NEUTROSOPHIC SYSTEMS WITH APPLICATIONS

AN INTERNATIONAL JOURNAL ON INFORMATICS, DECISION SCIENCE, INTELLIGENT SYSTEMS APPLICATIONS

ISSN (ONLINE): 2993-7159

ISSN (PRINT): 2993-7140

VOLUME 18

2024

https://sciencesforce.com

nswa@sciencesforce.com

Neutrosophic Systems with Applications

An International Journal on Informatics, Decision Science, Intelligent Systems Applications

Information for Authors and Subscribers

Neutrosophic Systems with Applications (NSWA) is an international academic journal, published monthly online and on paper by the Sciences Force publisher, Five Greentree Centre, 525 Route 73 North, STE 104 Marlton, New Jersey 08053, United States, that has been created for publications of advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as Informatics, Decision Science, Computer Science, Intelligent Systems Applications, etc.

The submitted papers should be professional, and in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e., notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory, every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjointed two by two. But, since in many cases, the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and intuitionistic fuzzy logic). In neutrosophic logic, a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^{-}0, 1^{+}[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of classical statistics.

What distinguishes neutrosophic from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

All submissions should be designed in MS Word format using our template file on the journal website.

A variety of scientific books in many languages can be downloaded freely from the Digital Library of Science:

To submit a paper, mail the file to the Editor-in-Chief on this email nswa@sciencesforce.com. To order printed issues, contact the Editor-in-Chief on this email nswa@sciencesforce.com.

Journal of Neutrosophic Systems with Applications is also supported by:

University of New Mexico and Zagazig University, Computer Science Department.

This journal is a non-commercial, academic edition. It is printed from private donations.

Publisher's Name: Sciences Force

The home page of the publisher is accessed on. https://sciencesforce.com/

The home page of the journal is accessed on. https://sciencesforce.com/nswa

Publisher's Address: Five Greentree Centre, 525 Route 73 North, STE 104 Marlton, New Jersey 08053.

Tel: +1 (509) 768-2249 Email: nswa@sciencesforce.com

Contents

Neutrosophic Systems with Applications, Vol. 18, 2024
https://doi.org/10.61356/j.nswa.2024.18262

An Innovative Approach on Yao's Three-Way Decision Model Using Intuitionistic Fuzzy Sets for Medical Diagnosis

Wajid Ali [1,*], Tanzeela Shaheen [1], Iftikhar Ul Haq [1], Florentin Smarandache [2], Hamza Ghazanfar Toor [3], and Faiza Asif [4]

[1] Department of Mathematics, Air University, E-9, Islamabad 44000, Pakistan. Emails: wajidali00258@gmail.com, tanzeela.shaheen@mail.au.edu.pk, chibmathteacher@gmail.com.
[2] University of New Mexico, 705 Gurley Ave., Gallup, NM 87301, USA; smarand@unm.edu.
[3] Biomedical Engineering Department, Riphah International University, Islamabad 44000, Pakistan; hamza.ghazanfar@riphah.edu.pk.
[4] Management and Finance Department, Leeds Beckett University Leeds United Kingdom; f.asif2456@student.leedsbeckett.ac.uk.

* Correspondence: wajidali00258@gmail.com.

Abstract: In the realm of medical diagnosis, intuitionistic fuzzy data serves as a valuable tool for representing information that is uncertain and imprecise. Nevertheless, decision-making based on this kind of knowledge can be quite challenging due to the inherent vagueness of the data. To address this issue, we employ power aggregation operators, which prove effective in combining several sources of data, such as expert thoughts and patient information. This allows for a more correct diagnosis; a particularly crucial aspect of medical practice where precise and timely diagnoses can significantly impact medication policy and patient results. In our research, we introduce a novel methodology to the three-way decision idea. Initially, we revamp the three-way decision model using rough set theory and incorporate interval-valued classes to handle intuitionistic fuzzy data. Secondly, we explore the use of intuitionistic fuzzy power weighted and intuitionistic fuzzy power weighted geometric aggregation operators to consolidate attribute values within the data system. Furthermore, we present a case study in the medical field to exhibit the validity and efficiency of our offered technique. This innovative method enables us to classify participants into three distinct zones based on their symptoms. The manuscript concludes with a summary of key points provided by the authors.

Keywords: Intuitionistic Fuzzy Sets; Aggregation Operators; Information System; Three-Way Decision; Medical Diagnosis; Decision Making; Optimization.

1. Introduction

Medical conditions can manifest with a variety of symptoms, which can complicate the process of accurate diagnosis. Some typical symptoms include elevated body temperature, fatigue, coughing, nausea, diarrhea, and skin eruptions. The identification of medical illnesses typically necessitates the involvement of healthcare professionals, who rely on a combination of patient history, physical assessments, laboratory tests, medical imaging, and other diagnostic measures to establish an analysis. This decision-making [1] process in medical diagnosis is influenced by numerous features, involving the patient's age, medical history, and genetic susceptibility. Physicians may also utilize diagnostic algorithms or decision trees to aid in the assessment of intricate medical conditions. In recent times, advancements in medical technology have considerably enhanced the precision and speed of medical diagnoses. This includes the integration of new imaging methods, genetic testing, and decision-making tools based on artificial intelligence [2-5].

The rough set model is a mathematical framework designed to systematically address situations involving incomplete and uncertain information. Zdzislaw Pawlak [6] first introduced this theory in the early 1980s to handle vague and doubtful information. In the context of medical diagnosis, a rough set model proves valuable in determining the presence or absence of specific diseases or conditions, even when dealing with incomplete or uncertain data sources [7], such as symptoms, medical history, test results, and other related information [8]. The core principle following the rough set model revolves around the segmentation of information into subsets, primarily driven by their attributes, such as symptoms or test outcomes. This partitioning procedure aids in the discernment of the critical features or factors most closely linked to a specific ailment or medical condition. Once the data is organized into these distinct subsets, rough set theory can be leveraged to unveil rules or shapes that facilitate making predictions about whether a given patient is afflicted by an actual disease or condition. A multitude of scientists have contributed to the advancement of innovative algorithms for disease diagnosis using this methodology. For instance, El-Bably and colleagues [8-10] started the concept of soft, rough approximation and implemented it in the realm of medical difficulty diagnosis. Hosny et al. [11] expanded the application of rough sets by introducing the maximal right neighborhood system and exploring its uses in the field of medicine. Additionally, Al-Shami et al. [12] defined maximal rough neighborhoods and employed this approach for the diagnosis of medical conditions.

Atanassov [13] established the theory of an intuitionistic fuzzy set (IFS), which represents an expansion of the conventional fuzzy set (FS). Within the realm of IFS, one can express both the degree of membership and non-membership of an element within a universal set. IFSs hold significant importance in the field of medicine, particularly in the context of disease identification and problem-solving. Researchers have extensively investigated the utilization of IFS in medical diagnosis, especially in scenarios characterized by substantial hesitation and flexibility in symptoms and test outcomes. The intuitionistic fuzzy set offers a valuable tool for capturing and conveying this uncertainty, thus enhancing the precision of diagnostic knowledge. For instance, in the context of diagnosing complex conditions such as cancer, Intuitionistic Fuzzy Sets (IFS) can portray the degree of confidence or vagueness related to the diagnosis, considering a wide range of diagnostic criteria like blood test results, imaging studies, and biopsy conclusions. This approach ultimately leads to more precise and dependable diagnoses while also facilitating the creation of personalized treatment plans. Jiang et al. [14] employed IFS in medical image fusion, utilizing entropy measures, whereas Mehmood et al. [15, 16] extended the concept of intuitionistic fuzzy sets and applied these principles to the field of medical diagnosis. De et al. [17] also delved into the application of IFS in medical diagnosis, much like the work of Davvaz et al. [18]. Szmidt et al. [19] investigated the utilization of IFS in intelligent data analysis for medical diagnosis. In the decision-making process involving IFS, aggregation operators play a crucial role in computing attribute values. As a result, several experts have proposed a variety of aggregation operators for this purpose. For instance, Xu et al. [20] developed and implemented power aggregation operators for IFS in Multi-Attribute Decision Making (MADM). In 2006, Xu et al. [21] introduced geometric aggregation operators tailored for IFS. Mehmood et al. [22] presented similarity measures and power aggregation operators based on Intuitionistic Hesitant Fuzzy Sets (IHFS). More recently, Senapati et al. [23] and Garg et al. [24] have explored novel operators in this context.

The three-way decision (TWD) concept represents a notable extension of the RS theory, initially introduced by Yao [25, 26]. In the realm of medical diagnosis, a three-way decision entails the assessment of three potential outcomes: positive, negative, or inconclusive. In the case of a Positive Outcome, when a medical diagnosis yields a positive result, it confirms that the patient indeed has the specific condition or disease under examination. This necessitates treatment for the diagnosed ailment, with healthcare professionals closely monitoring the patient's progress. Conversely, a Negative Outcome in a medical diagnosis indicates that the patient does not have the particular condition or disease under investigation. In such instances, the patient may not need any medication, and healthcare providers may need to explore other potential causes for the patient's symptoms. An

Inconclusive Outcome arises when the test results do not provide sufficient clarity to decide whether the patient possesses the condition or disease under scrutiny. In such situations, additional tests or evaluations may be necessary to succeed at a more definitive diagnosis. Lately, Li et al. [27, 28] applied TWD procedures to improve decision-making in medical diagnosis. Hu et al. [29, 30] introduced the notion of a lattice model for medical diagnosis, incorporating TWD. Jia and Fan [31] devised TWD models for multi-criteria situations, while Ye et al. [32] integrated the TWD concept into the emerging field of fuzzy information systems. In a similar vein, numerous scholars have explored this field, proposing innovative approaches across various extensions of fuzzy sets [33-35].

In our exploration of the literature, we discovered that TWD models prove highly beneficial for medical problem diagnosis. The fusion of IFS and TWD, as described in reference [36], yields a robust framework for addressing situations characterized by vagueness and ambiguity. It should be highlighted that aggregating the outcomes of numerous participants through TWD poses a significant challenge. Researchers have traditionally employed conventional methods to compute alternatives for TWD, as evidenced by references [37-40]. In the existing TWD model, as outlined in references [25, 37], an external concept becomes necessary for determining equivalence classes. Additionally, a threshold is employed to categorize the alternatives into three distinct regions.

The primary objective behind creating this piece of work is to create an innovative algorithm for the TWD by utilizing aggregation operators and enhancing the TWD decision process through interval-valued equivalence classes for Interval-Valued Fuzzy Sets (IVFS). This approach aims to address the existing deficiencies and challenges in TWD computation. The following is a depiction of the key contribution made by this analysis.

i. Establish the notion of intervals to represent the degrees of membership in Interval Fuzzy Sets (IFS) by utilizing the step size function.
ii. Create equivalence classes by leveraging intervals and refer to them as interval-valued classes.
iii. To address concerns related to computational efficiency and timesaving, we introduce the IFPWA and IFPWG aggregation operators specifically tailored for the TWD theory.
iv. Present an algorithm designed for the classification of diverse patients and the diagnosis of diseases using multiple symptom criteria.

The rest of the article follows this structure: Section 2 presents an overview of essential concepts, including IFS, power aggregation operators, and Three-way Decisions in Section 3, we define membership grade intervals using the step size function and create equivalence classes. We then proceed to adapt the TWD for IFS based on these intervals. Section 4 encompasses the development of a well-defined algorithm, including a flow chart, and an in-depth, step-by-step explanation of the approach. Section 5 delves into a case study where we apply the offered methodology to diagnose a medical issue and classify alternatives using power aggregation operators for IFS. We also extensively discuss the benefits and benefits of the suggested models. Finally, Section 6 encapsulates the authors' conclusion and their plan.

2. Preliminaries

In this section, we take a closer look at several fundamental concepts within intuitionistic fuzzy sets (IFS) and explore some notions about power aggregation operators.

2.1 Intuitionistic Fuzzy Sets and Aggregation Operators

Atanassov [13] proposed the theory of IFS as an expansion of FS. While FS offers the membership grade (MG) of an element within a specific set [0, 1], IFS simultaneously provides both MG and non-membership grade (NMG).

Definition 1: [13] An IFS T on set E is represented using the two mappings $l(e)$ and $m(e)$. Mathematically, this representation is expressed through the following structure:

$$T = \langle e, l_T(e), m_T(e)) | e \in E \rangle,$$

Where, $l_T(e): E \to [0,1]$ and $m_T(e): E \to [0,1]$ signifies the MG and NMG including the condition $0 \le l(e) + m(e) \le 1$, for each $e \in E$.

Definition 2: let $T = (l_T, m_T)$ be an IFN, then the score function and accuracy function are stated and represented as:

$$S(T) = l_T - m_T, \qquad S(T) \in [-1,1];$$
$$H(T) = l_T + m_T, \qquad H(T) \in [0,1].$$

Definition 3: Suppose $T_1 = (l_1, m_1)$, $T_2 = (l_2, m_2)$ be intuitionistic fuzzy sets (IFSs), then some basic operations are described as below:

i. $T_1 \oplus T_2 = (\{l_1 + l_2 - l_1 l_2\}, \{m_1 m_2\});$
ii. $T_1 \otimes T_2 = (\{l_1 l_2\}, \{m_1 + m_2 - m_1 m_2);$
iii. $\lambda T_1 = \left(1 - (1-l)^\lambda, m^\lambda\right), \lambda > 0;$
iv. $T_1^\lambda = \left((l)^\lambda, 1 - (1-m)^\lambda\right), \lambda > 0;$
v. $T_1^c = (m_1, l_1).$

Definition 4: [21] Suppose that $T_j = \left(l_j, m_j\right)$ is a collection of IFS and the $k_j = (k_1, k_2, \dots, k_m)^T$ is weight vector for T_j, and $\sum_{j=1}^m k_j = 1$. Then $IFPWP_k$ an operator is a mapping $IFPWP_k: T^m \to T$ where

$$IFPWP_k(T_1, T_2, \dots, T_m) = \frac{\oplus_{j=1}^m (k_j(1 + J(T_j)T_j)}{\sum_{j=1}^m k_j(1 + J(T_j))} =$$
$$\left(1 - \prod_{j=1}^m (1 - (l_j)^{\frac{k_j((1+J(T_j))}{\sum_{j=1}^m k_j(1+J(T_j))}}, \prod_{j=1}^m (m_j)^{\frac{k_j(1+J(T_j))}{\sum_{j=1}^m k_j(1+TJT_j))}}\right),$$

Where,

$$J(T_j) = \sum_{\substack{i=1 \\ i \ne j}}^m k_j Sup(T_j, T_i),$$
$$Sup(T_j, T_i) = 1 - d(T_j, T_i),$$
$$d(T_j, T_i) = \frac{1}{l} \sum_{\substack{i=1 \\ i \ne j}}^l \left(\left|l_i - l_j\right| + |m_i - m_j|\right).$$

Definition 5: For IFSs $T_i = (l_i, m_i)$ with k_j such that $k_j > 0$ and $\sum_{j=1}^m k_j = 1$. A mapping $IFPOWP_k$: $T^m \to T$, is stated as:

$$IFPOWP_k(T_1, T_2, \dots, T_m) = \frac{\oplus_{j=1}^m (k_j(1 + J(T_{\sigma(j)})T_{\sigma(j)})}{\sum_{j=1}^m k_j(1 + J(T_{\sigma(j)}))}$$
$$= \left(1 - \prod_{j=1}^m (1 - (l_{\sigma(j)})^{\frac{(k_j(1+J(T_{\sigma(j)}))}{\sum_{j=1}^m k_j(1+J(T_{\sigma(j)}))}}, \prod_{j=1}^m (m_{\sigma(j)})^{\frac{(k_j(1+J(T_{\sigma(j)}))}{\sum_{j=1}^m k_j(1+J(T_{\sigma(j)}))}}\right).$$

Definition 6: For a set of IFS $T_j = \left(l_j, m_j\right)$ and the weights k_j for T_j, and $\sum_{j=1}^m k_j = 1$. Then $IFPWG_k$ an operator is a mapping $IFPWG_k: T^m \to T$.

$$IFPWG_k(T_1, T_2, \dots, T_m) = \frac{\otimes_{j=1}^m (k_j(1 + J(T_j)T_j)}{\sum_{j=1}^m k_j(1 + J(T_j))} =$$
$$\left(\prod_{j=1}^m (l_j)^{\frac{k_j\left(1+J(T_j)\right)}{\sum_{j=1}^m k_j(1+J(T_j))}}, 1 - \prod_{j=1}^m (1 - (m_j)^{\frac{k_j((1+J(T_j))}{\sum_{j=1}^m k_j(1+J(T_j))}}\right).$$

Where, $J(T_j) = \sum_{\substack{i=1 \\ i \ne j}}^m k_j Sup(T_j, T_i)$.

Definition 7: For IFSs $T_i = (l_i, m_i)$ with their weights k_j such that $k_j > 0$ and $\sum_{j=1}^m k_j = 1$. A mapping $IFPOWG_k$: $T^m \to T$, is expressed as:

$$IFPOWG_k(T_1, T_2, \dots, T_m) = \frac{\otimes_{j=1}^m (k_j(1 + J(T_{\sigma(j)})T_{\sigma(j)})}{\sum_{j=1}^m k_j(1 + J(T_{\sigma(j)}))}$$
$$= \left(\prod_{j=1}^m (l_{\sigma(j)})^{\frac{(k_j(1+J(T_{\sigma(j)}))}{\sum_{j=1}^m k_j(1+J(T_{\sigma(j)}))}}, 1 - \prod_{j=1}^m (1 - (m_{\sigma(j)})^{\frac{(k_j(1+J(T_{\sigma(j)}))}{\sum_{j=1}^m k_j(1+J(T_{\sigma(j)}))}}\right).$$

2.2 Three-Way Decision Model Based on Rough Sets

Rough sets [6] constitute a mathematical theory that was formulated by Zdzislaw Pawlak, a Polish computer scientist, during the early 1980s. This notion deals with a structured methodology for addressing uncertainty and handling incomplete information within the realm of data analysis. The theory of three-way decision (TWD) [25] is an expansion of a rough set model, designed to accommodate the notion of "don't know" or "undetermined" elements in decision-making processes. In the traditional two-way decision-making framework, data is typically categorized into two sets: one that fulfills specific conditions and another that does not. The primary constraints associated with this approach are detailed below.

Definition 8: [38] Let $\mathcal{S} = (E, At, V_a, f)$ be an information system (IS), where $E = \{P_1, P_2, \dots P_m\}$ is the universe of discourse. $At = \{c_1, c_2, \dots, c_l\}$ is the set of the attributes, $V_a = \cup_{c \in Pt} \mathbb{V}_c$ is the range of values, $\mathbb{V}_c$ represents the value under attribute c, and $f = E \times At \to V_a$ is an information mapping function.

Furthermore, within these IS, two different categories of attributes exist condition attributes (C) and decision attributes (D), which collectively form the set of attributes denoted as $At = C \cup D$. These information systems are occasionally referred to as decision IS. To collect the parts in E based on the features in Pt, equivalence classes are constructed in rough sets (RSs). In this context, equivalence classes of the relations $BND(C)$ and $BND(D)$ will be called condition and decision classes, respectively.

Definition 9: Let $\mathcal{S} = (E, At, V_a, f)$ be an IS, and B is, $B \subseteq At$, an equivalence relation R is defined as:

$$R_B = \left\{ \left(P_{c_i}, P_{c_j}\right) \in E \times E \mid for\ all\ c \in B\ \left(P_{c_i} = P_{c_j}\right) \right\}.$$

With this relationship, the equivalence class of an element P_B in set E is established as follows,

$$\left[P_{c_i}\right]_B = \left\{ P_{c_j} \in E \mid \left(P_{c_i}, P_{c_i}\right) \in R_B \right\}$$

The main objective of an equivalence relation is to show the inability to distinguish objects. Using the equivalence relation denoted as R_B, the IS can be separated into three different segments by approximation classes.

Definition 10: Pawlak [6] started the concept of approximation classes for the approximation space $Appr(E, R)$ of E, defined for all $U \subseteq E$ as follows:

$$\underline{Appr}(U) = \{P \in E \mid [P]_B \subseteq U\},$$

$$\overline{Appr}(U) = \{P \in E \mid [P]_B \cap U \neq \emptyset\},$$

These categories are referred to as the lower approximation class denoted as $\underline{Appr}(U)$ and the upper approximation class indicated as $\overline{Appr}(U)$ with $[P]_B$ representing the equivalence class of P.

Definition 11: Using the classification of approximations, three distinct regions are defined in the following manner.

$$POS(U) = \underline{Appr}(U),$$

$$NEG(U) = E - \overline{Appr}(U),$$

$$BND(U) = \overline{Appr}(U) - \underline{Appr}(U).$$

Definition 12: Let $\mathcal{S} = (E, At, V_a, f)$ be an IS and a subset of attributes $B \subseteq At$, then the decision rules of $U \subseteq E$ and $z \in U$ are designed as:

$(\mathcal{A})$ If $q \vDash Des([P]_B)\ for\ [P]_B \in POS(U)$, then accept q,

$(\mathcal{R})$ If $q \vDash Des([P]_B)\ for\ [P]_B \in NEG(U)$, then reject q,

$(\mathcal{N})$ If $q \vDash Des([P]_B)\ for\ [P]_B \in BND(U)$, then neither accept nor reject q.

3. A Novel Three-Way Decision Model Based on Interval-Valued Classes

In this portion, we introduce an innovative approach to model TWD by creating intervals. These intervals lead to the creation of unique sets of interval-valued equivalence classes, which, in turn, are used to categorize participants into three different regions: POS (positive), NEG (negative), and BND (boundary), facilitating their classification.

To transform the information system into a discrete form, we replace traditional equivalence classes with interval-valued equivalence classes, guided by the step size function. This function assists in dividing the alternatives into intervals, and its definition is as follows.

Definition 13: For the collection of IFNs (T_i), the intervals ($\mathcal{J}_m$) for approximation classes established on MGs are defined and denoted as:

$$\mathcal{J}_m = [Mim(l_i), Mim(l_i) + h]$$

When the step size function (h) is established for the MGs of IFNs, it is defined by:

$$h = \frac{Max(l_i) - Mim(l_i)}{m}$$

Where m is the number of intervals $\mathcal{J}_m$ which we required.

According to the parental concept of TWD by Yao [25], by using the equivalence classes, we can provide the approximation classes. Continually, by the definition of intervals $\mathcal{J}_n$ in definition 13, nth interval-valued equivalence classes $[P]_B$ for the participants are developed as below:

Definition 14: The design of interval-valued equivalence classes $[P]_B$ for the alternatives P_i is structured such that:

$$[P]_B = \{P: P_i \in \mathcal{J}_m\}$$

Definition 15: The approximation classes within the approximation space $Appr\ (E, R)$ for all $U \subseteq E$, as defined by:

$$\underline{Appr}(U) = \{P \in E | [P]_B \subseteq U\},$$
$$\overline{Appr}(U) = \{P \in E | [P]_B \cap U \neq \emptyset\}.$$

Definition 16: Using the approximation classes outlined in definition 15, we can introduce three distinct regions as below.

$$POS(U) = \underline{Appr}(U),$$
$$NEG(U) = E - \overline{Appr}(U),$$
$$BND(U) = \overline{Appr}(U) - \underline{Appr}(U).$$

Definition 17: The three kinds of decision rules ($\mathcal{A}2 - \mathcal{N}2$) of $U \subseteq E$ for an IS $\mathcal{S} = (E, At, V_a, f)$ are described as:

($\mathcal{A}2$) If $q \vDash Des([P]_B) for\ [P]_B \in POS(U)$, then accept q,

($\mathcal{R}2$) If $q \vDash Des([P]_B) for\ [P]_B \in NEG(U)$, then reject q,

($\mathcal{N}2$) If $q \vDash Des([P]_B) for\ [P]_B \in BND(U)$, then neither accept nor reject q.

4. An Algorithm for the Proposed Model

This portion delves into the detailed application of $IFPWP_k$ and $IFPWG_k$ aggregation operators under IF information for three-way decision-making. We plan five stages for choosing the TWD rules for distinct partakers. Let $\mathrm{E} = \{P_1, P_2 \dots P_n\}$ represent the set of participants and consider $U = \{Yes, No\}$ denote the set of states indicating the decisions of participants, where $U \subseteq E$. The flowchart of the TWD model is displayed in Figure 1.

Step 1. Assess the data system with conditional and decision attributes using an intuitionistic fuzzy approach.

Step 2. For participants $P_i (i = 1,2, \dots, m)$ aggregate all the IF attributes $P_{ij} (j = 1,2, \dots, l)$ into a general result P_i utilizing $IFPWA_k$ and $IFPWG_k$ operators as below:

$$IFPWA_k(T_1, T_2, \dots, T_m) = \frac{\oplus_{j=1}^{m} (k_j (1 + J(T_j) T_j)}{\sum_{j=1}^{m} k_j (1 + J(T_j))} =$$
$$\left(1 - \prod_{j=1}^{m} (1 - (l_j)^{\frac{k_j((1+J(T_j))}{\sum_{j=1}^{m} k_j (1+J(T_j))}}, \prod_{j=1}^{m} (m_j)^{\frac{k_j(1+J(T_j))}{\sum_{j=1}^{m} k_j (1+J(T_j))}} \right),$$

and

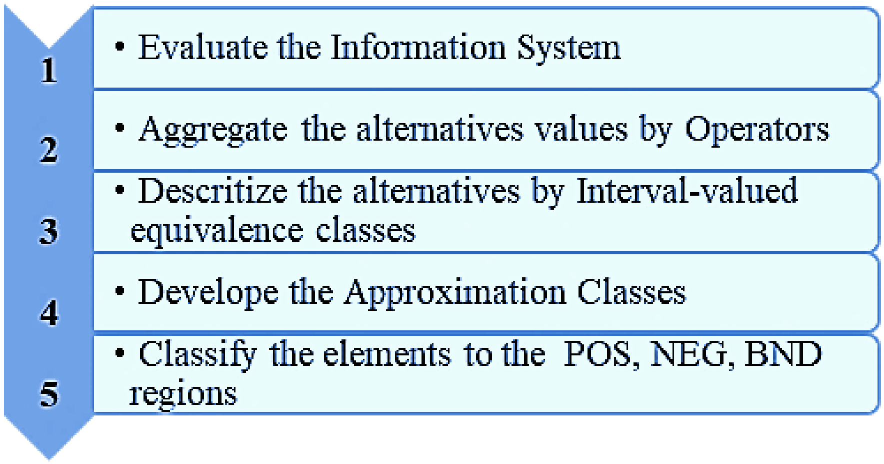

Figure 1. Five steps flow chart of interval-valued TWD model.

$$IFPWG_k(T_1, T_2, \dots, T_m) = \frac{\otimes_{j=1}^{m} (k_j(1 + J(T_j)T_j)}{\sum_{j=1}^{m} k_j(1 + J(T_j))} =$$

$$\left(\prod_{j=1}^{m} (l_j)^{\frac{k_j(1+J(T_j))}{\sum_{j=1}^{m} k_j(1+J(T_j))}}, 1 - \prod_{j=1}^{m} (1 - (m_j)^{\frac{k_j((1+J(T_j))}{\sum_{j=1}^{m} k_j(1+J(T_j))}}\right),$$

Step 3. Determine the interval-valued equivalence classes based on the interval specified in Definition 13.
Step 4. Discretize the upper approximation class and lower approximation class defined in Definition 15.
Step 5. Categorize the options into the POS, NEG, and BND regions based on their approximate classes.

5. Mathematical Model

Now, we offer an illustrative case that serves as a practical example for making decisions regarding the investigation of medical issues, with a focus on confirming or ruling out diseases in patients.

5.1 Explanation of the Problem

Medical diagnosis involves identifying the specific illness or condition that matches a person's symptoms. Healthcare professionals strive to make precise determinations by evaluating a patient's symptoms. It's a process where doctors select a particular disease based on the symptoms exhibited by an individual. The use of IFRS aids healthcare experts in handling complex linguistic concepts and minimizes inaccuracies. The effectiveness of IFRS in medical diagnosis is demonstrated in references [17, 28]. Figure 2 provides a visual representation of the medical diagnosis procedure.

Suppose that a collection of alternatives denoted as P_i (where $i = 1,2, \dots ,15$), participates in the investigation to diagnose the disease "Coronavirus."

Additionally, let I be the group of conditional attributes, specifically $B = \{B_1\ (Chest\ paim), B_2\ (Fever), B_3(Fatigue), B_4(Cough), \}$. Furthermore, the set U, represented as $U = \{P_1, P_2, P_4, P_{15}, P_{11}\}$, indicates the decision attributes that provide the concept of "Yes" for the disease. Let the experts diagnose the disease for all participants and classify their decisions using the weighty vector $k = \{0.2, 0.3, 0.4, 0.1\}$. We will now employ a step-by-step algorithm to provide a detailed explanation of this medical condition.

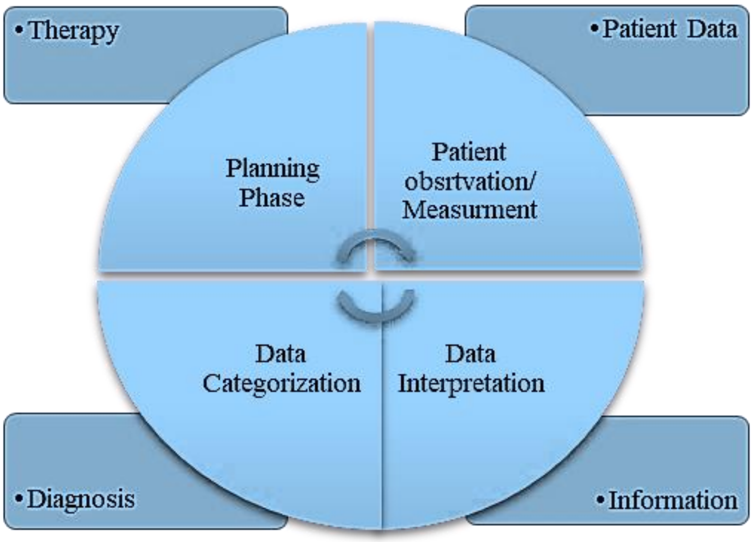

Figure 2. Medical diagnosis diagram.

Step 1: Table 1 represents the IF data of patients.

Table 1. An information Table of alternative conditions.

At	B_1	B_2	B_3	B_4	D
P_1	(0.1,0.3)	(0.4,0.5)	(0.1,0.5)	(0.1,0.5)	Yes
P_2	(0.4,0.5)	(0.5,0.4)	(0.5,0.3)	(0.2,0.6)	Yes
P_3	(0.2,0.3)	(0.2,0.4)	(0.6,0.2)	(0.4,0.5)	No
P_4	(0.4,0.2)	(0.1,0.2)	(0.7,0.4)	(0.3,0.1)	Yes
P_5	(0.5,0.3)	(0.5,0.2)	(0.3,0.2)	(0.4,0.2)	No
P_6	(0.6,0.2)	(0.7,0.1)	(0.4,0.1)	(0.4,0.4)	No
P_7	(0.7,0.1)	(0.2,0.2)	(0.5,0.2)	(0.5,0.2)	No
P_8	(0.3,0.4)	(0.3,0.3)	(0.6,0.2)	(0.2,0.3)	No
P_9	(0.4,0.2)	(0.5,0.2)	(0.7,0.2)	(0.3,0.5)	No
P_{10}	(0.5,0.2)	(0.8,0.1)	(0.2,0.3)	(0.4,0.3)	No
P_{11}	(0.6,0.2)	(0.9,0.1)	(0.5,0.3)	(0.5,0.4)	Yes
P_{12}	(0.8,0.1)	(0.0,0.9)	(0.6,0.4)	(0.2,0.2)	No
P_{13}	(0.9,0.1)	(0.3,0.2)	(0.4,0.3)	(0.4,0.3)	No
P_{14}	(0.1,0.2)	(0.2,0.2)	(0.6,0.3)	(0.3,0.4)	No
P_{15}	(0.8,0.1)	(0.1,0.3)	(0.3,0.4)	(0.4,0.2)	Yes

Step 2. For participants P_i calculate all the conditional attributes information applying $IFPWA_k$ and $IFPWG_k$ operators in the following:

$$IFPWA_k(T_1,T_2,\dots,T_m)=\frac{\oplus_{i=1}^{m}(k_j(1+J(T_i)T_i)}{\sum_{i=1}^{m}k_j(1+J(T_i))}=$$
$$\left(1-\prod_{i=1}^{m}(1-(l_i)^{\frac{k_j((1+J(T_i))}{\sum_{i=1}^{m}k_j(1+J(T_i))}},\prod_{i=1}^{m}(m_i)^{\frac{k_j(1+J(T_i))}{\sum_{i=1}^{m}k_j(1+J(T_i))}}\right),$$

and

$$IFPWG_k(T_1,T_2,\dots,T_m)=\frac{\otimes_{i=1}^{m}(k_j(1+J(T_i)T_i)}{\sum_{i=1}^{m}k_j(1+J(T_i))}=$$

$$\left(\prod_{i=1}^{m}(l_i)^{\frac{k_j(1+J(T_i))}{\sum_{j=1}^{m}k_j(1+J(T_i))}}, 1-\prod_{i=1}^{m}(1-(m_i)^{\frac{k_j((1+J(T_i))}{\sum_{i=1}^{m}k_j(1+J(T_i))}}\right).$$

The results are presented in Table 2.

Table 2. Aggregated results of all alternatives.

At	$IFPWA_k$	$IFPWG_k$
P_1	(0.203, 0.458)	(0.151, 0.470)
P_2	(0.466, 0.374)	(0.450, 0.393)
P_3	(0.429, 0.281)	(0.347, 0.306)
P_4	(0.500, 0.261)	(0.333, 0.292)
P_5	(0.409, 0.214)	(0.389, 0.217)
P_6	(0.545, 0.124)	(0.507, 0.142)
P_7	(0.471, 0.177)	(0.401, 0.183)
P_8	(0.452, 0.261)	(0.400, 0.275)
P_9	(0.581, 0.213)	(0.541, 0.226)
P_{10}	(0.523, 0.201)	(0.372, 0.227)
P_{11}	(0.703, 0.205)	(0.615, 0.236)
P_{12}	(0.507, 0.384)	(0.00, 0.618)
P_{13}	(0.535, 0.220)	(0.420, 0.239)
P_{14}	(0.411, 0.252)	(0.302, 0.262)
P_{15}	(0.395, 0.276)	(0.259, 0.312)

Step 3. Determine the interval-based equivalence classes applying the prescribed method with a step size of $n = 5$ as illustrated in Table 3.

Table 3. Interval-valued equivalence classes.

$IFPWA_k$	$[P_1] = \{P_1\}$ $[P_2] = \{P_2, P_3, P_4, P_5, P_7, P_8, P_{14}\}$ $[P_6] = \{P_6, P_9, P_{10}, P_{12}, P_{13}\}$ $[P_{11}] = \{P_{11}\}$ $[P_{15}] = \{P_{15}\}$
$IFPWG_k$	$[P_1] = \{P_1\}$ $[P_2] = \{P_2, P_5, P_7, P_8, P_{10}, P_{13}\}$ $[P_3] = \{P_3, P_4, P_{14}, P_{15}\}$ $[P_6] = \{P_6, P_9, P_{11}\}$ $[P_{12}] = \{P_{12}\}$

Step 4. Determine the lower approximation and upper approximation by Definition 15 for the given decision attributes $U = \{P_1, P_2, P_4, P_{15}, P_{11}\}$.

Table 4. Approximation classes.

IFPA	$\underline{Appr}(U) = \{P_1, P_{15}, P_{11}\}$ $\overline{Appr}(U) = \{P_1, P_{15}, P_3, P_5, P_7, P_8, P_4, P_{14}, P_2\}$
IFPG	$\underline{Appr}(U) = \{P_1\}$, $\overline{Appr}(U) =$ $\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{13}, P_{14}\}$

Step 5. Finally, the classification of the elements for POS, NEG, and BND regions respectively represented in Table 5.

Table 5. Classification of alternatives accordingly.

IFPA	IFPG
$POS(U) = \{P_1, P_{15}, P_{11}\}$ $NEG(U) = \{P_{12}, P_6, P_{13}, P_9, P_{10}\}$ $BND(U) = \{P_2, P_3, P_4, P_8, P_7, P_{10}\}$	$POS(U) = \{P_1\}$ $NEG(U) = \{P_{12}\}$ $BND(U) = \{P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{13}, P_{14}\}$

Thus, the findings indicate that individuals in the POS zone have tested positive for Coronavirus disease, those in the NEG regions are free from the virus, and individuals in the BND regions have not received confirmation. Additionally, we can categorize newcomers' choices based on the descriptions of previously tested elements.

5.2 Advantages Offered by the Proposed Model

The proposed approach offers several advantages as outlined below:

i. One of the key benefits of this approach is its broader applicability. It serves as a more generalized version of intuitionistic fuzzy sets. When the non-membership grades are set to zero, intuitionistic fuzzy sets transform into FSs.
ii. Power aggregation operators prove to be highly effective and straightforward tools for addressing decision-making problems in fuzzy environments. These operators facilitate the determination of attribute values for elements and can account for their significance when aggregating data.
iii. Many of the existing methods in the literature for TWD primarily adhere to conventional theories such as Yao's [37]. In contrast, our approach presents novel procedures for TWD, which encompass the development of power aggregation operators. Furthermore, we introduce interval-valued categories to categorize participants.
iv. In the context of medical diagnosis, especially in complex cases, such as the one presented here, accurate disease diagnosis is a critical concern for both experts and patients. To tackle this challenge, we have established an idea that accounts for distinct patient profiles and their disease attributes. Ultimately, decisions are made based on input from experts.

6. Conclusion and Future Work

In conclusion, it is imperative to categorize potential solutions and opt for the most practical choices. Decision-making can be quite challenging as it varies depending on the context. Therefore, it's crucial to weigh both the pros and cons of each option. Furthermore, effective decision-making is beneficial for your overall well-being and enhances the chances of identifying the most suitable choice. It's vital to determine the exact amount of essential information that decision-makers need. In the decision-making model, the most efficient strategy involves closely focusing on your objectives.

In the article, we initially explored the fundamental concept of three-way decisions introduced by Yao [25] and the utilization of power aggregation operators. We devised an innovative approach for discretizing the information table. For classifying participants, we employed interval-valued classes, creating three zones based on these classes. The use of aggregation operators is highly advantageous for consolidating results and combining attribute values into single values. Given the significance of these operators, we employed power aggregation operators. Furthermore, we developed an algorithm for disease identification utilizing the suggested method.

Moving forward, the outcomes of this study will be extended to encompass fuzzy and rough data [41- 44], and we will devise novel aggregation operators to address real-life issues.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contributions
All authors contributed equally to this research.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Funding

This research was not supported by any funding agency or institute.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

References

1. Sox, H. C. (2007). Medical decision making. ACP Press.
2. Chen, E. L., Chung, P. C., Chen, C. L., Tsai, H. M., & Chang, C. I. (1998). An automatic diagnostic system for CT liver image classification. IEEE transactions on biomedical engineering, 45(6), 783-794.
3. Szolovits, P., Patil, R. S., & Schwartz, W. B. (1988). Artificial intelligence in medical diagnosis. Annals of internal medi-cine, 108(1), 80-87.
4. Ledley, R. S., & Lusted, L. B. (1959). Reasoning foundations of medical diagnosis: symbolic logic, probability, and value theory aid our understanding of how physicians' reason. Science, 130(3366), 9-21.
5. Awotunde, J. B., Matiluko, O. E., & Fatai, O. W. (2014). Medical diagnosis system using fuzzy logic. African Journal of Computing & ICT, 7(2), 99-106.
6. Pawlak, Z., Grzymala-Busse, J., Slowinski, R., & Ziarko, W. (1995). Rough sets. Communications of the ACM, 38(11), 88-95.
7. Salama, A. S. (2010). Bitopological rough approximations with medical applications. Journal of King Saud Universi-ty-Science, 22(3), 177-183.
8. El-Bably, M. K., Abu-Gdairi, R., & El-Gayar, M. A. (2023). Medical diagnosis for the problem of Chikungunya disease using soft, rough sets. AIMS Mathematics, 8(4), 9082-9105.
9. Bai, J., Sun, B., Chu, X., Wang, T., Li, H., & Huang, Q. (2022). Neighborhood rough set-based multi-attribute prediction approach and its application of gout patients. Applied Soft Computing, 114, 108127.
10. Ejegwa, P. A., Onyeke, I. C., & Adah, V. (2020). An algorithm for an improved intuitionistic fuzzy correlation measure with medical diagnostic application. Annals of Optimization Theory and Practice, 3(3), 51-66.
11. Hosny, M. (2022). Generalization of rough sets using maximal right neighborhood systems and ideals with medical applications. AIMS Mathematics, 7(7), 13104-13138.
12. Al-Shami, T. M. (2022). Maximal rough neighborhoods with a medical application. Journal of Ambient Intelligence and Humanized Computing, 1-12.
13. Atanassov, K. T. (2012). On intuitionistic fuzzy sets theory (Vol. 283). Springer.
14. Jiang, Q., Huang, J., Jin, X., Wang, P., Zhou, W., & Yao, S. (2023). Medical Image Fusion Using a New Entropy Measure between Intuitionistic Fuzzy Sets Joint Gaussian Curvature Filter. IEEE Transactions on Radiation and Plasma Medical Sciences.
15. Mahmood, T., Ahmmad, J., Ali, Z., & Yang, M. S. (2023). Confidence Level Aggregation Operators Based on Intuition-istic Fuzzy Rough Sets with Application in Medical Diagnosis. IEEE Access, 11, 8674-8688.
16. Mahmood, T., Ali, W., Ali, Z., & Chinram, R. (2021). Power aggregation operators and similarity measures based on improved intuitionistic hesitant fuzzy sets and their applications to multiple attribute decision making. Computer Modeling in Engineering & Sciences, 126(3), 1165-1187.
17. De, S. K., Biswas, R., & Roy, A. R. (2001). An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy sets and Systems, 117(2), 209-213.
18. Davvaz, B., & Hassani Sadrabadi, E. (2016). An application of intuitionistic fuzzy sets in medicine. International journal of biomathematics, 9(03), 1650037.

19. Szmidt, E., & Kacprzyk, J. (2001). Intuitionistic fuzzy sets in intelligent data analysis for medical diagnosis. In Compu-tational Science-ICCS 2001: International Conference San Francisco, CA, USA, May 28—30, 2001 Proceedings, Part II 1 (pp. 263-271). Springer Berlin Heidelberg.
20. Xu, Z. (2011). Approaches to multiple attribute group decision making based on intuitionistic fuzzy power aggregation operators. Knowledge-Based Systems, 24(6), 749-760.
21. Xu, Z., & Yager, R. R. (2006). Some geometric aggregation operators are based on intuitionistic fuzzy sets. International journal of general systems, 35(4), 417-433.
22. Mahmood, T., Ali, W., Ali, Z., & Chinram, R. (2021). Power aggregation operators and similarity measures based on improved intuitionistic hesitant fuzzy sets and their applications to multiple attribute decision making. Computer Modeling in Engineering & Sciences, 126(3), 1165-1187.
23. Senapati, T., Chen, G., Mesiar, R., & Yager, R. R. (2023). Intuitionistic fuzzy geometric aggregation operators in the framework of Aczel-Alsina triangular norms and their application to multiple attribute decision making. Expert Sys-tems with Applications, 212, 118832.
24. Garg, H., Ali, Z., Mahmood, T., Ali, M. R., & Alburaikan, A. (2023). Schweizer-Sklar prioritized aggregation operators for intuitionistic fuzzy information and their application in multi-attribute decision-making. Alexandria Engineering Journal, 67, 229-240.
25. Yao, Y. (2012, August). An outline of a theory of three-way decisions. In International conference on rough sets and current trends in computing (pp. 1-17). Berlin, Heidelberg: Springer Berlin Heidelberg.
26. Yao, Y. (2021). The geometry of three-way decision. Applied Intelligence, 51(9), 6298-6325.
27. Li, Z., Zhang, P., Xie, N., Zhang, G., & Wen, C. F. (2020). A novel three-way decision method in a hybrid information system with images and its application in medical diagnosis. Engineering Applications of Artificial Intelligence, 92, 103651.
28. Li, Z., Xie, N., Huang, D., & Zhang, G. (2020). A three-way decision method in a hybrid decision information system and its application in medical diagnosis. Artificial Intelligence Review, 53, 4707-4736.
29. Hu, J., Chen, D., & Liang, P. (2019). A novel interval three-way concept lattice model with its application in medical diagnosis. Mathematics, 7(1), 103.
30. Hu, B. Q. (2014). Three-way decisions space and three-way decisions. Information sciences, 281, 21-52.
31. Jia, F., & Liu, P. (2019). A novel three-way decision model under multiple-criteria environment. Information Sciences, 471, 29-51.
32. Ye, J., Zhan, J., & Xu, Z. (2020). A novel decision-making approach based on three-way decisions in fuzzy information systems. Information Sciences, 541, 362-390.
33. Ali, W., Shaheen, T., Haq, I. U., Toor, H., Akram, F., Garg, H., ... & Hassan, M. M. (2023). Aczel-Alsina-based aggregation operators for intuitionistic hesitant fuzzy set environment and their application to multiple attribute decision-making process. *AIMS Math*, *8*(8), 18021-18039.
34. Zhang, Q., Xie, Q., & Wang, G. (2018). A novel three-way decision model with decision-theoretic rough sets using utility theory. Knowledge-Based Systems, 159, 321-335.
35. Liu, D., Liang, D., & Wang, C. (2016). A novel three-way decision model based on incomplete information system. Knowledge-Based Systems, 91, 32-45.
36. Yang, J., & Yao, Y. (2021). A three-way decision-based construction of shadowed sets from Atanassov intuitionistic fuzzy sets. Information Sciences, 577, 1-21.
37. Yao, Y., & Deng, X. (2011, August). Sequential three-way decisions with probabilistic rough sets. In IEEE 10th International Conference on Cognitive Informatics and Cognitive Computing (ICCI-CC'11) (pp. 120-125). IEEE.
38. Zhang, Q., Pang, G., & Wang, G. (2020). A novel sequential three-way decisions model based on penalty function. Knowledge-Based Systems, 192, 105350.
39. Dai, J., Chen, T., & Zhang, K. (2023). The intuitionistic fuzzy concept-oriented three-way decision model. Information Sciences, 619, 52-83.
40. Gora, P. (2023). Intuitionistic Fuzzy Modulus Similarity Measure. International Journal of Decision Support System Technology (IJDSST), 15(1), 1-22.

41. Ali, W., Shaheen, T., Haq, I. U., Alballa, T., Alburaikan, A., & El-Wahed Khalifa, H. A. (2023). A Novel Generalization of Q-Rung Orthopair Fuzzy Aczel Alsina Aggregation Operators and Their Application in Wireless Sensor Networks. *Sensors*, *23*(19), 8105.
42. Radenovic, S., Ali, W., Shaheen, T., Haq, I. U., Akram, F., & Toor, H. (2023). Multiple attribute decision-making based on bonferroni mean operators under square root fuzzy set environment. *Journal of Computational and Cognitive Engineering*, *2*(3), 236-248.
43. Haq, I. U., Shaheen, T., Ali, W., Toor, H., Senapati, T., Pilla, F., & Moslem, S. (2023). Novel Fermatean Fuzzy Aczel–Alsina Model for Investment Strategy Selection. *Mathematics*, *11*(14), 3211.
44. Ali, W., Shaheen, T., Toor, H. G., Alballa, T., Alburaikan, A., & Khalifa, H. A. E. W. (2023). An Improved Intuitionistic Fuzzy Decision-Theoretic Rough Set Model and Its Application. *Axioms*, *12*(11), 1003

Received: 05 Jan 2024, **Revised:** 11 Apr 2024,

Accepted: 12 May 2024, **Available online:** 15 May 2024.

Neutrosophic Systems with Applications, Vol. 18, 2024
https://doi.org/10.61356/j.nswa.2024.18263

A Robust Decision-Making Model for Medical Supplies via Selecting Appropriate Unmanned Aerial Vehicle

Amira Salam [1], Mai Mohamed [1,*], Rui Yong [2], and Jun Ye [2]

[1] Faculty of Computers and Informatics, Zagazig University, Zagazig 44519, Sharqiyah, Egypt;
Emails: sallam18@fci.zu.edu.eg; mmgaafar@zu.edu.eg.
[2] School of Civil and Environmental Engineering, Ningbo University, Ningbo, Zhejiang, China;
Emails: yongrui@nbu.edu.cn, yejun1@nbu.edu.cn;

* Correspondence: mmgaafar@zu.edu.eg.

Abstract: Recently, Unmanned Aerial Vehicles (UAVs) have been used in many fields, including the field of health care, especially in delivering the necessary medical equipment and supplies, due to the many advantages they have compared to other traditional methods and the presence of different types of UAVs, to improve healthcare and provide it with the medical supplies and equipment necessary to save the lives of patients. Choosing the appropriate UAV for a specific situation represents a problem facing decision-makers, which is considered a multi-criteria decision-making problem. Since the decision-making process is cumbersome and complex, and deals with uncertainty and ambiguity. In this research, we proposed multi-criteria decision-making (MCDM) model using CRITERIA (Criteria Importance through Intercriteria Correlation) and MARICA (Multi-Attribute Rating Analysis with Ideal Concepts) methods integrated with neutrosophic logic, which is considered a powerful tool in dealing with uncertainty and ambiguity. The CRITIC method calculates the weight of criteria, whereas it takes into account the correlations and relationships between the criteria, whether they are positive or negative, unlike other methods that consider the criteria separately, which allows for a more accurate and comprehensive analysis of the decision problem. The MARICA method is used also to rank the alternatives. It allows decision-makers to evaluate alternatives according to how well they perform across multiple criteria by considering several factors at once. This helps increase the effectiveness of judgments by taking into account all relevant factors. Moreover, MARICA is a user-friendly method that doesn't require complex mathematical calculations, making it accessible to anyone who wants to make sound choices. The UAV with the highest ranking is the one that will be chosen and represents the best among the alternatives. The proposed model proved its effectiveness by applying it to an experimental case.

Keywords: Unmanned Aerial Vehicle; MCDM; CRITIC; MARICA; Medical Supplies Delivery.

1. Introduction

Healthcare is crucial in saving human lives, and its demand has recently surged. Delivering medical supplies efficiently and reliably has become more important than ever, particularly after the outbreak of the coronavirus pandemic. This includes delivering necessary equipment and supplies to patients to provide them with the healthcare they need. Timely delivery of medical supplies is critical to saving lives, and traditional transportation and delivery operations often face obstacles in terms of delivering the package late or some damage, such as breakage and corruption, Therefore, a solution must be sought.

With the spread of information technology and the Internet of Things (IoT), which has contributed to the development of health care [1], unmanned aircraft systems have been included in the development of transportation and delivery operations, especially in urban areas, due to their characteristics [2]. UAVs have proven to be a highly versatile tool across various industries, playing a crucial role in addressing several pressing issues, it was used in agriculture [3], was used in animal wealth, as it contributed to the effective detection of and counting of livestock [4], was used in water management [5]. Similarly, in the medical field, UAVs have brought about a significant breakthrough in delivering medical equipment to remote areas that are difficult to access through conventional means. This has been instrumental in ensuring that medical aid reaches those in need in a timely and efficient manner [6]. UAVs are increasingly being recognized as a viable option for delivering medical resources and equipment. They offer several advantages over traditional delivery methods, such as their high speed, ease of deployment, and ability to access remote areas that are difficult to reach otherwise [7-9]. Additionally, drones are highly resistant to wind, making them suitable for delivering packages even in challenging weather conditions while ensuring the safety of the items being transported [10]. The authors discussed the limitations of prehospital blood transfusion in military settings, and the potential uses of UAVs for medical logistics [11]. Comparisons were made and it was proven that using UAVs to transport medical supplies to healthcare facilities is more cost-effective and environmentally friendly than using traditional techniques has been demonstrated [12].

Because there is a wide range of UAVs on the market, each with its own set of features, choosing the best UAV type to meet a given situation can be difficult and restrictive for decision-makers, they all have distinct goals and perspectives. To select the finest one, a methodical approach is therefore required between options based on the applied criteria. Therefore, choosing and evaluating UAVs and using them in the process of delivering medical supplies represents a challenge for multi-criteria decision-making.

MCDM is a technique that involves analyzing the various available options in a situation and has been used to choose the best UAV to be used to deliver medical supplies and equipment. Some authors aimed to highlight the evolution and significance of MCDM approaches in military healthcare by examining the literature's different applications of MCDM methods in the military and healthcare domains [13]. The interval-valued Pythagorean fuzzy VIKOR approach and the interval-valued Pythagorean fuzzy analytic hierarchy process were used to select UAVs for transporting medical supplies between disaster zones and warehouses [14]. The authors provided a comprehensive set of criteria for comparing various last-mile drone options, which used the interval-valued inferential fuzzy TOPSIS method which is a systematic decision-making strategy and handling uncertainty [15].

The aforementioned studies have demonstrated that utilizing MCDM technology enables one to arrive at informed decisions. Therefore, in this research we present a method to evaluate UAVs and choose the best among the alternatives, which are used in the operations of delivering and supplying medical supplies, using a new MCDM model in the context of neutrosophic logic.

This research aims to help decision-makers make the best decision based on an organized and effective methodology based on expert's opinions. Therefore, to select the best UAV for medical supply delivery, the problem was formulated as a MCDM problem.

Utilizing MCDM technology to evaluate the best UAV suitable for delivering the necessary medical supplies through:

- Applying the CRITIC (Criteria Importance Through Intercriteria Correlation) method, to calculate the weight of criteria and sub-criteria related to UAVs used for delivering medical supplies.
- Applying the MARICA (Multi-Attribute Rating Analysis with Ideal Concepts) method for ranking the alternatives depending on the weight calculated by CRITIC, this is in the context of the concept of truth, falsity, and indeterminacy (T,I, and$F$) membership.

Also, the proposed method to evaluate the best UAV is simple and has the great ability to deal with uncertainty phenomena and solve the ambiguous information that commonly arises in the decision-making process.

The remaining parts of our research are provided below for processing purposes. In section 2, a proposed methodology for selecting the best UAV among the alternatives that are used in medical supply delivery is described. In section 3, a case study for selecting the best UAV is solved to demonstrate the method's applicability in a neutrosophic environment. In section 4, the managerial implications are presented. This research's conclusions and recommendations for the future are presented in Section 5.

2. Methodology

Our model utilizes two MCDM techniques for selecting the best UAV among the alternatives that are used in medical supply delivery. We are using the CRITIC as an MCDM method to get weights of criteria, and we are using the MARICA method to rank the UAV according to the weights that are obtained from the CRITIC. Figure 1 shows, the flowchart of our model. Our model consisted of several steps as follows:

Step 1: (Define the experts based on the area of concern): Experts are people with great experience and have high knowledge in the field of UAV devices.

Step 2: Determined list of evaluation (criteria and sub-criteria) and alternatives based on expert opinions, let C be a set of criteria$C = \{c1, c2 \dots\dots cn\}$, where $c1, c2 \dots\dots cn$ are main criteria in each criteria Ci, $1 < i < n$ is formed by sub-criteria: $C1 = \{c11, c12 \dots.\}$, $C2 = \{c21, c22 \dots.\}$. Let's consider $A = \{A_1, A_2, A_3, A_4\}$ be a set of alternatives representing the UAV's type.

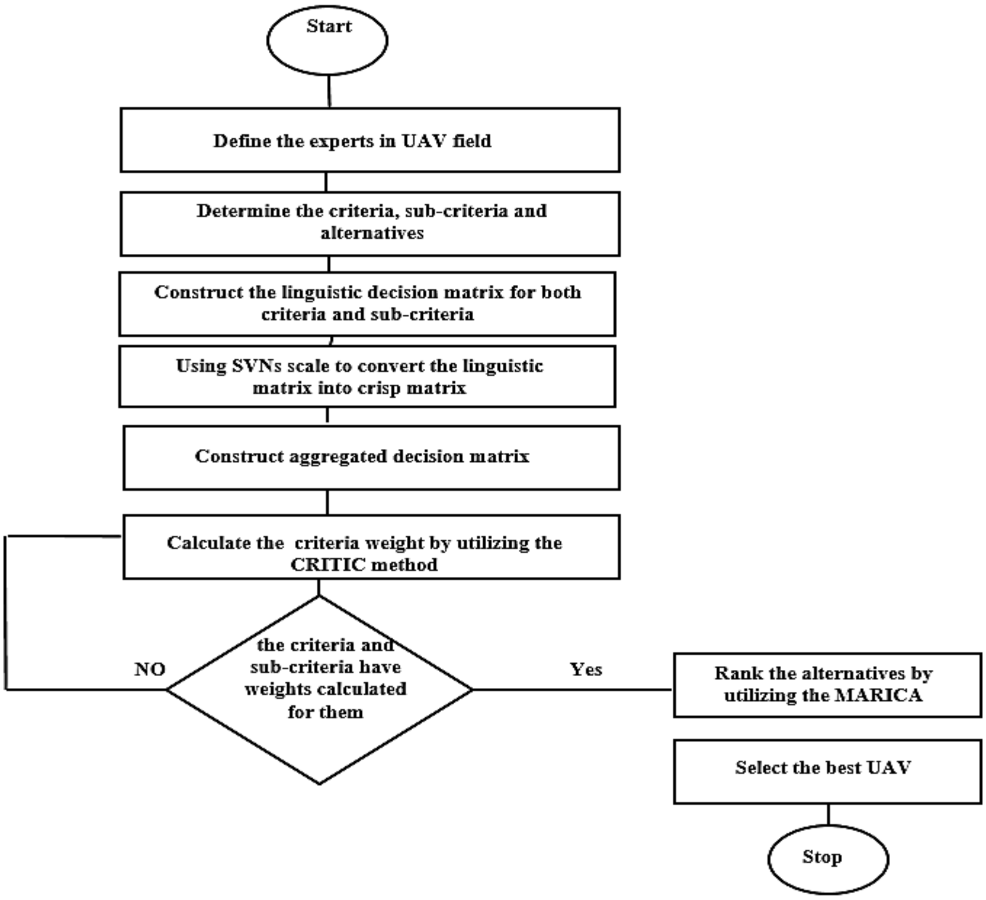

Figure 1. The flowchart of our model.

Step 3: (Expert decision matrix): When making decisions, we often encounter ambiguity, as all decisions usually involve uncertain or unclear information. However, simply using linguistic variables to address uncertainty is not enough. To tackle the problem of linguistic ambiguity, we propose the use of the neutrosophic group which is capable of dealing with the ambiguous information that commonly arises in the decision-making process. Thus, we use a single-valued neutrosophic scale (SVNs) to convert the linguistic scale into a corresponding numerical scale, using the terms used by experts to construct decision matrices. Each term used by experts has a set of characteristics including truth, indeterminacy, and falsity, collectively referred to as SVNS. As shown in Table 1. After collecting the SVNS data, it can be converted into a distinct value that is compatible with the proposed model. It is important to note that the neutrosophic matrix can be transformed into a crisp matrix using the scoring function represented in Eq. (1) [16].

$$Score\ Function = \frac{2+(Tr-F-Id)}{3} \quad (1)$$

Where Tr, F, Id refers to truth, false, and indeterminacy respectively.

Step 4: (Construct aggregated decision matrix): Because we have more than one expert and each of them has its decision matrix, the experts' matrices must be collected into one decision matrix called aggregated decision matrix by using Eq. (2).

$$Y_{ij} = \frac{\sum_{j=1}^{N} g_{ij}}{N} \quad (2)$$

Where g_{ij} represents the value of criterion in the matrix, N represents the number of experts.

Step 5: (CRITIC method): To determine criteria weights of relative importance. Where, the standard deviation score is used to measure the degree of variety and dispute, and determines the relationship between each attribute using the correlation coefficient between them. The CRITIC method was introduced by Diakoulaki in 1995 and can be summarized into the following steps [17].

Step 5.1: Normalized aggregated decision matrix by applying Eq. (3) as follows:

$$x_{ij}^{-} = \frac{x_{ij} - x_{worst}}{x_{best} - x_{worst}} \quad , i = 1,2 \ldots .m \, , j = 1,2 \ldots n \quad (3)$$

Where, x_{ij}^{-} is the normalized performance score of ith alternative on jth criteria, x_{worst} is the worst score of criteria j and the x_{best} is the best score of criteria j, where m is the number of alternatives and n is the number of criteria.

Step 5.2: Calculate the standard division of each criteria by applying Eq. (4) as follows:

$$\sigma_j = \sqrt{\frac{(\sum_{i=1}^{m} x_{ij} - x_j^{-})^{\wedge}2}{m-1}} \quad (4)$$

Where x_j^{-} the mean score of the criterion is j calculated from Eq. (3), and m is the number of alternatives.

Step 5.3: Determine the symmetric matrix of $n * n$ with the element r_{jk}, which is the linear correlation coefficient between the vector x_j and x_k , It can be seen that the more discordant the scores of the alternatives in criteria j and k, the lower the value r_{ik}.

Step 5.4: Calculate the measure of the conflict created by criterion j with respect to the decision situation defined by the rest of the criteria, by applying Eq. (5) as follows:

$$Con = \sum_{k=1}^{m} (1 - r_{jk}) \quad (5)$$

Step 5.5: Determine the quantity of the information in relation to each criterion, by applying Eq. (6) as follows:

$$C_j = \sigma_j * \sum_{k=1}^{m} (1 - r_{jk}) \quad (6)$$

Step 5.6: Determine the criteria weights by applying Eq. (7) as follows:

$$w_j = \frac{C_j}{\sum_{k=1}^{m} C_j} \quad (7)$$

Step 6: (MARICA method): We utilize the MARICA method to rank the alternatives, the MARICA method was introduced by Pamucar et al in 2014 [18]. By the MARICA method, the overall gap for each alternative is calculated by summing the gaps for each criterion, After that, the alternatives are ranked, and the alternative with the lowest value of the total gap is the best alternative that will be chosen, where, the alternative with the smallest overall gap is the one that has the most similar values to the ideal values of the criterion across the greatest number of criteria. The MARICA is implemented through the following:

Step 6.1: Calculating decision matrix, we used the aggregated matrix that we calculated before in step 4 as the decision matrix for the MARICA method.

Step 6.2: Establishment of preferences according to alternatives p_{Ai} choice.

$$p_{A_i} = \frac{1}{m} \; ; \; \sum_{i=1}^{m} p_{A_i} = 1 \quad , i = 1,2 \ldots m \tag{8}$$

Where m is the total number of alternatives, take into account that all preferences of the individual alternatives are equal:

$$p_{A_1} = p_{A_2} = \cdots p_{A_m} \tag{9}$$

Step 6.3: Calculation of the matrix element of theoretical evaluation T_p with size $(n \; x \; 1)$ as follows:

$$T_p = p_{A_i} \left[p_{A_1} * w_1 \quad p_{A_2} * w_2 \quad \ldots \quad p_{A_i} * w_n \right] \tag{10}$$

Where n is the number of criteria and w_n is the criteria weight coefficients that we calculated before by CRITIC method.

Step 6.4: Calculation of the actual evaluation matrix T_r as follows:

$$T_r = \begin{matrix} A_1 \\ A_2 \\ A_m \end{matrix} \begin{bmatrix} t_{r11} & \ldots & t_{r1n} \\ \vdots & \ldots & t_{r2n} \\ t_{rm1} & \ldots & t_{rmn} \end{bmatrix} \tag{11}$$

Where n is the number of criteria and m is the number of alternatives. The T_r is determined by multiplying the matrix elements of the theoretical evaluation T_p and the elements of the initial decision matrix (X) according to the expression:

- For criteria of (benefit type):

$$t_{rij} = t_{pij} \left(\frac{x_{ij} - x_i^-}{x_i^+ - x_i^-} \right) \tag{12}$$

- For criteria of (non-benefit type):

$$t_{rij} = t_{pij} \left(\frac{x_{ij} - x_i^+}{x_i^- - x_i^+} \right) \tag{13}$$

Step 6.5: Calculation of the total gap matrix (G): the elements of the matrix are obtained as the difference (gab) between the t_{rij} and the t_{pij}

$$G = \begin{bmatrix} t_{p11} - t_{r11} & \ldots & t_{p1n} - t_{r1n} \\ \vdots & \ldots & t_{p2n} - t_{r2n} \\ t_{pm1} - t_{rm1} & \ldots & t_{pmn} - t_{rmn} \end{bmatrix} \tag{14}$$

Step 6.6: Calculation of the final value of criterion functions (Q_i) by alternatives, calculated as follows:

$$Q_i = \sum_{j=1}^{n} g_{ij} \quad , i = 1,2 \ldots .m \tag{15}$$

Step 6.7: Ranking of the alternatives.

Table 1. Single-valued neutrosophic scale (SVNs) [16].

Variables of Linguistic	Abbreviation	SVNs		
		Tr	Id	F
Extremely Bad	EB	0.00	1.00	1.00
Very Very Bad	VVB	0.10	0.90	0.90
Very Bad	VB	0.20	0.85	0.80
Bad	B	0.30	0.75	0.70
Medium Bad	MB	0.40	0.65	0.60
Medium	M	0.50	0.50	0.50
Medium Good	MG	0.60	0.35	0.40
Good	G	0.70	0.25	0.30
Very Good	VG	0.80	0.15	0.20
Very Very Good	VVG	0.90	0.10	0.10
Extremely Good	EG	1.00	0.00	0.00

3. Case Study (Result and Analysis)

In our study, We will conduct an experiment study to evaluate our proposed model to choose the best UAV to deliver medical equipment, as there is a need to deliver the ICD device and blood bags from Dr. Magdy Yacoub Hospital in Aswan City to Dar Al Fouad Hospital in Cairo city to perform a heart surgery necessary to save a patient's life. This device is small in size, and the distance between Aswan and Cairo is about 906 kilometers, which takes an arrival time of about 11 hours using traditional methods. Thus, the UAV is used to transport the ICD device and the necessary blood bags to Dar Al Fouad Hospital in Cairo instead of traditional methods, due to the importance of time and the safe arrival of the package. Therefore the selection of suitable UAVs is a hard task.

We are introducing a new method to assist decision-makers in selecting the most appropriate UAV from a set of UAVs for delivering medical supplies taking into account factors such as time and package delivery integrity. We assume that there are four UAVs (alternatives), each with unique characteristics (criteria) that are denoted as $A = \{A_1, A_2, A_3, A_4\}$ and that there are four decision-makers with extensive knowledge of a particular subject.

Step 1: We assume that there are four experts $\{expert1, expert2,\ expert3,$ and $expert4\}$ as follows: Expert1&2: have a PhD degree in the aeronautical engineering field. Expert 3: have a PhD degree in the medical field. Expert 4: have a PhD degree in the machine learning field. The expert who possesses extensive experience and high knowledge in designing, operating, and maintaining UAVs. All of them have the same level of expertise. The experts will evaluate the judgment comparison of the main criteria based on their area of concern.

Step 2: The selection of a UAV involves assessing the importance of various criteria, which differ from one alternative to another. Hence, it is crucial to determine and define the criteria to be used in medical supply delivery. In this research, we will outline criteria that are collected from previous research [19, 20]. In this research, we divide criteria into main criteria and others branching from them(sub-criteria), three main criteria $\{C1, C2, C3\}$ that have been defined for choosing the best UAV for medical supply delivery, and each of them includes sub-criteria {payload, speed, distance, control system, safety, Clock synchronization and flight time} which denoted as $\{C11, C12, C13, C21, C22, C31, C32\}$ respectively, as shown in Table 2. The criteria and sub-criteria described as follows: the main criteria= $\{C1, C2, C3\}$, where $C1$ = performance, $C2$ = physical feature and $C3 =$ timing. The sub-criteria $C1 = \{C11, C12, C13\}$ = {payload, speed, distance}, $C2 = \{C21, C22\}$ = {control system, safety} and $C3 = \{C31, C32\}$= {Clock synchronization, flight time}. The performance criterion $(C1)$ can be determined by its payload which, refers to the maximum weight that the drone can carry, which affects the process of delivering medical equipment and supplies, as the UAV has a high payload, and can carry heavy equipment effectively and with low cost. Besides the speed of the UAV in delivering the package. Besides, the distance /criterion refers to the maximum distance that an unmanned aircraft can travel at one time. The physical feature criterion$(C2)$, includes safety, which refers to the protection system used in the UAV to ensure that the package arrives safely, in addition to the control system, which refers to how much manual labor is needed to operate the UAV. The timing criterion,$(C3)$ can determined by clock synchronization to ensure the success of UAV delivery operations, it is crucial to have real-time clock synchronization. This synchronization helps to prevent delays, errors, and other issues by ensuring accurate timing throughout the delivery process. By implementing real-time clock synchronization, UAV delivery companies can ensure smooth and efficient operations. Besides flight time which, refers to the maximum period of time that the UAV can fly in the air.

Table 2. The main criteria and sub-criteria of our model.

Main criteria	Sub-criteria
performance C1	Payload C11
	Speed C12
	Distance C13
Physical feature C2	Control system C21
	Safety C22
Timing C3	Clock synchronization C31
	Flight time C32

Step 3: Four experts start to evaluate the main criteria, as shown in Tables 3, 4, 5, and 6, then the expert's decision matrices will converted into crisp matrices by utilizing Eq. (1), using the scale in Table 1, as shown in Tables 7,8,9 and 10.
Step 4: All the crisp decision matrices must collected into one aggregated matrix by utilizing Eq. (2), as shown in Table 11.
Step 5: After collecting the expert decision matrices into one aggregated matrix, the CRITIC method will be utilized to get the criteria weights, firstly we construct the normalized matrix for the main criteria based on the CRITIC method by utilizing Eq. (3), as shown in Table 12. Table 13 shows, the standard division of each criterion by utilizing Eq. (4). Table 14 shows, the linear correlation coefficient symmetric matrix between each pair of the main criteria. Table 15 shows, the measure of the conflict by utilizing Eq. (5). Table 16 shows, the final weight of the main criteria by calculation of the quantity of the information in relation to each criterion by utilizing Eq. (6) and (7), where the timing (C3) is the highly preferred criterion to other criteria with weight equal to 0.03042516 and final ranking of the main criteria as $C3 > C2 > C1$, as shown in Figure 2. To calculate the weight of sub-criteria, we will repeat the steps from step three to step five, as we did in the main criteria, thus:
For the performance sub-criteria, after the experts evaluate the performance sub-criteria, we will convert the expert's decision matrices into a crisp matrix by utilizing Eq. (1), these matrices are collected into one aggregated matrix by applying Eq. (2) as shown in Table 17. We apply the CRITIC method on the aggregated matrix to get the performance sub-criteria weight, as shown in Table 18. Figure 3, shows that the payload $(C11)$is the highly preferred performance sub-criteria over the other performance sub-criteria with a weight equal to 0.441945, and the final ranking of the performance sub-criteria as$C11 > C13 > C12$, as shown in Figure 3.
For physical feature sub-criteria, Table 19, shows the aggregated matrix of physical feature sub-criteria by utilizing Eq. (2). Table 20, shows the calculation of the physical feature sub-criteria weight by the CRITIC method. Figure 4, shows that the $C22$ is the highly preferred physical feature sub-criteria over the other physical feature sub-criteria with a weight equal to 0.501386, and the final ranking of the physical feature sub-criteria as $C22 > C21$. For timing sub-criteria in level 2, Table 21, shows the aggregated matrix of the timing sub-criteria by utilizing Eq. (2). Table 22 shows the calculation of the timing sub-criteria weight in level 2 by the CRITIC method. As shown in Figure 5, $C31$ is the highly preferred sub-criteria in level 2 over the other timing sub-criteria with a weight equal to 0.501905. After completing the calculation of the weights of all sub-criteria, we can obtain the final weights for the criteria as shown in Table 23. Figure 6 shows that the $C31$ is the highly preferred criterion over the other criteria with a weight equal to 0.220895, $C31 > C32 > C22 > C21 > C11 > C13 > C12$. As shown, the time criterion followed by the safety criterion are the high priority based on the presented scenario.
Step 6: After calculating the weight of the main criteria and sub-criteria, apply the MARICA method to rank the alternatives and choose the best UAV suitable for our scenario. For the main criteria: firstly the aggregated matrix in Table 11 is represented as the decision matrix, then establishes the preferences according to alternatives p_{Ai} by utilizing Eq. (8), in our scenario $p_{Ai} = \frac{1}{4} = 0.25$. The theoretical evaluation matrix T_p is calculated by utilizing Eq. (10) using the weight of the main criteria in Table 16 that were calculated before by the CRITIC method, as shown in Table 24. Table 25 shows the actual theoretical evaluation matrix T_r by utilizing Eq. (12), note that all the criteria are benefit criteria. Table 26 shows, the total gap matrix by utilizing Eq. (14). Table 27 shows, the final value of criterion functions (Q_i) by alternatives that are calculated by utilizing Eq. (15). According to Figure 7, $A2$ is the one with the highest rank, whereas, the alternative with the lowest value of the total gap (Q_i), is the best alternative that will be chosen; thus, the alternatives ranked as$A2 > A4 > A3 > A1$. So, decision-makers will choose the $A2$ for medical supply delivery in our scenario. For the sub-criteria: apply the same MARICA method steps thus, Table 28 shows, the final value of criterion functions (Q_i) by alternatives that are calculated by utilizing Eq. (15) in the performance sub-criteria,

note that, we using Table 17 as the decision matrix of the performance sub-criteria and using the weight in Table 18 that was calculated before by the CRITIC method. Figure 8 shows that A2 is the one with the highest rank according to the performance sub-criteria, where, A2 is the lowest value of the total gap (Q_i) in performance sub-criteria, thus, the alternatives ranked as $2 > A4 > A3 > A1$. Then, A2 is the best alternative that will be chosen according to performance sub-criteria. Table 29 shows, the final value of criterion functions (Q_i) by alternatives that are calculated by utilizing Eq. (15) in the physical feature sub-criteria using Table 19 as the decision matrix of the physical feature sub-criteria and using the weight in Table 20 that was calculated before by the CRITIC method. Figure 9 shows that, also A2 is the one with the highest rank according to physical feature sub-criteria. Table 30 shows, the final value of criterion functions (Q_i) by alternatives according to timing sub-criteria. Figure 10 shows that, also A2 is the one with the highest rank according to timing sub-criteria. According to the previous results, the best UAV according to the proposed scenario is A2.

Table 3. Decision matrix of Expert1 for the main criteria.

Alternatives	Main Criteria		
	C1	C2	C3
A1	VB	MB	B
A2	VVG	G	VVG
A3	M	MG	G
A4	G	VG	MG

Table 4. Decision matrix of Expert2 for the main criteria.

Alternatives	Main Criteria		
	C1	C2	C3
A1	MB	B	VB
A2	EG	VG	VVG
A3	MG	G	M
A4	VG	G	MG

Table 5. Decision matrix of Expert3 for the main criteria.

Alternatives	Main Criteria		
	C1	C2	C3
A1	B	VB	MB
A2	G	VVG	VG
A3	G	M	G
A4	VG	MG	VG

Table 6. Decision matrix of Expert4 for the main criteria.

Alternatives	Main Criteria		
	C1	C2	C3
A1	MB	MB	VB
A2	EG	VG	VVG
A3	M	G	MG
A4	MG	VG	G

Table 7. Crisp decision matrix of Expert1 for the main criteria.

Alternatives	Main Criteria in Level 1		
	C1	C2	C3
A1	0.1833333	0.3833333	0.2833333
A2	0.9	0.716667	0.9
A3	0.5	0.616667	0.716667
A4	0.716667	0.816667	0.616667

Table 8. Crisp decision matrix of Expert2 for the main criteria.

Alternatives	Main Criteria in Level 1		
	C1	C2	C3
A1	0.3833333	0.2833333	0.616667
A2	1	0.816667	0.9
A3	0.616667	0.716667	0.5
A4	0.816667	0.716667	0.616667

Table 9. Crisp decision matrix of Expert3 for the main criteria.

Alternatives	Main Criteria in Level 1		
	C1	C2	C3
A1	0.2833333	0.61666667	0.383333333
A2	0.716667	0.9	0.81666667
A3	0.716667	0.5	0.71666667
A4	0.816667	0.61666667	0.81666667

Table 10. Crisp decision matrix of Expert4 for the main criteria.

Alternatives	Main Criteria in Level 1		
	C1	C2	C3
A1	0.3833333	0.3833333	0.616667
A2	1	0.816667	0.9
A3	0.5	0.716667	0.616667
A4	0.616667	0.816667	0.716667

Table 11. Aggregated matrix for the main criteria.

Alternatives	Main Criteria		
	C1	C2	C3
A1	0.30833333	0.41666667	0.475
A2	0.90416667	0.8125	0.879167
A3	0.58333333	0.6375	0.6375
A4	0.74166667	0.74166667	0.691667

Table 12. Normalized matrix for main criteria.

Alternatives	Main Criteria		
	C1	C2	C3
A1	0	0	0
A2	1	1	1
A3	0.46153846	0.55789473	0.402061524
A4	0.72727273	0.82105264	0.536082857

Table 13. The standard division of each main criterion.

Alternatives	Main Criteria		
	$C1$	$C2$	$C3$
$\boldsymbol{A1}$	0	0	0
$\boldsymbol{A2}$	1	1	1
$\boldsymbol{A3}$	0.46153846	0.55789473	0.402061524
$\boldsymbol{A4}$	0.72727273	0.82105264	0.536082857
$\boldsymbol{\sigma_j = \sqrt{\frac{(\sum_{i=1}^{m} x_{ij} - x_j^{-})^{\wedge}2}{m-1}}}$	0.42591852	0.43609109	0.412285252

Table 14. The linear correlation coefficient symmetric matrix.

	Main Criteria		
	C1	C2	C3
C1	1	0.9921657	0.97738703
C2	0.9921657	1	0.94881659
C3	0.97738703	0.94881659	1

Table 15. The measure of the conflict.

	Main Criteria			Con $= \sum_{k=1}^{m}(1 - r_{jk})$
	C1	C2	C3	
C1	0	0.0078343	0.02261297	0.0304473
C2	0.0078343	0	0.05118341	0.0590177
C3	0.02261297	0.05118341	0	0.0737964

Table 16. The weight of the main criteria.

Main criteria	$\sigma_j = \sqrt{\frac{(\sum_{i=1}^{m} x_{ij} - x_j^{-})^2}{m-1}}$	Con $= \sum_{k=1}^{m}(1 - r_{jk})$	$C_j = \sigma_j * \sum_{k=1}^{m}(1 - r_{jk})$	$w_j = \frac{C_j}{\sum_{k=1}^{m} C_j}$	Percentage weight
C1	0.4259185	0.03044726	0.01296805	0.18758855	18.70%
C2	0.4360911	0.0590177	0.02573709	0.372298287	37.20%
C3	0.4122853	0.07379637	0.03042516	0.440113164	44.00%

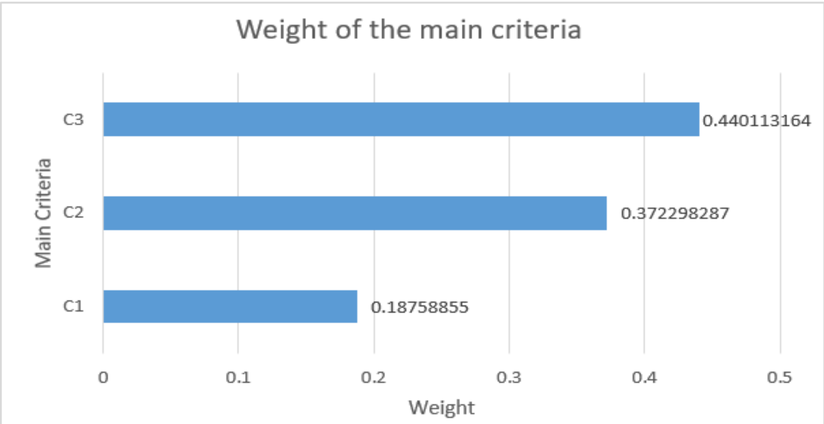

Figure 2. The weight of the main criteria by the CRITIC method.

Table 17. Aggregated matrix for the performance sub-criteria.

Alternatives	Performance Sub-criteria		
	C11	C12	C13
A1	0.14166667	0.25833333	0.20833333
A2	0.7875	0.95	0.74166667
A3	0.44583333	0.52916667	0.3875
A4	0.64166667	0.76666667	0.58333333

Table 18. The weight of the performance sub-criteria.

Performance Sub-criteria	$\sigma_j = \sqrt{\frac{(\sum_{i=1}^{m} x_{ij} - x_j^{-})^2}{m-1}}$	Con $= \sum_{k=1}^{m}(1 - r_{jk})$	$C_j = \sigma_j * \sum_{k=1}^{m}(1 - r_{jk})$	$w_j = \frac{C_j}{\sum_{k=1}^{m} C_j}$	Percentage weight
C11	0.4324346	0.01504891	0.006508	0.441945	44.10%
C12	0.4331873	0.0057793	0.002504	0.170018	17.00%
C13	0.4350459	0.01313395	0.005714	0.388037	38.80%

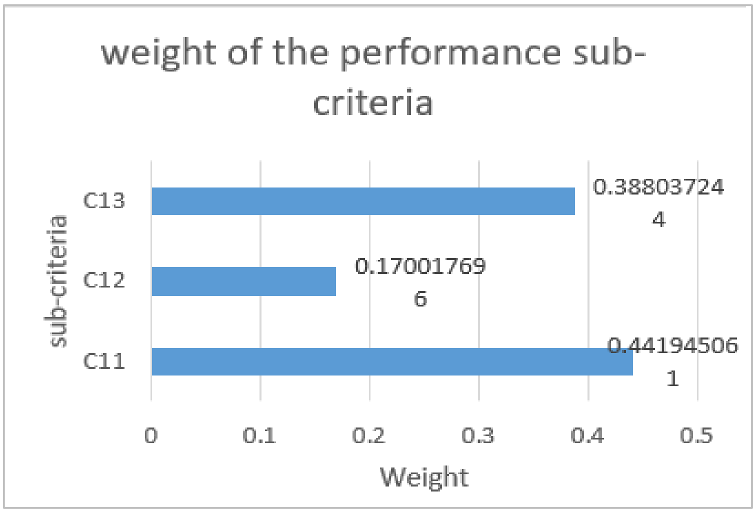

Figure 3. The weight of the performance sub-criteria by the CRITIC method.

Table 19. Aggregated matrix for the physical feature sub-criteria.

Alternatives	**The physical Feature Sub-criteria**	
	C21	C22
A1	0.2583333	0.14583333
A2	0.9041667	0.8125
A3	0.6916667	0.58333333
A4	0.7916667	0.70833333

Table 20. The weight of the physical feature sub-criteria.

Physical feature Sub-criteria	$\sigma_j = \sqrt{\frac{(\sum_{i=1}^{m} x_{ij} - x_j^{-})^2}{m-1}}$	Con $= \sum_{k=1}^{m}(1 - r_{jk})$	$C_j = \sigma_j * \sum_{k=1}^{m}(1 - r_{jk})$	$w_j = \frac{C_j}{\sum_{k=1}^{m} C_j}$	Percentage weight
C21	0.437296	0.0004492	0.000196	0.498614	49.80%
C22	0.439726	0.0004492	0.000198	0.501386	50.10%

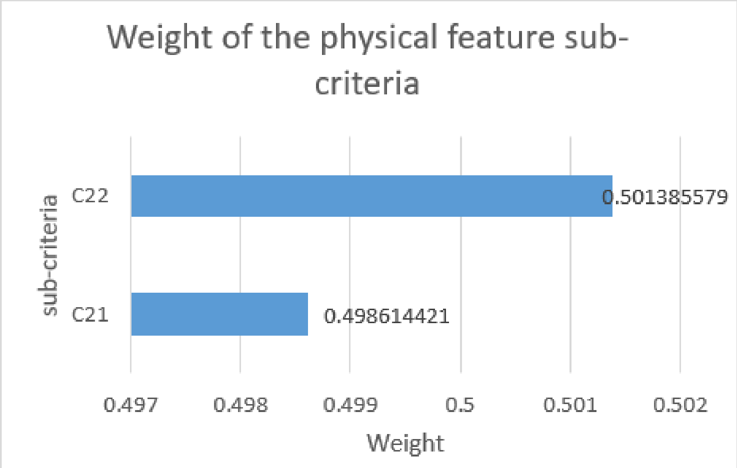

Figure 4. The weight of the physical feature sub-criteria by CRITIC method.

Table 21. Aggregated matrix for the timing sub-criteria.

Alternatives	**The Timing Sub-criteria**	
	C31	C32
A1	0.3	0.2625
A2	0.95	0.88333333
A3	0.58333333	0.58333333
A4	0.71666667	0.6625

Table 22. The weight of timing sub-criteria.

Physical feature Sub-criteria in Level 2	$\sigma_j = \sqrt{\frac{(\sum_{i=1}^{m} x_{ij} - x_j^-)^2}{m-1}}$	**Con** $= \sum_{k=1}^{m}(1 - r_{jk})$	$C_j = \sigma_j * \sum_{k=1}^{m}(1 - r_{jk})$	$w_j = \frac{C_j}{\sum_{k=1}^{m} C_j}$	**Percentage weight**
C31	0.41734	0.004582	0.001912	0.501905	50.19%
C32	0.414172	0.004582	0.001898	0.498095	49.80%

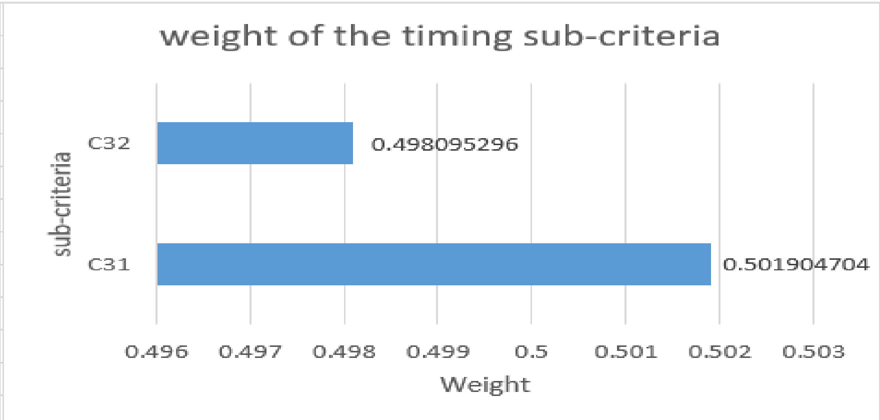

Figure 5. The weight of the timing sub-criteria by CRITIC method.

Table 23. The Final weight of the criteria.

Criteria	The Final weight
C11	0.082904
C12	0.031893
C13	0.072791
C21	0.185633
C22	0.186665
C31	0.220895
C32	0.219218

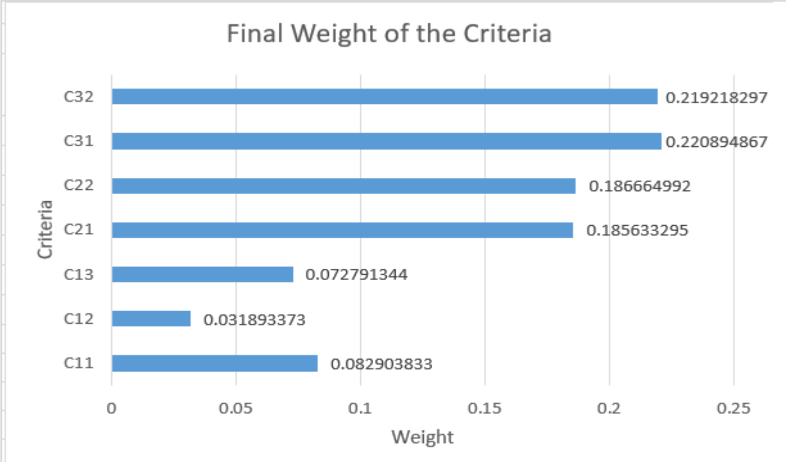

Figure 6. The rank of the final weight of the criteria.

Table 24. The theoretical evaluation matrix in the main criteria

		Main Criteria		
Weights		0.18758855	0.372298287	0.440113164
PA	0.25	C1	C2	C3
$\mathbf{T_p}$		0.04689714	0.093074572	0.110028291

Table 25. The actual theoretical evaluation matrix T_r in the main criteria.

	Main Criteria		
Weights	0.18758855	0.372298287	0.440113164
Alternatives	C1	C2	C3
A1	0	0	0
A2	0.046897138	0.093074572	0.110028291
A3	0.021644832	0.051925813	0.044238142
A4	0.034107009	0.076419123	0.058984281

Table 26. The total gap matrix G in the main criteria.

	Main Criteria		
Weights	0.18758855	0.372298287	0.440113164
Alternatives	C1	C2	C3
A1	0.046897138	0.093074572	0.110028291
A2	0	0	0
A3	0.025252305	0.041148758	0.065790149
A4	0.012790128	0.016655449	0.05104401

Table 27. The final value of criterion functions (Q_i) according to the main criteria.

Alternatives	Qi
A1	0.25
A2	0
A3	0.132191212
A4	0.080489588

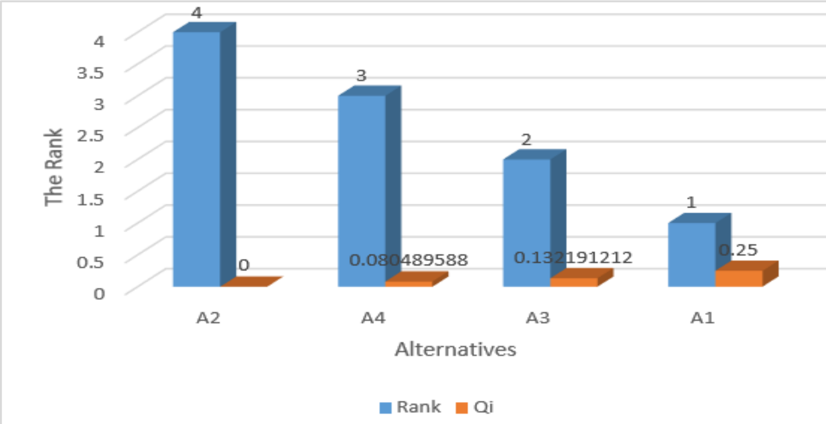

Figure 7. The Rank of alternatives according to the main criteria.

Table 28. The final value of criterion functions (Q_i) according to the performance sub-criteria

Alternatives	Qi
A1	0.25
A2	0
A3	0.148732
A4	0.042832

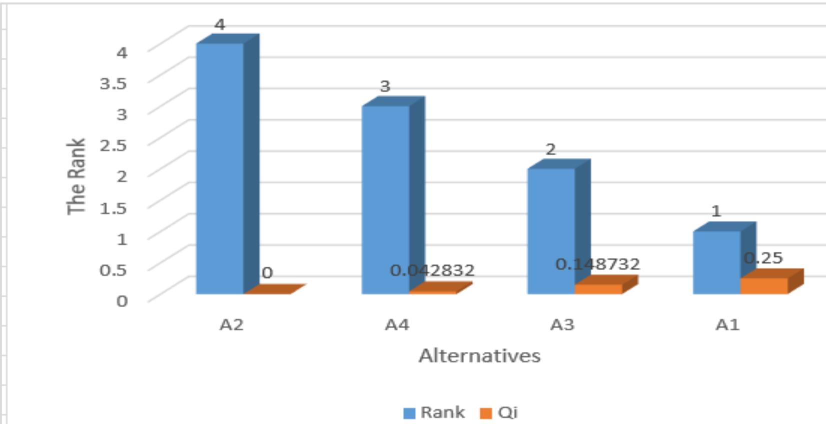

Figure 8. The Rank of alternatives according to the performance sub-criteria.

Table 29. The final value of criterion functions (Q_i) according to the physical feature sub-criteria

Alternatives	Qi
A1	0.25
A2	0
A3	0.084102878
A4	0.041299224

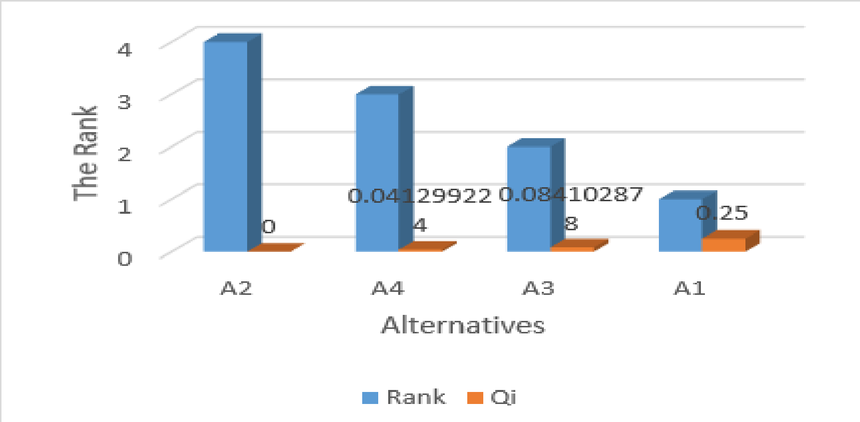

Figure 9. The Rank of alternatives according to the physical feature sub-criteria.

Table 30. The final value of criterion functions (Q_i) according to the timing sub-criteria

Alternatives	**Qi**
A1	0.25
A2	0
A3	0.130954026
A4	0.089336438

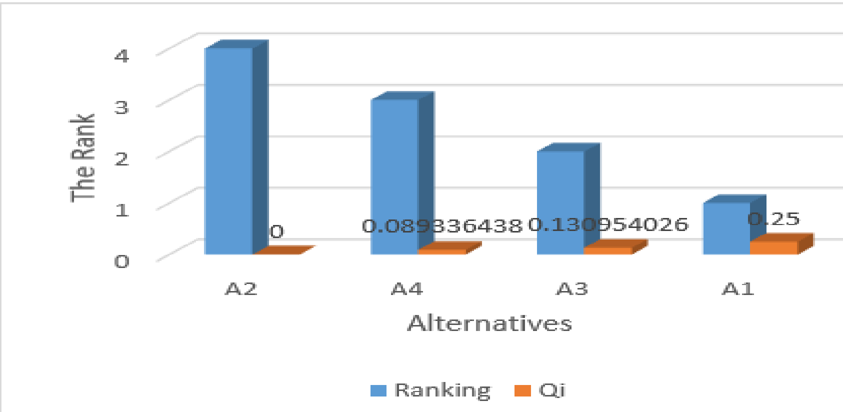

Figure 10. The Rank of alternatives according to the timing sub-criteria.

4. Managerial implications

Since the selection process is a complex and hard mission due to numerous and conflicting criteria that exist nowadays, so we need an efficient and effective MCDM technique. Therefore, in this research, we present a neutrosophic model to evaluate UAVs and choose the best among the alternatives, which are used in the operations of delivering and supplying medical supplies. The presented model can be a dominant guide for firms, organizations, and governments to make precise decisions about any medical, social, economic, and environmental problems.

5. Conclusion and Future Work

A new MCDM model was proposed to evaluate UAVs and choose the appropriate one among the set of UAVs for the process of delivering medical supplies to improve health care and contribute to saving patients. The experiment study results demonstrated that the proposed model is capable of dealing with ambiguity in decision problems effectively. In addition, it takes into account the inter-correlation between the criteria, whether positive or negative, and determines the priority of the criteria and weighting them effectively by applying the CRITICA method. Also, using the MARICA Method, allows for effective evaluation of alternatives, as it provides a symmetric framework and does not require complex mathematical calculations. According to our experimental study, the time and safety factors are the two criteria that are most preferred over the other criteria, and based on them, the best UAV was chosen by applying our model.

In our future work, we will use the CRITIRIA method along with another approach to evaluate alternatives and make comparisons between them.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contributions
All authors contributed equally to this research.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Funding
This research was not supported by any funding agency or institute.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

References

1. Kelly, J. T., Campbell, K. L., Gong, E., & Scuffham, P. (2020). The Internet of Things: Impact and implications for health care delivery. Journal of medical Internet research, 22(11), e20135.
2. Patella, S. M., Grazieschi, G., Gatta, V., Marcucci, E., & Carrese, S. (2020). The adoption of green vehicles in last mile logistics: A systematic review. Sustainability, 13(1), 6.
3. Velusamy, P., Rajendran, S., Mahendran, R. K., Naseer, S., Shafiq, M., & Choi, J.-G. (2021). Unmanned Aerial Vehicles (UAV) in precision agriculture: Applications and challenges. Energies, 15(1), 217.
4. Ocholla, I. A., Pellikka, P., Karanja, F. N., Vuorinne, I., Odipo, V., & Heiskanen, J. (2024). Livestock detection in African rangelands: Potential of high-resolution remote sensing data. Remote Sensing Applications: Society and Environment, 101139.
5. Acharya, B. S., Bhandari, M., Bandini, F., Pizarro, A., Perks, M., Joshi, D. R., Wang, S., Dogwiler, T., Ray, R. L., & Kharel, G. (2021). Unmanned aerial vehicles in hydrology and water management: Applications, challenges, and perspectives. Water Resources Research, 57(11), e2021WR029925.
6. Euchi, J. (2021). Do drones have a realistic place in a pandemic fight for delivering medical supplies in healthcare systems problems? In (Vol. 34, pp. 182-190): Elsevier.
7. Bor-Yaliniz, R. I., El-Keyi, A., & Yanikomeroglu, H. (2016). Efficient 3-D placement of an aerial base station in next generation cellular networks. 2016 IEEE international conference on communications (ICC),

8. Fotouhi, A., Qiang, H., Ding, M., Hassan, M., Giordano, L. G., Garcia-Rodriguez, A., & Yuan, J. (2019). Survey on UAV cellular communications: Practical aspects, standardization advancements, regulation, and security challenges. IEEE Communications surveys & tutorials, 21(4), 3417-3442.
9. San, K. T., Mun, S. J., Choe, Y. H., & Chang, Y. S. (2018). UAV delivery monitoring system. MATEC Web of Conferences.
10. Sorbelli, F. B., Corò, F., Das, S. K., & Pinotti, C. M. (2020). Energy-constrained delivery of goods with drones under varying wind conditions. IEEE Transactions on Intelligent Transportation Systems, 22(9), 6048-6060.
11. Lammers, D. T., Williams, J. M., Conner, J. R., Baird, E., Rokayak, O., McClellan, J. M., Bingham, J. R., Betzold, R., & Eckert, M. J. (2023). Airborne! UAV delivery of blood products and medical logistics for combat zones. Transfusion, 63, S96-S104.
12. Walia, S. S., Somarathna, K., Hendricks, R., Jackson, A., & Nagarur, N. (2018). Optimizing the emergency delivery of medical supplies with unmanned aircraft vehicles. IIE Annual Conference. Proceedings,
13. YILMAZ, B. Ö., TOZAN, H., & KARADAYI, M. A. (2020). Multi-criteria decision-making (MCDM) applications in the military healthcare field. Journal of Health Systems and Policies, 2(2), 149-181.
14. Tas, P. G., Kavus, B. Y., Ayyildiz, E., & Taskin, A. (2022). UAV Selection for Post-Disaster Medical Supply Distribution Using a Two-Level Interval-Valued Pythagorean Fuzzy AHP Integrated Interval-Valued Pythagorean Fuzzy VIKOR Methodology, and an Application. In Multi-Criteria Decision Analysis (pp. 267-279). CRC Press.
15. Nur, F., Alrahahleh, A., Burch, R., Babski-Reeves, K., & Marufuzzaman, M. (2020). Last mile delivery drone selection and evaluation using the interval-valued inferential fuzzy TOPSIS. Journal of Computational Design and Engineering, 7(4), 397-411.
16. Abdel-Basset, M., Gamal, A., Moustafa, N., Abdel-Monem, A., & El-Saber, N. (2021). A security-by-design decision-making model for risk management in autonomous vehicles. IEEE Access, 9, 107657-107679.
17. Diakoulaki, D., Mavrotas, G., & Papayannakis, L. (1995). Determining objective weights in multiple criteria problems: The critic method. Computers & Operations Research, 22(7), 763-770.
18. Pamučar, D., Vasin, L., & Lukovac, L. (2014). Selection of railway level crossings for investing in security equipment using hybrid DEMATEL-MARICA model. XVI international scientific-expert conference on railway, railcon,
19. Banik, D., Ibne Hossain, N. U., Govindan, K., Nur, F., & Babski-Reeves, K. (2023). A decision support model for selecting unmanned aerial vehicle for medical supplies: Context of COVID-19 pandemic. The International Journal of Logistics Management, 34(2), 473-496.
20. Pavithran, R., Lalith, V., Naveen, C., Sabari, S., Kumar, M. A., & Hariprasad, V. (2020). A prototype of Fixed Wing UAV for delivery of Medical Supplies. IOP Conference Series: Materials Science and Engineering,

Received: 22 Dec 2023, **Revised:** 07 Apr 2024,

Accepted: 08 May 2024, **Available online:** 15 May 2024.

Neutrosophic Systems with Applications, Vol. 18, 2024
https://doi.org/10.61356/j.nswa.2024.18261

Rough Fermatean Neutrosophic Sets and its Applications in Medical Diagnosis

P. Dhanalakshmi [1,*]

[1] Assistant Professor, CK College of Engineering and Technology, India.
Emails: pdhanamaths@gmail.com; p@ckcet.edu.in

* Correspondence: pdhanamaths@gmail.com.

Abstract: This paper introduces the concept of rough fermatean neutrosophic sets and investigates their properties. Additionally, a cosine similarity measure between these sets is proposed. By applying this measure to a medical diagnosis example, the paper illustrates how the method can be used in practical situations, highlighting its effectiveness in complex decision-making scenarios. This innovation holds promise for improving decision-making processes, especially in critical areas like medical diagnosis, where making accurate assessments amidst uncertainty is crucial.

Keywords: Neutrosophic Sets; Rough Neutrosophic Sets; Fermatean Sets; Rough Fermatean Sets; Rough Fermatean Neutrosophic Sets; Cosine Similarity Measure.

1. Introduction

The concept of neutrosophic sets [14] originated from the new branch of philosophy called neutrosophy, which means knowledge of neutral thought and this neutral represents the main distinction between fuzzy and intuitionistic fuzzy logic and set. It is a logic in which each proposition is estimated to have a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F) respectively, and which lies between [0, 1]. The neutrosophic set generalizes the classical set or crisp set proposed by Cantor, the fuzzy set proposed by Zadeh [20], the interval-valued fuzzy set proposed independently by Zadeh [21], Grattan-Guiness [6], the intuitionistic fuzzy set proposed by Atanassov [1], and interval-valued intuitionistic fuzzy set proposed by Atanassov and Gargov [2].

Authors explored neutrosophic sets & SVNS across fields like decision-making, image processing, medical diagnosis, and more[4,5,7-11,13, 15-19]. Senapati and Yager[22] discussed a numerical case to validate the rationality of the concept of fermatean fuzzy sets. It is also important to mention that the class of this type of fuzzy set has more ability to capture the uncertainties as compared to intuitionistic fuzzy sets and Pythagorean fuzzy sets. Fermatean neutrosophic sets are studied by C. Antony Crispin Sweety et al. [3].

Rough set theory, introduced by Pawlak [12] indeed offers a valuable framework for handling imprecise and uncertain information, which is common in many real-world scenarios. It extends the traditional crisp set theory to accommodate this kind of data, making it particularly useful in the study of intelligent systems where information may be incomplete or ambiguous.

The recent development of rough neutrosophic sets adds another layer of sophistication to this field. The fusion of rough set theory with neutrosophic sets in the form of rough neutrosophic sets offers a powerful mathematical tool for handling incomplete information. In rough neutrosophic sets, the rating of alternatives is expressed using upper and lower approximation operators, capturing the uncertainty inherent in the data. Moreover, the characterization of sets by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree allows for a nuanced representation of incomplete information, making rough neutrosophic sets a versatile tool for decision-making in uncertain environments.

The paper consists of four sections. The first two sections serve as the introduction and provide preliminary information. In the third section, we define rough fermatean neutrosophic sets and establish several operations associated with them. Section four presents the introduction of cosine similarity measures for rough fermatean neutrosophic sets. Lastly, a numerical example is solved to demonstrate the practicality, relevance, and effectiveness of the proposed methodologies.

2. Preliminaries

This section consists of the basic results of this paper, refer to [1-22].

3. Rough Fermatean Neutrosophic Sets

In this section, we have to introduce the Rough Fermatean Neutrosophic (RFN)set.

Definition 3.1. Let K be the universal set and Θ be an equivalence relation on K. Let F be the fermatean neutrosophic set of K. The lower and upper approximations of F in the approximation space (K, Θ) are defined as follows:

$$\Theta_{\blacksquare}(F) = \langle\left(g, \Theta_{\blacksquare}(F_t)(g), \Theta_{\blacksquare}(F_i)(g), \Theta_{\blacksquare}(F_f)(g)\right), g \in K\rangle$$

$$\Theta^{\blacksquare}(F) = \langle\left(g, \Theta^{\blacksquare}(F_t)(g), \Theta^{\blacksquare}(F_i)(g), \Theta^{\blacksquare}(F_f)(g)\right), g \in K\rangle$$

Where
$$\Theta_{\blacksquare}(F_t)(g) = \wedge_{s\in[g]_\Theta} F_t(s)$$
$$\Theta_{\blacksquare}(F_i)(g) = \vee_{s\in[g]_\Theta} F_i(s)$$
$$\Theta_{\blacksquare}(F_f)(g) = \vee_{s\in[g]_\Theta} F_f(s)$$

Also $\Theta^{\blacksquare}(F_t)(g) = \vee_{s\in[g]_\Theta} F_t(s)$
$$\Theta^{\blacksquare}(F_i)(g) = \wedge_{s\in[g]_\Theta} F_i(s)$$
$$\Theta^{\blacksquare}(F_f)(g) = \wedge_{s\in[g]_\Theta} F_f(s)$$

Where $0 \le (\Theta_{\blacksquare}(F_t)(g))^3 + (\Theta_{\blacksquare}(F_i)(g))^3 + (\Theta_{\blacksquare}(F_f)(g))^3 \le 2$ and
$$0 \le (\Theta^{\blacksquare}(F_t)(g))^3 + (\Theta^{\blacksquare}(F_i)(g))^3 + (\Theta^{\blacksquare}(F_f)(g))^3 \le 2$$

Example 3.2. Let $U = \{a, b, c, d\}$ be the universal set. Let F_1 be the FN set defined by $\left\{\frac{a}{0.6,0.1,0.7}, \frac{b}{0.5,0.8,0.4}, \frac{c}{0.7,0.5,0.3}, \frac{d}{0.4,1,0.8}\right\}$. Let Θ be a congruence relation on P such that congruence classes are the subsets given by $\{\{a\}, \{b, c, d\}\}$. Then the lower and upper approximations of F_1 are given by,

$$\Theta_{\blacksquare}(F_1)(x) = \left\{\frac{a}{0.6,0.1,0.7}, \frac{b}{0.4,1,0.8}, \frac{c}{0.4,1,0.8}, \frac{d}{0.4,1,0.8}\right\}, for\ all\ x \in U$$

and

$$\Theta^{\blacksquare}(F_1)(x) = \left\{\frac{a}{0.6,0.1,0.7}, \frac{b}{0.7,0.5,0.4}, \frac{c}{0.7,0.5,0.4}, \frac{d}{0.7,0.5,0.4}\right\}, for\ all\ x \in U$$

Example 3.3. Let $U = \{a, b, c, d\}$ be the universal set. Let F_2 be the fermatean Neutrosophic set defined by,

$$\left\{\frac{a}{0.3,0.7,0.5}, \frac{b}{0.4,0.6,0.7}, \frac{c}{0.8,0.3,0.7}, \frac{d}{0.7,0.2,0.4}\right\}$$

Let Θ be congruence relations on P such that congruence classes are the subsets given by $\{\{a, b, c\}, \{d\}\}$. Then the lower and upper approximations of F_2 are given by,

$$\Theta_{\blacksquare}(F_2)(x) = \left\{\frac{a}{0.3,0.7,0.7}, \frac{b}{0.3,0.7,0.7}, \frac{c}{0.3,0.7,0.7}, \frac{d}{0.7,0.2,0.4}\right\}, for\ all\ x \in U$$

and

$$\Theta^{\blacksquare}(F_2)(x) = \left\{\frac{a}{0.8,0.3,0.5}, \frac{b}{0.8,0.3,0.5}, \frac{c}{0.8,0.3,0.5}, \frac{d}{0.7,0.2,0.4}\right\}, for\ all\ x \in U$$

Definition 3.4. Let F be the RFN fuzzy set. Then the complement of F, F^c is defined as follows:

$$\Theta_{\blacksquare}(F^c)(g) = \{\Theta_{\blacksquare}(F_f)(g), 1 - \Theta_{\blacksquare}(F_i)(g), \Theta_{\blacksquare}(F_t)(g)\}$$

And
$$\Theta^{\blacksquare}(F^c)(g) = \{\Theta^{\blacksquare}(F_f)(g), 1 - \Theta^{\blacksquare}(F_i)(g), \Theta^{\blacksquare}(F_t)(g)\}$$

For all $g \in F$.

Definition 3.6. Let $\Theta(F_1)$ and $\Theta(F_2)$ be two RFN fuzzy sets. Then $\Theta(F_1) \subseteq \Theta(F_2)$ if and only if the following conditions hold:

$$\Theta_{\blacksquare}(F_{1t})(g) \leq \Theta_{\blacksquare}(F_{2t})(g)$$
$$\Theta_{\blacksquare}(F_{1i})(g) \geq \Theta_{\blacksquare}(F_{2i})(g)$$
$$\Theta_{\blacksquare}(F_{1f})(g) \geq \Theta_{\blacksquare}(F_{2f})(g)$$

and

$$\Theta^{\blacksquare}(F_{1t})(g) \leq \Theta^{\blacksquare}(F_{2t})(g)$$
$$\Theta^{\blacksquare}(F_{1i})(g) \geq \Theta^{\blacksquare}(F_{2i})(g)$$
$$\Theta^{\blacksquare}(F_{1f})(g) \geq \Theta^{\blacksquare}(F_{2f})(g)$$

Definition 3.7. Let$\Theta(F_1)$ and $\Theta(F_2)$ be two RFN fuzzy sets. Then$\Theta(F_1) \cup \Theta(F_2)$is defined as follows.

$$(\Theta_{\blacksquare}(F_{1t}) \cup \Theta_{\blacksquare}(F_{2t}))(h) = max\{\Theta_{\blacksquare}(F_{1t}), \Theta_{\blacksquare}(F_{2t})\}$$
$$(\Theta_{\blacksquare}(F_{1i}) \cup \Theta_{\blacksquare}(F_{2i}))(h) = min\{\Theta_{\blacksquare}(F_{1i}), \Theta_{\blacksquare}(F_{2i})\}$$
$$(\Theta_{\blacksquare}(F_{1f}) \cup \Theta_{\blacksquare}(F_{2f}))(h) = min\{\Theta_{\blacksquare}(F_{1f}), \Theta_{\blacksquare}(F_{2f})\}$$

and

$$(\Theta^{\blacksquare}(F_{1t}) \cup \Theta^{\blacksquare}(F_{2t}))(h) = max\{\Theta^{\blacksquare}(F_{1t}), \Theta^{\blacksquare}(F_{2t})\}$$
$$(\Theta^{\blacksquare}(F_{1i}) \cup \Theta^{\blacksquare}(F_{2i}))(h) = min\{\Theta^{\blacksquare}(F_{1i}), \Theta^{\blacksquare}(F_{2i})\}$$
$$(\Theta^{\blacksquare}(F_{1f}) \cup \Theta^{\blacksquare}(F_{2f}))(h) = min\{\Theta^{\blacksquare}(F_{1f}), \Theta^{\blacksquare}(F_{2f})\}$$

Example 3.7 Consider the RFN sets in Example 2.2 and 2.3. Then the union is given by,

$$(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2))(a) = (0.6,0.7,0.7)$$
$$(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2))(b) = (0.4,0.7,0.7)$$
$$(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2))(c) = (0.4,0.7,0.7)$$
$$(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2))(d) = (0.7,0.2,0.4)$$

and

$$(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2))(a) = (0.8,0.3,0.5)$$
$$(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2))(b) = (0.8,0.3,0.4)$$
$$(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2))(c) = (0.8,0.3,0.4)$$
$$(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2))(d) = (0.7,0.2,0.4)$$

Definition 3.8. Let $\Theta(F_1)$ and $\Theta(F_2)$ be two RFN fuzzy sets. Then $\Theta(F_1) \cap \Theta(F_2)$ is defined as follows.

$$(\Theta_{\blacksquare}(F_{1t}) \cap \Theta_{\blacksquare}(F_{2t}))(h) = min\{\Theta_{\blacksquare}(F_{1t}), \Theta_{\blacksquare}(F_{2t})\}$$
$$(\Theta_{\blacksquare}(F_{1i}) \cap \Theta_{\blacksquare}(F_{2i}))(h) = max\{\Theta_{\blacksquare}(F_{1i}), \Theta_{\blacksquare}(F_{2i})\}$$
$$(\Theta_{\blacksquare}(F_{1f}) \cap \Theta_{\blacksquare}(F_{2f}))(h) = max\{\Theta_{\blacksquare}(F_{1f}), \Theta_{\blacksquare}(F_{2f})\}$$

and

$$(\Theta^{\blacksquare}(F_{1t}) \cap \Theta^{\blacksquare}(F_{2t}))(h) = min\{\Theta^{\blacksquare}(F_{1t}), \Theta^{\blacksquare}(F_{2t})\}$$
$$(\Theta^{\blacksquare}(F_{1i}) \cap \Theta^{\blacksquare}(F_{2i}))(h) = max\{\Theta^{\blacksquare}(F_{1i}), \Theta^{\blacksquare}(F_{2i})\}$$
$$(\Theta^{\blacksquare}(F_{1f}) \cap \Theta^{\blacksquare}(F_{2f}))(h) = max\{\Theta^{\blacksquare}(F_{1f}), \Theta^{\blacksquare}(F_{2f})\}$$

Example 3.9. Consider the RFN set in Example 2.2 and 2.3. Then the intersection is given by,

$$(\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2))(a) = (0.3,1,0.7)$$
$$(\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2))(b) = (0.3,1,0.8)$$
$$(\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2))(c) = (0.3,1,0.8)$$
$$(\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2))(d) = (0.4,1,0.8)$$

and

$$(\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2))(a) = (0.6,1,0.7)$$
$$(\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2))(b) = (0.7,0.5,0.5)$$
$$(\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2))(c) = (0.7,0.5,0.5)$$
$$(\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2))(d) = (0.7,0.5,0.4)$$

Definition 3.10. If F_1 and F_2 be two RFN sets. Then we define the following

1. $\Theta(F_1) = \Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1) = \Theta_{\blacksquare}(F_2)$ and $\Theta^{\blacksquare}(F_1) = \Theta^{\blacksquare}(F_2)$.
2. $\Theta(F_1) \subseteq \Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1) \subseteq \Theta_{\blacksquare}(F_2)$ and $\Theta^{\blacksquare}(F_1) \subseteq \Theta^{\blacksquare}(F_2)$.
3. $\Theta(F_1) \cup \Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2)$and $\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2)$.
4. $\Theta(F_1) \cap \Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2)$ and $\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2)$.
5. $\Theta(F_1) + \Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1) + \Theta_{\blacksquare}(F_2)$ and $\Theta^{\blacksquare}(F_1) + \Theta^{\blacksquare}(F_2)$.
6. $\Theta(F_1)^{\circ}\Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1)^{\circ}\Theta_{\blacksquare}(F_2)$ and $\Theta^{\blacksquare}(F_1)^{\circ}\Theta^{\blacksquare}(F_2)$.

Proposition 3.11. If $\Theta(F_2), \Theta(F_2)$ and $\Theta(F_3)$are RFN sets. Then the following are straightforward from the definitions.

1. $\sim\Theta(F_1)\big(\sim\Theta(F_1)\big) = \Theta(F_1)$
2. $\Theta(F_1) \cup \Theta(F_2) = \Theta(F_2) \cup \Theta(F_1), \Theta(F_1) \cap \Theta(F_2) = \Theta(F_2) \cap \Theta(F_1)$
3. $\big(\Theta(F_1) \cup \Theta(F_2)\big) \cup \Theta(F_3) = \Theta(F_1) \cup \big(\Theta(F_2) \cup \Theta(F_3)\big)$ and $\big(\Theta(F_1) \cap \Theta(F_2)\big) \cap \Theta(F_3) = \Theta(F_1) \cap \big(\Theta(F_2) \cup \Theta(F_3)\big)$
4. $\big(\Theta(F_1) \cup \Theta(F_2)\big) \cap \Theta(F_3) = \big(\Theta(F_1) \cup \Theta(F_3)\big) \cap \big(\Theta(F_2) \cup \Theta(F_3)\big)$ and $\big(\Theta(F_1) \cup \Theta(F_2)\big) \cap \Theta(F_3) = \big(\Theta(F_1) \cup \Theta(F_3)\big) \cap \big(\Theta(F_2) \cup \Theta(F_1)\big)$

Proposition 3.12. If$\Theta(F_1) and \Theta(F_2)$are RFN sets. Then the following are satisfied.

1. $\sim\big(\Theta(F_1) \cup \Theta(F_2)\big) = \big(\sim\Theta(F_1)\big) \cap \big(\sim\Theta(F_2)\big)$
2. $\sim\big(\Theta(F_1) \cap \Theta(F_2)\big) = \big(\sim\Theta(F_1)\big) \cup \big(\sim\Theta(F_2)\big)$

Proof:

(i) $\sim\big(\Theta(F_1) \cup \Theta(F_2)\big) = \sim\big\{\big(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2)\big), \big(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2)\big)\big\}$

$$= \big\{\sim\big(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2)\big), \sim\big(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2)\big)\big\}$$

$$= \big\{\sim\big(\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2)\big), \sim\big(\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2)\big)\big\}$$

$$= \big(\sim\Theta(F_1)\big) \cap \big(\sim\Theta(F_2)\big)$$

(ii) Similarly, we prove this part.

Proposition 3.13. Let F_1 and F_2be two RFN sets. Then

(i) $F_1 \cup F_2 \supseteq \mathrm{F}_1 \cup \mathrm{F}_2$
(ii) $F_1 \cap F_2 \subseteq \mathrm{F}_1 \cap \mathrm{F}_2$

Proof:

$$(\Theta_{\blacksquare}(F_{1t}) \cup \Theta_{\blacksquare}(F_{2t}))(h) = max\{\Theta_{\blacksquare}(F_{1t}), \Theta_{\blacksquare}(F_{2t})\}$$
$$(\Theta_{\blacksquare}(F_{1i}) \cup \Theta_{\blacksquare}(F_{2i}))(h) = min\{\Theta_{\blacksquare}(F_{1i}), \Theta_{\blacksquare}(F_{2i})\}$$
$$(\Theta_{\blacksquare}(F_{1f}) \cup \Theta_{\blacksquare}(F_{2f}))(h) = min\{\Theta_{\blacksquare}(F_{1f}), \Theta_{\blacksquare}(F_{2f})\}$$

Consider$\Theta_{\blacksquare}(F_{1t} \cup (\Theta_{\blacksquare}(F_{1t}) \cup \Theta_{\blacksquare}(F_{2t}))(h) = max\{\Theta_{\blacksquare}(F_{1t}), \Theta_{\blacksquare}(F_{2t})\}$

$$(\Theta_{\blacksquare}(F_{1i}) \cup \Theta_{\blacksquare}(F_{2i}))(h) = min\{\Theta_{\blacksquare}(F_{1i}), \Theta_{\blacksquare}(F_{2i})\}$$
$$(\Theta_{\blacksquare}(F_{1f}) \cup \Theta_{\blacksquare}(F_{2f}))(h) = min\{\Theta_{\blacksquare}(F_{1f}), \Theta_{\blacksquare}(F_{2f})\}$$
$$(F_{1t} \cup F_{2t})(g) = \wedge_{s\in[g]_\Theta} F_{1t} \cup F_{2t}(s)$$
$$= \wedge_{s\in[g]_\Theta} (max\{F_{1t}, F_{2t}\})$$
$$\geq max\{\wedge_{s\in[g]_\Theta} F_{1t}(s), \wedge_{s\in[g]_\Theta} F_{2t}(s)\}$$
$$\geq max\{\wedge_{s\in[g]_\Theta} F_{1t}(s), \quad \wedge_{s\in[g]_\Theta} F_{2t}(s)\}$$

$$= max\{\Theta_{\blacksquare}(F_{1t})(g), \Theta_{\blacksquare}(F_{2t})(g)\}$$
$$= \Theta_{\blacksquare}(F_{1t})(g) \cup \Theta_{\blacksquare}(F_{2t})(g)$$

Similarly,

$$\Theta_{\blacksquare}(F_{1i} \cup F_{2i})(g) \leq \Theta_{\blacksquare}(F_{1i})(g) \cup \Theta_{\blacksquare}(F_{2i})(g)$$
$$\Theta_{\blacksquare}(F_{1i} \cup F_{2i})(g) \leq \Theta_{\blacksquare}(F_{1i})(g) \cup \Theta_{\blacksquare}(F_{2i})(g)$$

Thus, $\Theta_{\blacksquare}(F_1 \cup F_2) \supseteq \Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2)$

In the same way, we prove for upper approximation.

Hence, $F_1 \cup F_2 \supseteq \mathrm{F}_1 \cup \mathrm{F}_2$.

(iii) The proof is similar to the proof (i).

4. Application of Rough Fermatean Neutrosophic Sets

In this section, we introduce the application of rough fermatean neutrosophic sets. Also, study the cosine similarity measure of rough fermatean neutrosophic sets. Moreover, medical diagnosis problems are discussed for establishing the proposed model.

4.1 Cosine Similarity Measure of Rough Fermatean Neutrosophic Sets

Definition 4.1.1. $\Theta(F_1)$ *and* $\Theta(F_2)$ are RFN sets in $X = \{x_1, x_2 \dots \dots x_n\}$. A cosine similarity measure between $\Theta(F_1)$ *and* $\Theta(F_2)$ is defined as follows:

$$COS_{RFN}\big(\Theta(F_1), \Theta(F_2)\big) = \frac{1}{n}\sum_{i=1}^{n} \frac{\big(\delta\Theta(F_{1t})(x_i)\delta\Theta(F_{2t})(x_i) + \delta\Theta(F_{1i})(x_i)\delta\Theta(F_{2i})(x_i) + \delta\Theta(F_{1f})(x_i)\delta\Theta(F_{2f})(x_i)\big)}{\sqrt{(\delta\Theta(F_{1t})(x_i))^2 + (\delta\Theta(F_{1t})(x_i))^2 + (\delta\Theta(F_{1f})(x_i))^2}\sqrt{(\delta\Theta(F_{2t})(x_i))^2 + (\delta\Theta(F_{2i})(x_i))^2 + (\delta\Theta(F_{2f})(x_i))^2}}$$

Where

$$\delta\Theta(F_{1t})(x_i) = \frac{\big(\Theta_{\blacksquare}(F_{1t})(x_i) + \Theta^{\blacksquare}(F_{1t})(x_i)\big)}{2}$$

$$\delta\Theta(F_{1i})(x_i) = \frac{\big(\Theta_{\blacksquare}(F_{1i})(x_i) + \Theta^{\blacksquare}(F_{1i})(x_i)\big)}{2}$$

$$\delta\Theta(F_{1f})(x_i) = \frac{\big(\Theta_{\blacksquare}(F_{1f})(x_i) + \Theta^{\blacksquare}(F_{1f})(x_i)\big)}{2} \text{ and}$$

$$\delta\Theta(F_{2t})(x_i) = \frac{\big(\Theta_{\blacksquare}(F_{2t})(x_i) + \Theta^{\blacksquare}(F_{2t})(x_i)\big)}{2}$$

$$\delta\Theta(F_{2i})(x_i) = \frac{\big(\Theta_{\blacksquare}(F_{2i})(x_i) + \Theta^{\blacksquare}(F_{2i})(x_i)\big)}{2}$$

$$\delta\Theta(F_{2f})(x_i) = \frac{\big(\Theta_{\blacksquare}(F_{2f})(x_i) + \Theta^{\blacksquare}(F_{2f})(x_i)\big)}{2}$$

Proposition 4.1.2. A RFN fuzzy cosine similarity measure between $\Theta(F_1)$ *and* $\Theta(F_2)$ satisfies the following properties:

1. $0 \leq C_{RFN}\big(\Theta(F_1), \Theta(F_2)\big) \leq 1$
2. $C_{RFN}\big(\Theta(F_1), \Theta(F_2)\big) = 1 \Leftrightarrow \Theta(F_1) = \Theta(F_2)$
3. $C_{RFN}\big(\Theta(F_1), \Theta(F_2)\big) = C_{RFN}\big(\Theta(F_1), \Theta(F_2)\big)$

If we consider the weight ω_i of each element, x_i, a weighted RFNcosine similarity measure between RFN sets $\Theta(F_1)$ *and* $\Theta(F_2)$ is defined as follows:

$$COS_{RFN}\big(\Theta(F_1)\,,\Theta(F_2)\big) =$$

$$\frac{1}{n}\sum_{i=1}^{n}\omega_i \frac{\big(\delta\Theta(F_{1t})(x_i)\delta\Theta(F_{2t})(x_i)+\delta\Theta(F_{1i})(x_i)\delta\Theta(F_{2i})(x_i)+\delta\Theta(F_{1f})(x_i)\delta\Theta(F_{2f})(x_i)\big)}{\sqrt{(\delta\Theta(F_{1t})(x_i))^2+(\delta\Theta(F_{1t})(x_i))^2+(\delta\Theta(F_{1f})(x_i))^2}\sqrt{(\delta\Theta(F_{2t})(x_i))^2+(\delta\Theta(F_{2i})(x_i))^2+(\delta\Theta(F_{2f})(x_i))^2}}$$

$\omega_i \in [0,1], i = 1,2,3 \ldots n$ and $\sum_{i=1}^{n}\omega_i = 1$. If we take $\omega_i = \frac{1}{n}, i = 1,2, \ldots . n$ then$C_{WRFN}\big(\Theta(F_1)\,,\Theta(F_2)\big) = C_{RFN}\big(\Theta(F_1)\,,\Theta(F_2)\big)$.

The weighted RFN cosine similarity measure between two RFN sets $\Theta(F_1)\ and\ \Theta(F_2)$ also satisfies the following properties:

Proposition 4.1.3.

1. $0 \le C_{WRFN}\big(\Theta(F_1)\,,\Theta(F_2)\big) \le 1$
2. $C_{WRFN}\big(\Theta(F_1)\,,\Theta(F_2)\big) = 1 \Leftrightarrow \Theta(F_1) = \Theta(F_2)$
3. $C_{WRFN}\big(\Theta(F_1)\,,\Theta(F_2)\big) = C_{WRFN}\big(\Theta(F_1)\,,\Theta(F_2)\big)$

5. Methodology

This section explores the application of RFN sets in the realm of medical diagnosis. Specifically, within a given medical scenario, F represents the set of symptoms, D denotes the array of diseases, and P signifies the cohort of patients manifesting symptoms in S.

Consider Q as the RFN relation mapping patients to symptoms (P → S), and R as a RFN relation mapping symptoms to diseases (S → D). The methodology comprises three primary tasks:

1. Identifying symptoms.
2. Formulating medical insights using RFN sets.
3. Establishing diagnoses.

5.1 Algorithm for RFN Cosine Similarity Measure

In this section, we present an algorithm of cosine similarity measure in RFN environment to diagnose the disease of the patient. Let $F = \{f_1, f_2 \ldots \ldots . f_n\}$ be set of symptoms and $P = \{p_1, p_2 \ldots \ldots . p_n\}$be set of patients and $D = \{d_1, d_2 \ldots \ldots . d_n\}$be set of disease.

The procedure unfolds as follows:

1. Gather symptoms exhibited by patients, concurrently establishing the patient-symptom relationship denoted as Q.
2. Derive the relationship R between symptoms and diseases.
3. Perform computations.
4. Choose the highest cosine similarity measure value.
5. Determine that the disease D affects the patients in P.

6. Illustrative Example

In this section, we provide an illustrative example demonstrating the application of cosine similarity measures for RFN sets.

6.1 Example of Rough Fermatean Cosine Similarity Measure

We consider a practical perspective on a medical diagnosis scenario to clarify the proposed approach. Within the field of medical science, the primary objective is the diagnosis of diseases. Therefore, medical diagnosis is highly valued as an art dedicated to identifying an individual's pathological conditions of the body from all the available symptoms.

Let D = $\{d_1, d_2 \dots \dots . d_n\}$ represent the set of diseases, F = $\{f_1, f_2 \dots \dots . f_n\}$ denote the symptoms, and P = $\{p_1, p_2 \dots \dots . p_n\}$ represent the set of patients exhibiting symptoms in F. The relationship between symptoms and diseases is described in the form of RNF sets.

Table 1. Relationship between patients and diseases in the form of RNF sets.

	d_1	d_2	d_3	d_4
p_1	(0.8,0.8,0.8), (0.8,0.8,0.8)	(0.7,0.9,0.8), (0.8,0.7,0.7)	(0.7,0.9,0.8), (0.8,0.7,0.7)	(0.9,0.7,0.8), (0.9,0.7,0.8)
p_2	(0.7,0.8,0.8), (0.7,0.8,0.8)	(0.8,0.9,0.7), (0.85,0.9,0.65)	(0.8,0.9,0.7), (0.85,0.9,0.65)	(0.8,0.95,0.7), (0.8,0.95,0.7)
p_3	(0.7,0.8,0.9), (0.7,0.8,0.6)	(0.9,0.8,0.5), (0.9,0.8,0.5)	(0.5,0.4,0.9), (0.5,0.4,0.9)	(0.7,0.8,0.9) (0.7,0.8,0.6)
p_4	(0.6,0.8,0.9), (0.9,0.6,0.4)	(0.7,0.7,0.8), (0.7,0.7,0.8)	(0.9,0.9,0.7), (0.9,0.9,0.7)	(0.6,0.8,0.9) (0.9,0.6,0.4)

Table 2. Relationship between patients and symptoms in the form of RNF sets.

	d_1	d_2	d_3	d_4
f_1	(0.6,1,0.7) (0.6,1,0.7)	(0.5,0.8,0.4) (0.7,0.5,0.3)	(0.5,0.8,0.4) (0.7,0.5,0.3)	(0.4,0.1,0.8) (0.4,0.1,0.8)
f_2	(0.4,0.9,0.6) (0.4,0.9,0.6)	(0.3,1,0.8) (0.8,0.7,0.8)	(0.3,1,0.8) (0.8,0.7,0.8)	(0.3,1,0.8) (0.8,0.7,0.8)
f_3	(0.3,0.7,0.7) (0.8,0.2,0.4)	(0.7,0.2,0.4) (0.7,0.2,0.4)	(0.3,0.7,0.7) (0.8,0.2,0.4)	(0.4,0.6,0.7) (0.4,0.6,0.7)
f_4	(0.3,0.8,0.7) (0.7,0.2,0.4)	(0.6,0.5,0.8) (0.6,0.5,0.8)	(0.3,0.7,0.7) (0.7,0.2,0.4)	(0.3,0.7,0.7) (0.7,0.2,0.4)

Table 3. Final results.

	d_1	d_2	d_3	d_4
p_1	0.5983	0.8877	0.9927	0.9990
p_2	0.9834	0.9771	0.9771	0.9826
p_3	0.9902	0.8405	0.9571	0.9838
p_4	0.9951	0.9910	0.9876	0.9951

From Table 3 we conclude that patient $p_1$affected byd_4, p_2 affected by d_1, $p_3$affected by d_1 and p_4 affected by d_1 and d_4.

7. Conclusions

The concept of uncertainty plays a vital role in all science and engineering problems. Especially, Fuzzy theory, Intuitionistic fuzzy theory and then Neutrosophic theory are the most valuable tools for finding the optimum solution to medical diagnosis problems. In this work, we include one more concept called RFN sets in the list which has Pythagorean Neutrosophic, Single Valued Neutrosophic, and Bipolar Neutrosophic graphs. We also apply this new type of set in a decision-making problem. We are extending our research on this new concept to introduce rough Interval-valued Fermatean Neutrosophic sets and their application in our future work.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Funding

This research was not supported by any funding agency or institute.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

References

1. Atanassov, K., (1986). Intuitionistic fuzzy sets, Fuzzy Sets Syst, 20(1).87–96. https://doi.org/10.1016/S0165-0114 (86)80034-3.
2. Atanassov, K., & Gargov, G., (1989). Interval valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 31, 343-349. http://doi.org/10.1016/0165-0114(89)90205-4
3. Antony Crispin Sweety, C., &Jansi, R. (2021). Fermatean Neutrosophic sets, International journal of Advanced Research in Computer and Communication Engineering, 10(6), 24-27. DOI 10.17148/IJARCCE.2021.10605
4. Biswas, P., Pramanik, S., & Giri, B. C. (2014). Entropy based grey relational analysis method for multi-attribute decision-making under single valued neutrosophic assessments. Neutrosophic Sets and Systems, 2, 102-110. DOI: 10.5281/zenodo.22459
5. Deng, J. L. (1982). Control problems of grey system. System and Control Letters, 5, 288–294. http://dx.doi.org/10.1016/S0167-6911(82)80025-X
6. Grattan-Guiness, I. (1975). Fuzzy membership mapped onto interval and many-valued quantities. Z. Math. Logik. Grundladen Math. 22, 149-160. Doi 10.1002/malq.19760220120
7. Kharal, A. (2013). A neutrosophic multicriteria decision making method. New Mathematics and Natural Computation. Creighton University, USA. DOI: 10.1142/S1793005714500070
8. Majumdar, P., &Samant, SK., (2014). On similarity and entropy of neutrosophic sets. J Intell Fuzzy Syst. 26(3), 1245–1252. DOI: 10.3233/IFS-130810
9. Mondal, K., &Pramanik, S. (2014). A study on problems of Hijras in West Bengal based on neutrosophic cognitive maps. Neutrosophic Sets and Systems (NSS) 5, 21-26. DOI: 10.5281/zenodo.22436
10. Mondal, K., &Pramanik, S. (2014). Multi-criteria group decision making approach for teacher recruitment in higher education under simplified neutrosophic environment, Neutrosophic Sets and Systems, 6(2014), 28-34. DOI: 10.5281/zenodo.30208
11. Mondal, K., &Pramanik, S. (2015). Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, volume 7. DOI: 10.5281/zenodo.22582
12. Pawlak. Z. (1982). Rough sets. International Journal of Information and Computer Sciences, 11(5), 341-356. http://dx.doi.org/10.1007/BF01001956
13. Şahin, R., & Liu, PD., (2016). Correlation coefficient of single-valued neutrosophic fermatean fuzzy sets and its applications in decision making. Neural Comput Appl. DOI: 10.1007/s00521-015-2163-x
14. Smarandache, F., (1999) .A unifying field in logics. neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth. DOI: 10.6084/M9.FIGSHARE.1014204
15. Torra,V., (2010). Fermatean fuzzy sets. Int J Intell Syst. 25.529–539.https://doi.org/10.1002/int.20418
16. Torra, V., &Narukawa, Y., (2009). On fermatean fuzzy sets and decision. In: The 18th IEEE international conference on fuzzy systems. Jeju Island, 1378–1382. https://doi.org/10.1109/FUZZY.2009.5276884
17. Ye,J., (2018). Multiple-attribute decision-making method using similarity measures of single-valued neutrosophic fermatean fuzzy sets based on least common multiple cardinality, J Intell Fuzzy Syst 34(6),4203–4211. https://doi.org/10.3390/sym10080330

18. Ye, J. 2013. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. International Journal of General Systems, 42(4), 386-394. DOI: 10.1080/03081079.2012.761609
19. Ye, J. 2014. Single valued neutrosophic cross entropy for multicriteria decision making problems. Applied Mathematical Modeling, 38, 1170-1175.doi 10.1016/j.apm.2013.07.020
20. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8, 338-353. http://dx.doi.org/10.1016/S0019-9958(65)90241-X.
21. Zadeh, L. (1975). The concept of a linguistic variable and its application to approximate reasoning I, Information Sciences. 8, 199–249. https://doi.org/10.1016/0020-0255(75)90036-5
22. Tapan Senapati & Ronald R. Yager (2020), Fermatean fuzzy sets, Journal of ambient intelligence and humanized computing, 11,663-674. DOI:10.1007/s12652-019-01377-0

Received: 22 Jan 2024, **Revised:** 27 Apr 2024,

Accepted: 28 May 2024, **Available online:** 31 May 2024.

Neutrosophic Systems with Applications, Vol. 18, 2024
https://doi.org/10.61356/j.nswa.2024.18254

Application of Secant Span in Medical Diagnosis

R. Narmadhagnanam [1,*] and A. Edward Samuel [1]

[1] Ramanujan Research Centre P.G. & Research Department of Mathematics, Government Arts College (Autonomous), Kumbakonam, Tamil Nadu, India;
Emails: narmadhagnanam03@gmail.com; edwardsamuel.a@gacakum.ac.in.

* Correspondence: narmadhagnanam03@gmail.com.

Abstract: Many common and specific characteristics engrave most diseases. Water-borne diseases differ slightly in their characteristics. Erroneous diagnoses can be attributed to shared characteristics. Current approaches tend to rely on imprecise diagnoses and lack robust techniques for differentiating between characteristics. Every illness also presents with specific symptoms. To assist doctors in approaching a likely diagnosis, the suggested method is successful in determining the connection between a class of sickness and the people with a specific pathology to the indications. Among n-valued interval neutrosophic sets, a secant span is proposed in this paper and a few of its attributes are talked about here. The idea behind the aforementioned approach is a crucial mechanism for addressing doubts as well as flaws in the current approaches. The application of medical diagnosis is explained to figure out the illness that the people are experiencing. The diagnosis's outcome demonstrated how successful the suggested strategy was.

Keywords: Secant Span; Water-borne Diseases; Neutrosophic Sets; Erroneous Diagnoses.

Symbols

SP - A band that includes people with a specific pathology.
L - Collection of indications
SK - A class of sickness
H -n-valued interval neutrosophic connection from a band that includes people with a specific pathology to the collection of indications
J - Interval neutrosophic relation from the collection of indications to the class of sickness
G-Secant span

1. Introduction

The fuzzy sets created by Zadeh [1] can prove helpful in numerous real-life instances as a way of tackling uncertainty. Atanassov's [2] intuitionistic fuzzy sets allow for both truth- and falsity membership and various techniques are suggested and used in a few domains by Ejegwa et al & Edward and Narmadha [3, 4]. By presenting intuitionistic fuzzy multi-sets, Shinoj and Sunil [5] expanded on the idea of fuzzy multi-sets, In addition to this, Edward and Narmadha[6] presented a revolutionary technique. From a philosophical perspective, the neutrosophic set, as defined by Smarandache [7], can deal with ambiguous, imprecise, partial, and inconsistent information that exists in the real world. Said et al. [8] were the first to illustrate rough neutrosophic sets, while Edward and Narmadha [9-12] proposed several methods for these sets. Haibin et al. [13] were the first to illustrate single-valued neutrosophic sets, while Edward and Narmadha [14-16] proposed many methods for these sets. Single-valued neutrosophic multisets were first proposed by Shan Ye and Jun Ye [17] and in this regard, Edward and Narmadha [18] provided a revolutionary technique that was subsequently used in healthcare diagnosis. Said and Irfan[19] & Edward and Narmadha [20-22] offered numerous approaches in neutrosophic refined sets. The idea of n-valued neutrosophic sets is extended to the situation of n-valued interval neutrosophic sets by Broumi et al. [23] and plenty of methods were introduced by Edward and Narmadha [24–26] which were utilized in medical diagnosis. With greater accuracy than the other methods, the suggested approach was also able to effectively manage the shortcomings and restrictions of the earlier research. Indicators within a band of individuals with a particular pathology and assortment of illnesses are discovered to be related in this investigation. The results of this study will assist the researcher in precisely identifying the illness that affected a group of individuals with a particular disease. There are none of the usual restrictions associated with various research methods while using this one. A novel theory on image processing, cluster analysis, etc., has been created in this study without such restrictions. The article is organized as follows for the most part. The Stated concept and some of its characteristics are covered in section 2. The methodology, procedure, and hypothetical example of medical diagnosis are covered in sections 3, 4, and 5 respectively. In section 6, significance statements are provided. Section 7 provides a conclusion.

1.1 Main contributions

This study finds relationships between indicators among a group of people with certain pathologies and a range of disorders. The findings of this investigation will help the researcher pinpoint the exact ailment that afflicted a subset of people suffering from a specific condition. When employing this research method, there are none of the typical limitations that come with other approaches.

2. Stated Concept

2.1 Secant span

Between two n-valued interval neutrosophic sets

$$R=\left\{\begin{array}{l} z,\left(\left[\inf imum T_R^1(z),\sup remum T_R^1(z)\right],\left[\inf imum T_R^2(z),\sup remum T_R^2(z)\right],\ldots,\left[\inf imum T_R^q(z),\sup remum T_R^q(z)\right]\right),\\ \left(\left[\inf imum I_R^1(z),\sup remum I_R^1(z)\right],\left[\inf imum I_R^2(z),\sup remum I_R^2(z)\right],\ldots,\left[\inf imum I_R^q(z),\sup remum I_R^q(z)\right]\right),\\ \left(\left[\inf imum F_R^1(z),\sup remum F_R^1(z)\right],\left[\inf imum F_R^2(z),\sup remum F_R^2(z)\right],\ldots,\left[\inf imum F_R^q(z),\sup remum F_R^q(z)\right]\right): z\in Z \end{array}\right\}$$

&

$$S=\left\{\begin{array}{l} z,\left(\left[\inf imum T_S^1(z),\sup remum T_S^1(z)\right],\left[\inf imum T_S^2(z),\sup remum T_S^2(z)\right],\ldots,\left[\inf imum T_R^q(z),\sup remum T_S^q(z)\right]\right),\\ \left(\left[\inf imum I_S^1(z),\sup remum I_S^1(z)\right],\left[\inf imum I_S^2(z),\sup remum I_S^2(z)\right],\ldots,\left[\inf imum I_S^q(z),\sup remum I_S^q(z)\right]\right),\\ \left(\left[\inf imum F_S^1(z),\sup remum F_S^1(z)\right],\left[\inf imum F_S^2(z),\sup remum F_S^2(z)\right],\ldots,\left[\inf imum F_S^q(z),\sup remum F_S^q(z)\right]\right): z\in Z \end{array}\right\}$$

the secant, span is provided as

$$SEC_{NIS}(R,S)=$$

$$\frac{7}{k}\sum_{d=1}^{q}\left[\sum_{e=1}^{k}\sec\left[6+\left[\frac{\pi}{4}\left[\begin{array}{l} \left|\inf imum T_R^d(z_e)-\inf imum T_S^d(z_e)\right|+\left|\sup remum T_R^d(z_e)-\sup remum T_S^d(z_e)\right|+\\ \left|\inf imum I_R^d(z_e)-\inf imum I_S^d(z_e)\right|+\\ \left|\sup remum I_R^d(z_e)-\sup remum I_S^d(z_e)\right|+\\ \left|\inf imum F_R^d(z_e)-\inf imum F_S^d(z_e)\right|+\left|\sup remum F_R^d(z_e)-\sup remum F_S^d(z_e)\right| \end{array}\right]\right]\right]\right] \quad (1)$$

2.2 Proposition

i. $SEC_{NIS}(R,S)>0$

ii. $SEC_{NIS}(R,S)=SEC_{NIS}(S,R)$

iii. If $R\subseteq S\subseteq U$ then $SEC_{NIS}(R,U)\geq SEC_{NIS}(R,S)$ & $SEC_{NIS}(R,U)\geq SEC_{NIS}(S,U)$

Proof

i. The evidence is easy

ii. The evidence is easy

iii. By (1),

$$\inf imum\, T_R^d(z_e) \le \inf imum\, T_S^d(z_e) \le \inf imum\, T_U^d(z_e)$$
$$\sup remum\, T_R^d(z_e) \le \sup remum\, T_S^d(z_e) \le \sup remum\, T_U^d(z_e)$$
$$\inf imum\, I_R^d(z_e) \ge \inf imum\, I_S^d(z_e) \ge \inf imum\, I_U^d(z_e)$$
$$\sup remum\, I_R^d(z_e) \ge \sup remum\, I_S^d(z_e) \ge \sup remum\, I_U^d(z_e)$$
$$\inf imum\, F_R^d(z_e) \ge \inf imum\, F_S^d(z_e) \ge \inf imum\, I_U^d(F_e)$$
$$\sup remum\, F_R^d(z_e) \ge \sup remum\, F_S^d(z_e) \ge \sup remum\, F_U^d(z_e)$$

In this case, the secant span is a rising function.

$\therefore SEC_{NIS}(R,U) \ge SEC_{NIS}(R,S) \,\&\, SEC_{NIS}(R,U) \ge SEC_{NIS}(S,U)$

3. Methodology

This part delivered a clinical evaluation. Ensure that L generates the collection of indications[Temperature, Headache, Stomach pain, Cough, Chest pain], SK reflects a class of sickness[Viral fever, Malaria, Stomach problem, Chest problem] and SP symbolizes a band that includes people[Adrian, Caleb, Gabriel] with a specific pathology. Let H be an-valued interval neutrosophic connection from a band that includes people with a specific pathology to the collection of indications and let J be an interval neutrosophic relation from the collection of indications to the class of sickness. The key goals of the calculation method are as follows:

(i) Figuring out the indications.
(ii) Utilizing n-valued interval neutrosophic sets and interval neutrosophic sets to construct scientific knowledge.
(iii) An evaluation using the recently developed computing method.

4. Procedure

Step 1: Table 1 lists a band that includes people with a specific pathology to the collection of indications H.
Step 2: Table 2 lists the collection of indications to the class of sickness J.
Step 3: Tables 1 and 2 yield the calculation G which is reported in Table 3. In every row, the number with the lowest value was chosen to determine the likelihood that a band that includes people with a specific pathology was impacted by the class of sickness.

5. Hypothetical Example

Table 1. Applying step 1.

H	Temperature	Headache	Stomach Pain	Cough	Chest Pain
Adrian	[0.2,0.3],[0.3,0.4],[0.4,0.5] [0.0,0.2],[0.2,0.4],[0.4,0.6] [0.3,0.4],[0.2,0.3],[0.1,0.3]	[0.4,0.6],[0.2,0.4],[0.3,0.5] [0.2,0.4],[0.2,0.4],[0.1,0.2] [0.0,0.1],[0.1,0.2],[0.2,0.5]	[0.1,0.2],[0.2,0.3],[0.5,0.5] [0.0,0.3],[0.1,0.2],[0.3,0.5] [0.2,0.7],[0.4,0.6],[0.2,0.3]	[0.2,0.5],[0.2,0.4],[0.0,0.6] [0.0,0.7],[0.1,0.8],[0.2,0.7] [0.3,0.4],[0.2,0.5],[0.3,0.7]	[0.5,0.5],[0.2,0.6],[0.3,0.4] [0.2,0.5],[0.3,0.4],[0.2,0.5] [0.3,0.6],[0.2,0.5],[0.3,0.7]
Caleb	[0.2,0.4],[0.3,0.4],[0.4,0.6] [0.3,0.4],[0.5,0.5],[0.2,0.6] [0.1,0.7],[0.2,0.3],[0.2,0.4]	[0.1,0.8],[0.2,0.7],[0.3,0.7] [0.2,0.8],[0.1,0.9],[0.3,0.6] [0.1,0.6],[0.2,0.6],[0.3,0.5]	[0.1,0.4],[0.2,0.7],[0.3,0.6] [0.2,0.4],[0.0,0.9],[0.3,0.4] [0.1,0.8],[0.2,0.7],[0.3,0.6]	[0.0,0.9],[0.1,0.4],[0.2,0.5] [0.2,0.4],[0.2,0.5],[0.3,0.6] [0.4,0.6],[0.2,0.5],[0.3,0.6]	[0.2,0.4],[0.3,0.4],[0.2,0.7] [0.1,0.8],[0.2,0.5],[0.3,0.4] [0.2,0.7],[0.3,0.6],[0.2,0.6]
Gabriel	[0.1,0.4],[0.3,0.4],[0.1,0.1] [0.0,0.5],[0.3,0.6],[0.1,0.7] [0.0,0.2],[0.3,0.4],[0.4,0.5]	[0.1,0.2],[0.1,0.6],[0.3,0.4] [0.2,0.5],[0.3,0.4],[0.4,0.5] [0.3,0.5],[0.2,0.8],[0.3,0.6]	[0.0,0.3],[0.2,0.4],[0.5,0.5] [0.4,0.4],[0.2,0.7],[0.0,0.2] [0.3,0.6],[0.2,0.3],[0.3,0.3]	[0.2,0.4],[0.3,0.5],[0.2,0.6] [0.2,0.4],[0.3,0.5],[0.2,0.7] [0.1,0.8],[0.2,0.7],[0.3,0.4]	[0.0,0.3],[0.1,0.9],[0.2,0.8] [0.2,0.5],[0.3,0.5],[0.2,0.6] [0.3,0.4],[0.0,0.8],[0.3,0.3]

Table 2. Applying step 2.

J	Viral fever	Malaria	Stomach Problem	Chest problem
Temperature	[0.2,0.6],[0.4,0.5],[0.2,0.7]	[0.3,0.5],[0.2,0.6],[0.4,0.5]	[0.0,0.9],[0.1,0.3],[0.2,0.4]	[0.0,0.5],[0.2,0.6],[0.0,0.2]
Head Ache	[0.1,0.3],[0.2,0.6],[0.5,0.5]	[0.0,0.1],[0.2,0.3],[0.4,0.5]	[0.3,0.6],[0.2,0.7],[0.3,0.4]	[0.4,0.6],[0.4,0.5],[0.1,0.5]
Stomach Pain	[0.3,0.3],[0.2,0.3],[0.3,0.6]	[0.2,0.3],[0.1,0.6],[0.2,0.8]	[0.4,0.5],[0.2,0.3],[0.3,0.5]	[0.4,0.4],[0.3,0.4],[0.5,0.5]
Cough	[0.2,0.4],[0.3,0.5],[0.2,0.7]	[0.3,0.6],[0.1,0.9],[0.3,0.4]	[0.2,0.6],[0.3,0.5],[0.4,0.5]	[0.2,0.6],[0.1,0.1],[0.4,0.6]
Chest Pain	[0.1,0.6],[0.2,0.3],[0.1,0.6]	[0.1,0.7],[0.2,0.5],[0.0,0.3]	[0.3,0.3],[0.2,0.4],[0.1,0.8]	[0.3,0.5],[0.2,0.5],[0.1,0.7]

Table 3. Applying step 3.

G	Viral fever	Malaria	Stomach Problem	Chest problem
Adrian	7.0708	7.0740	7.0753	7.0742
Caleb	7.0676	7.0751	7.0711	7.0763
Gabriel	7.0659	7.0816	7.0707	7.0737

6. Significance Statements

The results of this study will assist us in precisely identifying the sickness that impacted the people. The technique used is devoid of the restrictions that are frequently present in other research. Without these restrictions, this work has produced new theories on processing pictures, pattern assessment, etc.

7. Conclusion

The connection between a band that includes people with a specific pathology to the indications and the class of sickness has been examined in this study and one method (secant span) has been used to determine which sickness may have impacted the people. This study's techniques are dependable and trustworthy, making them suitable for handling medical diagnosis issues with ease. Due to the method's increased diagnostic accuracy, it may be able to avoid the shortcomings and restrictions of earlier studies.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contributions
All authors contributed equally to this research.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Funding
This research was not supported by any funding agency or institute.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

References

1. Zadeh, L.A. (1965). Fuzzy sets. Information and Control, 8(3), 338-353.
2. Atanassov,K.T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20 (1), 87-96.https://doi.org/10.1016/S0165-0114(86)80034-3
3. Ejegwa, P.A., Akubo, A. J., & Joshua, O. M. (2014). Intuitionistic fuzzy sets and its application in career determination via normalized Euclidean distance method.European Scientific Journal, 10(15), 529-536. https://doi.org/10.30598/barekengvol17iss2pp0995-1006
4. Edward Samuel, A.,& Narmadhagnanam, R. (2018).Intuitionistic fuzzy sets in medical diagnosis. International Journal of Mathematical Archive, 9(1), 1-5. http://dx.doi.org/10.22457/ijfma.v16n1a3

5. Shinoj, T.K., & Sunil Jacob John. (2012). Intuitionistic fuzzy multi sets and its application in medical diagnosis.World Academy of Science, Engineering and Technology,6, 1418-1421
6. Edward Samuel, A.,& Narmadhagnanam, R. (2018).Intuitionistic Fuzzy Multi Set in Medical Diagnosis. International Journal of Fuzzy Mathematical Archive, Vol. 16, No. 1, 13-19. http://dx.doi.org/10.22457/ijfma.v16n1a3
7. Florentin Smarandache (1998). A unifying field in logics, neutrosophy: neutrosophic probability, set & logic. Rehoboth: American Research Press.
8. Said Broumi, Florentin Smarandache.,&Mamoni Dhar. (2014). Rough neutrosophic sets. Italian Journal of Pure and Applied Mathematics,32,493-502. 10.6084/M9.FIGSHARE.1502608
9. Edward Samuel, A.,& Narmadhagnanam, R.(2018).Utilization of Rough neutrosophic sets in medical diagnosis.International Journal of Engineering Science Invention,7(3), 1-5.
10. Edward Samuel, A.,& Narmadhagnanam, R.(2018). Neoteric techniques for Rough Neutrosophic set and their utilization in Medical Diagnosis. International Journal for Research in Engineering Application & Management,4(7), 253-258.
11. Edward Samuel, A.,& Narmadhagnanam, R.(2018).Rough Neutrosophic set in Medical Diagnosis. International Journal of Pure and Applied Mathematics, 120(8), 79-87.
12. Edward Samuel, A.,& Narmadhagnanam, R. (2019).Pi distance of Rough neutrosophic sets for medical diagnosis. Neutrosophic Sets and Systems, 28(1), 51-57.
13. Haibin Wang., Florentin Smarandache, Yan Qing Zhang.,&RajshekarSunderraman.(2010). Single valued neutrosophic sets.Multispace and Multistructure,4,410-413.
14. Edward Samuel, A.,& Narmadhagnanam, R. (2018).Cosine logarithmic distance of Single valued neutrosophic sets in medical diagnosis. International Journal of Engineering, Science and Mathematics, 7(6), 14-19.
15. Edward Samuel, A.,& Narmadhagnanam, R. (2018).Tangent inverse similarity measure of single valued neutrosophic sets in medical diagnosis. International Journal of Creative Research Thoughts,6(2), 77-79.
16. Narmadhagnanam, R.,& Edward Samuel, A. (2022).Sine exponential measure of single valued neutrosophic sets in medical diagnosis. Neutrosophic sets and systems,51,303-310.
17. Shan Ye.,&Jun Ye. (2014).Dice similarity measure between single valued neutrosophic multisets and its application in medical diagnosis, Neutrosophic sets, and systems, 6, 48-53.
18. Edward Samuel, A.,& Narmadhagnanam, R.(2018). Cosine Exponential Distance of Single Valued Neutrosophic Multi Set in Medical Diagnosis. International Journal of Engineering, Research and Application,8(6), 26-29.
19. Said Broumi,& Irfan Deli. (2016). Correlation measure for Neutrosophic refined set and its application in Medical Diagnosis.Palestine Journal of Mathematics,5 (1),135-143.DOI:10.5281/zenodo.23112
20. Edward Samuel, A., & Narmadhagnanam, R. (2017).Neutrosophic refined set in Medical Diagnosis.International Journal of Fuzzy Mathematical Archive,14(1),117-123.
21. Edward Samuel, A.,& Narmadhagnanam, R. (2018). Sine Logarithmic distance of Neutrosophic refined Set in Medical Diagnosis. Journal of Global Research in Mathematical Archive, 5(6), 1-5.
22. Edward Samuel, A.,& Narmadhagnanam, R. (2018).Contemporary techniques for Neutrosophic refined set and their exploitation in Medical Diagnosis. Journal of Emerging Technologies and Innovative Research,4(6), 25-29.
23. Said Broumi, Irfan Deli.,&FlorentinSmarandache. (2015). n-valued interval neutrosophic sets and their application in medical diagnosis. Critical Review,10, 45-69.DOI:10.6084/M9.FIGSHARE.1502596
24. Edward Samuel, A., & Narmadhagnanam, R. (2017). Innovative approaches for n-valued Interval Neutrosopic set and their Execution in Medical Diagnosis. Journal of Applied Sciences,17(9), 429-440.DOI: 10.3923/jas.2017.429.440
25. Edward Samuel, A.,& Narmadhagnanam, R. (2018).n-valued Interval Neutrosophic set in Medical Diagnosis. International Journal of Pure and Applied Mathematics,120 (8), 59-67.
26. Edward Samuel, A., & Narmadhagnanam, R. (2018). Execution of n-valued interval Neutrosophic set in Medical Diagnosis. International Journal of Mathematics, Trends and Technology,58(1), 66-70.DOI: https://doi.org/10.14445/22315373/IJMTT-V58P509

Received: 28 Jan 2024, **Revised:** 30 Apr 2024,
Accepted: 29 May 2024, **Available online:** 31 May 2024.

Neutrosophic Systems with Applications, Vol. 18, 2024
https://doi.org/10.61356/j.nswa.2024.18284

Ranking Cloud Service Providers using SWARA-MARCOS in Type-2 Neutrosophic Number Set Environment

Mai Mohamed [1,*], Shaimaa Ayman [1], Rui Yong [2], and Jun Ye [2]

[1] Faculty of Computers and Informatics, Zagazig University, Zagazig 44519, Sharqiyah, Egypt.
Emails: mmgaafar@zu.edu.eg, sh.ayman021@fci.zu.edu.eg.
[2] School of Civil and Environmental Engineering, Ningbo University, Ningbo, Zhejiang, China.
Emails: yongrui@nbu.edu.cn, yejun1@nbu.edu.cn.

* Correspondence: mmgaafar@zu.edu.eg.

Abstract: Cloud computing is a model for allowing suitable, on-demand network access to a shared store of resources such as servers, networks, storage, apps, and services, modified according to specific needs or requirements. The main goal of cloud technology development is to increase the use of resources that work together to achieve reliability at the lowest cost. Cloud service providers (CSPs) have gained popularity in recent years due to their accessibility and availability, as well as the growing quantity of cloud service providers (CSPs) that appear. Choosing (CSPs) has grown to be a challenging decision for many companies. The paper aims to rank a set of cloud service providers based on the multi-criteria decision-making (MCDM) method. The suggested method's applicability is verified by comparing the outcomes with two established methodologies: SWARA and MARCOS methods under the type-2 neutrosophic number set (T2NNS) environment to calculate the importance of evaluation criteria and ranking the alternatives of cloud providers. A sensitivity analysis was executed to check the robustness of this model by examining the effect of criteria weights on the ranking of the alternatives.

Keywords: Cloud Computing, Cloud Service Provider, Type-2 Neutrosophic Number Set, Multi-Criteria Decision-Making, SWARA, MARCOS.

1. Introduction

The rapid progress in information technology has led to the emergence of a novel approach in the field of distributed computing known as cloud computing, which has quickly gained huge popularity. High-performance computing was traditionally handled by costly grids, clusters, or supercomputers. There were drawbacks to each of these choices, such as higher infrastructure costs or less efficient use of available resources [1]. Cloud computing services through the Internet users can access collected computer resources including software applications, processing power, and storage. The collection of resources available to consumers on request is referred to as “the cloud”. As cloud computing grows, next-generation systems aim to become more pervasive, global, and present everywhere [2]. One benefit of cloud computing for businesses is its wide flexibility. It eliminates the need to spend money on physical infrastructure, allows for the quick reduction of resources when not in use, and allows at last minute changes without risking productivity. There are four types of cloud computing: public, private, community, and hybrid. Public cloud: depending on the service providers, this type of cloud is created and maintained by businesses in the education sector, and the government. It is accessible to the general public for usage like Azure, and AWS [3]. Private cloud: a single customer is the only one who has access to it, while education, business, and security agencies can use it privately like VMware, and IBM [4]. Community clouds: are used for business or security reasons, and they are developed and managed by a specific community. It is managed by one or two organizations like Salesforce, and Google Cloud [5]. The hybrid cloud: improves computing resource information and application availability by merging many cloud models, such as private, public, and community [6].

The rapid development of the software industry has encouraged many major cloud resource providers, referred to as cloud service providers (CSPs), to offer their services and become more competitive. By keeping pace with operating systems, software, and data architecture, cloud providers control user data. Now, users can pay for what they use as needed computing services, like Infrastructure as a Service (IaaS) as appear in Figure 1, Platform as a Service (PaaS), and Software as a Service (SaaS) from any location in the world, because of the cloud computing model. This model works especially well for companies whose resource requirements are as irregular as those whose needs change according to varies seasonally. As a result, some businesses and startups have become cloud users by totally renting cloud computing infrastructure from cloud providers instead of spending high beginning expenses on hiring specialized staff and buying customized computing equipment. To use the services, the user can make monthly or yearly payments. The increased flexibility that cloud computing offers enterprises is one of its main benefits.

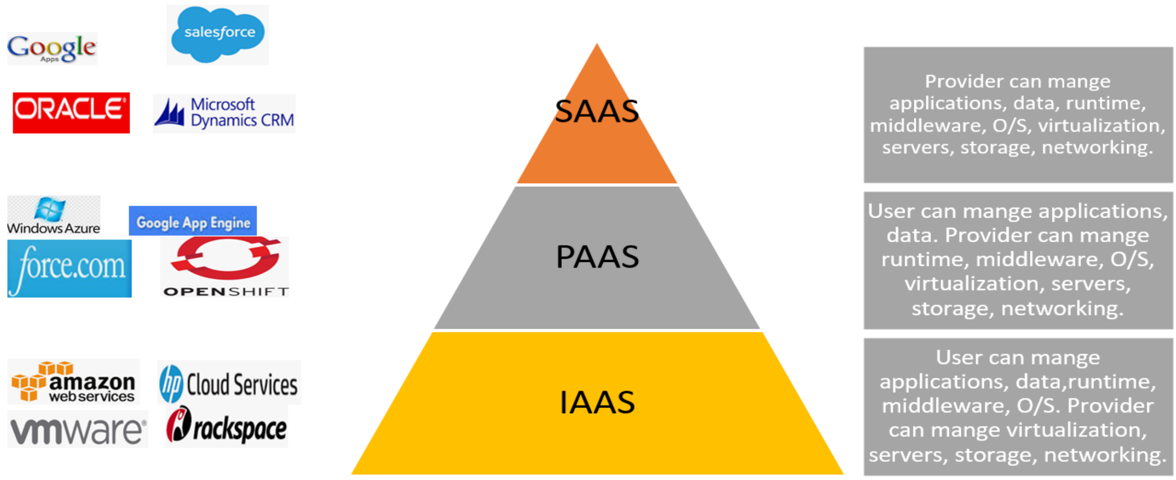

Figure 1. Cloud computing service models and their providers [7].

IAAS: provides companies with on-demand automatic server, storage, computing, and network infrastructure deployment. Companies may now control the networking, storage, and server saving them the trouble of building the IT infrastructure [8, 9].

PAAS: It helps companies manage the entire application without worrying about IT infrastructure. Cloud tools and services for developing applications, testing, and continuous deployment are provided by service providers.

SAAS: It allows companies to rent servers and subscribe to the programs when needed rather than purchase and maintain online applications. It achieves a growing level of acceptance in the marketplace [9].

The remainder of the paper is organized as follows: A summary of related work is given in Section 2. Section 3 explains the preliminary steps. Section 4 offers an overview of the suggested framework. In the framework of cloud computing, CSPs are ranked in Section 5 using SWARA and MARCOS based on the type-2 neutrosophic number sets (T2NNSs). Section 6 gives a case study. Section 7 presents a sensitivity analysis. Section 8 presents the conclusion.

2. Related Work

An extensive literature review has been done to understand the main concept study of cloud computing and the providers' services. Although there are a lot of definitions of cloud computing, the NIST definition is the most popular [10], which describes cloud computing as technologies that allow storage, networking, applications, and services to be quickly provisioned and released with little administrative effort. Using cloud service providers (CSPs) in the finance industry has several advantages, in 2021 Cloud Path survey indicates that bank managers understand enhanced company flexibility and freedom to adapt to market needs as the advantages of cloud-based operations [11]. In this study [12] they introduce an efficient analytical technique based on data envelopment analysis (DEA) to evaluate the Industry 4.0 CSPs' sustainability. They offer not only an effective strategy with solid backing from academia but also important management insights for practitioners to evaluate the sustainability of CSPs for Industry 4.0. Liu S et al. [13] selected the cloud service provider in MCDM using the SV-TOPSIS-SAW approach. Martens et al. [14] take risk and cost into account when developing a complex mathematical model for the cloud computing decision problem. Athraa et al. [15] introduced a hybrid MADM framework, FFS-FUCOM, and Grey-TOPSIS methodologies to rank CSPs using Quality of Service (QoS) attributes. Users can access the data at any time and need to be aware of the security protocols and how to defend themselves against threats like denial-of-service (DoS) attacks [16]. The taxonomic framework created by Rimal et al. [17] can categorize particular cloud providers. By the CSA CAIQ framework, cloud providers additionally make available service-specific capabilities of security, compliance, data governance, etc. through a public repository called STAR [18]. David et al. [19] PrPl was created for usage with both rented cloud machines and servers that users supply, Users use cloud services like Microsoft Azure and Amazon EC2 to store their private data. Each user has their virtual machine (VM) server running in the cloud. After that, content is divided up among user servers into groups, with the group's founder in charge of communication and membership.

Many criteria must be taken into evaluation for several CSPs. Several alternatives are evaluated in comparison to several criteria to determine which is the optimal alternative [20].

The set of criteria needed for the evaluation of cloud service providers (CSPs) are as follows:

C1- Downtime is a major drawback of cloud computing. Cloud providers may have technical problems such as data center maintenance requirements, poor Internet access, and power cuts. This can cause the internet service to go down temporarily.

C2- Speed: Make sure the cloud service provider has high-performance computing (HPC) servers if your company depends on super-fast cloud services.

C3- Security is challenging to access private data on the cloud because of the ability to safeguard all data from hackers. Strong encryption on files and databases is a feature of cloud computing that might lessen vulnerability to hacker attacks.

C4- Flexibility: Companies could use it to quickly expand storage and resources to meet demand. Similarly, resources can be quickly depleted if not in use on the cloud.

C5- Cost Reduction: For small and medium companies, cloud computing minimizes IT expenses, users can set up even basic apps, like email, and most of them are available for free with Google Apps.

Also, the set of alternative cloud service providers (CSPs) needed for the evaluation are as follows:

A1- Amazon Web Services: five times as much computing capacity as competing cloud service providers. Different data centers are available in different locations Offer the option to set up their security firewall to be either private or public based on requirements [1].

A2- Microsoft Azure: offers a competitive advantage in the commercial world because of its speed in important areas. It has an excellent disaster recovery mechanism that can operate in demanding environments [21].

A3- Google Cloud Platform: Compared to other cloud platforms, this one offers more affordable pricing, requiring users to pay only for the compute time they utilize [22].

A4- IBM Cloud: recovering from disasters rather quickly and integrating (IaaS) and (PaaS). The distribution of the workload is methodical to provide users with a satisfactory application response [23].

A5- Salesforce: The first real-time cloud platform for creating dependable, quick, and secure multitenant customized or business apps. Cloud solutions are provided for business services, marketing, sales, and other purposes [24].

A6- VMWare: the basic structure of VMware's cloud solution is its vCloud suite, which offers an API-based platform for managing and controlling clouds [25].

A7- Alibaba Cloud: is the proportion of applied resources to actual resources used and provides cloud computing DBaaS, SaaS, PaaS, and IaaS [26].

3. Preliminaries

This section defines the preliminary steps that were taken to design the framework. The theory of neutrosophic sets introduces T2NNS. Its definition is a generalization of the set definition found in set theory.

3.1 Type-2 Neutrosophic Number Set

A neutrosophic set has three membership functions to represent: the truth membership function (T), the indeterminacy membership function (I), and the falsity membership function (F) [27].

Definition 1. Suppose X is a universe of discourse, U is a neutrosophic set, T_U, I_U, F_U represent the degree of truth membership (T), the degree of indeterminacy membership (I), and the degree of falsity membership (F) of the element x.

$$U = \left\{\langle (T_{T_U}(\mathrm{x}), T_{I_U}(\mathrm{x}), T_{F_U}(\mathrm{x})), (I_{T_U}(\mathrm{x}), I_{I_U}(\mathrm{x}), I_{F_U}(\mathrm{x})), (F_{T_U}(\mathrm{x}), F_{I_U}(\mathrm{x}), F_{F_U}(\mathrm{x}))\rangle | \mathrm{x} \in \mathrm{X}\right\} \quad (1)$$

Where $\mathrm{X} \rightarrow [0,1]^3$, $\mathrm{x} \in \mathrm{X} : 0 \leq (T_{T_U}(\mathrm{x}), T_{I_U}(\mathrm{x}), T_{F_U}(\mathrm{x})) \leq 3$, $0 \leq (I_{T_U}(\mathrm{x}), I_{I_U}(\mathrm{x}), I_{F_U}(\mathrm{x})) \leq 3$, $0 \leq (F_{T_U}(\mathrm{x}), F_{I_U}(\mathrm{x}), F_{F_U}(\mathrm{x})) \leq 3$.

Definition 2. Let two T2NNSs U_1, U_2 be defined as the following operations:

Addition $$U_1 \oplus U_2 = \{\langle (T_{T_{U1}}(\mathrm{x}) + T_{T_{U2}}(\mathrm{x}) - T_{T_{U1}}(\mathrm{x}) . T_{T_{U2}}(\mathrm{x})), (T_{I_{U1}}(\mathrm{x}) + T_{I_{U2}}(\mathrm{x}) - T_{I_{U1}}(\mathrm{x}) . T_{I_{U2}}(\mathrm{x})), (T_{F_{U1}}(\mathrm{x}) + T_{F_{U2}}(\mathrm{x}) - T_{F_{U1}}(\mathrm{x}) . T_{F_{U2}}(\mathrm{x}))\rangle, (I_{T_{U1}}(\mathrm{x}) . I_{T_{U2}}(\mathrm{x}),\ I_{I_{U1}}(\mathrm{x}) . I_{I_{U2}}(\mathrm{x}),\ I_{F_{U1}}(\mathrm{x}) . I_{F_{U2}}(\mathrm{x})), (F_{T_{U1}}(\mathrm{x}) . F_{T_{U2}}(\mathrm{x}),\ F_{I_{U1}}(\mathrm{x}) . F_{I_{U2}}(\mathrm{x}),\ F_{F_{U1}}(\mathrm{x}) . F_{F_{U2}}(\mathrm{x}))\} \quad (2)$$

Multiplication $$U_1 \otimes U_2 = \Big\{(T_{T_{U1}}(\mathrm{x}). T_{T_{U2}}(\mathrm{x}),\ T_{I_{U1}}(\mathrm{x}). T_{I_{U2}}(\mathrm{x}),\ T_{F_{U1}}(\mathrm{x}) . T_{F_{U2}}(\mathrm{x})), \Big(\Big(I_{T_{U1}}(\mathrm{x}) + I_{T_{U2}}(\mathrm{x}) - I_{T_{U1}}(\mathrm{x}). I_{T_{U2}}(\mathrm{x})\Big),\ \Big(I_{I_{U1}}(\mathrm{x}) + I_{I_{U2}}(\mathrm{x}) - I_{I_{U1}}(\mathrm{x}). I_{I_{U2}}(\mathrm{x})\Big), \Big(I_{F_{U1}}(\mathrm{x}) + I_{F_{U2}}(\mathrm{x}) - I_{F_{U1}}(\mathrm{x}). I_F(\mathrm{x})\Big)\Big), \Big(\Big(F_{T_{U1}}(\mathrm{x}) + F_{T_{U2}}(\mathrm{x}) - F_{T_{U1}}(\mathrm{x}). F_{T_{U2}}(\mathrm{x})\Big),\ \Big(F_{I_{U1}}(\mathrm{x}) + F_{I_{U2}}(\mathrm{x}) - F_{I_{U1}}(\mathrm{x}). F_{I_{U2}}(\mathrm{x})\Big), \Big(F_{F_{U1}}(\mathrm{x}) + F_{F_{U2}}(\mathrm{x}) - F_{F_{U1}}(\mathrm{x}). F_F(\mathrm{x})\Big)\Big) \Big\} \quad (3)$$

Definition 3. The score functions $S(U)$ of a type-2 neutrosophic number (T2NN) is defined as:

$$S(U) = \frac{1}{12} \langle 8 + (T_{T_U}(\mathrm{x}) + 2(T_{I_U}(\mathrm{x})) + T_{F_U}(\mathrm{x})) - (I_{T_U}(\mathrm{x}) + 2(I_{I_U}(\mathrm{x})) + I_{F_U}(\mathrm{x})) - (F_{T_U}(\mathrm{x}) + 2(F_{I_U}(\mathrm{x})) + F_{F_U}(\mathrm{x}))\rangle \quad (4)$$

Definition 4. Aggregate the crisp value by using the average:

$$X_U = \frac{[(T_{T_U}(x), T_{I_U}(x), T_{F_U}(x)), (I_{T_U}(x), I_{I_U}(x), I_{F_U}(x)), (F_{T_U}(x), F_{I_U}(x), F_{F_U}(x)]}{n} \quad (5)$$

Where n number of experts.

3.2 SWARA

Determination of the important weight for each criterion of the decision-makers. Step-wise Weight Assessment Ratio Analysis (SWARA) is one of the MCDM methods that was introduced by Kersuliene et al. [28].

Definition 5. The criteria have been arranged in descending order according to their expected importance.

Definition 6. Evaluate the relative importance of the j criterion for the (j −1) criterion for each specific criterion. Begin with the second criterion so that criterion j is compared with the previous criterion (j −1). S_j is the comparative significance of mean value $0 \leq S_j \leq 1$ [28].

Definition 7. Calculating the coefficient K_j of comparative importance by Eq. (6).

$$K_j = \begin{cases} 1 & j = 1 \\ S_j + 1 & j > 1 \end{cases} \quad ; j = 1, \ldots\ldots, m \tag{6}$$

Definition 8. Calculating the initial weight of a criteria Q_j for every decision-maker by Eq. (7).

$$Q_j = \begin{cases} 1 & j = 1 \\ \frac{Q_{j-1}}{K_j} + 1 & j > 1 \end{cases} \quad ; j = 1, \ldots\ldots, m \tag{7}$$

Definition 9. Calculating the relative weights W_j of the criteria every decision-maker by Eq. (8) and the summation of this weight equal 1.

$$W_j = \frac{Q_j}{\sum_{j=1}^{m} Q_j} \tag{8}$$

where W_j is the relative weight of criterion j, and m is the number of criteria.

3.3 MARCOS

Measurement of Alternatives and Ranking according to the Compromise Solution (MARCOS) method introduced by Željko et al. [29]. It depends on establishing a combination between reference values and alternatives (ideal and anti-ideal alternatives). We used this method for Selecting and ranking the alternatives with respect to decision variables. Let A = ($A1$, $A2$,...Am) the number of alternatives, C = ($C1$, $C2$, $C3$, $C4$, $C5$) the numbers of criteria.

Definition 10. Constitute the initial T2NN decision-making matrix and calculate the AAI, and AI by applying Eqs. (10), and (11).

$$X = \begin{matrix} \text{AAI} \\ A_1 \\ \vdots \\ A_n \\ AI \end{matrix} \begin{bmatrix} x_{aa1} & \cdots & x_{aam} \\ x_{11} & \ddots & x_{1m} \\ \vdots & & \vdots \\ x_{n1} & \cdots & x_{nm} \\ x_{ai1} & \ldots & x_{aim} \end{bmatrix} \tag{9}$$

Where AAI = Anti-Ideal solution is the worst alternative, AI = Ideal solution is the best alternative depending on the nature of the criteria, n number of alternatives.

$$\text{AAI} = \min x_{ij} \; if \; j \in benefit \;\; and \; \max x_{ij} \; if \; j \in cost \tag{10}$$

$$\text{AI} = \max x_{ij} \; if \; j \in benefit \;\; and \; \min x_{ij} \; if \; j \in cost \tag{11}$$

Definition 11. Normalization of the initial matrix X. The normalized matrix's elements N = $[n_{ij}]_{n*m}$ obtained by applying Eqs. (12) and (13).

$$n_{ij} = \frac{x_{aj}}{x_{ij}} \qquad if \qquad j \in cost \tag{12}$$

$$n_{ij} = \frac{x_{ij}}{x_{aj}} \qquad if \qquad j \in benefit \tag{13}$$

Definition 12. Construct the weighted normalized decision matrix V $= [v_{ij}]_{n*m}$ Eq. (14).

$$v_{ij} = n_{ij} \times W_j \tag{14}$$

Definition 13. Calculate the utility degree of alternatives K_i by applying Eqs. (15), (16).

$$k_i^- = \frac{S_i}{S_{aai}} \tag{15}$$

$$k_i^+ = \frac{S_i}{S_{ai}} \tag{16}$$

Where S_i is the summation of elements in matrix V applied by Eq. (17).

$$S_i = \sum_{i=1}^{n} v_{ij} \quad (17)$$

Definition 14. Determine if the utility function of alternatives $f(k_i)$ by the following Equation:

$$f(k_i) = \frac{k_i^+ + k_i^-}{1 + \frac{1-f(k_i^+)}{f(k_i^+)} + \frac{1-f(k_i^-)}{f(k_i^-)}} \quad (18)$$

Where $f(k_i^+)$ is the utility function to the ideal solution, $f(k_i^-)$is the utility function of the anti-ideal solution.

Definition 15. Calculate the utility function to the ideal solution and anti-ideal solution by Eqs. (19), and (20).

$$f(k_i^+) = \frac{k_i^-}{k_i^+ + k_i^-} \quad (19)$$

$$f(k_i^-) = \frac{k_i^+}{k_i^+ + k_i^-} \quad (20)$$

Definition 16. Ranking of the optimal alternatives which depend on the final values of utility functions $f(k_i)$ in Eq. (18).

4. Case Study

A case study is performed on seven CSPs and five criteria shown in Table, according to five experts' CSPs judgments based on a scale shown in Table. After identifying CSPs and making the decision matrix a T2NN-MARCOS method is applied to rank the CSPs in descending order the $F(k_i)$ values. The following descriptions of steps are shown below as in Figure 2:

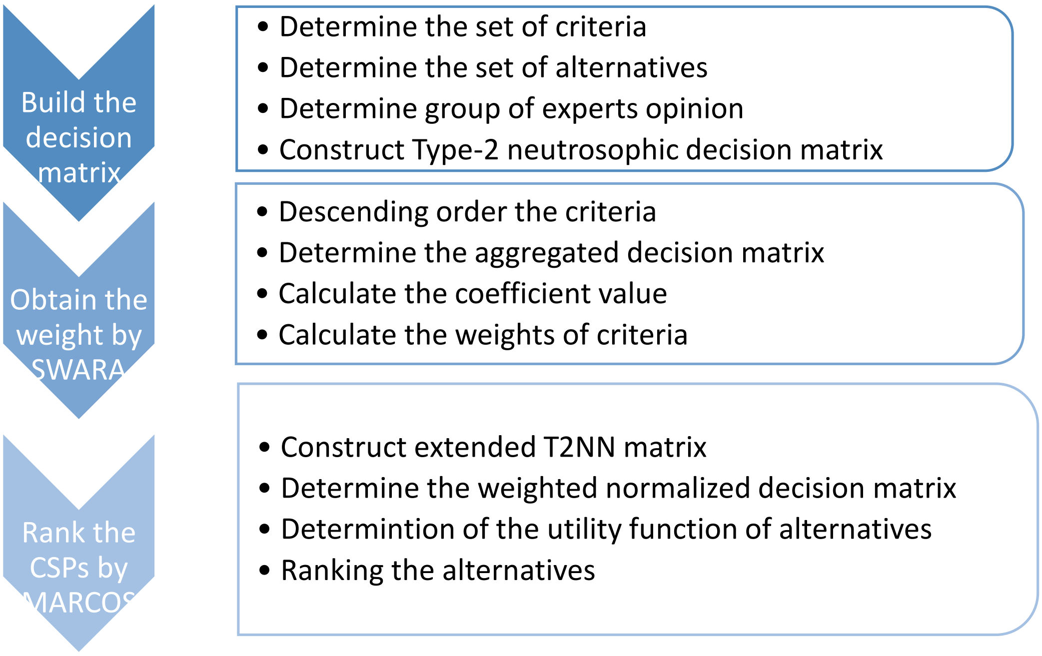

Figure 2. Framework of T2NN-SWARA-MARCOS.

Step 1. Identify the decision matrix.

The goal is to order the cloud service providers, first must determine the evaluation criteria. Suppose that the selected set of criteria is $C = (C1, C2, C3, C4, C5)$. As well as identifying the alternatives. Suppose that the selected set of CSPs is $A = (A1, A2, ...Am)$ where m number of CSPs, and Ex = (Ex_1, Ex_2, Ex_3, Ex_4, Ex5) be a set of experts.

Step 2. Construct the models by converting linguistic variables into crisp values. Transform the decision matrix into type-2 neutrosophic set values which are displayed in Table 1 [27].

Table 1. T2NN scale [27].

Linguistic variables	Type 2 neutrosophic number scale [(T_T, T_I, T_F), (I_T, I_I, I_F), (F_T, F_I, F_F)]	Score
Weakly important (WI)	⟨ (0.20, 0.30, 0.20), (0.60, 0.70, 0.80), (0.45, 0.75, 0.75) ⟩	0.291667
Equal important (EI)	⟨ (0.40, 0.30, 0.25), (0.45, 0.55, 0.40), (0.45, 0.60, 0.55) ⟩	0.425
Strong important (SI)	⟨ (0.65, 0.55, 0.55), (0.40, 0.45, 0.55), (0.35, 0.40, 0.35) ⟩	0.579167
Very strongly important (VSI)	⟨ (0.80, 0.75, 0.70), (0.20, 0.15, 0.30), (0.15, 0.10, 0.20) ⟩	0.804167
Absolutely important (AI)	⟨ (0.90, 0.85, 0.95), (0.10, 0.15, 0.10), (0.05, 0.05, 0.10) ⟩	0.9

Step 3. Based on the expected opinions of the expert's judgment, suppose that five experts start to judge the criteria in the scale of Table 2. Applying the score function equation and the scale that is shown in Table 1 to convert EXs' linguistic variables into crisp values by using Eq. (4).

Table 2. The crisp value of the expert's judgment.

EX1					
	C1	C2	C3	C4	C5
A1	0.292	0.9	0.804	0.579	0.579
A2	0.425	0.804	0.579	0.425	0.425
A3	0.579	0.292	0.425	0.9	0.579
A4	0.804	0.9	0.804	0.292	0.9
A5	0.579	0.425	0.579	0.292	0.292
A6	0.9	0.579	0.292	0.804	0.425
A7	0.579	0.425	0.9	0.579	0.804
EX2					
A1	0.425	0.804	0.9	0.292	0.292
A2	0.579	0.579	0.804	0.9	0.804
A3	0.804	0.9	0.292	0.579	0.292
A4	0.292	0.425	0.9	0.9	0.804
A5	0.579	0.579	0.425	0.804	0.579
A6	0.425	0.9	0.579	0.425	0.9
A7	0.9	0.579	0.425	0.804	0.579
EX3					
A1	0.425	0.9	0.804	0.292	0.9
A2	0.804	0.579	0.292	0.9	0.292
A3	0.579	0.579	0.9	0.425	0.804
A4	0.9	0.425	0.579	0.579	0.425
A5	0.425	0.9	0.804	0.292	0.579
A6	0.292	0.292	0.579	0.804	0.425
A7	0.579	0.579	0.9	0.425	0.804
EX4					
A1	0.804	0.425	0.579	0.9	0.579
A2	0.579	0.579	0.9	0.425	0.425
A3	0.425	0.9	0.579	0.579	0.9
A4	0.9	0.804	0.292	0.579	0.579
A5	0.292	0.425	0.425	0.425	0.292
A6	0.579	0.579	0.579	0.9	0.9
A7	0.804	0.425	0.579	0.9	0.579
EX5					
A1	0.9	0.292	0.579	0.425	0.425
A2	0.292	0.425	0.425	0.579	0.579
A3	0.804	0.579	0.579	0.425	0.425
A4	0.9	0.9	0.579	0.9	0.9
A5	0.425	0.804	0.292	0.425	0.804
A6	0.579	0.292	0.9	0.579	0.292
A7	0.9	0.292	0.579	0.425	0.425

Step 4. Obtaining the aggregate matrix by taking the average of the expert opinions by applying Eq. (5).
Step 5. Applying the SWARA method to calculate the weight of criteria ($C1$, $C2$, $C3$, $C4$, $C5$) as shown in Table 3. Ordering the criteria in a descending order from most important to least significant.

Table 3. Ranking criteria.

Criteria	Order
C3	1
C2	2
C4	3
C1	4
C5	5

Step 6. Evaluating the relative importanceS_j.

Table 4. Comparative importance of the criterion.

Questionnaire			
1	Security	15%	More important than Speed
2	Speed	30%	More important than Flexibility
3	Flexibility	10%	More important than Downtime
4	Downtime	25%	More important than Cost

Table 5. The relative importance

Order	Criteria	S_j
1	Security	----
2	Speed	0.15
3	Flexibility	0.3
4	Downtime	0.1
5	Cost	0.25

Step 7. Calculating the coefficient K_j by applying Eq. (6).

Table 6. The coefficient.

Criteria	K_j
Security	1
Speed	1.15
Flexibility	1.3
Downtime	1.1
Cost	1.25

Step 8. Finding the recalculated weight Q_j by applying Eq. (7), and the relative weights of the criteria W_j by applying Eq. (8) shown in Figure 3 and presented in Table 7.

Table 7. The weights of criteria.

Criteria	Q_j	W_j
Security	1	0.275253
Speed	0.869565217	0.239351
Flexibility	0.668896321	0.184116
Downtime	0.608087565	0.167378
Cost	0.486470052	0.133902

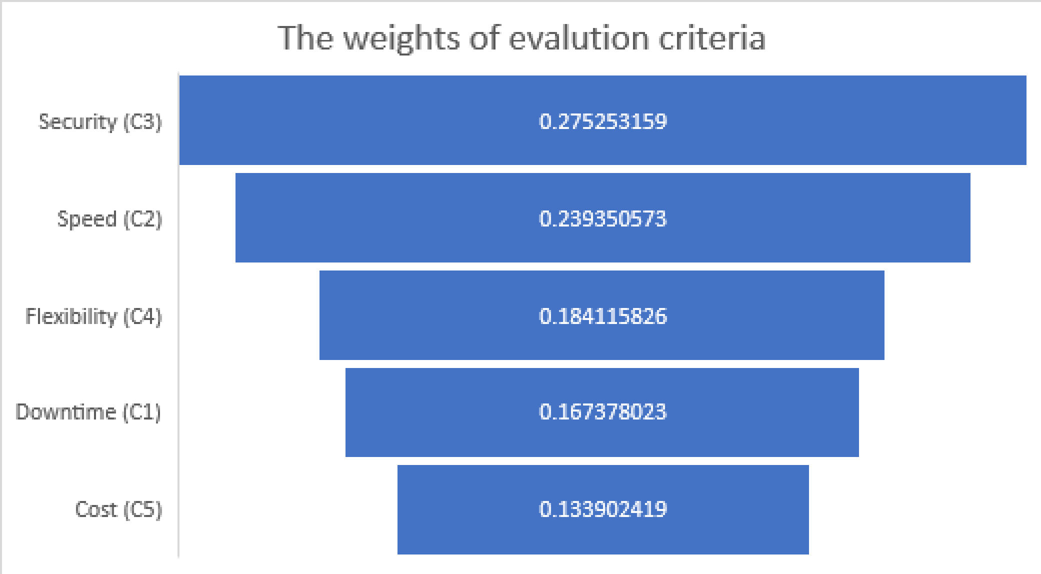

Figure 3. The final weights.

Step 9. After constituting the T2NN decision matrix, then calculate the AAI, and AI by applying Eqs. (10), and (11) as exhibited in Table 8.

Table 8. The T2NN decision matrix.

	C1	**C2**	**C3**	**C4**	**C5**
	min	max	max	max	min
$\boldsymbol{W_i}$	0.167378	0.239351	0.275253	0.18411583	0.133902
AAI	0.7592	0.3166	0.445	0.3616	0.7216
A1	0.5692	0.6642	0.7332	0.4976	0.555
A2	0.5358	0.5932	0.6	0.6458	0.505
A3	0.6382	0.65	0.555	0.5816	0.6
A4	0.7592	0.6908	0.6308	0.65	0.7216
A5	0.46	0.6266	0.505	0.4476	0.5092
A6	0.555	0.5284	0.5858	0.7024	0.5884
A7	0.4116	0.3166	0.445	0.3616	0.4374
AI	0.4116	0.6908	0.7332	0.7024	0.4374

Step 10. Constructing the normalized decision matrix N $= [n_{ij}]_{n*m}$ applying by Eqs. (12), and (13) as shown in Table 9.

Table 9. The normalized decision matrix.

	C1	C2	C3	C4	C5
	min	max	max	max	min
$\boldsymbol{W_i}$	0.167378	0.239351	0.275253	0.18411583	0.133902
AAI	0.54215	0.458309	0.606929	0.51480638	0.606153
A1	0.72312	0.961494	0.606929	0.70842825	0.788108
A2	0.768197	0.858715	1	0.91941913	0.866139
A3	0.644939	0.940938	0.818331	0.82801822	0.729
A4	0.54215	1	0.756956	0.92539863	0.606153
A5	0.894783	0.907064	0.860338	0.63724374	0.858995
A6	0.741622	0.76491	0.688762	1	0.743372
A7	1	0.458309	0.798963	0.51480638	1
AI	1	1	1	1	1

Step 11. Determination of the weighted normalized decision matrix V $= [v_{ij}]_{n*m}$ applying by Eq. (14), as presented in Table 10.

Table 10. The weighted normalized decision matrix.

	C1	C2	C3	C4	C5
	min	max	max	max	min
W_i	0.167378	0.239351	0.275253	0.184116	0.133902
AAI	0.090744	0.109697	0.167059	0.094784	0.081165
A1	0.121034	0.230134	0.167059	0.130433	0.10553
A2	0.128579	0.205534	0.275253	0.16928	0.115978
A3	0.107949	0.225214	0.225248	0.152451	0.097615
A4	0.090744	0.239351	0.208354	0.170381	0.081165
A5	0.149767	0.217106	0.236811	0.117327	0.115021
A6	0.124131	0.183082	0.189584	0.184116	0.099539
A7	0.167378	0.109697	0.219917	0.094784	0.133902
AI	0.167378	0.239351	0.275253	0.184116	0.133902

Step 12. Collecting the utility degree of alternatives K_i by applying Eq. (15), and (16) as shown in Table 11.

Table 11. The utility degree of alternatives.

A	k_i^-	k_i^+
A1	1.38778464	0.754189975
A2	1.64619715	0.894623954
A3	1.48767792	0.808476863
A4	1.45366922	0.78999487
A5	1.53838252	0.836032215
A6	1.43610899	0.780451787
A7	1.33532019	0.725678228

Step 13. Calculating the utility function to the ideal solution and anti-ideal solution by Eqs. (19), and (20). Determination of the utility function of alternatives $f(k_i)$ by Eq. (18) as shown in Table 12.

Table 12. The utility function of alternatives.

$f(k_i^-)$	$1-f(k_i^-)$	$f(k_i^+)$	$1-f(k_i^+)$	$f(k_i)$
0.3521	0.6479	0.6479	0.35210033	0.633056
0.417663	0.582337	0.768542	0.23145814	0.942637
0.377445	0.622555	0.694536	0.30546426	0.743283
0.368816	0.631184	0.678658	0.32134153	0.704475
0.390309	0.609691	0.718208	0.28179237	0.80368
0.364361	0.635639	0.67046	0.32953968	0.68496
0.338789	0.661211	0.623406	0.37659383	0.57962

Step 13. Ranking of the optimal alternatives which depend on the final values of utility functions $f(k_i)$ in Eq. (18), as presented in Table 13.

Table 13. Rank of alternatives based on T2NN-MARCOS.

A	Rank
A1	6
A2	1
A3	3
A4	4
A5	2
A6	5
A7	7

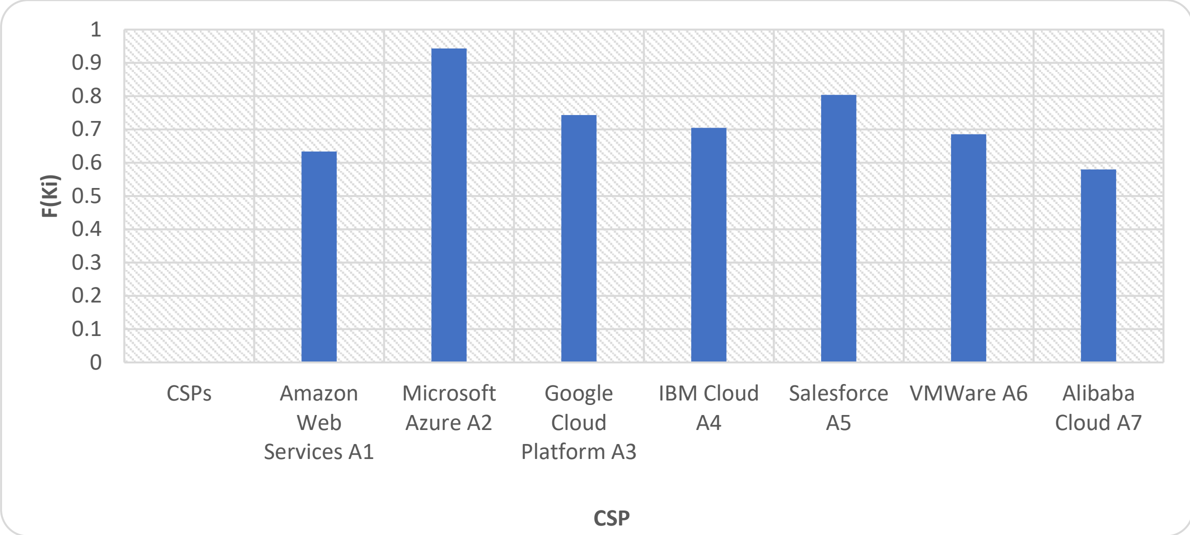

Figure 4. Rank of alternatives based on T2NN MARCOS.

The final rank of alternatives shown in Figure 4 using the T2NNs-MARCOS shows that Microsoft Azure which donated as A2 is the best CSP and A7 is the worst. The ranking of all alternatives is A2>A5>A3>A4>A6>A1>A7.

5. Sensitivity Analysis

In this part, a sensitivity analysis was performed on the alternative rank. As a result, we will show how different criterion weights will affect the final ranking of alternatives. A change has been made to the weight measurement values to indicate whether the alternative order will change, which suggests weights based on different criteria to rank the alternatives in different situations and show the stability of the rank. Table 14 shows the rank of alternatives after changing the weights. As shown in Figure 5, in case 1 if the weight of C1 is bigger than the weight of C3, alternative 2 is the best, and alternative 7 is the worst, A2>A5>A3>A6>A4>A1>A7.

Case 2 if the weight C1 is bigger than the weight C2, alternative 2 is the best, and alternative 1 is the worst, A2>A5>A3>A6>A7>A4>A1.

Case 3 if the weight C2 is bigger than the weight C3, alternative 2 is the best, and alternative 7 is the worst, A2>A5>A3>A4>A6>A1>A7.

Case 4 if the weight C3 is bigger than the weight C4, alternative 2 is the best, and alternative 7 is the worst, A2>A5>A3>A6>A4>A1>A7.

Case 5 if the weight C4 is bigger than the weight C5, alternative 2 is the best, and alternative 7 is the worst, A2>A5>A3>A4>A6>A1>A7.

Case 6 if the weight C5 is bigger than the weight C2, alternative 2 is the best, and alternative 7 is the worst, A2>A5>A3>A7>A6>A4>A1.

Case 7 if the weight C5 is bigger than the weight C3, alternative 2 is the best, and alternative 7 is the worst, A2>A5>A3>A6>A1>A4>A7.

Case 8 if the weight C4 is bigger than the weight C3, alternative 2 is the best, and alternative 7 is the worst, A2>A5>A3>A6>A4>A1>A7.

Case 9 if the weight C1 is bigger than the weight C4, alternative 2 is the best, and alternative 7 is the worst, A2>A5>A3>A4>A6>A1>A7.

Table 14. The rank of alternatives under sensitivity analysis.

	Original	Case 1 c1>c3	Case 2 c1>c2	Case 3 c2>c3	Case4 c3>c4	Case5 c4>c5	Case 6 c5>c2	Case 7 c5>c3	Case 8 c4>c3	Case 9 c1>c4
A1	6	6	7	6	6	6	7	5	6	6
A2	1	1	1	1	1	1	1	1	1	1
A3	3	3	3	3	3	3	3	3	3	3
A4	4	5	6	4	5	4	6	6	5	4
A5	2	2	2	2	2	2	2	2	2	2
A6	5	4	4	5	4	5	5	4	4	5
A7	7	7	5	7	7	7	4	7	7	7

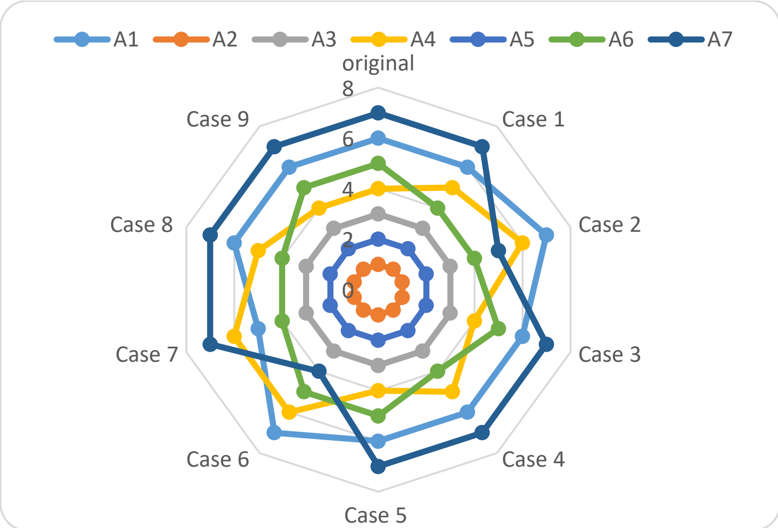

Figure 5. Cases in criteria weights changing.

6. Conclusion

Leading enterprise companies like Amazon, Microsoft, and Google now offer a wide range of cloud services in the form of specialized, dependable, and reasonably priced web apps. Individuals and organizations in various fields find these services attractive, such as healthcare, business, and education. So, we rank the popular Cloud Service Providers (CSPs) among potential cloud customers according to specific criteria or attributes related to the services they offer. This study shows that the proposed approach is an effective multi-criteria decision-making (MCDM) tool for the difficult analysis of selection among information sets. We provided a numerical example showing our suggested method of selecting Cloud Service Providers (CSPs) in cloud service management to select the optimal one. The proposed method was evaluated using type-2 neutrosophic (T2NN) based on two popular MCDM methods, SWARA and MARCOS. In the future, we plan to use different multi-criteria decision-making methods with more complex criteria.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contributions
All authors contributed equally to this research.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Funding
This research was not supported by any funding agency or institute.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

References

1. Ostermann, S., Iosup, A., Yigitbasi, N., Prodan, R., Fahringer, T., & Epema, D. (2010). A performance analysis of EC2 cloud computing services for scientific computing. In Cloud Computing: First International Conference, CloudComp 2009 Munich, Germany, October 19–21, 2009 Revised Selected Papers 1 (pp. 115-131). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-12636-9_9
2. Varghese, B., & Buyya, R. (2018). Next generation cloud computing: New trends and research directions. Future Generation Computer Systems, 79, 849-861. https://doi.org/10.1016/j.future.2017.09.020
3. Hu, H., Wen, Y., & Niyato, D. (2017). Public cloud storage-assisted mobile social video sharing: A supermodular game approach. IEEE journal on selected areas in communications, 35(3), 545-556. https://doi.org/10.1109/JSAC.2017.2659478
4. Doelitzscher, F., Sulistio, A., Reich, C., Kuijs, H., & Wolf, D. (2011). Private cloud for collaboration and e-Learning services: from IaaS to SaaS. Computing, 91, 23-42. https://doi.org/10.1007/s00607-010-0106-z
5. Goyal, S. (2014). Public vs private vs hybrid vs community-cloud computing: a critical review. International Journal of Computer Network and Information Security, 6(3), 20-29.
6. Li, Q., Wang, Z. Y., Li, W. H., Li, J., Wang, C., & Du, R. Y. (2013). Applications integration in a hybrid cloud computing environment: Modelling and platform. Enterprise Information Systems, 7(3), 237-271. https://doi.org/10.1080/17517575.2012.677479
7. Laghari, A. A., He, H., Halepoto, I. A., Memon, M. S., & Parveen, S. (2017). Analysis of quality of experience frameworks for cloud computing. IJCSNS, 17(12), 228.
8. Alqahtany, S., Clarke, N., Furnell, S., & Reich, C. (2014). A forensically-enabled IAAS cloud computing architecture. https://doi.org/10.4225/75/57b3e3a5fb87e
9. Lal, P., & Bharadwaj, S. S. (2016). Understanding the impact of cloud-based services adoption on organizational flexibility: An exploratory study. Journal of enterprise information management, 29(4), 566-588. https://doi.org/10.1108/JEIM-04-2015-0028
10. Mell, P., & Grance, T. (2011). The NIST definition of cloud computing.
11. Harmon, R. L., & Psaltis, A. (2021). The future of cloud computing in financial services: A machine learning and artificial intelligence perspective. In The Essentials of Machine Learning in Finance and Accounting (pp. 123-138). Routledge.
12. Azadi, M., Moghaddas, Z., Cheng, T. C. E., & Farzipoor Saen, R. (2023). Assessing the sustainability of cloud computing service providers for Industry 4.0: a state-of-the-art analytical approach. International Journal of Production Research, 61(12), 4196-4213. https://doi.org/10.1080/00207543.2021.1959666
13. Liu, S., Chan, F. T., & Ran, W. (2016). Decision making for the selection of cloud vendor: An improved approach under group decision-making with integrated weights and objective/subjective attributes. Expert Systems with Applications, 55, 37-47. https://doi.org/10.1016/j.eswa.2016.01.059
14. Martens, B., & Teuteberg, F. (2012). Decision-making in cloud computing environments: A cost and risk based approach. Information Systems Frontiers, 14, 871-893. https://doi.org/10.1007/s10796-011-9317-x
15. Al-Mashhadani, A. K., Salman, M. S., & Al-Yasiri, F. A. H. N. (2024). Quality of Service Attributes Based Hybrid Decision-Making Framework for Ranking Cloud Service Providers Under Fermatean Fuzzy Environment. Yugoslav Journal of Operations Research. http://dx.doi.org/10.2298/YJOR231215010A

16. Iqbal, S., Kiah, M. L. M., Dhaghighi, B., Hussain, M., Khan, S., Khan, M. K., & Choo, K. K. R. (2016). On cloud security attacks: A taxonomy and intrusion detection and prevention as a service. Journal of Network and Computer Applications, 74, 98-120. https://doi.org/10.1016/j.jnca.2016.08.016
17. Rimal, B. P., Choi, E., & Lumb, I. (2009, August). A taxonomy and survey of cloud computing systems. In 2009 fifth international joint conference on INC, IMS and IDC (pp. 44-51). Ieee. https://doi.org/10.1109/NCM.2009.218
18. Habib, S. M., Volk, F., Hauke, S., & Mühlhäuser, M. (2015). Computational trust methods for security quantification in the cloud ecosystem. The Cloud Security Ecosystem, 463-493. https://doi.org/10.1016/B978-0-12-801595-7.00021-5
19. Koll, D., Li, J., & Fu, X. (2017). The good left undone: Advances and challenges in decentralizing online social networks. Computer Communications, 108, 36-51. https://doi.org/10.1016/j.comcom.2017.04.008
20. Choosing cloud servises providers. https://cloud.google.com/learn/what-is-a-cloud-service-provider: p. https://cloud.google.com/learn/what-is-a-cloud-service-provider.
21. Redkar, T., Guidici, T., & Meister, T. (2011). Windows azure platform (Vol. 1). New York City: Apress.
22. Cloud, S., Google Cloud Platform. 2018.
23. McIntyre, J. (2012). IBM SmartCloud: Becoming a Cloud Service Provider. IBM Redbooks.
24. Choudharyor, N., & Arya, V. (2021). Salesforce IoT cloud platform. In Proceedings of Integrated Intelligence Enable Networks and Computing: IIENC 2020 (pp. 301-309). Springer Singapore. https://doi.org/10.1007/978-981-33-6307-6_31
25. Gallagher, S. (2013). VMware private cloud computing with vCloud Director. John Wiley & Sons.
26. Deng, L., Ren, Y. L., Xu, F., He, H., & Li, C. (2018). Resource utilization analysis of Alibaba cloud. In Intelligent Computing Theories and Application: 14th International Conference, ICIC 2018, Wuhan, China, August 15-18, 2018, Proceedings, Part I 14 (pp. 183-194). Springer International Publishing. https://doi.org/10.1007/978-3-319-95930-6_18
27. Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing, 77, 438-452. https://doi.org/10.1016/j.asoc.2019.01.035
28. Keršuliene, V., Zavadskas, E. K., & Turskis, Z. (2010). Selection of rational dispute resolution method by applying new step-wise weight assessment ratio analysis (SWARA). Journal of business economics and management, 11(2), 243-258. https://doi.org/10.3846/jbem.2010.12
29. Stević, Ž., Pamučar, D., Puška, A., & Chatterjee, P. (2020). Sustainable supplier selection in healthcare industries using a new MCDM method: Measurement of alternatives and ranking according to COmpromise solution (MARCOS). Computers & industrial engineering, 140, 106231. https://doi.org/10.1016/j.cie.2019.106231

Received: 25 Jan 2024, **Revised:** 30 Apr 2024,

Accepted: 29 May 2024, **Available online:** 02 Jun 2024.

NEUTROSOPHIC
SYSTEMS WITH APPLICATIONS

AN INTERNATIONAL JOURNAL ON INFORMATICS, DECISION SCIENCE, INTELLIGENT SYSTEMS APPLICATIONS

Sciences Force
Five Greentree Centre, 525 Route 73
North, STE 104 Marlton, New Jersey
08053.
www.sciencesforce.com

ISSN (ONLINE): 2993-7159
ISSN (PRINT): 2993-7140