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Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e., notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory, every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjointed two by two. But, since in many cases, the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and intuitionistic fuzzy logic). In neutrosophic logic, a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^{-}0, 1^{+}[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of classical statistics.

What distinguishes neutrosophic from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Fuzzy Inference Full Implication Method Based on Single Valued Neutrosophic t-representable t-norm: Purposes, Strategies, and a Proof-of-Principle Study

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Abstract: As a generalization of intuitionistic fuzzy sets, single-valued neutrosophic sets have certain advantages in solving indeterminate and inconsistent information. In this paper, we study the fuzzy inference full implication method based on single-valued neutrosophic t-representable t-norm. Firstly, single-valued neutrosophic fuzzy inference triple I principles for fuzzy modus ponens and fuzzy modus tollens are given. Then, single-valued neutrosophic R-type triple I solutions for FMP and FMT are given. Finally, the robustness of the full implication triple I method based on the left-continuous single-valued neutrosophic t-representable t-norm is investigated. As a special case of the main results, the sensitivity of full implication triple I solutions based on three special single-valued neutrosophic t-representable t-norms are given.

Keywords: Single Valued Neutrosophic Set; Single Valued Neutrosophic; t-representable t-norm; Full Implication Triple I Method.

1. Introduction

Fuzzy sets have been applied to deal with uncertain, vague, inaccurate information in the real world. However, it is widely known that fuzzy reasoning plays an important role in fuzzy set theory. Especially, the most basic forms of fuzzy reasoning are Fuzzy Modus Ponens (FMP for short) and Fuzzy Modus Tollens (FMT for short), which can be shown as follows [1, 2]:

FMP (A, B, A^*): given the fuzzy rule and premise A^*, attempt to reason a suitable fuzzy consequent B^*.

FMT (A, B, B^*): given the fuzzy rule and premise B^*, attempt to reason a suitable fuzzy consequent A^*.

In the above models, and$B, B^* \in F(Y)$, where and denote fuzzy subsets of the universes and respectively.

The most famous method to solve the above models is the Compositional Rule of Inference (CRI for short), which is presented by Zadeh [2, 3]. However, the CRI method lacks clear logic semantics and reductivity. To overcome this shortcoming, Wang [1] proposed the fuzzy reasoning full implication triple I method, which can bring fuzzy reasoning into the framework of logical semantic [4]. In recent years, many scholars have studied the fuzzy reasoning full implication method. Wang et al. [5] gave a unified form for fuzzy reasoning full implication method based on normal implication and regular implication. Pei [6] gave a unified form fuzzy reasoning full implication method based on residual implication induced by left continuous t-norms. Moreover, Pei [7] established the solid logical foundation for the fuzzy reasoning full implication method based on left continuous t-norms.

Liu et al. [8] gave the unified form of the solutions for fuzzy reasoning full implication method. Luo and Yao [9] studied the fuzzy reasoning triple I method based on Schweizer-Sklar operators.

Although fuzzy set theory has been successfully applied in many fields, there are some defects in dealing with fuzzy and incomplete information. Atanassov [10] introduced intuitionistic fuzzy sets (IFSs), which are represented by a membership and a non-membership function. Intuitionistic fuzzy sets can represent not only the positive and negative aspects of the given information but also the hesitant information. Meanwhile, Gorzalczany [11] and Turksen [12] proposed interval-valued fuzzy sets, which represent a subinterval in the membership function. Intuitionistic fuzzy sets and interval-valued fuzzy sets are equivalent [13]. In recent years, some research results on intuitionistic fuzzy reasoning and interval-valued fuzzy reasoning have been achieved. Zheng et al. [14] extended the triple I method on intuitionistic fuzzy sets. Li et al. [15] extended the CRI method on interval-valued fuzzy sets. Luo et al. [16-19] studied interval-value fuzzy reasoning full implication triple I method and reverse triple I method based on the interval-valued associated t-norm. Moreover, Luo et al.[20] studied fuzzy reasoning triple I method based on the interval-value t-representable t-norm.

Although an intuitionistic fuzzy set has some advantages in dealing with fuzzy and incomplete information, it has defects in dealing with fuzzy, incomplete, and inconsistent information. To deal with this case, Smarandache [21] proposed a neutrosophic set, which is represented by a truth-membership function, an indeterminacy-membership function, and a falsity-membership function. The neutrosophic set represents uncertain, incomplete, and inconsistent information in the real world. However, truth-membership, indeterminacy-membership, and falsity-membership functions are nonstandard fuzzy subsets, which are difficult to apply in practice. Smarandache [22] and Wang et al [23] proposed a single-valued neutrosophic set, the truth-membership, indeterminacy-membership, and falsity-membership degrees are a real number in the unit interval [0,1]. The single-valued neutrosophic set can be considered as a generalization intuitionistic fuzzy set. In recent years, Scholars have paid attention to the study of single-valued neutrosophic sets. Smarandache [21] studied a unifying field in logic. Smarandache [24] proposed n-norm and n-conorm in neutrosophic logic. Rivieccio [25] investigated neutrosophic logic. Alkhazaleh [26] gives some norms and conforms based on the neutrosophic set. Zhang et al. [27] gave a new inclusion relation for neutrosophic sets. Hu and Zhang [28] constructed the residuated lattices based on the neutrosophic t-norms and neutrosophic residual implications. So far, there is little research on fuzzy reasoning methods based on single-valued neutrosophic sets. In [29], Ghorai et al. studied the operations of the Cartesian product, composition, and union of two image fuzzy digraphs. In [30], Ghorai et al. proposed a bipolar fuzzy incidence graph and analyzed the properties of a bipolar fuzzy incidence graph. In [31], Ghorai et al. analyzed the properties of the complexity function and its importance in the network field and applied the complexity function to identify the period of COVID-19. Zhao et al. [32] study reverse triple I algorithms based on single-valued neutrosophic fuzzy inference.

Therefore, we consider researching the fuzzy reasoning triple I method based on a class single valued neutrosophic triangular norm. An important criterion for judging an algorithm is whether the algorithm has a logical basis. Therefore, this paper proposes a logic-based fuzzy reasoning algorithm based on a class single valued neutrosophic triangular norm. The algorithm proposed in this paper is a new neutrosophic set fuzzy inference algorithm with a logical basis.

1.1 The organization of the work

The organization of this paper is as follows: some basic concepts for single-valued neutrosophic sets are reviewed in section 2. In section 3, we give fuzzy inference triple I principles based on left-continuous single-valued neutrosophic t-representable t-norms for fuzzy modus ponens and fuzzy modus tollens, and the corresponding solutions of single-valued neutrosophic triple I methods. In section 4, the robustness of the triple I method based on left-continuous single-valued neutrosophic t-representable t-norm is investigated. Finally, the conclusions are given in Section 5.

2. Preliminaries

In this section, we review some basic concepts for triangular norm, triangular conorm, and single-valued neutrosophic set, which will be used in this article.

Definition 2.1. [33] A mapping T: $[0,1]\times[0,1]\rightarrow[0,1]$ is called a triangular norm (t-norm), if it satisfies associativity, commutativity, monotonicity, and boundary condition $T(x,1)$=x for any $x\in[0,1]$. A mapping S: is called a triangular conorm (t-conorm), if it satisfies associativity, commutativity, monotonicity, and boundary condition $S(x,0)$=x for any $x\in[0,1]$. A t-norm is called the dual t-norm of the t-conorm if $T(x,y)=1-S(1-x,1-y)$. Similarly, a t-conorm is called the dual t-conorm of the t-norm, if $S(x,y)=1-T(1-x,1-y)$.

Definition 2.2. [33] A t-norm T is called left-continuous (resp., right-continuous), if for any $(x_0,y_0)\in[0,1]^2$, and for each $\varepsilon>0$ there is a $\delta>0$ such that $T(x,y)>T(x_0,y_0)-\varepsilon$, whenever $(x,y)\in(x_0-\delta,x_0]\times(y_0-\delta,y_0]$ (resp., $T(x,y)<T(x_0,y_0)+\varepsilon$, whenever $(x,y)\in[x_0,x_0+\delta]\times[y_0,y_0+\delta]$).

Proposition 2.1. [33] A t-norm T is a left-continuous t-norm if and only if there exists a binary operation R_T such that (T,R_T) satisfies the residual principle, i.e., $T(x,z)\leq y$ iff $z\leq R_T(x,y)$ for all $x,y,z\in[0,1]$, where $R_T(x,y)=\sup\{z|T(x,z)\leq y\}$ is called a residual implication induced by t-norm T.

Proposition 2.2. [33] A t-conorm S is a right-continuous t-conorm if and only if there exists a binary operation R_S on L such that (S,R_S) forms a co-adjoint pair, i.e., $x\leq S(y,z)$ iff $R_S(x,y)\leq z$ for all $x,y,z\in[0,1]$, where $R_S(x,y)=\inf\{z|x\leq S(y,z)\}$ is called a coresidual implication induced by t-conorm S.

Example 1. Three important t-norms and their residual implication, t-conorms, and their coresidual implication [32, 33] are in Table 1.

Table 1. t-norms and their residual implications, t-conorms and their coresidual implications.

Name	t-norms	Residual Implications	t-conorms	Coresidual Implications
Łukasiewicz	$T_L(x,y)$ =$0\vee(x+y-1)$	$R_{T_L}(x,y)$ $=1\wedge(1-x+y)$	$S_L(x,y)$ $=(x+y)\wedge 1$	$R_{S_L}(x,y)$ $=(x-y)\vee 0$
Gougen	$T_{Go}(x,b)\ =\ xy$	$R_{T_{Go}}(x,y)=1\wedge\frac{y}{x}$	$S_{Go}(x,y)=\ x+y-xy$	$R_{S_{Go}}(x,y)=\frac{x-y}{1-y}\vee 0$
Gödel	$T_G(x,y)\ =\ x\wedge y$	$R_{T_G}(x,y)=\begin{cases}1, & if \quad x\leq y,\\ y, & if \quad x>y.\end{cases}$	$S_G(x,y)=x\vee y$	$R_{S_{Go}}(x,y)=\begin{cases}0, & if \quad x\leq y,\\ x, & if \quad x>y.\end{cases}$

Definition 2.3. [22] Let X be a universal set. A neutrosophic set A on X is characterized by three functions, i.e., a truth-membership function $t_A(x)$, an indeterminacy-membership function $i_A(x)$ and a falsity-membership function $f_A(x)$. Then, a neutrosophic set A can be defined as follows:

$$A=\{\langle x,t_A(x),i_A(x),f_A(x)\rangle\mid x\in X\},$$

where $t_A(x):X\rightarrow]^-0,1^+[$, $i_A(x):X\rightarrow]^-0,1^+[$, $f_A(x):X\rightarrow]^-0,1^+[$, such that $0^-\leq t_A(x)+i_A(x)+f_A(x)\leq 3^+$, $t_A(x),i_A(x),f_A(x)\in[0,1]$ and satisfy the condition $0\leq t_A(x)+i_A(x)+f_A(x)\leq 3$ for each x in X.

The family of all single valued neutrosophic sets is denoted by $SVNS(X)$.

Definition 2.4. [22] Let A, B be two single valued neutrosophic sets on universal X, the following relations are defined as follows:

(i). $A \subseteq B$ if and only $t_A(x) \leq t_B(x)$, $i_A(x) \geq i_B(x)$ and $f_A(x) \geq f_B(x)$ for all $x \in X$;

(ii). $A = B$ if and only $A \subseteq B$ and $B \subseteq A$;

(iii). $A \cap B = \langle \min(t_A(x), t_B(x)), \max(i_A(x), i_B(x)), \max(f_A(x), f_B(x)) \rangle$ for all $x \in X$ for all $x \in X$;

(iv). $A \cup B = \langle \max(t_A(x), t_B(x)), \min(i_A(x), i_B(x)), \min(f_A(x), f_B(x)) \rangle$ for all $x \in X$;

(v). $A^c = \{\langle f_A(x), 1 - i_A(x), t_A(x) \rangle | x \in X\}$.

Remark 2.1. For arbitrary single valued neutrosophic set $A \in SVNS(X)$, we can obtain:

(i). If $t_A(x) + i_A(x) + f_A(x) = 1$, then a single-valued neutrosophic set A reduces to an intuitionistic fuzzy set.

(ii). If $t_A(x) + i_A(x) + f_A(x) = 1$ and $i_A(x) = 0$, then a single-valued neutrosophic set A reduces to a fuzzy set.

The set of all single valued neutrosophic numbers denoted by $SVNN$, i.e. $SVNN = \{$Žt$, i, f$ž $\mid t, i, f \in [0,1]\}$. Let $\alpha = \langle t_\alpha, i_\alpha, f_\alpha \rangle$, $\beta = \langle t_\beta, i_\beta, f_\beta \rangle \in SVNN$, an ordering on $SVNN$ as $\alpha \leq \beta$ if and only if $t_\alpha \leq t_\beta, i_\alpha \geq i_\beta, f_\alpha \geq f_\beta$, $\alpha = \beta$ iff $\alpha \leq \beta$ and $\beta \leq \alpha$ .Obviously, $\alpha \wedge \beta = \langle t_\alpha \wedge t_\beta, i_\alpha \vee i_\beta, f_\alpha \vee f_\beta \rangle$, $\alpha \vee \beta = \langle t_\alpha \vee t_\beta, i_\alpha \wedge i_\beta, f_\alpha \wedge f_\beta \rangle$, $\wedge_{i \in I} \alpha_i = \langle \wedge_{i \in I} t_{\alpha_i}, \vee_{i \in I} i_{\alpha_i}, \vee_{i \in I} f_{\alpha_i} \rangle$, $\vee_{i \in I} \alpha_i = \langle \vee_{i \in I} t_{\alpha_i}, \wedge_{i \in I} i_{\alpha_i}, \wedge_{i \in I} f_{\alpha_i} \rangle 0^* = \langle 0,1,1 \rangle$ and $1^* = \langle 1,0,0 \rangle$ are the smallest element and the greatest element in $SVNN$, respectively. It is easy to verify that $(SVNN, \leq)$ is a complete lattice [29].

After introducing single-valued neutrosophic numbers, we will then introduce the properties of single-valued neutrosophic t-norm.

Definition 2.5. [28] A function $\mathcal{T}$: $SVNN \times SVNN \rightarrow SVNN$ is called a single-valued neutrosophic t-norm if the following four axioms are satisfied, for all $\alpha, \beta, \gamma \in SVNN$,

(i). $\mathcal{T}(\alpha, \beta) = \mathcal{T}(\beta, \alpha)$, (commutativity)

(ii). $\mathcal{T}((\alpha, \beta), \gamma) = \mathcal{T}(\alpha, (\beta, \gamma))$, (associativity)

(iii). $\mathcal{T}(\alpha, \gamma) \leq \mathcal{T}(\beta, \gamma)$ if $\alpha \leq \beta$, (monotonicity)

(iv). $\mathcal{T}(\alpha, 1^*) = \alpha$. (boundary condition)

Example 2. [32] The function $\mathcal{T}$: $SVNN \times SVNN \rightarrow SVNN$ defined by $\mathcal{T}(\alpha, \beta) = \langle T(t_\alpha, t_\beta), S(i_\alpha, i_\beta), S(f_\alpha, f_\beta) \rangle$ is a single-valued neutrosophic t-norm, which is called a single-valued neutrosophic t-representable t-norm, where T is a t-norm and S is its dual t-conorm on [0, 1]. $\mathcal{T}$ is called a left-continuous single valued neutrosophic t-representable t-norm if T is left-continuous and S is right-continuous.

Definition 2.6. [32] A single valued neutrosophic residual implication is defined by $\mathcal{R}_\mathcal{T}(\alpha, \beta) = \sup\{\gamma \in SVNN \mid \mathcal{T}(\gamma, \alpha) \leq \beta\}$, $\forall \alpha, \beta \in SVNN$, where $\mathcal{T}$ is a left-continuous single valued neutrosophic t-representable t-norm.

Proposition 2.3. [32] Let $\mathcal{T}$ be a single-valued neutrosophic t-representable t-norm, the following statements are equivalent:

(i). $\mathcal{T}$ is left-continuous;

(ii). $\mathcal{T}$ and $\mathcal{R}_\mathcal{T}$ form an adjoint pair, i.e., they satisfy the following residual principle

$$\mathcal{T}(\gamma, \alpha) \leq \beta \Leftrightarrow \gamma \leq \mathcal{R}_\mathcal{T}(\alpha, \beta), \alpha, \beta, \gamma \in SVNN.$$

Proposition 2.4. [32] Let $\alpha = \langle t_\alpha, i_\alpha, f_\alpha \rangle$, $\beta = \langle t_\beta, i_\beta, f_\beta \rangle \in SVNN$, then $\mathcal{R}_\mathcal{T}(\alpha,\beta) = \langle R_T(t_\alpha, t_\beta), R_S(i_\beta, i_\alpha), R_S(f_\beta, f_\alpha) \rangle$, which is the single-valued neutrosophic residual implication induced by left-continuous single-valued neutrosophic t-representable t-norm, where R_T is residual implication induced by left-continuous t-norm T, R_S is coresidual implication induced by right-continuous t-conorm S.

Proposition 2.5. Let $\mathcal{R}_\mathcal{T}$ be single valued neutrosophic residual implication induced by left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, then

(i). $\mathcal{R}_\mathcal{T}(\alpha,\beta) = 1^*$ **iff** $\alpha \leq \beta$;
(ii). $\gamma \leq \mathcal{R}_\mathcal{T}(\alpha,\beta)$ iff $\alpha \leq \mathcal{R}_\mathcal{T}(\gamma,\beta)$;
(iii). $\mathcal{R}_\mathcal{T}(1^*,\alpha) = \alpha$;
(iv). $\mathcal{R}_\mathcal{T}(\alpha, \mathcal{R}_\mathcal{T}(\mathcal{R}_\mathcal{T}(\alpha,\beta),\beta) = 1^*$;
(v). $\mathcal{R}_\mathcal{T}(\bigvee_{i\in I} \beta_i, \alpha) = \bigwedge_{i\in I} \mathcal{R}_\mathcal{T}(\beta_i,\alpha)$;
(vi). $\mathcal{R}_\mathcal{T}(\beta, \bigwedge_{i\in I} \alpha) = \bigwedge_{i\in I} \mathcal{R}_\mathcal{T}(\beta,\alpha_i)$;
(vii). $\mathcal{R}_\mathcal{T}$ is antitone in the first variable and isotone in the second variable.

After introducing the properties of single-valued neutrosophic t-representable t-norm, to better understand its usage, we will use the following examples to introduce three important single-valued neutrosophic t-representable t-norms and their residual implications.

Example 3. [32] The following are three important single-valued neutrosophic t-representable t-norms and their residual implications.

(i). The single valued neutrosophic Łukasiewicz t-norm and its residual implication:

$$\mathcal{T}_L(\alpha,\beta) = \langle (t_\alpha + t_\beta - 1) \vee 0, (i_\alpha + i_\beta) \wedge 1, (f_\alpha + f_\beta) \wedge 1 \rangle$$
$$\mathcal{R}_{\mathcal{T}_L}(\alpha,\beta) = \langle 1 \wedge (1 - t_\alpha + t_\beta), (i_\beta - i_\alpha) \vee 0, (f_\beta - f_\alpha) \vee 0 \rangle.$$

(ii). The single valued neutrosophic Gougen t-norm and its residual implication:

$$\mathcal{T}_{Go}(\alpha,\beta) = \langle t_\alpha t_\beta, i_\alpha + i_\beta - i_\alpha i_\beta, f_\alpha + f_\beta - f_\alpha f_\beta \rangle.$$

$$\mathcal{R}_{\mathcal{T}_{Go}}(\alpha,\beta) = \begin{cases} \langle 1,0,0 \rangle, & if \quad t_\alpha \leq t_\beta, i_\beta \leq i_\alpha, f_\beta \leq f_\alpha, \\ \langle 1,0,\dfrac{f_\beta - f_\alpha}{1-f_\alpha} \rangle, & if \quad t_\alpha \leq t_\beta, i_\beta \leq i_\alpha, f_\alpha < f_\beta, \\ \langle 1,\dfrac{i_\beta - i_\alpha}{1-i_\alpha},0 \rangle, & if \quad t_\alpha \leq t_\beta, i_\alpha < i_\beta, f_\beta \leq f_\alpha, \\ \langle 1,\dfrac{i_\beta - i_\alpha}{1-i_\alpha},\dfrac{f_\beta - f_\alpha}{1-f_\alpha} \rangle, & if \quad t_\alpha \leq t_\beta, i_\alpha < i_\beta, f_\alpha < f_\beta, \\ \langle \dfrac{t_\beta}{t_\alpha},0,0 \rangle, & if \quad t_\beta < t_\alpha, i_\beta \leq i_\alpha, f_\beta \leq f_\alpha, \\ \langle \dfrac{t_\beta}{t_\alpha},0,\dfrac{f_\beta - f_\alpha}{1-f_\alpha} \rangle, & if \quad t_\beta < t_\alpha, i_\beta \leq i_\alpha, f_\alpha < f_\beta, \\ \langle \dfrac{t_\beta}{t_\alpha},\dfrac{i_\beta - i_\alpha}{1-i_\alpha},0 \rangle, & if \quad t_\beta < t_\alpha, i_\alpha < i_\beta, f_\beta \leq f_\alpha, \\ \langle \dfrac{t_\beta}{t_\alpha},\dfrac{i_\beta - i_\alpha}{1-i_\alpha},\dfrac{f_\beta - f_\alpha}{1-f_\alpha} \rangle, & if \quad t_\beta < t_\alpha, i_\alpha < i_\beta, f_\alpha < f_\beta. \end{cases}$$

(iii). The single valued neutrosophic t-norm and its residual implication:

$$\mathcal{T}_G(\alpha,\beta) = \langle t_\alpha \wedge t_\beta, i_\alpha \vee i_\beta, f_\alpha \vee f_\beta \rangle.$$

$$\mathcal{R}_{\mathcal{T}_G}(\alpha,\beta)=\begin{cases}\langle 1,0,0\rangle, & if \quad t_\alpha\le t_\beta, i_\beta\le i_\alpha, f_\beta\le f_\alpha,\\ \langle 1,0,f_\beta\rangle, & if \quad t_\alpha\le t_\beta, i_\beta\le i_\alpha, f_\alpha< f_\beta,\\ \langle 1,i_\beta,0\rangle, & if \quad t_\alpha\le t_\beta, i_\alpha< i_\beta, f_\beta\le f_\alpha,\\ \langle 1,i_\beta,f_\beta\rangle, & if \quad t_\alpha\le t_\beta, i_\alpha< i_\beta, f_\alpha< f_\beta,\\ \langle t_\beta,0,0\rangle, & if \quad t_\beta< t_\alpha, i_\beta\le i_\alpha, f_\beta\le f_\alpha,\\ \langle t_\beta,0,f_\beta\rangle, & if \quad t_\beta< t_\alpha, i_\beta\le i_\alpha, f_\alpha< f_\beta,\\ \langle t_\beta,i_\beta,0\rangle, & if \quad t_\beta< t_\alpha, i_\alpha< i_\beta, f_\beta\le f_\alpha,\\ \langle t_\beta,i_\beta,f_\beta\rangle, & if \quad t_\beta< t_\alpha, i_\alpha< i_\beta, f_\alpha< f_\beta.\end{cases}$$

To further demonstrate the robustness of single-valued neutrosophic t-norm, we will now introduce a distance metric d.

Definition 2.7. [34] A metric space is an ordered pair (X,d), where X is a set and d is a metric on X, i.e., a function d: $X\times X\to[0,+\infty)$ such that for any $x,y,z\in X$, the following holds:
$(D1)$ $d(x,y)\ge$ **0;**
$(D2)$ $d(x,y)$ **= 0 if and only if** $x=y$**;**
$(D3)$ $d(x,y)\le d(x,z)+d(y,z)$**.**
The function d is called a distance.

3. Single-Valued Neutrosophic Fuzzy Inference Triple I Method

In this section, we will study the single-valued neutrosophic fuzzy inference triple I method based on left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$. Suppose $\mathcal{R}$ is a single-valued neutrosophic residuated implication induced by left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$. A single valued neutrosophic set A on universe X is called normal if there exists $x_0\in X$ such that $A(x_0)=1^*$. A single valued neutrosophic set A on universe X is called co-normal if there exists $x_0\in X$ such that $A(x_0)=0^*$.

Definition 3.1. (Single valued neutrosophic fuzzy inference triple I principle for FMP) Suppose that $\mathcal{R}$ is a single-valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$, $A,A^*\in SVNS(X)$ and $B\in SVNS(Y)$. Let $P(x,y)=\mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(A^*(x),1^*))$, and $\mathrm{B}(A,B,A^*)=\{C\in SVNS(Y) \mid \mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(A^*(x),C(y)))=P(x,y),x\in X,y\in Y\}$.

If there exist the smallest element of the set $\mathrm{B}(A,B,A^*)$ (denoted by B^*), then B^* is called the single-valued neutrosophic fuzzy inference triple I solution for FMP.

Definition 3.2. (Single valued neutrosophic fuzzy inference triple I principle for FMT) Suppose that $\mathcal{R}$ is a single-valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$. $A\in SVNS(X)$ and $B,B^*\in SVNS(Y)$. Let $Q(x,y)=\mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(0^*,B^*(x)))$, and $\mathrm{A}(A,B,B^*)=\{D\in SVNS(X) \mid \mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(D(x),B^*(x)))=Q(x,y),x\in X,y\in Y\}$.

If there exists the greatest element of the set $\mathrm{A}(A,B,B^*)$ (denoted by A^*), then A^* is called the single-valued neutrosophic fuzzy inference triple I solution for FMT.

After introducing the single-valued neutrosophic fuzzy inference triple I principle for FMP and FMT, we can now derive the single-valued neutrosophic fuzzy inference triple I solution of FMP and FMT.

Theorem 3.1. Let $A,A^*\in SVNS(X)$, $B\in SVNS(Y)$, $\mathcal{R}$ be single valued neutrosophic residual implication induced by a left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single-valued neutrosophic fuzzy inference triple I solution B^* of FMP is as follows:

$$B^*(y) = \sup_{x\in X}\mathcal{T}(A^*(x), \mathcal{R}(A(x), B(y)))(\forall y \in Y) \tag{1}$$

Proof:

Firstly, we prove $B^* \in \mathrm{B}(A, B, A^*)$. It follows from equation (1), we have $\mathcal{T}(A^*(x), \mathcal{R}(A(x), B(y))) \leq B^*$ (y). By the residuation property, we obtain $\mathcal{R}(A(x), B(y))) \leq \mathcal{R}(A^*(x), B^*(y)))$. Therefore, $\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), B^*(y))) = 1^*$, i.e., $B^* \in \mathrm{B}(A, B, A^*)$.

Secondly, we prove that B^* is the smallest single valued neutrosophic fuzzy subset of $\mathrm{B}(A, B, A^*)$. Suppose C is an arbitrary single-valued neutrosophic fuzzy subset in $\mathrm{B}(A, B, A^*)$, i.e.$\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), C(y))) = 1^*$.

By the residuation property, then $\mathcal{R}(A(x), B(y)) \leq \mathcal{R}(A^*(x), C(y))$. we have $\mathcal{T}(A^*(x), \mathcal{R}(A(x), B(y)) \leq C(y)$, hence $B^* \leq C$, i.e., B^* is the smallest single valued neutrosophic fuzzy subset of $\mathrm{B}(A, B, A^*)$, and B^* is the single-valued neutrosophic fuzzy inference triple I solution for FMP.

After obtaining the solution for single valued neutrosophic fuzzy inference triple I solution of FMP, we can now obtain the single-valued neutrosophic residual implication induced by a left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$ triple I solution for FMP.

Corollary 3.1. Let $\mathcal{R}$ be single valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single-valued neutrosophic fuzzy inference triple I solution $B^* = \{\langle y, t_{B^*}(y), i_{B^*}(y), f_{B^*}(y)\rangle \mid y \in Y\}$ for FMP can be shown as follows:

$t_{B^*}(y) = \vee_{x\in X} T\,(t_{A^*}(x), R_T(t_A(x), t_B(y)))(\forall y \in Y)$,

$i_{B^*}(y) = \wedge_{x\in X} S\,(i_{A^*}(x), R_S(i_B(y), i_A(x)))(\forall y \in Y)$,

$f_{B^*}(y) = \wedge_{x\in X} S\,(f_{A^*}(x), R_S(f_B(y), f_A(x)))(\forall y \in Y)$.

Corollary 3.2. Let $\mathcal{R}$ be the single-valued neutrosophic Łukasiewicz residual implication $\mathcal{R}_{\mathcal{T}_L}$, then the single-valued neutrosophic fuzzy inference triple I solution $B^* = \{\langle y, t_{B^*}(y), i_{B^*}(y), f_{B^*}(y)\rangle \mid y \in Y\}$ of FMPas follows:

$t_{B^*}(y) = \vee_{x\in X}\{\,[t_{A^*}(x) + ((1 - t_A(x) + t_B(y)) \wedge 1) - 1] \vee 0\}(\forall y \in Y)$,

$i_{B^*}(y) = \wedge_{x\in X}\{\,[i_{A^*}(x) + ((i_B(y) - i_A(x)) \vee 0)] \wedge 1\}(\forall y \in Y)$,

$f_{B^*}(y) = \wedge_{x\in X}\{\,[f_{A^*}(x) + ((f_B(y) - f_A(x)) \vee 0)] \wedge 1\}(\forall y \in Y)$.

Corollary 3.3. Let $\mathcal{R}$ be the single-valued neutrosophic Gougen residual implication $\mathcal{R}_{\mathcal{T}_{Go}}$, then the single-valued neutrosophic fuzzy inference triple I solution $B^* = \{\langle y, t_{B^*}(y), i_{B^*}(y), f_{B^*}(y)\rangle \mid y \in Y\}$ of FMP as follows:

$t_{B^*}(y) = \vee_{x\in X}\{\,t_{A^*}(x) \cdot (\frac{t_{B(y)}}{t_{A(x)}} \wedge 1)\}(\forall y \in Y)$,

$i_{B^*}(y) = \wedge_{x\in X}\{\,i_{A^*}(x) + [\frac{i_B(y) - i_A(x)}{1 - i_A(x)} \vee 0] - i_{A^*}(x) \cdot [\frac{i_B(y) - i_A(x)}{1 - i_A(x)} \vee 0]\}(\forall y \in Y)$,

$f_{B^*}(y) = \wedge_{x\in X}\{\,f_{A^*}(x) + [\frac{f_B(y) - f_A(x)}{1 - f_A(x)} \vee 0] - f_{A^*}(x) \cdot [\frac{f_B(y) - f_A(x)}{1 - f_A(x)} \vee 0]\}(\forall y \in Y)$.

Corollary 3.4. Let $\mathcal{R}$ be the single-valued neutrosophic $Gödel$ residual implications $\mathcal{R}_{\mathcal{T}_G}$, then the single-valued neutrosophic fuzzy inference triple I solution $B^* = \{\langle y, t_{B^*}(y), i_{B^*}(y), f_{B^*}(y)\rangle \mid y \in Y\}$ of FMP as follows:

$t_{B^*}(y) = \vee_{x\in X}\{\,(t_{A^*}(x) \wedge R_{T_G}(t_A(x), t_B(y)))\}(\forall y \in Y)$,

$i_{B^*}(y) = \wedge_{x\in X}\{\,(i_{A^*}(x) \vee R_{S_G}(i_B(y), i_A(x)))\}(\forall y \in Y)$,

$f_{B^*}(y) = \wedge_{x\in X}\{\,(f_{A^*}(x) \vee R_{S_G}(f_B(y), f_A(x)))\}(\forall y \in Y)$.

Theorem 3.2. Let $A \in SVNS(X)$, $B, B^* \in SVNS(Y)$, $\mathcal{R}$ be single valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single-valued neutrosophic fuzzy inference triple I solution A^* of FMT is as follows:

$$A^*(x) = \bigwedge_{y \in Y} \mathcal{R}\,(\mathcal{R}(A(x), B(y)), B^*(y))(\forall x \in X) \tag{2}$$

Proof:

Firstly, we prove $A^* \in \mathrm{A}(A, B, B^*)$. It follows from equation (2), we obtain $A^*(x) \leq \mathcal{R}(\mathcal{R}(A(x), B(y)), B^*(y))$. By the residuation property, we have $\mathcal{T}(A^*, \mathcal{R}(A(x), B(y))) \leq B^*$, and $\mathcal{R}(A(x), B(y)) \leq \mathcal{R}(A^*(x), B^*(y))$. Therefore, $\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), B^*(y))) = 1^*$, i.e., $A^* \in \mathrm{A}(A, B, B^*)$.

Secondly, we show that A^* is the greatest single valued neutrosophic fuzzy subset of $\mathrm{A}(A, B, B^*)$. Suppose D is an arbitrary single-valued neutrosophic fuzzy subset in $\mathrm{A}(A, B, B^*)$, i.e.,$\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(D(x), B^*(y))) = 1^*$, then $\mathcal{R}(A(x), B(y)) \leq \mathcal{R}(D(x), B^*(y))$ by the residuation property. We have $\mathcal{T}(D(x), \mathcal{R}(A(x), B(y)) \leq B^*(y)$ and $D(x) \leq \mathcal{R}(\mathcal{R}(A(x), B(y)), B^*(y))$ by Proposition 2.3, hence $D \leq A^*$, i.e., A^* is the greatest single valued neutrosophic fuzzy subset of $\mathrm{A}(A, B, B^*)$, and A^* is the single-valued neutrosophic fuzzy inference triple I solution for FMT.

After obtaining the solution for single valued neutrosophic fuzzy inference triple I solution of FMT, we can now obtain the single-valued neutrosophic residual implication induced by a left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$ triple I solution for FMT.

Corollary 3.5. Let $\mathcal{R}$ be a single-valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single-valued neutrosophic fuzzy inference triple I solution $A^* = \{\langle x, t_{A^*}(x), i_{A^*}(x), f_{A^*}(x)\rangle \mid x \in X\}$ for FMT can be shown as follows:

$t_{A^*}(x) = \bigwedge_{y \in Y} R_T\,(R_T(t_A(x), t_B(y)), t_{B^*}(y))(\forall x \in X)$,
$i_{A^*}(x) = \bigvee_{y \in Y} R_S\,(i_{B^*}(y), R_S(i_B(y), i_A(x)))(\forall x \in X)$,
$f_{A^*}(x) = \bigvee_{y \in Y} R_S\,(f_{B^*}(y), R_S(f_B(y), f_A(x)))(\forall x \in X)$.

Corollary 3.6. Let $\mathcal{R}$ be the single-valued neutrosophic Łukasiewicz residual implication $\mathcal{R}_{\mathcal{T}_L}$, then the single-valued neutrosophic fuzzy inference triple I solution $A^* = \{\langle x, t_{A^*}(x), i_{A^*}(x), f_{A^*}(x)\rangle \mid x \in X\}$ for FMT as follows:

$t_{A^*}(x) = \bigwedge_{y \in Y}\{\,[1 - ((1 - t_A(x) + t_B(y)) \wedge 1) + t_{B^*}(y)] \wedge 1\}(\forall x \in X)$,
$i_{A^*}(x) = \bigvee_{y \in Y}\{\,[i_{B^*}(y) - ((i_B(y) - i_A(x)) \vee 0)] \vee 0\}(\forall x \in X)$,
$f_{A^*}(x) = \bigvee_{y \in Y}\{\,[f_{B^*}(y) - ((f_B(y) - f_A(x)) \vee 0)] \vee 0\}(\forall x \in X)$.

Corollary 3.7. Let $\mathcal{R}$ be the single-valued neutrosophic Gougen residual implication $\mathcal{R}_{\mathcal{T}_{Go}}$, then the single-valued neutrosophic fuzzy inference triple I solution $A^* = \{\langle x, t_{A^*}(x), i_{A^*}(x), f_{A^*}(x)\rangle \mid x \in X\}$ for FMT as follows:

$t_{A^*}(x) = \bigwedge_{y \in Y}\{\frac{t_{B^*}(y)}{(\frac{t_B(y)}{t_A(x)} \wedge 1)} \wedge 1\}(\forall x \in X)$,

$i_{A^*}(x) = \bigvee_{y \in Y}\{\frac{i_{B^*}(y) - (\frac{i_B(y) - i_A(x)}{1 - i_A(x)} \vee 0)}{1 - (\frac{i_B(y) - i_A(x)}{1 - i_A(x)} \vee 0)} \vee 0\}(\forall x \in X)$,

$f_{A^*}(x) \bigvee_{y \in Y}\{\frac{f_{B^*}(y) - (\frac{f_B(y) - f_A(x)}{1 - f_A(x)} \vee 0)}{1 - (\frac{f_B(y) - f_A(x)}{1 - f_A(x)} \vee 0)} \vee 0\}(\forall x \in X)$.

Corollary 3.8. Let B be the single-valued neutrosophic $Gödel$ residual implications $\mathcal{R}_{\mathcal{T}_G}$, then the single-valued neutrosophic fuzzy inference triple I solution $A^* = \{\langle x, t_{A^*}(x), i_{A^*}(x), f_{A^*}(x)\rangle \mid x \in X\}$ for FMT as follows:

$t_{A^*}(x) = \wedge_{x\in X}\{ R_{T_G}(R_{T_G}(t_A(x), t_B(y)), t_{B^*}(y))\}(\forall x \in X),$

$i_{A^*}(x) = \vee_{x\in X}\{ R_{S_G}(i_{B^*}(y), R_{S_G}(i_B(y), i_A(x)))\}(\forall x \in X),$

$f_{A^*}(x) = \vee_{x\in X}\{ R_{S_G}(f_{A^*}(x), R_{S_G}(f_B(y), f_A(x)))\}(\forall x \in X).$

To prove the single-valued neutrosophic fuzzy inference triple I method is recoverable, we define reducibility.

Definition 3.3. [4] A method for FMP is called recoverable if $A^* = A$ implies $B^* = B$. similarly, a method for FMT is called recoverable if $B^* = B$ implies $A^* = A$.}

Theorem 3.3. The single-valued neutrosophic fuzzy inference triple I method for FMP is reductive if A is a normal single-valued neutrosophic set.

Proof:
Suppose $A^* = A$ and there exists an element $x_0 \in X$ such that $A(x_0) = A^*(x_0) = \langle 1,0,0\rangle = 1^*$. Then we have

$$\begin{aligned} B^*(y) = & \bigvee_{x\in X} \mathcal{T}\,(A^*(x), \mathcal{R}(A(x), B(y))) \\ \geq & \quad \mathcal{T}(A^*(x_0), \mathcal{R}(A(x_0), B(y))) \\ = & \quad \mathcal{T}(1^*, \mathcal{R}(1^*, B(y))) = B(y). \end{aligned}$$

On the other hand, by Proposition 2.6 (5) for any $y \in Y$,

$$\mathcal{R}(B^*(y), B(y)) = \mathcal{R}(\bigvee_{y\in Y} \mathcal{T}\,(\mathcal{R}(A(x), B(y)), A^*(x)), B(y)) = \bigwedge_{y\in Y} \mathcal{R}\,(\mathcal{T}(\mathcal{R}(A(x), B(y)), A(x)), B(y)) = 1^*,$$

we have, $B^*(y) \leq B(y)$.
Therefore, $B^* = B$. This shows that the single-valued neutrosophic fuzzy inference triple I method for FMP is recoverable.

Theorem 3.4. The single-valued neutrosophic fuzzy inference triple I method for FMT is reductive if single-valued neutrosophic residual implication $\mathcal{R}$ satisfies $\mathcal{R}(\mathcal{R}(A, 0^*), 0^*) = A$, and B is a co-normal single-valued neutrosophic set.
Proof:
Suppose $B^* = B$ is a co-normal single-valued neutrosophic set, i.e. there exists an element $y_0 \in Y$ such that $B^*(y_0) = B(y_0) = \langle 0,1,1\rangle = 0^*$, then we have:

$$\begin{aligned} A^*(x) = & \bigwedge_{y\in Y} \mathcal{R}\,(\mathcal{R}(A(x), B(y)), B^*(y)) \\ \leq & \quad \mathcal{R}(\mathcal{R}(A(x), B(y_0)), B^*(y_0)) \\ = & \quad \mathcal{R}(\mathcal{R}(A(x), 0^*), 0^*) = A(x). \end{aligned}$$

On the other hand, by Proposition 2.5(3) and (4) for any $x \in X$,

$$\mathcal{R}(A(x), A^*(x)) = \mathcal{R}(A(x), \bigwedge_{y\in Y} \mathcal{R}\,(\mathcal{R}(A(x), B(y)), B^*(y))) = \bigwedge_{y\in Y} \mathcal{R}\,(A(x), \mathcal{R}(\mathcal{R}(A(x), B(y)), B(y))) = 1^*,$$

we have $A(x) \leq A^*(x)$.
Therefore, $A^* = A$. This shows that the single-valued neutrosophic fuzzy inference triple I method for FMT is recoverable.

4. Robustness of Single-Valued Neutrosophic Fuzzy Inference Triple I Method

In this section, we introduce a new distance between single-valued neutrosophic sets. Through this distance, we can prove the robustness of the single-valued neutrosophic fuzzy inference triple I method. We study the robustness of the single-valued neutrosophic fuzzy inference triple I method based on left-continuous single-valued neutrosophic t-representable t-norms with this new distance.

Theorem 4.1. Let $X=\{x_1,x_2,\dots,x_n\}$, for all $A,B\in SVNS(X)$, then

$$d(A,B)=\max\{\bigvee_{x_i\in X}|t_A(x_i)-t_B(x_i)|,\bigvee_{x_i\in X}|i_A(x_i)-i_B(x_i)|,\bigvee_{x_i\in X}|f_A(x_i)-f_B(x_i)|\}$$

is a metric on $SVNS(X)$ and $(SVNS(X),d)$ is a metric space. d is called a distance on $SVNS(X)$.

Proof: By Definition 2.7, (1) (2) are obvious for any $A,B\in SVNS(X)$. Therefore, we only prove (3). For any $A,B,C\in SVNS(X)$

$$\begin{aligned}
&d(A,B)\\
&=\max\{\bigvee_{x_i\in X}|t_A(x_i)-t_B(x_i)|,\bigvee_{x_i\in X}|i_A(x_i)-i_B(x_i)|,\bigvee_{x_i\in X}|f_A(x_i)-f_B(x_i)|\}\\
&=\max\{\bigvee_{x_i\in X}|t_A(x_i)-t_C(x_i)+t_C(x_i)-t_B(x_i)|,\\
&\bigvee_{x_i\in X}|i_A(x_i)-i_C(x_i)+i_C(x_i)-i_B(x_i)|,\bigvee_{x_i\in X}|f_A(x_i)-f_C(x_i)+f_C(x_i)-f_B(x_i)|\}\\
&\le\max\{\bigvee_{x_i\in X}|t_A(x_i)-t_C(x_i)|,\bigvee_{x_i\in X}|i_A(x_i)-i_C(x_i)|,\bigvee_{x_i\in X}|f_A(x_i)-f_C(x_i)|\}\\
&+\max\{\bigvee_{x_i\in X}|t_C(x_i)-t_B(x_i)|,\bigvee_{x_i\in X}|i_C(x_i)-i_B(x_i)|,\bigvee_{x_i\in X}|f_C(x_i)-f_B(x_i)|\}\\
&\le d(A,C)+d(C,B)
\end{aligned}$$

Therefore, d is a metric on $SVNS(X)$, and $(SVNS(X),d)$ is a metric space.

Definition 4.1. Suppose that $\mathfrak{F}$ is a n-tuple mapping form to $SVNN^n$ to $SVNN$, $\forall\varepsilon\in(0,1)$. For any $\langle \mathrm{t,i,f}\rangle=(\langle t_1,i_1,f_1\rangle,\langle t_2,i_2,f_2\rangle,\dots,\langle t_n,i_n,f_n\rangle)\in SVNN^n$,

$$\triangle_{\mathfrak{F}}(\langle \mathrm{t,i,f}\rangle,\varepsilon)=\vee\{d(\mathfrak{F}\langle \mathrm{t,i,f}\rangle,\mathfrak{F}\langle \mathrm{t',i',f'}\rangle)|\langle \mathrm{t',i',f'}\rangle\in SVNN^n,d(\langle \mathrm{t,i,f}\rangle,\langle \mathrm{t',i',f'}\rangle)\le\varepsilon\}$$

is called the sensitivity of the point $\langle \mathrm{t,i,f}\rangle$, where $d(\langle \mathrm{t,i,f}\rangle,\langle \mathrm{t',i',f'}\rangle)=\max\{\vee_j|t_j-t'_j|,\vee_j|i_j-i'_j|,\vee_j|f_j-f'_j|\}$.

Definition 4.2. The biggest ε sensitivity of $\mathfrak{F}$ denoted by $\triangle_{\mathfrak{F}}(\varepsilon)=\vee_{\langle \mathrm{i,t,f}\rangle\in SVNN^n}\triangle_{\mathfrak{F}}(\langle \mathrm{t,i,f}\rangle,\varepsilon)$ is called sensitivity of $\mathfrak{F}$.

Definition 4.3. Let $\mathfrak{F}$ and $\mathfrak{F}'$ be two n-tuple single-valued neutrosophic fuzzy connectives. We say that $\mathfrak{F}$ at least as robust as $\mathfrak{F}'$ at point $\langle \mathrm{t,i,f}\rangle$, if $\forall\varepsilon\in(0,1)$, $\triangle_{\mathfrak{F}}(\langle \mathrm{t,i,f}\rangle,\varepsilon)\le\triangle_{\mathfrak{F}'}(\langle \mathrm{t,i,f}\rangle,\varepsilon)$. We say that $\mathfrak{F}$ is more robust than $\mathfrak{F}'$ at point $\langle \mathrm{t,i,f}\rangle$, if there exists $\varepsilon>0$ such that $\triangle_{\mathfrak{F}}(\langle \mathrm{t,i,f}\rangle,\varepsilon)<\triangle_{\mathfrak{F}'}(\langle \mathrm{t,i,f}\rangle,\varepsilon)$.

Definition 4.4. Let $\mathfrak{F}$ and $\mathfrak{F}'$ be two n-tuple single-valued neutrosophic fuzzy connectives. We say that $\mathfrak{F}$ at least as robust as $\mathfrak{F}'$, if $\forall\varepsilon\in(0,1)$, $\triangle_{\mathfrak{F}}(\varepsilon)\le\triangle_{\mathfrak{F}'}(\varepsilon)$. We say that $\mathfrak{F}$ is more robust than $\mathfrak{F}'$ if there exists $\varepsilon>0$ such that $\triangle_{\mathfrak{F}}(\varepsilon)<\triangle_{\mathfrak{F}'}(\varepsilon)$.

Proposition 4.1. For a binary single valued neutrosophic fuzzy connectives $\mathfrak{F}$: $SVNN\times SVNN\to SVNN$, we can obtain:

(i). Let $\mathfrak{F}$ be a left-continuous single valued neutrosophic t-representable t-norm on $SVNN$, $\mathcal{T}(\alpha,\beta)=\langle T(t_\alpha,t_\beta),S(i_\alpha,i_\beta),S(f_\alpha,f_\beta)\rangle$ for all $\alpha=\langle t_\alpha,i_\alpha,f_\alpha\rangle$, $\beta=\langle t_\beta,i_\beta,f_\beta\rangle\in SVNN$, then

$$\begin{aligned}\triangle_{\mathcal{T}}(\varepsilon)&=\bigvee_{(\alpha,\beta)\in SNVS^2}\triangle_{\mathcal{T}}((\alpha,\beta),\varepsilon)\\&=\bigvee_{(\alpha,\beta)\in SNVS^2}\{\vee\{|T(t_\alpha,t_\beta)-T(t'_\alpha,t'_\beta)|,\ |S(i_\alpha,i_\beta)-S(i'_\alpha,i'_\beta)|,\ |S(f_\alpha,f_\beta)-S(f'_\alpha,f'_\beta)|\ |d(\mathcal{T}(\alpha,\beta),\mathcal{T}(\alpha',\beta'))\le\varepsilon\}\}\\&=\bigvee_{(\alpha,\beta)\in SNVS^2}\left\{\begin{array}{c}T(t_\alpha,t_\beta)-T(t_\alpha+\varepsilon,t_\beta+\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha+\varepsilon,i_\beta+\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha+\varepsilon,f_\beta+\varepsilon)|,\\|T(t_\alpha,t_\beta)-T(t_\alpha+\varepsilon,t_\beta-\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha+\varepsilon,i_\beta-\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha+\varepsilon,f_\beta-\varepsilon)|,\\|T(t_\alpha,t_\beta)-T(t_\alpha+\varepsilon,t_\beta)|,|S(i_\alpha,i_\beta)-S(i_\alpha+\varepsilon,i_\beta)|,|S(f_\alpha,f_\beta)-S(f_\alpha+\varepsilon,f_\beta)|,\\|T(t_\alpha,t_\beta)-T(t_\alpha,t_\beta+\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha,i_\beta+\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha,f_\beta+\varepsilon)|,\\|T(t_\alpha,t_\beta)-T(t_\alpha,t_\beta-\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha,i_\beta-\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha,f_\beta-\varepsilon)|,\\|T(t_\alpha,t_\beta)-T(t_\alpha-\varepsilon,t_\beta+\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha-\varepsilon,i_\beta+\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha-\varepsilon,f_\beta+\varepsilon)|,\\|T(t_\alpha,t_\beta)-T(t_\alpha-\varepsilon,t_\beta-\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha-\varepsilon,i_\beta-\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha-\varepsilon,f_\beta-\varepsilon)|,\\|T(t_\alpha,t_\beta)-T(t_\alpha-\varepsilon,t_\beta)|,|S(i_\alpha,i_\beta)-S(i_\alpha-\varepsilon,i_\beta)|,|S(f_\alpha,f_\beta)-S(f_\alpha-\varepsilon,f_\beta)|\end{array}\right\}\end{aligned}$$

(ii). Let $\mathfrak{F}$ be single valued neutrosophic residuated implication $\mathcal{R}_{\mathcal{T}}$ induced by left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, $\mathcal{R}_{\mathcal{T}}(\alpha,\beta)$ = $\langle R_T(t_\alpha,t_\beta)),R_S(i_\beta,i_\alpha),R_S(f_\beta,f_\alpha)\rangle$ for all $\alpha=\langle t_\alpha,i_\alpha,f_\alpha\rangle$, $\beta=\langle t_\beta,i_\beta,f_\beta\rangle\in SVNN$, then

$$\begin{aligned}\triangle_{\mathcal{R}_{\mathcal{T}}}(\varepsilon)&=\bigvee_{(\alpha,\beta)\in SNVS^2}\triangle_{\mathcal{R}_{\mathcal{T}}}((\alpha,\beta),\varepsilon)\\&=\bigvee_{(\alpha,\beta)\in SNVS^2}\left\{\bigvee\{|R_T(t_\alpha,t_\beta)-R_T(t'_\alpha,t'_\beta)|,|R_S(i_\beta,i_\alpha)-R_S(i'_\beta,i'_\alpha)|,|R_S(f_\beta,f_\alpha)-R_S(f'_\beta,f'_\alpha)||d(\mathcal{T}(\alpha,\beta),\mathcal{T}(\alpha',\beta'))\le\varepsilon\}\right\}\\&=\bigvee_{(\alpha,\beta)\in SNVS^2}\left\{\begin{array}{c}|R_T(t_\alpha,t_\beta)-R_T(t_\alpha+\varepsilon,t_\beta+\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta+\varepsilon,i_\alpha+\varepsilon)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta+\varepsilon,f_\alpha+\varepsilon)|,\\|R_T(t_\alpha,t_\beta)-R_T(t_\alpha+\varepsilon,t_\beta-\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta+\varepsilon,i_\alpha-\varepsilon)|,|R_S(f_\beta+\varepsilon,f_\alpha)-R_S(f_\beta,f_\alpha-\varepsilon)|,\\|R_T(t_\alpha,t_\beta)-R_T(t_\alpha+\varepsilon,t_\beta)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta+\varepsilon,i_\alpha)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta+\varepsilon,f_\alpha)|,\\|R_T(t_\alpha,t_\beta)-R_T(t_\alpha,t_\beta+\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta,i_\alpha+\varepsilon)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta,f_\alpha+\varepsilon)|,\\|R_T(t_\alpha,t_\beta)-R_T(t_\alpha,t_\beta-\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta,i_\alpha-\varepsilon)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta,f_\alpha-\varepsilon)|,\\|R_T(t_\alpha,t_\beta)-R_T(t_\alpha-\varepsilon,t_\beta+\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta-\varepsilon,i_\alpha+\varepsilon)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta-\varepsilon,f_\alpha+\varepsilon)|,\\|R_T(t_\alpha,t_\beta)-R_T(t_\alpha-\varepsilon,t_\beta-\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta-\varepsilon,i_\alpha-\varepsilon)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta-\varepsilon,f_\alpha-\varepsilon)|,\\|R_S(i_\beta,i_\alpha)-R_S(i_\beta-\varepsilon,i_\alpha)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta-\varepsilon,f_\alpha)|\end{array}\right\}\end{aligned}$$

where R_T is residual implication induced by left-continuous t-norm T, R_S is coresidual implication induced by right-continuous t-conorm S.

Corollary 4.1. The ε sensitivity of the single-valued neutrosophic Łukasiewicz t-representable t-norm is $\triangle_{\mathcal{T}_L}(\varepsilon)=2\varepsilon\wedge 1$.

Corollary 4.2. The ε sensitivity of the single-valued neutrosophic Łukasiewicz residual implication is $\triangle_{\mathcal{R}_{\mathcal{T}_L}}=2\varepsilon\wedge 1$.

Definition 4.5. Let A and A' be two single valued neutrosophic fuzzy sets on universal X. If $\parallel A-A'\parallel=\vee_{x\in X}d(A(x),A'(x))\le\varepsilon$ for all $x\in X$, then A' is called ε-perturbation of A denoted by $A'\in O(A,\varepsilon)$.

Theorem 4.2. Let A, A', B, B', A^* and A'^* be single-valued neutrosophic fuzzy sets. If $\parallel A-A'\parallel\le\varepsilon$, $\parallel B-B'\parallel\le\varepsilon$, $\parallel A^*-A'^*\parallel\le\varepsilon$, B^* and B'^* are the single-valued neutrosophic fuzzy inference triple I solutions of $FMP(A,\ B,\ A^*)$ and $FMP(A',\ B',\ A'^*)$ given in Theorem 3.1 respectively, then the ε sensitivity of the single-valued neutrosophic fuzzy inference triple I solution B^* for FMP is

$$\triangle_{B^*}(\varepsilon)=\parallel B^*-B'^*\parallel\le\triangle_{\mathcal{T}}(\triangle_{\mathcal{R}}(\varepsilon)).$$

Proof: Let $A,A',A^*,A'^*\in SNVS(X)$, $B,B'\in SNVS(Y)$. If $\parallel A-A'\parallel\le\varepsilon$, $\parallel B-B'\parallel\le\varepsilon$, $\parallel A^*-A'^*\parallel\le\varepsilon$, then we have,

$$\triangle_{B^*}(\varepsilon) = \| B^* - B'^* \|$$
$$= \bigvee_{y\in Y} d\,(B^*(y), B'^*(y))$$
$$= \bigvee_{y\in Y} d\,(\bigvee_{x\in X} \mathcal{T}\,(\mathcal{R}(A(x),B(y)),A^*(x)), \bigvee_{x\in X} \mathcal{T}\,(\mathcal{R}(A'(x),B'(y)),A'^*(x)))$$
$$\leq \bigvee_{y\in Y}\bigvee_{x\in X} d\,(\mathcal{T}(\mathcal{R}(A(x),B(y)),A^*(x)),\mathcal{T}(\mathcal{R}(A'(x),B'(y)),A'^*(x)))$$
$$\leq \triangle_{\mathcal{T}}\,(\triangle_{\mathcal{R}}\,(\varepsilon))$$

Corollary 4.3. Suppose $\mathcal{R}$ is residuated implication induced by single valued neutrosophic Łukasiewicz t-representable t-norm $\mathcal{T}$, then $\triangle_{B^*}(\varepsilon) = 3\varepsilon \wedge 1$.

Proof: Let $A^*(x) = \langle t_1, i_1, f_1\rangle$, $A(x) = \langle t_2, i_2, f_2\rangle$, $B(y) = \langle t_3, i_3, f_3\rangle$, $A'^*(x) = \langle t_1', i_1', f_1'\rangle$, $A'(x) = \langle t_2', i_2', f_2'\rangle$, $B'(y) = \langle t_3', i_3', f_3'\rangle$. Suppose that $\| A - A' \| \leq \varepsilon$, $\| B - B' \| \leq \varepsilon$, $\| A^* - A'^* \| \leq \varepsilon$, according to Proposition 4.1, then we have:

$$d(\mathcal{T}(A^*(x),\mathcal{R}(A(x),B(y))),\mathcal{T}(A'^*(x),\mathcal{R}(A'(x),B'(y))))$$
$$= max\{|(0 \vee (t_1 + R_T(t_2,t_3) - 1)) - (0 \vee (t_1' + R_T(t_2',t_3') - 1))|,$$
$$|((i_1 + R_S(i_3,i_2)) \wedge 1) - ((i_1' + R_S(i_3',i_2')) \wedge 1)|,$$
$$|((f_1 + R_S(f_3,f_2)) \wedge 1) - ((f_1' + \mathcal{R}_S(f_3',f_2')) \wedge 1)|\}$$
$$\leq max\{|(0 \vee (t_1 + R_T(t_2,t_3) - 1)) - (0 \vee ((t_1+\varepsilon) + R_T(t_2,t_3) + \triangle_{\mathcal{R}}(\varepsilon) - 1))|,$$
$$|((i_1 + R_S(i_3,i_2)) \wedge 1) - ((i_1 + \varepsilon + R_S(i_3,i_2) + \triangle_{\mathcal{R}}(\varepsilon)) \wedge 1)|,$$
$$|((f_1 + R_S(f_3,f_2)) \wedge 1) - ((f_1 + \varepsilon + R_S(f_3,f_2) + \triangle_{\mathcal{R}}(\varepsilon)) \wedge 1)|\}$$
$$\leq \varepsilon + \triangle_{\mathcal{R}}(\varepsilon)$$

For Łukasiewicz implication, for all $A^*(x) = \langle t_1, i_1, f_1\rangle$, we can tack $A'^*(x) = \langle t_1+\varepsilon, i_1+\varepsilon, f_1+\varepsilon\rangle$, $\mathcal{R}(\langle t_2,i_2,f_2\rangle,\langle t_3,i_3,f_3\rangle) = \langle 1,0,0\rangle$, $\mathcal{R}(\langle t_2',i_2',f_2'\rangle,\langle t_3',i_3',f_3'\rangle) = \langle 1-\triangle_{\mathcal{R}}(\varepsilon), \triangle_{\mathcal{R}}(\varepsilon), \triangle_{\mathcal{R}}(\varepsilon)\rangle$ satisfy the above equation, i.e. $\triangle_{B^*}(\varepsilon) = \varepsilon + \triangle_{\mathcal{R}}(\varepsilon)$. Therefore, $\triangle_{B^*}(\varepsilon) = 3\varepsilon \wedge 1$ by Corollary 4.2.

Corollary 4.4. Suppose $\mathcal{R}$ is a residuated implication induced by single valued neutrosophic Goguen t-representable t-norm $\mathcal{T}$, then $\triangle_{B^*}(\varepsilon) = \varepsilon + (1-\varepsilon)\,\triangle_{\mathcal{R}}(\varepsilon)$.

Proof: Let $A^*(x) = \langle t_1, i_1, f_1\rangle$, $A(x) = \langle t_2, i_2, f_2\rangle$, $B(y) = \langle t_3, i_3, f_3\rangle$, $A'^*(x) = \langle t_1', i_1', f_1'\rangle$, $A'(x) = \langle t_2', i_2', f_2'\rangle$, $B'(y) = \langle t_3', i_3', f_3'\rangle$. Suppose that $\| A - A' \| \leq \varepsilon$, $\| B - B' \| \leq \varepsilon$, $\| A^* - A'^* \| \leq \varepsilon$, according to Proposition 4.1, then we have:

$$d(\mathcal{T}(A^*(x),\mathcal{R}(A(x),B(y))),\mathcal{T}(A'^*(x),\mathcal{R}(A'(x),B'(y))))$$
$$= |t_1 \cdot \mathcal{R}_T(t_2,t_3) - t_1' \cdot \mathcal{R}_T(t_2',t_3')|$$
$$\vee\,|(i_1 + \mathcal{R}_S(i_3,i_2) - i_1 \cdot \mathcal{R}_S(i_3,i_2)) - (i_1' + \mathcal{R}_S(i_3',i_2') - i_1' \cdot \mathcal{R}_S(i_3',i_2'))|$$
$$\vee\,|(f_1 + \mathcal{R}_S(f_3,f_2) - f_1 \cdot \mathcal{R}_S(f_3,f_2)) - (f_1' + \mathcal{R}_S(f_3',f_2') - f_1' \cdot \mathcal{R}_S(f_3',f_2')|$$
$$\leq |t_1 \cdot \mathcal{R}_T(t_2,t_3) - (t_1-\varepsilon)\cdot(\mathcal{R}_T(t_2,t_3) - \triangle_{\mathcal{R}}(\varepsilon))|$$
$$\vee\,|(i_1 + \mathcal{R}_S(i_3,i_2) - i_1 \cdot \mathcal{R}_S(i_3,i_2)) - ((i_1+\varepsilon) + (\mathcal{R}_S(i_3,i_2) - \triangle_{\mathcal{R}}(\varepsilon)) - ((i_1+\varepsilon)\cdot(\mathcal{R}_S(i_3,i_2) - \triangle_{\mathcal{R}}(\varepsilon)))|$$
$$\vee\,|(f_1 + \mathcal{R}_S(f_3,f_2) - f_1 \cdot \mathcal{R}_S(f_3,f_2)) - ((f_1+\varepsilon) + (\mathcal{R}_S(f_3,f_2) - \triangle_{\mathcal{R}}(\varepsilon)) - ((f_1+\varepsilon)\cdot(\mathcal{R}_S(f_3,f_2) - \triangle_{\mathcal{R}}(\varepsilon)))|$$
$$\leq \varepsilon + (1-\varepsilon)\,\triangle_{\mathcal{R}}(\varepsilon)$$

For Goguen implication, we can take $A^*(x) = \langle t_1, i_1, f_1\rangle = \langle 1,0,0\rangle$, $A'^*(x) = \langle 1-\varepsilon, \varepsilon, \varepsilon\rangle$, $\mathcal{R}(\langle t_2,i_2,f_2\rangle,\langle t_3,i_3,f_3\rangle) = \langle 1,0,0\rangle$, satisfy the above equation, i.e. $\triangle_{B^*}(\varepsilon) = \varepsilon + (1-\varepsilon)\,\triangle_{\mathcal{R}}(\varepsilon)$.

Corollary 4.5. Suppose $\mathcal{R}$ is residuated implication induced by single valued neutrosophic $Gödel$ t-representable t-norm $\mathcal{T}$, then $\triangle_{B^*}(\varepsilon) = \triangle_{\mathcal{R}}(\varepsilon)$.

Proof: According to Theorem 4.1, we have $\triangle_{B^*}(\varepsilon) \leq \triangle_{\mathcal{T}}(\triangle_{\mathcal{R}}(\varepsilon))$, since $\mathcal{T}$ is single-valued neutrosophic $Gödel$ t-norm, then we have $\triangle_{B^*}(\varepsilon) \leq \triangle_{\mathcal{R}}(\varepsilon)$. Let $A^*(x) = 1^*$, then $B^*(y) = \bigvee_{x\in X} \mathcal{T}\,(1^*, \mathcal{R}(A(x),B(y))) = \bigvee_{x\in X} \mathcal{R}\,(A(x),B(y))$, i.e., $\triangle_{B^*}(\varepsilon) \geq \triangle_{\mathcal{R}}(\varepsilon)$. Therefore, $\triangle_{B^*}(\varepsilon) = \triangle_{\mathcal{R}}(\varepsilon)$.

Theorem 4.3. Let $A,\ A',\ B,\ B',\ B^*$ and B'^* be single-valued neutrosophic fuzzy sets. If $\parallel A - A' \parallel \leq \varepsilon$, $\parallel B - B' \parallel \leq \varepsilon$, $\parallel B^* - B'^* \parallel \leq \varepsilon$, A^* and A'^* are single-valued neutrosophic $\mathcal{R}$-type triple I solutions of FMT $(A,\ B,\ B^*)$ and FMT$(A',\ B',\ B'^*)$ given in Theorem 3.2 respectively, then the ε sensitivity of the single-valued neutrosophic $\mathcal{R}$-type triple I solution A^* for FMT is

$$\triangle_{A^*}(\varepsilon) = \parallel A^* - A'^* \parallel \leq \triangle_{\mathcal{R}}(\triangle_{\mathcal{R}}(\varepsilon)).$$

Proof: Let $A, A' \in SNVS(X)$, $B, B', B^*, B'^* \in SNVS(Y)$. If $\parallel A - A' \parallel \leq \varepsilon$, $\parallel B - B' \parallel \leq \varepsilon$, $\parallel B^* - B'^* \parallel \leq \varepsilon$, then we have,

$$\begin{aligned}\triangle_{A^*}(\varepsilon) &= \parallel A^* - A'^* \parallel \\ &= \bigvee_{x\in X} d\,(A^*(x), A'^*(x)) \\ &= \bigvee_{x\in X} d\,(\bigwedge_{y\in Y} \mathcal{R}\,(\mathcal{R}(A(x),B(y)),B^*(y)), \bigwedge_{y\in Y} \mathcal{R}\,(\mathcal{R}(A'(x),B'(y)),B'^*(y))) \\ &\leq \bigvee_{x\in X}\bigvee_{y\in Y} d\,(\mathcal{R}(\mathcal{R}(A(x),B(y)),B^*(y)), \mathcal{R}(\mathcal{R}(A'(x),B'(y)),B'^*(y))) \\ &\leq \triangle_{\mathcal{R}}(\triangle_{\mathcal{R}}(\varepsilon))\end{aligned}$$

Corollary 4.6. Suppose $\mathcal{R}$ is residuated implication induced by single valued neutrosophic Łukasiewicz t-representable t-norm $\mathcal{T}$, then $\triangle_{A^*}(\varepsilon) = 3\varepsilon \wedge 1$.

Proof: Let $B^*(y) = \langle t_1, i_1, f_1\rangle$, $A(x) = \langle t_2, i_2, f_2\rangle$, $B(y) = \langle t_3, i_3, f_3\rangle$, $B'^*(y) = \langle t_1', i_1', f_1'\rangle$, $A'(x) = \langle t_2', i_2', f_2'\rangle$, $B'(y) = \langle t_3', i_3', f_3'\rangle$. Suppose that $\parallel A - A' \parallel \leq \varepsilon$, $\parallel B - B' \parallel \leq \varepsilon$, $\parallel A^* - A'^* \parallel \leq \varepsilon$, according to Proposition 4.1, then we have:

$$\begin{aligned}& d(\mathcal{R}(\mathcal{R}(A(x),B(y)),B^*(y)), \mathcal{R}(\mathcal{R}(A'(x),B'(y)),B'^*(y))) \\ &= max\{|(1 \wedge (1 - R_T(t_2,t_3) + t_1)) - (1 \wedge (1 - R_T(t_2',t_3') + t_1'))|, \\ &\quad |((i_1 - R_S(i_3,i_2)) \vee 0) - ((i_1' - R_S(i_3',i_2')) \vee 0)|, \\ &\quad |((f_1 - R_S(f_3,f_2)) \vee 0) - ((f_1' - \mathcal{R}_S(f_3',f_2')) \vee 0)|\} \\ &\leq max\{|(1 \wedge (1 - R_T(t_2,t_3) + t_1)) - (1 \wedge (1 - (R_T(t_2,t_3) - \triangle_{\mathcal{R}}(\varepsilon)) + (t_1 + \varepsilon)))|, \\ &\quad |((i_1 - R_S(i_3,i_2)) \vee 0) - ((i_1 - \varepsilon - (R_S(i_3,i_2) + \triangle_{\mathcal{R}}(\varepsilon))) \vee 0)|, \\ &\quad |((f_1 - R_S(f_3,f_2)) \vee 0) - ((f_1 - \varepsilon - (R_S(f_3,f_2) + \triangle_{\mathcal{R}}(\varepsilon))) \vee 0)|\} \\ &\leq \varepsilon + \triangle_{\mathcal{R}}(\varepsilon)\end{aligned}$$

For the single-valued neutrosophic Lukasiewicz implication, for all $A^*(x) =$ Žt_1, i_1, f_1ž, we can tack $A'^*(x) = \langle t_1 + \varepsilon, i_1 - \varepsilon, f_1 - \varepsilon\rangle$, $\mathcal{R}(\langle t_2, i_2, f_2\rangle, \langle t_3, i_3, f_3\rangle) = \langle 1,0,0\rangle$, $\mathcal{R}(\langle t_2', i_2', f_2'\rangle, \langle t_3', i_3', f_3'\rangle) = \langle 1 - \triangle_{\mathcal{R}}(\varepsilon), \triangle_{\mathcal{R}}(\varepsilon), \triangle_{\mathcal{R}}(\varepsilon)\rangle$ satisfy the above equation, i.e. $\triangle_{A^*}(\varepsilon) = \varepsilon + \triangle_{\mathcal{R}}(\varepsilon)$. Therefore, $\triangle_{A^*}(\varepsilon) = 3\varepsilon \wedge 1$ by Corollary 4.2.

Corollary 4.7. Suppose $\mathcal{R}$ is a residuated implication induced by single valued neutrosophic Goguen t-representable t-norm $\mathcal{T}$, then $\triangle_{A^*}(\varepsilon) = \frac{\varepsilon}{1-\triangle_{\mathcal{R}}(\varepsilon)}$.

Proof: Let $B^*(y) = \langle t_1, i_1, f_1\rangle$, $A(x) = \langle t_2, i_2, f_2\rangle$, $B(y) = \langle t_3, i_3, f_3\rangle$, $B'^*(y) = \langle t_1', i_1', f_1'\rangle$, $A'(x) = \langle t_2', i_2', f_2'\rangle$, $B'(y) = \langle t_3', i_3', f_3'\rangle$. Suppose that $\parallel A - A' \parallel \leq \varepsilon$, $\parallel B - B' \parallel \leq \varepsilon$, $\parallel A^* - A'^* \parallel \leq \varepsilon$, according to Proposition 4.1, then we have:

$$d(\mathcal{R}(\mathcal{R}(A(x),B(y)),B^*(y)),\mathcal{R}(\mathcal{R}(A'(x),B'(y)),B'^*(y)))$$
$$= max\{|(1\wedge(\frac{t_1}{R_T(t_2,t_3)}))-(1\wedge(\frac{t_1'}{R_T(t_2',t_3')})|,$$
$$|\frac{(i_1-R_S(i_3,i_2))\vee 0}{1-R_S(i_3,i_2)}-\frac{(i_1'-R_S(i_3',i_2'))\vee 0}{1-R_S(i_3',i_2')}|,$$
$$|\frac{(f_1-R_S(f_3,f_2))\vee 0}{1-R_S(f_3,f_2)}-\frac{(f_1'-R_S(f_3',f_2'))\vee 0}{1-R_S(f_3',f_2')}|,$$
$$\leq max\{|(1\wedge(\frac{t_1}{R_T(t_2,t_3)}))-(1\wedge(\frac{t_1-\varepsilon}{R_T(t_2,t_3)+\Delta_{\mathcal{R}}(\varepsilon)}))|,$$
$$|\frac{(i_1-R_S(i_3,i_2))\vee 0}{1-R_S(i_3,i_2)}-\frac{((i_1-\varepsilon)-(R_S(i_3,i_2)+\Delta_{\mathcal{R}}(\varepsilon)))\vee 0}{1-(R_S(i_3,i_2)+\Delta_{\mathcal{R}}(\varepsilon)}|,$$
$$|\frac{(f_1-R_S(f_3,f_2))\vee 0}{1-R_S(f_3,f_2)}-\frac{((f_1-\varepsilon)-(R_S(f_3,f_2)+\Delta_{\mathcal{R}}(\varepsilon)))\vee 0}{1-(R_S(f_3,f_2)+\Delta_{\mathcal{R}}(\varepsilon)}\}$$
$$\leq\frac{\varepsilon}{1-\Delta_{\mathcal{R}}(\varepsilon)}$$

For the single-valued neutrosophic Goguen implication, we can tack $B^*(y)=\langle\varepsilon,1,1\rangle$, $B'^*(y)=\langle 0,1-\varepsilon,1-\varepsilon\rangle$, $\mathcal{R}(\langle t_2,i_2,f_2\rangle,\langle t_3,i_3,f_3\rangle)=\langle 1-\Delta_{\mathcal{R}}(\varepsilon),0,0\rangle$, satisfy the above equation, i.e. $\Delta_{A^*}(\varepsilon)=\frac{\varepsilon}{1-\Delta_{\mathcal{R}}(\varepsilon)}$.

Corollary 4.8. Suppose $\mathcal{R}$ is residuated implication induced by single valued neutrosophic $Gödel$ t-representable t-norm $\mathcal{T}$, then $\Delta_{A^*}(\varepsilon)=1$.

Proof: Let $B^*(y)=\langle t_1,i_1,f_1\rangle$, $A(x)=\langle t_2,i_2,f_2\rangle$, $B(y)=\langle t_3,i_3,f_3\rangle$, $B'^*(y)=\langle t_1',i_1',f_1'\rangle$, $A'(x)=\langle t_2',i_2',f_2'\rangle$, $B'(y)=\langle t_3',i_3',f_3'\rangle$. Suppose that $\parallel A-A'\parallel\leq\varepsilon$, $\parallel B-B'\parallel\leq\varepsilon$, $\parallel A^*-A'^*\parallel\leq\varepsilon$, according to Proposition 4.1, then we have:

$$d(\mathcal{R}(\mathcal{R}(A(x),B(y)),B^*(y)),\mathcal{R}(\mathcal{R}(A'(x),B'(y)),B'^*(y)))$$
$$= max\{|R_T(R_T(t_2,t_3),t_1)-R_T(R_T(t_2',t_3'),t_1'))|,$$
$$|R_S(i_1,R_S(i_3,i_2))-R_S(i_1',R_S(i_3',i_2'))|,$$
$$|R_S(f_1,R_S(f_3,f_2))-R_S(f_1',\mathcal{R}_S(f_3',f_2'))|\}$$
$$\leq 1$$

For the single-valued neutrosophic $Gödel$ implication, we can tack $B^*(y)=\langle\varepsilon,1,1\rangle$,$A(x)=\langle\frac{\varepsilon}{2},1-\varepsilon,1-\varepsilon\rangle$, $B(y)=\langle\frac{\varepsilon}{4},1-\frac{\varepsilon}{2},1-\frac{\varepsilon}{2}\rangle$, $B'^*(y)=\langle 0,1-\varepsilon,1-\varepsilon\rangle$, $A'(x)=\langle\varepsilon,1-2\varepsilon,1-2\varepsilon\rangle$, $B'(y)=\langle\frac{\varepsilon}{2},1-\varepsilon,1-\varepsilon\rangle$, then $\mathcal{R}(A(x),B(y))=\langle\frac{\varepsilon}{4},0,0\rangle$, $\mathcal{R}(A'(x),B'(y))=\langle\frac{\varepsilon}{2},0,0\rangle$, satisfy the above equation, i.e., $\Delta_{A^*}(\varepsilon)=1$.

5. Conclusions

In this paper, we extend the fuzzy inference triple I method on single-valued neutrosophic sets. Single valued neutrosophic fuzzy inference triple I Principle for and are proposed. Moreover, the single-valued neutrosophic fuzzy inference triple I solutions for and are given respectively. The reductivity and the robustness of the single-valued neutrosophic fuzzy inference triple I methods are studied.

This article only conducts research on fuzzy reasoning algorithms at the theoretical level and has not been applied in databases; when using t-representable t-norm, this article only considers the case of $R_T=R_S$, without analyzing and demonstrating the case of $R_T\neq R_S$.

The logical basis of a fuzzy inference method is very important. In the future, we will consider building the strict logic foundation for the triple I method based on left-continuous single-valued neutrosophic t-representable t-norms, and bring the single-valued neutrosophic fuzzy inference method within the framework of logical semantic. Not only that, analyze and discuss the case of $R_T \neq R_S$ for the algorithm, and apply the algorithm to pattern recognition in the database.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Pairing New Approach of Tree Soft with MCDM Techniques: Toward Advisory an Outstanding Web Service Provider Based on QoS Levels

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Abstract: Web services (WSs) have become dynamic because of technological advancements and internet usage. Hence, selecting a WS provider among a variety of WS providers that perform the same function is a critical process. However, the crucial point is that various consumers may have varied needs when it comes to the quality attributes of services, such as cost, response time, throughput, security, availability, etc. These aspects of Web services are known as quality of service (QoS), or non-functional characteristics. Hence, this issue is the robust motivator for conducting this study. The objective of this study is to evaluate a set of WSs that provide various services for various consumers and organizations. This evaluation is conducted based on a set of QoS attributes. Hence, we are applying a new approach to describe this problem in the form of leaves or branches of a tree or hierarchy. This approach is represented in a soft tree set. Also, we leveraged Multi-Criteria Decision Making (MCDM) techniques such as entropy and weighted sum methods under the authority of the Single Value Neutrosophic (SVN) Scale. The entropy technique analyzes attributes or leaves in each level contained in the tree's soft approach, obtaining attributes' weights. These weights are used to rank and recommend optimal WS providers through the application of these weights in WSM. The results of implementing entropy-WSM in a tree-soft approach indicated that WS_2 is the optimal provider. In contrast, WS_3 is the worst provider.

Keywords: Tree Soft Set; Single Value Neutrosophic; Multi-Criteria Decision Making; Quality of Service.

1. Introduction

Presently, Web services (WSs) with equivalent functionality are contrasted, taking into account non-functional characteristics that might affect the quality of service that WS provides [1]. With the use of extensible markup language (XML)-based protocols like web services description language (WSDL), universal description discovery and integration (UDDI), and simple object access protocol (SOAP), WS, based on [2], is described as a software component that facilitates interoperability among loosely coupled systems over the Internet. As stated by [3], one of the most difficult and important tasks in service-oriented architecture (SOA) is choosing WS that will best meet the demands of WS users. The World Wide Web Consortium (W3C) described WSs in [4] as software systems established with the purpose of enabling ubiquitous machine-to-machine communication across a network. In order to process requests, complete workflows, and complete intricate transactions, Web services communicate with one another and with other systems. In order to serve business objectives and data consolidation for any firm [5], WSs are generally acknowledged as the most effective standards-based technique to build SOA.

According to Figure 1, SOA consists of various parties, each of whom is responsible for an important role.

(i). Service Provider: that provides various services for a variety of consumers.

(ii). Service Consumer: that request variety of services based on several of consumers from several of service providers.

(iii). Service Broker: which represents as intermediary between N of providers, N of consumers and register for supporting consumer to get services from the responsible provider. This provider offers needed service for consumers who need this service.

(iv). Service Register: that contains all providers or as register of N of providers. This register response to request of broker about provider which provides requested service then register recommend suitable provider for broker.

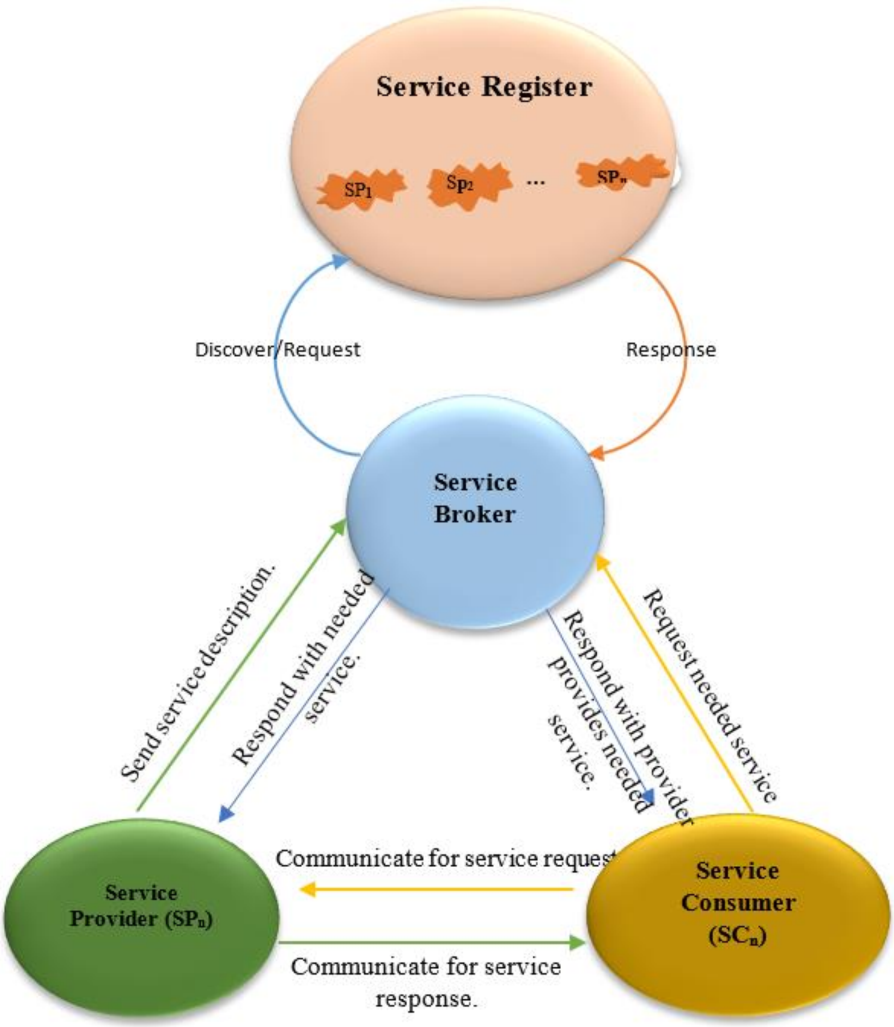

Figure 1. Service oriented architecture framework.

Despite the abundance of functionally equivalent online services, there exist differences in quality of services (QoS) amongst them. Because of this exponential increase, it is now difficult to choose the required Web service from the many that offer the same functionality. Scholars as Subbulakshmi et al. [6] classified QoS into (1) functional which described attributes associated with the kind, name of operation, and the semantics and format of the data they receive or produce.(2) non-functional which includes response time, availability, throughput, dependability, security, Latency….etc. In this context, QoS is a key differentiator between various web services and is used to characterize the non-functional aspects of web services [7]. Hence, QoS in [8] provided by the WS to the end-user is described by the QoS parameters. The end user inquiry indicates the needed WS's anticipated quality. Accordingly, the process of selecting appropriate WS for satisfying consumer's requirements is conducted based on QoS criteria. From the perspective of [9] WS selection is formulating through leveraging techniques of MCDM which have ability to treat with the conflict of QoS's criteria. The problem of selecting WS based on QoS is described in hierarchy architecture.

Hence, this study embraces perspective in [9] to be a motivator for constructing tree soft evaluator model. The notion of tree soft is highlighted and embraced by Smarandache [10] where this notion considers the first approach represents the selection problem in form of leaves in the levels of tree. The constructed tree soft evaluator model treats with work hierarchically through employing soft tree sets with MCDM techniques toward choosing optimal WS based on hierarchical of QoS's criteria.

2. Previous perspectives and studies

This section clarifies the prior studies and perspectives that embraced the techniques that contributed to our study. Hence, this section reflects and aggregates various studies based on surveys conducted to apply techniques to solve the problem of WS selection.

2.1 MCDM as solver techniques in WS selection: prior works

Plenty of studies have used MCDM techniques to select WS according to QoS's criteria [11]. For instance, a QoS assessment indicator system for SPs in KI-C is built into [12]. The weights of the assessment indicators are also determined using the Decision-Making Trial and Assessment Laboratory (DEMATEL) technique. The rank-sum ratio (RSR) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) are used to assess and grade the SPs, respectively. In the same vein [3], WSs' weights are calculated by deploying the Analytic Hierarchy Process (AHP) used for weight calculation and have been ranked by employing the Technique of Order Preference by Similarity to Ideal Solution (TOPSIS) method. Also, TOPSIS was applied with fuzzy in [13] for enhancing QoS-conscious semantic WS selection and ranking. Other MCDM techniques, as in [14], where the service selection problem is formulated and an integrated decision model using fuzzy AHP techniques and WASPAS, or weighted aggregated sum product assessment, is constructed for solving this problem,. Trustworthy cloud service providers are obtained in [15] through the fuzzy PROMETHEE method based on Shannon entropy.

Generally, we are exploiting the ability of MCDM techniques to treat multi-attributes and criteria and representing these attributes in the form of leaves in a tree by applying the tree soft approach.

2.2 General perspective of tree soft set: fundamental principles

The approach of tree soft set is introduced by Smarandache [10] who is founded of this approach. This approach was founded based on a soft set idea. Tree soft is described and defined by Smarandache as:

Let U be a universe of discourse, and H a non-empty subset of U, with P(H) the powerset of H.

Let A be a set of attributes (parameters, factors, etc.), $A= \{A_1, A_2, \dots , A_n\}$, for integer $n \geq 1$, where A_1, $A_2, \dots , A_n$ are considered attributes of first level (since they have one-digit indexes).

Each attribute Ai, $1 \leq i \leq n$, is formed by sub-attributes:

$A_1 = \{A_{1,1}, A_{1,2}, \dots\}$ $A_2 = \{A_{2,1} , A_{2,2} , \dots \}$ $A_n = \{A_{n,1} , A_{n,2} , \dots \}$

where the above $A_{i,j}$ are sub-attributes (or attributes of second level) (since they have two-digit indexes). Again, each sub-attribute $A_{i,j}$ is formed by sub-sub-attributes (attributes of third level): $A_{i,j,k}$

And so on, as much refinement as needed into each application, up to sub-sub-…-sub-attributes (or attributes of m-level (or having m digits into the indexes):

$A_{i1,i2,\dots,im}$

Therefore, a graph-tree is formed, that we denote as Tree(A), whose root is A (considered of level zero), then nodes of level 1, level 2, up to level m.
We call leaves of the graph-tree, all terminal nodes (nodes that have no descendants).
Then the TreeSoft Set is:

$$F: P(Tree(A)) \rightarrow P(H)$$

Tree(A) is the set of all nodes and leaves (from level 1 to level m) of the graph-tree, and P(Tree(A)) is the powerset of the Tree(A).

All node sets of the TreeSoft Set of level m are:

$$Tree(A) = \{A_{i1} \mid i1= 1, 2, ...\}$$

3. Methodology of selection process

Herein, the study took advantage of surveys conducted for prior studies, which produced the outcomes in the previous section. Hence, we are exploiting entropy as a technique of MCDM to obtain QoS's weights, which are represented in a soft tree in a hierarchy form toward selecting the optimal WS. The process of selection is performed based on several steps.
Step 1. Construct the tree set.

- Determining influential attributes/criteria of QoS as main attributes (A_n) in level 1 in form $\{A_1, A_2, \ldots A_n\}$. the inherent attributes /criteria of main in level 1 form in level 2 which entails sub-attributes related to level 1 as $\{A_{1i}, A_{2i}, .. A_{ni}\}$.
- Set of candidates of WSs as $\{WS_1, WS_2, \ldots WS_n\}$ are recommended to contribute to selection process.

Step 2. Analyzing and valuing attributes of level 1 and 2.

- LEDM: Linguistic expert's Decision Matrices are constructed for evaluating WS_n over attributes (A_n) in level 1 $\{A_1, A_2 \ldots A_n\}$. Also, Linguistic expert's Decision Matrices are constructed for evaluating WS_n over attributes (A_{ni}) in level 2 $\{A_{1i}, A_{2i}, .. A_{ni}\}$.
- Constructed decision matrices are valuing based on scale of single value Neutrosophic sets (SVNSs).
- Entropy technique starts to implement in constructed decision matrices for WS_n over attributes (A_n) in level 1 and WS_n over attributes (A_{ni}) in level 2 through following sub-steps:

Step 2.1. The various decision matrices are transformed into crisp matrices through Eq. (1).

$$\boldsymbol{s}(Q_{ij}) = \frac{(2+\mathcal{g}-\partial-\wp)}{3} \tag{1}$$

Where: $\mathcal{g}$, ∂, $\wp$ refers to truth, false, and indeterminacy respectively.
Step 2.2. Eq.(2) is employed in crisp matrices to aggregate it into single decision matrix.

$$D_{Mt_{ij}} = \frac{(\sum_{j=1}^{N} Q_{ij})}{N} \tag{2}$$

Where:
Q_{ij} refers to value of criterion in matrix, N refers to number of decision makers.
Step 2.3. Normalizing the aggregated decision matrix by Eq. (3).

$$X_{ij} = \frac{D\_Mt_{ij}}{\sum_{j=1}^{m} D\_Mt_{ij}} \tag{3}$$

Where:
$\sum_{j=1}^{m} D\_Mt_{ij}$ represents sum of each criterion in aggregated matrix per column.
Step 2.4. Entropy for normalized matrix computes by Eq. (4).

$$e_j = -h \sum_{i=1}^{m} x_{ij} \ln X_{ij} \tag{4}$$

Where:

$$h = \frac{1}{\ln(WS)} \tag{5}$$

WS refers to number of alternatives.

Step 2.5. Compute weight vectors through employing Eq. (6).

$$w_j = \frac{1-e_j}{\sum_{j=1}^{n}(1-e_j)} \quad (6)$$

Step 3. Selecting optimal web services.

- WSM is an essential technique in soft tree for attributes with various levels. This technique is exploiting generated aggregated matrix toward rank WSs based on leaves of soft tree (i.e. Attributes).

Step 3.1. Eqs. (7), and (9) are employed in aggregated matrix.

$$\mathrm{Norm}_{\mathrm{Agg\_matij}} \frac{\mathrm{C_{ij}}}{\mathrm{sum(C_{ij})}} \quad \text{, For Benficial key indicators} \quad (7)$$

$$\mathrm{Z} = \frac{1}{\mathrm{C_{Ij}}} \quad (8)$$

$$\mathrm{Norm}_{\mathrm{Agg\_matij}} = \frac{\mathrm{Z}}{\mathrm{sum(Z)}} \quad \text{, For Non − Benficial key indicators} \quad (9)$$

Where:

$\mathrm{C_{ij}}$ indicates to each element in the aggregated matrix.

Step 3.2. The obtained QoS criteria's weights of entropy technique are applied in the following Eq. (10) to generate weighted matrix.

$$\mathrm{weighted\_matrix_{ij}} = \mathrm{weight_i} * \mathrm{Norm}_{\mathrm{Agg_{matij}}} \quad (10)$$

Where:

$\mathrm{weighted\_matrix_{ij}}$ is weighted decision matrix.

Step 3.3. Global score computes through Eq. (11).

$$\mathrm{V}(\mathrm{weighted\_matrix_{ij}}) = \sum_{j=1}^{n} \mathrm{weighted\_matrix_{ij}} \quad (11)$$

Where: $\mathrm{V}(\mathrm{w\_matrix_{ij}})$ is global score values.

4. Real case study

To validate the accuracy of our methodology for selecting optimal WS. This process is performed by applying the constructed soft tree model-based hybrid mathematical techniques.

Herein, four WSs contributed to our case study. Also, criteria and attributes are determined to be leaves of the soft tree model, as shown in Figure 2. In this problem of selecting optimal WS, there are three experts related to our search field who rate determined candidates over determined attributes in soft tree's levels.

4.1 Entropy based tree soft set: Calculating attributes Level 1's weights.

- Firstly, LEDM are produced through using SVNSs scale in Ref. [16] and these matrices are transformed into crisp matrices based on Eq. (1).
- Eq. (2) contributes to develop Table 1 which represents an aggregated matrix for attributes {A_1, A_2}.
- This matrix is normalized by Eq. (3) to produce Table 2.
- entropy (e_j) is calculated by utilizing Eq.(4) to generate Table 3 and Figure 3 showcases vector weight's QoS criteria/attributes. According to this Figure we noticed that A_1 is the highest criterion with highest value of weight while A_2 is least one.

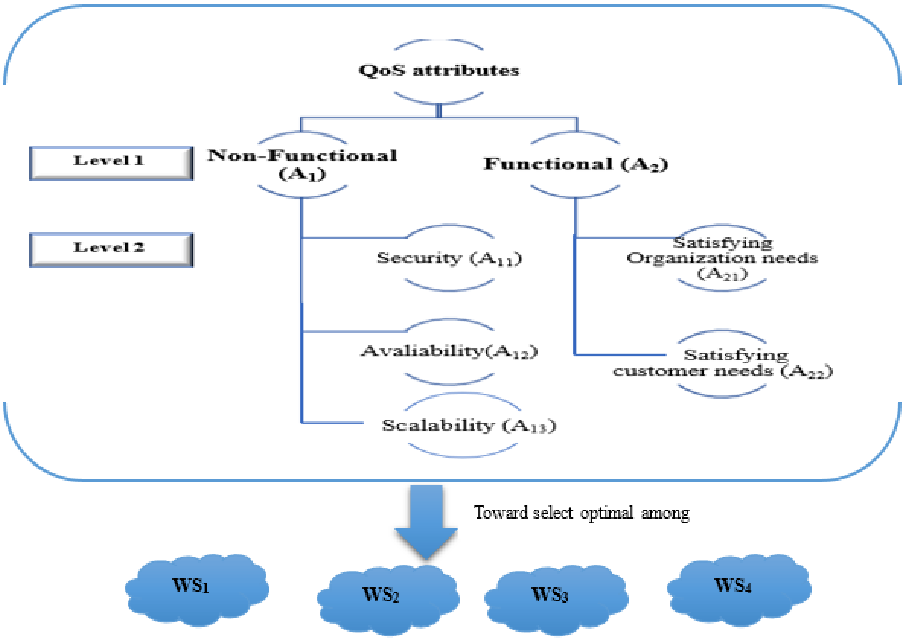

Figure 2. Determined leaves in soft tree model.

Table 1. An aggregated matrix of attributes A_1, A_2 at level 1.

	A_1	A_2
WS_1	0.477777778	0.455555556
WS_2	0.522222222	0.588888889
WS_3	0.222222222	0.672222222
WS_4	0.366666667	0.427777778

Table 2. Normalized matrix of attributes A_1, A_2 at level 1.

	A_1	A_2
WS_1	0.300699301	0.2124352
WS_2	0.328671329	0.2746114
WS_3	0.13986014	0.3134715
WS_4	0.230769231	0.1994819

Table 3. Entropy of normalized matrix of attributes A_1, A_2 at level 1.

	A_1	A_2
WS_1	-0.361333665	-0.329087269
WS_2	-0.365711611	-0.354907298
WS_3	-0.275120609	-0.363641621
WS_4	-0.338385477	-0.32157114
$\sum_{i=1}^{m} X_{ij}$	-1.340551363	-1.369207329
$-h\sum_{i=1}^{m} X_{ij} \ln X_{ij}$	0.966537533	0.987198484

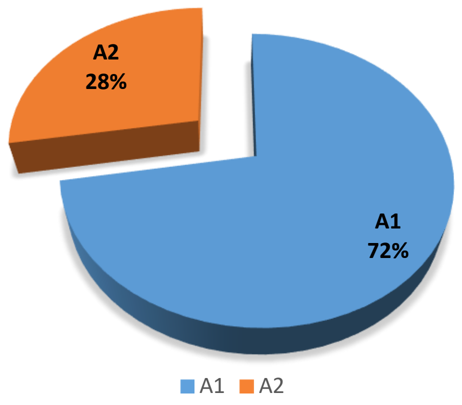

Figure 3. Weights of attributes in Level 1.

4.2 Entropy based tree soft set: Calculating attributes Level 2's weights

- The previous steps in sub-section 4.1 are repeated to obtain weights of attributes at level 2.

4.2.1 Calculating non-functional attributes' weights at level 2.

- Table 4 showcases an aggregated matrix for {WS_1, WS_2, WS_3, WS_4} over attributes {A_{11}, A_{12}, A_{13}}.
- Table 5 generated through normalizing the aggregated matrix.
- Entropy is represented in Table 6.
- Final weights for {A_{11}, A_{12}, A_{13}} are illustrated in Figure 4. Attribute A_{11} at level 2 is the best one which represents security non-functional. Otherwise, attribute A12 is availability non-functional considers the worst one.

Table 4. An aggregated matrix of non-functional attributes A_{11}: A_{13} at level 2.

	A_{11}	A_{12}	A_{13}
WS_1	0.611111111	0.566666667	0.538888889
WS_2	0.633333333	0.555555556	0.666666667
WS_3	0.255555556	0.533333333	0.4
WS_4	0.255555556	0.666666667	0.661111111

Table 5. Normalized matrix of non-functional attributes A_{11}: A_{13} at level 2.

	A_{11}	A_{12}	A_{13}
WS_1	0.348101266	0.244019139	0.237745098
WS_2	0.360759494	0.23923445	0.294117647
WS_3	0.14556962	0.229665072	0.176470588
WS_4	0.14556962	0.28708134	0.291666667

Table 6. Entropy of Normalized matrix of non-functional attributes A_{11}: A_{13} at level 2.

	A_{11}	A_{12}	A_{13}
WS_1	-0.367337985	-0.344191098	-0.341534194
WS_2	-0.367810093	-0.342179724	-0.35993395
WS_3	-0.280527334	-0.337867921	-0.306106069
WS_4	-0.280527334	-0.358274552	-0.35937524
$\sum_{i=1}^{m} X_{ij}$	-1.296202746	-1.382513296	-1.366949453

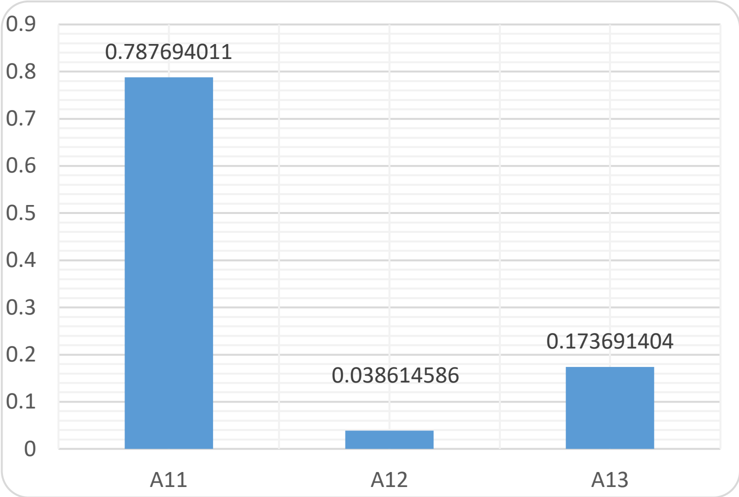

Figure 4. Weights of non-functional attributes A_{11}:A_{13} in Level 2.

4.2.2 Calculating functional attributes' weights at level 2.

- Table 7 showcases an aggregated matrix for {WS_1, WS_2, WS_3, WS_4} over attributes {A_{21}, A_{22}}.
- Table 8 generated through normalizing the aggregated matrix.
- Entropy is represented in Table 9.
- Final weights for {A_{21}, A_{22}} are illustrated in Figure 5. Attribute A_{21} at level 2 is the best one which represents satisfying organization needs. Otherwise, attribute A_{22} is satisfying customer needs considers the worst one.
- Figure 6 represents final weights for tree's attributes from A_1 until A_{11}: A_{22}. According to this Figure the security (A_{11}) is optimal with weight =0.57whilst availability (A_{12}) is least with weight = 0.028.

Table 7. An aggregated matrix of functional attributes A_{21}: A_{22} at level 2.

	A_{21}	A_{22}
WS_1	0.444444444	0.361111111
WS_2	0.727777778	0.7
WS_3	0.255555556	0.472222222
WS_4	0.316666667	0.605555556

Table 8. Normalized matrix of functional attributes A_{21}: A_{22} at level 2.

	A_1	A_2
WS_1	0.25477707	0.168831169
WS_2	0.417197452	0.327272727
WS_3	0.146496815	0.220779221
WS_4	0.181528662	0.283116883

Table 9. Entropy of normalized matrix of functional attributes A_{21}: A_{22} at level 2

	A_1	A_2
WS_1	-0.348373593	-0.300326349
WS_2	-0.364712203	-0.365551013
WS_3	-0.281383991	-0.333507342
WS_4	-0.30974993	-0.357263907

$\sum_{i=1}^{m} X_{ij}$	-1.304219716	-1.35664861
$-h\sum_{i=1}^{m} X_{ij} \ln X_{ij}$	0.940342416	0.978143648

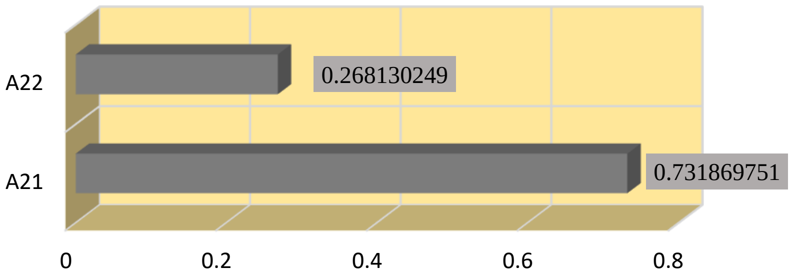

Figure 5. Weights of functional attributes A21:A22 in Level 2.

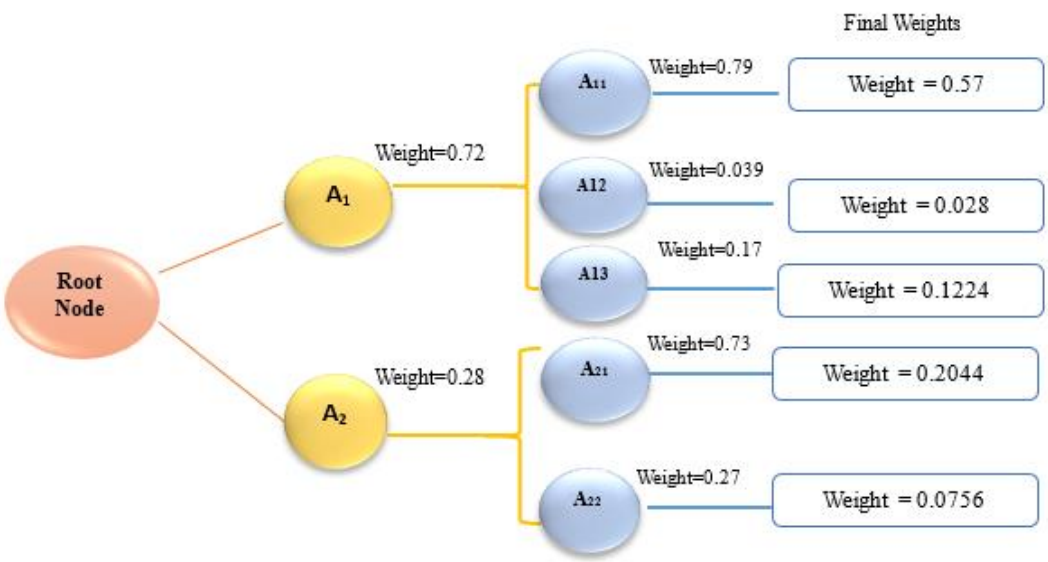

Figure 6. Final weights of attributes in tree soft.

4.3 WSM based tree soft set: Selection of optimal WS

- We are exploiting an aggregated matrix that generated from entropy based tree soft for selecting best WS based on QoS attributes/criteria described in hierarchy form in tree soft.

4.3.1 Recommending best WS from candidates over A_1:A_2

- Weighted decision matrix is constructed based on Eq. (10) as listed in Table 10.
- Final ranking for WSs from WS_1 to WS_4 which is illustrated in Figure 7. We demonstrated that WS_2 is the optimal one.

Table 10. Weighted matrix of attributes A_1: A_2 at level 1.

	A_1	A_2
WS_1	0.217494038	0.058782077
WS_2	0.237726041	0.075986587
WS_3	0.101160017	0.086739406
WS_4	0.166914029	0.055197804

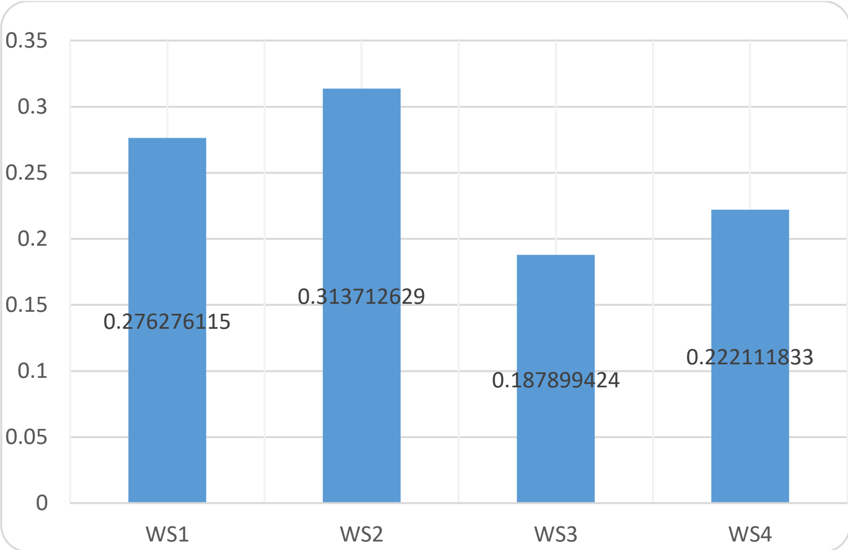

Figure 7. Ranking web services based on attributes in Level 1.

4.3.2 Recommending best WS from candidates over A_{11}:A_{13} non-functional in Level 2.

- Eq. (10) helped in obtaining weighted decision matrix from normalized matrix and entropy's weights (explained in sub section 4.2.1) and the produced weighted matrix is obtained in Table 11.
- Ranking of WSs candidates are illustrated in Figure 8 where WS_2 is the optimal one.

Table 11. Weighted matrix of attributes A_{11}: A_{13} at level 2.

	A_{11}	A_{12}	A_{13}
WS_1	0.198417722	0.006832536	0.0291
WS_2	0.205632911	0.006698565	0.036
WS_3	0.082974684	0.006430622	0.0216
WS_4	0.082974684	0.008038278	0.0357

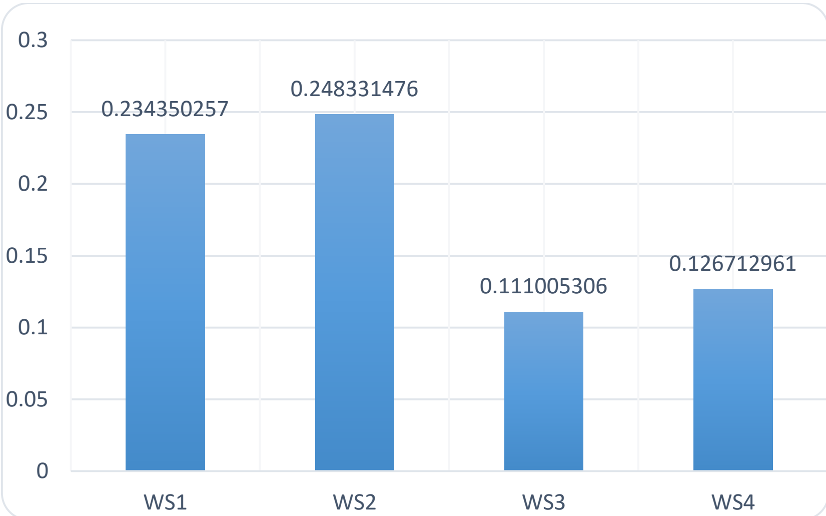

Figure 8. Ranking web services based on non-functional attributes in Level 2.

4.3.3 Recommending best WS from candidates over A_{21}:A_{22} functional in Level 2.

- Normalized matrix and entropy's weights are exploited to produce weighted matrix via Eq. (10) is obtained in Table 12.

- Ranking of WSs candidates are illustrated in Figure 9 where WS_2 is the optimal one.

Table 12. Weighted matrix of attributes A_{21}: A_{22} at level 2.

	A_{21}	**A_{22}**
WS_1	0.052076433	0.012763636
WS_2	0.085275159	0.024741818
WS_3	0.029943949	0.016690909
WS_4	0.037104459	0.021403636

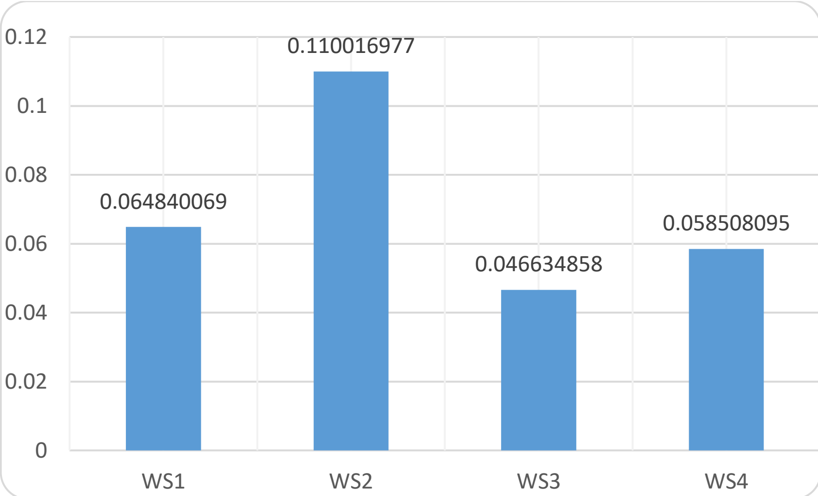

Figure 9. Ranking web services based on functional attributes in Level 2.

5. Conclusions

Making use of web services to complete complicated tasks online is becoming increasingly useful. Hence, it is important to select an appropriate WS that satisfies the customer's and organization's needs.

Plenty of prior studies which are relevant for our scope are analyzed selecting optimal WSs or service providers (SPs) through QoS. The selection process based on QoS conducted in this study according to functional and non-functional attributes.

The problem of selecting optimal WS or SPs represents in selection according to set of attributes fall under functional and non-functional. Also, these attributes are branched into sub-attributes. Accordingly, this problem can represent in hierarchy form. Hence, this study exploited surveys conducted for previous studies and volunteering tree soft approach for first time to describe this problem into set of levels. Each level entails a set of attributes. Also, MCDM techniques are employed in WSs selection tree soft to analyze attributes in each level and recommend the optimal WS among set of candidates.

Herein, entropy technique implemented in WSs selection tree soft to obtaining attributes' weights in each level through preferences of experts who related to our scope. The rating is performed through applying SVN scale as in [16]. The results of implementation of entropy indicated that security (A_{11}) is optimal attribute otherwise availability (A_{12}) is least based on its final values of its weights. After that WSM is leveraged the generated weights of attributes to rank WSs candidates and recommend the best and worst WS. In our case, there is an agreement on recommending WS_2 as optimal candidate based on its ranking in various levels of tree from level A_1 to level A_{22}. In contrast to WS $_3$ is the worst one.

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Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors.

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Finding a Basic Feasible Solution for Neutrosophic Linear Programming Models: Case Studies, Analysis, and Improvements

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Abstract: Since the inception of operations research, linear programming has received the attention of researchers in this field due to the many areas of its use. The focus was on the methods used to find the optimal solution for linear models. The direct simplex method, with its three basic stages, begins by writing the linear model in standard form and then finding a basic solution that is improved according to the simplex steps until We get the optimal solution, but we encounter many linear models that do not give us a basic solution after we put it in a standard form, and here we need to solve a rule through which we reach the optimal solution. For these models, researchers and scholars in the field of operations research introduced the simplex method with an artificial basis, which helped to Find the optimal solution for linear models, given the importance of this method and as a complement to the previous research we presented using the concepts of neutrosophic science. In this research, we will reformulate the simplex algorithm with an artificial basis using concepts of neutrosophic science.

Keywords: Linear Programming; Simplex Method; Neutrosophic Science; Simplex Neutrosophic Method; Artificial Variable.

1. Introduction

The great scientific development that our contemporary world has witnessed has led to the emergence of what is called operations research. This name refers to the group of scientific methods used in analyzing problems and searching for optimal solutions. Operations research is considered one of the modern sciences whose applications have achieved wide success in various fields of life. One of the methods of operations research is the linear programming method that allows us to model, analyze, and solve a wide range of issues that have resulted from the great scientific development that our contemporary world is witnessing [1]. In all previous studies, we have reached the optimal solution for solvable models, and this solution has a specific value resulting from specific data provided by the study. The field studies that were conducted are linked to the conditions that existed, but the reality of the situation indicates that the conditions surrounding the work environment are not fixed and the future cannot be predicted. These specific values for profits and available resources are subject to instantaneous change. Out of interest in keeping pace with scientific development, we have in this research reformulated one of the most important methods used to find the optimal solution for linear models: the simplex method with an artificial basis using the concepts of neutrosophic science, the science that has proven its ability to provide the best solution in many fields of science. Therefore, researchers' interest has focused on providing studies and research in various fields according to the concepts of this science [2-6].

The purpose of solving linear models is to choose the optimal solution from the set of acceptable solutions. This is done based on a base solution that is improved using the direct simplex algorithm that was presented according to the concepts of neutrosophic science in the research [7]. It consists of three basic stages:

(i). The stage of converting the imposed model into an equivalent systematic form [8].
(ii). The stage of converting the regular form into a basic form to obtain the non-negative basic solutions.
(iii). The stage of searching for the ideal solution requires from among the non-negative basic solutions [7].

Therefore, the process of searching for the optimal solution does not begin until after obtaining a base solution, but in many linear models we face great difficulty in obtaining the base solution, so the simplex method with an artificial base was proposed, where a base is formed consisting of a set of artificial variables that are not negativity is added to constraints that do not contain a basic variable, thus obtaining the basic solution. Then we improve it using the direct simplex algorithm until we obtain the optimal solution. In this research, we will reformulate the simplex algorithm with an artificial basis to find the optimal solution for linear models for which it is difficult to obtain a basic solution, using the concepts of neutrosophic.

2. Problem statement

Find the optimal solution for the following neutrosophic linear model:

$$Max\, Z = NC_1x_1 + NC_2x_2 + \cdots + NC_nx_n + NC_0$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + \varepsilon_1 = Nb_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \varepsilon_2 = Nb_2$$
$$a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + \varepsilon_3 = Nb_3$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + \varepsilon_m = Nb_m$$
$$x_1, x_2, \dots, x_n \geq 0$$

where $NC_j = C_j \pm \varepsilon_j$, $Nb_i = b_i \pm \delta_i$, a_{ij}, $j = 1,2,\dots,n$, $i = 1,2,\dots,m$ are constants having set or interval values according to the nature of the given problem, x_j are decision variables. It is worth mentioning that the coefficients subscribed by the index N are of neutrosophic values. The objective function coefficients $NC_1, NC_2, \dots, NC_n$ have neutrosophic meaning are intervals of possible values: That is, $Nc_j \in [\lambda_{j1}, \lambda_{j2}]$, where $\lambda_{j1}, \lambda_{j2}$ are the upper and the lower bounds of the objective variables x_j respectively, $j = 1,2,\dots,n$. Also, we have the values of the right-hand side of the inequality constraints $Nb_1, Nb_2, \dots, Nb_m$ are regarded as neutrosophic interval values:
$Nb_i \in [\mu_{i1}, \mu_{i2}]$, here, μ_{i1}, μ_{i2} are the upper and the lower bounds of the constraint $i = 1,2,\dots,m$.

In the previous model, we note that the number of variables is n and the number of constraints is m, and this model is in the standard form.
We move to the second stage, which is to find a basic solution. Here we use the simplex algorithm with an artificial base, which is represented by the following:

(i). From the standard form, we form an artificial base form by adding to the left side of each of the constraint equations a non-negative artificial variable ε_i. Thus, we form a base consisting of the non-negative variables $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m$.
(ii). Since the artificial variables are introduced into constraints that were originally linear equations, these variables must take the value of zero so that the linear constraints are not affected.

(iii). Therefore, we must move all of them from the base until they become non-base variables, and to be able to make this transition, we use the direct simplex algorithm.

(iv). We introduce these variables into the objective function with the likes of M (where M is a sufficiently large positive number that is at least greater than any $|Nc_j|$) and preceded by a minus sign (because the objective function is a maximization function) so that we do not transfer them back to the base variables again.

(v). We obtain the following basic form of the neutrosophic linear model:

$$Max\, Z = NC_1x_1 + NC_2x_2 + \cdots + NC_nx_n - M\varepsilon_1 - M\,\varepsilon_2 - \cdots - M\,\varepsilon_m + NC_0$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + \varepsilon_1 = Nb_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \varepsilon_2 = Nb_2$$
$$a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + \varepsilon_3 = Nb_3$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + \varepsilon_m = Nb_m$$

$$x_j \geq 0\ ,\varepsilon_i > 0\,, Nb_i > 0\,; j = 1,2,\dots,n\ and\ i = 1,2,\dots,m$$

(vi). After obtaining the basic solution, we use the direct simplex algorithm to improve this solution to reach the optimal solution. Therefore, we arrange the previous information in Table 1.

Table 1. General data of the model.

Variables / Basic	x_1	x_2	....	x_n	ε_1	ε_2	....	ε_m	b_i
ε_1	a_{11}	a_{12}	....	a_{1n}	1	0	....	0	b_1
ε_2	a_{21}	a_{22}	…	a_{2n}	0	1	0	0	b_2
....	…	…	…	…	…	…	…	…	…
ε_m	a_{m1}	a_{m2}	…	a_{mn}	0	0	…	1	b_m
objective function	NC_1	NC_2	....	NC_n	$-M$	$-M$	…	$-M$	$Z - NC_0$

We get rid of the artificial variables. Here we study the constants b_i corresponding to the artificial variables and choose the largest of them, let it be b_t corresponding to the variable ε_t and we consider its row to be the pivot row. Then we determine the pivot element in it by dividing the elements of the objective function row (elements NC_j) by the elements of the ε_t row and then we take the smallest positive ratio θ where:

$$\theta = \underset{j}{\text{Min}}\left[\frac{NC_j}{a_{tj}} > 0\right] = \frac{NC_s}{a_{ts}}$$

where $a_{tj} > 0$, then the pivot element is a_{ts}, and we exchange the variables x_s and ε_t, According to the direct neutrosophic Simplex algorithm instructions, see [7]. We repeat step (vi) until we get rid of all the artificial variables and obtain a normal base consisting of the basic variables.
After getting rid of the artificial variables, we return to working according to the direct neutrosophic simplex algorithm.

3. Examples

3.1 Example: All constraints are of type equals

Find the ideal solution for the following linear model:

$$Max\, Z = -12x_1 + [6,9]x_2 + 3x_3$$

Constraints:

$$8x_1 - x_2 + 4x_3 = [4,6]$$

$$6x_1 - 3x_2 + 3x_3 = [-12,-9]$$

$$x_1, x_2, x_3 \geq 0$$

Solution:

1. We convert the model to the standard form, multiply the second equation by (-1) and we obtain the following model:
   Find a rule solution for the following neutrosophic linear model:
   $$Max\, Z = -12x_1 + [6,9]x_2 + 3x_3$$
   Constraints:
   $$8x_1 - x_2 + 4x_3 = [4,6]$$
   $$-6x_1 + 3x_2 - 3x_3 = [9,12]$$
   $$x_1, x_2, x_3 \geq 0$$

2. We add the artificial variables and enter them into the objective function with a capital letter M preceded by a minus sign. Here we take $M = 15$.
   Find a rule solution for the following neutrosophic linear model:
   $$Max\, Z = -12x_1 + [6,9]x_2 + 3x_3 - 15\varepsilon_1 - 15\varepsilon_2$$
   Constraints:
   $$8x_1 - x_2 + 4x_3 = [4,6]$$
   $$-6x_1 + 3x_2 - 3x_3 = [9,12]$$
   $$x_1, x_2, x_3, \varepsilon_1, \varepsilon_2 \geq 0$$

   We arrange the previous information in Table 2.

Table 2. Artificial base.

Basic \ Variables	x_1	x_2	x_3	ε_1	ε_2	b_i
ε_1	8	-1	4	1	0	[4,6]
ε_2	-6	3	-3	0	1	[9,12]
objective function	-12	[6,9]	3	-15	-15	$Z - 0$

Since the rule is artificial, we study the constants $b_i$and find that the largest of them belong to the group [9,12] corresponding to the variable ε_2. Therefore, we divide the objective function row by the positive elements in the ε_2 row and calculate the index θ, and we find that:

$$\theta = \underset{j}{\text{Min}}\left[\frac{[6,9]}{3}\right] = \frac{[6,9]}{3}$$

Thus, the pivot element is (3) corresponding to x_2. Therefore, we replace x_2 with ε_2, then the variable x_2 becomes a base variable and ε_2 comes out of the base. We perform the necessary calculations and obtain Table 3.

Table 3. The first change table in the base.

Basic \ Variables	x_1	x_2	x_3	ε_1	ε_2	b_i
ε_1	6	0	3	1	1/3	$[\mathbf{7}, \mathbf{10}]$
x_2	-2	1	-1	0	1/3	$[\mathbf{3}, \mathbf{4}]$
objective function	[0,6]	0	[9,12]	-15	$[-18, -17]$	$\boldsymbol{Z} - [\mathbf{18}, \mathbf{36}]$

The artificial variable ε_1 is still present in the base, so we perform another substitution, adopting the pivot line as the line opposite it. To determine the pivot column, we calculate the index θ, we find:

$$\theta = \underset{j}{\text{Min}} \left[\frac{[0,6]}{6}, \frac{[9,12]}{3} \right] \in \frac{[0,6]}{6}$$

Thus, the pivot element is (6) corresponding to x_1, so we move x_1 to the base instead of ε_1, so we get the following Table 4.

Table 4. The second change in the base.

Basic \ Variables	x_1	x_2	x_3	ε_1	ε_2	b_i
x_1	1	0	$\frac{1}{2}$	$\frac{3}{6}$	$\frac{1}{18}$	$\left[\frac{7}{6}, \frac{10}{6}\right]$
x_2	0	1	0	$\frac{1}{3}$	$\frac{4}{9}$	$\left[\frac{16}{3}, \frac{22}{3}\right]$
objective function	0	0	9	$[-18, -15]$	$\left[-18, \frac{-50}{3}\right]$	$Z - [18,46]$

From the previous table, we note that the base variables x_1, x_2, and thus we have obtained an initial solution for the linear model, which gives us the following rule solution:

$$\left(x_1 \in \left[\frac{7}{6}, \frac{10}{6}\right], x_2 \in \left[\frac{16}{3}, \frac{22}{3}\right], x_3 = 0, \varepsilon_1 = 0, \varepsilon_2 = 0 \right)$$

But it is clear from the table that this solution is not the ideal solution because, in the target function line, there is a positive value corresponding to the variable x_3. Therefore, we apply the direct simplex algorithm to improve the basic solution. We obtain the ideal solution from Table 5.

Table 5. The optimal solution for the model.

Variables / Basic	x_1	x_2	x_3	ε_1	ε_2	b_i
x_3	2	0	1	1	$\frac{1}{9}$	$\left[\frac{7}{3},\frac{10}{3}\right]$
x_2	0	1	0	$\frac{1}{3}$	$\frac{4}{9}$	$\left[\frac{16}{3},\frac{22}{3}\right]$
objective function	-18	0	0	$[-27,-24]$	$\left[-19,\frac{-53}{3}\right]$	$Z-[39,76]$

The optimal solution for the linear model:

$$x_1 = 0, x_2 \in \left[\frac{16}{3},\frac{22}{3}\right], x_3 \in \left[\frac{7}{3},\frac{10}{3}\right], \varepsilon_1 = 0, \varepsilon_2 = 0$$

In this solution, the goal function takes its greatest value, which is:

$$Z \in [39,76]$$

The solution can be verified by substituting the constraints and the objective function statement, we note that the values in the ideal solution of the previous linear model are neutrosophic values.

3.2 Example: Constraints are mixed

Find the ideal solution for the following linear model:

$$Min\, Z = -3x_1 + [8,10]x_2 + [0,6]x_3$$

Constraints:

$$x_1 - 2x_2 + x_3 \leq [3,7]$$
$$-4x_1 + x_2 + 2x_3 \geq [9,6]$$
$$2x_1 - x_3 = 1$$
$$x_1, x_2, x_3 \geq 0$$

Converting this model to standard form the problem becomes:

Find the ideal solution for the following linear model:

$$Min\, Z = -3x_1 + [8,10]x_2 + [0,6]x_3 + 0y_1 + 0y_2$$

Constraints:

$$x_1 - 2x_2 + x_3 + y_1 = [3,7]$$
$$-4x_1 + x_2 + 2x_3 - y_2 = [9,6]$$
$$2x_1 - x_3 = 1$$
$$x_1, x_2, x_3, y_1, y_2 \geq 0$$

The variable y_1 in the first constraint is a basic variable, and since there are no other basic variables, we add artificial variables to the second and third restrictions and enter them into the objective function in sufficiently positive times because the model is a minimization model, and thus we obtain the following basic form:
Find the ideal solution for the following linear model:

$$Min\,Z = -3x_1 + [8,10]x_2 + [0,6]x_3 + 0y_1 + 0y_2 + 12\varepsilon_1 + 12\varepsilon_2$$

Constraints:

$$x_1 - 2x_2 + x_3 + y_1 = [3,7]$$

$$-4x_1 + x_2 + 2x_3 - y_2 + \varepsilon_1 = [9,6]$$

$$2x_1 - x_3 + \varepsilon_2 = 1$$

$$x_1, x_2, x_3, y_1, y_2, \varepsilon_1, \varepsilon_2 \geq 0$$

We follow the same steps mentioned in Example 1 to remove the artificial variables from the base and insert the basic variables. After obtaining the base solution, we use the direct simplex method to find the optimal solution.

Important Notes:

- If the row ε_i does not include a positive element and $b_t > 0$, this indicates a conflict of constraints and the problem is unsolvable.
- If we cannot find a positive ratio $\frac{NC_j}{a_{tj}}$, we calculate the largest negative ratio θ' where:

$$\theta' = Max\left[\frac{NC_j}{a_{tj}} < 0\right] = \frac{NC_s}{a_{ts}}$$

where $a_{tj} > 0$, so a_{ts} is the pivot element and it is a positive element.

4. Conclusions

In this study, we presented one of the important methods for finding the optimal solution for neutrosophic linear models, which is the synthetic simplex method that we resort to when we are unable to find a rule solution. We found that the optimal solution that we obtained is neutrosophic values, indeterminate values, perfectly defined, belonging to a field that represents its minimum. The smallest value that the objective function can take and the highest alone represent the highest value of the objective function, which is proportional to the conditions surrounding the system's operating environment, which can be represented by the linear model.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Extended Event Calculus using Neutrosophic Logic: Method, Implementation, Analysis, Recent Progress and Future Directions

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Abstract: Brains do not reason as digital computers do. Computers reason in clear steps with statements that are either true or false, while humans reason with vague terms of common sense. Neutrosophy is a new branch of philosophy and machine intelligence that deals with neutralities, specifically the idea of indeterminacy that is evident and experienced in our everyday lives. Indeterminacy is interpreted as everything that falls between a concept, idea, statement, declaration, etc. and its opposite. The fundamental thesis of neutrosophy is to employ neutrosophic logic, an extension of fuzzy logic, to incorporate fuzzy truth into complex schemes of formal reasoning. Event calculus is a logical formalism used to describe and reason about events and their consequences over time. It is considered a valuable mathematical tool in the field of artificial intelligence (AI) for depicting dynamic systems where events occur and have temporal relationships with each other. However, previous studies in AI have neglected to adequately address the complexity of time. In this context, our work aims to introduce a neutrosophic event-based calculus as a logic formalism to handle situations where there is insufficient knowledge or ambiguity regarding the occurrence or consequences of certain events in a system. In particular, neutrosophic event calculus examines causality between ideas and the connection between tasks and actions in the presence of time. Due to the lack of related studies in the existing literature, we believe that our work will contribute to the field of knowledge representation by proposing an alternative to current forms of logic. We aim to demonstrate the capacity of neutrosophic event calculus in the context of knowledge representation and reasoning.

Keywords: Neutrosophy; Neutrosophic Logic; Event Calculus; Logical Formalism; Knowledge Representation and Reasoning; Artificial Intelligence.

1. Introduction

Fuzziness originated as vagueness in the late 19th century. A concept is considered vague if its boundaries are blurred, meaning not all statements can be categorized as true or false to the same extent. Logician Bertrand Russell was the first to identify vagueness in symbolic logic [1]. Concept A is vague if it violates Aristotle's law of excluded middle, meaning A or not-A does not hold. Russell realized in his 1923 article "Vagueness" that we may need to relax Aristotle's law to handle paradoxes and account for the vagueness of factual statements. This article marks the beginning of formal fuzzy logic.

Polish logician Jan Lukasiewicz made the next major breakthrough after Russell. In the 1920s, he developed the first fuzzy or multivalued logic [2]. In a 1937 article in Philosophy of Science, quantum philosopher Max Black applied multivalued logic to lists or sets of objects and drew the first fuzzy curves [3]. These sets A are such that each object x partially belongs to A and not-A, making them properly vague or fuzzy. Kaplan and Schott [4], along with other logicians in the 1950s [5], introduced the min and max operations to define a fuzzy set algebra.

In 1965, Zadeh published his influential paper "Fuzzy Sets" [6], which introduced the term "fuzzy" to mean "vague" in technical literature for the first time. Zadeh's paper applied Lukasiewicz's logic to each object in a set to establish a complete fuzzy set algebra and extend the convex separation theorem of pattern recognition. Zadeh introduced the concept of objects in a class being seen as a continuum of grades of membership. He explained the grade of membership function, including its union and intersection operations. When the nodes and edges of a linked graphical or network system are unclear, fuzzy graphs (FG) can provide intuition. Within this framework, the determination of vertex degree and membership values is always necessary to determine the strengths of vertices in a FG. The Randic index can be used to identify the most significant vertices and the most loaded pathways [7]. Bipolar FGs, which represent two opposite ways of thinking, such as forward and backward, effect and side effect, cooperation and competition, gain and loss etc, can be used to provide qualitative solutions in decision-making problems in real life. The article of [8] introduces the concept of a bipolar fuzzy incidence graph (BFIG) and its matrix representation and it also discusses the characteristics of a bipolar fuzzy incidence subgraph. Researchers in [9] have applied the concept of competitive graphs (CG) using ϕ-tolerances (TCG) in a picture fuzzy (PF) environment which is not well studied in the literature. PF-TCG models are more successful than other models in solving specific scheduling and resource allocation problems in operations research. Three special types of picture fuzzy ϕ-tolerance CGs are introduced and applied to two real-life applications in railway network and medical science, using ϕ as max, min, and sum functions.

In the mid-90s, Smarandache began utilizing non-standard analysis with a tri-component logic/set/probability theory, starting from a philosophical exploration of multi-valued logics. As a result, he developed neutrosophic logic, as fuzzy logic alone is believed to be unable to demonstrate indeterminacy [10]. According to the definition provided in [10], "Neutrosophic logic is a logic variety that generalizes fuzzy logic, paraconsistent logic, intuitionistic logic, and other logic variants. The degree of membership (T) of each set element is the first part of neutrosophic logic, indeterminacy (I) is the middle part, and falsehood (F) is the third part, respectively."

Neutrosophic logic is significant and has been applied in various research areas in recent years. Within our research framework, particularly in the field of knowledge representation, scholars in [11] have explored how neutrosophic logic can be integrated into situation analysis to propose a framework that addresses the multiple aspects of uncertainty and information inherent in the situation analysis environment, effectively dealing with the ontological and epistemological challenges of situation analysis. Additionally, the work in [12] deserves mention as the first study to introduce neutrosophic modal logic in the related literature. Neutrosophic modal logic is a formal logic that incorporates neutrosophic modalities and is governed by a set of neutrosophic axioms and rules.

It is evident from our understanding that the natural world is constantly evolving or changing. Therefore, processes, whether natural or technical, are dynamic, and abstract concepts must embrace change to be useful. Consequently, the concept of evolving representations over time is crucial. The notion of time and its explanation within the limits of our perception has been a concern for humanity since ancient times. The study of temporal logic originated with Aristotle and the Megarian and Stoic schools in ancient Greece. It is worth noting that as early as 350 B.C., Aristotle argued that actions are justified by a logical connection between goals and knowledge of the outcome of the action. In the modern era, Findley [13] was the first to propose a standardized calculus for reasoning based on time, but the most significant impact is attributed to the seminal work of Arthur Prior published in 1967 [14].

Event calculus is a logical formalism used to reason about dynamic systems and events in the fields of AI and philosophy. Kowalski and Sergot introduced the event calculus as a logic programming paradigm for modeling events and their consequences, particularly in database systems [15]. Shanahan [16] proposed further improvements based on first-order predicate calculus, which can describe a wide range of phenomena, including acts with indirect consequences, activities with non-deterministic effects, compound actions, concurrent actions, and continuous change.

1.1 Motivation

- The main consideration that we had in mind concerning the motivation of our study was to propose a proper formalism that would integrate and manipulate the concepts of uncertainty and incomplete information in the context of event calculus. Event calculus's constraints, in terms of uncertainty, stem from its fundamental character as a deterministic logic-based formalism. Traditional event calculus lacks explicit techniques for dealing with uncertainty or partial or probabilistic information. This is achieved by proposing a hybrid logical framework that incorporates neutrosophic logic with event calculus.
- Another concern that led us to our current research work is that time representation and comprehension in AI have been identified as areas that require further study. While there are several temporal formalisms, they frequently simplify time in ways that do not convey its entire complexity in real-world circumstances. In everyday life, time is sometimes uncertain, and events may have fuzzy or probabilistic temporal features. Furthermore, the dynamics of systems fluctuate over time, and describing such changing temporal dynamics is difficult. Within this concern, developing formalisms that explicitly account for uncertain or probabilistic temporal information, enabling AI systems to reason effectively in the face of incomplete or uncertain temporal knowledge, is considered crucial.
- At this point, one could possibly ask why we should use event calculus instead of, for example, situation calculus [17], which is also a prominent logic-based formalism that deals with similar situations. Our answer comes with the observation that when there is a single agent doing instantaneous and discrete actions, situation analysis formalism works effectively. But in cases when actions have durations and can overlap, situation analysis becomes fairly cumbersome. As a result, we address these concerns using an alternative formulation based on event calculus, which could be considered a time-based formalism rather than a state-based one. Furthermore, it allows reasoning in terms of time intervals instead of states, which is a more realistic approach when dealing with real-world problems because being able to handle temporal aspects and causal relationships makes it useful in modeling and reasoning about dynamic environments.
- Lastly, we should remark that, in the framework of classical event calculus, formulating inference rules can be complex and cumbersome, especially for domains with a large number of interconnected occurrences. Developing rules that correctly capture all conceivable time links can be difficult. In response to this, we chose to integrate in our model a simpler representation of inference rules in the form of IF…THEN… that approaches human intuition and allows the modification or updating of rules as new information becomes available, allowing systems to react to changing situations or uncertainty in real-time.

1.2 Novelties

In this paper, we suggest the Neutrosophic Event Calculus (NEC), which is a temporal reasoning framework that integrates neutrosophic logic to manage uncertainty, indeterminacy, and inadequate knowledge. The main characteristics of the NEC framework include the following:

- Inference rules: Our model enables the user to generate inference rules, aiding decision-makers in understanding and resolving difficulties by facilitating efficient problem-solving in complex and unpredictable contexts.

- Handling Uncertainty: In the presence of uncertainty, NEC provides a depiction of events and temporal connections. It is appropriate for circumstances in which the precise result or temporal relationships between events are unknown.
- Indeterminate Events and Consequences: NEC can describe events and their consequences when the truth value or result is unknown, expressing scenarios in which the repercussions of events are uncertain or not completely known.
- Inadequate Knowledge Expressiveness: It provides a framework for reasoning about events and their temporal linkages, even when knowledge is inadequate or the truth value of assertions is uncertain.

1.3 Contributions

The main contributions of the current manuscript can be summarized as follows:

- To the best of our knowledge, this is the first study in the related literature that integrates neutrosophic logic with event calculus. In this way, we seek to suggest a new formalism that will operate as a solid theoretical basis in the field of knowledge representation, especially in the way that a logical agent should make decisions or act in response to the effects of actions.
- In the field of neutrosophic logic, we introduce for the first time the neutrosophic event calculus to enrich the related mathematical toolbox, considering the advances of neutrosophic theories and their applications first discussed in [18]. We consider that our study will aid towards direction of the intersection of computer science and neutrosophic calculus/logic.
- In this light, we hope to spark research interest in the academic community, aiming at their need to comprehend not only logic-based formalisms in the process of designing complex computer programs based on sound engineering principles but also defining a mathematical framework for examining, on a logical basis, research problems in various fields.

1.4 Structure of the paper

The remainder of the article is as follows: The current article was written with the intention of being as self-contained as possible. As a result, section 2 summarizes the fundamental concepts and ideas required to understand the basic concepts of neutrosophic logic and event calculus in order to build our theory and propose our logic formalism, namely neutrosophic event calculus (NEC). In section 3, we present an illustrative example to examine NEC's applicability and expressiveness in a real-world situation. Next, in section 4, we discuss why and where our formalism could find fertile research ground, and in the last section, we highlight, from a scientific perspective, NEC's usefulness and importance, which could pave the way for academics and practitioners. Lastly, we propose future research work in which NEC could play a pivotal role in the context of knowledge representation and reasoning.

2. Materials and Methods

In this section we firstly present the basic concepts and definitions of neutrosophic logic and event calculus that will provide the necessary knowledge needed so as to describe our proposed formalism, namely neutrosophic event calculus (NEC). For a deeper investigation on neutrosophic logic the interested reader is referred to the works of [19-20] and for a detailed review on event calculus and its extensions we propose the works of [15-16].

As a first step, and beginning with the consideration of what kind of reasoning we will adopt in our paper, we made a decision to extend first order predicate logic and specifically neutrosophic predicate logic by providing appropriate predicates and functions for describing the type of action-related information we're interested in, as well as offering a set of axioms restricting the set of models we desire.

2.1 Neutrosophic logic

Neutrosophic logic, which is presented as a general framework for logical approaches, is a branch of classical and fuzzy logic that deals with the idea of indeterminacy, allowing for the representation of three different types of components: truth, falsity, and indeterminacy. More specifically, truth component (T) represents the degree to which a statement or proposition is true. Falsehood component (F) denotes the degree to which a statement or proposition is false and indeterminacy component (I) represents the degree of indeterminacy, uncertainty, or incompleteness associated with a statement or proposition.

In this framework, a formula φ is characterized by a triplet of truth-values, called the *neutrosophical value* defined as [11]:

$$NL(\varphi) = (T(\varphi), I(\varphi), F(\varphi)) \quad (1)$$

where $(T(\varphi), I(\varphi), F(\varphi)) \subset ||-0, 1+||3$, $||-0, 1+||$ being an interval of hyperreals.

2.1.1 Neutrosophic predicate logic

Neutrosophic predicate is a generalization of neutrosophic propositional logic and of classical predicate logic. As a neutrosophic formal syntax, neutrosophical predicate logic addresses *neutrosophic predicates*, *neutrosophic variables*, and *neutrosophic quantifiers*, which are predicates, variables and quantifiers respectively that deal with indeterminacy [18]. In other words, instead of the classical binary true or false values, neutrosophic predicates allow for truth-membership, false-membership, and indeterminacy-membership degrees.

Let us consider the following simple example:

$P(\Theta)$ = "Θ is a logician academic", where Θ is a human being. The neutrosophic truth-value of $P(\Theta)$ is (T, I, F) where T, I, F are subsets of the interval [0, 1]. Then we say that the predicate "is a logician academic" takes one variable, namely "Θ".

2.2 Event calculus

In this subsection and throughout our paper we will use the basic event calculus version which has all the characteristics of a full version and is considered efficacious for the scope of our current work. Different formulations of event calculus have been proposed in the literature that are suitably established to cope with specific research problems such as continuous change and mathematical modelling [21], with ramifications [22], with representing agent beliefs [23] and to deal with programming constructs and compound events [24].

The event calculus is a logical system that deduces what is true given what happens when and what actions take place. The "what happens when" section provides a timeline of events, while the "what actions do" segment outlines the outcomes of acts [25]. The basic ontology of the event calculus includes *actions or events*[1], *fluents* and *time points*. A fluent is anything whose value fluctuates with time.

In the event calculus, fluents apply to points in time, rather than states, and the calculus is designed to allow reasoning in terms of time intervals. The event calculus axiom says that a fluent is true at a point in time if the fluent was initiated by an event at some past time and not terminated by an intervening event. Table 1 depicts the basic predicates used in the simple event calculus.

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[1] we use the terms action and event interchangeably.

Table 1. Event calculus predicates [15].

Formula	Meaning
Initiates (a, b, t)	Fluent b starts to hold after action a at time t
Terminates(a, b, t)	Fluent b ceases to hold after action a at time t
Initially$_P$(b)	Fluent b holds from time 0
$t_1 < t_2$	Time point t_1 is before time point t_2
Happens(a, t)	Action a occurs at time t
HoldsAt(b, t)	Fluent b holds at time t
Clipped(t_1,b,t_2)	Fluent b is terminated between times t_1 and t_2

Fluents are reified in the event calculus, as is apparent from Table 1. That is, fluents are first-class objects that may be quantified and appear as parameters to predicate statements.

Typically the axiom of the event calculus consists of the following:

$$T(f, t_2) \Leftrightarrow \exists\ e, t\ Happens(e, t) \wedge Initiates(e, f, t) \wedge (t < t_2) \wedge Clipped(t, f, t_2)$$

$$Clipped(t, f, t_2) \Leftrightarrow \exists\ e, t_1\ Happens(e, t_1) \wedge Terminates(e, f, t_1) \wedge (t<t_1) \wedge (t_1<t_2)$$

The above axiom gives us functionality similar to that of calculus of states but with the ability to talk about time points and intervals. In this manner we can say for example *Clipped(10:00, TurnOff(TV),11:00)* so as to indicate that the TV appliance switched off at some time between 10:00 and 11:00.

It is worth noting that, according to the above axiom, a fluent does not hold during the event that originates it but does hold during the event that ends it. In other words, fluents retain open intervals on the left and closed intervals on the right.

2.3 Neutrosophic event calculus

In this subsection we introduce for the first time in the literature the Neutrosophic Event Calculus (NEC) as an extension of the Event Calculus, which integrates neutrosophic logic into its logical framework. In this context, NEC enables the modelling and reasoning of systems containing aspects of uncertainty, imprecision, or missing knowledge. In order to achieve the latter, it expands standard event calculus by including neutrosophic features, allowing for the representation and manipulation of ambiguous, uncertain, or conflicting information inside the logical framework used for thinking about events and their consequences throughout time.

Like in classical event calculus, our ontology includes actions or events, fluents and time points which are considered the basic concepts of our framework but this time they are enriched and applied in neutrosophic logic based environment. This means that they are allowed to be T% true, F% false, and I% indeterminate. This leads us to adopt the notation presented in [16] which indicates that in a neutrosophic model each neutrosophic proposition $\mathcal{P}$ has a neutrosophic truth-value (T_{wn}, I_{wn}, F_{wn}) respectively to each neutrosophic world $w_N \in G_N$, where $T_{wn,}\ I_{wn}\ F_{wn}$ are subsets of [0, 1] and G_N is a neutrosophic frame which is a non-empty neutrosophic set, whose elements are called possible neutrosophic worlds. In order to capture the aforesaid information and based on the definition of the neutrosophic formula given with Eq. (1), we add a parameter, namely *neutrosophic degree (nd),* in the predicates so as to define its neutrosophical value. Parameter *neutrosophic degree,* which is based on expert's knowledge, expertise and available or historical data, could be expressed as a neutrosophic numerical value or as a linguistic variable or even phrase to ease human intuition. For example, we could assign the term "*very low*" with a neutrosophic degree such as *(0.1, 0.8, 0.9),* thus indicating that the examined concept has 10% chance to occur, 90% chance not to occur and 80% indeterminate chance to happen. Furthermore, we could have replaced the argument *nd* with an expression such as "possibly 30 units "so as to refer to the possible number of units that we could place for a specific order. So it is obvious that this degree reflects the level of ambiguity or lack of clarity about the truth value of the statement or event in question.

Another key feature that is introduced in our model, as opposed to classical event calculus, is that we can generate several inference rules based on the predicates according to the problem domain in a much simpler form aiming to offer a significantly more flexible version. This is achieved by using the form of IF…THEN… to derive new facts or conclusions based on existing facts and rules in the knowledge base. This feature is really useful as it enables us to insights into potential relationships between events or states, thus aiding in decision-making by capturing possible cause-and-effect relationships amidst uncertainties. In this manner, it also provides us with a significant advantage over the notation adopted in the classical event calculus. It enables us to reason in a high level language which is remarkably akin to human language that is easily understood by non-experts in the field.

The selection of basic predicates goes together in hand with the selection of ontology. Table 2 shows the predicates used in NEC.

Table 2. Neutrosophic event calculus predicates.

Formula	Meaning
$\boldsymbol{Initiates_N}$**(a, b, t, nd)**	Fluent *b neutrosophically* starts to hold after action *a* at time *t*
$\boldsymbol{Terminates_N}$**(a, b, t, nd)**	Fluent *b neutrosophically* ceases to hold after action *a* at time *t*
$\boldsymbol{InitiallyP_N}$**(b, nd)**	Fluent *b neutrosophically* holds from time 0
$\mathbf{t_1 < t_2}$	Time point t_1 is before time point t_2
$\boldsymbol{Happens_N}$**(a, t, nd)**	Action a *neutrosophically* occurs at time *t*
$\boldsymbol{HoldsAt_N}$**(b, t, nd)**	Fluent *b neutrosophically* holds at time *t*
$\boldsymbol{Clipped_N}$**($\mathbf{t_1}$,b,$\mathbf{t_2}$, nd)**	Fluent *b* will *neutrosophically* be terminated between times t_1 and t_2

Within our context the above predicates have the following meaning:

- *Initiates*: interpreting this statement requires admitting that once action *α* happens, there is a transition or commencement of fluent *b*'s truth value from a possibly false or indeterminate state to a state where it begins to hold or becomes true. Because of the statement's neutrosophic character, the degree of certainty or truthfulness about this transition may fluctuate, incorporating degrees of truth, falsity, and indeterminacy at the same time.
- *Terminates:* this relationship might encompass various degrees of termination, acknowledging uncertainty or indeterminacy regarding the exact impact or timing of action 'a' on the termination of fluent 'b'. The neutrosophic truth values associated with this predicate would capture the degrees of truth, falsehood, and indeterminacy regarding the termination of fluent *b* by action *a* at time *t*.
- *Initially:* the truth value associated with *b* holding at time 0 could encompass elements of truth, falsehood, and indeterminacy simultaneously. The statement indicates that, at the onset (time 0), the truth status of *b* is considered, taking into account any uncertainties or degrees of indeterminacy associated with its truth value.
- *Happens:* while the statement indicates the occurrence of action 'a' at time 't' in classical event calculus, the neutrosophic interpretation accounts for the uncertainty or imprecision surrounding the actual occurrence of the action at that specific time, allowing for varying degrees of truth or falsehood associated with this event-fluent relation.
- *HoldsAt:* the statement fluent *b* holds at time *t* signifies the status or truth value of fluent *b* at a specific time point denoted by *t.* Therefore, it accommodates the inherent uncertainty or indeterminacy, allowing for different degrees of truth, falsehood, or indeterminacy associated with the fluent's state at that moment in the temporal domain.

- *Clipped:* in a neutrosophic context, it allows for degrees of truth, falsehood, and indeterminacy regarding the certainty of this termination within that specific time range, i.e. between time t_1 and t_2. It acknowledges that it might not be definitively true or false; instead, it could have varying levels of certainty or uncertainty regarding the fluent's termination within the specified time interval.

Following the above terminology we can now re formulate the simple example that we examined in subsection 2.2. This would showcase the core idea behind neutrosophic event calculus and how this could be applied in more complex problems as shown in the next section. We would like to indicate that a TV appliance *might* be switched off at some time between 10:00 and 11:00 with a neutrosophic degree (0.8, 0.2, 0.1), i.e. 80% chance that the TV appliance will switch off, 10% chance not to switch off and 20% indeterminate chance to happen. In this context this could be expressed as: $Clipped_N$(10.00, *TurnOff(TV)*,11.00, (0.8, 0.2, 0.1))

From the above it can be concluded that when combined with neutrosophic truth values, these predicates enable a more nuanced representation of uncertainty, imprecision, and indeterminacy within the neutrosophic event calculus, allowing for more flexible and realistic modelling of dynamic systems where complete information may not be available or certain. In the next section we will showcase the robustness of our method by examining an illustrative example.

3. Implementation of Neutrosophic Event Calculus

In order to study the effectiveness and usefulness of our proposed formalism (NEC), let us examine the following example taken from the logistics/supply chain domain which is based in a real world scenario. For the sake of brevity we will restrict the solution of the given example to the inventory and supply chain management levels. However, our approach will be efficiently being demonstrated

Example 1. Due to unknown circumstances like as weather, transportation delays, and various customs processes across nations, a global corporation confronts difficulty in properly anticipating delivery schedules. This ambiguity has an impact on inventory management, production planning, and customer satisfaction.

Step 1. Let us first explain and list the NEC's predicates used in the above example.

- **$HoldsAt_N$(InventoryLevel, t, neutrosophic_ degree):** Represents the uncertain inventory level of a product at a specific time.
- **$Initiates_N$(OrderPlacement, Product_X, t, neutrosophic_ degree):** Indicates the initiation of an uncertain order for a specific quantity of a product at a specific time.
- **$Happens_N$(WeatherImpact, t, neutrosophic_ degree):** Represents the potential impact of weather on transportation at a specific time.
- **$HoldsAt_N$(CustomsProcessDelay, t, neutrosophic_ degree):** Indicates the potential delay in customs processes at a specific time.
- **$Terminates_N$ (DelayedDelivery, Product_X, t, neutrosophic_degree):** Represents the uncertain termination or delay in delivery of a certain quantity of a product at a specific time.
- **$Clipped_N$(t_1, DelayedDeliver, t_2, neutrosophic_ degree):** Refers to delay in delivery of a certain quantity of a product clipped between minimum and maximum time values.

We should note the key role of the argument *neutrosophic_degree* in our model which allows the representation of uncertainty, permitting to reason about prospective outcomes or occurrences without the need for precise, deterministic values.

Step 2. We proceed by giving possible numerical values or linguistic expressions where appropriate to the above predicates based on expert(s) judgement(s).

$HoldsAt_N$(InventoryLevel, ProductX, 11:00 a.m., possibly 100 units).

$Initiates_N$(OrderPlacement, 11:00 a.m., ProductX, possibly 70 units).
$Happens_N$(WeatherImpact, 11:00 a.m., possibly moderate).
$HoldsAt_N$(CustomsProcessDelay, 11:00 a.m., possibly 3 days).
$Terminates_N$(DelayedDelivery, ProductX, 11:00 am, possibly 25 units).
$Clipped_N$(1 day, DelayedDeliver, 4 days, possibly 25 units).

Step 3. Now we are ready to write the inference rules that best accommodate our example in the inventory and supply chain management levels.

1. Inventory Level:
   - **IF** $HoldsAt_N$ (InventoryLevel, ProductX, 11:00 a.m., possibly 100 units) **THEN** $Initiates_N$(OrderPlacement, 11:00 a.m., ProductX, possibly 70 units).
   - **IF** $Happens_N$ (WeatherImpact, 11:00 a.m., possibly moderate) **THEN** $HoldsAt_N$(CustomsProcessDelay, 11:00 a.m., possibly 3 days).
   - **IF** $Terminates_N$ (DelayedDelivery, ProductX, 11:00 a.m., possibly 25 units) **THEN** $Clipped_N$(1 day, DelayedDelivery, 4 days, possibly 25 units).

According to the first rule, a given inventory level signals the prospective beginning of an order placement, showing that observed stock levels impact the choice to initiate an order. The second rule provides a probable relationship between moderate weather impacts and predicted delays in customs processes, meaning that moderate weather may cause customs delays. The third rule leverages the 'Clipped' predicate to guarantee that the inferred delivery delay remains within a reasonable range (between 1 and 4 days), while acknowledging the uncertainty indicated by the possibility of terminating delayed delivery for Product X.

2. Supply chain management Level:
   - **IF** $HoldsAt_N$ (InventoryLevel, ProductX, 11:00 a.m., possibly *100* units) **THEN** $Initiates_N$(OrderPlacement, 11:00 a.m., ProductX, possibly *60* units)// if there is a probability of InventoryLevel being high then initiate OrderPlacement conservatively.
   - **IF** $Happens_N$ (WeatherImpact, 11:00 a.m., (0.8, 0.3, 0.2)) **THEN** $HoldsAt_N$(CustomsProcessDelay, 11:00 a.m., possibly *4* days) // If weather conditions show a likelihood of impact then anticipate CustomsProcessDelay prudently.
   - **IF** $Terminates_N$(DelayedDelivery, ProductX, 11:00 a.m., possibly 70 units) **THEN**

$Clipped_N$ (1 day, DelayedDelivery, 4 days, possibly 30 units)// if there is a chance of DelayedDelivery then prepare for potential clipping.

The usefulness of the NEC formalism in the context of the given real world case study could be summarized as follows:

- Inventory Management: It aids in order placement decisions based on unknown inventory levels, guaranteeing appropriate supply without relying on accurate information.
- Supply Chain Management: Assists in simulating the influence of unknown occurrences such as weather or customs delays on supply schedules, allowing for proactive management and planning.
- Clipped Predicate Utility: The usage of predicates such as 'Clipped' enables the definition of realistic limitations for inferred delays, ensuring they remain within practical and reasonable ranges.

4. Discussion

It is the first time in related literature that an extended formalism like ours has been proposed. The aim of establishing a new logical approach, known as NEC, stems from the need to propose a suitable formalism that can effectively represent relationships between events, fluents, and their properties, considering the inherent indeterminacy encountered in real-world problems. This indeterminacy is addressed using neutrosophic logic.

NEC provides a versatile means of representing events, their initiation, termination, and attributes, while accounting for the uncertainty inherent in complex systems. This adaptability enables more accurate modeling of real-world settings with unpredictable and imprecise occurrences. In this way, NEC's ability to accommodate and reason about uncertain information is particularly valuable in scenarios where traditional logic-based approaches struggle to provide accurate representations.

We believe that our suggested approach could act as a useful mathematical toolbox when dealing with the following real situations:

- Modelling uncertainty: It is advantageous when capturing and reasoning about systems where uncertainty exists. Many real-world circumstances contain partial or unclear knowledge, which NEC enables for a more realistic description of similar situations.
- Decision-making under uncertainty: It is valuable for decision-making processes when information is inadequate or uncertain. It aids in making informed judgments even in uncertain contexts by concurrently recording degrees of truth, untruth, and indeterminacy.
- Dynamic system analysis: It analyzes and predicts the behavior of dynamic systems that are influenced by events and changes over time. This is especially important in industries such as engineering, finance, logistics, and artificial intelligence, where understanding dynamic relationships is critical.
- Risk assessment and management: It aids in the assessment and management of risks in systems with a high degree of uncertainty. It can provide a more thorough risk assessment by taking into account varying degrees of truth and falsity.
- Artificial intelligence and Robotics: It can be used in AI and robotics to represent settings with noisy or unclear sensor input, allowing these systems to make more nuanced judgments.

5. Conclusions

Neutrosophic Event Calculus offers a logic-based framework that extends traditional event-based reasoning to include indeterminacy, uncertainty, and imprecision. This research study has demonstrated the promise of this approach in various research disciplines. We suggest that this integrated approach could offer a more realistic portrayal of dynamic systems, where events, fluents, and their interactions are susceptible to different degrees of truth, untruth, and indeterminacy. Its capacity to deal with inadequate or ambiguous data offers possibilities for more complex and accurate modeling of real-world systems.

Future research objectives for expanding the NEC include refining neutrosophic logic to strengthen the basis of NEC, particularly in dealing with degrees of truth, falsehood, and indeterminacy, as well as reasoning from effects to causes [26, 27]. To ensure the formalism's soundness and completeness, it is necessary to examine examples, counterexamples, and logical arguments to establish its sufficiency and consistency. Additionally, validation through real-world case studies is required to demonstrate its practical application. Furthermore, the combination of NEC with other computational models, such as fuzzy logic, probabilistic methodology, or machine learning techniques, may result in more adaptive and robust modeling approaches, thereby extending its applicability across other domains.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Neutrosophic Statistical Analysis of Temperatures of Cities in the Southeastern Anatolia Region of Turkey

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Abstract: In the paper, neutrosophic statistical analysis of temperature data of different cities in the southeastern Anatolia region of Turkey is given. The neutrosophic mean and neutrosophic coefficient of variation are computed using the temperature data. From the analysis, it is concluded that the temperatures of Mardin and Şanlıurfa cities are more consistent than the other cities in Turkey. In addition, the neutrosophic results are compared with results under classical statistics. Based on the comparative study, it can be concluded that neutrosophic statistical results are more adequate, flexible, and more informative than classical statistics.

Keywords: Neutrosophic Sets; Neutrosophic Statistics; Temperature; Indeterminacy.

1. Introduction

The geographical features of the world have changed several times in the process until the emergence of people at this stage of history. In certain periods, depending on the deterioration of the natural balance between the elements of our world due to various reasons, there have been great changes in the climate. As a matter of fact, in the period from the beginning of human history to the present, the natural and human environment that lived in the glacial and interglacial periods, when the earth was covered with glaciers, was greatly affected. Certainly, human influences have also contributed to these changes, which are related to natural factors, since the middle of the 19th century.

Today, it is accepted by almost all climate scientists that there is a deterioration in the world climate system. It is clearly stated that if various activities of the people causing the deterioration of the natural balance continue without taking the necessary precautions, these deteriorations in the climate will increase and there will be climatic changes due to global warming, the result of which may be very negative. Because, due to human reasons, the increase in greenhouse gas accumulations and particles in the atmosphere, the destruction of the natural environment, and the depletion of the ozone layer, will cause a global temperature increase.

Turkey is one of the countries that will be most affected by possible climate change within its complex climate structure, especially due to global warming. It is naturally surrounded by seas on three sides and has a faulty topography. Different regions of Turkey will be affected by climate change differently and to varying extents due to its orographic characteristics. For example, arid and semi-arid regions such as Southeast and Central Anatolia, which are under the threat of desertification rather than temperature increase, and semi-humid Aegean and Mediterranean regions that do not have sufficient water will be more affected. The climatic changes that will occur will cause changes in the natural habitats of animals and plants in agricultural activities, and important problems will arise in terms of water resources, especially in our regions mentioned above.

In recent years, many heat strokes have been recorded, causing many problems in the environment. Animals die because of water due to environmental change. Statistical methods are widely applied for estimation and estimation of temperature. Several researchers have also studied different aspects of temperature. Öztürk [1] examined global climate change and its possible effects on Turkey. Kaygusuz [2] studied the energy policy and climate change in Turkey. Afzal et al. [3] presented the analysis of resistance depending on the temperature variance of conducting material under the neutrosophic statistical analysis. Further, Janjua et al. [4] worked on the climate variability and wheat crop under a neutrosophic environment and Shahzadi [5] presented a neutrosophic statistical analysis of temperature data from five different cities in Pakistan. Various studies on this concept with applications can be seen in [6, 7].

Inspired by the reasons mentioned above, and the analysis presented by [5], the main focus of the paper was to neutrosophic statistical analysis of temperature data of different cities. This study is the first in this concept for the southeastern Anatolia region of Turkey. It is expected that neutrosophic statistical results are more adequate and informative than classical statistics.

1.1 Organization of this paper

This paper is organized as follows: a literature review and some definitions and notations are given in the next section. Section 3, collected temperature data from different cities in the southeastern Anatolia region of Turkey like Adıyaman, Batman, Diyarbakır, Gaziantep, Kilis, Mardin, Siirt, Şanlıurfa, Şırnak, and the data is reported in Table 1 which presents low and high values of the temperature data. We performed the neutrosophic statistical analysis using the temperature data and calculated the neutrosophic mean of temperature, the neutrosophic standard deviation, and the neutrosophic coefficient variation in Section 4. Section 5 contained a comparative discussion about neutrosophic statistical analysis and classical statistical analysis. At last, a conclusion of this work was given in Section 6.

2. Methodology

Let X_{iN} are the neutrosophic numbers having X_{iL} lower values and X_{iU} higher values, so the neutrosophic formula for the ith interval:

$$X_{iN} = X_{iL} + X_{iU} I_N \quad (i = 1,2,3, \dots, n_N)$$

Here $I_N \in [I_L, I_U]$ and $X_N \in [X_L, X_U]$ is a random neutrosophic variable having a size $n_N \in [n_L, n_U]$. The variable $X_{iN} \in [X_{iL}, X_{iU}]$ has two parts: lower value X_{iL} a classical part, and upper-value $X_{iU} I_N$ an indeterminate part having an indeterminacy interval $I_N \in [I_L, I_U]$.

Similarly, Chen et al. [8, 9] and Aslam [10] the neutrosophic average of temperature data $\overline{X}_N \in [\overline{X}_L, \overline{X}_U]$ can be calculated as

$$\overline{X}_N = \overline{X}_L + \overline{X}_U I_N \ ; \ I_N \in [I_L, I_U]$$

where

$$\overline{X}_U = \frac{1}{n_L} \sum_{i=1}^{n_L} X_{iL} \ ,$$
$$\overline{X}_L = \frac{1}{n_U} \sum_{i=1}^{n_U} X_{iU}.$$

NNs and neutrosophic statistics are first proposed by Smarandache [11-13]. However, it is difficult to use Smarandache's neutrosophic statistics for engineering applications. Thus, Ye et al. [14] presented some new operations of NNs to make them suitable for engineering applications.

Let NNs be $z_1 = a_1 + b_1 I$ and $z_2 = a_2 + b_2 I$ for $I_N \in [I_L, I_U]$. Then, Ye et al. [14] proposed their basic operations:

$$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)I = [a_1 + a_2 + b_1 I_L + b_2 I_L, a_1 + a_2 + b_1 I_U + b_2 I_U];$$

$$z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)I = [a_1 - a_2 + b_1 I_L - b_2 I_L, a_1 - a_2 + b_1 I_U - b_2 I_U]; \quad (1)$$

$$z_1 \times z_2 = a_1 a_2 + (a_1 b_2 + a_2 b_1)I + (b_1 b_2)I^2$$

$$= \begin{bmatrix} min\begin{pmatrix} (a_1 + b_1 I_L)(a_2 + b_2 I_L), (a_i + b_i I_L)(a_2 + b_2 I_U) \\ (a_1 + b_1 I_U)(a_2 + b_2 I_L), (a_i + b_i I_U)(a_2 + b_2 I_U) \end{pmatrix} \\ max\begin{pmatrix} (a_1 + b_1 I_L)(a_2 + b_2 I_L), (a_i + b_i I_L)(a_2 + b_2 I_U) \\ (a_1 + b_1 I_U)(a_2 + b_2 I_L), (a_i + b_i I_U)(a_2 + b_2 I_U) \end{pmatrix} \end{bmatrix}$$

$$\frac{z_1}{z_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{[a_1 + b_1 I_L, a_1 + b_1 I_U\ ]}{[a_2 + b_2 I_L, a_2 + b_2 I_U]}$$

$$= \begin{bmatrix} min\left(\frac{a_1+b_1 I_L}{a_2+b_2 I_U}, \frac{a_1+b_1 I_L}{a_2+b_2 I_L}, \frac{a_1+b_1 I_U}{a_2+b_2 I_U}, \frac{a_1+b_1 I_U}{a_2+b_2 I_L}\right) \\ max\left(\frac{a_1+b_1 I_L}{a_2+b_2 I_U}, \frac{a_1+b_1 I_L}{a_2+b_2 I_L}, \frac{a_1+b_1 I_U}{a_2+b_2 I_U}, \frac{a_1+b_1 I_U}{a_2+b_2 I_L}\right) \end{bmatrix}.$$

Then, these basic operations are different from the ones introduced in [12] and this makes them suitable for engineering applications. Based on Eq. (1), we can give the neutrosophic statistical algorithm of the neutrosophic average value and standard deviation of NNs.
Let $z_i = a_i + b_i I$ (i = 1,2,…, n) be a group of NNs (neutrosophic numbers) for $I_N \in [I_L, I_U]$ Then their neutrosophic average value and standard deviation can be calculated by the following neutrosophic statistical algorithm:

Step 1. Calculate the neutrosophic average value of a_i (i = 1,2,…, n):

$$\overline{a} = \frac{1}{n}\sum_{i=1}^{n} a_i \quad (2)$$

Step 2. Calculate the neutrosophic average value of b_i (i = 1,2,…, n):

$$\overline{b} = \frac{1}{n}\sum_{i=1}^{n} b_i \quad (3)$$

Step 3. Obtain the neutrosophic average value:

$$\overline{z}_N = \overline{a} + \overline{b} I_N \ ; \ I_N \in [I_L, I_U] \quad (4)$$

Step 4. Get the differences between z_i (i = 1,2,…, n) and $\overline{z}$:

$$z_i - \overline{z} = a_i - \overline{a} + (b_i - \overline{b})I_N \ , \ I_N \in [I_L, I_U]$$

Step 5. Calculate the square of all the differences between z_i (i = 1,2,…, n) and $\overline{z}$:

$$(z_i - \overline{z})^2 = \begin{bmatrix} min\begin{pmatrix} (a_i + b_i I_L)(\overline{a} + \overline{b} I_L), (a_i + b_i I_L)(\overline{a} + \overline{b} I_U) \\ (a_i + b_i I_U)(\overline{a} + \overline{b} I_L), (a_i + b_i I_U)(\overline{a} + \overline{b} I_U) \end{pmatrix} \\ max\begin{pmatrix} (a_i + b_i I_L)(\overline{a} + \overline{b} I_L), (a_i + b_i I_L)(\overline{a} + \overline{b} I_U) \\ (a_i + b_i I_U)(\overline{a} + \overline{b} I_L), (a_i + b_i I_U)(\overline{a} + \overline{b} I_U) \end{pmatrix} \end{bmatrix}, \ I_N \in [I_L, I_U]$$

Step 6. Calculate the neutrosophic standard deviation:

$$\sigma_z = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(z_i - \overline{z})^2} \ [8].$$

The neutrosophic variance can be computed by;

$$\sigma_z^2 = \frac{1}{n}\sum_{i=1}^{n}(z_i - \overline{z})^2 \ , \ \text{i} = 1,2,\ldots,\text{n}$$

where $\sigma_z^2 \in [\sigma_{zL}^2, \sigma_{zU}^2]$. The neutrosophic form of $\sigma_z^2 \in [\sigma_{zL}^2, \sigma_{zU}^2]$ can be written as

$$a_S + b_S I_{NS} \ ; \ I_{NS} \in [I_{LS}, I_{US}].$$

The neutrosophic coefficient of variation (CV_N) can be applied to see the consistency of the temperature in the different cities of Turkey. A city having a smaller value of CV_N means more consistent than the other city in temperature. The CV_N can be computed by;

$$CV_N = \frac{\sigma_z}{\bar{X}_N} x100 \ ; \ CV_N \in [CV_L, CV_U].$$

The neutrosophic form of CV_N is

$$a_\vartheta + b_\vartheta I_{N\vartheta} \ ; \ I_{N\vartheta} \in [I_{L\vartheta}, I_{U\vartheta}] \ [10].$$

Note that, $z_i = X_N$, $a_i = X_L$ and $b_i = X_U$. We will use the symbols a_i and b_i to present the lower and upper values, respectively throughout the paper.

3. Data Collection

We collected temperature data from different cities in the southeastern Anatolia region of Turkey like Adıyaman, Batman, Diyarbakır, Gaziantep, Kilis, Mardin, Siirt, Şanlıurfa and Şırnak. We aim to investigate which city on average has the best temperature and which city temperature is more consistent. We collected data for February 2023 from the website https://www.gismeteo.com/. The data is reported in Table 1. Table 1 presents low and high values of the temperature data. The temperature data given in the interval cannot be analyzed using classical statistics. The interval data can be analyzed using neutrosophic statistics. The neutrosophic statistical analysis for the temperature data is shown in Section 3.

Table 1. The low and high values of the temperature data.

		Adıyaman		**Batman**		**Diyarbakır**		**Gaziantep**		**Kilis**		**Mardin**		**Siirt**		**Şanlıurfa**		**Şırnak**	
Day	Date	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
Monday	6	1	4	2	6	2	5	-2	4	1	5	4	6	0	4	2	6	-1	2
Tuesday	7	-4	2	1	6	-2	4	-6	1	0	4	1	6	0	2	0	6	-4	1
Wednesday	8	-1	3	-2	3	-4	1	-3	2	1	6	0	6	-4	0	2	6	-6	1
Thursday	9	-2	4	-1	5	-5	3	-4	3	1	6	1	7	-4	3	1	6	-6	2
Friday	10	-1	5	-1	6	-5	4	-1	5	3	8	1	8	-3	4	1	10	-6	3
Saturday	11	0	4	1	5	-3	1	-2	3	3	5	3	8	-1	4	3	5	-4	3
Sunday	12	3	4	3	6	-1	3	0	4	2	6	4	9	1	4	4	7	1	5
Monday	13	1	9	2	9	-1	4	-1	8	2	11	4	11	1	8	3	12	-2	3
Tuesday	14	2	10	1	8	-4	6	0	10	4	12	2	11	-1	7	4	11	-4	4
Wednesday	15	3	10	1	8	-3	6	1	7	5	11	4	12	-1	8	5	12	-3	6
Thursday	16	4	12	5	13	-1	10	1	11	5	13	7	14	4	11	5	13	2	7
Friday	17	2	9	3	10	0	9	2	10	6	13	6	15	1	9	5	13	2	9
Saturday	18	-1	7	1	9	1	9	2	10	4	11	4	12	-1	6	3	11	0	8
Sunday	19	-1	8	1	9	1	10	2	12	4	13	4	13	-1	7	2	13	0	8
Monday	20	0	6	1	8	1	8	3	9	5	11	4	12	-1	6	3	10	0	8
Tuesday	21	-1	6	1	8	0	8	2	9	4	11	4	12	-1	6	2	10	1	7
Wednesday	22	-1	9	0	9	0	10	2	12	4	13	3	13	-2	7	2	12	0	8
Thursday	23	0	10	1	10	1	11	3	13	4	14	3	14	-1	7	3	14	0	8
Friday	24	2	11	2	11	3	12	5	14	6	15	5	15	0	9	5	15	1	10
Saturday	25	3	12	3	12	4	13	6	17	7	17	5	16	2	10	5	17	2	11
Sunday	26	4	12	3	12	4	12	7	16	9	17	7	16	1	10	6	16	2	12

4. Results and Interpretation

We performed the neutrosophic statistical analysis using the temperature data. The neutrosophic mean of temperature is shown in Table 2. The neutrosophic standard deviation is shown in Table 3. The neutrosophic coefficient variation is shown in Table 4. For example, the neutrosophic average temperature value and the standard deviation of Adıyaman city are calculated. Then, we give the following calculational steps based on the neutrosophic statistical algorithm.

Step 1. By Eq. (2), calculate the average value of the determinate low temperature of the city corresponding to the first column as follows:

$$\overline{a}_1 = \frac{1}{n}\sum_{i=1}^{n} a_{i1} = \frac{1}{21}\sum_{i=1}^{21} a_{i1}$$

$$= \frac{(1-4-1-2-1+0+3+1+2+3+4+2-1-1+0-1-1+0+2+3+4)}{21} = 0{,}666$$

Step 2. By Eq. (3), calculate the average value of the determinate high temperature of the city corresponding to the first column as follows:

$$\overline{b}_1 = \frac{1}{n}\sum_{i=1}^{n} b_{i1} = \frac{1}{21}\sum_{i=1}^{21} b_{i1}$$

$$= \frac{(4+2+3+4+5+4+4+9+10+10+12+9+7+8+6+6+9+10+11+12+12)}{21}$$

$$= 7.476$$

Step 3. By Eq. (4), obtain the neutrosophic average temperature value of Adıyaman city:
$\overline{z}_1 = \overline{a}_1 + \overline{b}_1 I_N = 0{,}666 + 7.476 I_N,\ I_N \in [0{,}0.91]$. The neutrosophic mean of temperature is shown in Table 2.

Table 2. The neutrosophic mean of temperature.

Cities	$\overline{z}_i$	$\overline{a}_i + \overline{b}_i I_N,\ I_N \in [I_L, I_U]$
Adıyaman	$[0.66, 7.47]$	$0.66 + 7.47I_N, I_N \in [0{,}0.91]$
Batman	$[1.33, 8.23]$	$1.33 + 8.23I_N, I_N \in [0{,}0.83]$
Diyarbakır	$[-0.57, 7.09]$	$-0.57 + 7.09I_N,\ I_N \in [0{,}1.08]$
Gaziantep	$[0.80, 8.57]$	$0.80 + 8.57I_N, I_N \in [0{,}0.90]$
Kilis	$[3.80, 10.57]$	$3.80 + 10.57I_N, I_N \in [0{,}0.63]$
Mardin	$[3.61, 11.23]$	$3.61 + 11.23I_N,\ I_N \in [0{,}0.67]$
Siirt	$[-0.52, 6.28]$	$-0.52 + 6.28I_N, I_N \in [0{,}1.08]$
Şanlıurfa	$[3.14, 10.71]$	$3.14 + 10.71I_N, I_N \in [0{,}0.70]$
Şırnak	$[-1.19, 6]$	$-1.19 + 6I_N,\ I_N \in [0{,}1.19]$

Step 4. Obtain the differences between z_i $(i = 1,2,\ldots,n)$ and $\overline{z}_i$ of Adıyaman city:

$$z_1 - \overline{z}_1 = a_1 - \overline{a}_1 + (b_1 - \overline{b}_1)I_N = (1 - 0.66) + (4 - 7.47)I_N$$

$$z_1 - \overline{z}_1 = 0.34 + (-3.47)I_N\ ,\ I_N \in [0{,}1.09]$$

$$\vdots$$

$$z_{21} - \bar{z}_1 = 3.34 + 4.53I_N \ , \ I_N \in [0,0.26].$$

Step 5. Calculate the square of all the differences between z_i $(i = 1,2,\dots,n)$ and $\bar{z}_i$:

$$(z_i - \bar{z}_1)^2 = \begin{bmatrix} min\left((a_1 - \bar{a}_1)^2, (a_1 - \bar{a}_1)((a_1 - \bar{a}_1) + 1.09x(b_1 - \bar{b}_1)), ((a_1 - \bar{a}_1) + 1.09x(b_1 - \bar{b}_1))^2\right) \\ max\left((a_1 - \bar{a}_1)^2, (a_1 - \bar{a}_1)((a_1 - \bar{a}_1) + 1.09x(b_1 - \bar{b}_1)), ((a_1 - \bar{a}_1) + 1.09x(b_1 - \bar{b}_1))^2\right) \end{bmatrix}$$

$$= \left[(a_1 - \bar{a}_1)^2, \left((a_1 - \bar{a}_1) + 1.09x(b_1 - \bar{b}_1)\right)^2\right] = [0.116, 11.847], \ I_N \in [0,1.09]$$

$$\vdots$$

$$(z_{21} - \bar{z}_1)^2 = [11.15, 20.41], \ I_N \in [0,0.26].$$

Step 6. Calculate the neutrosophic standard deviation:

$$\sigma_{z1} = \sqrt{\frac{1}{21}\sum_{i=1}^{21}(z_i - \bar{z}_1)^2} = \left[\sqrt{\frac{1}{21}(0.116 + \dots + 11.15)}, \sqrt{\frac{1}{21}(11.847 + \dots + 20.43)}\right]$$

The neutrosophic standard deviation is shown in Table 3. Also, the neutrosophic coefficient variation is shown in Table 4.

Table 3. The neutrosophic standard deviation.

Cities	σ_z	$a_s + b_s I_{Ns}, \ I_{Ns} \in [I_{Ls}, I_{Us}]$
Adıyaman	[2.04, 3.06]	$2.04 + 3.06I_N, I_N \in [0,0.33]$
Batman	[1.54, 2.56]	$1.54 + 2.56I_N, I_N \in [0,0.40]$
Diyarbakır	[2.58, 6.47]	$2.58 + 6.47I_N, I_N \in [0,0.60]$
Gaziantep	[3.13, 7.08]	$3.13 + 7.08I_N, I_N \in [0,0.55]$
Kilis	[2.14, 4.49]	$2.14 + 4.49I_N, I_N \in [0,0.52]$
Mardin	[1.83, 3.77]	$1.83 + 3.77I_N, I_N \in [0,0.51]$
Siirt	[1.87, 4.55]	$1.87 + 4.55I_N, I_N \in [0,0.58]$
Şanlıurfa	[1.54 , 3.69]	$1.54 + 3.69I_N, I_N \in [0,0.58]$
Şırnak	[2.71, 6.35]	$2.71 + 6.35I_N, I_N \in [0,0.57]$

Table 4. The neutrosophic coefficient variation.

Cities	CV_N	$a_v + b_v I_{Nv}, \ I_{Ns} \in [I_{Lv}, I_{Uv}]$
Adıyaman	[40.96, 309.09]	$40.96 + 309.09I_N, I_N \in [0,0.87]$
Batman	[31.10, 115.79]	$31.10 + 115.79I_N, I_N \in [0,0.73]$
Diyarbakır	[−452.63, 91.25]	$-452.63 + 91.25I_N, I_N \in [0,5.96]$
Gaziantep	[82.61, 391.25]	$82.61 + 391.25I_N, I_N \in [0,0.79]$
Kilis	[42.48, 56.31]	$42.48 + 56.31I_N, I_N \in [-0.24,0]$
Mardin	[33.57,50.69]	$33.57 + 50.69I_N, I_N \in [0,0.33]$
Siirt	[−359.61, 72.45]	$-359.61 + 72.45I_N, I_N \in [0,5.96]$
Şanlıurfa	[34.45, 49.04]	$34.45 + 49.04I_N, I_N \in [0,0.297]$
Şırnak	[−227.73, 105.83]	$-227.73 + 105.83I_N, I_N \in [0,3.15]$

The measures of indeterminacy associated with the coefficient of variation are also shown in Table 4. Based on the analysis, it can be concluded that the values of the coefficient of variation of temperature in Mardin and Şanlıurfa are minimal. Therefore, the temperatures of Mardin and Şanlıurfa cities are more consistent than the other cities in Turkey. Figures 1-3, present the neutrosophic average values, standard deviations, and coefficient variations of temperatures in different cities, respectively.

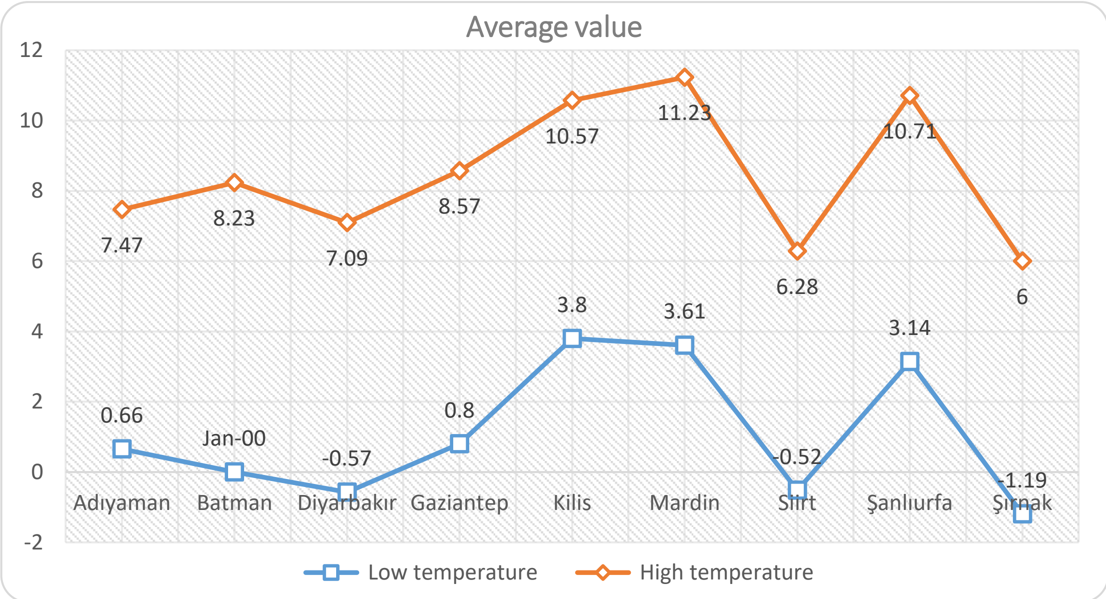

Figure 1. The neutrosophic average values of temperatures in different cities.

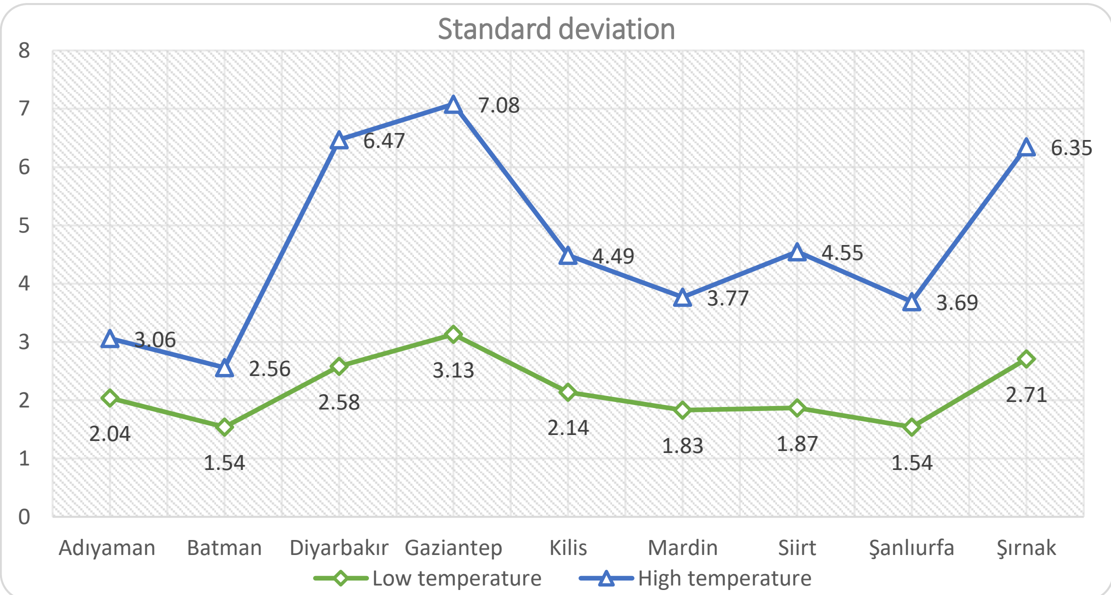

Figure 2. The neutrosophic standard deviations of temperatures in different cities

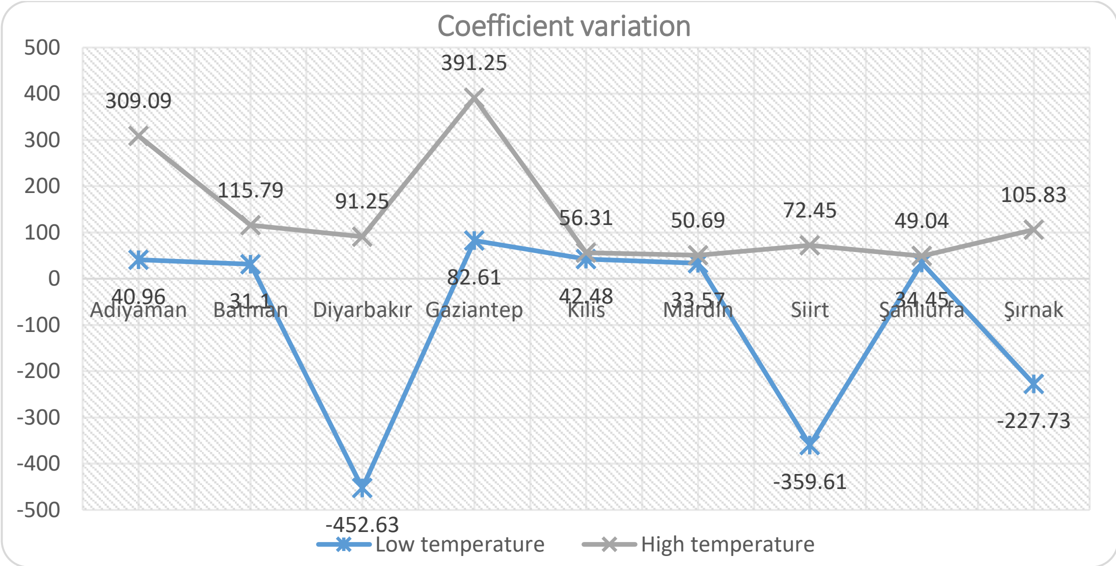

Figure 3. The neutrosophic coefficient variations of temperatures in different cities.

5. Comparative study

The neutrosophic statistical analysis is the generalization of the classical statistical analysis. The neutrosophic statistical analysis reduces to classical statistical analysis when no indeterminacy is found in the data or data is not recorded in the intervals. Note here that temperature data is always recorded in intervals and therefore adequately analyzed by the neutrosophic statistics. We now compare the results obtained using neutrosophic statistics with the results of classical statistics. The neutrosophic forms of the temperatures of Mardin and Şanlıurfa cities are $CV_N = 33.57 + 50.69I_N$ and $CV_N = 34.45 + 49.04I_N$. The first values (determinate) 33.57 and 34.45 of this neutrosophic show the analysis from the classical statistics while the second part $50.69I_N$ and $49.04I_N$ of the neutrosophic forms show the indeterminate part. From the analysis, it can be seen that the values CV_N ranges from 33.57% to 50.69% and 34.45% to 49.04% with the measure of indeterminacy or uncertainties at 0.33 and 0.297. Note that when I_{N_v}, the neutrosophic statistical results reduce to the results under classical statistics.

6. Conclusion

In this work, we applied neutrosophic statistical analysis to temperature data of different cities in Turkey. Based on the comparative study, it can be concluded that neutrosophic statistical results are more adequate, flexible, and more informative than classical statistics. Serious steps should be taken to reduce global warming by planting more trees, especially in Mardin and Şanlıurfa cities. The neutrosophic statistical analysis can be applied to analyze the interval data more adequately than classical statistics. Also, in future studies, the work can be extended to all regions of Turkey. Furthermore, this calculation can be used for humidity, amount of rainfall, etc. for different regions of Turkey.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

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Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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