from the standpoint of operations [4]. To be more specific, in terms of the repercussions that are caused by such disinformation and media hype, supply chain experts are expected to balance the risk of prospective stock-outs against the risk of keeping supplies of the product. In point of fact, subsequent studies have shown that the "bullwhip effect," which was caused by the spread of false information about Covid-19, swiftly led to an excess of inventory, hoarding, and significant problems with the management of inventories.

Businesses generally develop business continuity plans in addition to risk management methods as a means of mitigating the effects of interruptions in order to respond to issues of this kind. The deployment of vendor-managed inventory contractual agreements and the creation of leagile supply chains that boost the performance of the company in spite of uncertainties are two typical concrete techniques that serve as a precaution against such risks and are examples of typical risk mitigation measures. However, research has demonstrated that new technologies, such as artificial intelligence and corporate data analytics among others, are essential to ensuring the continuation of a firm, particularly in the face of external shocks. These days, supply chains are made better by sensors and actuators like RFIDs, GPS and POS, tags, and other smart devices. Since all of these things transmit and receive data, the Internet of Things has the potential to be an avenue via which accurate predictions may be made.

To this day, there is an ever-expanding interest in the usage and application of artificial intelligence (AI) and big data analytics (BDA) for risk management and establishing and sustaining resilience in supply chains. This interest can be seen in both the public and private sectors [5]. In spite of considerable curiosity, there are still certain areas that aren't completely understood. In a recent extensive assessment of supply chain resilience, the emphasis was on research carried out over the previous ten years. The study went into depth on the many kinds of disruptions, as well as their effect on the supply chain and recovery techniques for reducing them, while technology was looked at on a very abstract level. Other studies have concentrated on determining and categorizing the many AI approaches that are used for risk management, as well as assessing the various AI strategies that are employed as components of supply chain resilience.

This study is being driven by the overall research questions:

RQ1: What is the impact of AI and BDA on achieving a sustainable and resilient supply chain?

The answer to the previous question is in the following research question.

RQ2: What are the influenced criteria related to embracing AI and BDA in supply chain (SC) to gain competitive advantages toward sustainability?

RQ3: What are appraiser methods which volunteer for appraising enterprises based on influenced criteria extracted from embracing AI and BDA techniques?

2. Earlier Studies Related to our Scope

Herein, we exhibit various perspectives through conducting survey for previous studies which rely on embrace digital technologies in manufacturers' SC toward sustainability and to be resilience manufacturers.

2.1 Digital Technologies: Industry 5.0 a Paradigm

From perspective of [6] the focus on a human-centric approach, technological integration, cross-sector collaboration, and a shared goal of using technology for a better future have all been key inspirations for Industry 5.0 (I 5.0), which has drawn heavily from Society 5.0. in same vein [7] argued that in order to advance industrial productivity and socio-environmental values, I 5.0 should expand on several aspects of I 4.0, such as the widespread adoption of disruptive technical breakthroughs.

According to prior studies, I 5.0 aims focus on set of principles. For instance, [8] where I 5.0 focus on human-centricity through striking a balance between the use of digital technology to specifically adapt corporate operations to the demands of employees and the adaptation of human resources to the digitalization of society. This value aim suggests that rather than the opposite, digital technology should benefit society. Another principle entailed in circularity in [9] through embracing techniques as AI, digital twin (DT), BDA, etc. which already have the ability to encourage resource efficiency, reduce waste, make it easier to integrate greener energy, and promote cleaner manufacturing facilities.

Hence, we focused on studying the extent influence of embracing technologies of I 5.0 in manufacturing process and operation toward sustainable and resilience manufacturer.

2.2 Sustainable Manufacturer Based on Industry 5.0

Technologies of I 5.0 are contributing to achieve sustainability of manufacturer through covering various directions as mentioned in [10] and exhibits in Figure 1.

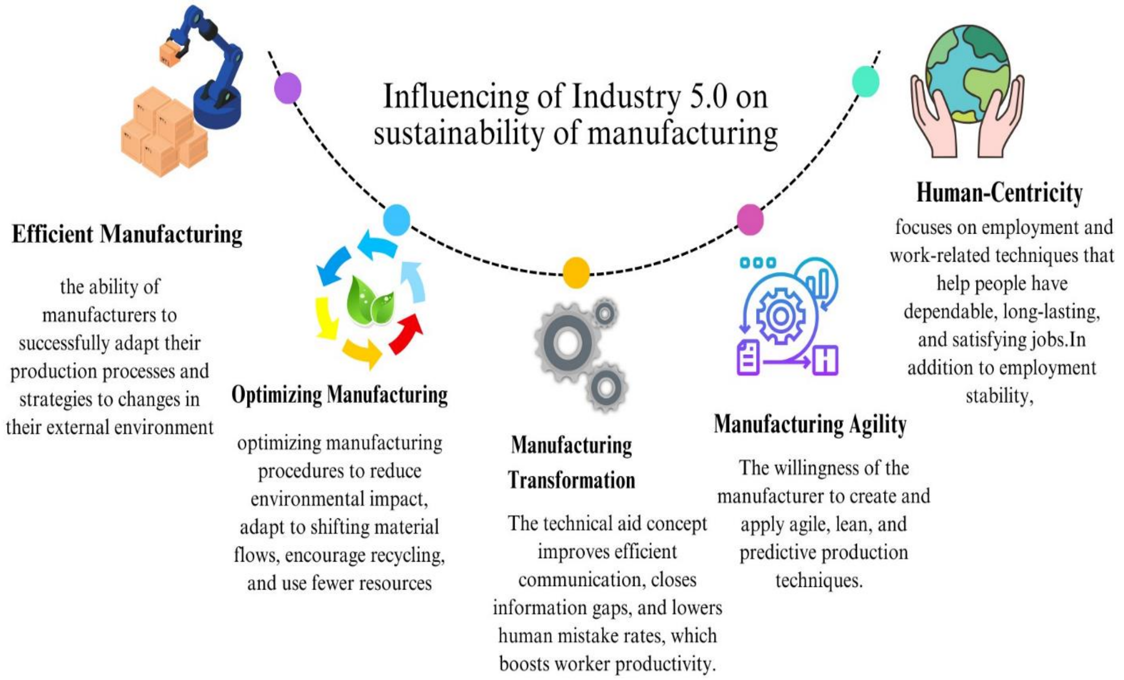

Figure 1. Role of Industry 5.0 on sustainability of manufacturing.

According to the survey conducted, appraising process for manufacturing's sustainability is vital. This process is conducting for manufacturers which embrace the notion of I 5.0 technologies in its chain whether inside and outside partners also, in its operations.

Herein, we focus on provide suitable methodology to appraise these manufacturers. Therefore, we constructed appraiser model in next section for making suitable decision for determining the most sustainable digital manufacturer. In this study, multi-criteria decision-making (MCDM) techniques have been volunteered under authority of uncertainty theory (i.e., neutrosophic) to appraise the alternatives of digital manufacturers and recommend optimal one to be sustainable digital manufacturers.

3. Appraisal Decision Model

Herein, we showcase methodology for appraising the manufacturers which embrace BDA-AI techniques whether inside or outside its chain. The appraisal process has been conducted for nominees of enterprises based on a set of criteria. Whereas the appraisal of enterprises is influenced by several direct and indirect factors, just as with a typical decision-making problem. Thereby, MCDM techniques are adopted and bolstered by uncertainty theory referred to neutrosophic theory to bolster MCDM techniques' capacity to cope with ambiguous situations and in complete data Process. Hence, this study mingles SVNSs as subset of neutrosophic theory with the Best Worst Method (BWM) - the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) as techniques of MCDM to generate ADM.

Consequently, the appraisal process in this study divides into set of stages:

Stage 1: *Insightful survey*.

This stage entails the vital data that is collected through various methods such as field expeditions and conducted questionnaires for enterprises.

Firstly, we identify the most influential criteria based on prior studies.

Secondly, we prepared questionnaires in order to rate the identified criteria via managers and experts who related to our search scope.

Stage 2: *Calculation identified criteria's weights.*

Valuation of the identified criteria is an important stage and achieve through calculating criteria's weights. Herein, we are employing entropy technique to work under SVNSs as branch of neutrosophic theory for generating criteria's weights through following set of steps:

- Different neutrosophic decision matrices have been constructed based on preferences for each member of panel based on scale is listed in [11].
- These neutrosophic matrices are transforming into crisp matrices through employing Eq. (1).

$$\boldsymbol{s}(\mathrm{Q_{ij}}) = \frac{(2+\mathrm{Tr}-\mathrm{Fl}-\mathrm{Id})}{3} \tag{1}$$

  Where Tr, Fl, Id refers to truth, false, and indeterminacy respectively.

- These matrices are volunteering Eq. (2) to aggregate it into single decision matrix.

$$\mathrm{Y_{ij}} = \frac{(\sum_{j=1}^{N} \mathrm{Q_{ij}})}{\mathrm{N}} \tag{2}$$

  Where Q_{ij} refers to value of criterion in matrix, N refers to number of decision makers.

- Eq. (3) is utilized to normalize the aggregated decision matrix.

$$\mathrm{Norm_{ij}} = \frac{\mathrm{y_{ij}}}{\sum_{j=1}^{m} \mathrm{y_{ij}}} \tag{3}$$

  Where $\sum_{j=1}^{m} \mathrm{y_{ij}}$ represents sum of each criterion in aggregated matrix per column.

- We are computing entropy based on Eq. (4).

$$\mathrm{e_{j}} = -\mathrm{h} \sum_{i=1}^{m} \mathrm{Norm_{ij}} \ln \mathrm{Norm_{ij}} \tag{4}$$

$$h = \frac{1}{\ln(m)} \tag{5}$$

  M refers to number of alternatives.

- Compute weight vectors through employing Eq. (6).

$$\mathrm{w_j} = \frac{1-e_j}{\sum_{j=1}^{n}(1-e_j)} \tag{6}$$

Stage 3: *Recommending optimal digital manufacturer.*

In this stage, we are merging weighted sum model (WSM) with SVNSs for achieving the purpose of this stage through recommending the optimal digital manufacturer. So, we follow set of steps for achieving this purpose.

- The aggregated decision matrix which is generated for previous stage is normalized according to following Eqs.

$$Norm_{Agg\_matij} = \frac{y_{ij}}{sum(y_{ij})} \quad ,For\ Benficial\ key\ indicators \tag{7}$$

$$Z = \frac{1}{y_{Ij}} \tag{8}$$

$$Norm_{Agg\_matij} = \frac{Z}{sum(Z)} \quad ,For\ Non-Benficial\ key\ indicators \tag{9}$$

Where:

y_{ij} indicates to each element in the aggregated matrix.

- The obtained key indicators' weights of entropy technique are applied in the following Eq. (10) to generate weighted matrix.

$$w\_matrix_{ij} = weight_i * Norm_{Agg_{matij}} \quad (10)$$

Where:

w_matrix_{ij} is weighted decision matrix.

- Utilizing Eq. (11) contributes to calculate global score. Based on values of $V(w\_matrix_{ij})$, ranking process for alternatives of perform and obtain optimal and worst manufacturer.

$$V(w\_matrix_{ij}) = \sum_{j=1}^{n} w\_matrix_{ij} \quad (11)$$

Where:

$V(w\_matrix_{ij})$ is global score values.

4. Empirical case study

We are applying our constructed ADM on real case study to ensure its validity. We communicated with manufacturing enterprises that related and embraced our study's notion. These manufacturing enterprises are in Egypt with different activities. The first manufacturing enterprise (ME1) that Medical and prosthetic devices, the second manufacturing enterprise (ME2) produces textile products, the third manufacturing enterprise (ME3) Produces electrical appliances and the manufacturing enterprise (ME4) is responsible for cable production.

We applied the mentioned stages of ADM to these enterprises manufacturing as follows:

Firstly, we identified the most influenced criteria which related to AI and BDA techniques toward the sustainability of these manufacturing. These criteria are summarized in the following Table 1 based on the study of [12].

Table 1. Influenced Criteria based on utilization of Big Data Analytics and Artificial Intelligence.

Criteria	Description
Proactivity (C_1)	Enterprises can use real-time crucial information made available by BDA to take corrective action.
Enhancing manufacturer performance (C_2)	The ability of AI to process information may be used to enhance and improve the manufacturing process [13].
Transparency (C_3)	All manufacturer's partners in its chain have ability for accessing to information [14].
Accurate forecasting(C_4)	Applying various AI techniques for analyzing real-time and historical data to forecast future behavior.
Vicinity to clients and suppliers (C_5)	Using agent-based simulation, AI techniques are being used to manage urban freight transportation [15].

After that, decision makers are contributing to rate the four alternatives based on the identified criteria based on scale in Ref [11].

Secondly, Entropy is utilized with support of SVNSs to generate vector of criteria's weights through applying several of equations are listed in following steps.

Step 1: Three neutrosophic decision matrices are created based on preferences for three decision makers.

Step 2: The constructed neutrosophic decision matrices transformed into deneutrosophic decision matrices based on Eq. (1).
Step 3: We aggregated these matrices into single deneutrosopic decision matrix through Eq. (2) as listed in Table 2.
Step 4: We are employing Eq. (3) to normalize the aggregated decision matrix to generate Table 3.
Step 5: We calculate entropy (e_j) through utilizing Eq. (4) to generate Table 4 and vector weight's criteria are produced in Figure 2. According to this Figure we noticed that C_1 is the highest criterion with highest value of weight followed by C_5 while C_2 is least one.

Table 2. Aggregated decision matrix.

	C_1	C_2	C_3	C_4	C_5
ME_1	0.6111	0.2611	0.8056	0.5389	0.6667
ME_2	0.5222	0.5000	0.7500	0.1611	0.7611
ME_3	0.6389	0.7389	0.4333	0.5944	0.4278
ME_4	0.6000	0.5222	0.3667	0.6333	0.5611

Table 3. Normalized decision matrix based on entropy-SVNSs.

	C_1	C_2	C_3	C_4	C_5
ME_1	0.2576	0.1291	0.3420	0.2795	0.2759
ME_2	0.2201	0.2473	0.3184	0.0836	0.3149
ME_3	0.2693	0.3654	0.1840	0.3084	0.1770
ME_4	0.2529	0.2582	0.1557	0.3285	0.2322

Table 4. Calculation of entropy.

	C_1	C_2	C_3	C_4	C_5
ME_1	-0.3494	-0.2643	-0.3669	-0.3563	-0.3553
ME_2	-0.3332	-0.3455	-0.3644	-0.2074	-0.3639
ME_3	-0.3533	-0.3679	-0.3115	-0.3628	-0.3065
ME_4	-0.3477	-0.3496	-0.2895	-0.3657	-0.3390
$\sum_{i=1}^{m} \mathbf{Norm_{ij}}$	-1.3836	-1.3273	-1.3323	-1.2922	-1.3647
$-h\sum_{i=1}^{m} \mathbf{Norm_{ij}} \ln \mathbf{Norm_{ij}}$	0.3459	0.3318	0.3331	0.3231	0.3412

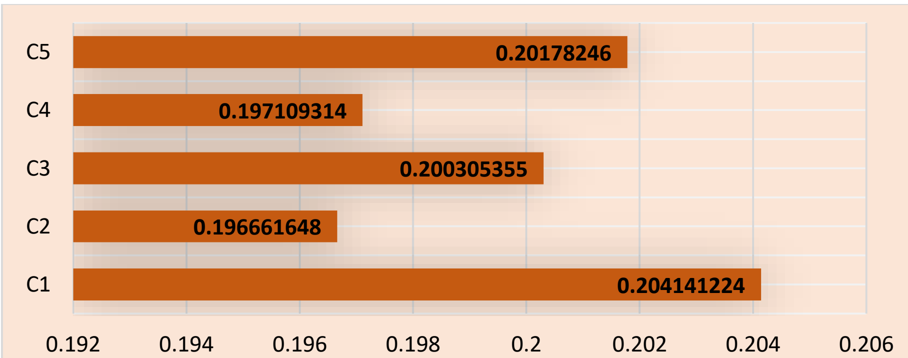

Figure 2. Weights of criteria based on Entropy-SVNSs.

Thirdly, WSM based on SVNSs is volunteering to recommend the most sustainable digital manufacturer to be the optimal one.

Step 1: We normalize the aggregated matrix in Table 2 through utilizing Eq. (7) and Table 5 is generated as normalized matrix.

Step 2: Eq. (10) plays vital role to generated weighted decision matrix through multiply entropy's weights by normalized matrix has been showcased in Table 6.

Step 3: The alternatives (An) are rating according to Eq. (11) through calculating global score for each alternative and recommend optimal one. According to Figure 3, we concluded that ME_2 otherwise ME_3.

Table 5. Normalized decision matrix based on WSM-SVNSs.

	C_1	C_2	C_3	C_4	C_5
ME_1	0.3454	0.2310	0.4028	0.2438	0.1549
ME_2	0.3775	0.2986	0.2000	0.1875	0.3216
ME_3	0.1606	0.2535	0.2000	0.2750	0.1549
ME_4	0.1165	0.2169	0.1972	0.2938	0.3685

Table 6. Weighted decision matrix based on WSM-SVNSs.

	C_1	C_2	C_3	C_4	C_5
ME_1	0.4778	0.4556	0.8056	0.4333	0.3667
ME_2	0.5222	0.5889	0.4000	0.3333	0.7611
ME_3	0.2222	0.5000	0.4000	0.4889	0.3667
ME_4	0.1611	0.4278	0.3944	0.5222	0.8722

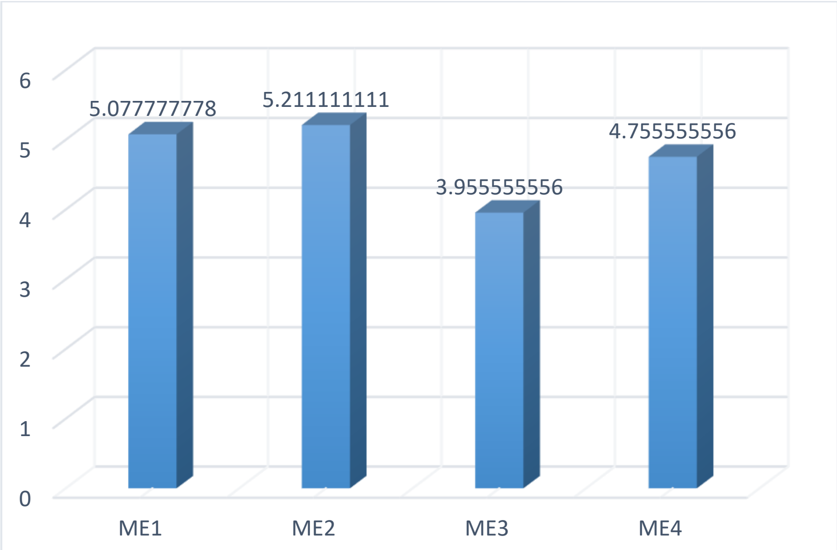

Figure 3. Ranking alternatives of manufacturers based on WSM-SVNSs.

5. Conclusions

In this study, we attempted to answer the question of why I 5.0 is important for SC especially the manufacturer as a partner in SC. Also, this study attempted to highlight the role of I 5.0 during disruptions in the business environment as the COVID-19 pandemic. Hence, the earlier studies endeavored to illustrate how I 5.0 agenda may help manufacturing be more sustainable.

Manufacturing enterprises are always under pressure to change their production methods in order to compete in the market. Using technology such as BDA in SC, especially manufacturer cases supports it to enhance decision making and support it to be proactive through analyzing real-time data and historical to predict what will happen and recognize what will be done and suitable actions.

Herein, we constructed ADM for appraising the manufacturing enterprises. The criteria in this process are vital factors in the appraisal. So, we determined the most influenced criteria which related to AI-BDA as technologies of I 5.0. In ADM MCDM techniques are supported by uncertainty theory where each technique has a vital function. For instance, Entropy is merged with SVNSs to generate a vector of criteria's weights. The results from these techniques indicated that C1 has the highest weight value of 0.204 while C2 is the lowest one with a value of 0.196.

The generated vector of weights contributes to recommending optimal and sustainable ME. In this stage, WSM under SVNSs are utilized for rating alternatives (ME n) which embrace BDA and AI techniques. The results from these techniques indicated that ME2 is optimal one and other manufacturers are ranking as ME2>ME1>ME4>ME3

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Received: 02 June 2023, **Revised:** 05 Oct 2023,

Accepted: 12 Oct 2023, **Available online:** 20 Oct 2023.

https://doi.org/10.61356/j.nswa.2023.72

Some Types of Neutrosophic Filters in Basic Logic Algebras

A. Ibrahim [1,*], S. Karunya Helen Gunaseeli [2] and Said Broumi [3,4]

[1] Assistant Professor, P.G. and Research Department of Mathematics, H.H. the Rajah's College, Pudukkottai, Affiliated to Bharathidasan University, Trichirappalli, Tamilnadu, India; dribrahimaadhil@gmail.com.

[2] Research Scholar, P.G. and Research Department of Mathematics, H.H. the Rajah's College, Pudukkottai, Affiliated to Bharathidasan University, Trichirappalli, Tamilnadu, India; karjes821@gmail.com.

[3] Laboratory of Information Processing, Faculty of Science Ben M'Sik, University of Hassan II, Casablanca, Morocco; s.broumi@flbenmsik.ma.

[4] Regional Center for the Professions of Education and Training (C.R.M.E.F), Casablanca- Settat, Morocco.

* Correspondence: dribrahimaadhil@gmail.com.

Abstract: The purpose of this article is to study the neutrosophication of prime and Boolean filters in Basic Logic (BL) algebras. We establish the notions of the neutrosophic prime and Boolean filters of BL-algebras with suitable examples and examine a few of their properties. As a result, we can determine the necessary and sufficient conditions and extension properties of both the neutrosophic prime and Boolean filters of BL-algebras. We obtain, C is a neutrosophic prime filter if and only if $\mathrm{T_C}(\mathrm{g_1} \to \mathrm{h_1}) = \mathrm{T_C}(1)$ or $\mathrm{T_C}(\mathrm{h_1} \to \mathrm{g_1}) = \mathrm{T_C}(1)$, $\mathrm{I_C}(\mathrm{g_1} \to \mathrm{h_1}) = \mathrm{I_C}(1)$ or $\mathrm{I_C}(\mathrm{h_1} \to \mathrm{g_1}) = \mathrm{I_C}(1)$, $\mathrm{F_C}(\mathrm{g_1} \to \mathrm{h_1}) = \mathrm{F_C}(1)$ or $\mathrm{F_C}(\mathrm{h_1} \to \mathrm{g_1}) = \mathrm{F_C}(1)$. Also, we prove $\mathrm{C_2}$ is a neutrosophic Boolean filter if $\mathrm{C_1} \subseteq \mathrm{C_2}$ and $\mathrm{T_{C_1}}(1) = \mathrm{T_{C_2}}(1)$, $\mathrm{I_{C_1}}(1) = \mathrm{I_{C_2}}(1)$, $\mathrm{F_{C_1}}(1) = \mathrm{F_{C_2}}(1)$,where $\mathrm{C_1}$ is a neutrosophic Boolean filter and C_2 is a neutrosophic filter. In addition, by combining both filters we instigate the concept of the neutrosophic prime Boolean filter of BL-algebras with illustration. In the future, the above study can be extended to soft multiset. Moreover, these filters can be applied to various digital image processing techniques.

Keywords: Basic Logic Algebra; Neutrosophic Filter; Neutrosophic Prime Filter; Neutrosophic Boolean Filter; Neutrosophic Prime Boolean Filter.

1. Introduction

The neutrosophic set was first introduced by Smarandache [1] in 1998, and its central idea is to explain the conception of 'uncertainty' using three mutually independent features. The neutrosophic set is now receiving a lot of attention for its potential to resolve a variety of real-world issues, including uncertainty and indeterminacy. Many novel neutrosophic theories [1, 2] such as the neutrosophic cubic, rough, and soft sets, are also put forth. The algebraic characteristics of the truth-value structure of each many-valued logic serve as a unique identifier [3]. A residuated lattice [4] is a common algebraic construction. The most well-known classes of residuated lattices include Basic Logic (BL), MTL, MV-algebras, and others.

A logical system's structure can be investigated by applying filters with special properties, as is well known. Additionally, there is a significant impact of filter qualities on the algebraic structure properties. The authors [5] introduced the concept of neutrosophic filters in BL-algebras and investigated a few of their associated features in a few instances. Further, the authors [6] discussed many of its properties and extended them to neutrosophic fantastic filters.

By using the prime filters of BL-algebras, Hajek [7] demonstrated the completeness of BL-algebras. In BL-algebras, Turunen [8] proposed the idea of Boolean filters. S. Yahya Mohamed and P.

Umamaheshwari [9] introduced the vague prime and Boolean filter of BL-algebras. However prime and Boolean filters in neutrosophic sets have not been studied so far. This motivated the authors to develop this article. In this article, we explore the ideas of neutrosophic prime and Boolean filters in BL-algebras and a few of their characteristics.

Our major contributions:

- In Section 2, a literature review of a few definitions and concepts regarding the neutrosophic set and filter of BL-algebras is conferred.
- In Section 3, we explore the idea of a neutrosophic prime filter and its features.
- In Section 4, we illustrate the idea of a neutrosophic Boolean and prime Boolean filters with examples.

2. Preliminaries

In this part, a few of the definitions and findings from the literature are referred to evolve the major conclusions.

Definition 2.1: [10, 11] A BL-algebra $(\mathcal{G}, \vee, \wedge, \circ, \rightarrow, 0,1)$ of type (2, 2, 2, 2, 0, 0) such that the subsequent requirements are persuaded for all $g_1, h_1, i_1 \in \mathcal{G}$,

(i) $(\mathcal{G}, \vee, \wedge, 0,1)$ is a bounded lattice,

(ii) $(\mathcal{G}, \circ, 1)$ is a commutativemonoid,

(iii) $'\circ'$ and $'\rightarrow'$form an adjointpair, that is, $i_1 \leq g_1 \rightarrow h_1$if and only if $g_1 \circ i_1 \leq h_1$

(iv) $g_1 \wedge h_1 = g_1 \circ (g_1 \rightarrow h_1)$,

(v) $(g_1 \rightarrow h_1) \vee (h_1 \rightarrow g_1) = 1$.

Proposition 2.2: [8, 10] the succeeding requirements are persuaded in BL-algebra $\mathcal{G}$ for all $g_1, h_1, i_1 \in \mathcal{G}$,

(i) $h_1 \rightarrow (g_1 \rightarrow i_1) = g_1 \rightarrow (h_1 \rightarrow i_1) = (g_1 \circ h_1) \rightarrow i_1$,

(ii) $1 \rightarrow g_1 = g_1$,

(iii) $g_1 \leq h_1$ if and only if $g_1 \rightarrow h_1 = 1$,

(iv) $g_1 \vee h_1 = ((g_1 \rightarrow h_1) \rightarrow h_1) \wedge ((h_1 \rightarrow g_1) \rightarrow g_1)$,

(v) $g_1 \leq h_1$ implies $h_1 \rightarrow i_1 \leq g_1 \rightarrow i_1$,

(vi) $g_1 \leq h_1$ implies $i_1 \rightarrow g_1 \leq i_1 \rightarrow h_1$,

(vii) $g_1 \rightarrow h_1 \leq (i_1 \rightarrow g_1) \rightarrow (i_1 \rightarrow h_1)$,

(viii) $g_1 \rightarrow h_1 \leq (h_1 \rightarrow i_1) \rightarrow (g_1 \rightarrow i_1)$,

(ix) $g_1 \leq (g_1 \rightarrow h_1) \rightarrow h_1$,

(x) $g_1 \circ (g_1 \rightarrow h_1) = g_1 \wedge h_1$,

(xi) $g_1 \circ h_1 \leq g_1 \wedge h_1$

(xii) $g_1 \rightarrow h_1 \leq (g_1 \circ i_1) \rightarrow (h_1 \circ i_1)$,

(xiii) $g_1 \circ (h_1 \rightarrow i_1) \leq h_1 \rightarrow (g_1 \circ i_1)$,

(xiv) $(g_1 \rightarrow h_1) \circ (h_1 \rightarrow i_1) \leq g_1 \rightarrow i_1$,

(xv) $(g_1 \circ g_1^*) = 0$.

Note. In the above sequence, $\mathcal{G}$ is used to intend the BL-algebras, and the operations$'\vee'$, $'\wedge'$,$'\circ'$ have preference on the way to the operations $'\rightarrow'$.

Note. In a BL-algebra $\mathcal{G}$, $'*'$is a complement defined as $g_1^* = g_1 \rightarrow 0$ for all $g_1 \in \mathcal{G}$.

Definition 2.3: [12, 13] A neutrosophic subset C of the universe U is a triple (T_C, I_C, F_C) where $T_C: U \rightarrow [0,1]$, $I_C: U \rightarrow [0,1]$ and $F_C: U \rightarrow [0,1]$ represents truth membership, indeterminacy and false membership functions, respectively where $0 \leq T_C(g_1) + I_C(g_1) + F_C(g_1) \leq 3$, for all $g_1 \in U$.

Definition 2.4: [5] A neutrosophic set C of a BL-algebra $\mathcal{G}$ is called a neutrosophic filter, if it persuades the following:

(i) $T_C(g_1) \leq T_C(1)$, $I_C(g_1) \geq I_C(1)$ and $F_C(g_1) \geq F_C(1)$,

(ii) min $\{T_C(g_1 \to h_1), T_C(g_1)\} \leq T_C(h_1)$, min$\{I_C(g_1 \to h_1), I_C(g_1)\} \geq I_C(h_1)$ and min $\{F_C(g_1 \to h_1), F_C(g_1)\} \geq F_C(h_1)\}$ for all $g_1, h_1 \in \mathcal{G}$.

Proposition 2.5: [5] Let C be a neutrosophic set of BL-algebras $\mathcal{G}$. C is a neutrosophic filter of $\mathcal{G}$ if and only if

(i) If $g_1 \leq h_1$ then $T_C(g_1) \leq T_C(h_1)$, $I_C(g_1) \geq I_C(h_1)$ and $F_C(g_1) \geq F_C(h_1)$,

(ii) $T_C(g_1 \circ h_1) \geq$ min $\{T_C(g_1), T_C(h_1)\}$, $I_C(g_1 \circ h_1) \leq$ min$\{I_C(g_1), I_C(h_1)\}$ and $F_C(g_1 \circ h_1) \leq$ min $\{F_C(g_1), F_C(h_1)\}$ for all $g_1, h_1 \in \mathcal{G}$.

Proposition 2.6: [5, 6] Let C be a neutrosophic set of BL-algebras $\mathcal{G}$. Let C be a neutrosophic filter of $\mathcal{G}$ for all $g_1, h_1, i_1 \in \mathcal{G}$ then the following hold.

(i) $T_C(g_1 \to h_1) = T_C(1)$, then $T_C(g_1) \leq T_C(h_1)$, $I_C(g_1 \to h_1) = I_C(1)$, then $I_C(g_1) \geq I_C(h_1)$, $F_C(g_1 \to h_1) = F_C(1)$, then $F_C(g_1) \geq F_C(h_1)$

(ii) $T_C(g_1 \wedge h_1) = \min\{T_C(g_1), T_C(h_1)\}$, $I_C(g_1 \wedge h_1) = \min\{I_C(g_1), I_C(h_1)\}$, $F_C(g_1 \wedge h_1) = \min\{F_C(g_1), F_C(h_1)\}$

(iii) $T_C(g_1 \circ h_1) = \min\{T_C(g_1), T_C(h_1)\}$, $I_C(g_1 \circ h_1) = \min\{I_C(g_1), I_C(h_1)\}$, $F_C(g_1 \circ h_1) = \min\{F_C(g_1), F_C(h_1)\}$

(iv) $T_C(0) = \min\{T_C(g_1), T_C(g_1{}^*)\}$, $I_C(0) = \min\{I_C(g_1), I_C(g_1{}^*)\}$, $F_C(0) = \min\{F_C(g_1), F_C(g_1{}^*)\}$.

3. Neutrosophic Prime filter

In this segment, we put forward the concept of a neutrosophic prime filter and confer its features with illustrations.

Definition 3.1 Let C be a non-constant neutrosophic filter of a BL-algebra $\mathcal{G}$. C is called a neutrosophic prime filter, if $T_C(g_1 \vee h_1) \leq \min\{T_C(g_1), T_C(h_1)\}$,

$I_C(g_1 \vee h_1) \geq \min\{I_C(g_1), I_C(h_1)\}$,

$F_C(g_1 \vee h_1) \geq \min\{F_C(g_1), F_C(h_1)\}$ for all $g_1, h_1 \in \mathcal{G}$.

Example 3.2: Let $C = \{0, g_1, h_1, i_1, j_1, 1\}$. The binary operations are specified by Tables 1 and 2.

Table 1. '$\circ$' Operation.

$\circ$	**0**	g_1	h_1	i_1	j_1	**1**
0	0	0	0	0	0	0
g_1	0	h_1	h_1	j_1	0	g_1
h_1	0	h_1	h_1	0	0	h_1
i_1	0	j_1	0	i_1	j_1	i_1
j_1	0	0	0	j_1	0	j_1
1	0	g_1	h_1	i_1	j_1	1

Table 2. '$\to$' Operation.

$\to$	**0**	g_1	h_1	i_1	j_1	**1**
0	1	1	1	1	1	1
g_1	j_1	1	g_1	i_1	i_1	1
h_1	i_1	1	1	i_1	i_1	1
i_1	h_1	g_1	h_1	1	g_1	1
j_1	g_1	1	g_1	1	1	1
1	0	g_1	h_1	i_1	j_1	1

Consider $C = \{(0, [0.6,0.3,0.3]), (g_1, [0.5,0.3,0.3]), (h_1, [0.5,0.4,0.4]),$
$(i_1, [0.5,0.4,0.4]), (j_1, [0.5,0.4,0.4]), (1, [0.6,0.3,0.3])\}.$

It is evident that C assures the Definition 3.1.
Hence, C is a neutrosophic prime filter of $\mathcal{G}$.

Example 3.3: Let $D = \{0, g_1, h_1, i_1, j_1, 1\}$. The binary operations are specified by Tables 1 and 2.
Consider $D = \{(0, [0.6,0.3,0.3]), (g_1, [0.4,0.3,0.3]), (h_1, [0.5,0.4,0.4]),$
$(i_1, [0.5,0.4,0.4]), (j_1, [0.5,0.4,0.4]), (1, [0.6,0.3,0.3])\}.$

Here, D is not a neutrosophic prime filter of $\mathcal{G}$.

Since, $T_D(h_1) = 0.5 \nleq 0.4 = \min\{T_D(g_1), T_D(h_1)\}$.

Proposition 3.4: Let C be a non-constant neutrosophic prime filter of $\mathcal{G}$ if and only if,
$(T_C)_{T_C(1)} = \{g_1 / T_C(g_1) \geq T_C(1), g_1 \in \mathcal{G}\}$,
$(I_C)_{I_C(1)} = \{g_1 / I_C(g_1) \leq I_C(1), g_1 \in \mathcal{G}\}, (F_C)_{F_C(1)} = \{g_1 / F_C(g_1) \leq F_C(1), g_1 \in \mathcal{G}\}$ is a prime filter.
Proof: Let C be a neutrosophic prime filter of $\mathcal{G}$.
Obviously, $(T_C)_{T_C(1)} = \{g_1 / T_C(g_1) \geq T_C(1), g_1 \in \mathcal{G}\}$.
Since, C is a non-constant neutrosophic filter $T_C(0) \leq T_C(1)$.
That is $0 \notin (T_C)_{T_C(1)}$.
Hence, $(T_C)_{T_C(1)}$ is a prime filter.
Conversely, if $(T_C)_{T_C(1)}$ is a prime filter.
Then, $g_1 \to h_1 \in (T_C)_{T_C(1)}$ (or) $h_1 \to g_1 \in (T_C)_{T_C(1)}$ for $g_1, h_1 \in \mathcal{G}$.
This means that, $(g_1 \vee h_1) \to h_1 = g_1 \to h_1 \in (T_C)_{T_C(1)}$ (or)
$(g_1 \vee h_1) \to g_1 = h_1 \to g_1 \in (T_C)_{T_C(1)}$.
Then, $T_C\big((g_1 \vee h_1) \to h_1\big) = T_C(1)$.
From the Definition 2.4, we have

$$T_C(h_1) \geq T_C\big((g_1 \vee h_1) \to h_1\big) \wedge T_C(g_1 \vee h_1) = T_C(g_1 \vee h_1)$$
$$T_C(g_1) \geq T_C\big((g_1 \vee h_1) \to g_1\big) \wedge T_C(g_1 \vee h_1) = T_C(g_1 \vee h_1)$$
$$\text{Therefore, } T_C(g_1) \wedge T_C(h_1) \geq T_C(g_1 \vee h_1).$$

Similarly, we can prove for I_C , F_C.
Hence, C is a neutrosophic prime filter.
Proposition 3.5: Let C be non-constant neutrosophic filter of $\mathcal{G}$. C is a neutrosophic prime filter if and only if $T_C(g_1 \to h_1) = T_C(1)$ or $T_C(h_1 \to g_1) = T_C(1)$, $I_C(g_1 \to h_1) = I_C(1)$ or $I_C(h_1 \to g_1) = I_C(1)$, $F_C(g_1 \to h_1) = F_C(1)$ or $F_C(h_1 \to g_1) = F_C(1)$.
Proof: Let C be a non-constant neutrosophic filter of $\mathcal{G}$.
From the Proposition 3.4, C is a neutrosophic prime filter
if and only if $(T_C)_{T_C(1)}$ is a prime filter.
if and only if $g_1 \to h_1 \in (T_C)_{T_C(1)}$ (or) $h_1 \to g_1 \in (T_C)_{T_C(1)}$
if and only if $T_C(g_1 \to h_1) = T_C(1)$ (or) $T_C(h_1 \to g_1) = T_C(1)$.
Similarly, we can prove for I_C , F_C.
Proposition 3.6: Let C_1 be a non-constant neutrosophic prime filter of $\mathcal{G}$ and C_2 be a non-constant neutrosophic filter of $\mathcal{G}$. If $C_1 \subseteq C_2$, then $T_{C_1}(1) = T_{C_2}(1)$, $I_{C_1}(1) = I_{C_2}(1), F_{C_1}(1) = F_{C_2}(1)$ then C_2 is also a neutrosophic prime filter.

Proof: Let C_1 be a neutrosophic prime filter of $\mathcal{G}$.
Then, from the Proposition 3.5, $T_{C_1}(g_1 \to h_1) = T_{C_1}(1)$ or $T_{C_1}(h_1 \to g_1) = T_{C_1}(1)$ for all $g_1, h_1 \in \mathcal{G}$.
If $T_{C_1}(g_1 \to h_1) = T_{C_1}(1)$ by $C_1 \subseteq C_2$ and $T_{C_1}(1) = T_{C_2}(1)$, we have $T_{C_2}(g_1 \to h_1) = T_{C_2}(1)$. Similarly, if $T_{C_1}(h_1 \to g_1) = T_{C_1}(1)$, then $T_{C_2}(h_1 \to g_1) = T_{C_2}(1)$.
Similarly, it can be proved for I_{C_2}, F_{C_2}.
From the Proposition 3.5, we have C_2 is a neutrosophic prime filter.

4. Neutrosophic Boolean and Neutrosophic prime Boolean filters

In this segment, we put forward the notion of neutrosophic Boolean and prime Boolean filters and confer their features with illustrations.
Definition 4.1: Let C be a neutrosophic filter of $\mathcal{G}$. C is called a neutrosophic Boolean filter if $T_C(g_1 \vee g_1^*) = T_C(1)$, $I_C(g_1 \vee g_1^*) = I_C(1)$, $F_C(g_1 \vee g_1^*) = F_C(1)$ for all $g_1 \in \mathcal{G}$.

Example 4.2: Let $C = \{0, g_1, h_1, i_1, 1\}$. The binary operations are specified by Tables 3 and 4.
Consider $C = \{(0, [0.8,0.2,0.2]), (g_1, [0.8,0.2,0.2]), (h_1, [0.6,0.3,0.3]),$
$(i_1, [0.6,0.3,0.3]), (1, [0.8,0.2,0.2])\}$.
It is evident that C assures the Definition 4.1. Hence, C is a neutrosophic Boolean filter of $\mathcal{G}$.

Table 3.' ∘ 'Operation.

∘	**0**	g_1	h_1	i_1	**1**
0	0	0	0	0	0
g_1	0	g_1	i_1	i_1	g_1
h_1	0	i_1	h_1	i_1	h_1
i_1	0	i_1	i_1	i_1	i_1
1	0	g_1	h_1	i_1	1

Table 4.' → 'Operation.

→	**0**	g_1	h_1	i_1	**1**
0	1	1	1	1	1
g_1	0	1	h_1	h_1	1
h_1	0	g_1	1	g_1	1
i_1	0	1	1	1	1
1	0	g_1	h_1	i_1	1

Example 4.3: Let $D = \{0, g_1, h_1, i_1, 1\}$. The binary operations are specified by the Tables 3 and 4.
Consider $D = \{(0, [0.6,0.3,0.3]), (g_1, [0.8,0.2,0.2]), (h_1, [0.6,0.3,0.3]),$
$(i_1, [0.6,0.3,0.3]), (1, [0.8,0.2,0.2])\}$.
Here, D is not a neutrosophic Boolean filter of $\mathcal{G}$.
Because, $T_D(g_1 \vee g_1^*) = T_D(0) = 0.6 \neq 0.8 = T_D(1)$.
Proposition 4.4 Let C be a neutrosophic Boolean filter of $\mathcal{G}$if and only if
$T_C((g_1 \to g_1^*) \to g_1^*) = T_C((g_1^* \to g_1) \to g_1) = T_C(1)$,
$I_C((g_1 \to g_1^*) \to g_1^*) = I_C((g_1^* \to g_1) \to g_1) = I_C(1)$,
$F_C((g_1 \to g_1^*) \to g_1^*) = F_C((g_1^* \to g_1) \to g_1) = F_C(1)$for all $g_1 \in \mathcal{G}$.
Proof: Let C be a neutrosophic Boolean filter of $\mathcal{G}$.
From the Definition 4.1, we know that $T_C(g_1 \vee g_1^*) = T_C(1)$.
Then, by (iv) of the Proposition 2.2,
we have $T_C(g_1 \vee g_1^*) = T_C(((g_1 \to g_1^*) \to g_1^*) \wedge ((g_1^* \to g_1) \to g_1))$

$= T_C\big((g_1 \to g_1{}^*) \to g_1{}^*\big) \wedge T_C\big((g_1{}^* \to g_1) \to g_1\big)$ [From (ii) of the proposition 2.6]
$= T_C(1)$

So, $T_C\big((g_1 \to g_1{}^*) \to g_1{}^*\big) = T_C\big((g_1{}^* \to g_1) \to g_1\big) = T_C(1)$for all $g_1 \in \mathcal{G}$.
Similarly, we can prove for I_C , $F_{C.}$
Similarly, the converse part can be proved.

Proposition 4.5: Let $C_1 \subseteq C_2$ and $T_{C_1}(1) = T_{C_2}(1), I_{C_1}(1) = I_{C_2}(1), F_{C_1}(1) = F_{C_2}(1)$, where C_1 is a neutrosophic Boolean filter and C_2 is a neutrosophic filter. Then C_2 is a neutrosophic Boolean filter.
Proof: Let $C_1$and C_2 be two neutrosophic filters of $\mathcal{G}$.
If C_1 is neutrosophic Boolean filter, then $T_{C_1}(g_1 \vee g_1{}^*) = T_{C_1}(1)$ for all $g_1 \in \mathcal{G}$.
Since, $C_1 \subseteq C_2$ and $T_{C_1}(1) = T_{C_2}(1)$, it follows that $T_{C_2}(g_1 \vee g_1{}^*) \geq T_{C_2}(1)$.
From (i) of the Definition 2.4, we have$T_{C_2}(g_1 \vee g_1{}^*) \leq T_{C_2}(1)$.
Hence,$T_{C_2}(g_1 \vee g_1{}^*) = T_{C_2}(1)$.
Similarly, we can prove for I_{C_2}, F_{C_2}.
Thus, C_2 is a neutrosophic Boolean filter.

Proposition 4.6: Let C be a neutrosophic Boolean filter of $\mathcal{G}$ if it persuades,
$T_C(g_1) = T_C(g_1{}^* \to g_1), I_C(g_1) = I_C(g_1{}^* \to g_1), F_C(g_1) = F_C(g_1{}^* \to g_1)$for all$g_1 \in \mathcal{G}$.
Proof: Let C be a neutrosophic Boolean filter of $\mathcal{G}$.
By the Definition 2.4, $T_C(g_1{}^* \to g_1) \geq \min\{T_C(g_1 \to (g_1^* \to g_1), T_C(g_1)\}$
$= \min\{T_C(1), T_C(g_1)\}$[Since, $g_1^* = g_1 \to 0$]
$\geq T_C(g_1)$ and from the Definition 2.4,
$T_C(g_1) \geq \min\{T_C\big((g_1 \vee g_1^*) \to g_1\big), T_C(g_1 \vee g_1^*)\}$
$= \min\{T_C\big((g_1 \to g_1) \wedge (g_1^* \to g_1)\big), T_C(1)\}$
$= \min\{T_C(1 \wedge (g_1^* \to g_1)), T_C(1)\}$
Therefore, $T_C(g_1) \geq T_C(g_1{}^* \to g_1)$.
Then, $T_C(g_1) = T_C(g_1{}^* \to g_1)$for all$g_1 \in \mathcal{G}$.
Similarly,$I_C(g_1) = I_C(g_1{}^* \to g_1)$,$F_C(g_1) = F_C(g_1{}^* \to g_1)$.

Definition 4.7: A neutrosophic filter N is called a neutrosophic prime Boolean filter if it is both a neutrosophic Boolean filter and a neutrosophic prime filter. The set of all neutrosophic prime Boolean filters of $\mathcal{G}$ is denoted by $NPB(\mathcal{G})$.

Example 4.8: Consider the Example 3.2.
Then, from the Example 3.2 C is a neutrosophic prime filter.
Also, by the Definitions 4.1 and 4.7 it is evident that C is a neutrosophic Boolean and prime Boolean filters of $\mathcal{G}$ respectively.

Example 4.9 Let $D = \{0, g_1, h_1, i_1, j_1, 1\}$. The binary operations are specified by the Tables 1 and 2.
Consider $D = \{(0, [0.5,0.2,0.2]), (g_1, [0.3,0.2,0.2]), (h_1, [0.4,0.3,0.3]),$
$(i_1, [0.4,0.3,0.3]), (j_1, [0.4,0.3,0.3]), (1, [0.5,0.2,0.2])\}$.

Here, by the Definition 4.1 D is a neutrosophic Boolean filter of $\mathcal{G}$.

But D is not a neutrosophic prime filter of $\mathcal{G}$.Since,$T_D(h_1) = 0.4 \nleq 0.3 = \min\{T_D(g_1), T_D(h_1)\}$.

Hence, D is not a $NPB(\mathcal{G})$.

Proposition 4.10 Let C and D be two neutrosophic filters of $\mathcal{G}$. Let $C \subseteq D$ such that $N_C(1) = N_D(1)$. If C is a neutrosophic prime Boolean filter of $\mathcal{G}$ then so is D.

Proof: Let C be a neutrosophic prime Boolean filter of $\mathcal{G}$.

Since, C is a neutrosophic Boolean filter $N_C(g_1) = N_C(1)$ (or) $N_C(g_1^*) = N_C(1)$for all $g_1 \in \mathcal{G}$.

By $C \subseteq D$and$N_C(1) = N_D(1)$, we get $N_D(g_1) = N_D(1)$ (or) $N_D(g_1^*) = N_D(1)$

Hence, D is a neutrosophic Boolean filter.

Since, C is a neutrosophic prime filter $N_C(h_1 \to g_1) = N_C(1)$ (or) $N_C(g_1 \to h_1) = N_C(1)$ for all $g_1, h_1 \in \mathcal{G}$.

By $C \subseteq D$and $N_C(1) = N_D(1)$, we get $N_D(h_1 \to g_1) = N_D(1)$ (or) $N_D(g_1 \to h_1) = N_D(1)$

Hence, D is a neutrosophic prime filter. Therefore, D is a neutrosophic prime Boolean filter.

5. Discussion

The key findings from this article are as follows:

- C_2 is a neutrosophic prime filter of $\mathcal{G}$, if C_1 is a non-constant neutrosophic prime filter of $\mathcal{G}$ where $C_1 \subseteq C_2, T_{C_1}(1) = T_{C_2}(1),\ I_{C_1}(1) = I_{C_2}(1), F_{C_1}(1) = F_{C_2}(1)$. [Extension property]
- C is a neutrosophic Boolean filter of $\mathcal{G}$ if it persuades, $T_C(g_1) = T_C({g_1}^* \to g_1), I_C(g_1) = I_C({g_1}^* \to g_1),\ F_C(g_1) = F_C({g_1}^* \to g_1)$for all$g_1 \in \mathcal{G}$.
- Suppose C and D are two neutrosophic filters of $\mathcal{G}$ and $C \subseteq D$ such that $N_C = N_D(1)$where C is a neutrosophic prime Boolean filter of $\mathcal{G}$ then so is D. [Extension property]

6. Conclusions

In the current study, we have put forward the notions of the neutrosophic Boolean and prime filters of a BL-algebra and looked into a few associated features. Additionally, we have inspected a few necessary and adequate criteria for those filters. Also, we have acquired an extension property for both the neutrosophic Boolean and prime filters. Finally, by combining both filters, the notion of a neutrosophic prime Boolean filter is presented with examples. This work stands out in studying the characteristics of prime and Boolean filters in BL-algebras as it mainly concentrates on their neutrosophic nature. In the future, the above study can be extended to deductive, ultra, and transitive filters and used to rectify problems in many other fields. Also, these filters can be applied to various medical diagnoses and image-processing techniques.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Received: 25 May 2023, **Revised:** 25 Sep 2023,

Accepted: 12 Oct 2023, **Available online:** 20 Oct 2023.

Neutrosophic Systems with Applications, Vol. 11, 2023
https://doi.org/10.61356/j.nswa.2023.81

Similarity Measure of Plithogenic Cubic Vague Sets: Examples and Possibilities

S. Anitha [1,*] and A. Francina Shalini [2]

[1] Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore 641018, India; anitharaj.ps@gmail.com.

[2] Associate Professor, Department of Mathematics, Nirmala College for Women, Coimbatore 641018, India; francshalu@gmail.com.

* Correspondence: anitharaj.ps@gmail.com.

Abstract: The crisp, fuzzy, intuitionistic fuzzy and neutrosophic sets are the extension of the plithogenic set, in which elements are characterized by the number of attributes and each attribute can assume many values. To achieve more accuracy and precise exclusion, a contradiction or dissimilarity degree is specified between each attribute and the values of the dominating attribute. A plithogenic cubic vague set is a combination of a plithogenic cubic set and a vague set. The key tool for resolving problems with pattern recognition and clustering analysis is the similarity measure. In this research, we characterize and investigate the similarities between two Plithogenic Cubic Vague sets (PCVSs) for $(z \equiv F)$, $(z \equiv IF)$ and $(z \equiv N)$. Also, examples are given to examine similarities in the pattern recognition application problems.

Keywords: Plithogenic Set, Plithogenic Cubic Vague Set, Pattern Recognition, Similarity Measure.

1. Introduction

Zadeh introduced fuzzy set a mathematical theory to deal with uncertainties [1]. It is characterized by the membership value and sometimes it is difficult to assign the value for a fuzzy set. Interval-valued fuzzy set was introduced by Zadeh to overcome this problem. Intuitionistic fuzzy sets (IF) and interval-valued intuitionistic fuzzy sets introduced by Atanassov et al. [2,3] are appropriate to handle this situation. However, it is not enough to handle the unreliable information existing in the belief system. Zulkifli et al. [4] proposed the interval-valued intuitionistic fuzzy vague sets (IVIFVS). Florentin Smarandache [5] introduced a neutrosophic set and provided a mathematical tool to handle difficulties involving inconsistent and indeterminate data. New ideas on neutrosophic sets are introduced by Anitha et al. [6-8]. The interval-valued neutrosophic set was introduced by Jun Ye [9]. Hazwani Hashimcet et al. [10] proposed Interval Neutrosophic Vague Sets. Banik et al. [11,12] studied the MCGDM problem in a pentagonal neutrosophic environment and a novel integrated neutrosophic cosine operator-based linear programming. Haque et al. [13-15] elevated decision-making ideas in interval neutrosophic environment, generalized spherical fuzzy environment, and linguistic generalized spherical fuzzy environment.

The vague set was developed by Gau and Buehrer [16]. The idea of similarity measure of fuzzy sets was introduced by Wang [17] and gave a computational formula. Since then it has attracted many researchers. Fei et al. [18] introduced the similarity between two intuitionistic fuzzy sets. Similarity measures of neutrosophic sets were given by Broumi et al. [19]. Ali et al. [20] introduced neutrosophic cubic set-based decision-making. Shawkat Alkhazaleh [21] studied neutrosophic vague set in 2015. Similarities between vague sets were introduced by Chen S.M [22]. The idea of a cubic set was introduced by Jun [23]. The idea of a cubic vague set was introduced by Khaleed et al. [24] by incorporating a cubic set and a vague set. He also presented a decision-making method based on the similarity measure of cubic vague set.

Smarandache introduced plithogenic set and it may have elements characterized by four or more attributes [25]. A plithogenic multi-criteria decision-making approach to estimate the sustainable supply chain risk management based on order preference and criteria importance through the correlation method is proposed by Abdel and Rehab [26]. Alkhazaleh introduced plithogenic soft set and measured the similarity between two plithogenic soft sets using a set-theoretic approach [27]. Anitha et al. [28] introduced the idea of plithogenic cubic vague set.

In this paper, we introduce the concept of similarity measure between two Plithogenic Cubic Vague sets (PCVSs) $((z \equiv F), (z \equiv IF), (z \equiv N))$. It has the novelty to precisely characterize and model data for real-life occurrences. Since cubic set fails to capture the false membership part to measure the alternative in the decision making method. PCVS has the ability to handle uncertainties and vague information considering the truth and false membership values as the elements are characterized by one or more attributes therefore it is possible to describe the problem. One of the best tool to solve it is similarity measure. Similarity measure of PCVS is a vital concept for measuring entropy in the data. The flow of this paper is as follows. An algorithm to determine the similarities between two PCVS $((z \equiv F), (z \equiv IF), (z \equiv N))$ for a pattern recognition problem is proposed. To illuminate the proposed measure numerical examples are provided.

The organization of the paper is as follows: Section 2 provides some preliminaries for the proposed concept. Section 3 covers the application of the plithogenic cubic vague set and it is divided into three subsections. In 3.1 algorithm and examples of the plithogenic fuzzy cubic vague set, in 3.2 plithogenic intuitionistic fuzzy cubic vague set and in 3.3 plithogenic neutrosophic cubic vague set were presented. In Section 4 discussion is made for the proposed measure. Finally, Section 5 concludes this paper and provides the direction for future studies.

2. Preliminaries

Definition 2.1: [23] A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $A_{NV} = \{\langle x, \hat{T}_{A_{NV}}, \hat{I}_{A_{NV}}, \hat{F}_{A_{NV}} \rangle\, x \in X\}$ whose truth membership, indeterminacy membership and falsity membership functions are defined as $\hat{T}_{A_{NV}}(x) = [T^-, T^+]$, $\hat{I}_{A_{NV}}(x) = [I^-, I^+]$, $\hat{F}_{A_{NV}}(x) = [F^-, F^+]$, where $T^+ = 1 - F^-$, $F^+ = 1 - T^-$ and $0^- \le T^- + I^- + F^- \le 2^+$.

Definition 2.2: [16] An interval valued neutrosophic vague set A_{INV} also known as INVS in the universe of discourse E. An IVNVS is characterized by truth membership, indeterminacy membership and falsity membership functions is defined as:

$A_{INV} = \{< e, [\hat{V}_A^L(e), \hat{V}_A^U(e)], [\widehat{W}_A^L(e), \widehat{W}_A^U(e)], [\hat{X}_A^L(e), \hat{X}_A^U(e)] > | e \in E\}$,

$\hat{V}_A^L(e) = [V^{L^-}, V^{L^+}]$, $\hat{V}_A^U(e) = [V^{U^-}, V^{U^+}]$, $\widehat{W}_A^L(e) = [W^{L^-}, W^{L^+}]$, $\widehat{W}_A^L(e) = [W^{U^-}, W^{U^+}]$, $\hat{X}_A^L(e) = [X^{L^-}, X^{L^+}]$, $\hat{X}_A^U(e) = [X^{U^-}, X^{U^+}]$ where

$V^{L^+} = 1 - X^{L^-}$, $X^{L^+} = 1 - V^{L^-}$, $V^{U^+} = 1 - X^{U^-}$, $X^{U^+} = 1 - V^{U^-}$ and

$0^- \le V^{L^-} + V^{U^-} + W^{L^-} + W^{U^-} + X^{L^-} + X^{U^-} \le 4^+$, $0^- \le V^{L^+} + V^{U^+} + W^{L^+} + W^{U^+} + X^{L^+} + X^{U^+} \le 4^+$.

Definition 2.3: [2] Let U be a universal set. The set $\mathbb{A}_p{}^v = \{\langle x, A_V(x), \lambda_V(x)\rangle : x \in X\}$ is called plithogenic fuzzy cubic vague set in which A_V is an interval valued plithogenic fuzzy vague set in X and λ_V is the fuzzy vague set in X.

Definition 2.4: [2] Let U be a universal set. The set $\mathbb{A}_p{}^v = \{\langle x, A_V(x), \lambda_V(x)\rangle : x \in X\}$ is called plithogenic intuitionistic fuzzy cubic vague set in which A_V is an interval valued plithogenic intuitionistic fuzzy vague set in X and λ_V is the intuitionistic fuzzy vague set in X.

Definition 2.5: [2] Let U be a universal set. The set $\mathbb{A}_p{}^v = \{\langle x, A_V(x), \lambda_V(x)\rangle : x \in X\}$ is called plithogenic neutrosophic cubic vague set in which A_V is an interval valued plithogenic neutrosophic vague set in X and λ_V is the neutrosophic vague set in X.

3. Application of Plithogenic Cubic Vague sets in Pattern Recognition Problem

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation as well as the experimental conclusions that can be drawn.

Here, we introduce the concept of similarity measure between two Plithogenic Cubic Vague sets (PFCVSs) $(z \equiv F)$, PIFCVSs $(z \equiv IF)$, (PNCVSs $(z \equiv N)$ and further results on similarity measure. An example is given to exhibit the effectiveness of the proposed method.

3.1 Plithogenic Fuzzy Cubic Vague Set

Definition 3.1.1: Let $\mathbb{A}_{p_1}^v$ and $\mathbb{A}_{p_2}^v$ be any two Plithogenic Fuzzy Cubic Vague sets (PFCVSs). Then,

(1) $0 \le \left|S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v\right)\right| \le 1$,

(2) $S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v\right) = S\left(\mathbb{A}_{p_2}^v, \mathbb{A}_{p_1}^v\right)$,

(3) $S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v\right) = 1 \Leftrightarrow \mathbb{A}_{p_1}^v = \mathbb{A}_{p_2}^v$,

(4) $\mathbb{A}_{p_1}^v \subseteq \mathbb{A}_{p_2}^v \subseteq \mathbb{A}_{p_3}^v \Longrightarrow S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_3}^v\right) \le S\left(\mathbb{A}_{p_2}^v, \mathbb{A}_{p_3}^v\right)$

Definition 3.1.2: Let X $= \{x_1, x_2, x_3\}$, $\mathbb{A}_{p_1}^v = \langle A_V^1, \lambda_V^1\rangle$ and $\mathbb{A}_{p_2}^v = \langle A_V^2, \lambda_V^2\rangle$ be two Plithogenic Fuzzy Cubic Vague Sets (PFCVSs) in X. The similarity measure between $\mathbb{A}_{p_1}^v and\, \mathbb{A}_{p_2}^v$ is given by $S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v\right)$, where

$$S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v\right) = \frac{1}{6n}\sum_{i=1}^{n}\Big(\left|T_{A^1{}_V}^{L^-}(x_i) - T_{A^2{}_V}^{L^-}(x_i)\right| + \left|T_{A^1{}_V}^{U^-}(x_i) - T_{A^2{}_V}^{U^-}(x_i)\right| + \left|T_{A^1{}_V}^{L^+}(x_i) - T_{A^2{}_V}^{L^+}(x_i)\right| + \left|T_{A^1{}_V}^{U^+}(x_i) - T_{A^2{}_V}^{+}(x_i)\right| + \left|T_{\lambda^1{}_V}^{-}(x_i) - T_{\lambda^2{}_V}^{-}(x_i)\right| + \left|T_{\lambda^1{}_V}^{+}(x_i) - T_{\lambda^2{}_V}^{+}(x_i)\right|\Big)$$

Algorithm:

Step 1. Construct PFCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$ as ideal pattern.

Step 2. Then construct PFCVSs $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j\rangle$, $j = 1,2 \ldots n$ for sample patterns which are to be known.

Step 3. Compute the similarities between ideal pattern $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$ and the sample pattern $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j\rangle$ using definition 3.1.2.

Step 4. The sample pattern $\mathbb{A}_{p_j}^v$ is considered to belong to ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}_{p_j}^v\right) \le 0.5$ and sample pattern $\mathbb{A}_{p_j}^v$ is not to be known for an ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}_{p_j}^v\right) > 0.5$.

Example 3.1.3: Consider a simple pattern recognition problem involving three sample patterns and an ideal pattern. Let X $= \{x_1, x_2, x_3\}$. The patterns indicated as pattern 1, pattern 2 and pattern 3 are the selected three sample patterns, whereas pattern 4 is the selected ideal pattern. Also, let $\mathbb{A}_p{}^v$ be PFCVS set of ideal pattern and pattern $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j\rangle$ be the PFCVSs of three sample patterns.

Step 1. Create an ideal PFCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$ on X as,

$$\mathbb{A}_p{}^v = <\{\frac{\langle[0.4,0.6],[0.5,0.5]\rangle}{x_1}, \frac{\langle[0.3,0.8],[0.5,0.6]\rangle}{x_2}, \frac{\langle[0.2,0.6],[0.3,0.6]\rangle}{x_3}\},\{\frac{[0.2,0.5]}{x_1}, \frac{[0.4,0.7]}{x_2}, \frac{[0.3,0.6]}{x_3}\}>$$

Step 2. Construct PFCVSs $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j\rangle$, $j = 1,2,3$ for the sample patterns as;

$$\mathbb{A}_{p_1}^v = <\{\frac{\langle[0.2,0.6],[0.3,0.6]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.5,0.5]\rangle}{x_2}, \frac{\langle[0.3,0.8],[0.5,0.6]\rangle}{x_3}\},\{\frac{[0.1,0.7]}{x_1}, \frac{[0.3,0.7]}{x_2}, \frac{[0.1,0.9]}{x_3}\}>$$

$$\mathbb{A}_{p_2}^v = <\{\frac{\langle[0.1,0.7],[0.2,0.4]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.1,0.2]\rangle}{x_2}, \frac{\langle[0.3,0.8],[0.5,0.6]\rangle}{x_3}\},\{\frac{[0.5,0.6]}{x_1}, \frac{[0.1,0.7]}{x_2}, \frac{[0.1,0.5]}{x_3}\}>$$

$$\mathbb{A}_{p_3}^v = <\{\frac{\langle[0.3,0.8],[0.5,0.6]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.1,0.2]\rangle}{x_2}, \frac{\langle[0.4,0.6],[0.5,0.5]\rangle}{x_3}\},\{\frac{[0.1,0.9]}{x_1}, \frac{[0.1,0.7]}{x_2}, \frac{[0.3,0.7]}{x_3}\}>$$

Step 3. Compute S the degree of similarity between ideal pattern $\mathbb{A}_p{}^v$ and the sample pattern $\mathbb{A}_{p_j}^v$, then the results obtained are:

$$S(\mathbb{A}_p{}^v, \mathbb{A}_{p_1}^v) = 0.13$$
$$S(\mathbb{A}_p{}^v, \mathbb{A}_{p_2}^v) = 0.19$$
$$S(\mathbb{A}_p{}^v, \mathbb{A}_{p_3}^v) = 0.17$$

Step 4. $S(\mathbb{A}_p{}^v, \mathbb{A}_{p_1}^v) \leq 0.5$, $S(\mathbb{A}_p{}^v, \mathbb{A}_{p_2}^v) \leq 0.5$ and $S(\mathbb{A}_p{}^v, \mathbb{A}_{p_3}^v) \leq 0.5$, the sample pattern whose corresponding PFCVS sets are denoted as $\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v$ and $\mathbb{A}_{p_3}^v$ are known as similar patterns of the family of ideal pattern whose PFCVS is represented by $\mathbb{A}_p{}^v$.

3.2 Plithogenic Intuitionistic Fuzzy Cubic Vague Set

Definition 3.2.1: Let $\mathbb{A}_{p_1}^v$ and $\mathbb{A}_{p_2}^v$ be two Plithogenic Intuitionistic Fuzzy Cubic Vague Set (PIFCVSs). Then,

(1) $0 \leq |S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v)| \leq 1$,

(2) $S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v) = S(\mathbb{A}_{p_2}^v, \mathbb{A}_{p_1}^v)$,

(3) $S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v) = 1 \Leftrightarrow \mathbb{A}_{p_1}^v = \mathbb{A}_{p_2}^v$,

(4) $\mathbb{A}_{p_1}^v \subseteq \mathbb{A}_{p_2}^v \subseteq \mathbb{A}_{p_3}^v \Longrightarrow S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_3}^v) \leq S(\mathbb{A}_{p_2}^v, \mathbb{A}_{p_3}^v)$

Definition 3.2.2: Let X $= \{x_1, x_2, x_3\}$, $\mathbb{A}_{p_1}^v = \langle A_V^1, \lambda_V^1 \rangle$ and $\mathbb{A}_{p_2}^v = \langle A_V^2, \lambda_V^2 \rangle$ be two Plithogenic Intuitionistic Fuzzy Cubic Vague Sets (PIFCVSs) in X. The similarities between $\mathbb{A}_{p_1}^v$ *and* $\mathbb{A}_{p_2}^v$ is given as $S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v)$, where

$$S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v) = \frac{1}{12n}\sum_{i=1}^{n}\Big(\left|T_{A^1{}_V}^{L^-}(x_i) - T_{A^2{}_V}^{L^-}(x_i)\right| + \left|T_{A^1{}_V}^{U^-}(x_i) - T_{A^2{}_V}^{U^-}(x_i)\right| + \left|F_{A^1{}_V}^{L^-}(x_i) - F_{A^2{}_V}^{L^-}(x_i)\right| +$$
$$\left|F_{A^1{}_V}^{U^-}(x_i) - F_{A^2{}_V}^{U^-}(x_i)\right| + \left|T_{A^1{}_V}^{L^+}(x_i) - T_{A^2{}_V}^{L^+}(x_i)\right| + \left|T_{A^1{}_V}^{U^+}(x_i) - T_{A^2{}_V}^{+}(x_i)\right| + \left|F_{A^1{}_V}^{L^+}(x_i) - F_{A^2{}_V}^{L^+}(x_i)\right| +$$
$$\left|F_{A^1{}_V}^{U^+}(x_i) - F_{A^2{}_V}^{U^+}(x_i)\right| + \left|T_{\lambda^1{}_V}^{-}(x_i) - T_{\lambda^2{}_V}^{-}(x_i)\right| + \left|F_{\lambda^1{}_V}^{-}(x_i) - F_{\lambda^2{}_V}^{-}(x_i)\right| + \left|T_{\lambda^1{}_V}^{+}(x_i) - T_{\lambda^2{}_V}^{+}(x_i)\right| +$$
$$\left|F_{\lambda^1{}_V}^{+}(x_i) - F_{\lambda^2{}_V}^{+}(x_i)\right|\Big)$$

Algorithm:

Step 1. Construct an ideal PIFCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V \rangle$.

Step 2. Then construct PIFCVSs $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j \rangle$, $j = 1,2 \ldots n$ for sample patterns.

Step 3. Calculate the similarities between ideal pattern $\mathbb{A}_p{}^v = \langle A_V, \lambda_V \rangle$ and sample pattern $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j \rangle$ using definition 3.2.2.

Step 4. The sample pattern $\mathbb{A}_{p_j}^v$ is considered to belong to ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}_{p_j}^v\right) \leq 0.5$ and sample pattern $\mathbb{A}_{p_j}^v$ is not to be known for ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}_{p_j}^v\right) > 0.5$.

Example 3.3.3: Let us consider three simple pattern which are to be known. Let X $= \{x_1, x_2, x_3\}$. Similarly let $\mathbb{A}_p{}^v$ be PIFCVS set of ideal pattern and pattern $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j \rangle$ be the PIFCVSs of sample patterns.

Step 1. Create ideal PIFCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V \rangle$ on X as,

$$\mathbb{A}_p{}^v = <\{\frac{\langle[0.4,0.6],[0.5,0.5]),([0.4,0.6],[0.5.0.5])\rangle}{x_1}, \frac{\langle[0.3,0.8],[0.5,0.6]),([0.2,0.7],[0.4.0.5])\rangle}{x_2}, \frac{\langle[0.2,0.6],[0.3,0.6]),([0.4,0.8],[0.4.0.7])\rangle}{x_3}\},$$
$$\{\frac{[0.2,0.5],[0.5,0.8]}{x_1}, \frac{[0.4,0.7],[0.3,0.6]}{x_2}, \frac{[0.3,0.6],[0.4,0.7]}{x_3}\} >$$

Step 2. Construct PIFCVSs $\mathbb{A}_{p_j}^{v} = \langle A_V^j, \lambda_V^j \rangle$

$$\mathbb{A}_{p_1}^{v} = <\{\frac{\langle[0.2,0.6],[0.3,0.6]),([0.4,0.8],[0.4.0.7]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.5,0.5]),([0.4,0.6],[0.5.0.5]\rangle}{x_2}, \frac{\langle[0.3,0.8],[0.5,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.1,0.7],[0.2,0.8]}{x_1}, \frac{[0.3,0.7],[0.3,0.6]}{x_2}, \frac{[0.1,0.9],[0.1,0.6]}{x_3}\} >$$

$$\mathbb{A}_{p_2}^{v} = <\{\frac{\langle[0.1,0.7],[0.2,0.4]),([0.3,0.9],[0.6.0.8]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.1,0.2]),([0.4,0.6],[0.8.0.9]\rangle}{x_2}, \frac{\langle[0.3,0.8],[0.5,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.5,0.6],[0.4,0.5]}{x_1}, \frac{[0.1,0.7],[0.2,0.8]}{x_2}, \frac{[0.1,0.5],[0.2,0.5]}{x_3}\} >$$

$$\mathbb{A}_{p_3}^{v} = <\{\frac{\langle[0.3,0.8],[0.5,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.1,0.2]),([0.4,0.6],[0.8.0.9]\rangle}{x_2}, \frac{\langle[0.4,0.6],[0.5,0.5]),([0.4,0.6],[0.5.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.1,0.9],[0.1,0.6]}{x_1}, \frac{[0.1,0.7],[0.2,0.8]}{x_2}, \frac{[0.3,0.7],[0.3,0.6]}{x_3}\} >$$

Step 3. Calculate the degree of similarity S between the ideal pattern $\mathbb{A}_p{}^{v}$ and the sample pattern $\mathbb{A}_{p_j}^{v}$, then the results obtained are

$$S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_1}^{v}) = 0.13$$
$$S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_2}^{v}) = 0.19$$
$$S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_3}^{v}) = 0.17$$

Step 4. $S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_1}^{v}) \leq 0.5$, $S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_2}^{v}) \leq 0.5$ and $S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_3}^{v}) \leq 0.5$, the sample pattern whose corresponding PIFCVS sets are denoted as $\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v}$ and $\mathbb{A}_{p_3}^{v}$ are known as similar patterns of the family of ideal pattern whose PIFCVS is denoted as $\mathbb{A}_p{}^{v}$.

3.3 Plithogenic Neutrosophic Fuzzy Cubic Vague Set

In this part, we will observe the similarities of two Plithogenic Neutrosophic Cubic Vague Set (PNCVSs) of pattern recognition problem.

Definition 3.3.1: Let $\mathbb{A}_{p_1}^{v}$ and $\mathbb{A}_{p_2}^{v}$ be any two Plithogenic Neutrosophic Cubic Vague Set (PNCVSs). Then,

(1) $0 \leq |S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v})| \leq 1$,

(2) $S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v}) = S(\mathbb{A}_{p_2}^{v}, \mathbb{A}_{p_1}^{v})$,

(3) $S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v}) = 1 \Leftrightarrow \mathbb{A}_{p_1}^{v} = \mathbb{A}_{p_2}^{v}$,

(4) $\mathbb{A}_{p_1}^{v} \subseteq \mathbb{A}_{p_2}^{v} \subseteq \mathbb{A}_{p_3}^{v} \Longrightarrow S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_3}^{v}) \leq S(\mathbb{A}_{p_2}^{v}, \mathbb{A}_{p_3}^{v})$

Definition: **3.3.2** Let X $= \{x_1, x_2, x_3\}$, $\mathbb{A}_{p_1}^{v} = \langle A_V^1, \lambda_V^1 \rangle$ and $\mathbb{A}_{p_2}^{v} = \langle A_V^2, \lambda_V^2 \rangle$ be two Plithogenic Neutrosophic Cubic Vague Set (PNCVSs). The similarity measure between $\mathbb{A}_{p_1}^{v}$ *and* $\mathbb{A}_{p_2}^{v}$ is given by $S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v})$, where

$$S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v}) = \frac{1}{18n}\sum_{i=1}^{n}\Big(\left|T_{A^1{}_V}^{L^-}(x_i) - T_{A^2{}_V}^{L^-}(x_i)\right| + \left|T_{A^1{}_V}^{U^-}(x_i) - T_{A^2{}_V}^{U^-}(x_i)\right| + \left|I_{A^1{}_V}^{L^-}(x_i) - I_{A^2{}_V}^{L^-}(x_i)\right| +$$

$$\left|I_{A^1{}_V}^{U^-}(x_i) - I_{A^2{}_V}^{U^-}(x_i)\right| + \left|F_{A^1{}_V}^{L^-}(x_i) - F_{A^2{}_V}^{L^-}(x_i)\right| + \left|F_{A^1{}_V}^{U^-}(x_i) - F_{A^2{}_V}^{U^-}(x_i)\right| + \left|T_{A^1{}_V}^{L^+}(x_i) - T_{A^2{}_V}^{L^+}(x_i)\right| +$$

$$\left|T_{A^1{}_V}^{U^+}(x_i) - T_{A^2{}_V}^{+}(x_i)\right| + \left|I_{A^1{}_V}^{L^+}(x_i) - I_{A^2{}_V}^{L^+}(x_i)\right| + \left|I_{A^1{}_V}^{U^+}(x_i) - I_{A^2{}_V}^{U^+}(x_i)\right| + \left|F_{A^1{}_V}^{L^+}(x_i) - F_{A^2{}_V}^{L^+}(x_i)\right| +$$

$$\left|F^{U^+}_{A^1_V}(x_i)-F^{U^+}_{A^2_V}(x_i)\right|+\left|T^-_{\lambda^1_V}(x_i)-T^-_{\lambda^2_V}(x_i)\right|+\left|I^-_{\lambda^1_V}(x_i)-I^-_{\lambda^2_V}(x_i)\right|+\left|F^-_{\lambda^1_V}(x_i)-F^-_{\lambda^2_V}(x_i)\right|+$$

$$\left|T^+_{\lambda^1_V}(x_i)-T^+_{\lambda^2_V}(x_i)\right|+\left|I^+_{\lambda^1_V}(x_i)-I^+_{\lambda^2_V}(x_i)\right|+\left|F^+_{\lambda^1_V}(x_i)-F^+_{\lambda^2_V}(x_i)\right|\Big)$$

Algorithm:

Step 1. Construct the ideal PNCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$.

Step 2. Then construct PNCVSs $\mathbb{A}^v_{p_j} = \langle A^j_V, \lambda^j_V\rangle$

Step 3. Calculate the similarities between ideal pattern $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$ and the sample pattern $\mathbb{A}^v_{p_j} = \langle A^j_V, \lambda^j_V\rangle$ using definition 3.3.2.

Step 4. The sample pattern $\mathbb{A}^v_{p_j}$ is considered to belong to the ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}^v_{p_j}\right) \le 0.5$ and sample pattern $\mathbb{A}^v_{p_j}$ is not to be known for an ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}^v_{p_j}\right) > 0.5$.

Example 3.3.3: Here we consider the example (3.1.3) for PNCVSs.

Step 1. Construct an ideal PNCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$ on X as,

$$\mathbb{A}_p{}^v =$$

$$<\{\frac{\langle[0.4,0.6],[0.5,0.5]),([0.1,0.8],[0.2,0.6]),([0.4,0.6],[0.5.0.5]\rangle}{x_1},\frac{\langle[0.3,0.8],[0.5,0.6]),([0.3,0.5],[0.4,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_2},$$

$$\frac{\langle[0.2,0.6],[0.3,0.6]),([0.4,0.8],[0.2,0.5]),([0.4,0.8],[0.4.0.7]\rangle}{x_3}\},$$

$$\{\frac{[0.2,0.5],[0.3,0.4],[0.5,0.8]}{x_1},\frac{[0.4,0.7],[0.2,0.3],[0.3,0.6]}{x_2},\frac{[0.3,0.6],[0.4,0.5],[0.4,0.7]}{x_3}\}>$$

Step 2. Construct PNCVSs $\mathbb{A}^v_{p_j} = \langle A^j_V, \lambda^j_V\rangle$

$$\mathbb{A}^v_{p_1} =$$

$$<\{\frac{\langle[0.2,0.6],[0.3,0.6]),([0.4,0.8],[0.2,0.5]),([0.4,0.8],[0.4.0.7]\rangle}{x_1},\frac{\langle[0.4,0.6],[0.5,0.5]),([0.3,0.8],[0.2,0.6]),([0.4,0.6],[0.5.0.5]\rangle}{x_2},$$

$$\frac{\langle[0.3,0.8],[0.5,0.6]),([0.3,0.5],[0.4,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.1,0.7],[0.3,0.8],[0.2,0.8]}{x_1},\frac{[0.3,0.7],[0.2,0.9],[0.3,0.6]}{x_2},\frac{[0.1,0.9],[0.2,0.7],[0.1,0.6]}{x_3}\}>$$

$$\mathbb{A}^v_{p_2} =$$

$$<\{\frac{\langle[0.1,0.7],[0.2,0.4]),([0.4,0.5],[0.3,0.6]),([0.3,0.9],[0.6.0.8]\rangle}{x_1}\ \frac{\langle[0.4,0.6],[0.1,0.2]),([0.4,0.5],[0.3,0.6]),([0.4,0.6],[0.8.0.9]\rangle}{x_2},$$

$$\frac{\langle[0.3,0.8],[0.5,0.6]),([0.3,0.5],[0.4,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.5,0.6],[0.5,0.6],[0.4,0.5]}{x_1},\frac{[0.1,0.7],[0.3,0.8],[0.2,0.8]}{x_2},\frac{[0.1,0.5],[0.2,0.8],[0.2,0.5]}{x_3}\}>$$

$$\mathbb{A}^v_{p_3} =$$

$$<\{\frac{\langle[0.3,0.8],[0.5,0.6]),([0.3,0.5],[0.4,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_1},\frac{\langle[0.4,0.6],[0.1,0.2]),([0.4,0.5],[0.3,0.6]),([0.4,0.6],[0.8.0.9]\rangle}{x_2},$$

$$\frac{\langle[0.4,0.6],[0.5,0.5]),([0.3,0.8],[0.2,0.6]),([0.4,0.6],[0.5.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.1,0.9],[0.2,0.7],[0.1,0.6]}{x_1},\frac{[0.1,0.7],[0.3,0.8],[0.2,0.8]}{x_2},\frac{[0.3,0.7],[0.2,0.9],[0.3,0.6]}{x_3}\}>$$

Step 3. Calculate S, the degree of similarity between the ideal pattern $\mathbb{A}_p{}^{v}$ and the sample pattern $\mathbb{A}_{p_j}^{v}$, then the results obtained are

$$S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_1}^{v}) = 0.14$$
$$S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_2}^{v}) = 0.19$$
$$S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_3}^{v}) = 0.16$$

Step 4. $S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_1}^{v}) \leq 0.5$, $S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_2}^{v}) \leq 0.5$ and $S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_3}^{v}) \leq 0.5$, the sample pattern whose corresponding PNCVS sets are denoted by $\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v}$ and $\mathbb{A}_{p_3}^{v}$ are known as the similar patterns of the family of ideal pattern whose PNCVS is denoted by $\mathbb{A}_p{}^{v}$.

4. Discussion

Consider the problem given above to demonstrate the advantage of our projected method of Plithogenic Cubic Vague Set (PCVs) comparing to the set proposed by Jun et al. [17]. Cubic set fails to capture the false membership part to measure the alternative in the decision making method, therefore it is not possible to describe the problem. The elements of PCVS are characterized by one or more attributes and it has the ability to handle uncertainties and vague information considering the truth and false membership values. We have considered a simple pattern recognition problem with three sample patterns and an ideal pattern. We assume that $S\left(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_j}^{v}\right) \geq 0.5$ is the ideal pattern and the aim is to find which one among the three sample belongs to the ideal pattern. All three sample pattern of Plithogenic Cubic Vague sets (PFCVSs) $(z \equiv F)$, PIFCVSs $(z \equiv IF)$, (PNCVSs $(z \equiv N)$ is recognized as similar patterns of the family of ideal pattern in the above examples.

5. Conclusion

In this paper, we projected a method to measure the similarities between two PCVSs (z≡F, z≡IF, z≡N) and studied some of its properties. Examples are provided to prove the application of similarity measure of Plithogenic Fuzzy Cubic Vague Set (PFCVS), Plithogenic Intuitionistic Fuzzy Cubic Vague Set (PIFCVS) and Plithogenic Neutrosophic Cubic Vague Set (PNCVS) separately in pattern recognition problem. In consecutive research, investigation of AND and OR operations, PCVS to groups, rings and its application in other fields will be carried out.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors.

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Received: 05 Jun 2023, **Revised:** 24 Sep 2023,
Accepted: 16 Oct 2023, **Available online:** 21 Oct 2023.

Neutrosophic Systems with Applications, Vol. 11, 2023
https://doi.org/10.61356/j.nswa.2023.95

Foundation of the SuperHyperSoft Set and the Fuzzy Extension SuperHyperSoft Set: A New Vision

Florentin Smarandache [1]

[1] University of New Mexico, Mathematics, Physics, and Natural Sciences Division 705 Gurley Ave., Gallup, NM 87301, USA; smarand@unm.edu.

* Correspondence: smarand@unm.edu.

Abstract: We introduce for the first time the SuperHyperSoft Set and the Fuzzy and Fuzzy Extension SuperHyperSoft Set. Through a theorem we prove that the SuperHyperSoft Set is composed from many HyperSoft Sets.

Keywords: Soft Set, HyperSoft Set, SuperHyperSoft Set, Fuzzy Soft Set, Fuzzy SuperHyperSoft Set, Fuzzy Extension Soft Set, Neutrosophic SuperHyperSoft Set, Fuzzy Extension SuperHyperSoft Set.

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1. Definition of Soft Set

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the power set of $\mathcal{U}$, and a set of attributes A. Then, the pair (F, $\mathcal{U}$), where $F: A \to \mathcal{P}(\mathcal{U})$ is called a **Soft Set** over $\mathcal{U}$ [1].

2. Definition of HyperSoft Set

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the power set of $\mathcal{U}$.
Let a_1, a_2, ..., a_n, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets A_1, A_2, ..., A_n, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, A_1 \times A_2 \times A_n)$, where:

$F: A_1 \times A_2 \times \ldots \times A_n \to \mathcal{P}(\mathcal{U})$ is called a **HyperSoft Set** over $\mathcal{U}$ [2].

3. Numerical Example of HyperSoft Set

Let $\mathcal{U} = \{x_1, x_2, x_3, x_4\}$ and a set $\mathcal{M} = \{x_1, x_3\} \subset \mathcal{U}$.
Let the attributes be: a_1 = size, a_2 = color, a_3 = gender, a_4 = nationality, and their attributes' values respectively:

Size = A_1 = {small, medium, tall},

Color = A_2 = {white, yellow, red, black},

Gender = A_3 = {male, female},

Nationality = A_4 = {American, French, Spanish, Italian, Chinese}.

Let the function be:

$F: A_1 \times A_2 \times A_3 \times A_4 \to \mathcal{P}(\mathcal{U})$. This is a HyperSoft Set.

Let's assume:

$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1, x_3\}$, which means that both x_1 and x_3 are: tall, white, female, and Italian.

4. Definition of SuperHyperSoft Set

The **SuperHyperSoft Set** is an extension of the HyperSoft Set. As for the SuperHyperAlgebra, SuperHyperGraph, SuperHyperTopology and in general for SuperHyperStructure and Neutrosophic SuperHyperStructure (that includes indeterminacy) in any field of knowledge, "Super" stands for working on the powersets (instead of sets) of the attribute value sets.

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the powerset of $\mathcal{U}$.
Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$.

Let $\mathcal{P}(A_1), \mathcal{P}(A_2), \ldots, \mathcal{P}(A_n)$ be the powersets of the sets $A_1, A_2, \ldots, A_n$ respectively. Then the pair $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n)$, where $\times$ meaning Cartesian product, or: $F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}(\mathcal{U})$ is called a SuperHyperSoft Set.

5. Example of SuperHyperSoft Set

If we define the function: $F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3) \times \mathcal{P}(A_4) \rightarrow \mathcal{P}(\mathcal{U})$. we get a *SuperHyperSoft Set*.
Let's assume, from the previous example, that:
$F(\{\text{medium, tall}\}, \{\text{white, red, black}\}, \{\text{female}\}, \{\text{American, Italian}\}) = \{x_1, x_2\}$, which means that: $F(\{\text{medium or tall}\} \text{ and } \{\text{white or red or black}\} \text{ and } \{\text{female}\} \text{ and } \{\text{American or Italian}\}) = \{x_1, x_2\}$.

Therefore, the SuperHyperSoft Set offers a larger variety of selections, so x_1 and x_2 may be: either medium, or tall (but not small), either white, or red, or black (but not yellow), mandatory female (not male), and either American, or Italian (but not French, Spanish, Chinese).

In this example there are: Card{medium, tall} · Card{white, red, black} · Card{female} · Card{American, Italian} = 2·3·1·2 =12 possibilities, where Card{ } means cardinal of the set { }.

This is closer to our everyday life, since for example, when selecting something, we have not been too strict, but accepting some variations (for example: medium or tall, white or red or black, etc.).

6. Fuzzy-Extension-SuperHyperSoft Set

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}\left(\mathcal{U}\left(x(d^0)\right)\right)$ where $x(d^0)$ is the fuzzy or any fuzzy-extension degree of appurtenance of the element x to the set $\mathcal{U}$.

Fuzzy-Extensions mean all types of fuzzy sets [3], such as: Fuzzy Set, Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type), Fermatean Fuzzy Set, q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, n-HyperSpherical Fuzzy Set, Neutrosophic Set, Spherical Neutrosophic Set, Refined Fuzzy/Intuitionistic Fuzzy/Neutrosophic/other fuzzy extension Sets, Plithogenic Set, etc.

7. Example of Fuzzy Extension SuperHyperSoft Set

In the previous example, taking the degree of a generic element $x(d^0)$ as neutrosophic, one gets the Neutrosophic SuperHyperSoft Set.

Assume, that: $F(\{\text{medium, tall}\}, \{\text{white, red, black}\}, \{\text{female}\}, \{\text{American, Italian}\}) = \{x_1(0.7, 0.4, 0.1), x_2(0.9, 0.2, 0.3)\}$.

Which means that: x_1 with respect to the attribute values $(\{\text{medium or tall}\} \text{ and } \{\text{white or red or black}\} \text{ and } \{\text{female}\}, \text{and } \{\text{American or Italian}\})$ has the degree of appurtenance to the set 0.7, the indeterminate degree of appurtenance 0.4, and the degree of non-appurtenance 0.1.

While x_2 has the degree of appurtenance to the set 0.9, the indeterminate degree of appurtenance 0.2, and the degree of non-appurtenance 0.3.

8. Theorem

The SuperHyperSoft Set is equivalent to a union of the HyperSoft Sets.

Proof

Let's consider the SuperHyperSoft:

$$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}(\mathcal{U})$$

Assume that the non-empty sets

$B_1 \subseteq A_1, B_2 \subseteq A_2, \ldots, B_n \subseteq A_n$ and $F(B_1, B_2, \ldots, B_n) \in P(U)$

$B_1 = \{b_{11}, b_{12}, \ldots\}, B_2 = \{b_{21}, b_{22}, \ldots\}, \ldots, B_n = \{b_{n1}, b_{n2}, \ldots\}$, therefore

$F(\{b_{11}, b_{12}, \ldots\}, \{b_{21}, b_{22}, \ldots\}, \ldots, \{b_{n1}, b_{n2}, \ldots\})$ can be decomposed in many $F(b_{1k_1}, b_{2k_2}, \ldots, b_{nk_n})$ $\in P(U)$ which are actually HyperSoft Sets.

If we reconsider the previous example, then:

({medium *or* tall} *and* {white *or* red *or* black} *and* {female} *and* {American *or* Italian}) produces *12* possibilities:

1. medium, white, female, American;
2. medium, white, female, Italian;
3. medium, red, female, American;
4. medium, red, female, Italian;
5. medium, black, female, American;
6. medium, black, female, Italian;
7. tall, white, female, American;
8. tall, white, female, Italian;
9. tall, red, female, American;
10. tall, red, female, Italian;
11. tall, black, female, American;
12. tall, black, female, Italian.

Whence F of each of them is equal to $\{x_1, x_2\}$, or:

F(medium, white, female, American) = $\{x_1, x_2\}$

F(medium, white, female, Italian) = $\{x_1, x_2\}$

F(tall, black, female, Italian) = $\{x_1, x_2\}$ and all 12 are HyperSoft Sets.

9. Conclusion

A new type of soft set has been introduced, called SuperHyperSoft Set and an application has been presented. Further work to do is to define the operations (union, intersection, complement) of the SuperHyperSoft Sets.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Received: 01 Aug 2023, **Revised:** 25 Oct 2023,

Accepted: 30 Oct 2023, **Available online:** 07 Nov 2023.

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SYSTEMS WITH APPLICATIONS