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Considerations of HyperNeutrosophic Set and ForestNeutrosophic Set in Livestock Applications and Proposal of New Neutrosophic Sets

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Abstract

The Neutrosophic Set offers a robust mathematical framework for handling uncertainty by incorporating three key membership functions: truth, indeterminacy, and falsity [36]. To address the growing complexity of real-world problems, advanced extensions such as the HyperNeutrosophic Set and the SuperHyperNeutrosophic Set have been developed. These higher-dimensional models provide an enhanced representation of uncertainty and are formally defined in [20, 21, 22, 24, 23, 37, 19]. In addition, the TreeNeutrosophic Set and ForestNeutrosophic Set are well-established extensions of the Neutrosophic Set, broadening its applicability to structured and hierarchical data[16].

This paper explores the practical applications of the Neutrosophic Set in livestock management [28, 11, 2], highlighting its potential to enhance decision-making processes in this field. Additionally, we introduce several new variations of the Neutrosophic Set and the ForestNeutrosophic Set [16], further extending their theoretical framework and practical applicability.

Keywords: Set Theory, SuperhyperNeutrosophic set, Neutrosophic Set, HyperNeutrosophic set, Forest-Neutrosophic Set

1 | Preliminaries and Definitions

This section introduces fundamental concepts and notations that form the foundation of this study. We build upon classical set-theoretic frameworks and extend them into more sophisticated structures.

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1.1 | Neutrosophic, HyperNeutrosophic, and n-SuperHyperNeutrosophic Sets

Addressing uncertainty, vagueness, and imprecision in decision-making has led to the development of various mathematical frameworks. Among these, *Fuzzy Sets*, introduced by Zadeh [43, 44, 49, 47, 46, 45, 48], are widely recognized. Another significant advancement is *Intuitionistic Fuzzy Sets*, explored extensively by Atanassov [5, 7, 6, 4, 8]. Additionally, *Vague Sets* [29, 25] have contributed valuable perspectives.

To provide a more refined approach for modeling indeterminacy, *Neutrosophic Sets* were introduced by Smarandache [35, 36, 39]. These sets extend fuzzy logic by incorporating an explicit indeterminacy component alongside truth and falsity values. This allows for a richer representation of uncertain or incomplete information.

For more complex and higher-dimensional scenarios, *HyperNeutrosophic Sets* and *n-SuperHyperNeutrosophic Sets* have been developed [18, 15, 20, 21, 23, 17, 22, 19, 37]. These structures enable multi-layered uncertainty modeling, making them highly effective for intricate decision-making problems.

Definition 1 (Set)**.** [26] A *set* is a well-defined collection of distinct objects, called *elements*. If x is an element of a set A, it is written as $x \in A$. Sets are typically represented using curly braces.

Definition 2 (Base Set)**.** [14] A *base set* S is the fundamental set from which more complex structures, such as powersets and hyperstructures, are derived. It is formally defined as:

$$S = \{x \mid x \text{ is an element within a specified domain}\}.$$

Any element appearing in constructions such as $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ originates from S.

Definition 3 (Powerset)**.** [14, 34] The *powerset* of a set S, denoted by $\mathcal{P}(S)$, is the collection of all subsets of S, including the empty set and S itself:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 4 (n-th Powerset)**.** (cf. [38, 14])

The n-th powerset of a set H, denoted as $P_n(H)$, is defined recursively as:

$$P_1(H) = P(H), \quad P_{n+1}(H) = P(P_n(H)), \quad \text{for } n \geq 1.$$

The corresponding n-th non-empty powerset, denoted $P_n^*(H)$, is defined as:

$$P_1^*(H) = P^*(H), \quad P_{n+1}^*(H) = P^*(P_n^*(H)).$$

Here, $P^*(H)$ represents the standard powerset with the empty set removed.

Definition 5 (Neutrosophic Set)**.** [35, 36] Let X be a non-empty set. A *Neutrosophic Set (NS) A* on X is defined by three membership functions:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1].$$

For each $x \in X$, these functions represent the degrees of truth ($T_A(x)$), indeterminacy ($I_A(x)$), and falsity ($F_A(x)$), respectively. These values satisfy the following constraint:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 6 (HyperNeutrosophic Set)**.** [15, 37] Let X be a non-empty set. A *HyperNeutrosophic Set (HNS) $\tilde{A}$* on X is a mapping:

$$\tilde{\mu} : X \to \mathcal{P}([0,1]^3),$$

where $\mathcal{P}([0,1]^3)$ is the collection of all non-empty subsets of the unit cube. For each $x \in X$, $\tilde{\mu}(x)$ contains sets of triples (T, I, F) satisfying:

$$0 \leq T + I + F \leq 3.$$

Definition 7 (n-SuperHyperNeutrosophic Set)**.** (cf. [15, 18]) Let X be a non-empty set. An *n-SuperHyperNeutrosophic Set (n-SHNS)* is a higher-order extension of Neutrosophic Sets, defined as:

$$\tilde{A}_n : \mathcal{P}_n(X) \to \mathcal{P}_n([0,1]^3),$$

where:

- $\mathcal{P}_1(X) = \mathcal{P}(X)$ is the power set of X, and for $k \geq 2$,
$$\mathcal{P}_k(X) = \mathcal{P}(\mathcal{P}_{k-1}(X)).$$
- $\mathcal{P}_n([0,1]^3)$ is constructed similarly for the unit cube.

For each $A \in \mathcal{P}_n(X)$ and $(T, I, F) \in \tilde{A}_n(A)$, the constraint:
$$0 \leq T + I + F \leq 3$$
is satisfied.

Definition 8 (TreeNeutrosophic Set)**.** [15] A *TreeNeutrosophic Set* F is defined as:
$$F : \mathcal{P}(\mathrm{Tree}(A)) \to ([0,1]^3)^U,$$
where each attribute set $S \in \mathcal{P}(\mathrm{Tree}(A))$ is mapped to a neutrosophic membership triple:
$$F(S)(x) = (T_S(x), I_S(x), F_S(x)),$$
with $0 \leq T_S(x) + I_S(x) + F_S(x) \leq 3$.

Definition 9 (ForestNeutrosophic Set)**.** [16] Given a collection of TreeNeutrosophic Sets $\{F_t\}_{t\in T}$, the *Forest-Neutrosophic Set* is defined as:
$$\mathbf{F} : \mathcal{P}(\mathrm{Forest}(\{A^{(t)}\})) \to ([0,1]^3)^U.$$
For each X, it is given by:
$$\mathbf{F}(X)(x) = (\max_t T_t(X)(x),\ \max_t I_t(X)(x),\ \max_t F_t(X)(x)).$$

2 | Some Examples of Neutrosophic Sets in Livestock

In this section, we examine examples of Neutrosophic Sets applied to livestock.

Example 10 (Neutrosophic Set for Evaluating Livestock Health)**.** Livestock health refers to the overall well-being of farm animals, including disease prevention, nutrition, reproduction, and management for optimal productivity and welfare (cf.[41, 1, 30, 27]). Consider a livestock set
$$X = \{\text{Cow A},\ \text{Cow B},\ \text{Pig C}\}.$$
We wish to assess whether each animal is in optimal health. For each $x \in X$, assign a triple
$$(T(x),\ I(x),\ F(x))$$
where:

- $T(x)$ is the degree of certainty that x is in optimal health,
- $I(x)$ represents the uncertainty in the assessment, and
- $F(x)$ is the degree of certainty that x is not in optimal health.

For instance, we set:
$$\begin{aligned} T(\text{Cow A}) &= 0.9, & I(\text{Cow A}) &= 0.05, & F(\text{Cow A}) &= 0.05, \\ T(\text{Cow B}) &= 0.7, & I(\text{Cow B}) &= 0.2, & F(\text{Cow B}) &= 0.1, \\ T(\text{Pig C}) &= 0.3, & I(\text{Pig C}) &= 0.4, & F(\text{Pig C}) &= 0.3. \end{aligned}$$
This Neutrosophic Set clearly indicates the health state of each animal by capturing not only the degree of health (truth) but also the uncertainty and degree of unhealthiness.

Example 11 (Neutrosophic Set for Livestock Fertility Evaluation)**.** (cf.[40]) Let
$$X = \{\text{Cow1}, \text{Cow2}, \text{Bull1}\}$$
be a set of livestock animals. To evaluate fertility, we assign to each animal a neutrosophic membership triple, where:

- $T(x)$ represents the degree of fertility,
- $I(x)$ represents the uncertainty in the fertility assessment, and
- $F(x)$ represents the degree of non-fertility.

For example, we might set:

$$\begin{aligned} &T(\text{Cow1}) = 0.85, \quad &I(\text{Cow1}) = 0.10, \quad &F(\text{Cow1}) = 0.05, \\ &T(\text{Cow2}) = 0.60, \quad &I(\text{Cow2}) = 0.25, \quad &F(\text{Cow2}) = 0.15, \\ &T(\text{Bull1}) = 0.90, \quad &I(\text{Bull1}) = 0.05, \quad &F(\text{Bull1}) = 0.05. \end{aligned}$$

This Neutrosophic Set captures the fertility status of each animal along with the associated uncertainties.

Example 12 (HyperNeutrosophic Set for Evaluating Livestock Nutrition)**.** Livestock nutrition involves providing balanced diets, essential nutrients, and proper feeding strategies to ensure animal health, growth, reproduction, and optimal agricultural productivity (cf.[32, 42, 3, 12]). Let the same set $X = \{\text{Cow A}, \text{ Cow B}, \text{ Pig C}\}$ be evaluated on their nutritional condition. Suppose that multiple experts (e.g., a veterinarian and a nutritionist) provide assessments. In a HyperNeutrosophic Set, each animal $x \in X$ is assigned a set of triples:

$$\tilde{\mu}(x) \subseteq [0, 1]^3.$$

For example, we might have:

$$\begin{aligned} \tilde{\mu}(\text{Cow A}) &= \{(0.85,\ 0.10,\ 0.05),\ (0.90,\ 0.05,\ 0.05)\}, \\ \tilde{\mu}(\text{Cow B}) &= \{(0.70,\ 0.20,\ 0.10),\ (0.75,\ 0.15,\ 0.10)\}, \\ \tilde{\mu}(\text{Pig C}) &= \{(0.40,\ 0.30,\ 0.30),\ (0.35,\ 0.35,\ 0.30)\}. \end{aligned}$$

Here, each ordered triple corresponds to one expert's evaluation. For example, for Cow A the veterinarian might assign (0.85, 0.10, 0.05) while the nutritionist gives (0.90, 0.05, 0.05). This approach allows us to capture multiple perspectives in the nutritional assessment.

Example 13 (HyperNeutrosophic Set for Livestock Disease Resistance Assessment)**.** (cf.[31, 10, 9, 33]) Consider a set

$$X = \{\text{Cow1}, \text{Sheep1}, \text{Pig1}\}$$

of livestock. To evaluate each animal's disease resistance, we collect opinions from multiple experts. For each animal, the HyperNeutrosophic Set assigns a set of membership triples:

$$\begin{aligned} \tilde{\mu}(\text{Cow1}) &= \{(0.80,\ 0.15,\ 0.05),\ (0.85,\ 0.10,\ 0.05)\}, \\ \tilde{\mu}(\text{Sheep1}) &= \{(0.70,\ 0.20,\ 0.10),\ (0.65,\ 0.25,\ 0.10)\}, \\ \tilde{\mu}(\text{Pig1}) &= \{(0.55,\ 0.30,\ 0.15),\ (0.60,\ 0.25,\ 0.15)\}. \end{aligned}$$

This example demonstrates how multiple expert evaluations can be combined into a HyperNeutrosophic Set for assessing disease resistance.

Example 14 (n-SuperHyperNeutrosophic Set for Evaluating Group Productivity (with $n = 2$))**.** Assume that we now want to evaluate the productivity of groups of livestock. Let the set of animals be

$$X = \{\text{Cow A}, \text{ Cow B}, \text{ Pig C}\},$$

and consider a grouping represented as:

$$A = \Big\{ \{\text{Cow A, Cow B}\},\ \{\text{Pig C}\} \Big\} \in \mathcal{P}_2(X).$$

Here, $\{\text{Cow A, Cow B}\}$ represents the cow group and $\{\text{Pig C}\}$ represents the pig group. An n-SuperHyperNeutrosophic Set (with $n = 2$) assigns to A a set of evaluation triples:

$$\tilde{A}_2(A) \subseteq \mathcal{P}_2([0, 1]^3).$$

For example, let

$$\tilde{A}_2(A) = \{(0.80,\ 0.15,\ 0.05),\ (0.75,\ 0.20,\ 0.05)\}.$$

The first triple might represent a combined productivity evaluation of the cow group (e.g., milk production) and the pig group (e.g., meat production) with 80% confidence, 15% uncertainty, and 5% disapproval, while the second evaluation reflects another expert's assessment. This recursive evaluation approach is particularly useful when group-level assessments are required.

Example 15 (2-SuperHyperNeutrosophic Set for Group Productivity Evaluation)**.** Let

$$X = \{\text{Cow1}, \text{Cow2}, \text{Sheep1}, \text{Sheep2}\}$$

be a set of livestock. Suppose we form two groups:

$$G_1 = \{\text{Cow1}, \text{Cow2}\} \quad \text{and} \quad G_2 = \{\text{Sheep1}, \text{Sheep2}\}.$$

We define the grouping as an element of $\mathcal{P}_2(X)$:

$$A = \{G_1, G_2\}.$$

An evaluation of the overall productivity of these groups might be given by the 2-SuperHyperNeutrosophic Set:

$$\tilde{A}_2(A) = \{(0.80,\ 0.10,\ 0.10),\ (0.75,\ 0.15,\ 0.10)\}.$$

Here, each triple may represent a different expert's assessment of the productivity, thereby capturing group-level performance and uncertainty.

Example 16 (TreeNeutrosophic Set for Multi-Attribute Livestock Evaluation)**.** Suppose each animal is evaluated based on a set of attributes organized in a tree structure. Let the attribute set be:

$$\text{Tree}(A) = \{\text{Health},\ \text{Nutrition},\ \text{Productivity}\}.$$

For the universe of animals $U = \{\text{Cow A},\ \text{Cow B},\ \text{Pig C}\}$, a TreeNeutrosophic Set is defined as:

$$F : \mathcal{P}(\text{Tree}(A)) \to \left([0, 1]^3\right)^U.$$

For any combination of attributes $S \subseteq \text{Tree}(A)$, we assign to each animal x a triple $F(S)(x) = (T_S(x), I_S(x), F_S(x))$. For instance, for the attribute combination $S = \{\text{Health},\ \text{Productivity}\}$, we might define:

$$\begin{aligned} F(S)(\text{Cow A}) &= (0.90,\ 0.05,\ 0.05), \\ F(S)(\text{Cow B}) &= (0.80,\ 0.10,\ 0.10), \\ F(S)(\text{Pig C}) &= (0.50,\ 0.30,\ 0.20). \end{aligned}$$

This example shows how different combinations of attributes can be evaluated simultaneously using a tree-structured approach.

Example 17 (TreeNeutrosophic Set for Multi-Attribute Livestock Evaluation)**.** Let

$$X = \{\text{Cow1}, \text{Sheep1}, \text{Goat1}\}$$

be a set of animals. Suppose we consider multiple attributes organized in a tree structure:

$$\text{Tree}(A) = \{\text{Growth}, \text{Behavior}, \text{Feeding Efficiency}\}.$$

For a selected attribute combination, say

$$S = \{\text{Growth}, \text{Feeding Efficiency}\},$$

we define the TreeNeutrosophic Set F by assigning each animal a neutrosophic membership triple:

$$\begin{aligned} F(S)(\text{Cow1}) &= (0.85,\ 0.10,\ 0.05), \\ F(S)(\text{Sheep1}) &= (0.75,\ 0.15,\ 0.10), \\ F(S)(\text{Goat1}) &= (0.80,\ 0.10,\ 0.10). \end{aligned}$$

This set effectively combines different attributes to evaluate each animal.

Example 18 (ForestNeutrosophic Set for Integrated Livestock Evaluation)**.** Now, assume that two different TreeNeutrosophic Sets provide distinct evaluations of the livestock. For example:

- F_1: Based on physical health.
- F_2: Based on production performance.

Using the same universe $U = \{\text{Cow A}, \text{Cow B}, \text{Pig C}\}$ and for the attribute combination $S = \{\text{Health}, \text{Productivity}\}$, suppose we have:

$$\begin{aligned}
F_1(S)(\text{Cow A}) &= (0.90,\ 0.05,\ 0.05), \quad F_2(S)(\text{Cow A}) = (0.85,\ 0.10,\ 0.05),\\
F_1(S)(\text{Cow B}) &= (0.70,\ 0.20,\ 0.10), \quad F_2(S)(\text{Cow B}) = (0.75,\ 0.15,\ 0.10),\\
F_1(S)(\text{Pig C}) &= (0.60,\ 0.25,\ 0.15), \quad F_2(S)(\text{Pig C}) = (0.55,\ 0.30,\ 0.15).
\end{aligned}$$

The ForestNeutrosophic Set $\mathbf{F}$ aggregates these evaluations by taking the maximum of the corresponding components for each animal:

$$\begin{aligned}
\mathbf{F}(S)(\text{Cow A}) &= \Big(\max\{0.90, 0.85\},\ \max\{0.05, 0.10\},\ \max\{0.05, 0.05\}\Big)\\
&= (0.90,\ 0.10,\ 0.05),\\
\mathbf{F}(S)(\text{Cow B}) &= \Big(\max\{0.70, 0.75\},\ \max\{0.20, 0.15\},\ \max\{0.10, 0.10\}\Big)\\
&= (0.75,\ 0.20,\ 0.10),\\
\mathbf{F}(S)(\text{Pig C}) &= \Big(\max\{0.60, 0.55\},\ \max\{0.25, 0.30\},\ \max\{0.15, 0.15\}\Big)\\
&= (0.60,\ 0.30,\ 0.15).
\end{aligned}$$

This integration provides a comprehensive evaluation that combines different assessment perspectives.

Example 19 (ForestNeutrosophic Set for Integrating Livestock Health and Productivity)**.** Let

$$X = \{\text{Cow1}, \text{Sheep1}, \text{Pig1}\}$$

be a set of livestock. Consider two distinct TreeNeutrosophic Sets:

- F_1 evaluates animal health,
- F_2 evaluates productivity.

Suppose for an attribute combination S, the evaluations are:

$$\begin{aligned}
F_1(S)(\text{Cow1}) &= (0.90,\ 0.05,\ 0.05), \quad F_2(S)(\text{Cow1}) = (0.85,\ 0.10,\ 0.05),\\
F_1(S)(\text{Sheep1}) &= (0.80,\ 0.10,\ 0.10), \quad F_2(S)(\text{Sheep1}) = (0.82,\ 0.08,\ 0.10),\\
F_1(S)(\text{Pig1}) &= (0.70,\ 0.20,\ 0.10), \quad F_2(S)(\text{Pig1}) = (0.75,\ 0.15,\ 0.10).
\end{aligned}$$

The ForestNeutrosophic Set $\mathbf{F}$ is then defined by taking the maximum of the corresponding components:

$$\begin{aligned}
\mathbf{F}(S)(\text{Cow1}) &= (0.90,\ 0.10,\ 0.05),\\
\mathbf{F}(S)(\text{Sheep1}) &= (0.82,\ 0.10,\ 0.10),\\
\mathbf{F}(S)(\text{Pig1}) &= (0.75,\ 0.20,\ 0.10).
\end{aligned}$$

This integrated approach provides a comprehensive assessment by merging evaluations from different criteria.

3 | Additional Concepts: GraphicNeutrosophic Set and ClusterNeutrosophic Set

In this section, we introduce new variations of the Neutrosophic Set. Specifically, we examine the GraphicNeutrosophic Set and the ClusterNeutrosophic Set. The relevant definitions and theorems are presented below.

Definition 20 (Graph and Its Power Set)**.** (cf.[13]) Let $G = (V, E)$ be a finite graph, where V is the set of vertices (each representing an attribute) and $E \subseteq V \times V$ is the set of edges (representing relationships among these attributes). Define a *subgraph* H of G as a graph $H = (V_H, E_H)$ with $V_H \subseteq V$ and $E_H \subseteq E \cap (V_H \times V_H)$. The *power set of* G, denoted $\mathcal{P}(G)$, is defined as the collection of all subgraphs of G:

$$\mathcal{P}(G) = \{\, H \mid H \text{ is a subgraph of } G \,\}.$$

Definition 21 (GraphicNeutrosophic Set)**.** Let U be a universe of discourse, and let $G = (V, E)$ be a graph representing a set of attributes and their interrelationships. A *GraphicNeutrosophic Set* is a mapping

$$F : \mathcal{P}(G) \to \mathcal{N}(U),$$

where $\mathcal{N}(U)$ denotes the set of all neutrosophic subsets of U. For each subgraph $H \in \mathcal{P}(G)$, $F(H)$ is a neutrosophic set on U; that is, for every $x \in U$,

$$F(H)(x) = (T_H(x),\, I_H(x),\, F_H(x))$$

with

$$T_H(x),\, I_H(x),\, F_H(x) \in [0,1] \quad \text{and} \quad 0 \leq T_H(x) + I_H(x) + F_H(x) \leq 3.$$

Intuitively, $F(H)(x)$ indicates the degree to which x exhibits the combined attributes represented by the subgraph H.

Example 22 (GraphicNeutrosophic Set on Personal Attributes)**.** Let

$$U = \{\text{Alice}, \text{Bob}, \text{Charlie}, \text{Diana}\}$$

and consider a graph

$$G = (V, E)$$

with

$$V = \{\text{Smart}, \text{Friendly}, \text{Athletic}\}.$$

Assume some (unspecified) relationships among these attributes. Define a GraphicNeutrosophic Set $F : \mathcal{P}(G) \to \mathcal{N}(U)$ as follows:

- For the subgraph H_1 containing only the vertex {Smart}, set:

$$F(H_1)(\text{Alice}) = (0.8,\, 0.1,\, 0.1), \quad F(H_1)(\text{Bob}) = (0.6,\, 0.2,\, 0.1),$$

$$F(H_1)(\text{Charlie}) = (0.9,\, 0.05,\, 0.05), \quad F(H_1)(\text{Diana}) = (0.7,\, 0.15,\, 0.15).$$

- For the subgraph H_2 with only the vertex {Friendly}, define:

$$F(H_2)(\text{Alice}) = (0.7,\, 0.2,\, 0.1), \quad F(H_2)(\text{Bob}) = (0.8,\, 0.1,\, 0.1),$$

$$F(H_2)(\text{Charlie}) = (0.6,\, 0.3,\, 0.1), \quad F(H_2)(\text{Diana}) = (0.9,\, 0.05,\, 0.05).$$

- For the subgraph H_3 with vertices $\{\text{Smart}, \text{Friendly}\}$ (and the corresponding edge), we define an aggregation (for instance, via a neutrosophic intersection operator where truth is taken as the minimum, and indeterminacy and falsity as the maximum):

$$F(H_3)(x) = \Big(\min\{T_{H_1}(x), T_{H_2}(x)\},\, \max\{I_{H_1}(x), I_{H_2}(x)\},\, \max\{F_{H_1}(x), F_{H_2}(x)\}\Big).$$

  For example, for $x = \text{Alice}$:

$$F(H_3)(\text{Alice}) = (\min\{0.8, 0.7\},\, \max\{0.1, 0.2\},\, \max\{0.1, 0.1\}) = (0.7,\, 0.2,\, 0.1).$$

Theorem 23 (Monotonicity of GraphicNeutrosophic Set)**.** *Assume that for any subgraphs $H_1, H_2 \in \mathcal{P}(G)$ with $H_1 \subseteq H_2$ (i.e., $V(H_1) \subseteq V(H_2)$ and $E(H_1) \subseteq E(H_2)$), the aggregation operator used in F satisfies:*

$$T_{H_2}(x) \leq T_{H_1}(x), \quad I_{H_2}(x) \geq I_{H_1}(x), \quad F_{H_2}(x) \geq F_{H_1}(x)$$

for all $x \in U$. Then the GraphicNeutrosophic Set F is monotonic with respect to the inclusion of subgraphs.

Proof: If $H_1 \subseteq H_2$, then every attribute in H_1 is also present in H_2. Under the common neutrosophic intersection operator, the truth membership is computed as the minimum of the truth values over the involved attributes, while the indeterminacy and falsity are computed as the maximum. Hence, adding extra attributes (which might lower the minimum truth and raise the maximum indeterminacy and falsity) yields:

$$T_{H_2}(x) = \min_{v \in V(H_2)} T_v(x) \leq \min_{v \in V(H_1)} T_v(x) = T_{H_1}(x),$$

and similarly,

$$I_{H_2}(x) \geq I_{H_1}(x), \quad F_{H_2}(x) \geq F_{H_1}(x).$$

Thus, the mapping F is monotonic with respect to the inclusion relation on $\mathcal{P}(G)$. □

Definition 24 (ClusterNeutrosophic Set)**.** Let $\{F_i\}_{i\in I}$ be a finite family of neutrosophic sets over a universe U, where each F_i is a mapping

$$F_i : U \to [0,1]^3,$$

with

$$F_i(x) = (T_i(x),\, I_i(x),\, F_i(x))$$

satisfying

$$0 \le T_i(x) + I_i(x) + F_i(x) \le 3, \quad \forall x \in U.$$

Suppose that the index set I is partitioned into disjoint clusters $\{C_j\}_{j\in J}$ such that for $j \neq k$, $C_j \cap C_k = \emptyset$. Then a *ClusterNeutrosophic Set* is a mapping

$$G : \{C_j \mid j \in J\} \to \mathcal{N}(U),$$

where for each cluster $C \subseteq I$ and each $x \in U$,

$$G(C)(x) = \Big(\max_{i\in C} T_i(x),\, \min_{i\in C} I_i(x),\, \min_{i\in C} F_i(x)\Big).$$

This aggregation rule is one possible choice that reflects taking the union (for truth) and intersection (for indeterminacy and falsity) in a neutrosophic context.

Example 25 (ClusterNeutrosophic Set: Candidate Evaluation)**.** Let

$$U = \{\text{Candidate1, Candidate2, Candidate3, Candidate4}\}$$

be a set of candidates. Assume two experts provide evaluations in the form of neutrosophic sets:

$$F_1 : U \to [0,1]^3, \quad \text{with } F_1(\text{Candidate1}) = (0.9,\, 0.05,\, 0.05),\ F_1(\text{Candidate2}) = (0.7,\, 0.1,\, 0.2),\ ...$$

$$F_2 : U \to [0,1]^3, \quad \text{with } F_2(\text{Candidate1}) = (0.85,\, 0.1,\, 0.05),\ F_2(\text{Candidate2}) = (0.75,\, 0.05,\, 0.2),\ ...$$

Let $I = \{1,2\}$ and define a single cluster $C_1 = \{1,2\}$. Then the ClusterNeutrosophic Set $G(C_1)$ is given by:

$$G(C_1)(x) = \Big(\max\{T_1(x),\, T_2(x)\},\, \min\{I_1(x),\, I_2(x)\},\, \min\{F_1(x),\, F_2(x)\}\Big),$$

for each $x \in U$. For instance, if

$$F_1(\text{Candidate1}) = (0.9,\, 0.05,\, 0.05) \quad \text{and} \quad F_2(\text{Candidate1}) = (0.85,\, 0.1,\, 0.05),$$

then

$$G(C_1)(\text{Candidate1}) = (\max\{0.9, 0.85\},\, \min\{0.05, 0.1\},\, \min\{0.05, 0.05\}) = (0.9,\, 0.05,\, 0.05).$$

Example 26 (ClusterNeutrosophic Set: Product Quality Assessment)**.** Let

$$U = \{\text{Product1, Product2, Product3}\}$$

be a set of products. Suppose three independent quality assessments are given by neutrosophic sets:

$$F_1(\text{Product1}) = (0.8,\, 0.1,\, 0.1), \quad F_2(\text{Product1}) = (0.85,\, 0.05,\, 0.1), \quad F_3(\text{Product1}) = (0.8,\, 0.08,\, 0.12).$$

Let the indices be $I = \{1,2,3\}$ and form a single cluster $C_1 = \{1,2,3\}$. Then,

$$G(C_1)(\text{Product1}) = \Big(\max\{0.8, 0.85, 0.8\},\, \min\{0.1, 0.05, 0.08\},\, \min\{0.1, 0.1, 0.12\}\Big) = (0.85,\, 0.05,\, 0.1).$$

Theorem 27 (Idempotence of Cluster Aggregation)**.** *If for every $i \in C \subseteq I$, the neutrosophic set F_i is identical, i.e., $F_i(x) = (T(x), I(x), F(x))$ for all $x \in U$, then*

$$G(C)(x) = (T(x), I(x), F(x))$$

for every $x \in U$.

Proof: Since $F_i(x) = (T(x), I(x), F(x))$ for all $i \in C$ and all $x \in U$, we have:

$$\max_{i\in C} T_i(x) = T(x), \quad \min_{i\in C} I_i(x) = I(x), \quad \min_{i\in C} F_i(x) = F(x).$$

Thus, by the definition of $G(C)(x)$,

$$G(C)(x) = (T(x), I(x), F(x)),$$

which proves the claim. □

Theorem 28 (Monotonicity of ClusterNeutrosophic Aggregation)**.** *Let $C_1 \subseteq C_2 \subseteq I$ be two clusters. Then, for every $x \in U$,*

$$\max_{i \in C_1} T_i(x) \le \max_{i \in C_2} T_i(x),$$
$$\min_{i \in C_1} I_i(x) \ge \min_{i \in C_2} I_i(x),$$
$$\min_{i \in C_1} F_i(x) \ge \min_{i \in C_2} F_i(x).$$

Proof: Since $C_1 \subseteq C_2$, the set $\{T_i(x) \mid i \in C_1\}$ is a subset of $\{T_i(x) \mid i \in C_2\}$. Therefore,

$$\max_{i \in C_1} T_i(x) \le \max_{i \in C_2} T_i(x).$$

Similarly, for the indeterminacy and falsity components, taking the minimum over a larger set yields a value less than or equal to that over a smaller set, i.e.,

$$\min_{i \in C_1} I_i(x) \ge \min_{i \in C_2} I_i(x), \quad \min_{i \in C_1} F_i(x) \ge \min_{i \in C_2} F_i(x).$$

Thus, the monotonicity properties hold. □

4 | Additional Concepts: New ForestNeutrosophic Sets

In this section, we explore several new variations of the ForestNeutrosophic Set. We hope that further research on these concepts will advance in the future.

Definition 29 (Weighted ForestNeutrosophic Set)**.** Let $\{F_t\}_{t \in T}$ be a collection of TreeNeutrosophic Sets, where for each $t \in T$,

$$F_t : \mathcal{P}(\mathrm{Tree}(A^{(t)})) \to \left([0,1]^3\right)^U.$$

Let $\{w_t\}_{t \in T}$ be a set of nonnegative weights satisfying

$$w_t \ge 0 \quad \text{for all } t \in T \quad \text{and} \quad \sum_{t \in T} w_t = 1.$$

Then, the *Weighted ForestNeutrosophic Set* is defined by

$$\mathbf{F}_w(X)(x) = \left(\max_{\substack{t \in T \\ X \cap \mathrm{Tree}(A^{(t)}) \neq \emptyset}} \Big\{ w_t \cdot T_t(X)(x) \Big\}, \max_{\substack{t \in T \\ X \cap \mathrm{Tree}(A^{(t)}) \neq \emptyset}} \Big\{ w_t \cdot I_t(X)(x) \Big\}, \max_{\substack{t \in T \\ X \cap \mathrm{Tree}(A^{(t)}) \neq \emptyset}} \Big\{ w_t \cdot F_t(X)(x) \Big\} \right),$$

for each $X \subseteq U$ and $x \in U$, where $T_t(X)(x)$, $I_t(X)(x)$, and $F_t(X)(x)$ denote the truth, indeterminacy, and falsity membership values given by F_t, respectively.

Theorem 30. *For every $X \subseteq U$ and $x \in U$, the weighted forest neutrosophic triple $\mathbf{F}_w(X)(x)$ satisfies*

$$\mathbf{F}_w(X)(x) \in [0,1]^3.$$

Proof: For any fixed $t \in T$, since

$$F_t(X)(x) = \Big(T_t(X)(x),\ I_t(X)(x),\ F_t(X)(x) \Big) \in [0,1]^3,$$

and because $w_t \in [0,1]$, it follows that

$$w_t \cdot T_t(X)(x) \in [0,1], \quad w_t \cdot I_t(X)(x) \in [0,1], \quad \text{and} \quad w_t \cdot F_t(X)(x) \in [0,1].$$

Taking the maximum over a collection of values in $[0,1]$ (for each coordinate) yields a number in $[0,1]$. Hence, every component of $\mathbf{F}_w(X)(x)$ lies in $[0,1]$, so

$$\mathbf{F}_w(X)(x) \in [0,1]^3.$$

□

Definition 31 (Probabilistic ForestNeutrosophic Set)**.** Let $\{F_t\}_{t\in T}$ be a collection of TreeNeutrosophic Sets with

$$F_t : \mathcal{P}(\mathrm{Tree}(A^{(t)})) \to \left([0,1]^3\right)^U,$$

and let $\{p_t\}_{t\in T}$ be a probability distribution on T, i.e.,

$$p_t \geq 0 \quad \text{for all } t \in T \quad \text{and} \quad \sum_{t\in T} p_t = 1.$$

Then, the *Probabilistic ForestNeutrosophic Set* is defined as

$$\mathbf{F}_p(X)(x) = \left(\sum_{\substack{t\in T \\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} p_t \cdot T_t(X)(x), \quad \sum_{\substack{t\in T \\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} p_t \cdot I_t(X)(x), \quad \sum_{\substack{t\in T \\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} p_t \cdot F_t(X)(x) \right),$$

for every $X \subseteq U$ and $x \in U$.

Theorem 32. *For every $X \subseteq U$ and $x \in U$, the probabilistic forest neutrosophic triple $\mathbf{F}_p(X)(x)$ is a convex combination of the respective membership values, and therefore*

$$\mathbf{F}_p(X)(x) \in [0,1]^3.$$

Proof: For any $t \in T$, each of the membership values $T_t(X)(x)$, $I_t(X)(x)$, and $F_t(X)(x)$ lies in $[0,1]$. Since $\{p_t\}_{t\in T}$ is a probability distribution, the sum

$$\sum_{\substack{t\in T \\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} p_t \cdot T_t(X)(x)$$

is a convex combination of numbers in $[0,1]$, and thus it lies in $[0,1]$. The same reasoning applies to the other two components. Consequently, the triple

$$\mathbf{F}_p(X)(x) \in [0,1]^3.$$

□

Definition 33 (Temporal ForestNeutrosophic Set)**.** Let $\{F_t^\tau\}_{t\in T}$ be a time-indexed family of TreeNeutrosophic Sets such that for each $t \in T$ and for every time instant τ in a time domain $\mathcal{T}$,

$$F_t^\tau : \mathcal{P}(\mathrm{Tree}(A^{(t)})) \to \left([0,1]^3\right)^U.$$

Then, the *Temporal ForestNeutrosophic Set* is defined as

$$\mathbf{F}_T(X,\tau)(x) = \left(\max_{\substack{t\in T \\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} \{T_t^\tau(X)(x)\}, \quad \max_{\substack{t\in T \\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} \{I_t^\tau(X)(x)\}, \quad \max_{\substack{t\in T \\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} \{F_t^\tau(X)(x)\} \right),$$

for every $X \subseteq U$, $x \in U$, and $\tau \in \mathcal{T}$.

Theorem 34. *Suppose that for every $t \in T$ and $x \in U$, the functions $T_t^\tau(X)(x)$, $I_t^\tau(X)(x)$, and $F_t^\tau(X)(x)$ are continuous with respect to τ. Then the Temporal ForestNeutrosophic Set $\mathbf{F}_T(X,\tau)(x)$ is continuous in τ.*

Proof: The maximum of a finite set of continuous functions is continuous. Since for each fixed $X \subseteq U$ and $x \in U$, the functions

$$\tau \mapsto T_t^\tau(X)(x), \quad \tau \mapsto I_t^\tau(X)(x), \quad \tau \mapsto F_t^\tau(X)(x)$$

are continuous for every $t \in T$, it follows that

$$\tau \mapsto \max_{\substack{t\in T \\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} \{T_t^\tau(X)(x)\},$$

$$\tau \mapsto \max_{\substack{t\in T \\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} \{I_t^\tau(X)(x)\}, \quad \text{and} \quad \tau \mapsto \max_{\substack{t\in T \\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} \{F_t^\tau(X)(x)\}$$

are continuous in τ. Therefore, the vector-valued function $\mathbf{F}_T(X,\tau)(x)$ is continuous with respect to τ. □

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Conflicts of Interest

The authors declare that there is no conflict of interest in this research.

Ethical Approval

This article does not contain any studies involving human participants or animals conducted by any of the authors.

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