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HyperNeutrosophic Set and Forest HyperNeutrosophic Set with Practical Applications in Agriculture

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Abstract

The Neutrosophic Set provides a flexible mathematical framework for handling uncertainty by incorporating three distinct membership functions: truth, indeterminacy, and falsity [50]. In response to the growing complexity of real-world problems, advanced extensions such as the Hyperneutrosophic Set and the SuperHyperneutrosophic Set have been introduced. These extensions offer a higher-dimensional approach to modeling uncertainty and can be formally defined in [33, 34, 35, 37, 36, 51, 32].
Since the Hyperneutrosophic Set and the SuperHyperneutrosophic Set have only been recently defined, there remains a lack of concrete examples demonstrating their applicability. This paper aims to bridge this gap by exploring practical examples of these concepts in the field of agriculture. Additionally, we introduce two novel extensions, the Forest Hyperneutrosophic Set and the Forest SuperHyperneutrosophic Set, which further generalize the existing models to accommodate hierarchical and interconnected uncertainty structures in complex systems.

Keywords: Set Theory, SuperhyperNeutrosophic set, Neutrosophic Set, HyperNeutrosophic set

1 | Preliminaries and Definitions

This section provides an overview of the fundamental concepts and definitions essential for the discussions in this paper. The analysis utilizes classical set-theoretic operations and extends them into advanced frameworks.

1.1 | Neutrosophic, HyperNeutrosophic, and n-SuperHyperNeutrosophic Sets

To address uncertainty, vagueness, and imprecision in decision-making processes, numerous set-theoretic frameworks have been developed. These frameworks include Fuzzy Sets, which were introduced in foundational works such as those by Zadeh [56, 57, 64, 63, 62, 58, 59, 61, 60]. Another prominent framework is Intuitionistic Fuzzy

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Sets, extensively studied by Atanassov and others [4, 7, 5, 3, 8, 6]. Vague Sets, introduced and developed by researchers, also contribute significantly to this domain [42, 39, 1, 9, 45].

Neutrosophic Sets, first introduced by Smarandache, offer a powerful means of capturing indeterminacy, allowing for more nuanced decision-making models [49, 50, 31, 26, 17, 23, 16, 53, 25, 28, 15, 27, 24]. Neutrosophic Sets generalize Fuzzy Sets by introducing an additional component: indeterminacy, alongside truth and falsity [49, 50, 47, 48]. This enhancement allows for a richer and more precise representation of uncertainty and ambiguity.

To address increasingly complex scenarios, HyperNeutrosophic Sets and n-SuperHyperNeutrosophic Sets have been developed. These advanced models are particularly suited for high-dimensional and intricate problem spaces [29, 19].

Definition 1 (Base Set)**.** A *base set* S is the foundational set from which complex structures such as powersets and hyperstructures are derived. It is formally defined as:

$$S = \{x \mid x \text{ is an element within a specified domain}\}.$$

All elements in constructs like $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ originate from the elements of S.

Definition 2 (Powerset)**.** [18, 44] The *powerset* of a set S, denoted $\mathcal{P}(S)$, is the collection of all possible subsets of S, including both the empty set and S itself. Formally, it is expressed as:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 3 (n-th Powerset)**.** (cf.[52, 18, 20, 13, 46])

The n-th powerset of a set H, denoted $P_n(H)$, is defined iteratively, starting with the standard powerset. The recursive construction is given by:

$$P_1(H) = P(H), \quad P_{n+1}(H) = P(P_n(H)), \quad \text{for } n \geq 1.$$

Similarly, the n-th non-empty powerset, denoted $P_n^*(H)$, is defined recursively as:

$$P_1^*(H) = P^*(H), \quad P_{n+1}^*(H) = P^*(P_n^*(H)).$$

Here, $P^*(H)$ represents the powerset of H with the empty set removed.

Definition 4 (Neutrosophic Set)**.** [49, 50, 47, 54, 53] Let X be a non-empty set. A *Neutrosophic Set (NS) A* on X is characterized by three membership functions:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 5 (HyperNeutrosophic Set)**.** (cf.[19, 51, 22, 14, 30, 29]) Let X be a non-empty set. A *HyperNeutrosophic Set (HNS) $\tilde{A}$* on X is a mapping:

$$\tilde{\mu} : X \to \mathcal{P}([0,1]^3),$$

where $\mathcal{P}([0,1]^3)$ is the family of all non-empty subsets of the unit cube $[0,1]^3$. For each $x \in X$, $\tilde{\mu}(x) \subseteq [0,1]^3$ is a set of neutrosophic membership triplets (T, I, F) that satisfy:

$$0 \leq T + I + F \leq 3.$$

Definition 6 (n-SuperHyperNeutrosophic Set)**.** (cf.[19, 14, 30, 29]) Let X be a non-empty set. An *n-SuperHyperNeutrosophic Set (n-SHNS)* is a recursive generalization of Neutrosophic Sets and HyperNeutrosophic Sets. It is defined as a mapping:

$$\tilde{A}_n : \mathcal{P}_n(X) \to \mathcal{P}_n([0,1]^3),$$

where:

- $\mathcal{P}_1(X) = \mathcal{P}(X)$, the power set of X, and for $k \geq 2$,
$$\mathcal{P}_k(X) = \mathcal{P}(\mathcal{P}_{k-1}(X)),$$
representing the k-th nested family of non-empty subsets of X.
- $\mathcal{P}_n([0,1]^3)$ is defined similarly for the unit cube $[0,1]^3$.

For each $A \in \mathcal{P}_n(X)$ and $(T, I, F) \in \tilde{A}_n(A)$, the following condition is satisfied:
$$0 \leq T + I + F \leq 3,$$
where T, I, F represent the degrees of truth, indeterminacy, and falsity for the n-th level subsets of X.

2 | Some Examples of Neutrosophic Set

This section outlines the main results presented in this paper.

2.1 | Neutrosophic Set in Agriculture

We provide several examples related to the Neutrosophic Set in Agriculture.

Example 7 (Neutrosophic Set in Agriculture (Crop Suitability))**. Context and Intuition:** Crop Suitability refers to the assessment of land, climate, and soil conditions to determine the feasibility of growing specific crops effectively[2, 41, 38, 12]. Let $X = \{\text{Plot A}, \text{Plot B}, \text{Plot C}\}$ represent three agricultural plots. We want to assess their suitability for growing a particular vegetable (say, organic tomatoes). A *Neutrosophic Set* A on X is given by three membership functions:
$$T_A(x), \quad I_A(x), \quad F_A(x) \quad \text{for each } x \in X,$$
which measure the degrees of *truth* (suitability), *indeterminacy* (uncertainty), and *falsity* (unsuitability), respectively. These functions must satisfy
$$0 \ \leq \ T_A(x) + I_A(x) + F_A(x) \ \leq \ 3.$$

Specific Membership Values:
We assign:
$$\begin{aligned} &T_A(\text{Plot A}) = 0.85, \quad I_A(\text{Plot A}) = 0.1, \quad F_A(\text{Plot A}) = 0.05,\\ &T_A(\text{Plot B}) = 0.60, \quad I_A(\text{Plot B}) = 0.30, \quad F_A(\text{Plot B}) = 0.10,\\ &T_A(\text{Plot C}) = 0.40, \quad I_A(\text{Plot C}) = 0.45, \quad F_A(\text{Plot C}) = 0.15. \end{aligned}$$
Interpretation:

- Plot A is very likely suitable ($T = 0.85$) and only slightly uncertain ($I = 0.10$), possibly due to consistent past yields.
- Plot B shows moderate suitability ($T = 0.60$) with higher uncertainty ($I = 0.30$), e.g. incomplete soil analysis or pending irrigation upgrades.
- Plot C has fairly low confirmed suitability ($T = 0.40$) and the highest uncertainty ($I = 0.45$), reflecting ambiguous data (e.g. new farmland or mixed historical records).

Example 8 (Neutrosophic Set in Agriculture (Pest Infestation Risk))**. Context and Intuition:** Pest Infestation Risk refers to the likelihood of pest outbreaks affecting crops, influenced by environmental factors, pest species, and agricultural management practices (cf.[55, 10, 43, 40, 11]). Using the same set $X = \{\text{Plot A}, \text{Plot B}, \text{Plot C}\}$, we now define a Neutrosophic Set B that indicates each plot's propensity to suffer from a particular pest infestation.

Membership Functions:
For each $x \in X$:
$$\begin{aligned} T_B(x) &: \text{likelihood that pests will be a problem},\\ I_B(x) &: \text{uncertainty in pest predictions},\\ F_B(x) &: \text{likelihood that pests will } \textit{not} \text{ be a problem}. \end{aligned}$$

Again, $T_B(x) + I_B(x) + F_B(x) \leq 3$.

Specific Values:

$$\begin{aligned}
&T_B(\text{Plot A}) = 0.25, \quad I_B(\text{Plot A}) = 0.50, \quad F_B(\text{Plot A}) = 0.25,\\
&T_B(\text{Plot B}) = 0.70, \quad I_B(\text{Plot B}) = 0.20, \quad F_B(\text{Plot B}) = 0.10,\\
&T_B(\text{Plot C}) = 0.55, \quad I_B(\text{Plot C}) = 0.25, \quad F_B(\text{Plot C}) = 0.20.
\end{aligned}$$

Interpretation:

- Plot A has a low-but-non-negligible pest risk ($T = 0.25$) and a high uncertainty ($I = 0.50$) since historical records of pest infestation are incomplete.
- Plot B shows a high risk ($T = 0.70$), consistent with repeated infestations in prior seasons.
- Plot C has moderate risk ($T = 0.55$) and a fair amount of uncertainty ($I = 0.25$), reflecting partial data on pest patterns.

2.2 | HyperNeutrosophic Set in Agriculture

We provide several examples related to the HyperNeutrosophic Set in Agriculture.

Example 9 (HyperNeutrosophic Set in Agriculture (Yield Scenarios))**. Context and Intuition:** Now, let us move to a *HyperNeutrosophic Set*. We retain $X = \{\text{Plot A, Plot B, Plot C}\}$. A HyperNeutrosophic Set $\widetilde{B}$ is defined by

$$\widetilde{B} : X \to \mathcal{P}([0,1]^3),$$

meaning that for each $x \in X$, $\widetilde{B}(x)$ is a *non-empty subset* of $[0,1]^3$. Each point (T, I, F) in $\widetilde{B}(x)$ satisfies $0 \leq T + I + F \leq 3$, but now each element x can have *multiple* possible triplets to represent differing conditions.

Example Definition:
Interpret (T, I, F) as "favorable yield," "uncertainty," and "unfavorable yield" degrees under various climate or fertilizer scenarios:

$$\widetilde{B}(\text{Plot A}) = \Big\{(0.75,\ 0.20,\ 0.05),\ (0.65,\ 0.25,\ 0.10)\Big\},$$
$$\widetilde{B}(\text{Plot B}) = \Big\{(0.45,\ 0.35,\ 0.20)\Big\}, \quad \widetilde{B}(\text{Plot C}) = \Big\{(0.80,\ 0.10,\ 0.10),\ (0.75,\ 0.15,\ 0.10)\Big\}.$$

Interpretation:

- Plot A has two possible triplets, e.g. different irrigation levels or nutrient programs, each giving a slightly different yield forecast.
- Plot B is represented by a single triplet, suggesting simpler or more stable yield predictions.
- Plot C has two plausible yield states, potentially corresponding to variations in weather patterns.

Example 10 (HyperNeutrosophic Set in Agriculture (Pest Management Outcomes))**. Context and Intuition:** Using a HyperNeutrosophic Set $\widetilde{C}$, we consider the *degree of successful pest control* across multiple pest species and weather combinations.

Definition:

$$\widetilde{C}(\text{Plot A}) = \Big\{(0.60,\ 0.30,\ 0.10),\ (0.50,\ 0.40,\ 0.10)\Big\},$$
$$\widetilde{C}(\text{Plot B}) = \Big\{(0.35,\ 0.40,\ 0.25),\ (0.30,\ 0.45,\ 0.25),\ (0.25,\ 0.50,\ 0.25)\Big\},$$
$$\widetilde{C}(\text{Plot C}) = \Big\{(0.80,\ 0.10,\ 0.10)\Big\}.$$

Here, (T, I, F) might be interpreted as:

- T: Degree to which pest control is fully effective;

- I: Degree of uncertainty (e.g. unexpected pest behavior, incomplete data);
- F: Degree to which pest control *fails*.

Having multiple triplets per plot (Plot A, Plot B) reflects different strategies or environmental conditions.

2.3 | n-SuperHyperNeutrosophic Set in Agriculture

We examine several examples related to the n-SuperHyperNeutrosophic Set in Agriculture.

Example 11 (n-SuperHyperNeutrosophic Set in Agriculture (Nested Uncertainty))**. Context and Intuition:** Consider the base set $X = \{\text{Plot A}, \text{Plot B}\}$. We define the *first powerset*:

$$\mathcal{P}(X) = \{\emptyset, \{\text{A}\}, \{\text{B}\}, \{\text{A}, \text{B}\}\}.$$

Then the *second powerset* is $\mathcal{P}_2(X) = \mathcal{P}(\mathcal{P}(X))$, which is the set of *all subsets* of $\{\emptyset, \{\text{A}\}, \{\text{B}\}, \{\text{A}, \text{B}\}\}$.

An n-SuperHyperNeutrosophic Set with $n = 2$ on X is:

$$\tilde{A}_2 : \mathcal{P}_2(X) \longrightarrow \mathcal{P}_2([0,1]^3).$$

This means for every subset of $\mathcal{P}(X)$, $\tilde{A}_2$ assigns *a family of subsets* of $[0,1]^3$. Each triplet (T, I, F) still respects $0 \leq T + I + F \leq 3$, but now we handle deeply nested uncertainty and multi-level interactions.

Concrete Illustration:
Pick two subsets from $\mathcal{P}_2(X)$:

- $S_1 = \{\{\text{A}\}\}$. Then we could define:

  $$\tilde{A}_2(S_1) = \Big\{\{\,(0.80,\ 0.15,\ 0.05)\},\ \{\,(0.70,\ 0.20,\ 0.10)\}\Big\}.$$

  This indicates that for the single-subset set $\{\{\text{A}\}\}$, we have two possible sets of triplets, each describing different *secondary* scenarios (e.g. microclimate changes or supplementary fertilization). One scenario has $(T = 0.80, I = 0.15, F = 0.05)$; another has $(T = 0.70, I = 0.20, F = 0.10)$.

- $S_2 = \{\{\text{A}\},\ \{\text{B}\}\}$. We might define:

  $$\tilde{A}_2(S_2) = \Big\{\{\,(0.55,\ 0.30,\ 0.15),\ (0.50,\ 0.35,\ 0.15)\}\Big\}.$$

  Hence for the set that simultaneously considers $\{\text{A}\}$ and $\{\text{B}\}$, we store a single set of two triplets capturing synergy or correlation in uncertain yield or pest-control outcomes across both plots.

This richer structure is valuable for modeling multi-layered agricultural decisions, where each “level” of the powerset might encode different grouping or combined management strategies, and each set of triplets captures variable environmental assumptions.

3 | Additional Result: Forest Hyperneutrosophic set

The ForestNeutrosophic Set is a concept that applies the idea of the ForestSoft Set to the framework of Neutrosophic Sets. In this section, we introduce two new frameworks—the *Forest HyperNeutrosophic Set* and the *Forest n-SuperHyperNeutrosophic Set*—that extend the concepts of HyperNeutrosophic and n-SuperHyperNeutrosophic Sets, respectively, while also generalizing the notion of a “Forest Neutrosophic Set” within these higher-level structures.

Definition 12 (ForestNeutrosophic Set)**.** [21] Let $\{F_t\}_{t\in T}$ be TreeNeutrosophic Sets. The *ForestNeutrosophic Set*

$$\mathbf{F} : P\big(\text{Forest}(\{A^{(t)}\})\big) \;\to\; \big([0,1]^3\big)^U$$

is given, for each X in the domain, by

$$\mathbf{F}(X)(x) = \Big(\max_{t:X\cap \text{Tree}(A^{(t)})\neq\emptyset} T_t(X)(x),\quad \max_{t:X\cap \text{Tree}(A^{(t)})\neq\emptyset} I_t(X)(x),\quad \max_{t:X\cap \text{Tree}(A^{(t)})\neq\emptyset} F_t(X)(x)\Big).$$

Definition 13 (Forest HyperNeutrosophic Set)**.** Let X be a non-empty set, and let $\mathcal{F} \subseteq \mathcal{P}(X)$ be any collection of non-empty subsets of X that forms a *forest* under a suitable hierarchy or partial order.[1] A *Forest HyperNeutrosophic Set* on $\mathcal{F}$ is a mapping

$$\widetilde{H} \;:\; \mathcal{F} \;\longrightarrow\; \mathcal{P}([0,1]^3)$$

such that for each $F \in \mathcal{F}$, every triplet $(T, I, F) \in \widetilde{H}(F)$ satisfies

$$0 \;\leq\; T + I + F \;\leq\; 3.$$

Here, as in the usual HyperNeutrosophic framework, T, I, F represent (respectively) *truth*, *indeterminacy*, and *falsity* degrees, but now indexed by the forest structure $\mathcal{F}$.

Theorem 14 (Generalization Property of the Forest HyperNeutrosophic Set)**.** *A Forest HyperNeutrosophic Set generalizes both the standard HyperNeutrosophic Set* and *any Forest Neutrosophic Set.*

Proof: **(1) Generalization of the HyperNeutrosophic Set.** Recall that a HyperNeutrosophic Set $\tilde{A}$ on a base set X is given by

$$\tilde{\mu} : X \;\to\; \mathcal{P}([0,1]^3).$$

This can be seen as a *special case* of Definition 13 by letting $\mathcal{F}$ be the singleton collection $\{X\}$. Indeed, in that scenario, for each $F = X \in \mathcal{F}$, one simply sets $\widetilde{H}(F) \equiv \tilde{\mu}(x)$ for each $x \in X$. Because there is exactly one subset in $\mathcal{F}$ (namely X itself), the forest structure trivially holds (no nested or disjoint subsets are involved). Hence, every HyperNeutrosophic Set arises as a particular instance of a Forest HyperNeutrosophic Set.

(2) Generalization of the Forest Neutrosophic Set. A "Forest Neutrosophic Set" (in the sense of combining multiple "TreeNeutrosophic Sets" or similar constructs) typically maps each forest node (subset) to a *single* neutrosophic membership triplet in $[0,1]^3$. If we specialize Definition 13 by requiring that $\widetilde{H}(F)$ be a *singleton set* of the form $\{\,(T, I, F)\}$ for each $F \in \mathcal{F}$, then we recover exactly the notion of a Forest Neutrosophic Set. Thus, the Forest HyperNeutrosophic Set (allowing *sets* of triplets) strictly generalizes the simpler Forest Neutrosophic model. □

Definition 15 (Forest n-SuperHyperNeutrosophic Set)**.** Let X be a non-empty set, and let $\mathcal{F} \subseteq \mathcal{P}(X)$ be a forest (as above). For a positive integer n, define $\mathcal{P}_n(\mathcal{F})$ recursively in the same manner as standard n-th powersets but restricted to $\mathcal{F}$. In other words,

$$\mathcal{P}_1(\mathcal{F}) \;=\; \mathcal{F}, \quad \mathcal{P}_{k+1}(\mathcal{F}) \;=\; \mathcal{P}(\mathcal{P}_k(\mathcal{F})) \quad (\text{for } k \geq 1),$$

where $\mathcal{P}(\cdot)$ is the usual powerset operator, taken over subsets of $\mathcal{F}$.

A *Forest n-SuperHyperNeutrosophic Set* is then defined as a mapping

$$\widetilde{H}_n : \mathcal{P}_n(\mathcal{F}) \;\longrightarrow\; \mathcal{P}_n([0,1]^3),$$

such that for every $A \in \mathcal{P}_n(\mathcal{F})$ and each $(T, I, F) \in \widetilde{H}_n(A)$, the usual neutrosophic condition holds:

$$0 \;\leq\; T + I + F \;\leq\; 3.$$

Theorem 16 (Generalization Property of the Forest n-SuperHyperNeutrosophic Set)**.** *A Forest n-SuperHyperNeutrosophic Set generalizes both the standard n-SuperHyperNeutrosophic Set* and *the Forest HyperNeutrosophic Set.*

Proof: **(1) Generalization of the n-SuperHyperNeutrosophic Set.** In the usual n-SuperHyperNeutrosophic framework, we have a mapping

$$\tilde{A}_n : \mathcal{P}_n(X) \;\to\; \mathcal{P}_n([0,1]^3).$$

If we choose $\mathcal{F} = \{\emptyset, X\}$ (the most trivial forest that contains only the entire set X and possibly the empty set if desired), then $\mathcal{P}_1(\mathcal{F}) = \mathcal{F}$ essentially collapses to the entire set X-level. Hence, the domain $\mathcal{P}_n(\mathcal{F})$ merges with $\mathcal{P}_n(X)$, thereby reproducing exactly the original n-SuperHyperNeutrosophic definition.

[1] For instance, one may regard $\mathcal{F}$ as a family of subsets that are pairwise "tree-like" or acyclic under set-inclusion. The exact nature of "forest" may depend on the underlying application or classification scheme.

(2) Generalization of the Forest HyperNeutrosophic Set. Let us consider the case $n = 1$. Then a Forest n-SuperHyperNeutrosophic Set (with $n = 1$) is precisely

$$\widetilde{H}_1 : \mathcal{P}_1(\mathcal{F}) = \mathcal{F} \quad \longrightarrow \quad \mathcal{P}_1([0,1]^3) = \mathcal{P}([0,1]^3).$$

This is exactly the mapping in Definition 13, namely a Forest HyperNeutrosophic Set, because $\mathcal{P}_1(\mathcal{F}) = \mathcal{F}$. Thus, the n-SuperHyperNeutrosophic structure reduces to the Forest HyperNeutrosophic structure when $n = 1$.

Putting both points together, we see that the Forest n-SuperHyperNeutrosophic Set strictly contains the classical n-SuperHyperNeutrosophic Set (as a special restriction on the domain) and also contains the Forest HyperNeutrosophic Set (by setting $n = 1$). This completes the proof. □

3.1 | Forest Neutrosophic Set in Agriculture

We provide several examples related to the Forest Neutrosophic Set in Agriculture.

Example 17 (Forest Neutrosophic Set in Agriculture)**. Context and Intuition:** Suppose we have three orchards, each cultivating different fruit trees:

$$\mathcal{F} = \{F_1, F_2, F_3\}.$$

Each F_i is itself a non-empty subset of a larger farmland X. For instance:

$$F_1 = \{\text{Plot A}\}, \quad F_2 = \{\text{Plot B}, \text{Plot C}\}, \quad F_3 = \{\text{Plot D}\},$$

where these sets form a *forest* under set inclusion (i.e. they are pairwise disjoint or minimally overlapping, creating a structure that has no cycles).

A *Forest Neutrosophic Set* assigns to each orchard F_i a *single* triplet $(T_i,\, I_i,\, F_i)$, representing the degrees of truth, indeterminacy, and falsity for some agricultural criterion (for example, overall suitability for an organic certification).

Definition in This Example: For each $F_i \in \mathcal{F}$, define:

$$\mathbf{F}(F_1) = (0.80, 0.10, 0.10), \quad \mathbf{F}(F_2) = (0.60, 0.20, 0.20), \quad \mathbf{F}(F_3) = (0.50, 0.40, 0.10).$$

These satisfy $0 \le T_i + I_i + F_i \le 3$. Hence:

- F_1 has high truth degree (0.80), indicating strong suitability for the certification, with only moderate uncertainty (0.10) and low falsity (0.10).
- F_2 is moderately suitable (0.60), with slightly higher falsity (0.20) and some uncertainty (0.20).
- F_3 shows a balanced scenario of medium truth (0.50) and fairly high indeterminacy (0.40), reflecting less reliable data for orchard Plot D.

This construction provides a straightforward *Forest Neutrosophic Set*, mapping each orchard (forest node) to one triplet in $[0,1]^3$.

Example 18 (Forest Neutrosophic Set in Agriculture)**. Context and Intuition:** Consider a forest structure $\mathcal{F} = \{F_1, F_2, F_3\}$ where:

$$F_1 = \{\text{Field A, Field B}\}, \quad F_2 = \{\text{Field C}\}, \quad F_3 = \{\text{Field D}\}.$$

The sets are pairwise disjoint, forming a "forest" of fields. Let us define a *Forest Neutrosophic Set* to capture, for each orchard, the *likelihood of successful pest management* during the next harvest season:

$$\mathbf{F}(F_1) = (0.75,\ 0.15,\ 0.10), \quad \mathbf{F}(F_2) = (0.40,\ 0.40,\ 0.20), \quad \mathbf{F}(F_3) = (0.55,\ 0.25,\ 0.20).$$

Each triplet (T, I, F) must satisfy $0 \le T + I + F \le 3$.

- For F_1, truth degree of 0.75 indicates a high likelihood of effective pest control. Uncertainty is modest (0.15), perhaps due to partial knowledge of local pest species.
- For F_2, truth is only 0.40, with a large uncertainty (0.40), reflecting difficulty predicting pest infestations in Field C.

- For F_3, moderate truth (0.55) and moderate uncertainty (0.25) suggest better but not conclusive data for Field D.

This example again shows how a Forest Neutrosophic Set succinctly encodes a single neutrosophic membership value for each distinct orchard or forest node.

Example 19 (Forest HyperNeutrosophic Set in Agriculture)**. Context and Intuition:** We extend the previous scenario to a *Forest HyperNeutrosophic Set*, allowing each orchard to have *multiple* possible triplets. Suppose $\mathcal{F} = \{F_1, F_2\}$ where:

$$F_1 = \{\text{Plot A, Plot B}\}, \quad F_2 = \{\text{Plot C}\}.$$

Each orchard (subset) is mapped to a *set* of membership triplets in $[0,1]^3$.

Specific Definition: Define $\widetilde{H} : \mathcal{F} \to \mathcal{P}([0,1]^3)$ by:

$$\widetilde{H}(F_1) = \{(0.80,\ 0.10,\ 0.10),\ (0.70,\ 0.20,\ 0.10)\},$$

$$\widetilde{H}(F_2) = \{(0.50,\ 0.40,\ 0.10),\ (0.45,\ 0.40,\ 0.15)\}.$$

Here, each triplet (T, I, F) satisfies $0 \le T + I + F \le 3$.

Interpretation:

- For orchard F_1, we have two potential scenarios describing some agricultural criterion (for instance, the success of irrigation strategies).

  $$(0.80, 0.10, 0.10) \quad \text{vs.} \quad (0.70, 0.20, 0.10).$$

  Each scenario accounts for different rainfall conditions or technology improvements.

- For orchard F_2, we again have two possible sets of truth-indeterminacy-falsity values, reflecting *multiple* plausible management strategies or uncertain climatic outcomes.

Thus, $\widetilde{H}$ is *hyper*-valued at each forest node, capturing more nuanced possibilities than a single triplet would.

Example 20 (Forest HyperNeutrosophic Set in Agriculture)**. Context and Intuition:** Let $\mathcal{F} = \{F_1, F_2, F_3\}$ be three subsets of farmland, each orchard focusing on different crop types. We want to model the *likelihood of pest control success* in multiple distinct weather patterns. Because each orchard might face different pest species and climate forecasts, we assign *several* triplets to each node.

Definition:

$$\widetilde{H}(F_1) = \Big\{(0.65, 0.25, 0.10),\ (0.60, 0.30, 0.10),\ (0.50, 0.35, 0.15)\Big\},$$

$$\widetilde{H}(F_2) = \Big\{(0.40, 0.40, 0.20),\ (0.45, 0.35, 0.20)\Big\},$$

$$\widetilde{H}(F_3) = \Big\{(0.75, 0.15, 0.10)\Big\}.$$

Each triplet (T, I, F) must satisfy $T + I + F \le 3$.

Interpretation:

- F_1 has three possible outcomes; for instance, one might correspond to normal rainfall, another to severe drought, and another to mild but extended rainy periods.
- F_2 has two scenarios capturing differences in pest types or pesticide availability.
- F_3 is currently modeled by a single scenario (e.g. well-studied orchard with stable conditions), but it could be expanded if more variability were discovered.

Example 21 (Forest n-SuperHyperNeutrosophic Set in Agriculture)**. Context and Intuition:** Consider again a farmland X with several subregions. Let $\mathcal{F} \subseteq \mathcal{P}(X)$ be a forest of those subregions (each node in $\mathcal{F}$ might be an orchard or field). A *Forest n-SuperHyperNeutrosophic Set* allows *multiple levels* of power set nesting, capturing *deeply nested* uncertainties across orchard groupings.

Illustration: Assume $\mathcal{F} = \{F_1, F_2\}$ where

$$F_1 = \{\text{Plot A}\}, \quad F_2 = \{\text{Plot B}, \text{Plot C}\}.$$

Then $\mathcal{P}_1(\mathcal{F}) = \mathcal{F}$ and $\mathcal{P}_2(\mathcal{F}) = \mathcal{P}(\mathcal{F})$ is the set

$$\{\emptyset,\ \{F_1\},\ \{F_2\},\ \{F_1, F_2\}\}.$$

A *Forest 2-SuperHyperNeutrosophic Set* is

$$\widetilde{H}_2 : \mathcal{P}_2(\mathcal{F}) \longrightarrow \mathcal{P}_2([0,1]^3).$$

Concrete Assignments:

- For $\{F_1\} \in \mathcal{P}_2(\mathcal{F})$, define

$$\widetilde{H}_2(\{F_1\}) = \Big\{\{\,(0.80,\ 0.15,\ 0.05)\},\ \{\,(0.75,\ 0.20,\ 0.05)\}\Big\}.$$

  Each of the two elements in this image is itself a subset of $[0,1]^3$. One subset is $\{(0.80, 0.15, 0.05)\}$ and the other is $\{(0.75, 0.20, 0.05)\}$. They could represent different orchard-level scenarios, refined at the second powerset stage.

- For $\{F_1, F_2\} \in \mathcal{P}_2(\mathcal{F})$, define

$$\widetilde{H}_2(\{F_1, F_2\}) = \Big\{\{\,(0.60,\ 0.30,\ 0.10),\ (0.55,\ 0.35,\ 0.10)\}\Big\}.$$

  This might capture scenarios where both orchards are managed together (e.g. a *combined strategy* for irrigation or pest control), leading to sets of triplets that represent synergy or correlation at the second level.

Such a structure models *layered* uncertainty: at the first power-set level, we have orchard-by-orchard data. At the second level, we examine subsets of orchards $\{F_1, F_2\}$ and assign sets of triplets for *that grouping*, enabling multi-level planning in agricultural contexts.

Example 22 (Forest n-SuperHyperNeutrosophic Set in Agriculture)**. Context and Intuition:** Let $\mathcal{F} = \{F_1, F_2, F_3\}$, where each F_i is a disjoint orchard. We aim to capture not only orchard-level uncertainties but also *how combinations of orchards* might jointly affect supply logistics, pest control at scale, or water-sharing resources.

Structure:

$$\mathcal{P}_1(\mathcal{F}) = \mathcal{F} = \{F_1, F_2, F_3\}, \quad \mathcal{P}_2(\mathcal{F}) = \mathcal{P}(\{F_1,\ F_2,\ F_3\}),$$

and so forth up to $\mathcal{P}_n(\mathcal{F})$ for higher nesting. Define

$$\widetilde{H}_n : \mathcal{P}_n(\mathcal{F}) \longrightarrow \mathcal{P}_n([0,1]^3).$$

Example Assignments for $n = 2$:

- For $S = \{F_1, F_2\} \in \mathcal{P}_2(\mathcal{F})$:

$$\widetilde{H}_2(S) = \Big\{\{\,(0.70, 0.20, 0.10)\},\ \{\,(0.65, 0.25, 0.10)\}\Big\}.$$

  This might represent synergy in orchard F_1 and F_2 if they share a pest management system or irrigation pipeline.

- For $S = \{F_2, F_3\} \in \mathcal{P}_2(\mathcal{F})$:
$$\widetilde{H}_2(S) = \Big\{ \{ (0.50, 0.40, 0.10),\ (0.55, 0.35, 0.10) \} \Big\}.$$
Multiple triplets in one subset can reflect slight variations (e.g. different rainfall patterns). By grouping F_2 and F_3, we capture complexities of orchard adjacency or shared pest vectors.
- For singletons like $\{F_3\} \in \mathcal{P}_2(\mathcal{F})$, we can store simpler sets of triplets, e.g.
$$\widetilde{H}_2(\{F_3\}) = \Big\{ \{ (0.75, 0.15, 0.10) \} \Big\}.$$

These higher-level power set mappings allow multi-orchard analysis and multi-scenario planning in a single unified framework.

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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