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*"***Neutrosophic Systems with Applications***"* has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.
The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.
Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.
This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e., notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.
Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.
In a classical way <A>, <neutA>, <antiA> are disjointed two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.
Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.
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Contents

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Evaluation of Cyber Insecurities of the Cyber Physical System Supply Chains Using α-Discounting MCDM

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Abstract: Recently, supply chains (SCs) are applying information technology to enable data sharing among suppliers, instant access to information, and complete tracking of products. With more Cybersecurity risks present, such as theft of information, service interruptions, and financial resources risks, the vulnerability of systems is increased. The management of supply chain Cybersecurity, which encompasses information systems, software, and infrastructure, is the emphasis of the supply chain's safety measure. There are several serious danger that attack supply chain systems. Most SC Cybersecurity procedures are used to reduce the threats posed by vulnerabilities to those processes. Researchers have mostly concentrated on supply chain-related cyber physical system (CPS) issues. This study makes attempts to classify and evaluates the Cybersecurity insecurities of supply chains. In addition, this work provides an update of the analytic hierarchy process (AHP) method called α-discounting multi-criteria decision-making (α-D MCDM), which enables a more uniform assessment of supply chain cyber insecurities. This paper suggests using the α-D MCDM in various ways to address various supply chain evaluation problems.

Keywords: Supply Chain; Cybersecurity Risks; Cyber Physical System; α-D MCDM.

1. Introduction

A supply chain (SC), which is a combination of various entities that coordinate their procedures, targets and some system elements with those of suppliers, customers, and other external organizations. A SC consists of all operations associated with the movement of products, services, and information from suppliers to consumers [1]. Supply chain management (SCM) aims to deliver the appropriate item to the appropriate customer at the optimal cost, at the correct place, and at the optimal time. In order to increase process effectiveness as well as cost enhancement, businesses are now utilizing information technologies (IT) in their processes [2]. According to Singh et al. [3], the efficient use of IT tools guarantees an ongoing development of supply chains.

Cyber-physical systems (CPS) are systems made up of physical ingredients, network infrastructures, embedded hardware, software, and connections between devices and sensors for transferring data. The development of CPS with SC operations has changed how supply chains operate in numerous ways over time [4]. An organization's information systems and information technologies, which improves supply chain productivity, may also be the source of security risks as well as weaknesses. The organization and business relationships through every phase of the supply chain is required for efficient and achievable supply chain management (SCM). Integrating technology into corporate operations improves overall productivity and even costs optimizing. Cyber threats are one of the difficulties brought on by utilizing CPS in supply chain processes [5].

Modern industrial demands, such as decentralization and systems connectivity cannot be satisfied by the conventional supply chain architecture. In contrast, the utilization of CPS and the internet of things (IoT) leads to the production system being intelligently connected, which improves manufacturing, efficiency, and productivity increases [6]. Data authenticity, consistency, and security are some of the issues that come with growing connection, the volume of data, and their sensitive nature. Due to several factors, including software flaws and vulnerabilities discovered in any supply chain through data transfer, cyber-attacks could have consequences on supply chain processes [7].

In this paper, the objective is to categorize the cyber insecurities of cyber SC regarding to supply, operation, and customer. Firstly, cyber supply chain definitions are discussed and how it may improve the SCs performance and efficiency. Secondly, we describe the expansion of analytic hierarchy process (AHP), which, by addressing AHP's imperfections in order to evaluate the categories of the cyber insecurities that may attack supply chain. Thirdly, we put forth the concept of a multi-criteria decision-making (MCDM) framework that supports management in assessing supply chain cyber vulnerabilities by combining the α-discounting (α-D) with various MCDM techniques.

This research is structured as follows: Section 2 reviews earlier papers on the cyber supply chain and cyber-attacks that could target the SC phases. In Section 3, discussion of cyber supply chain insecurities is presented. The suggested concept of evaluating cyber supply chain vulnerabilities based on α-D MCDM with various MCDM is presented in Section 4. The conclusion and future directions are made clear in Section 5.

2. Literature Reviews

Supply chains are now integrated with organizations through digital communication channels as a result of digitalization. In supply chains, all members become as powerful due to shared knowledge and security mechanisms along the supply chain, as stated by Pandey et al. [8]. An organization can achieve its strategic goals by utilizing the secure network infrastructure that supply chain Cybersecurity offers. While the way that organizations and industries function has changed significantly, as a result of the application of CPS in the field of SCs. However, CPS supply networks also brought forth a number of difficulties, including a lack of security measures and risk management [9].

2.1 Cyber Supply Chain

The quality of services provided in the field of SC has steadily improved due to technological applications. Cheung et al. [10] investigated the Cybersecurity measures in SCM. Several major findings and relevant research initiatives related to Cybersecurity in logistics and SCM are discussed [10]. The research of Yeboah-Ofori et al. [11] tries to analyze and predict risks in order to improve Cybersecurity in the field of SCs. They used Cyber Threat Intelligence (CTI) to investigate and anticipate attacks based on CTI features [11].

Luo and Choi [12] focused their research on how firms make investments in Cybersecurity at a high cost. Because cyber-attacks pose a threat to e-commerce supply chains and its participants. Customers who buy things online run the danger of having their personal information hacked [12]. Pandey et al. [8] attempt to classify the Cybersecurity threats that arise as a result of supply chains working in cyber physical systems. The research provides a framework comprised of various cyber-attacks spanning information flows in global supply chains [8].

2.2 Cybersecurity and Supply Chain Risk

In order to evaluate the influence of Cybersecurity on digital operations in the UAE pharmaceutical business, the research of Del Giorgio Solfa [13] examined empirical data. The results confirmed the strong positive association between supply chain risk and Cybersecurity in relation to digital operations [13]. The main goal of Melnyk et al.'s study from 2022 is to create a foundation for future research on supply chain Cybersecurity [14]. A need for greater research on Cybersecurity throughout the supply chain is made in the paper's conclusion. An exploratory research technique was used, which drew on a number of sources to construct the research framework [14].

In order to investigate how supply chain managers view the components of cyber supply chain risk management and the degree to which this is aligned with increased cyber supply chain resilience, Creazza et al. [15] studied the subject of supply chain security. In order to better respond to cyber threats, this study revealed that Logistics Service Providers can play a significant role as administrators of the Cybersecurity process. The study also emphasizes how crucial it is to prioritize humans while enhancing supply chain cyber resilience. Using a data fusion technique, Hossain et al. [16] established a paradigm that takes into account supply chains' resilience, sustainability, and Cybersecurity to determine how effectively they operate without interruption. A healthcare supply chain is used to verify the suggested framework [16]. In cyber supply chain risk analysis, SC weaknesses are frequently disregarded. To help with risk assessment and to investigate the intricate problems related to the demands for protecting hardware, firmware, software, and system data over the whole SC lifecycle, a novel SC cyber-attack framework is presented [17].

3. Cybersecurity Risks in Supply Chains

3.1 Cyber Physical System Supply Chains

Factors that make it difficult to model CPS effectively include the variety of systems and programming, the absence of representation of real-time operating systems, and timing-related system responsiveness [18]. The foundation of CPS is the fusion of both traditional and technological procedures. CPS encompass machines, structures, vehicles, and other means of transportation as well as logistical, management procedures, and internet-based services [19]. While devices are used to respond to industrial or organisational changes and connect with other components, sensors help CPS gather, organise, and analyse data. CPS can be employed to handle a variety of concerns, including manufacturing, logistics, quality control, planning, and scheduling operations within the supply chain [20].

3.2 Cybersecurity Risks Categories Occurring along Supply Chains

Cyber supply chain systems based on CPS are frequently vulnerable to cyber-attacks notwithstanding their advantages in terms of safety and dependability. At a time, there are more and more advanced cyber-attacks that have a variety of negative effects on al supply chain operations and businesses. Attacks against emerging CPS can also have a negative effect, particularly on those that function in the logistics and SCM sectors [21]. Supply, operations, and demand are the three key supply chain stages that can be used to categorise cyber supply chain insecurities as shown in Table 1.

Table 1. Cybersecurity risks of cyber supply chains and their categorization.

Risks categories	Risk types
Supply risks	Lack of availability of providers
	Vendor credentials hacked
	Vulnerability of the supplier's connection
	Malware-induced source code alteration
	Provision of tainted software
Operations risks	Disruption of the manufacturing facility
	Unexpected breakdown of the manufacturing's operations
	Missing coding errors
	Invalid product specifications
	Information leakage
Demand risks	Theft of inventions
	Altering information
	Access of Client information without permission
	Deceptive communication
	Data destruction
	Unlicensed payment processors

- *Supply Insecurities*

Supply risks are the incident related to incoming supplies that could lead to supplier failures. The firm's difficulties to satisfy client demand is the result of these failings. Prior to the final manufacturing, suppliers frequently give the companies with the necessary parts. Therefore, it's essential to effectively manage the supply chain of Cybersecurity products in accordance with the requirements of the Cybersecurity strategy [22].

- *Operational Insecurities*

Operations risk is defined as the potential for an occurrence that has an impact on the firm's capability to provide goods and services, productivity, and its financial performance. These risks arise from a major breakdown in the access restrictions on supply chain operations, which gives the attacker the ability to interrupt business [23].

- *Demand Insecurities*

Demand risk is defined as the potential of a situation involving outgoing transactions that could change the possibility of clients placing orders with the business. Demand risk results from the unanticipated change in markets and business breakdown. The public's opinions are impacted by the supply risks in CPS, and the associated demand also creates the demand risks [24].

4. Application of α-D MCDM to evaluate Cybersecurity Risks of Supply Chain

4.1 α-D MCDM Definitions

In this research, we examine a novel method that extends Saaty's AHP and is known as the α-D MCDM. This method can be applied to any set of preferences that can be transformed into a set of homogeneous linear equations [25]. It is helpful not only for preferences that are pairwise comparisons of criteria as AHP does, but also for preferences of any n-wise (with $n \geq 2$) assessments of criteria that can be expressed as linear homogeneous formulas.

The overall aim of α-D MCDM is to change the null-solution of linear homogeneous system, into a non-null solution system, by reduce or raise the coefficients in the right-hand side [26].

Additionally, this approach has an edge in that it can convert those MCDM issues that the AHP has categorized as inconsistent into a consistent form. Taking a decision among the options available to a decision maker is not an easy task since most often, numerous criteria with diverse orientations are used in place of a single criterion with a single direction in the decision-making process. That's why, α-D MCDM is a good choice in such evaluation problems.

4.2 Application of α-D MCDM

The MCDM techniques in the literature have benefits and drawbacks. AHP is constrained in the way some issues are structured. The most beneficial advantage of the α-D MCDM that is not limited by the number of comparisons of criteria. By decreasing or increasing the linear evaluation equation coefficients at/to specific amounts, α-D MCDM can solve the problem of converts an inconsistency of the problem. The following are the procedure steps for α-D MCDM [27]:

1. Let $X = \{X_1, X_2, X_3, \dots, X_n\}$,$n \geq 2$, be a problem structure components. The group of preferences is $R = \{R_1, R_2, R_3, \dots, R_m\}$, $m \geq 1$. Each preference R_m represent the relationship to a certain criteria X_n as follows$R_m = f_i(X_1, X_2, X_3, \dots, X_n)$. Let us build a basic belief assignment (bba) for the weights of the problem components. $m: X \to [0, 1]$, where $m(X_i) = x_i$,$0 < x_i < 1$.
$$\sum_{i=1}^{n} m(X_i) = \sum_{i=1}^{n} x_i = 1$$
2. In order to get the variable x_i in accordance with preferences R, build $m \times n$ linear homogeneous matrix $A = (a_{ij})$ as follows
$$\begin{Bmatrix} x_{1,1}w_1 + x_{1,2}w_2 + \dots + x_{1,n}w_n = 0 \\ \dots \\ x_{m,1}w_1 + x_{m,2}w_2 + \dots + x_{m,n}w_n = 0 \end{Bmatrix}$$
$$A = \begin{bmatrix} x_{1,1} & \dots & x_{1,n} \\ \dots & \dots & \dots \\ x_{m,1} & \dots & x_{m,n} \end{bmatrix}$$
3. Calculate the determinant $\det(A)$ of the matrix A. If $\det(A) = 0$, then the system is consistent. Otherwise, it's inconsistent.
4. After examine the problem consistency, if the problem is inconsistent, then do the following steps of α discounting:
   - Introduce a new matrix called A(α) by increasing or decreasing the right hand side with α, then compute α that makes the determinant equal 0 using the Fairness principle (equalize all parameters). Then, solve the system.
   - Substitute the secondary variables by 1 and then, normalize the result.

4.3 α-D MCDM in the Evaluation of Cyber Insecurities Categories of Cyber Supply Chains

α-D MCDM outperforms AHP in the evaluation of n-wise comparisons. According to the literature, we used the α-D MCDM in this study to quantify the cyber insecurities of cyber SCs. In order to use this approach, we consult with a SCM specialist who can provide us with advice on the relative importance of each category of supply chain cyber threats.

Let's propose that supply risks is x, operations risks is y, and demand risk is z. The following is the expert's preference:

i. Supply risks is as important as 2 times of operations risks plus 3 times of demand risks.
ii. Operations demand is 4 times as important as supply risks.
iii. Demand risks is 5 times as important as supply risks.

$$\begin{cases} x = 2y + 3z \\ y = 4x \\ z = 5x \end{cases} \qquad A = \begin{bmatrix} 1 & -2 & -3 \\ -4 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$$

As det ≠ 0, so right-side coefficient must be parameterized.

$$\begin{cases} x = 2\alpha_1 y + 3\alpha_2 z \\ y = 4\alpha_3 x \quad ; \text{where } \alpha_1, \ \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6 > 0 \, . \\ z = 5\alpha_4 x \end{cases}$$

The α-D MCDM outperforms AHP in the evaluation of n-wise comparisons. According to the literature, we used the α-D MCDM in this study to quantify the cyber insecurities of cyber SCs. In order to use this approach, we consult with a SCM specialist who can provide us with advice on the relative importance of each category of supply chain cyber threats.

Then, we will solve the system:

$x = 2\alpha_1(4\alpha_3 x) + 3\alpha_2(5\alpha_4 x)$

$1 = 8\alpha_1\alpha_3 + 15\alpha_2\alpha_4$ Set 1 to the secondary variable

Let $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha > 0$

$1 = 8\alpha^2 + 15\alpha^2$ (Parametric equation)

$\alpha = \sqrt{23/23}$

$S = [1 \quad 4\alpha_3 \quad 5\alpha_4 \,]$ (Priority vector)

$S = \left[1 \quad \frac{4\sqrt{23}}{23} \quad \frac{5\sqrt{23}}{23}\right]$

Normalized priority vector to find the weight of each cyber insecurities category.

$W = [0.3476, \quad 0.2899, \quad 0.3625]$

The α-D MCDM method was used to evaluate the three cyber insecurities of supply chains, and based on expert preferences, demand risks were found to be the superior element with a weight of 0.3625. The supply risks and operation risks are ranked second and third, with weights of 0.3476 and 0.2899, respectively as presented in Figure 1.

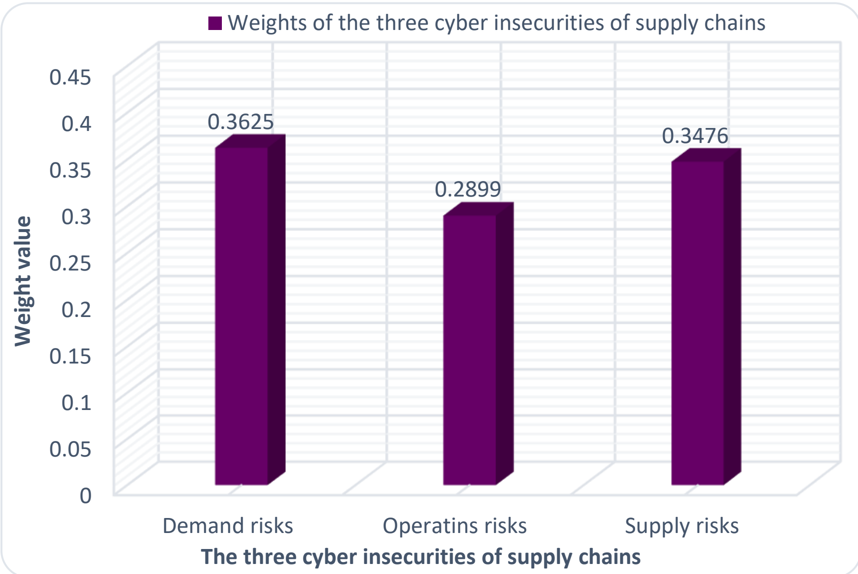

Figure 1. Weights of the three cyber insecurities of supply chains.

5. Conclusion and Future Works

Managing cyber insecurities in SCs is a significant concern for organizations seeking to remain competitive in today's market. The digital transformation of the supply chain has resulted in a platform with fewer silos. Risks that attacks data are higher than ever. While new technologies have provided up new supply chain management opportunities, they have also produced potential security holes that cybercriminals may exploit. Thus, in this study the cyber insecurities that facing the cyber supply chains have been highlighted. According to the literature, the cyber supply chain insecurities are categorized into three types: supply risks, operational risks, and demand risks. Also, the α-D MCDM method was discussed and applied to evaluate the three categories of cyber supply chain insecurities in sufficient manner.

Our future plan is to apply an integrated MCDM framework to evaluate the overall cyber insecurities that face the cyber supply chains as a result of the noticeable trend towards fourth and fifth generation technologies for industry. The integrated framework that suggested in the future studies is recommended to be as integration between α discounting method and other MCDM method to evaluate the main insecurities and its corresponding risks.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Intelligent Healthcare: Evaluation Potential Implications of Metaverse in Healthcare Based on Mathematical Decision-Making Framework

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Abstract: The Metaverse has the ability to restructure and change the manner in which individuals connect with one another as well as the activities that they carry out on a daily basis. One definition of a Metaverse describes it as "a virtual, digital, and three-dimensional universe formed by the integration of various cutting-edge technologies and virtual places." In this piece, we will explore the potential applications of the Metaverse in the medical field. Our conversations on the notion of the Metaverse and the primary technologies that make it possible are both thought-provoking and in-depth. In this article, we study and analyze the possible uses of the Metaverse in the healthcare industry, including telemedicine and telehealth, medical education and training, medical marketing, healthcare supply chain, healthcare facilities, as well as fitness and wellness. The primary obstacles that prevent the broad implementation of the Metaverse in healthcare are discussed in this paper. Finally, evaluating implications of leveraging Metaverse in healthcare is discussed. Best Worst Method (BWM) is deployed with Triangular Neutrosophic Sets (TriNSs) to estimate criteria's weights where five criteria have been determined for alternatives of Metaverse. These alternatives are evaluated and ranked through weighted sum model (WSM) with help of TriNSs. The findings of this process demonstrated that Metaverse alternative 1 (MeT1) Health Surveillance is optimal according to global score. Antithesis Metaverse alternative 3 (MeT3) Virtual surgical is least one.

Keywords: Metaverse; Healthcare; Virtual reality; Blockchain; Artificial intelligence; Cutting-edge technologies; Neutrosophic.

1. Introduction

One of the most important factors that helps ensure the overall, physical, social, and mental well-being of the whole human population around the globe is access to quality healthcare [1]. Any healthcare system should have as its major goal to direct its resources towards actions that would promote, repair, maintain, and enhance healthcare services. Additionally, it makes a significant contribution toward the effective growth and industrialization of a nation's economy. Because of this, the sector has seen quick expansion and revolution as a result of being highly exposed to the progression of technology in the interest of enhancing the experience of contact between carers, patients, and associated stakeholder groups. The advent of digital healthcare has been a significant catalyst for transformation in the healthcare sector. The introduction of digital health services that make use of the internet and digital tools have had a significant effect on the contact that takes place between patients and medical professionals on a very broad scale. These changes were made visible as a result of the development of technologies such as blockchain, augmented reality (AR), and virtual reality (VR) [1]. Even though there has been a lot of progress made in the healthcare industry, there are still a lot of problems that need to be solved in this industry. Some of these problems include the overwhelming burden of long-term chronic diseases, rising expenses, an aging population, a lack of qualified healthcare workers, and limited resources that are available. Due to the prevalence of these problems, there is a growing need for medical treatments that may be delivered directly to the homes of patients. The most recent pandemic, COVID-19, has placed immense strain on the global healthcare system and the underlying labour, infrastructure, and supply chain management. COVID-19 has been the key cause for the acceleration of fast change across the healthcare ecosystem, and it has driven the players in this sector to seek adaption of the technologies that are utilized in this industry [2].

The period after the epidemic has been accompanied by significant changes in the fundamental underpinnings of the healthcare field. For instance, consumers of the current age have recently begun to take an active role in the decision-making process on matters pertaining to their healthcare, which has been followed by an enthusiastic adoption of virtual healthcare systems and other relevant digital advancements. Additionally, there has been an active drive for the utilization of inter-operable data and data analytics; unprecedented cooperation in the creation of treatments, which have forced governments, healthcare providers, and other stakeholders to adapt and innovate. However, there are still significant obstacles to overcome, and the way in which these obstacles are addressed will determine how far we go toward realizing the potential of the healthcare industry [3]. The patients and their experiences are the driving force behind innovations in this sector due to the fact that the consumers' ever-changing objectives and aspirations are what drive innovation. Their key preferences include the development of technologically enabled, on-demand, and seamless patient-clinician contact, with the goal of assuring the delivery of patient-centric services across geographical borders and socioeconomic groupings.

The health journey of every single patient is distinct, and it is essential to recognize this fact in order to tailor the particular services and bring every encounter to the level of a personalized healthcare experience. It is now very necessary to use cutting-edge digital tools and services in order to maximize customer happiness. This will make it possible to keep better track of patient's health conditions and ensure that they take their drugs as prescribed. Customers in the healthcare industry are becoming more open to the idea of disclosing their private information [4]. As a result, there is a growing need for organizations to offer interoperability across different organizations and to maintain customers' confidence by displaying dependability, openness, and empathy in their business practices. The current plan calls for shifting the emphasis away from healthcare and towards health and well-being, which will hopefully motivate changes in the design of service offers and delivery methods. Therefore, organizations are pushing the adoption of virtual care, remote monitoring, digital diagnostics, decision support systems, at-home prescription delivery systems, and self-service apps for educational and social support purposes. Utilizing technologies such as artificial intelligence, cloud computing, augmented reality, and virtual reality, this digital revolution has had a tremendous influence on the healthcare ecosystem by enhancing its working capacity, access to services, and the experience that patients and clinicians have with one another.

The purpose of this essay is to provide a visionary framework for the Metaverse that addresses current difficulties in the healthcare industry by using a variety of sophisticated and upcoming technologies. We provide an overarching summary of the Metaverse, including information on its system architecture and the technologies that make it possible. We investigate the possible uses of the Metaverse in the medical field, which have the potential to revolutionize the industry and provide virtual healthcare services of a higher quality. We describe the many ways in which enabling technologies for the Metaverse may be incorporated expressly for applications in the healthcare industry. In addition, we show how the system architecture for such applications may be built and organized in a variety of different ways. In addition, we explore the possible associated difficulties that the Metaverse may provide for applications in the healthcare industry. In conclusion, we discuss potential avenues for further study.

2. Metaverse Overview

In Neal Stephenson's science fiction book titled "Snow Crash," which was published in 1992, the word "Metaverse" was first used. The story envisioned the creation of a virtual environment in which individuals might utilize digital avatars to explore and communicate with one another. The Metaverse is a new idea and a technology that is always developing, and its definition is subject to change depending on the people who are engaged and the application that it is designed to serve. A computer simulation that enables digital avatars of participants to interact with each other in a realistic, shared, and life-like environment is referred to as the Metaverse. In its most basic form, the Metaverse may be understood as a broad term. Although the idea of a Metaverse may be implemented in a variety of applications for a wide variety of reasons, the components of the system architecture are, for the most part, consistent. In general, it is made up of its physical world, its virtual world (the Metaverse), its interface and access, and the Metaverse enabling technologies. These four components make up its primary makeup.

- The real-world, physical environment is augmented with a computer-generated visual overlay in real-time via the use of augmented reality. It employs the use of lenses, glasses, or cellophanes. The purpose of the Metaverse is to supplement the knowledge that is already available in the natural world. Examples include the mobile game Pokémon Go and animated three-dimensional medical presentations.
- The inner word is expanded upon in another way via the practice of lifelogging. In contrast to augmented reality, the use of smart gadgets to capture ordinary lives and upload them to the internet is becoming more common. Instagram, Facebook, Twitter, and even apps that track your health are some examples.
- A simulation of our actual environment is what we mean when we talk about a "mirror world." The actual look, information, and structure are brought into a virtual area, which makes it possible to carry out tasks via the internet or mobile apps.
- The inner world is simulated using a technology known as virtual reality, which is an online simulation of a high-tech, three-dimensional reality complete with avatars and a tool for quick communication. The cultural, physical, and social qualities of the avatar are not reflective of reality, and the avatar may be customized to reflect the user's preferences. The avatar is able to carry in conversations with other entities and complete objectives. Online multiplayer video games, virtual hospitals and clinics, and online consultation spaces are a few examples.

3. Enabling Metaverse Technologies for Healthcare

Figure 1 provides a concise overview of the primary technologies that enable the Metaverse, and the following paragraphs provide a short summary of the roles played by these technologies.

3.1 Extended reality

In the realm of extended reality (XR), technologies such as AR, VR, holograms, and mixed reality (MR) are combined with artificial intelligence (AI), computer vision, and linked devices such as mobile phones, wearables, and head-mounted displays [5]. This innovative technology is revolutionizing the way services are provided and boosting their quality in a variety of industries. It does this by merging speech recognition, gesture recognition, motion tracking, vision, and haptics. People have always believed that VR will primarily be beneficial to the entertainment business. It was to be anticipated that participating in an immersive experience would in no way detract from a user's enjoyment of a movie or video game. Despite this, the use of XR has far exceeded these estimates. It is being used in an expanding range of sectors, ranging from the medical field to the industrial industry. XR will develop to its greatest potential and continue to advance as it is present in the Metaverse. Combining VR with AR technology enables users to experience a feeling of virtual presence inside the Metaverse. One example of such a technology is holographic communication, which provides users with an immersive experience in a setting that is relatively hassle-free. Holographic communications have the potential to materialize the three-dimensional digital representations that are used in the process. Integrating 3D capture, hologram synthesis, transport, and display, as well as other aspects of 3D technology makes this technology conceivable.

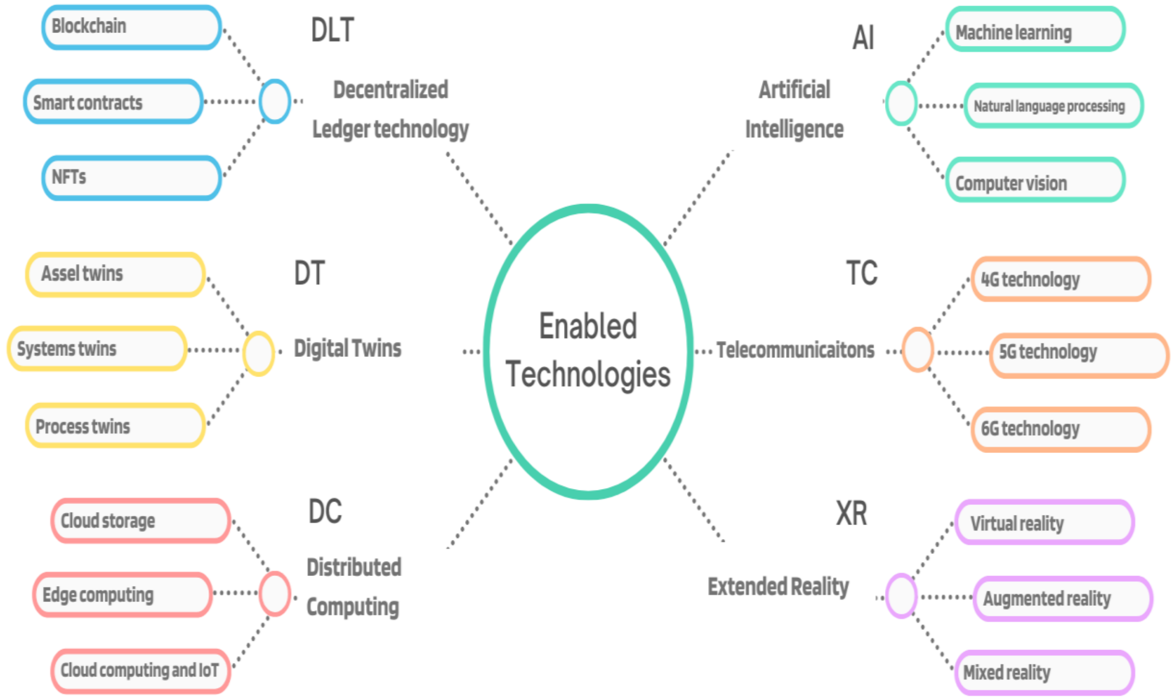

Figure 1: Metaverse technologies for healthcare.

3.2 *Artificial Intelligence*

The creation and administration of technology that can learn on its own to make judgments and carry out activities in place of people is the primary focus of AI, which is also referred to as machine intelligence [6]. Machine learning, computer vision, natural language understanding (NLU), and natural language processing are all examples of technologies that fall under the umbrella of AI. AI is a collection of technologies that encompasses any software or hardware component that enables these areas of study. The infrastructure of the Metaverse will be improved with the aid of AI, which will also improve the quality of the 3D immersive experience and the built-in services of virtual worlds. The use of AI technology will also contribute to the enhancement of the overall quality of the Metaverse environment.

3.3 *Distributed Computing*

Users in the Metaverse engage in real-time interaction with high-quality 3D representations as well as programs that are very sensitive to latency and demand a significant amount of bandwidth. In addition, Internet of Things devices and sensors that are installed in the real world are utilized to provide data to the Metaverse so that the status of digitized items may be kept up to date. As a result, in order to provide consumers with a seamless and immersive experience, it is necessary to handle an enormous quantity of data at very fast rates. In addition, developing and deploying a customized Metaverse is a process that is time-consuming, expensive, and difficult since it necessitates the use of innovative and cutting-edge technology to process and store the data that is created by the Metaverse.

The use of distributed computing enables a variety of approaches to problems involving the processing and storage of massive amounts of data. The first step in providing all of the necessary services for the Metaverse is to use cloud computing and cloud storage in combination with one another. The person responsible for deploying the Metaverse will first upload all of the required data to a cloud storage location, which will then make the data available to cloud computing services. After then, the Metaverse will be hosted by a specialized cloud computing service, which will also provide it with all of the necessary services to ensure that it continues to function normally. Second, because of the increase in demands for processing power, cloud computing will have difficulty consolidating and storing all of the resources that are involved. As a result, this kind of data will need distribution and relocation to be brought closer to the place where it will be consumed. Edge computing and Internet of Things devices may work together to guarantee that users get essential data in real-time without any delays. This can be accomplished by installing edge devices in distant regions at the edge of the network and closer to customers. Edge computing and IoT devices can also work together to ensure that users receive the required data in real-time.

3.4 Digital Twin

A virtual representation that functions in the same way as the digital equivalent of a physical thing or activity in real-time is referred to as a digital twin. The term "digital twin" was first used in David Gelernter's book "Mirror Worlds," which was published in the year 1991. Michael Grieves, writing in the year 2002, was another person to discuss the idea. The United States National Aeronautics and Space Administration (NASA) came up with the first operational definition of a digital twin in 2010. This was done with the intention of improving the physical-model simulation of spacecraft. The ever-increasing sophistication of product design and engineering practices has led to the development of digital twins. The term "digital twin" refers to a digital representation of an item, activity, or service that is physically existent in the world. A digital twin is a digital clone of a physical entity, such as a piece of equipment, a medical gadget, or even a bigger object, such as skyscrapers or even whole towns. Digital twins may also be created for larger things, such as entire cities. On the other side, the technology known as the Metaverse creates a virtual environment in which everything and everyone behaves in a manner that is analogous to how they behave in the real world. Because they generate a digital copy of every physical item in the Metaverse, digital twins are an essential component in the Metaverse's infrastructure.

Establishing a digital twin of the whole hospital in the Metaverse in order to analyze the needs enables one to investigate various operational strategies, staffing models, and care delivery formats. These virtual models in the Metaverse may be helpful in a variety of situations, including those involving a shortage of beds, the spread of infectious diseases, the scheduling of physicians, and the availability of operating rooms. By using a Metaverse that is equipped with digital twins, it is possible to enhance the treatment of patients, the costs, and the performance of the personnel. Because the healthcare industry is both very complex and highly sensitive, having this information is essential for making strategic choices. It is possible to create a completely risk-free environment by completely simulating a hospital inside the Metaverse utilizing digital twins. The creation of personalized artificial organs will likewise benefit from the Metaverse's use of digital twins. A digital twin enabled Metaverse may also assist in the performance of brain and heart operations by allowing doctors to practice their craft in virtual simulations of surgical procedures before carrying out difficult treatments in the real world.

3.5 Telecommunications

In order to offer users with an immersive experience that is both data-intensive and time-sensitive, the Metaverse needs continual synchronization in real time between the real and virtual goals. In addition, the Metaverse calls for the transmission of 3D virtual objects and services, which creates a major demand on the current infrastructure for 5G telecommunications. Advanced telecommunications technologies, such as 6G technology, are necessary to allow much faster data rates and capacity in order to minimize network breakdowns or delays. This is the case even though these technologies already exist.

4. Metaverse Applications in Healthcare

The "handicraft workshop model" of healthcare, in which a patient's diagnosis and course of treatment might vary depending on which physician treats them and which hospital they go to, is considerably improved by the use of the Metaverse to provide complete medical care. In the event of a complete healthcare scenario, judgments will be taken on the basis of the proposals given by the expert as well as the findings acquired from the many different technologies that are made possible by the Metaverse. In the field of medicine, the Metaverse may be used for a wide variety of purposes, including research, physical examination, diagnosis, and even insurance.

4.1 Medical Diagnosis

The process of assessing the medical status of a patient based on the symptoms presented by the patient is referred to as a medical diagnosis. With the assistance of a variety of cutting-edge technologies including augmented reality, virtual reality, extended digital twins, blockchain, 5G, and many others, the adoption of the Metaverse in the field of medicine substantially assists in the accurate diagnosis of a patient's numerous medical ailments [6]. An essay on the "expert consensus on the Metaverse in medicine" outlines how and why the Metaverse may be used in various verticals in healthcare for the purpose of giving excellent comprehensive healthcare for all individuals. The Metaverse may also be seen as an improvement on the current medical IoT, in that it overcomes the constraints of the medical IoT with regard to human-computer interaction, connectivity, and integration with and between real and virtual worlds.

4.2 Surveillance of the Patient

The combination of Telepresence, digital twinning, and blockchain will bring about remarkable advances in medical care, particularly in the area of patient monitoring. These advances will be made possible by the Metaverse. The provision of medical services using Telepresence in medicine, commonly known as telemedicine, may be done remotely. It is possible to employ test dummies of patients in emergency circumstances. This allows for the reactions of patients to procedures or medications to be understood far in advance of actually administering them to real patients. Because medical data is the most sensitive and significant of all data types, the use of blockchain technology may help ensure that it is safely stored and transferred. This ensures that the data will not be altered and that it will not be placed in jeopardy.

4.3 Medical Education

In the annals of the history of medical education, the Metaverse represents an extraordinary watershed moment. In the realm of medical education, the Internet of Things (IoT), blockchain, artificial intelligence, augmented reality, and virtual reality are the forerunners of the Metaverse. It was stated in that the unique identifying tag in blockchain helps identify data in blockchain-based Metaverse. This contribution of AI, blockchain, and the Metaverse to the field of medical care was discussed. The Metaverse is an artificial intelligence and blockchain-based platform that enables the creation of a digital virtual world that is unconstrained by the constraints of the physical world. Even when they are working in a hectic clinical setting, medical students are still able to concentrate on the session at hand, take part in the conversation, communicate with one another in more depth, and do so with a greater sense of fun.

4.4 Surgeries

The Metaverse is rapidly emerging as a significant technology in the field of medicine, particularly in the field of surgery. Surgeons are presently adopting a variety of instruments, such as VR headsets and haptic gloves, to simulate genuine surgical operations, which improves both their level of readiness and their level of productivity in the operating room. By removing barriers to data availability for surgeons, AR might make it easier for them to conduct surgery. Surgeons may have quick, convenient, and hands-free access to patient information using AR, which works by putting 3D virtual representations onto the body of the patient. Within the Metaverse, instructors and lecturers might demonstrate complex surgical procedures to students in a three-dimensional setting. Additionally, the Metaverse has the potential to be used in the delivery of counselling services to post-operative patients.

5. Motivations of Study

Even though the healthcare industry has advanced quickly, a number of problems still exist and discussed by prior studies as [7]. These problems are represented in the unwavering burden of chronic illnesses that last a lifetime, rising healthcare expenditures, an ageing population, a shortage of healthcare workers, and the scarcity of resources.

Also, [8] stated that the COVID-19 pandemic has put a great deal of strain on the labour, infrastructure, and supply chain management associated with the global healthcare industry. Hence, other scholars expressed their opinion about solutions for these problems. For instance, [9] where suggested that it is now essential to implement cutting-edge digital tools and services to maximize customer happiness, allowing for better tracking, health status monitoring, and medication adherence.in same vein [10] expressed that the organizations are pushing for the use of digital diagnostics, virtual care, at-home prescription delivery systems, remote monitoring, decision support systems, and self-service applications for social assistance and education. Lately, the Metaverse offers a plethora of opportunities for application in clinical treatment. In the surgeon's field of vision, real-time advice may be given through immersive experiences that are re-created from surgical procedures. AR will make information within the operating room's sterile environment accessible, improving the flexibility and precision of surgery. In addition to collaborative medical operations, the Metaverse will provide simultaneous planning, teaching, and training.

Based on the suggestion of previous researchers, the importance of applying Metaverse in the medical sector was demonstrated. Hence, this study suggested a mathematical decision-making framework (MDMF) to evaluate alternatives of Metaverse entities which contribute to reconstruction as well as medical administration also, its implications on human health and clinical practice.

Toward evaluation process, MCDM techniques have been leveraged to evaluate alternatives based on set of criteria.in this study, judgments and evaluations exposed to uncertainty situations in ambiguity environment. Hence, neutrosophic theory has been exploited for supporting MCDM techniques toward accurate evaluations and judgments.

6. Methodology: Mathematical Decision-Making Framework

This section demonstrates the followed methodology through employing BWM techniques in cooperation with TriNSs as branch of neutrosophic theory. The methodology of evaluation has been implemented according to several stages.

Stage 1: Determination of main factors

First factor: Estimating Set of several alternatives of Metaverse entities are determined as MeTs = {MeT_1, MeT_2,…etc.}.Herein, we estimated these alternatives as:

i. **MeT_1: Health Surveillance:** Patients according to [11] will be able to keep an eye on their health without physically visiting a hospital or scheduling doctor's appointments thanks to AI-based services in the Metaverse.
ii. **MeT_2: Leveraging a virtual environment for consultation:** people may be able to have virtual consultations with physicians or other healthcare providers [12].
iii. **MeT_3: Virtual surgical:** With the development of Metaverse technology, medical professionals may now operate virtually with great accuracy and little mistake from humans [12].

iv. **MeT4: Healthcare Education:** The patient in the virtual mode is carried to a batch of medical students. Otherwise, conventional methods [5].

Second Factor: Estimate various criteria related to employ Metaverse in healthcare sector as set of C= {C_1, C_2…etc.}.in this study the criteria included:

i. **C_1: Confidentiality and privacy:** It is vital to the healthcare industry. Certain medical data that is obtained within the Metaverse has to be safeguarded by current and upcoming privacy laws [2].
ii. **C2: Real-time guidance:** allow access to the information easily.
iii. **C3: patient care management:** To provide doctors a better view of the patient's interior anatomy, augmented reality (AR) can enable doctors to display medical pictures, such as CT (Computed Tomography) scans, directly onto patients and in line with their bodies even when the patients move.
iv. **C4: Need for skilled users:** Technology professions require knowledge and experience [13].
v. **C5: Bolster clinician judgement and guarantee more accurate treatments:** Combining other technologies as AI for enhance planning, instruction, and training concurrently using a cooperative platform for cooperative medical treatments and education [2].

Third Factor: Forming expert panel which consists of four members who related to study's scope.

Stage 2: Determination of criteria's weights

BWM based on TriNSs is applied to determine best and worst criterion based on its values of weights. The following steps are clarifying process of obtaining criteria's weights.

Step 1: the relationships between best and worst criterion with other are clarified where these relations are exhibited in the same vein of [14].

Expert panel is rating the best criterion_{Best} over other ceriteria_j as $\widetilde{\text{criterion}}_{\text{Best}} = (\widetilde{\text{criterion}}_{\text{Best }1}, \ldots, \widetilde{\text{criterion}}_{\text{Best }4})$ based on scale listed in Table 1.

Also [14] identify the relation between other criteria to criterion_{Worst} . Consequently, expert panel rating criteria_j over least desired/important criterion_{Worst} as $\widetilde{\text{criterion}}_{\text{Worst}} = (\widetilde{\text{criterion}}_{1\text{ Worst}}, \ldots, \widetilde{\text{criterion}}_{6\text{ Worst}})$ according to Table 1.

Step 2: Utilizing Eq. (1) to convert expert panel's evaluation from TriNSs scale into crisp values. Following that, Eq. (2) where crisp matrices are aggregate into an aggregated matrix.

$$s(x_{ij}) = \frac{(l_{ij}+m_{ij}+u_{ij})}{9} * (2 + \theta - \lambda - \gamma) \quad (1)$$

where $\boldsymbol{s}(x_{ij})$refers to score function. $\theta,\ \lambda,\ \gamma$ refers to truth, false, and indeterminacy respectively.

$$\text{Agg}_{ij} = \frac{(\sum_{j=1}^{S} x_{ij})}{M} \quad (2)$$

where x_{ij} refers to value of criterion in matrix, M refers to number of panel.

Table 1. Linguistic triangular neutrosophic scale

Crisp Scale	Explanation	TriNSs Scale
1	Equally Essential	<<1,1,1>;0.5,0.5,0.5>>
2	Slightly Moderately	<<1,2,3>;0.4,0.6,0.65>>
3	slightly Essential	<<2,3,4>;0.3,0.75,0.7>>
4	Minor To Strong	<<3,4,5>;0.35,0.6,0.4>>
5	Mighty Essential	<<4,5,6>;0.8,0.15,0.2>>
6	Slightly Strong Essential	<<5,6,7>;0.7,0.25,0.3>>
7	High Strong Essential	<<6,7,8>;0.9,0.1,0.1>>
8	Very High Strong Essential	<<7,8,9>;0.85,0.1,0.15>>
9	Absolutely High Essential	<<9,9,9>;0.1,0.0,0.0>>

Step 3: Find the optimal weights for determining criteria according to following Eqs:

$$\mathrm{minmax_j} = \left\{\left|\frac{w_B}{w_j} - \mathrm{criteria_{Bestj}}\right|, \left|\frac{w_j}{w_w} - \mathrm{criteria_{jworst}}\right|\right\}$$

$$\begin{aligned} &\mathrm{s.t} \\ &\sum w_j = 1 \\ &w_j \geq 0 \text{ for all j} \end{aligned} \tag{3}$$

$\mathrm{minmax_j}$ is converted to a linear model as:

$$\begin{aligned} &\min \varepsilon^L \\ &\mathrm{s.t} \\ &\left|w_B - \mathrm{criteria_{Bestj}} w_j\right| \leq \varepsilon^L, \text{for all j} \\ &\left|w_j - \mathrm{criteria_{jworst}} w_{worst}\right| \leq \varepsilon^L, \text{for all j} \\ &\sum w_j = 1 \\ &w_j \geq 0 \text{ for all j} \end{aligned} \tag{4}$$

Where w_{Best} is the weight of best criterion. w_{worst} is the weight of the worst indicator.

Stage 3: Finding optimal alternative.

Herein WSM based on TriNSs for ranking the estimated alternatives and obtain optimal one according to following steps.

Step 4: Each member in panel rate alternatives based on set of criteria and decision matrix is constructed for each member based on scale are listed in Table 1.

Step 5: we are exploiting Eq. (1) to deneutrosophic the matrices after that Eq. (2) aggregated these matrices into one decision matrix.

Step 6: The new developed decision matrix in pervious step normalize based on Eqs. (5-7).

$$\mathrm{Nor_{Agg}} = \frac{Y_{ij}}{\mathrm{sum}(Y_{ij})} \quad , For\ Benficial\ criteria \tag{5}$$

$$N = \frac{1}{Y_{Ij}} \tag{6}$$

$$\mathrm{Nor_{Aggj}} = \frac{N}{\mathrm{sum}(N)} \quad , For\ Non-Benficial\ criteria \tag{7}$$

Where Y_{ij} indicates to each element in the aggregated matrix.

Step 7: The obtained criteria′ weights of BWM are applied in the following Eq. (8) to generate weighted decision matrix.

$$\mathrm{Dec\_mat_{ij}} = \mathrm{weight_i} * \mathrm{Nor_{Agg}} \tag{8}$$

Where $\mathrm{Dec\_mat_{ij}}$ is weighted decision matrix.

Step 8: Utilizing Eq. (9) contributes to calculate global score. Based on values of $V(\mathrm{Dec\_mat_{ij}})$, ranking process for set of En (n) perform and obtain optimal and worst alternative.

$$V(\mathrm{Dec\_mat_{ij}}) = \sum_{j=1}^{n} \mathrm{Dec\_mat_{ij}} \tag{9}$$

Where $V(\mathrm{Dec\_mat_{ij}})$ is global score values.

7. Empirical Case Study

Our constructed MDMF is applied on real healthcare organization which embrace our study's notion. The purpose of this step is to validate our MDMF. We communicated to four experts to contribute to evaluation process.

7.1 Scoring alternatives and criteria

- Each member in constructed panel is forming his/her decision matrix based on determined the best and worst criterion where C2 is best criterion and C3 is worst criterion based on scale in Table 1.
- The constructed TriN decision matrices are converted to crisp decision matrices through Eq. (1).
- Eq. (2) played vital role toward aggregating crisp matrices into one decision matrix.

7.2 Estimation criteria's weights

- Eqs. (3-4) are applied to obtain the final criteria' weight.
- Figure 2 showcases the final criteria's weights where C_2 is optimal with highest weight value. Otherwise, C_3 is worst where weight value is least.

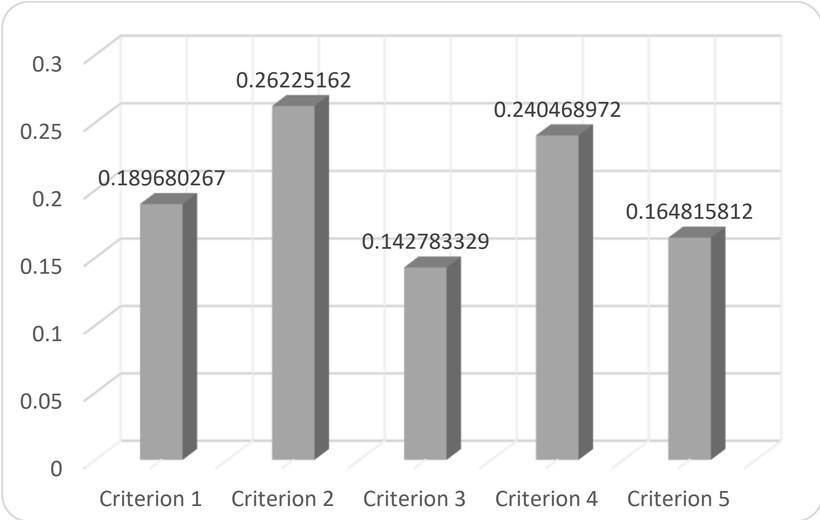

Figure 2. Final criteria weights based on BWM-TriNSs

7.3 Finding optimal alternative

- Four experts are rating alternatives based on criteria through Table 1. Hence, four TriNS decision matrices are constructed.
- Eq. (1) is converting TriNS decision matrices to crisp matrices and these matrices are aggregated as in Table 2 through Eq. (2).
- We normalized the aggregated matrix by employing Eq. (5) where all criteria in this study are beneficial criteria. Table 3 represents normalized matrix.
- The normalized decision matrix is multiply by weights obtained from BWM-TriNSs as in Eq. (8) to generate weighted decision matrix as listed in Table 4.
- Finally, Figure 3 illustrates the ranking of MeT alternatives based on value of global score. These values recommended that MeT_1 is optimal otherwise MeT_3 is the worst one.

Table 2. An aggregated decision matrix.

	C_1	C_2	C_3	C_4	C_5
MeT_1	2.4875	0.85	1.8	1.395833333	2.941666667
MeT_2	1.325	0.588888889	0.4	0.333333333	0.761111111
MeT_3	0.5	0.5	0.4	0.488888889	0.366666667
MeT_4	3.275	0.427777778	0.394444444	0.522222222	0.872222222

Table 3. Normalized decision matrix.

	C_1	C_2	C_3	C_4	C_5
MeT_1	0.327841845	0.35915493	0.601113173	0.509376584	0.595278246
MeT_2	0.174629325	0.248826291	0.133580705	0.121642169	0.154019112
MeT_3	0.065897858	0.211267606	0.133580705	0.178408515	0.074198988
MeT_4	0.431630972	0.180751174	0.131725417	0.190572732	0.176503654

Table 4. Weighted normalized decision matrix.

	C_1	C_2	C_3	C_4	C_5
MeT_1	0.062185129	0.094188962	0.08582894	0.122489264	0.098111268
MeT_2	0.033123737	0.065255098	0.019073098	0.029251167	0.025384785
MeT_3	0.012499523	0.055405272	0.019073098	0.042901712	0.012229167
MeT_4	0.081871878	0.047402288	0.018808194	0.045826829	0.029090593

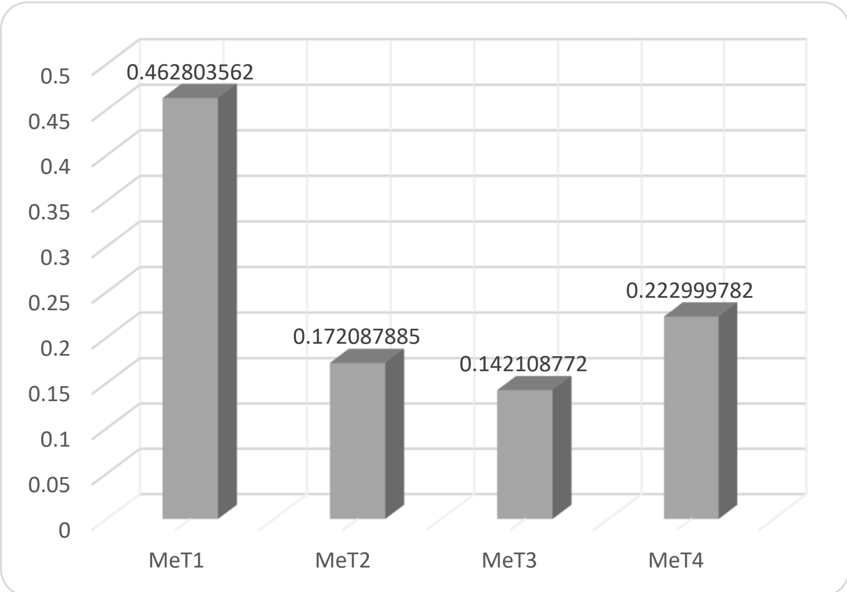

Figure 3. Final ranking for alternatives based on WSM-TriNSs.

8. Conclusions

In this paper, we provided an overview of the potential uses of the Metaverse in the medical field. We spoke about the Metaverse and its most important characteristics. We also introduced the primary technologies that make the Metaverse possible and outlined the roles those technologies play. We investigated the many ways in which the Metaverse may revolutionize medical practice. For the effective incorporation of the Metaverse into healthcare, we have outlined the essential components and prerequisites that must be addressed. We spoke about a few of the key problems that need to be overcome in order to make sure that the Metaverse can be successfully implemented in the healthcare industry. The following is a list of many potential future research directions.

- The technologies that make the Metaverse possible will have a significant impact on the direction in which it will develop in the future. It is exceedingly difficult to handle the related

expenses of integrating the Metaverse with current systems and giving equitable access to all users if the creators of these technologies do not properly align themselves with one another.
- It is anticipated that the incorporation of the Metaverse into healthcare would result in the emergence of new societal problems, particularly for those who participate in the Metaverse to an unhealthy degree. As a result, there has to be an established strategy on how to strike a healthy balance between online and offline activities.
- The Metaverse has to have well-established norms, standards, and codes of ethics before it can be utilized by a significant number of people.

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Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors.

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An Integrated Neutrosophic Approach for the Urban Energy Internet Assessment under Sustainability Dimensions

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Abstract: The urgent issue of climate change has resulted in an increasing preoccupation with the transition to low-carbon energy. The urban energy internet (UEI) enables the efficient utilization of renewable energy via the integration of contemporary energy grids, smart energy services, and cyber-physical systems. Consequently, the assessment of urban energy is vital for the construction of low-carbon cities. This study intends to provide an integrated decision-making approach for addressing assessment challenges related to UEI, with a focus on sustainability. The introduced approach adopts the opinions of three experts to express their opinions through semantic terms in evaluating sustainability factors and evaluating UEI projects. Determining the most suitable project for sustainable UEI is a difficult task. In addition, there are multiple sustainability dimensions, including: economic, social, technical, environmental and resource dimension. This paper presents an integrated approach through which experts can use semantic terms to express their opinions to evaluate the priorities of sustainability dimensions that affect the sustainability of UEI projects. Therefore, this study applies a hybrid multi-criteria decision making (MCDM) approach that takes into account multiple dimensions. Also, the uncertainty in the study is handled by type 2 neutrosophic numbers (T2NNs). Initially, the CRiteria Importance through Inter-criteria Correlation (CRITIC) method was used to determine the relative importance of the sustainability dimensions used in the study. Also, the evaluation based on distance from average solution (EDAS) method was used to rank the selected alternatives. The results indicate that the technical dimension is the most important with a weight of 0.384. Finally, a sensitivity analysis was conducted to prove the validity and reliability of the developed framework with differences in weights.

Keywords: Urban Energy Internet; Neutrosophic; T2NN; Sustainability; Uncertainty; MCDM; T2NN-CRITIC; T2NN-EDAS.

1. Introduction

Presently, the primary cause of energy issues is overdependence on traditional fossil fuels. The excessive dependence on a single source, together with inefficiencies and the squandering of resources in the generation, distribution, and utilization of energy, results in the depletion of resources and the degradation of the environment. In addition, the increasing worldwide need for energy is also restricted by restrictions on the exploration and use of conventional energy sources. The energy internet has arisen as a solution to meet the current challenges in the energy sector [1].

The urban energy internet (UEI) is an application of the energy internet that is specifically implemented at the city level [2]. It involves the practical application of the idea and technology of the energy internet to the energy system of a city. Scholars are presently giving significant attention to and showing interest in researching the UEI. Research in this area encompasses several subjects, such as the theoretical foundation and core concepts, the utilization of cutting-edge technology in their execution, and the development and enhancement of intelligent energy systems. These studies have played a role in the development of the UEI.

It is the primary goal of the UEI to achieve efficient energy allocation, intelligent management, and green low-carbon supply inside the city [3]. This will result in increased energy utilization efficiency, decreased carbon emissions, and the promotion of sustainable urban development strategies. In spite of the increased interest in UEI and the potential benefits it may provide, there is still a significant research vacuum in the assessment of UEI from the point of view of sustainability. For this reason, it is essential to build a comprehensive collection of assessment index systems and to make use of plausible scientific frameworks in order to evaluate the UEI from the point of view of sustainability [4]. It is possible to frame this issue as a complicated multi-criteria decision-making (MCDM) model because of its complexity [5].

By relying only on accurate information throughout the review process, it is often not possible to adequately portray the complexities of the actual world. It is the inherent ambiguity and subjectivity that are inherent to human decision-making processes that are the source of this insufficiency for humans. It has been shown that neutrosophic set theory is an effective solution to the issue of ambiguity in MCDM [6]. Therefore, the study relies mainly on presenting a multi-criteria decision-making approach consisting of the CRITIC and EDAS methods. The criteria importance through inter-criteria correlation (CRITIC) method is used to evaluate and prioritize sustainability dimensions [7]. Also, the evaluation based on distance from average solution (EDAS) is applied to evaluate and rank alternatives for urban energy internet [8]. In addition, the study is being conducted in a neutrosophic environment and using type 2 neutrosophic numbers (T2NNs).

The rest of this study is planned as follows: Section 2 introduces sustainability dimensions for the urban energy internet. Section 3 describes some definitions and preliminaries related to type 2 neutrosophic numbers and the applied approach. Section 4 shows the application of the introduced approach. In Section 5, the conclusions are demonstrated.

2. Sustainability dimensions

In this section, we present the dimensions of sustainability that have a direct impact on achieving sustainability in the urban energy internet. Figure 1 shows the five dimensions used, which are economic, social, technical, environmental and resource. Also, a detailed explanation of the five dimensions is provided as follows.

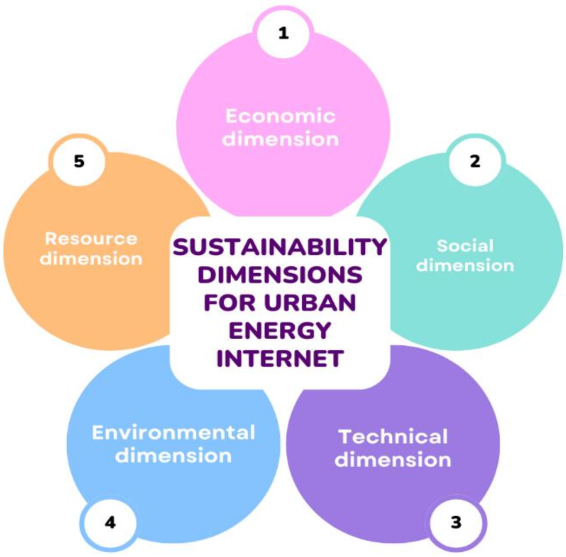

Figure 1. Sustainability dimensions for urban energy internet.

2.1 Economic dimension

The economic feature of an energy internet system pertains to its capacity to function and create sustainable revenues in the long run. Key elements that influence the economic viability of an energy internet system include initial investment expenses, governmental financial aid and inducements, and ongoing operational expenditures. Financial sustainability and return on investment are crucial factors to examine when analyzing energy internet systems, since they determine the viability and profitability of investments in energy infrastructure.

2.2 Social dimension

The social dimension encompasses the incorporation of social considerations into the design and functioning of urban energy systems, with the goal of attaining social fairness, engagement, and inclusiveness. The urban energy internet promotes social innovation and participation by offering open innovation platforms and cooperation methods to use the intellect and inventive capacities of different stakeholders. Moreover, it is crucial for the general populace to comprehend the importance of energy matters and the advantages of using environmentally friendly energy sources and energy-conserving strategies. By implementing educational initiatives and raising awareness via campaigns, society may be motivated to adopt environmentally conscious behaviors and contribute to the shift towards sustainable energy.

2.3 Technical dimension

During the planning, construction, and operation of energy infrastructure, the utilization of environmentally friendly and cutting-edge technology is referred to as the technical dimension. A few examples of these technologies include artificial intelligence, predictive analytics, smart grid technology, and sophisticated energy management systems. Increasing energy efficiency, lowering environmental effects, and contributing to the transformation of energy systems that are more sustainable and resilient are all things that may be accomplished via the use of these modern and environmentally friendly technologies.

2.4 Environmental dimension

Improving the influence that energy systems have on environmental components, including water resources, land, and biodiversity, is what is meant by the environmental dimension. Urban energy has the potential to reduce air pollution and environmental degradation, protect the health of ecosystems, and, as a result, promote environmental sustainability. This is accomplished through the utilization of clean energy and energy strategies that are favorable to the environment.

2.5 Resource dimension

The term "resource dimension" refers to the rational use and management of resources throughout the processes of energy generation, transmission, storage, and utilization. This is done in order to guarantee a sustainable supply and maximize resource efficiency. Internet advocates for the widespread use of renewable energy sources such as solar energy, wind energy, hydropower, and biomass. Urban energy internet advocates for this cause. As a result of their sustainable and renewable properties, these energy sources lessen reliance on limited resources and lessen the negative effects on the environment.

3. Study approach

In this section, the steps of the proposed methodology for conducting a study to evaluate the urban energy internet are presented as exhibited in Figure 2. The presented approach is based on two MCDM methods: CRITIC and TOPSIS. This study is also being conducted in a neutrosophic environment, specifically using T2NNs. In this regard, this section is divided into two parts. The first part provides some preliminaries and definitions about T2NNs. The second part presents the proposed approach for evaluating the urban energy internet.

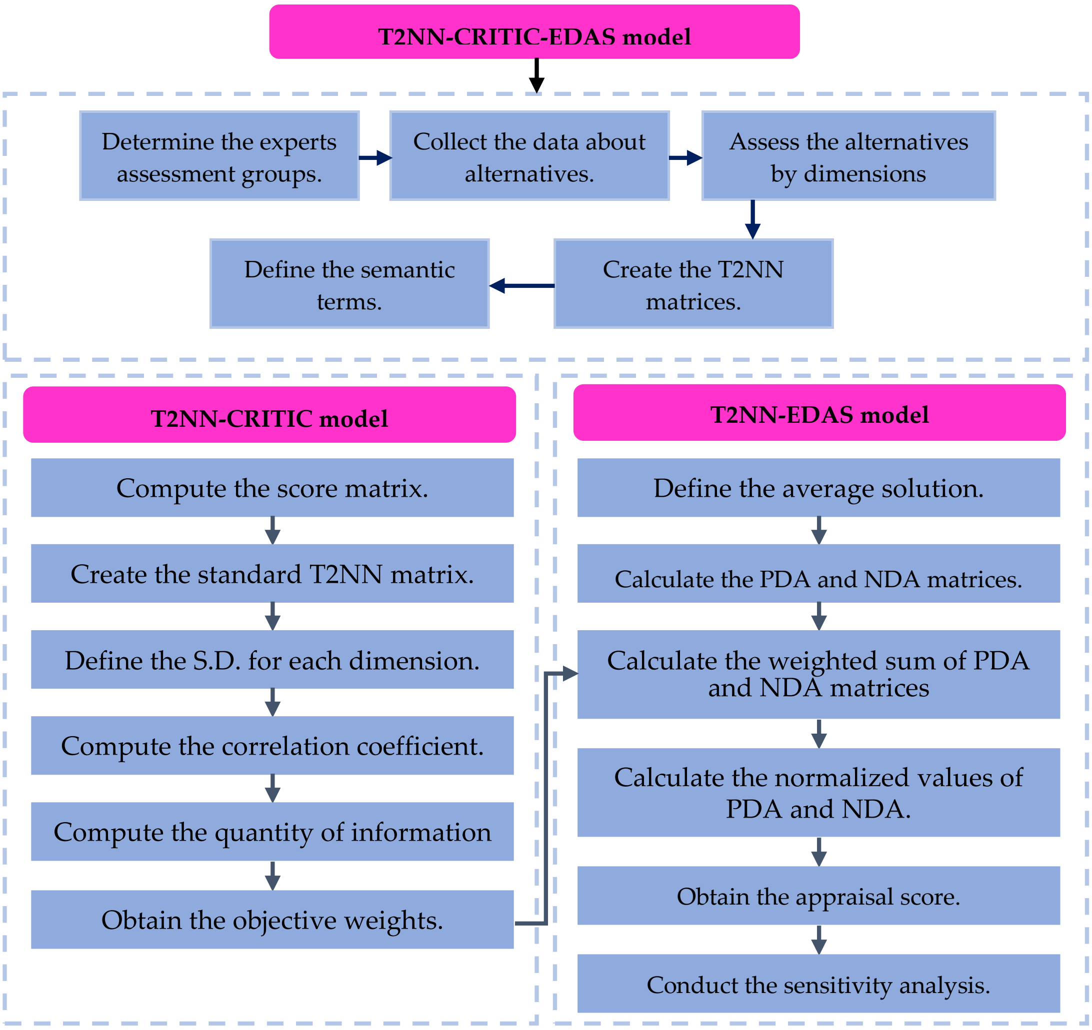

Figure 2. Flowchart of the applied approach.

3.1 Preliminaries and definitions

In this part, a set of definitions and preliminaries related to type 2 neutrosophic numbers are presented.

Definition 1. [9] Let X as the finite universe of discourse and $D[0,1]$, as the set of all triangular neutrosophic sets on $D[0,1]$. A type-2 neutrosophic number set (T2NNS) characterized by $\tilde{Z}$ can be well-defined in X as an object having the form:

$$\tilde{Z} = \{\langle x, \tilde{T}_{\tilde{Z}}(y), \tilde{I}_{\tilde{Z}}(y), \tilde{F}_{\tilde{Z}}(x) | \; x \in X\rangle\}, \tag{1}$$

where, $\tilde{T}_{\tilde{Z}}(x) : X \to D[0,1], \tilde{I}_{\tilde{Z}}(x) : X \to D[0,1], \tilde{F}_{\tilde{Z}}(x) : X \to D[0,1]$. The T2NNS $\tilde{T}_{\tilde{Z}}(x)$ $=\left(T_{T_{\tilde{Z}}}(x), T_{I_{\tilde{Z}}}(x), T_{F_{\tilde{Z}}}(x)\right)$, $\tilde{I}_{\tilde{Z}}(x) = \left(I_{T_{\tilde{Z}}}(x), I_{I_{\tilde{Z}}}(x), I_{F_{\tilde{Z}}}(x)\right)$, $\tilde{F}_{\tilde{Z}}(x) = \left(F_{T_{\tilde{Z}}}(x), F_{I_{\tilde{Z}}}(x), F_{F_{\tilde{Z}}}(x)\right)$, represent the truth, indeterminacy, and falsity memberships of x in $\tilde{Z}$, respectively.

The following conditions are satisfied by the membership parameters:

$$0 \leq \tilde{T}_{\tilde{Z}}(x)^3 + \tilde{I}_{\tilde{Z}}(x)^3 + \tilde{F}_{\tilde{Z}}(x)^3 \leq 3, \; \forall \; x \in X. \tag{2}$$

For ease of simplicity,

$\tilde{Z} = \langle\left(T_{T_{\tilde{Z}}}(x), T_{I_{\tilde{Z}}}(x), T_{F_{\tilde{Z}}}(x)\right), \left(I_{T_{\tilde{Z}}}(x), I_{I_{\tilde{Z}}}(x), I_{F_{\tilde{Z}}}(x)\right), \left(F_{T_{\tilde{Z}}}(x), F_{I_{\tilde{Z}}}(x), F_{F_{\tilde{Z}}}(x)\right)\rangle$ is determined as the T2NN.

Definition 2. [9] Let $\tilde{Z} = \left\langle \left(T_{T_{\tilde{Z}}}(x), T_{I_{\tilde{Z}}}(x), T_{F_{\tilde{Z}}}(x)\right), \left(I_{T_{\tilde{Z}}}(x), I_{I_{\tilde{Z}}}(x), I_{F_{\tilde{Z}}}(x)\right), \left(F_{T_{\tilde{Z}}}(x), F_{I_{\tilde{Z}}}(x), F_{F_{\tilde{Z}}}(x)\right) \right\rangle$ be a T2NN. The score function of the T2NN $\tilde{Z}$ is computed as:

$$S(\tilde{Z}) = \frac{1}{12}\left\langle 8 + \left(T_{T_{\tilde{Z}}}(x) + 2\left(T_{I_{\tilde{Z}}}(x)\right) + T_{F_{\tilde{Z}}}(x)\right) - \left(I_{T_{\tilde{Z}}}(x) + 2\left(I_{I_{\tilde{Z}}}(x)\right) + I_{F_{\tilde{Z}}}(x)\right) - \left(F_{T_{\tilde{Z}}}(x) + 2\left(F_{I_{\tilde{Z}}}(x)\right) + F_{F_{\tilde{Z}}}(x)\right) \right\rangle \tag{3}$$

Definition 3. [9] Let $\tilde{Z}_S = \left\langle \left(T_{T_{\tilde{Z}_S}}(x), T_{I_{\tilde{Z}_S}}(x), T_{F_{\tilde{Z}_S}}(x)\right), \left(I_{T_{\tilde{Z}_S}}(x), I_{I_{\tilde{Z}_S}}(x), I_{F_{\tilde{Z}_S}}(x)\right), \left(F_{T_{\tilde{Z}_S}}(x), F_{I_{\tilde{Z}_S}}(x), F_{F_{\tilde{Z}_S}}(x)\right) \right\rangle$ $(S = 1, 2, \ldots, p)$ is a set of T2NNs, and w = $(\mathrm{w}_1, \ldots, \mathrm{w}_S, \ldots, \mathrm{w}_q)^T$ be the weight vector of them with $\mathrm{w}_j \in [0, 1]$ and $\sum_{S=1}^{p} \mathrm{w}_S = 1$. A type 2 neutrosophic numbers weighted average (T2NNWA) operator is determined as follows:

$$\text{T2NNW}A_{\mathrm{w}}\ (\tilde{Z}_1,\ \tilde{Z}_S,\ \ldots, \tilde{Z}_p) = \mathrm{w}_1\tilde{Z}_1 \oplus \mathrm{w}_S\tilde{Z}_S \oplus \ldots \oplus \mathrm{w}_p\tilde{Z}_p = \bigoplus_{S=1}^{p} (\mathrm{w}_S\tilde{Z}_S) = \tag{4}$$

$$\left\langle \begin{array}{c} \left(1-\prod_{S=1}^{p}\left(1-T_{T_{\tilde{Z}_S}}(x)\right)^{\mathrm{w}_S}, 1-\prod_{S=1}^{p}\left(1-T_{I_{\tilde{Z}_S}}(x)\right)^{\mathrm{w}_S}, 1-\prod_{S=1}^{p}\left(1-T_{F_{\tilde{Z}_S}}(x)\right)^{\mathrm{w}_S}\right), \\ \left(\prod_{S=1}^{p}\left(I_{T_{\tilde{Z}_S}}(x)\right)^{\mathrm{w}_S}, \prod_{S=1}^{p}\left(I_{I_{\tilde{Z}_S}}(x)\right)^{\mathrm{w}_S}, \prod_{S=1}^{p}\left(I_{F_{\tilde{Z}_S}}(x)\right)^{\mathrm{w}_S}\right), \\ \left(\prod_{S=1}^{p}\left(F_{T_{\tilde{Z}_S}}(x)\right)^{\mathrm{w}_S}, \prod_{S=1}^{p}\left(F_{I_{\tilde{Z}_S}}(x)\right)^{\mathrm{w}_S}, \prod_{S=1}^{p}\left(F_{F_{\tilde{Z}_S}}(x)\right)^{\mathrm{w}_S}\right) \end{array} \right\rangle \tag{5}$$

3.2 The suggested model

In this section, the steps of the proposed approach to solve the urban energy internet selection problem are presented. The proposed approach consists of two MCDM methods, namely the DEMATEL method and the TOPSIS method. The proposed approach is implemented using T2NNs. Accordingly, the T2NN- CRITIC method is applied to evaluate and determine the weights of the dimensions used in the evaluation process. In addition, the T2NN-TOPSIS method is used to evaluate and rank the selected alternatives in the study.

Step 1. The problem is studied and the sustainability factors affecting the arrangement of five urban energy internet projects are identified. Also, the experts participating in the study are identified provided that they belong to the business and academic fields, as presented in Table 1.

Table 1. Information of experts participating in the study.

K	Expert	Experience (Years)	Occupation	Background	Academic degree
1	Expert_1	20	Academia	Energy management	Ph.D.
2	Expert_2	15	Industry	Energy management	Ph.D.
3	Expert_3	15	Industry	Energy management	M.Sc.

Step 2. The dimensions are determined through an analysis of the related literature, in addition to the opinions of the participating experts. $D_j = (D_1, D_2, \ldots, D_n)$, with *j = 1, 2, …, n*. Let $w = (w_1, w_2, \ldots, w_n)$ be the vector set used for outlining the dimensions weights, where $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Finally, the set $A_i = \{A_1, A_2, \ldots, A_m\}$, having i = 1, 2, …, m alternatives, is measured by n dimensions of set $D_j = \{D_1, D_2, \ldots, D_n\}$, with *j = 1, 2, …, n*.

Step 3. A set of semantic terms and their corresponding T2NNs are defined for experts to use in the evaluation process, whether the dimensions or the selected alternatives, as exhibited in Table 2.

Table 2. T2NN semantic terms for weighing dimensions and assessing alternatives.

Semantic terms	Acronyms	Type-2 neutrosophic number
Extremely low	ETL	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.80, 0.70)\rangle$
Low	LLO	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.75, 0.65)\rangle$
Medium low	MDM	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$
Medium	MEM	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$
Medium high	MGH	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.25, 0.15)\rangle$
High	HIH	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$
Extremely high	ELH	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$

3.2.1 T2NN-CRITIC method

Step 4. Construct a pairwise comparison matrix for dimensions by all experts to show their preferences for these dimensions. A pairwise decision matrix is created by using semantic terms that presented in Table 2, then by using T2NNs exhibited in Table 2 according to Eq. (6). Suppose a set of n dimensions is represented by $D = \{D_1, D_2, \ldots, D_n\}$ is assessed by a set of experts K_t(t = 1, 2... l) who presented their assessment report for each criterion D_j(j = 1, 2... n).

$$\widetilde{M} = \begin{array}{c} \\ K_1 \\ \vdots \\ K_t \end{array} \begin{array}{ccc} D_1 & \cdots & D_n \\ \left[\begin{array}{c} \left(T_{T_{\tilde{z}_1^{(1)}}}(x), T_{I_{\tilde{z}_1^{(1)}}}(x), T_{F_{\tilde{z}_1^{(1)}}}(x)\right), \\ \left(I_{T_{\tilde{z}_1^{(1)}}}(x), I_{I_{\tilde{z}_1^{(1)}}}(x), I_{F_{\tilde{z}_1^{(1)}}}(x)\right), \\ \left(F_{T_{\tilde{z}_1^{(1)}}}(x), F_{I_{\tilde{z}_1^{(1)}}}(x), F_{F_{\tilde{z}_1^{(1)}}}(x)\right) \end{array}\right] & \cdots & \left[\begin{array}{c} \left(T_{T_{\tilde{z}_t^{(t)}}}(x), T_{I_{\tilde{z}_t^{(t)}}}(x), T_{F_{\tilde{z}_t^{(t)}}}(x)\right), \\ \left(I_{T_{\tilde{z}_t^{(t)}}}(x), I_{I_{\tilde{z}_t^{(t)}}}(x), I_{F_{\tilde{z}_t^{(t)}}}(x)\right), \\ \left(F_{T_{\tilde{z}_t^{(t)}}}(x), F_{I_{\tilde{z}_t^{(t)}}}(x), F_{F_{\tilde{z}_t^{(t)}}}(x)\right) \end{array}\right] \\ \vdots & \cdots & \vdots \\ \left[\begin{array}{c} \left(T_{T_{\tilde{z}_1^{(1)}}}(x), T_{I_{\tilde{z}_1^{(1)}}}(x), T_{F_{\tilde{z}_1^{(1)}}}(x)\right), \\ \left(I_{T_{\tilde{z}_1^{(1)}}}(x), I_{I_{\tilde{z}_1^{(1)}}}(x), I_{F_{\tilde{z}_1^{(1)}}}(x)\right), \\ \left(F_{T_{\tilde{z}_1^{(1)}}}(x), F_{I_{\tilde{z}_1^{(1)}}}(x), F_{F_{\tilde{z}_1^{(1)}}}(x)\right) \end{array}\right] & \cdots & \left[\begin{array}{c} \left(T_{T_{\tilde{z}_t^{(t)}}}(x), T_{I_{\tilde{z}_t^{(t)}}}(x), T_{F_{\tilde{z}_t^{(t)}}}(x)\right), \\ \left(I_{T_{\tilde{z}_t^{(t)}}}(x), I_{I_{\tilde{z}_t^{(t)}}}(x), I_{F_{\tilde{z}_t^{(t)}}}(x)\right), \\ \left(F_{T_{\tilde{z}_t^{(t)}}}(x), F_{I_{\tilde{z}_t^{(t)}}}(x), F_{F_{\tilde{z}_t^{(t)}}}(x)\right) \end{array}\right] \end{array} \tag{6}$$

Step 5. Transform the T2NNs to real values according to Eq. (3).
Step 6. Compute the normalized decision matrix for dimensions according to Eq. (7) and Eq. (8).
For benefit dimensions:

$$x_{ij}^{*} = \frac{x_{ij} - min(x_{ij})}{max(x_{ij}) - min(x_{ij})} \quad \text{i = 1, 2...m and j = 1, 2...n.} \tag{7}$$

For cost dimensions:

$$x_{ij}^{*} = 1 - \frac{x_{ij} - min(x_{ij})}{max(x_{ij}) - min(x_{ij})} \quad \text{i = 1, 2...m and j = 1, 2...n.} \tag{8}$$

Step 7. Computation of the values of the matrix's standard deviation and linear correlation per column. Then, determining the amount of information of dimensions according to Eq. (9).

$$u_j = \sigma_j \;.\; \sum_{q=1}^{m}\left(1 - r_{jq}\right) \tag{9}$$

where σ_j is the standard deviation of the indices, and r_{jq} is linear correlation coefficient for the dimensions.
Step 8. Determine the dimensions weights according to Eq. (10).

$$w_j = \frac{u_j}{\sum_{q=1}^{m} u_q} \tag{10}$$

3.2.2 T2NN-EDAS method

Step 9. Construct an evaluation decision matrix $\left[\tilde{Q}_{ij}{}^{s}\right]_{m\times n}$ among dimensions determined and the alternatives by all experts according to Eq. (11) using semantic terms and T2NNs exhibited in Table 2.

$$\tilde{Q} = \begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & \cdots & D_n \end{array} \\ \begin{bmatrix} \begin{bmatrix} \left(T_{T_{\tilde{Q}_{11}}{}^{(s)}}(x), T_{I_{\tilde{Q}_{11}}{}^{(s)}}(x), T_{F_{\tilde{Q}_{11}}{}^{(s)}}(x)\right), \\ \left(I_{T_{\tilde{Q}_{11}}{}^{(s)}}(x), I_{I_{\tilde{Q}_{11}}{}^{(s)}}(x), I_{F_{\tilde{Q}_{11}}{}^{(s)}}(x)\right), \\ \left(F_{T_{\tilde{Q}_{11}}{}^{(s)}}(x), F_{I_{\tilde{Q}_{11}}{}^{(s)}}(x), F_{F_{\tilde{Q}_{11}}{}^{(s)}}(x)\right) \end{bmatrix} & \cdots & \begin{bmatrix} \left(T_{T_{\tilde{Q}_{1n}}{}^{(s)}}(x), T_{I_{\tilde{Q}_{1n}}{}^{(s)}}(x), T_{F_{\tilde{Q}_{1n}}{}^{(s)}}(x)\right), \\ \left(I_{T_{\tilde{Q}_{1n}}{}^{(s)}}(x), I_{I_{\tilde{Q}_{1n}}{}^{(s)}}(x), I_{F_{\tilde{Q}_{1n}}{}^{(s)}}(x)\right), \\ \left(F_{T_{\tilde{Q}_{1n}}{}^{(s)}}(x), F_{I_{\tilde{Q}_{1n}}{}^{(s)}}(x), F_{F_{\tilde{Q}_{1n}}{}^{(s)}}(x)\right) \end{bmatrix} \\ \vdots & \cdots & \vdots \\ \begin{bmatrix} \left(T_{T_{\tilde{Q}_{m1}}{}^{(s)}}(x), T_{I_{\tilde{Q}_{m1}}{}^{(s)}}(x), T_{F_{\tilde{Q}_{m1}}{}^{(s)}}(x)\right), \\ \left(I_{T_{\tilde{Q}_{m1}}{}^{(s)}}(x), I_{I_{\tilde{Q}_{m1}}{}^{(s)}}(x), I_{F_{\tilde{Q}_{m1}}{}^{(s)}}(x)\right), \\ \left(F_{T_{\tilde{Q}_{m1}}{}^{(s)}}(x), F_{I_{\tilde{Q}_{m1}}{}^{(s)}}(x), F_{F_{\tilde{Q}_{m1}}{}^{(s)}}(x)\right) \end{bmatrix} & \cdots & \begin{bmatrix} \left(T_{T_{\tilde{Q}_{mn}}{}^{(s)}}(x), T_{I_{\tilde{Q}_{mn}}{}^{(s)}}(x), T_{F_{\tilde{Q}_{mn}}{}^{(s)}}(x)\right), \\ \left(I_{T_{\tilde{Q}_{mn}}{}^{(s)}}(x), I_{I_{\tilde{Q}_{mn}}{}^{(s)}}(x), I_{F_{\tilde{Q}_{mn}}{}^{(s)}}(x)\right), \\ \left(F_{T_{\tilde{Q}_{mn}}{}^{(s)}}(x), F_{I_{\tilde{Q}_{mn}}{}^{(s)}}(x), F_{F_{\tilde{Q}_{mn}}{}^{(s)}}(x)\right) \end{bmatrix} \end{bmatrix} \end{array}, \quad (11)$$

Step 10. Transform the T2NNs to real values according to Eq. (3).
Step 11. Calculate the average value for each dimension according to Eq. (12).

$$AV_j = \sum_{i=1}^{n} x_{ij}/n \quad (12)$$

Step 12. Compute the positive distance from average matrix $PDA = \left[PDA_j\right]_{n\times m}$ according to Eq. (13), and the negative distance from average matrix $NDA = \left[NDA_j\right]_{n\times m}$ according to Eq. (14), accordingly with the type of criterion.

$$PDA_{ij} = max\left(0, \left(x_{ij} - AV_j\right)\right)/AV_j \ , \ NDA_{ij} = max\left(0, \left(AV_j - x_{ij}\right)\right)/AV_j \quad (13)$$

$$PDA_{ij} = max\left(0, \left(AV_j - x_{ij}\right)\right)/AV_j \ , \ NDA_{ij} = max\left(0, \left(x_{ij} - AV_j\right)\right)/AV_j \quad (14)$$

Step 13. Compute the weighted sum of PDA according to Eq. (15) and NDA according to Eq. (16) for each of the m alternatives according to n dimensions.

$$SP_i = \sum_{j=1}^{m} w_j PDA_{ij} \quad (15)$$

$$SN_i = \sum_{j=1}^{m} w_j NDA_{ij} \quad (16)$$

Step 14. Obtain the normalized values for each SP_i and SN_i according to Eq. (17) and Eq. (18).

$$NSP_i = SP_i/max_{i\,\in\{1,\ldots,n\}}(SP_i) \quad (17)$$

$$NSN_i = 1 - SN_i/max_{i\,\in\{1,\ldots,n\}}(SN_i) \quad (18)$$

Step 15. Determine the assessment score AS_i for each alternative according to Eq. (19).

$$AS_i = 0.5\ \left(NSP_i + NSN_i\right) \quad (19)$$

Step 16. Order the alternatives from largest to smallest according to AS_i. The best alternative has the greatest AS_i.

4. Application

4.1 Application of the suggested approach

In this part, the steps of the T2NN-CRITIC-EDAS approach are applied to evaluate and rank five Urban Energy Internet projects.

Step 1. Initially, the problem was studied in detail and its main goal was set, which was to arrange five urban energy internet projects.

Step 2. Three experts in the field of energy management were agreed to assist the authors in evaluating the sustainability dimensions used in the evaluation process as well as ranking the five urban energy projects selected for the study. The five sustainability dimensions used are: economic (D_1), social (D_2),

technical (D_3), environmental (D_4) and resource (D_5). Also, the five selected urban energy internet projects are symbolized by A_1, A_2, A_3, A_4, and A_5.
Step 3. A decision matrix for dimensions was constructed by all experts to show their preferences for these dimensions. The decision matrix is created by applying semantic terms in Table 2 and then by T2NNs in Table 2 according to Eq. (6), as presented in Table 3 and Table 4, respectively.
Step 4. The T2NNs were transformed to real values according to Eq. (3). Then, the normalized decision matrix was calculated by applying Eq. (7) as exhibited in Table 5.
Step 5. The values of the matrix's standard deviation and linear correlation per column were calculated. Then, the amount of information of dimensions was identified according to Eq. (9) as shown in Table 5.
Step 6. The dimensions weights were obtained according to Eq. (10) as exhibited in Table 6 and shown in Figure 3.

Table 3. Assessment matrix of dimensions by the three experts using semantic terms.

Dimensions	$\mathbf{D_1}$	$\mathbf{D_2}$	$\mathbf{D_3}$	$\mathbf{D_4}$	$\mathbf{D_5}$
$\mathbf{Expert_1}$	ETL	LLO	ETL	MEM	ETL
$\mathbf{Expert_2}$	ELH	MDM	ETL	HIH	ELH
$\mathbf{Expert_3}$	HIH	MEM	LLO	MGH	MEM

Table 4. Assessment matrix of dimensions by the three experts using T2NN.

Ds.	$\mathbf{Expert_1}$	$\mathbf{Expert_2}$
$\mathbf{D_1}$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.80, 0.70)\rangle$	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$
$\mathbf{D_2}$	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.75, 0.65)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$
$\mathbf{D_3}$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.80, 0.70)\rangle$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.80, 0.70)\rangle$
$\mathbf{D_4}$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$
$\mathbf{D_5}$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.80, 0.70)\rangle$	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$
Ds.	$\mathrm{Expert_3}$	
$\mathbf{D_1}$	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$	
$\mathbf{D_2}$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	
$\mathbf{D_3}$	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.75, 0.65)\rangle$	
$\mathbf{D_4}$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.25, 0.15)\rangle$	
$\mathbf{D_5}$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	

Table 5. Normalized decision matrix of dimensions.

Criteria	$\mathbf{D_1}$	$\mathbf{D_2}$	$\mathbf{D_3}$	$\mathbf{D_4}$	$\mathbf{D_5}$
$\mathbf{Expert_1}$	0.0000	0.0000	0.0000	0.0000	0.0000
$\mathbf{Expert_2}$	1.0000	0.5618	0.0000	1.0000	1.0000
$\mathbf{Expert_3}$	0.8321	1.0000	1.0000	0.5588	0.4877
$\boldsymbol{\sigma_j}$	0.5355	0.5013	0.5774	0.5012	0.5001

Table 6. Final weights for sustainability dimensions.

Ds.	$\mathbf{D_1}$	$\mathbf{D_2}$	$\mathbf{D_3}$	$\mathbf{D_4}$	$\mathbf{D_5}$	$\boldsymbol{\sigma_j}$	$\boldsymbol{u_j}$	$\boldsymbol{w_j}$
$\mathbf{D_1}$	1.0000	0.8198	0.3581	0.9558	0.9285	0.5355	0.5022	0.121
$\mathbf{D_2}$	0.8198	1.0000	0.8282	0.6152	0.5486	0.5013	0.5956	0.144
$\mathbf{D_3}$	0.3581	0.8282	1.0000	0.0678	-0.0142	0.5774	1.5936	0.384
$\mathbf{D_4}$	0.9558	0.6152	0.0678	1.0000	0.9966	0.5012	0.6839	0.165
$\mathbf{D_5}$	0.9285	0.5486	-0.0142	0.9966	1.0000	0.5001	0.7703	0.186

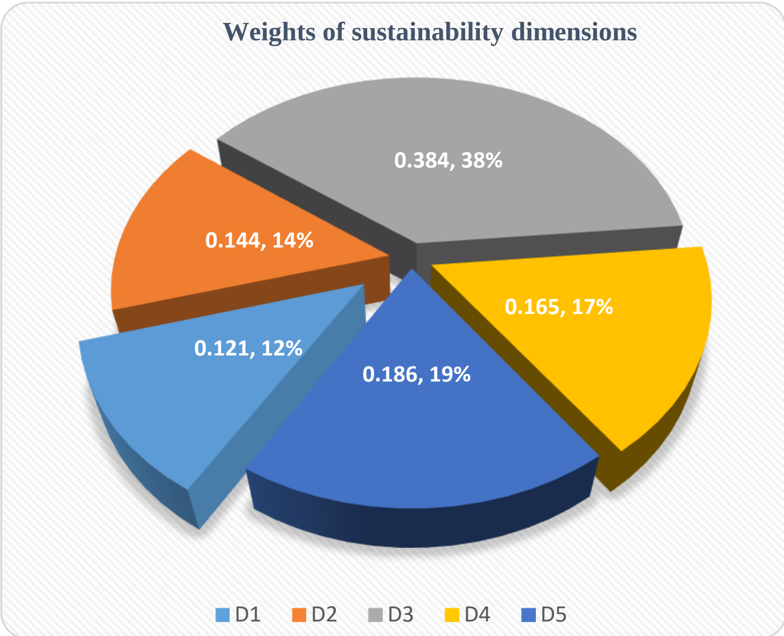

Figure 3. Final weights of dimensions using the T2NN-CRITIC method.

Step 7. An assessment decision matrix among the five dimensions determined and the five projects for urban energy internet was constructed by all experts according to Eq. (11) using semantic terms as shown in Table 7, and then by applying T2NNs.
Step 8. An assessment decision matrix among the five dimensions determined and the five projects for urban energy internet was constructed by all experts according to Eq. (11) using semantic terms as shown in Table 7, and by using T2NNs as presented in Table 8.
Step 9. The T2NNs were transformed to real values by applying Eq. (3) as shown in Table 9. Then, the average value for each dimension was computed according to Eq. (12) as exhibited in Table 9.
Step 10. The PDA and NDA of the five urban energy internet projects were calculated according to Eq. (13) and Eq. (14), respectively, as presented in Table 10.
Step 11. The weighted sum of PDA and NDA of five urban energy internet projects were computed according to Eq. (15) and Eq. (16), respectively, as presented in Table 11.
Step 12. The normalized values for the weighted sum of PDA and NDA of the five urban energy internet projects were computed according to Eq. (17) and Eq. (18), respectively, as presented in Table 12.
Step 13. The assessment score, AS_i for the five urban energy internet projects was computed by applying Eq. (19) as presented in Table 12. Finally, the five urban energy internet projects were ordered according toAS_i as presented in Table 12 and shown in Figure 4.

Table 7. Assessment matrix for five projects according to five sustainability dimensions.

Alternatives	Dimensions				
	D_1	D_2	D_3	D_4	D_5
A_1	MDM	MDM	LLO	MGH	ELH
A_2	MEM	ETL	LLO	HIH	MDM
A_3	ELH	HIH	MGH	ELH	MDM
$\mathbf{A_4}$	LLO	MEM	MDM	MDM	HIH
$\mathbf{A_5}$	MDM	MDM	ELH	MGH	HIH

Table 8. Assessment matrix for five projects according to five sustainability dimensions using T2NN.

Alt.	D_1	D_2
A_1	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$
A_2	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$	$\langle(0.20, 0.20, 0.10); (0.65, 0.80, 0.85); (0.45, 0.80, 0.70)\rangle$
A_3	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$
A_4	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.75, 0.65)\rangle$	$\langle(0.50, 0.45, 0.50); (0.40, 0.35, 0.50); (0.35, 0.30, 0.45)\rangle$
A_5	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$
Alt.	D_3	D_4
A_1	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.75, 0.65)\rangle$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.25, 0.15)\rangle$
A_2	$\langle(0.35, 0.35, 0.10); (0.50, 0.75, 0.80); (0.50, 0.75, 0.65)\rangle$	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$
A_3	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.25, 0.15)\rangle$	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$
A_4	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$
A_5	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$	$\langle(0.60, 0.45, 0.50); (0.20, 0.15, 0.25); (0.10, 0.25, 0.15)\rangle$
Alt.	D_5	
A_1	$\langle(0.95, 0.90, 0.95); (0.10, 0.10, 0.05); (0.05, 0.05, 0.05)\rangle$	
A_2	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$	
A_3	$\langle(0.40, 0.30, 0.35); (0.50, 0.45, 0.60); (0.45, 0.40, 0.60)\rangle$	
A_4	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$	
A_5	$\langle(0.70, 0.75, 0.80); (0.15, 0.15, 0.25); (0.10, 0.15, 0.15)\rangle$	

Table 9. Decision matrix for five projects according to five sustainability dimensions.

Alt.	$\mathbf{D_1}$	$\mathbf{D_2}$	$\mathbf{D_3}$	$\mathbf{D_4}$	$\mathbf{D_5}$
$\mathbf{A_1}$	0.458	0.458	0.308	0.708	0.929
$\mathbf{A_2}$	0.575	0.238	0.308	0.813	0.458
$\mathbf{A_3}$	0.929	0.813	0.708	0.929	0.458
$\mathbf{A_4}$	0.308	0.575	0.458	0.458	0.813
$\mathbf{A_5}$	0.458	0.458	0.929	0.708	0.813
$\boldsymbol{AV_j}$	0.546	0.508	0.542	0.723	0.694

Table 10. Obtaining of the positive and negative distances from average.

Alt.	PDA					NDA				
	D_1	D_2	D_3	D_4	D_5	D_1	D_2	D_3	D_4	D_5
A_1	0.000	0.000	0.000	0.000	0.338	0.161	0.099	0.432	0.021	0.000
A_2	0.054	0.000	0.000	0.124	0.000	0.000	0.532	0.432	0.000	0.340
A_3	0.703	0.599	0.306	0.285	0.000	0.000	0.000	0.000	0.000	0.340
A_4	0.000	0.131	0.000	0.000	0.171	0.435	0.000	0.155	0.367	0.000
A_5	0.000	0.000	0.713	0.000	0.171	0.161	0.099	0.000	0.021	0.000

Table 11. Obtaining the weighted sum of the positive and negative distances from average.

Alt.	Weighted PDA					Weighted NDA				
	D_1	D_2	D_3	D_4	D_5	D_1	D_2	D_3	D_4	D_5
A_1	0.000	0.000	0.000	0.000	0.063	0.019	0.014	0.166	0.003	0.000
A_2	0.007	0.000	0.000	0.020	0.000	0.000	0.077	0.166	0.000	0.063
A_3	0.085	0.086	0.117	0.047	0.000	0.000	0.000	0.000	0.000	0.063
A_4	0.000	0.019	0.000	0.000	0.032	0.053	0.000	0.060	0.061	0.000
A_5	0.000	0.000	0.274	0.000	0.032	0.019	0.014	0.000	0.003	0.000

Table 12. Ranking of five urban energy internet projects by applying the EDAS method.

Alt.	$\mathbf{SP_i}$	$\mathbf{SN_i}$	$\mathbf{NSP_i}$	$\mathbf{NSN_i}$	$\mathbf{AS_i}$	**Rank**
$\mathbf{A_1}$	0.063	0.203	0.187	0.336	0.262	4
$\mathbf{A_2}$	0.027	0.306	0.080	0.000	0.040	5
$\mathbf{A_3}$	0.336	0.063	1.000	0.793	0.897	1
$\mathbf{A_4}$	0.051	0.173	0.151	0.435	0.293	3
$\mathbf{A_5}$	0.306	0.037	0.911	0.878	0.895	2

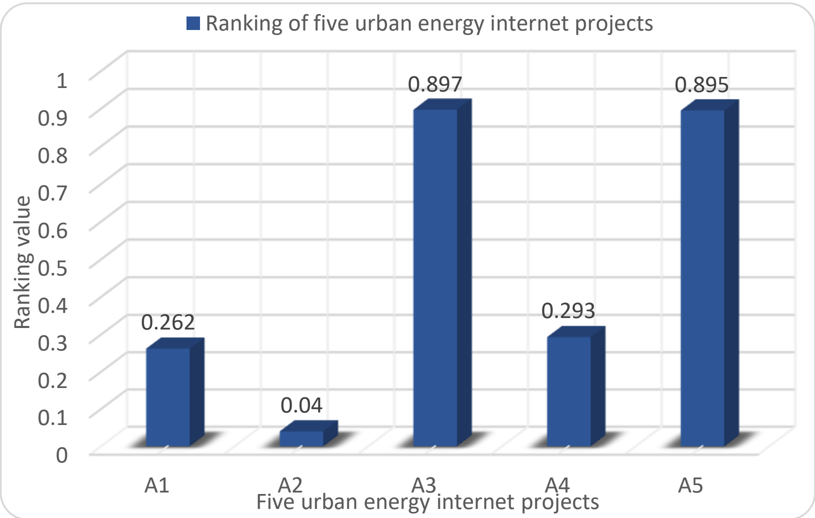

Figure 4. Final ranking of five urban energy internet projects using EDAS.

4.2 Findings and discussion

In this part, the results obtained from applying the T2NN-CRITIC-EDAS approach to evaluate five urban energy internet projects are discussed. Initially, the T2NN-CRITIC method was applied to evaluate five dimensions of urban energy internet sustainability. In this regard, the results indicate that the technical dimension is the dimension with the highest weight and most influence. Next, the T2NN-EDAS method was applied to rank five urban energy internet projects. The results indicate that the A_3 project is the highest in the ranking.

4.3 Sensitivity analysis

In this part, a sensitivity analysis model is presented by changing the weights for the technical dimension. The results of the sensitivity analysis in Figure 5 indicate that there are some slight changes in the order of the alternatives chosen in the study. The changes that occurred in the ranking of the alternatives are that the fifth alternative became the highest ranked when the weight of the technical dimension became equal to 0.5 until its weight became 1. Accordingly, the sensitivity analysis on the model used leads to some slight changes in the ranking of the selected alternatives, which means that changing the weights is important in the order of alternatives.

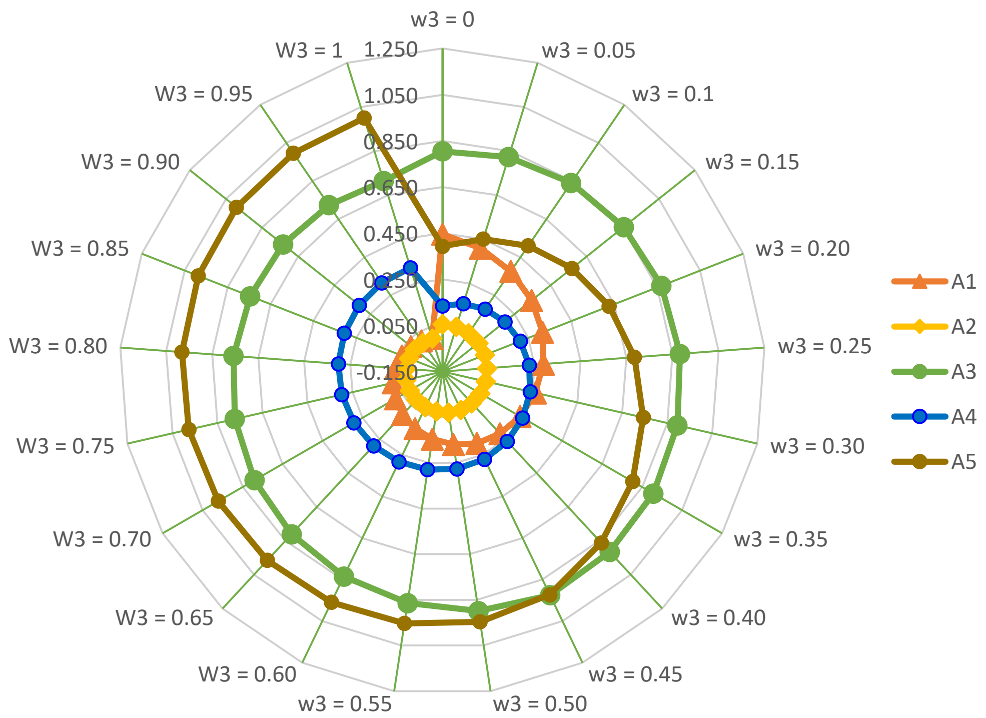

Figure 5. Influence of the w_3 on the ranking of urban energy internet projects.

5. Conclusions

The development and deployment of the UEI prioritize the achievement of sustainable energy supply, improved energy efficiency, and enhanced energy security. Nevertheless, the absence of research on the assessment of UEI projects has hindered the advancement of urban energy internet, despite the diligent endeavors of researchers and academics investigating its development and organization. In addition, scholars have acknowledged the need of addressing the evaluation problem by using neutrosophic set theory and its expansions, considering the inherent uncertainty in the actual world. This study presents an MCDM model to solve the problem of evaluating UEI projects in light of sustainability. The study was conducted under a neutrosophic environment and using T2NNs. The presented model relied entirely on the CRITIC and EDAS methods. Initially, the CRITIC method was used to evaluate five dimensions of sustainability and determine the most prioritized and weighted. In addition, the EDAS method was applied to rank five UEI projects selected for study. Also, a sensitivity analysis was conducted in order to show the importance of changing the weights on the results of the presented model. In future studies, the sustainability factors used in the evaluation process could be increased and the number of experts participating in the study could be increased. In addition, the study can be conducted under different environments to deal with uncertainty.

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Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors.

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An Exhaustive Review of Neutrosophic Logic in Addressing Image Processing Issues

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Abstract: Since the importance of images in our lives and the advancements in computer data gathering methods, anyone can collect a large number of images, but most of them cannot be processed manually. Image processing therefore becomes appealing since various types of data may be represented and processed digitally. Image processing has become the most popular processing method, employed in security camera films, healthcare images, images from remote sensors, and naturalistic image/videos because of fast computers and processors. In order to raise cognitive function and speed up decision-making, image processing is crucial to many information access systems. Since ambiguity now permeates every part of the world, including images, discussing the neutrosophic logic forms the central idea of this discussion, as it is able to handle this ambiguity. To apply the neutrosophic logic, this requires converting the image into neutrosophic reasoning. When using neutrosophic reasoning for image retrieval, average recall and precision measures improve over other approaches. As the image processing field covers several tracks such as image segmentation, noise reduction, image classification, and others. Because there are so many research articles published in this field every year, we thought it would be appropriate to introduce a survey study on this subject. As a result, this study offers a comprehensive assessment of the literature on applying neutrosophic logic to image processing problems that have surfaced during the previous five years (2019–2023).

Keywords: Neutrosophic Logic; Image Processing; Image Segmentation; Noise Reduction; Image Enhancement.

1. Introduction

The complex process of vision involves many different parts of both the brain and the eye cooperating. One of the most important senses regarding humanity's survival and adaptation has always been eyesight. The visual system is used by humans to view, gather, analyze, and comprehend visual information, as well as draw conclusions from it. The ultimate objective of the field of image processing is to automate the collection and processing of visual data. The process of converting an image into a digital format and carrying out specific operations on it in order to extract valuable information is known as image processing. We have seen a broad expansion in the number of image processing applications and methodologies throughout the years. In numerous applications including face recognition [1, 2], object identification [3, 4], medical images, and noise removal [5, 6], it is a crucial preliminary step. Image processing process includes a set of steps, which can be indicated in Figure 1. As mentioned above, the field of image processing encompasses a number of areas, including noise reduction, image clustering, and image segmentation. As a result, numerous approaches, including metaheuristics, machine learning, and deep learning, have been used to address these issues. Recently, neutrosophic logic was added to them, because of its high ability to deal with blurring.

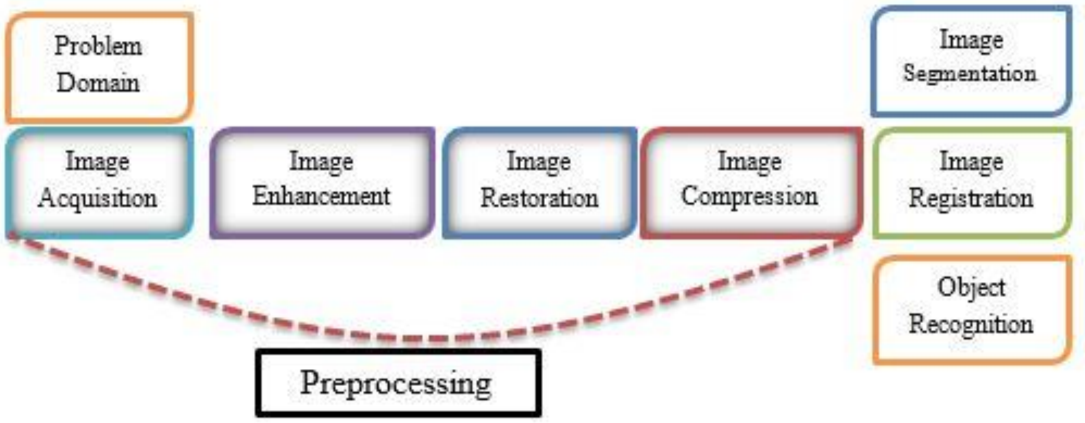

Figure 1. Phases in image processing preprocessing.

Neutrosophic set studies the nature, origins, and scope of impartialities as well as how they interact with other mental spectra. Neutrosophic sets are comparatively recent extensions of intuitionistic fuzzy sets. In Neutrosophic logic, a proposition is assigned three values, such as truth (T), indeterminacy (I), and falsity (F), to each proposition. Any set that falls between [0, 1] and has a grade of truth, indeterminacy, and falsity for each element of the cosmos is called a neutrosophic set. So, how can neutrosophic logic deal with digital images? – The digital image should be converted to a neutrosophic domain throughout a set of equations in [7], it also indicated in the following section. The conversion process from digital image to a neutrosophic image has opened the door for many researchers to solve image problems, because of its ability to handle ambiguity. Based on the previous, Neutrosophic logic has been a critical tool for image noise removal. So, Neutrosophic logic has been used and adapted by numerous scholars to solve image-related difficulties, which gave us the motivation to present a research paper that includes the publications on this era from 2019 through 2023. The primary contributions of this essay can be outlined as follows:

- Introducing a few significant difficulties with image processing that use neutrosophic logic.
- Survey the frequency of neutrosophic logic employed for image segmentation.
- Survey the frequency of neutrosophic logic employed for image classification.
- Survey the frequency of neutrosophic logic employed for image clustering.

2. Neutrosophic Logic

At the end of the nineties, Smarandache introduced neutrosophic theory for the first time [8]. Neutrosophic set is considered the generalization of fuzzy set. According to Zadeh [9], In order to account for database uncertainty, fuzzy sets were introduced. Fuzzy sets use a single-valued membership function in [0, 1] to display acceptance, rejection, and uncertainty portions. They cannot, however, each represent indeterminacy separately. Smarandache in 1998 established neutrosophic logic as a result. The membership functional values of truth, falseness, and indeterminism for every attribute are examples of single-valued neutrosophic. These three functions are independent of one another, meaning that they do not influence one another. To talk about the use of neutrosophic technology in image processing, we should discuss the fuzzy set concept. Fuzzy set concept can be discussed throughout the following. Suppose that we have a universal set U and C is a subset of$U$. Subset C contains a set of elements$u$. Each element in set C has a membership degree indicates the belonging degree to set C, and this membership degree lies between [0, 1].

Moving to neutrosophic set, we can say that, it highlights three functions: membership, non-membership, and indeterminacy. To form a neutrosophic set C in universal set U, three membership functions (truth (T_C (u) ,falseness F_C (u), indeterminacy I_C (u)) are needed for each element u in C , each membership function from these lies in range of [0,1]. The Figure 2 indicates the relationship between four sets: classical set, fuzzy set, intuitionistic fuzzy set, and neutrosophic sets.

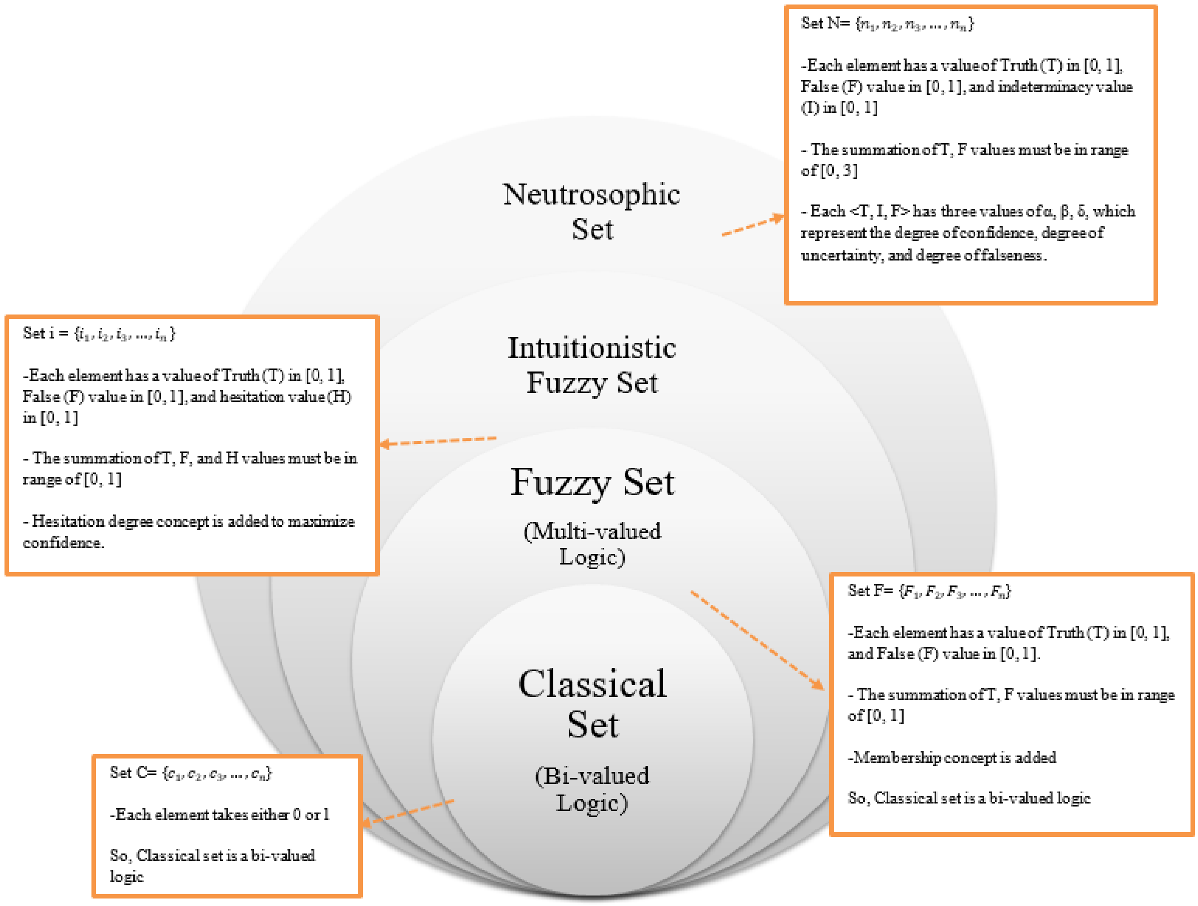

Figure 2. Generalization from classical set to neutrosophic set.

3. Image processing problems

Image processing process covers different areas such as: image segmentation, image enhancement, and image noise removal. The following subsections will cover each problem formulation and also the research trends in each topic.

3.1 Neutrosophic logic image segmentation

The well-known proverb "A picture is worth a thousand words" suggests that analyzing an image can reveal more information than analyzing written content. Image segmentation, or the division of an image into its component objects or region of interest, is the primary research field in computer vision. In general, it groups comparable parts of the image pixels together. It is a stage of pre-processing for a lot of image-based applications, such as recognition of patterns, object detection, healthcare imaging, and biometric authentication [10]. The digital image should be converted to a neutrosophic domain throughout a set of equations from 1 to 4.

Suppose that U be a discourse universe and let A be a set of bright pixels included in U. Truth (T), Indeterminacy (I), and Falseness (F) are the three subsets that define a neutrosophic image. In an image, a pixel P is identified as P(T, I, F) and is a part of A in the following manner: Where t changes in T, i changes in I, and f changes in F, each bright pixel set has t% truth, i% undetermined(indeterminate), and f% falseness. Every component gets a value between 0 and 1. The image domain's pixel P (a, b) is converted to the domain of neutrosophic as follows: PNS (a, b) = {T (a, b), I (a, b), F (a, b)}. The probabilities associated with the white, indeterminate, and non-white sets are represented by T (a, b), I (a, b), F (a, b), can be defined as follows:

$$T(a,b) = \frac{g_{a,b} - g_{min}}{g_{max} - g_{min}}, \tag{1}$$

$$I(a,b) = 1 - \frac{h_{a,b} - h_{min}}{h_{max} - h_{min}} \tag{2}$$

$$F(a,b) = 1 - T(a,b) \tag{3}$$

$$h_{a,b} = |e(a,b)| \tag{4}$$

Where $g_{a,b}$ represents the value of the pixel and the homogeneous value of T at (a,b) is denoted by $h_{a,b}$ and is represented by the local gradient value $e(a,b)$, which is derived by applying the Sobel operation.

After converting the digital image into a neutrosophic field, researchers raced to compile many research papers in this regard, including those related to image segmentation. So, it gave us a motivation to introduce this survey paper. From 2019 to 2023, some of neutrosophic image segmentation research articles were listed in Table 1.

Table 1. An overview of some Neutrosophic logic for image segmentation.

No	Ref	Main Idea	Data	Merit	Shortcomings	De-noising method
1	[11]	This research suggests an image segmentation module with neutrosophic that is based on a set of steps: First, use min/max normalization to reduce noise. Secondly, the image non-linearities are counted using activation functions. Third, membership functions are calculated to identify various regions and create neutrosophic sets. Lastly, the neutrosophic sets and Dice's coefficients are merged to guarantee an accurate assessment of the uncertainty surrounding the missing data and its indeterminacy for the purpose of segmenting images.	- Crow images	- Less time and computing power - Efficient for segmenting and processing images. - This method can be applied to any number of images and any type of typical problem (blurred images).	- No requirement for training	- Min-Max Normalization

2	[12]	In this study, The picture is initially converted into a neutrosophic image in the suggested technique. It is suggested to use a combined FCM based on particle swarm optimization (PSO) to increase the effectiveness of global search. Ultimately, the suggested approach is utilized for the segmentation of neutrosophic images. The results of the studies demonstrate that the new method is more effective than the FCM algorithm in removing picture noise and improving the clarity of the segmentation region border.	- Lena image - rice grain image - Warship image	The algorithm has the following attributes: - An adequate mage segmentation effectiveness -A powerful global search capability. -Workable and has strong noise reduction capabilities.	- No appreciable decrease in the program's execution time	- NSS
3	[13]	This study suggests a method for segmenting images by fusing the neutrosophic set (NS) theory with a saliency map. The methodology of this study as follows: first, applying a filter to address the image's weak edges, second, after creating saliency map, gray scale and local entropy maps are created, saliency is converted to NS space. The segmentation results are obtained using threshold.	- The experiment's picture source was the MSRA10K image library	- The suggested approach may effectively suppress multiple primary noises. - Effective at handling massive amounts of picture data.	- There is a limit to objective performance.	- Ns domain + Filter
4	[14]	Two offset-based methods were presented in this research. For image segmentation, the preliminary one operates on neutrosophic offsets. For edge identification, the second is built on neutrosophic offuninorms.	- Geometric image and brain image	- Effectiveness - Noise reduction - Calculation simplification through using offset		- Neutrosophic offsets - Neutrosophic offuninorms

5	[15]	In this study, we have combined the notion of weak continuity constraints with an NS-based multiclass segmentation technique. The border information is handled by the weak continuity constraints, which aid in precisely localizing the segmentation boundaries. By taking into account the location of segment boundaries, the suggested method assists in overcoming the drawbacks of existing methods for picture segmentation in the NS domain. Without requiring any prior knowledge of the number of classes, the accurate threshold values are computed repeatedly in the proposed method by reducing the energy function.	- Synthetic images - Non-destructive testing (NDT) images - Kaggle 2018 Data Science Bowl dataset	On both synthetic and a range of natural and non-destructive testing photos, the procedure worked fairly well. The approach exhibits superior quantitative and qualitative performance. The suggested solution performs better on the extremely uncertain Kaggle nuclei photos.	- Intricately textured photos are not processed. - Slower in comparing with a non-repetitive procedure.	- The weak continuity constraints in the NS domain.
6	[16]	An innovative method based on neutrosophic logic for segmenting dental radiography images. Using neutrosophic logic, the first region of interest is chosen. Using the local binary pattern, gradient feature, entropy feature, and patch level feature, the input dental radiography image is transformed into the neutrosophic domain. Localizing the initial region of interest is aided by the application of neutrosophic reasoning. Then, to segment a more precise region of	- Panoramic Dental X-rays with Segmented Mandibles' and 'Digital Dental X-ray Database for Caries Screening.	- Elevated degree of performance	- The suggested method's dependence on image resolution is one of its main shortcomings. It cannot operate in the same manner with images of varying resolutions. It is necessary to adjust system parameters to a specific image resolution.	-NSS

		interest, a fuzzy c means method is used.				
7	[17]	This paper proposes an adaptive region growth strategy for segmenting Brest ultrasound scans based on the neutrosophic set (NSSRG). Each of the pixels in the BUS image is characterized by computing the level of homogeneity score along with similarity set score once the images are converted into the NS domain. The seed regions are created using an adaptive Otsu-based thresholding approach and morphological techniques, after which an adaptable region growing methodology is adjusted depending on the neutrosophic set for the creation of potential tumor regions. To reach the ultimate segmentation result, a deep convolutional neural network based on VGG-16 net is employed for false positive reduction.	This study uses a dataset of clinical 384 BUS pictures that were gathered using a 4-5 MHz linear probe by VIVID 7 (GE, Horten, Norway).	- For the segmentation of breast tumors on BUS pictures, it is reliable and efficient, particularly when the tumors have blurry and low contrast borders.	- Not mentioned	-NSS
8	[18]	This work enhanced the general effectiveness of skin lesion segmentation in dermoscopic images and proposed novel concepts for the NS subgroup. The suggested definition was tested using various filter mixtures, including the standard Sobel with median filter combination. Furthermore, several combinations and methods of segmentation such as GC	- Ninety dermoscopic images from the ISIC2016 skin lesion dermoscopic images were used.	- High segmentation accuracy	- High computational time.	- Neutrosophic filters.

		and k-means were investigated.				
9	[19]	Having defined their aggregate operators and procedures, this paper then utilize the linguistic neutrosophic cubic set (LNCS) for image processing. Using three membership degrees, noisy grey scale images were converted into the LNCS domain for this investigation, and aggregation procedures were then used to sum the images. The Lena image and three other test photographs have their noise clearly defined using the suggested method.	- grey-scale Lena image - images with Edge detection - images with minute object's detailing	- Uses less memory and operates more quickly than the current techniques	-Not mentioned	- Linguistic neutrosophic cubic set
10	[20]	This research aims to identify brain tumors as benign or malignant in order to create an effective automatic brain tumor segmentation system. The neutrosophic set - expert maximum fuzzy-sure entropy (NS-EMFSE) approach was used to segment brain tumors. After Alexnet extracted the segmented image features using CNN architectures, SVM and KNN classifiers were used to classify the images.	- MRI images of 80 benign and 80 malignant brain tumors	-High accuracy -The module is effective in terms of in terms of Sensitivity, Precision, Accuracy	- Not mentioned	- The neutrosophic set - expert maximum fuzzy-sure entropy (NS-EMFSE) approach
11	[21]	This research proposes a gradient-based structural similarity based spuerpixel-neutrosophic C-means clustering for color image segmentation. To obtain precise target contours, the SLIC method combines numerous morphological	-three datasets are used (EORSSD, BSDS500, and MSRC)	-This technique can efficiently and accurately segment pictures into clean and noisy ones.	-The number of super pixel must be known in advance	- gradient-based structural similarity based spuerpixel-neutrosophic C-means clustering

		reconstructions. There are fewer clusters while the target contour is kept intact. Furthermore, the NCM algorithm is enhanced by using adaptive neighborhood information, resulting in a more accurate determination of doubtful super pixels.				
13	[22]	In order to segment images, this research suggests a Neutrosophic C-means Clustering with local information and noise distance-based kernel metric (NKWNLICM). First, in order to strengthen the robustness of noise picture segmentation, fuzzy spatial information and noisy distance information are added to the NCM model. After then, the distance across pixels is calculated using the kernel function. The classification performance is additionally improved by mapping low-dimensional data into high-dimensional data. Finally, a new definition of the fuzzy factor is based on the separation between the center pixel and its surrounding pixels.	- Berkeley University image library	- improved segmentation outcomes for photos with noise	- Its segmentation performance has little advantage since the local data of the image, the spread of the noise, plus the kernel function are incorporated for image segmentation modeling.	- Local fuzzy information and noise distance
14	[23]	This work presents a novel method for handling uncertainty based on the neutrosophic set (NS) in order to segment nondestructive testing NDT image. By encoding an image as a true, false, and indeterminate subset, the NS controls the uncertainty. The two processes $\alpha -$ mean and $\beta -$ enhancement are	- NDT images from [24]	- manage the ambiguities for the segmentation	-Not mentioned	- NS + bat algorithm

		necessary for accurate depiction of NS values. Using the bat algorithm (BA), we can determine the appropriate values of α and β based on the statistics of the image.				
15	[25]	This work presents the segmentation and location identification of brain tumor tissue features in MRIs using a newly designed (type-2 neutrosophic set) T2NS. T2NSEIF, an image segmentation technique, was proposed based on the proposed T2NS. The suggested T2NSEIF approach also included integrations of the T2NSE and image fusion principles. The suggested approach employed T2NS for the neutrosophication of MRI grey levels, and T2NSE was used to quantify the inherent uncertainties.	- A collection of 3064 T1-weighted contrast-enhanced pictures [26] is used to pick three distinct brain tumors: meningioma, glioma, and pituitary	-High performance -consumes less CPU time.	-Not mentioned	-Type Two Neutrosophic set(T2NS)
16	[27]	The residue procedure, which indicates the residual values of neutrosophic membership intensities, was used in the article. This paper will investigate a brand-new concept for picture thresholding known as RNS. Three different RNS technique kinds will be available: minimum, average, and maximal. This approach takes into consideration the principles of current thresholding methods in neutrosophic solvation. In order to give an integrated visionary route	- A fingerprint picture.	- achieves a higher mean accuracy rating.	- Not mentioned	- Residue Neutrosophic Set

		segmentation methodology, this article employs unique methodologies.				
17	[28]	The two key components of the new MR image segmentation method reported in this study were the HSV color system and the neutrosophic-entropy based clustering algorithm (NEBCA). In this study, the NEBCA was utilized to segment the MR regions, and the HSV color scheme was employed to improve the visual representation of the features inside the segmented regions.	- 30 distinct Parkinson's disease (PD) MR pictures.	- Less time in terms of CPU time -efficiently segment several MR image regions	- The proposed method is validated only with MR images of PD patients.	- Neutrosophic-entropy based clustering algorithm (NEBCA).
18	[29]	A novel fuzzy clustering technique that utilizes non-local information is presented here to enhance the image segmentation algorithm's capacity to process boundaries and its anti-noise effectiveness. To find the fuzzy subset's clustering center, the suggested method first employs the data distribution of a deterministic subset. Also, the neutrosophic fuzzy mean clustering technique incorporates the fuzzy non-local pixel correlation.	- Three types of images are used as synthetic image, medical image and natural image	-Effective image segmentation method.	- The study of the bias field in the original image between the true value and the observed value is not done.	- Fuzzy clustering approach using neutrosophic and non-local information (NLNFC).
19	[30]	This study suggested a hybrid segmentation method for automatically identifying liver tumors from abdominal CT scans that combines the watershed algorithm, fast fuzzy c-means algorithm, and neutrosophic sets.	- Over 105 patients with CT scans of their abdomens and more than 150 slices per patient are included in the dataset [31]	- Precise precision, reduced time spending, and reduced noise sensitivity. - has outstanding precision as well as efficiency on noisy and non-	- Many CT scans are not processed in order to assess performance. - This method's primary flaw is that it requires a lot of computer power and is not suitable	-NSS

				uniform pictures, reducing over-segmentation.	for real-time applications.	
20	[32]	This paper segments the dermoscopic images in the neutrosophic set (NS) domain and suggests an efficient method for dermoscopic skin lesion segmentation employing a neutrosophic set-based kernel graph cut (NKGC).	- International Skin Imaging Collaboration (ISIC) 2016 skin lesion dataset.	- best average segmentation accuracy	-Not mentioned	- Neutrosophic set-based kernel graph cut (NKGC).
21	[33]	In this article, the author suggested a novel exemplar-based image completion method that finds the best match to fill the hole by segmenting an image using neutrosophic sets and applying segmentation information along with similarity measures that take neighborhood and similarity information into account.	-Two datasets (Kaggle dataset, Dhar and Kundu (2021)	-The suggested approach performs both qualitatively and quantitatively well for photos that are corrupted.	-Noise is not considered in this research	- Noise is not take into consideration in this study
22	[34]	This study combines two potent methodologies, such as expert maximum fuzzy-sure entropy (EMFSE) and neutrosophic set (NS), to present an effective edge detection method known as neutrosophic set – expert maximum fuzzy-sure entropy (NS-EMFSE).	- MRI images with 100 different brain tumors	- Efficient segmentation tool.	- Not mentioned	- Maximum fuzzy-sure entropy (EMFSE) + neutrosophic set

3.2 Neutrosophic logic in image classification and clustering

The effectiveness of Neutrosophic set categorization can be attributed to the use of straightforward processes. To control the gain noise and indeterminacy, the NS classifier makes use of neutrosophic logic. With regard to image classification and clustering, a set of research papers were listed in Table 2.

Table 2. An overview of some neutrosophic logic for image classification / clustering.

No	Ref	Main Idea	Data	Merit	Shortcomings	Task
1	[35]	This work suggests a novel method of integrating genetic algorithms and neutrosophic logic in a rule-based classification system to diagnose COVID19 patients based on chest X-ray pictures.	1885 image were collected from three repositories -Github-COVID chest X-ray -Kaggle-COVID radiography - Radiopaedia	-The module performs well in terms of accuracy, precision, sensitivity, and specificity.	- Time complexity	Classification Medical : Chest
2	[36]	In order to control uncertainty when classifying multimodal input, this work uses neutrosophic fuzzy sets for information retrieval tasks. This work aims to identify the images using neutrosophic classification algorithms, drawing inspiration from previous methods of embedding text over images. It makes use of both image and text data. Neutrosophic Convolutional Neural Networks (NCNNs) are used to train feature representations of the generated images for classification tasks.	- Two large scale multi-modal categorization datasets	- Good classification tool	- Not mentioned	Classification Text image
3	[37]	This study solves the texture classification problem by reinterpreting the innovative neutrosophic set-based Completed Local Binary Pattern (CLBP) hybrid approaches based on the neutrosophic set and a novel feature extraction method by using neutrosophic set components. The original input image has been replaced in the suggested technique	- CURET dataset - Poly-class texture datasets - A LOT and STex datasets	-It is suitable for real-time applications. -Good classification accuracy. -The suggested approach has produced excellent classification	-The suggested method's inability to be used directly to color images due to theoretical and mathematical issues is one of its drawbacks. -Time cost	Classification Texture problem

		with the neutrosophic truth and false components of the image. As a result, there are less noise effects and a more meaningful image.		results at a reasonable computing cost.		
4	[38]	A unique, reliable, automatic, and intelligent system for COVID-19 classification utilizing chest X-rays has been proposed in this study. The suggested pipeline combines neutrosophic and machine learning methodologies in a hybrid fashion.	- A public database in Github have been built [39] - The Kaggle Chest X-ray [40]	-The module performs well in terms of The classification accuracy, precision, sensitivity, and specificity.	- The shortcoming of our suggested pipeline is that it only provides a clear-cut positive or negative COVID-19	Classification Medical : Chest
5	[41]	This study presents the Breast Cancer Classification Strategy, a novel approach to classify breast cancer utilizing Neutrosophic approaches (NTs) and machine learning approaches.	-Brest cancer images	-Performs well in terms of accuracy, precision, recall, and F-measure	-Not mentioned	Classification Medical: Breast
6	[42]	In order to determine the final mammographic micro-calcification clusters MCCs, this study used a density-based clustering approach in conjunction with a DCNN classifier to construct an automatic detection method. As the FP MCs reduction stage, the DCNN1 classifier was utilized since it is specifically trained to distinguish between individual MCs. To quicken learning, a novel adaptive NB technique based on NS theory is used.	- A publicly accessible INbreast database and Nanfang Hospital (NFH) in Guangzhou, China are the primary sources of FFDMs of solely MCs [43]	- Plays a vital part in enhancing the efficiency of the automatic identification and categorization of MCCs.	-Time consume.	Classification Medical: Breast
7	[44]	In order to categorize BUS pictures more precisely, the author offer in this research unique Neutrosophic Gaussian Mixture Models (NGMMs) that combine a Deep Neural Network (DNN), neutrosophic logic, and an enhanced Expectation Maximization (EM) algorithm.	-Two public datasets of Breast Ultrasound Images.	- Greatest classification outcomes	- Not mentioned	- Classification Medical: Breast

8	[45]	A novel CVOID-X system based on deep learning classifiers has been suggested in this study to automatically recognize or COVID-19. The COVID-19 diagnosis neural network employing Neutrosophic classifier is a model based on deep learning designed for extracting visual features during volumetric tests for COVID-19 identification. The suggested system makes it easier for people to communicate with medical facilities so that the right COVID-19 patient can be contacted promptly.	-Covid-19 evaluated dataset; for example, the images in the dataset were taken using various imaging clinics' equipment and image collection settings.	- Efficient tool that can be used to other significant areas of healthcare like hepatitis, diabetes, and cancer. - Good classification accuracy.	- Architectures that need a lot of heterogeneous devices, like those for healthcare applications, still have problems with interoperability, data processing, CPU control, memory and disc resources, and big data.	Classification Medical: Chest: COVID-19
9	[46]	This research proposes a new clustering approach using Neutrosophic Set (NS) to extract/segment the lesion/region of interest in mammography pictures. Because NS can manage indeterminate information, the images' level of uncertainty is decreased. In our study, a new method based on Shannon entropy and standard deviation is used to compute indeterminate degree.⊚ Next, using the neutrosophic similarity function, an image based on neutrosophic similarity is created, which is subsequently grouped to identify lesions or tumors.	-Fifty images were used to validate the suggested strategy, and these images were obtained from Google.	-The suggested strategy is more accurate and efficient at segmenting the mammography pictures.	--Not mentioned	Clustering Medical: Breast
10	[47]	An innovative skin lesion segmentation approach, known as the optimized clustering estimation for neutrosophic graph cut algorithm (OCE-NGC), was presented in this paper.⊚First, the HBCE process is optimized using the	-Dermoscopic skin lesion image dataset from the International Skin Imaging Collaboration	- The experimental findings demonstrated the recommended OCE-NGC approach's superiority.	-The unsuccessful cases that involve the use of different colors in the same lesion areas are not addressed.	Clustering Medical: Skin lesions

		genetic algorithm (GA). Next, using the neutrosophic c-means (NCM) algorithm, the skin lesion dermoscopic pictures are mapped onto the neutrosophic set (NS) domain. Lastly, during the procedure of segmentation, a cost function of the graph cut (GC) algorithm is defined in the NS domain.	(ISIC 2016) for segmentation (The training and evaluation datasets for this dataset contain 379 and 900 dermoscopic pictures, respectively).			
11	[48]	This study examines the cluster ability of the breast cancer dataset using the commonly utilized c-means clustering algorithm, as well as its improved fuzzy and neutrosophic variations. A range of measures are employed in this comparison study to objectively assess the clustering effectiveness of the breast cancer dataset. The best clustering effectiveness is obtained by the proposed neutrosophic c-means clustering with regard to of silhouette score, precision, and Rand index.	- WDBC datasets, which are breast cancer datasets. Prior data was collected from the University of Wisconsin Hospitals.	- The results of the study demonstrate the advantage of neutrosophic c-means clustering in grouping together comparable cases of breast cancer.	-A multidimensional dataset.	Clustering Medical: breast lesions
12	[49]	A novel edge detection method based on the NS scheme category has been proposed in this research. The frame of picture fusion with edge detection allowed for this to be accomplished. The indeterminate conditions and uncertainties present in the photographs can be handled by this approach. The Sobel operator and the BS function are used by the NS framework to replicate images. A new binarized image is created by integrating the subgroups and the calculated	- MR Brain images.	- The suggested strategy is more accurate and efficient	- Image de-noising are not handled	Clustering Medical: MR Brain

		thresholds after that. After the binarized picture has undergone morphological procedures, the edges of the image are obtained. Every entropy is subjected to the same method, which displays many discovered edges.				

4. Conclusions

Neutrosophic logic provides a potent tool for describing the image with imprecise information. The value of neutrosophic theory in image segmentation, image classification/clustering, and noise reduction is discussed in this work. Because the neutrosophic set can handle indeterminacy stronger than the fuzzy/non-fuzzy sets, it has been found that the outcomes obtained using the neutrosophic set are significantly better. When neutrosophic sets are used in image processing, accuracy improves because of more precise segmentation, which leads to more precise outcomes, including correctly diagnosing medical images. Some real-world applications for this include the diagnosis of skin lesions, brain tumors, breast cancer, and dental X-ray pictures, among others. Neutrosophic set produces superior results, even in photos with weak contrast and hazy borders. The work mentioned above demonstrates how more image processing applications can benefit from the use of Neutrosophic based techniques. It also aids in the resolution of issues if a membership function is imprecisely defined because human error is absent.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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An Efficient Optimal Solution Method for Neutrosophic Transport Models: Analysis, Improvements, and Examples

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Abstract: Transport issues aim to determine the number of units that will be transferred from the production centers to consumption areas so that the cost of transportation is as low as possible, taking into account the conditions of supply and demand. Due to the great importance of these issues and to obtain more accurate results that take into account all circumstances, we conducted two research studies. In the first research, we presented a formulation of neutrosophic transport issues, and in the second research, we presented some ways to find a preliminary solution to these issues, but we do not know whether the preliminary solution is optimal or not, so we will present in this research a study whose purpose is to shed light on some important methods used to improve the optimal solution to transportation issues and then reformulating them using the concepts of neutrosophic science, a science that leaves nothing to chance or circumstances but rather provides solutions with neutrosophic values. Unspecified values take into account the best and worst conditions.

Keywords: Neutrosophic Science; Optimal Solution; Transportation; Stepping-Stone Method; Modified Distribution Method.

1. Introduction

The linear programming method is one of the most important operations research methods that companies and institutions have benefited from in their workflows by building linear models for which science has provided ways to find the optimal solution. Given this importance, we have reformulated these models in two previous studies and found one of the most important ways to solve them using the concepts of neutrosophic science [1, 2]. The issue of transportation is one of the most important issues that have been dealt with using the linear programming method because transportation problems appear frequently in practical life. We need to transfer materials from production centers to consumption centers to secure the regions; we need the lowest possible cost, To solve these recurring and daily problems, we use operations research methods, specifically the linear programming method, where we transform the issue data into a classical linear mathematical model when the data are classical and neutrosophic model when the data are neutrosophic. And in the research [3], there is a full explanation of the neutrosophic transport issues. Since these models are linear neutrosophic, we can get an optimal solution by using the direct simplex neutrosophic method described in the research [1].

But we know that these models have special characteristics, in terms of restrictions and objective function, which enabled scientists and researchers to find special methods, the methods in which we get preliminary solutions, and we explained how to obtain a preliminary solution for the neutrosophic transport in the research [4], and we recall that the neutrosophic transport issues are transport issues in which the required quantities and the available quantities are neutrosophic values of the form $Na_i \in a_i + \varepsilon_i$, where ε_i the indeterminacy of the produced quantities can take the forms $\varepsilon_i \in [\lambda_{i1}, \lambda_{i2}]$. In addition, the required quantities of neutrosophic values of the form $Nb_j \in b_j + \delta_j$, where δ_j is the indeterminacy of the quantities produced we take it as one of the forms $\delta_j \in [\mu_{j1}, \mu_{j2}]$. and the cost of transportation is neutrosophic values form $Nc_{ij} \in c_{ij} + \alpha_{ij}$, where α_{ij} is the indeterminacy of the cost of transportation, we take it as one of the form $c_{2j} \in \left\{\alpha_{1_{2j}} \alpha_{2_{2j}}\right\}$.In order to review the basic concepts of neutrosophic science and its stages of development and the most important topics of operations research that have been reformulated using the concepts of this science, the following research can be found [5-16].

2. Methods

The purpose of this research, as we mentioned in the abstract, is to highlight some of the methods used according to classical logic to improve the preliminary solution to transportation issues, where we will present:

i. The Stepping-Stone Method
ii. Modified Distribution Method

As stated in some references [17-20], with a focus on the scientific basis on which these methods were based, and then we will reformulate them using the concepts of neutrosophic science and use them to improve the solution of neutrosophic models after finding a preliminary solution for them using one of the methods mentioned in the reference [4].

2.1 The Stepping-Stone Method

To reach the optimal solution in this way, we follow the following steps:

i. We find the preliminary solution by one of the three aforementioned methods, then we calculate the total cost according to the preliminary solution.
ii. We identify the basic variables from the non-basic variables from the preliminary solution table.
iii. We determine the indirect cost by finding closed paths, as each closed path has its beginning and end as a non-basic variable and consists of horizontal and vertical lines whose pillars are basic variables, as it happens that there are two basic variables in the way of the path, so we deviate from the basic non-basic variable and in general, the closed path represents in the following Figure 1:

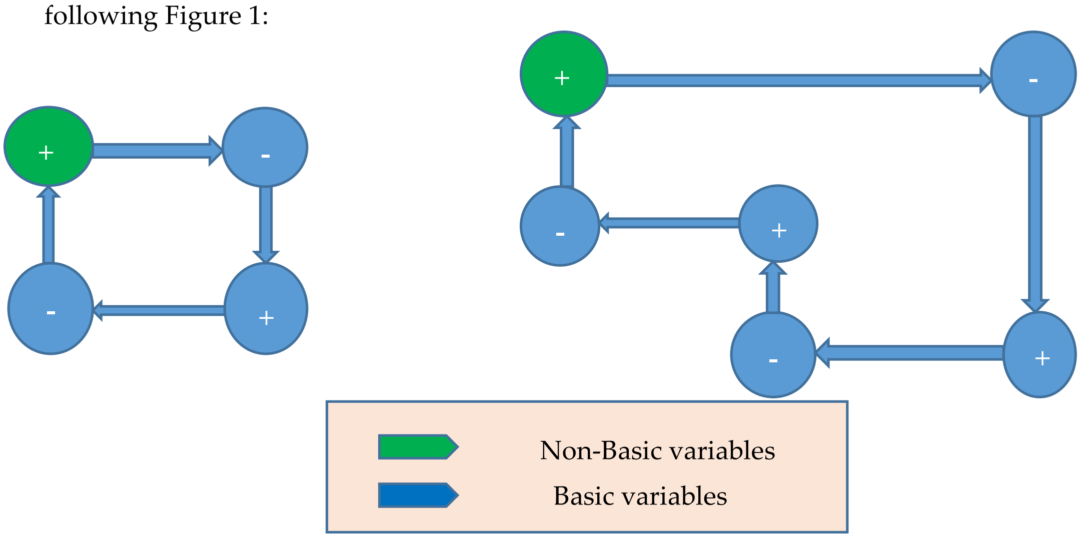

Figure 1. A representation of closed pathways.

We calculate the indirect cost of each non-basic variable by giving the cost of the non-basic variable a positive sign, and the cost of the basic variables we give it alternating negative and then positive signs, and so on. If the basic variables are positive or zero, this means that the solution that we got is optimal and we stop. But if at least one of the indirect costs is negative, then we must develop the solution by choosing one of the non-basic variables to become basic and the exit of one of the basic variables.

Note:

To determine the basic internal variable, we take the non-basic variable that achieved the most negative in the indirect cost, and to make the solution the best possible, we try to pass in it the largest possible amount, we explain the above through the following example 1:

Example 1:

The following Table 1 represents the cost of transporting goods from sources $A_i;\ i = 1, 2, 3, 4$ to distribution centers $B_j;\ j = 1,2,3,4$ it is required to use the mobile quarantine method to improve the solution and obtain the ideal solution**:**

Table 1. Issue data.

Consumption center / Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0	150
A_2	{3, 4.5}	{1, 2}	{5, 8}	200
A_3	6	2	4	325
A_4	1	7	9	25
Required quantities	180	320	200	700 / 700

In this example, the cost of transportation of the product at the production center A_2 is neutrosophic values we take in forms $c_{2j} \in \left\{\alpha_{1_{2j}}\alpha_{2_{2j}}\right\}$.

The solution:

We find the initial solution using the least cost method; we get the following preliminary solution Table 2.

Table 2. Preliminary solution.

Consumption center / Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0 150	**150**
A_2	{3,4.5}	{1,2} 200	{5,8}	**200**
A_3	6 155	2 120	4 50	**325** **170** **50**
A_4	1 25	7	9	**25**
Required quantities	180 155	320 120	200 50	700 / 700

We note that the number of occupied squares is equal to $m + n - 1 = 6$

The total transportation cost according to the preliminary solution is:

$$Z_1 = 0 \times 150 + \{1,2\} \times 200 + 6 \times 155 + 2 \times 120 + 4 \times 50 + 1 \times 25$$

For $c_{22} = 1 \Rightarrow Z_1 = 1595$

For $c_{22} = 2 \Rightarrow Z_1 = 1795$

That is, against this preliminary solution, we have a neutrosophic value for the total transportation cost:

$$Z_1 \in \{1595,1795\}$$

Now we see whether this solution is an optimal solution or not?

For this we define basic variables and non-basic variables, it is clear that

The basic variables are:

$$x_{13}, x_{22}, x_{31}, x_{32}, x_{33}, x_{41}$$

The non-basic variables are:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

We have six basic variables and six non-basic variables, so we get six closed paths are formed it is in Figure 2:

Note: *The non-basic variables are green.*

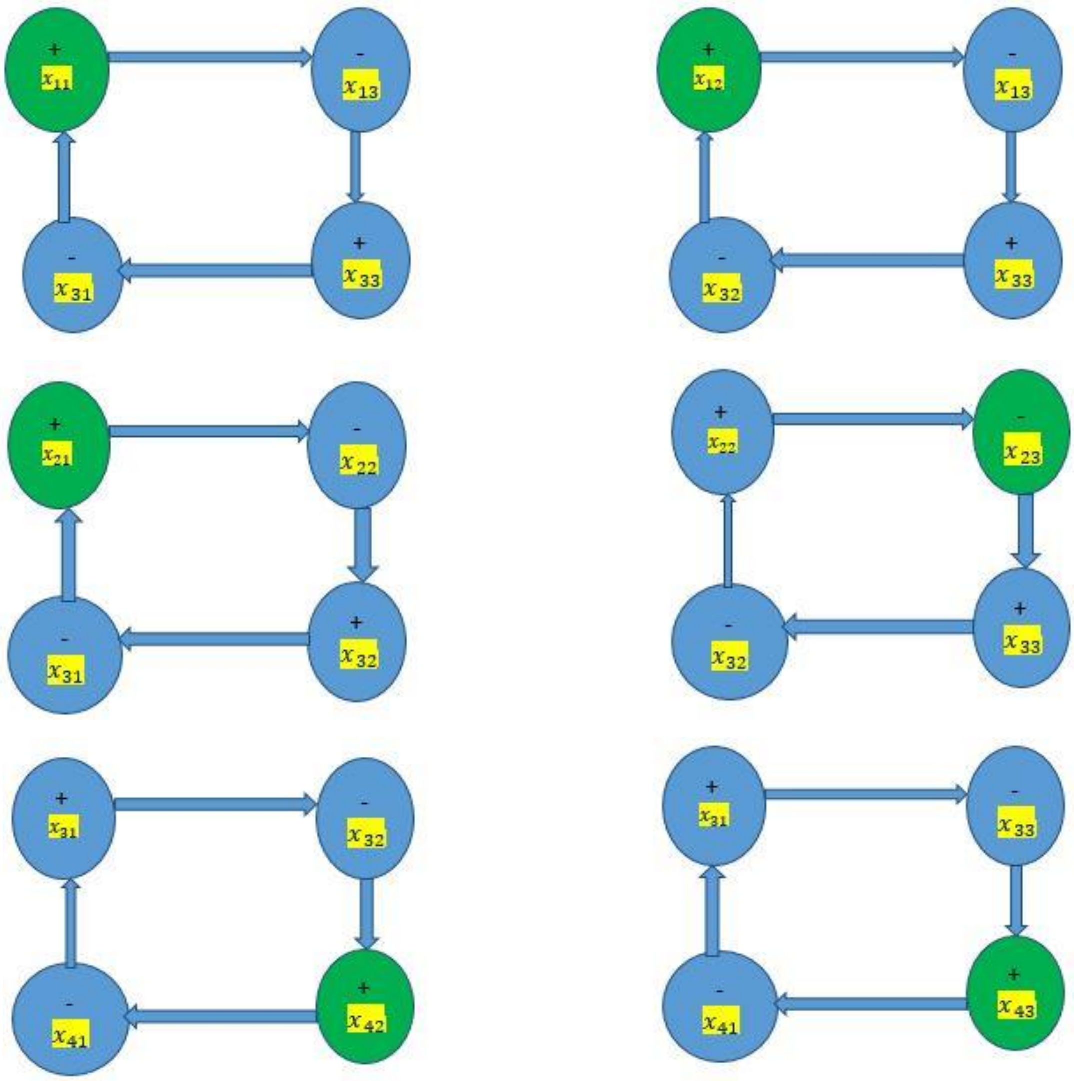

Figure 2. Possible closed paths after finding the initial solution.

The following Table 3, shows how the path is formed:

Table 3. Closed path identification.

Consumption center / Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0 150	150
A_2	{3,4.5}	{1,2} 200	{5,8}	200
A_3	6 155	2 120	4 50	325
A_4	1 25	7	9	25
Required quantities	180	320	200	700 700

We calculate the indirect cost, we find:
From the previous table and according to the drawn path, we explain how to calculate the indirect cost:
We start from the room (A_1B_1) we have the cost $c_{11} = 2$, we take it with a positive sign because it is the room of the non-basic variable, then to the room (A_1B_3) we have the cost $c_{13} = 0$ We take here the minus sign and the variable in this room is a basic variable then to the room (A_3B_3)we have the cost $c_{33} = 4$, we take here the sign is positive, and the variable in this room is a basic variable then to the room (A_3B_1) we have the cost $c_{31} = 6$ We take here the minus sign, and the variable in this room is a basic variable, So the indirect cost of the non-essential variable is x_{11} is :

$$x_{11}: 2 - 0 + 4 - 6 = 0$$

In the same way, we calculate the cost for all non-essential variables we find:

$$x_{12} : 4 - 0 + 4 - 2 = 6$$

$$x_{21} : \{3,4.5\} - \{1,2\} + 2 - 6 = \{-2, -3, -0.5, -1,5\}$$

$$x_{23} : \{5,8\} - 4 + 2 - \{1,2\} = \{2,1,5,4\}$$

$$x_{42} : 7 - 1 + 6 - 2 = 10$$

$$x_{43} : 9 - 1 + 6 - 4 = 10$$

We note that the indirect cost corresponding to the basic variable x_{21} is a negative amount and it is the only one, so we enter this variable and it becomes one of the basic variables and we exit instead of x_{31}
We notice that we can pass the quantity $x_{21} = 155$, then it becomes:

$$x_{31} = 0, \quad x_{32} = 275 \ , \ x_{22} = 45$$

We get the following Table 4:

Table 4. The first improvement.

Consumption center / Production centers	$\boldsymbol{B_1}$	$\boldsymbol{B_2}$	$\boldsymbol{B_3}$	Available Quantities
A_1	2	4	0 150	150
A_2	$\{3, 4.5\}$ 155	$\{1, 2\}$ 45	$\{5, 8\}$	200
A_3	6	2 275	4 50	325
A_4	1 25	7	9	25
Required quantities	180	320	200	700 / 700

We note that the transportation cost is according to the previous solution:

$$Z_2 \in (0 \times 150 + \{3,4.5\} \times 155 + \{1,2\} \times 45 + 2 \times 275 + 4 \times 50 + 1 \times 25)$$

For $c_{21} = 3$ *and* $c_{22} = 1 \Rightarrow Z_2 = 1285$

For $c_{21} = 3$ *and* $c_{22} = 2 \Rightarrow Z_2 = 1330$

For $c_{21} = 4.5$ *and* $c_{22} = 1 \Rightarrow Z_2 = 1517.5$

For $c_{21} = 4.5$ *and* $c_{22} = 2 \Rightarrow Z_2 = 1562.5$

Therefore:

$$Z_2 \in \{1285 , 1330 , 1517.5 ,1562.5 \}$$

$$\forall\, Z_2 \in \{1285 , 1330 , 1517.5 ,1562.5 \} \Longrightarrow Z_2 < Z_1 \in \{1595,1795\}$$

That is, this solution is better than the previous one, the question now is whether this solution is the optimal solution, for this we define the basic variables and the non-basic variables we find:

The basic variables are:

$$x_{41} ,x_{33} ,x_{32} ,x_{22} ,x_{21} ,x_{13}$$

The non-basic variables are:

$$x_{43} ,x_{42} ,x_{31} ,x_{23} ,x_{12} ,x_{11}$$

We have six basic variables and six non-basic variables, so we get six closed paths; we form the closed paths for the six non-basic variables in Figure 3:

Note: *The non-basic variables are green.*

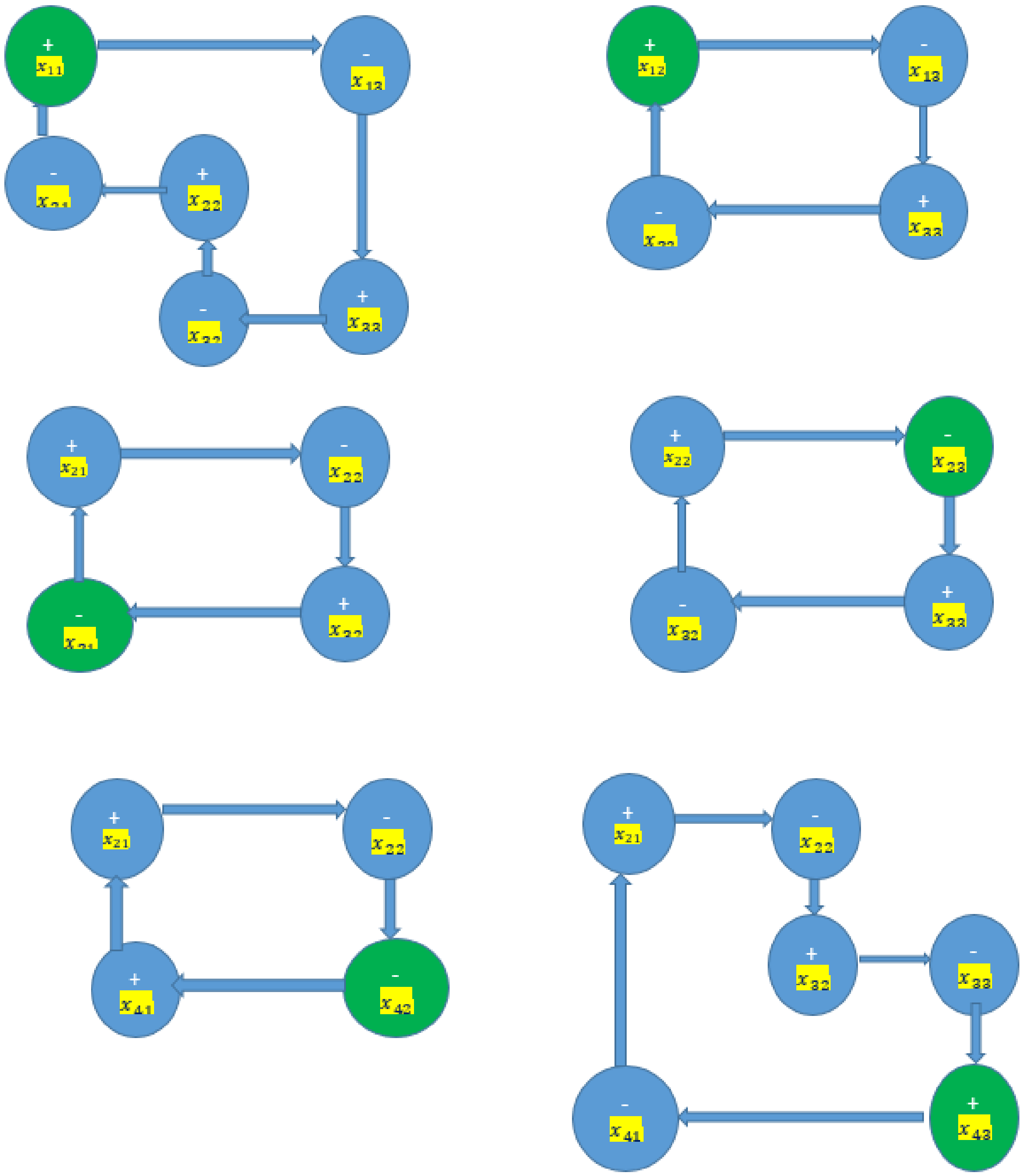

Figure 3. Possible closed paths after the first optimization.

We calculate the indirect cost:

$$x_{11} : 2 - 0 + 4 - \{1,2\} + 1 - 3 = \{3,2\}$$
$$x_{12} : 4 - 0 + 4 - 2 = 8$$
$$x_{32} : 6 - 2 + \{1,2\} - \{3,4.5\} = \{2,0.5,3,1.5\}$$
$$x_{42} : 7 - \{1,2\} + \{3,4.5\} - 1 = \{8,9.5,7,8.5\}$$
$$x_{43} : 9 - 4 + 2 - \{1,2\} + \{3,4.5\} - 1 = \{8,9.5,7,8.5\}$$

We note that the indirect cost for each non-basic variable is positive, and therefore we cannot introduce any non-basic variable to the basic rule. Therefore, the solution that we obtained in the first

improvement is an optimal vinegar, and the minimum transportation cost is the one that we obtained previously:
Therefore the optimal solution is:

$x_{13} = 150$, $x_{21} = 155$, $x_{22} = 45$, $x_{32} = 275$, $x_{33} = 50$, $x_{41} = 25$

The minimum cost of transportation is:

$$Z_2 \in \{1285 , 1330 , 1517.5 ,1562.5 \}$$

2.2 *Modified Distribution Method*

This method is another method of finding the optimal solution for transportation issues, and it is also similar to the previous method (the mobile stone method) conjunction.
To find the optimal solution to the transportation issue according to this method, we follow the following steps:

i. We find the initial solution in one of the previously mentioned ways
ii. We define the essential variables and non-basic variables for the solution
iii. We associate with each line $\boldsymbol{i}$ multiplied by $\boldsymbol{u_i}$, and with each column j multiplied, we call it v_j, so it is:

For each basic variable x_{ij} we have:

$$u_i + v_j = c_{ij} \qquad (*)$$

Where $\boldsymbol{c_{ij}}$ is the cost from A_i to B_j:
Since the number of basic variables is $m + n - 1$, we get the $m + n - 1$ equation from the previous Figure $(*)$ and by solving these equations we must find the values of u_i , v_j which have $m + n$ so we must give one of these multipliers an optional value, then we solve these equations according to this value.
After we found the values u_i , v_j , for each non-basic variable x_{ij} we calculate the quantities $c_q = c_{ij} - u_i - v_j$
In a similar way to the moving stone method, but if one of these quantities is negative, then we must introduce a non-basic variable to the set of basic variables and output instead of a basic variable, and the primary variable entered is chosen in the same previous way:

Example 2:

Let's take the previous example, where we found the preliminary solution according to the cost method:

Table 5. The preliminary solution.

	Consumption center / Production centers	$\boldsymbol{v_1}$	$\boldsymbol{v_2}$	$\boldsymbol{v_3}$	Available Quantities
		B_1	B_2	B_3	
u_1	A_1	2	4	0 150	150
u_2	A_2	{3,4.5}	{1,2} 200	{5,8}	200
u_3	A_3	6 155	2 120	4 50	325
u_4	A_4	1 25	7	9	25
Required quantities		180	320	200	700 700

Transportation cost is: $Z_1 = 1595$

Basic variables:

$$x_{13}, x_{22}, x_{31}, x_{32}, x_{33}, x_{41}$$

Non- basic variables:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

Multipliers is: $\boldsymbol{u_i}$; i = 1,2,3,4 and v_j ; j = 1,2,3.

For basic variables, we have:

For x_{13}, we have $u_1 + v_3 = 0$

For x_{22}, we have $u_2 + v_2 = \{1,2\}$

For x_{31}, we have $u_3 + v_1 = 6$

For x_{32}, we have $u_3 + v_2 = 2$

For x_{33}, we have $u_3 + v_3 = 4$

For x_{41}, we have $u_4 + v_1 = 1$

It is six equations with seven unknowns. To solve them, we impose $u_1 = 0$, so we find the rest of the variables:

$$u_1 = 0, u_2 = 3, u_3 = 4, \quad u_4 = -1$$

$$v_1 = 2, v_2 = -2, v_3 = 0$$

For non-basic variables, we have:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

For x_{11} it is: $\bar{c}_{11} = c_{11} - u_1 - v_1 = 2 - 0 - 2 = 0$

For x_{12} it is: $\bar{c}_{12} = c_{12} - u_1 - v_2 = 4 - 0 + 2 = 6$

For x_{21} it is: $\bar{c}_{21} = c_{21} - u_2 - v_1 = \{3,4.5\} - 3 - 2 = \{-2, -3.5\}$

For x_{23} it is: $\bar{c}_{23} = c_{23} - u_2 - v_3 = \{5,8\} - 3 - 0 = \{2, 5\}$

For x_{42} it is: $\bar{c}_{42} = c_{42} - u_4 - v_2 = 7 + 1 + 2 = 10$

For x_{43} it is: $\bar{c}_{43} = c_{43} - u_4 - v_3 = 9 + 1 - 0 = 10$

We note that the quantity $\bar{c}_{21} = -2$ is a negative value, and therefore the initial solution that we got is not optimal, we must develop this solution, and for that, we form the closed path for the non-basic variable x_{21}, so we find it from the Figure 4:

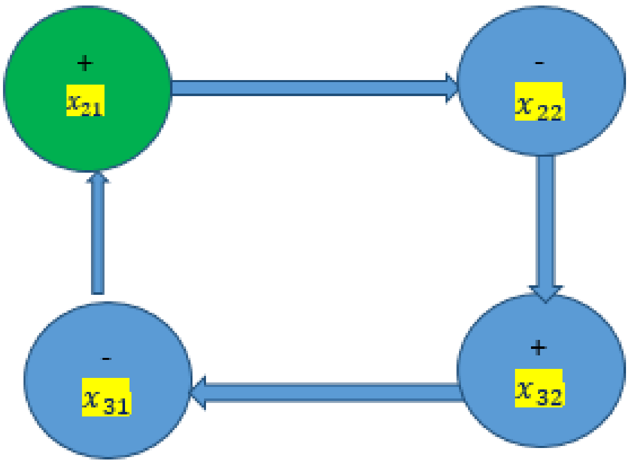

Figure 4. Possible closed pathways for the non-basic variable x_{21}.

We enter x_{21} into the set of basic variables, by giving it the value $x_{21} = 155$, and we remove the variable x_{31} so it becomes a non-basic variable, and then it becomes $x_{22} = 45$ and $x_{32} = 275$, so we get the following Table 6:

Table 6. The first improvement.

Production centers \ Consumption center		v_1 B_1	v_2 B_2	v_3 B_3	Available quantities
u_1	A_1	2	4	0 150	150
u_2	A_2	{3,4.5} 155	{1,2} 45	{5,8}	200
u_3	A_3	6	2 275	4 50	325
u_4	A_4	1 25	7	9	25
Required quantities		180	320	200	700 \ 700

We enter x_{21} into the set of basic variables, by giving it the value $x_{21} = 155$, and we remove the variable x_{31} so it becomes a non-basic variable, and then it becomes $x_{22} = 45$ and $x_{32} = 275$, so we get the following Table 6:
The new transfer cost is:

$$Z_2 \in \{1285\ ,1330\ ,1517.5\ ,1562.5\ \}$$

$$\forall\ Z_2 \in \{1285\ ,1330\ ,1517.5\ ,1562.5\ \} \Longrightarrow Z_2 < Z_1 \in \{1595,1795\}$$

This solution is better than the previous solution, but is it the optimal solution?
For basic variables, we have:
For x_{13} we have $u_1 + v_3 = 0$
For x_{21} we have $u_2 + v_1 = 3$
For x_{22} we have $u_2 + v_2 = 1$
For x_{32} we have $u_3 + v_2 = 2$
For x_{33} we have $u_3 + v_3 = 4$
For x_{41} we have $u_4 + v_1 = 1$
We assume $u_1 = 0$ and solve the system of equations we find:

$$u_1 = 0,\ u_2 = 3,\ u_3 = 4,\ u_4 = 1$$

$$v_3 = 0,\ v_2 = -2,\ \ ,\ v_1 = 0,$$

For non-basic variables:
For x_{11} it is: $\bar{c}_{11} = c_{11} - u_1 - v_1 = 2 - 0 - 0 = 2$
For x_{12} it is: $\bar{c}_{12} = c_{12} - u_1 - v_2 = 4 - 0 + 2 = 6$
For x_{21} it is: $\bar{c}_{21} = c_{21} - u_2 - v_1 = \{3,4.5\} - 3 + 0 = \{0,1.5\}$
For x_{23} it is: $\bar{c}_{23} = c_{23} - u_2 - v_3 = \{5,8\} - 3 + 0 = \{2,5\}$
For x_{42} then: $\bar{c}_{42} = c_{42} - u_4 - v_2 = 7 - 1 + 2 = 8$
For x_{43} it is: $\bar{c}_{43} = c_{43} - u_4 - v_3 = 9 - 1 - 0 = 8$
We note that all quantities $\bar{c}_{ij}$ are positive quantities, so the solution that we got is optimal, and the minimum cost of transportation is:

$$Z_2 \in \{1285\ ,1330\ ,1517.5\ ,1562.5\ \}$$

$$\forall\ Z_2 \in \{1285\ ,1330\ ,1517.5\ ,1562.5\ \} \Longrightarrow Z_2 < Z_1 \in \{1595,1795\}$$

Therefor the optimal solution is:

$x_{13} = 150$, $x_{21} = 155$, $x_{22} = 45$, $x_{32} = 275$, $x_{33} = 50$, $x_{41} = 25$

The minimum cost of transportation is:

$$Z_2 \in \{1285\ , 1330\ , 1517.5\ ,1562.5\ \}$$

3. Conclusion

In many practical issues, we encounter cases in which we are unable to provide confirmed data on the reality of the state of the system under study; the data is affected by circumstances surrounding the working environment, and this matter affects the future and may cause large losses. In the example that was presented in this research, we noticed the study gave us a neutrosophic transport cost suitable for all conditions because it was obtained through neutrosophic data.

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Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors.

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SuperHyperFunction, SuperHyperStructure, Neutrosophic SuperHyperFunction and Neutrosophic SuperHyperStructure: Current understanding and future directions

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Abstract: The *n-th* PowerSet of a Set {or $P^n(S)$} better describes our real world, because a system *S* (which may be a company, institution, association, country, society, set of objects/plants/animals/beings, set of concepts/ideas/propositions, etc.) is formed by sub-systems, which in their turn by sub-sub-systems, and so on. We prove that the SuperHyperFunction is a generalization of classical Function, SuperFunction, and HyperFunction. And the SuperHyperAlgebra, SuperHyperGraph are part of the SuperHyperStructure. Almost all structures in our real world are Neutrosophic SuperHyperStructures since they have indeterminate/incomplete/uncertain/conflicting data.

Keywords: n-th PowerSet; Classical Function; HyperFunction; SuperFunction; SuperHyperFunction; Classical Operation; HyperOperation; SuperHyperOperation; Classical Axiom; HyperAxiom; SuperAxiom; SuperHyperAxiom; Classical Algebra; HyperAlgebra; SuperHyperAlgebra; Neutrosophic SuperHyperAlgebra; SuperHyperGraph; SuperHyperTopology; Classical Structure; HyperStructure; SuperHyperStructure; Neutrosophic SuperHyperStructure.

1. Introduction

In general, a system *S* (that may be a company, association, institution, society, country, etc.) is formed by sub-systems S_i {or $P(S)$, the PowerSet of *S*}, and each sub-system S_i is formed by sub-sub-systems S_{ij} {or $P(P(S)) = P^2(S)$} and so on. That's why the n-th PowerSet of a Set *S* {defined recursively and denoted by $P^n(S) = P(P^{n-1}(S))$} was introduced, to better describes the organization of people, beings, objects etc. in our real world.

The n-th PowerSet, introduced by Smarandache [2] in 2016, was used in defining the SuperHyperOperation, SuperHyperAxiom, and their corresponding Neutrosophic SuperHyperOperation, Neutrosophic SuperHyperAxiom in order to build the SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra. In general, in any field of knowledge, one in fact encounters *SuperHyperStructures*.

2. The n-th PowerSet of a Set better describes our Real World

(i) ***The n-th PowerSet*** of a set better describes our real world, because a system *S* (which may be a company, institution, association, country, society, set of objects/plants/animals/beings, set of concepts/ideas/propositions, etc.) is formed by sub-systems S_i, which in their turn are formed by sub-sub-systems S_{i_j}, and so on, up to a needed structural-level *n* of the system. Afterwards, between the sub-systems, sub-sub-systems and so on there are various inter-relationships (similar to the operations and axioms in general algebraic structures).

*

Let's recall the definition of the *n-th* PowerSet of a Set [2], proposed by Smarandache in 2016.

(ii) ***Definition of the n^th^ PowerSet of a Set*** *S* {denoted as $P^n(S)$, where the empty-set ϕ is allowed, and it represents the indeterminacy/uncertainty} is done recursively:
Let *S* be a set.

$P^0(S) \overset{def}{=} S$, be definition.

$P^1(S) = P(S)$ is the PowerSet of *S*, we call it the *1st*-PowerSet of S;

$P^2(S) = P(P(S))$ is the PowerSet of the PowerSet of *S*, or the *2nd*-PowerSet of *S*;

$P^3(S) = P(P^2(S)) = P(P(P(S)))$ is the PowerSet of the PowerSet of the PowerSet of S, or the *3rd*-PowerSet of *S*;

and so on,

$P^n(S) = P(P^{n-1}(S)) = ... = \underbrace{P(P(...P(S)...))}_{n-times}$, where P is repeated n times, and the empty-set is allowed.

(iii) ***Example of 2nd-PowerSet of a set S, where the empty-set is allowed.***
Let's consider an easy example to be able to distinguish between several types of functions, algebras, and structures.
Let the set $S = \{1, 2\}$.
Then, the 1st-PowerSet of *S*, with the empty set ϕ included, is: $P(S) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$ and this $P(S)$ is used for the neutrosophic versions of functions, operations (and operators), axioms, algebras, and structures.
The 2-nd PowerSet of *S*, with the empty set ϕ included, is:

$$P^2(S) = P(P(S)) = P(\{\phi, \{1\}, \{2\}, \{1, 2\}\}) =$$
$$\{\phi; \{1\}; \{2\}; \{1, 2\};$$
$$\{\{\phi, \{1\}\}; \{\phi, \{2\}\}; \{\phi, \{1, 2\}\};$$
$$\{\{1\}, \{2\}\}; \{\{1\}, \{1, 2\}\}; \{\{2\}, \{1, 2\}\};$$
$$\{\phi, \{1\}, \{2\}\}; \{\phi, \{1\}, \{1, 2\}\}; \{\phi, \{2\}, \{1, 2\}\}; \{\{1\}, \{2\}, \{1, 2\}\};$$
$$\{\phi, \{1\}, \{2\}, \{1, 2\}\}\}.$$

(iv) ***Definition of the n^th^ PowerSet of a Set S without the empty-set*** {denoted as $P_*^n(S)$ where the empty-set ϕ is not allowed} is also done recursively:
Let *S* be a non-empty set.

$P_*^0(S) \overset{def}{=} S$, be definition.

$P_*^1(S) = P_*(S)$ is the PowerSet of *S*, without the empty-set, we call it the *1st*-PowerSet of *S*;

$P_*^2(S) = P_*(P_*(S))$ is the PowerSet of the PowerSet of *S*, without the empty-set, or the *2nd* PowerSet of *S*;

$P_*^3(S) = P_*(P_*^2(S)) = P_*(P_*(P_*(S)))$ is the PowerSet of the PowerSet of the PowerSet of *S*, without the empty-set, or the *3rd*-PowerSet of *S*;

and so on,

$$P_*^n(S) = P_*(P_*^{n-1}(S)) = \ldots = \underbrace{P_*(P_*(\ldots P_*(S)\ldots))}_{n-times}$$, where P is repeated n times,

and the empty set is not allowed.

(v) ***Example of 2nd-PowerSet of a set S, without the empty-set***

Let's consider an easy example to be able to distinguish between several types of functions, algebras, and structures.

Let $S = \{1, 2\}$ be a set, then the 1st-PowerSet of the set *S*, without the empty set ϕ, is $P_*(S) = \{\{1\}, \{2\}, \{1, 2\}\}$.

The 2nd-PowerSet of the set *S*, without the empty set ϕ, is:

$$P_*^2(S) = P_*(P_*(S)) = P_*(\{\{1\}, \{2\}, \{1, 2\}\}) =$$
$$\{\{1\}; \{2\}; \{1, 2\};$$
$$\{\{1\}, \{2\}\}; \{\{1\}, \{1, 2\}\}; \{\{2\}, \{1, 2\}\};$$
$$\{\{1\}, \{2\}, \{1, 2\}\}\}.$$

What is the distinction between, for example, $A = \{1, 2\}$ and $B = \{\{1\}, \{2\}\}$?

In *A*, the elements *1* and *2* are totally dependent of each other and form together a sub-system called *A*; while in *B*, each of *{1}* and *{2}* are partially independent of each other and as such they are individual sub-sub-systems, and partially dependent of each other and united into a sub-system called *B* (a sub-system of sub-sub-systems).

In the real world, we may consider, for example, *A* as a group of two researchers, denoted by *1* and *2*, that work together (totally dependent on each other) for a common project.

But in *B*, researchers *{1}* and *{2}* work each of them separately for the projects *p1* and respectively *p2* (so they are independent from the point of view of these projects), but the researchers work together for the third common project *p3* (so they are dependent from the point of view of project *p3*).

3. Functions of One Variable

(i) ***Classical Function of One Variable***

The domain and codomain of the function is just *S*.

$f : S \to S$

Example of Classical Function of One Variable

Let's take, as above, *S = {1, 2}*.

$f(1) = 2$ (a single-value) $\in S$;

$f(2) = 1$ (a single-value) $\in S$.

(ii) ***HyperFunction of One Variable***

This is part of the HyperStructures [1], when the domain *S* remains unchanged, while the codomain of the function becomes the PowerSet $P_*(S)$.

$f : S \to P_*(S)$

Example of HyperFunction of One Variable

$f(1) = \{1, 2\}$ (a set-value) $\in P_*(S)$;

$f(2) = 1 \in P_*(S)$.

(iii) ***SuperFunction of One Variable***

This is an extension of the HyperFunction, when the domain *S* remains the same, but the codomain of the function becomes *n*th-PowerSet of the set *S*, i.e. $P_*^n(S)$, $n \ge 2$.

$f : S \to P_*^n(S)$, where integer $n \ge 2$.

(iv) ***Example of SuperFunction of One Variable***

Let's take the easiest case when *n* = 2, the domain of the function remains the same *S*, but one has the 2nd-PowerSet of the set S as codomain of the function:

$f : S \to P_*^2(S)$

$f(1) = \{\{1\},\{1,2\}\} \in P_*^2(S)$;

$f(2) = \{\{1\},\{2\}\} \in P_*^2(S)$.

(v) ***SuperHyperFunction of One Variable***

$f : P_*^r S \to P_*^n(S)$, for integers $r, n \ge 0$.

It is part of the SuperHyperStructure [2, 3].

(vi) ***Example of SuperHyperFunction of One Variable***

$f : P_*(S) \to P_*^2(S)$.

$f(\{1\}) = \{\{1\},\{2\},\{1,2\}\} \in P_*^2(S)$.

$f(\{2\}) = \{\{1\},\{2\}\} \in P_*^2(S)$.

$f(\{1,2\}) = \{\{2\},\{1,2\}\} \in P_*^2(S)$

(vii) ***Theorem 1:***

The SuperHyperFunction of One Variable is the most general form of functions of one variables.

Proof:

For r = 0, and n = 0, one gets the classical Function.

For r = 0, and n = 1, one gets the HyperFunction.

For r = 0, and n ≥ 2, one gets the SuperFunction.

4. Functions of Many Variables

Straightforward generalization of the functions, from one variable to many variables, are provided below.

(i) ***Classical Function of Many Variables***

$f : S^m \to S$, for integer m ≥ 2.

(ii) ***Example of Classical Function of Two Variables***

Let's consider some elementary case, when *m* = 2.

$f : S^2 \to S$

First, $S^2 = \{1,2\} \times \{1.2\} = \{(1,1),(1,2),(2,1),(2,2)\}$

$f(1,1) = 2$ (a single-value) $\in S$

$f(1,2) = 2 \in S$

$f(2,1) = 1 \in S$

$f(2,2) = 1 \in S$

(iii) ***HyperFunction of Many Variables***

This is part of the HyperStructures.

$f : S^m \to P_*(S)$

(iv) ***Example of HyperFunction of Two Variables***

$$f: S^2 \rightarrow P_*(S)$$

$f(1,1) = \{1, 2\}$ (a set-value) $\in P_*(S)$

$$f(1,2) = \{1\} \in P_*(S)$$
$$f(2,1) = \{1, 2\} \in P_*(S)$$
$$f(2,2) = \{2\} \in P_*(S)$$

(v) ***SuperFunction of Many Variables***

$f: S^m \rightarrow P_*^n(S)$, where the integers *m*, $n \geq 2$. When the 2-nd PowerSet of the Set *S* is involved.

(vi) ***Example of SuperFunction of Two Variables***

Let's take *m* = *n* = 2 the simplest case.

$$f: S^2 \rightarrow P_*^2(S)$$

$$f(1,1) = \{\{1\}, \{2\}, \{1, 2\}\} \in P_*^2(S)$$
$$f(1,2) = \{\{1\}, \{2\}\} \in P_*^2(S)$$
$$f(2,1) = \{\{2\}, \{1, 2\}\} \in P_*^2(S)$$
$$f(2,2) = \{\{1\}, \{1, 2\}\} \in P_*^2(S)$$

(vii) ***SuperHyperFunction of Many Variable***

$f: (P_*^r S)^m \rightarrow P_*^n(S)$, for integers $m \geq 2$ and $r, n \geq 0$.

It is part of the SuperHyperStructure.

(viii) ***Example of SuperHyperFunction of Two Variables***

Let's take *m = 2*, *r = 1*, and *n = 2*.

$$f: (P_*(S))^2 \rightarrow P_*^2(S)$$

Table 1. Values of the above SuperHyperFunction of two variable f(x, y).

x / y	**{1}**	**{2}**	**{1, 2}**
{1}	{{1}, {2}}	{1}	{{1}, {1 2}}
{2}	{{2}, {1, 2}}	{{1}, {1, 2}}	{2}
{1, 2}	{1, 2}		{{1}. {2}, {1, 2}}

For example, *f({1}, {1, 2}) = {{1}, {1 2}}*.

(ix) ***Theorem 2:***

Similarly, the SuperHyperFunction of Many Variables is a generalization of the Classical Function, HyperFunction, and SuperFunction of Many Variables.

Proof is the same as in the previous theorem, by keeping the same *m* (number of variables) value:

For $r = 0$, and $n = 0$, one gets the classical Function of Many Variables.

For $r = 0$, and $n = 1$, one gets the HyperFunction of Many Variables.

For $r = 0$, and $n \geq 2$, one gets the SuperFunction of Many Variables.

5. Definition of SuperHyperFunction (SHF_m) of $m \geq 2$ Variables

$f_{SH}^{SH}: P_*^{r_1}(S) \times P_*^{r_2}(S) \times \ldots \times P_*^{r_k}(S) \to P_*^{n}(S)$, where integers $r_1, r_2, \ldots, r_k, n \geq 0$ and *SH* stands for *SuperHyper*, the upper *SH* is for the function's domain, and the bottom *SH* is for the function's codomain, meaning that both are some PowerSets of PowerSets etc. of the set S.

For any $x_1 \in P_*^{r_1}(S), x_2 \in P_*^{r_2}(S), \ldots, x_k \in P_*^{r_k}(S)$, one has $f_{SH}^{SH}(x_1, x_2, \ldots, x_k) \in P_*^{n}(S)$.

This is a generalization of all previous functions.

6. Operations / HyperOperations / SuperHyperOperations and Axioms / HyperAxioms / SuperHyperAxioms

Let $m \geq 1$ be an integer.

The **Operations** (and operators) can be treated as *m-ary* functions, while the **Axioms** as logical propositions involving the *m-ary* operations.

Similarly, the **HyperOperations** can be treated as *m-ary* HyperFunctions, while the **HyperAxioms** as logical propositions involving the *m-ary* HyperOperations. And last, the **SuperHyperOperations** can be treated as *m-ary* SuperHyperFunctions, while the **SuperHyperAxioms** (***HSAx***) as logical propositions involving *m-ary* SuperHyperOperations.

7. Structure / HyperStructure / SuperHyperStructure

(i) The classical **Structure** is a structure built on a set *S*, endowed with classical Operations ($\#_C$) $\#_C: S^m \to S$, for integer $m \geq 1$, and classical Axioms (A_C), which are axioms that act on the set S endowed with classical Operations.

(ii) The **HyperStructure** {defined by F. Marty [1] in 1934}, is a structure built on a set *S*, endowed with HyperOperations ($\#_H$), $\#_H: S^m \to P_*(S)$, for integer $m \geq 1$, and HyperAxioms (A_H), which are axioms that act on the set S endowed with HyperOperations. "Hyper" stands for the codomain of the operations, which is $P_*^n(S)$ instead of *S* that is for the classical structure.

(iii) The **SuperStructure** {defined by F. Smarandache [2] in 2016}, is a structure built on $P_*^n(S)$, that is the n^{th}-PowerSet of the Set *S*, without the empty-set, endowed with SuperOperations, $\#_S: (P_*^n(S))^m \to P_*^q(S)$, for integers $n \geq 0, q \geq 0$, and SuperAxioms, which are axioms that act on the set $P_*^n(S)$ endowed with SuperOperations. "Super" stands for the codomain of the SuperOperations, which is $P_*^q(S)$, instead of *S* that is for the classical structure or of $P_*^n(S)$ that is for HyperStructure, or for the domain of the SuperOperations, which is $P_*^n(S)$.

(iv) The **SuperHyperStructure** {defined by F. Smarandache [2, 3] in 2016 and 2019}, is a structure built on the n[th]-PowerSet of the Set *S*, $P_*^n(S)$, endowed with SuperHyperOperations and SuperHyperAxioms.

8. The most general form of the SuperHyperAlgebra (SHA) endowed with One Operation and Many Axioms

is:

$(P_*^n(S);\#_{SHO}^m;SHAx_1,SHAx_2,...,SHAx_q)$ where S is a non-empty set, $P_*^n(S)$ is the *n[th]*-PowerSet of the set S, for $n\geq 2$, and $\#_{SHO}^m$ is an ***m-ary*** **SuperHyperOperation (*SHO*)**, acting on $P_*^n(S)$:

$\#_{SHO}^m:\underbrace{P_*^n(S)\times P_*^n(S)\times...\times P_*^n(S)}_{m-times}\rightarrow P_*^n(S)$, where $P_*^n(S)$ is repeated *m times* into the operation domain, and integer $m\geq 1$, and *q* is the number of **SuperHyperAxioms**.

9. The most general form of the SuperHyperAlgebra with Many Operations and Many Axioms

is:

$(P_*^n(S);\#_{SHO1}^{m_1},\#_{SHO2}^{m_2},...,\#_{SHOr}^{m_r};SHAx_1,SHAx_2,...,SHAx_q)$ where the m_i -ary SuperHyperOperations are defined as follows:

$\#_{SHO}^{m_i}:\underbrace{P_*^n(S)\times P_*^n(S)\times...\times P_*^n(S)}_{m_i-times}\rightarrow P_*^n(S)$, with $P_*^n(S)$ being repeated m_i times into the operation domain, $m_i\geq 1$, for $1\leq i\leq r$, and $r\geq 2$ is the number of m_i - *ary* SuperHyperOperations ($\#_{SHO1}^{m_1},\#_{SHO2}^{m_2},...,\#_{SHOr}^{m_r}$), and $q\geq 1$ is the number of SuperHyperAxioms ($SHAx_1,SHAx_2,...,SHAx_q$).

10. SuperHyperTopology

SuperHyperTopology [5] is a topology built on a SuperHyperAlgebra $(P_*^n(S),\#)$, for integer *n* ≥ 2, and it is a collection of SuperHyperSubsets from $P_*^n(S)$ that satisfy the axioms of classical topology.

11. Neutrosophic SuperHyperStructure et al.

All of the above *non-neutrosophic concepts* can easily be extended to the *neutrosophic framework*, therefore:

the Neutrosophic HyperFunction / SuperFunction / SuperHyperFunction of one or many variables, and the Neutrosophic HyperOperation / SuperHyperOperation, and the Neutrosophic HyperAxiom / SuperHyperAxiom, and the Neutrosophic SuperHyperAlgebra / SuperHyperTopology, and, in general, the Neutrosophic Super/Hyper/SuperHyperStructure are built in the same corresponding ways as the above non-neutrosophic concepts, with the only distinction that all $P^k{}_*(S)$, which do not include the empty-set, are replaced by $P^k(S)$, which do include the empty-set (leaving room for indeterminate/incomplete/uncertainty/conflicting data), for all integers $k\geq 1$.

12. Applications

We need to work with the n[th]-PowerSet of a set to better describe the organization of our real world. A system (set) S is composed by sub-systems (the elements of the $P_*(S)$, the PowerSet of S, let's denote them by S_1, S_2, …), and the sub-systems by sub-sub-systems (let's denote them by S_{11}, S_{12}, … respectively S_{21}, S_{22}, …), and so on.

As future possible research work will be to investigate the applicability of many types of SuperHyperStructures and Neutrosophic SuperHyperStructures in the real world.

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Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors.

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