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Introduction to Possibility Plithogenic Soft Sets

Nivetha Martin [1,*]

[1] Department of Mathematics, Arul Anandar College (Autonomous), Karumathur, India; nivetha.martin710@gmail.com.

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Abstract

Plithogenic sets are versatile in nature, and they are widely applied in decision making. However, the generalization of Plithogenic sets is essential to make the process of deriving optimal solutions more comprehensive. Hence, this paper introduces the concept of Possibility Plithogenic Soft Sets (PPSS) with the aim of developing a generalized form of Plithogenic soft sets. The types of PPSS,union and intersection operations, and similarity measures with possibility degrees are discussed. A practical application of Possibility Plithogenic Soft Sets is also presented in this research work to illustrate the advantages of PPSS in constructing solutions to decision-making problems.

Keywords: Plithogenic Soft Sets, Possibility Degree, Decision-Making.

1 | Introduction

Smarandache [1] is the pioneer of Plithogenic theory and has introduced the concept of Plithogenic sets of the form (P,a, V,d,c) with the set P, the attribute a, the attribute values V, the degree of appurtenance d and the degree of contradiction c. The theory of Plithogeny primarily deals with attributes and it can be referred also as the attribute theory. Plithogenic sets are also termed as the generalization of crisp, fuzzy, intuitionistic, and neutrosophic sets based on the degree of appurtenance. Plithogenic sets shall also be characterized as attribute-driven sets as these sets primarily deal with attributes and attribute values. Soft sets developed by Molodtsov [2] are yet another kind of set that deals with parameters or attributes. Maji et al developed Fuzzy soft sets [3], intuitionistic fuzzy soft sets [4], and neutrosophic soft sets [5] which integrate fuzzy sets, intuitionistic fuzzy sets, and neutrosophic sets with soft sets respectively. Alkhazaleh [6] has discussed Plithogenic soft sets encompassing the generalized representations of deterministic, fuzzy, intuitionistic, and neutrosophic soft sets.

As an extension of soft sets, the concept of the Possibility of soft sets is introduced as the generalization of soft sets. Alkhazaleh et al [7] introduced the possibility of fuzzy soft sets. Bashir et al [8] conceptualized the possibility of intuitionistic fuzzy soft sets. Karaaslan [9] defined the possibility of neutrosophic soft sets. Researchers have also discussed the operations and applications of possibility soft sets. The aforementioned research works have motivated to develop the theory of the Possibility of Plithogenic Soft sets as an extension of Plithogenic soft sets. Alkhazaleh [6] has contributed highly to the theoretical developments of Plithogenic soft sets, its operations, and similarity measures, however, the applications are not very focused. This paper identifies this as a lacuna and hence proposes the theoretical framework of the Possibility of Plithogenic soft

Corresponding Author: nivetha.martin710@gmail.com

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sets as the generalization of Plithogenic soft sets. There is a need to develop this new theory of PPSS to determine optimal solutions to decision-oriented problems based on degrees of possibility. A decision-making problem is generally attribute-centered and a decision-maker is constrained with the set of attributes. Plithogenic soft sets are highly competent in handling complex decision-making scenarios and henceforth the inclusion of possibility theory with Plithogenic soft sets will enhance the efficiency of these sets in deriving solutions to the problems. This research work conceptualizes the theoretical aspects of the Possibility of Plithogenic soft sets

The remaining sections of the paper are listed as follows. The state of the art of the work is discussed in section 2. The theory of the Possibility of Plithogenic soft sets is sketched out in section 3. The application of PPSS is discussed in section 4. The possibility of plithogenic soft sets is compared with Plithogenic soft sets in section 5 with few inferences and the last section concludes the research work with few insights of future research works.

2 | Literature Review

This section presents the state of art of the research works related to Plithogenic sets and Possibility soft sets. The applications and the theoretical developments are also well articulated in this section.

Plithogenic sets are discussed under various circumstances of multi-criteria decision-making. Plithogenic sets-based decision-making methods are developed using Plithogenic operators. Researchers applied plithogenic sets in several areas of decision-making, to mention a few, Plithogenic single valued fuzzy sets in obesity analysis [10], Plithogenic based sentiment analysis in product ranking [11], Plithogenic MACBETH MAIRCA [12] in studying environmental sustainability, Plithogenic VIKOR [13] in supply chain management, Neutrosophic Plithogenic AHP [14] is applied in selection of higher education program, Plithogenic sets are also integrated with other decision-making approaches. Martin et al [15] introduced Plithogenic Cognitive Maps, Plithogenic sociogram [16]. In addition to plithogenic sets of 5-tuple, the extended plithogenic sets of 7-tuple is also introduced considering both the dominant and recessive attribute values of an attribute [17]. Plithogenic sets are also applied in constructing algebraic structures. Merkepci and Abobala [18] developed Plithogenic rings. Ali and Hasan [19] introduced the theory of Plithogenic vector space. Zeina et al [20] discussed Plithogenic probability. Soueycatt et al [21] deliberated algebraic properties based on Plithogenic sets. Alhasan [22] discoursed on Plithogenic integrals, Moscoso et al [23] discoursed on Plithogenic statistics, and Bharathi and Leo [24] discussed Plithogenic product graphs. The Plithogeny theory is also blended with hypersoft sets and Smarandache introduced the Plithogenic hypersoft sets [25]. Rana [26] discoursed Plithogenic Fuzzy Hypersoft set Matrix, operators, and applications in medical diagnosis. Hema [27] developed the theory of interval-valued Plithogenic hypersoft sets. Thus, the theory of Plithogeny is integrated with several mathematical concepts and this has contributed to the development of Plithogenic theory and applications.

The Possibility of soft sets with different representations is discussed under the environments of fuzzy, intuitionistic, and neutrosophic and finds several applications in decision-making. The recent applications are presented in Table 1.

From the above table, it is very clear that the Possibility of soft sets is discussed in various environments and with different kinds of representations of sets such as bipolar sets, Pythagorean sets, vague sets, cubic sets, interval-valued sets, and many others. The following research gaps are identified as follows,

- The theory of the Possibility Plithogenic Soft sets does not exist in literature.
- The applications of Plithogenic soft sets are very limited.

To bridge these gaps, this research work develops the theory of the Possibility Plithogenic Soft sets based on the works of Alkhazaleh. The applications of PPSS are also discussed to showcase the viability of such kinds of extended sets.

Table 1. Applications of possibility soft sets with different representations.

Authors & Year	Nature of Possibility Soft Sets	Areas of Application
Al-Sharqi et al. [28]	Possibility interval fuzzy soft	Medical Diagnosis
Al-Qudah and Al-Sharqi [29]	Possibility interval-valued neutrosophic soft set	Individual decision-making on residence
Ahmed et al. [30]	Possibility Fermatean fuzzy soft set	Selection of Eco-system
Ali [31]	Possibility Fuzzy Soft	Selection of Agricultural land
Kirişci [32]	New Possibility Soft Sets	Education
Al-Sharqi et al. [33]	Possibility of neutrosophic soft expert sets	Medical Diagnosis
Jayanthi [34]	Interval-valued possibility intuitionistic fuzzy soft sets	Treatment of drug addict patients
Palanikumar & Arulmozhi [35]	Possibility of spherical fuzzy soft set	Education
Palanikumar et al. [36]	Novel Possibility Pythagorean Cubic Fuzzy Soft Sets	Selection problem
Priya et al. [37]	Possibility of neutrosophic bipolar fuzzy soft sets	Selection problem
Palanikumar & Arulmozhi [38]	Novel possibility Pythagorean interval-valued fuzzy soft set	Education
Palanikumar et al. [39]	Possibility of Pythagorean Neutrosophic Vague Soft Sets	Robotic engineering selection
Kuo et al. [40]	Gra-Based Possibility soft set	Selection of Pest Control Methods

3 | Possibility Plithogenic Soft Sets

This section presents the conceptualization of the Possibility of Plithogenic Soft sets based on the theoretical developments of Plithogenic Soft sets developed by Alkhazaleh [6]. The basic definitions of Plithogenic sets, Soft sets, and other basic preliminaries shall be referred from [1]. Some of the core definitions are only presented in this section.

3.1 | Possibility Plithogenic Soft Sets

Alkhazaleh [6] defines Plithogenic soft sets as the generalization of crisp, fuzzy, intuitionistic, and neutrosophic soft sets.

Let us consider U, the universe of discourse and U^z denotes the set of all crisp sets of U for z= C, the set of all fuzzy sets of U for z = F, the set of all intuitionistic fuzzy sets of U for z = I, and set of all neutrosophic sets for z = N.

Let $a_1,a_2,a_3,..a_n$ be the n attributes and let V1, V2,.., Vn be the set of attribute values of each attribute ai, i = 1,….,n with $V_i \cap V_j = \emptyset$ for j ≠ i, and i,j ∈ {1,2,..,n}. Let Vi = $\{v_{i1},v_{i2},..v_{im}\}$ and let $\Upsilon = V_1 \times V_2 \times V_3 \times \ldots V_n$. Let d = $(d_1,d_2,\ldots d_n)$ be the dominant attribute value of each of the attributes ai with $c(d_i,v_{ij})$ as the contradiction degree. The contradiction degree function is defined as c : $V_i \times V_i \rightarrow [0,1]$.

The pair (F_P^z, Υ) where $F_P^z\colon \Upsilon \rightarrow [0,1]_d \times U^z$ is called a Plithogenic Soft set over U.

The Plithogenic soft sets are characterized as crisp, fuzzy, intuitionistic, and neutrosophic based on z = C, F, I, and N respectively.

Based on this definition of Plithogenic soft sets, the Possibility of Plithogenic soft sets is developed as the generalization of Plithogenic soft sets with possibility degree.

A Possibility Plithogenic soft set over U is a set of ordered pairs of the form ((F_{PP}^z, Υ) defined by $F_{PP}^z\colon \Upsilon \rightarrow [0,1]_d \times U^z \times I^U$, where I^U is the collection of all fuzzy subsets of U.

This can also be represented as

$$F_{PP}^{z} = \left\{\left(e_h, \left\{\left(\frac{(u_l, c_d)}{F_P^z(e_h)(u_l)}, \mu(e_h)(u_l)\right): u_l \epsilon\ U\right\}\right): e_h \epsilon\ \Upsilon\right\}$$

In this expression, $\mu(e_h)(u_l)$ represents the possible degree of u_l with respect to e_h.

To understand this, let us consider a generalized example, Let U = {u_1,u_2,u_3, }be the elements of discourse, and Let the attributes be a_1, a_2, and a_3. Hence V_1 = {v_{11},v_{12},v_{13}}, V2 = {v_{21},v_{22},v_{23}}, V_3 ={ v_{31},v_{32},v_{33}}. Let the dominant attribute values be d = (v_{11},v_{21},v_{33}) Let $H \sqsubseteq \Upsilon$ and H = {e_1,e_2,e_3}, where e_1 = (v_{11},v_{22},v_{31}), e_2 =(v_{13},v_{22},v_{33}) . Then

$$F_{PP}^{z}(e_1) = \left\{\frac{(u_1, c(d, e_1)}{F_P^z(e_1)(u_1)}, \mu(e_1)(u_1), \frac{(u_2, c(d, e_1)}{F_P^z(e_1)(u_2)}, \mu(e_1)(u_2), \frac{(u_3, c(d, e_1)}{F_P^z(e_1)(u_3)}, \mu(e_1)(u_3)\right\}$$

$$F_{PP}^{z}(e_2) = \left\{\frac{(u_1, c(d, e_2)}{F_P^z(e_2)(u_1)}, \mu(e_2)(u_1), \frac{(u_2, c(d, e_2)}{F_P^z(e_2)(u_2)}, \mu(e_2)(u_2), \frac{(u_3, c(d, e_2)}{F_P^z(e_2)(u_3)}, \mu(e_2)(u_3)\right\}$$

This example is more generalized in nature and the same shall be applied in discussing the types of Possibility Plithogenic Soft sets by substituting specific values.

3.2 | Possibility Plithogenic Crisp Soft Set

A possibility plithogenic crisp soft set is defined as F_{PP}^{C}: H → $[0,1]_d \times U^C \times I^U$, where $H \sqsubseteq \Upsilon$. Let us consider an example of supplier selection. U = { s_1,s_2,s_3} be the set of suppliers, the attributes considered are a_1 = Quality, a_2= Price, a_3 = Delivery Time. The attribute values are

Quality A_1 = {low, medium, high}, Price A_2 = {cheap, budgetary, expensive}, Delivery Time A_3 = { slow, moderate, fast}.

Let H = { e_1 = (medium, cheap, moderate), e_2 = (high, budgetary, fast), e_3 = (high, expensive, fast)}

Let the dominant attribute values d = (high, cheap, fast)

$$F_{PP}^{C}(e_1) = \left\{\frac{(s_1, (0.3,0,0.6)_d)}{(1,1,1)}, 0.7, \frac{(s_2, c(0.3,0,0.6)_d)}{(1,1,1)}, 0.8, \frac{(s_3, (0.3,0,0.6)_d)}{(1,1,1)}, 0.6\right\}$$

$$F_{PP}^{C}(e_2) = \left\{\frac{(s_1, (0,0.6,0)_d)}{(1,1,1)}, 0.5, \frac{(s_2, c(0,0.6,0)_d)}{(1,1,1)}, 0.6, \frac{(s_3, (0,0.6,0)_d)}{(1,1,1)}, 0.7\right\}$$

$$F_{PP}^{C}(e_3) = \left\{\frac{(s_1, (0,0.3,0)_d)}{(1,1,1)}, 0.4, \frac{(s_2, c(0,0.3,0)_d)}{(1,1,1)}, 0.7, \frac{(s_3, (0,0.3,0)_d)}{(1,1,1)}, 0.6\right\}$$

The possibility Plithogenic crisp soft sets comprise the following approximations:

$$(F_{PP}^{C}, \Upsilon) = \left\{\begin{array}{c} \left(e_1, \left\{\frac{(s_1, (0.3,0,0.6)_d)}{(1,1,1)}, 0.7, \frac{(s_2, c(0.3,0,0.6)_d)}{(1,1,1)}, 0.8, \frac{(s_3, (0.3,0,0.6)_d)}{(1,1,1)}, 0.6\right\}\right), \\ \left(e_2, \left\{\frac{(s_1, (0,0.6,0)_d)}{(1,1,1)}, 0.5, \frac{(s_2, c(0,0.6,0)_d)}{(1,1,1)}, 0.6, \frac{(s_3, (0,0.6,0)_d)}{(1,1,1)}, 0.7\right\}\right), \\ \left(e_3, \left\{\frac{(s_1, (0,0.3,0)_d)}{(1,1,1)}, 0.4, \frac{(s_2, c(0,0.3,0)_d)}{(1,1,1)}, 0.7, \frac{(s_3, (0,0.3,0)_d)}{(1,1,1)}, 0.6\right\}\right) \end{array}\right\}$$

3.3 | Possibility Plithogenic Fuzzy Soft Set

A possibility plithogenic fuzzy soft set is defined as F_{PP}^{F}: H → $[0,1]_d \times U^F \times I^U$ and based on the example of

The possibility of Plithogenic fuzzy soft sets comprises the following approximations

$$
(F_{PP}^{F}, \Upsilon) = \left\{ \begin{array}{l} \left(e_1, \left\{\frac{(s_1, (0.3,0,0.6)_d)}{(0.6,0.4,0.7)}, 0.6, \frac{(s_2, c(0.3,0,0.6)_d)}{(0.5,0.7,0.2)}, 0.7, \frac{(s_3, (0.3,0,0.6)_d)}{(0.4,0.5,0.7)}, 0.5\right\}\right), \\ \left(e_2, \left\{\frac{(s_1, (0,0.6,0)_d)}{(0.2,0.4,0.7)}, 0.3, \frac{(s_2, c(0,0.6,0)_d)}{(0.7,0.5,0.8)}, 0.6, \frac{(s_3, (0,0.6,0)_d)}{(0.7,0.5,0.6)}, 0.8\right\}\right), \\ \left(e_3, \left\{\frac{(s_1, (0,0.3,0)_d)}{(0.7,0.9,0.5)}, 0.6, \frac{(s_2, c(0,0.3,0)_d)}{(0.7,0.5,0.3)}, 0.8, \frac{(s_3, (0,0.3,0)_d)}{(0.2,0.7,0.6)}, 0.7\right\}\right) \end{array} \right\}
$$

The above representation shall be put into a matrix form with fuzzy values pertaining to each of the attribute value subjected to each attribute.

$$
F_{PPa_1}^{F} = \begin{bmatrix} (0.6,0.6) & (0.5,0.7) & (0.4,0.5) \\ (0.2,0.3) & (0.7,0.6) & (0.7,0.8) \\ (0.7,0.6) & (0.7,0.8) & (0.2,0.7) \end{bmatrix}
$$

$$
F_{PPa_2}^{F} = \begin{bmatrix} (0.4,0.6) & (0.7,0.7) & (0.5,0.5) \\ (0.4,0.3) & (0.5,0.6) & (0.5,0.8) \\ (0.9,0.6) & (0.5,0.8) & (0.7,0.7) \end{bmatrix}
$$

$$
F_{PPa_3}^{F} = \begin{bmatrix} (0.7,0.6) & (0.2,0.7) & (0.7,0.5) \\ (0.7,0.3) & (0.8,0.6) & (0.6,0.8) \\ (0.5,0.6) & (0.3,0.8) & (0.6,0.7) \end{bmatrix}
$$

3.4 | Possibility Plithogenic Intuitionistic Soft Set

A possibility plithogenic intuitionistic fuzzy soft set is defined as $F_{PP}^{I}: \mathrm{H} \rightarrow [0,1]_d \times U^I \times I^U$ and based on the example of

The possibility of Plithogenic intuitionistic fuzzy soft sets comprises the following approximations: $(F_{PP}^{I}, \Upsilon) =$

$$
\left\{ \begin{array}{l} \left(e_1, \left\{\frac{(s_1,(0.3,0,0.6)_d)}{((0.6,0.2),(0.4,0.5),(0.7,0.2))}, 0.5, \frac{(s_2,c(0.3,0,0.6)_d)}{((0.5,0.4),(0.7,0.2).(0.2,0.7)}, 0.6, \frac{(s_3,(0.3,0,0.6)_d)}{((0.4,0.5),(0.5,0.4),(0.7,0.2))}, 0.7\right\}\right), \\ \left(e_2, \left\{\frac{(s_1,(0,0.6,0)_d)}{((0.2,0.7),(0.4,0.5),(0.7,0.3))}, 0.4, \frac{(s_2,c(0,0.6,0)_d)}{((0.7,0.1),(0.5,0.3),(0.8,0.1))}, 0.5, \frac{(s_3,(0,0.6,0)_d)}{((0.7,0.2),(0.5,0.1),(0.6,0.2))}, 0.8\right\}\right), \\ \left(e_3, \left\{\frac{(s_1,(0,0.3,0)_d)}{((0.7,0.2),(0.9,0.1),(0.5,0.2))}, 0.7, \frac{(s_2,c(0,0.3,0)_d)}{((0.7,0.2),(0.5,0.2),(0.3,0.4))}, 0.8, \frac{(s_3,(0,0.3,0)_d)}{((0.2,0.7),(0.7,0.2),(0.6,0.2))}, 0.6\right\}\right) \end{array} \right\}
$$

The above representation shall be put into a matrix form with intuitionistic values pertaining to each of the attribute values subjected to each attribute.

$$
F_{PPa_1}^{F} = \begin{bmatrix} ((0.6,0.2), 0.5) & ((0.5,0.4), 0.6) & ((0.4,0.5), 0.7 \\ ((0.2,0.7), 0.3) & ((0.7,0.1), 0.6) & ((0.7,0.2), 0.8) \\ ((0.7,0.2), 0.6) & ((0.7,0.2), 0.8) & ((0.2,0.7), 0.6) \end{bmatrix}
$$

$$
F_{PPa_2}^{F} = \begin{bmatrix} ((0.4,0.5), 0.5) & ((0.7,0.2), 0.6) & ((0.5,0.4), 0.7 \\ ((0.4,0.5), 0.3) & ((0.5,0.3), 0.6) & ((0.5,0.1), 0.8) \\ ((0.9,0.1), 0.6) & ((0.5,0.2), 0.8) & ((0.7,0.2), 0.6) \end{bmatrix}
$$

$$
F_{PPa_3}^{F} = \begin{bmatrix} ((0.7,0.2), 0.5) & ((0.2,0.7), 0.6) & ((0.7,0.2), 0.7 \\ ((0.7,0.3), 0.3) & ((0.8,0.1), 0.6) & ((0.6,0.2), 0.8) \\ ((0.5,0.2), 0.6) & ((0.3,0.4), 0.8) & ((0.6,0.2), 0.6) \end{bmatrix}
$$

3.5 | Possibility Plithogenic Neutrosophic Soft Set

A possibility plithogenic neutrosophic soft set is defined as $F_{PP}^{N}: \mathrm{H} \rightarrow [0,1]_d \times U^N \times I^U$ and based on the example of

The possibility of Plithogenic intuitionistic fuzzy soft sets comprises the following approximations: $(F_{PP}^{N}, \Upsilon) =$

$$\left\{\begin{array}{l}\left(e_1,\left\{\frac{(s_1,(0.3,0,0.6)_d)}{((0.6,0.1,0.2),(0.4,0.2,0.3),(0.7,0.1,0.2))},0.7,\frac{(s_2,c(0.3,0,0.6)_d)}{((0.5,0.1,0.2),(0.7,0.1,0.2),(0.2,0.1,0.6)},0.7,\frac{(s_3,(0.3,0,0.6)_d)}{((0.4,0.1,0.5),(0.5,0.1,0.3),(0.7,0.1,0.1))},0.8\right\}\right),\\ \left(e_2,\left\{\frac{(s_1,(0,0.6,0)_d)}{((0.2,0.1,0.7),(0.4,0.1,0.5),(0.7,0.2,0.3))},0.5,\frac{(s_2,c(0,0.6,0)_d)}{((0.7,0.1,0.2),(0.5,0.2,0.3),(0.8,0.1,0.1))},0.6,\frac{(s_3,(0,0.6,0)_d)}{((0.7,0.1,0.2),(0.5,0.1,0.3)(0.6,0.2,0.1))},0.8\right\}\right),\\ \left(e_3,\left\{\frac{(s_1,(0,0.3,0)_d)}{((0.7,0.1,0.2),(0.9,0.1,0.1),(0.5,0.2,0.3))},0.6,\frac{(s_2,c(0,0.3,0)_d)}{((0.7,0.1,0.2),(0.5,0.1,0.2),(0.3,0.1,0.2))},0.7,\frac{(s_3,(0,0.3,0)_d)}{((0.2,0.2,0.3),(0.7,0.1,0.1),(0.6,0.1,0.2))},0.9\right\}\right)\end{array}\right\}$$

The above representation shall be put into a matrix with neutrosophic values pertaining to each of the attribute values subjected to each attribute.

$$F_{PPa_1}^{F} = \begin{bmatrix} ((0.6,0.1,0.2),0.7) & ((0.5,0.1,0.2),0.7) & ((0.4,0.1,0.5),0.8) \\ ((0.2,0.1,0.7),0.5) & ((0.7,0.1,0.2),0.6) & ((0.7,0.1,0.2),0.8) \\ ((0.7,0.1,0.2),0.6) & ((0.7,0.1,0.2),0.7) & ((0.2,0.2,0.3),0.9) \end{bmatrix}$$

$$F_{PPa_2}^{F} = \begin{bmatrix} ((0.4,0.2,0.3),0.7) & ((0.7,0.1,0.2),0.7) & ((0.5,0.1,0.3),0.8) \\ ((0.4,0.1,0.5),0.5) & ((0.5,0.2,0.3),0.6) & ((0.5,0.1,0.3),0.8) \\ ((0.9,0.1,0.1),0.6) & ((0.5,0.1,0.2),0.7) & ((0.7,0.1,0.1),0.9) \end{bmatrix}$$

$$F_{PPa_3}^{F} = \begin{bmatrix} ((0.7,0.1,0.2),0.7) & ((0.2,0.1,0.6),0.7) & ((0.7,0.1,0.1),0.8) \\ ((0.7,0.2,0.3),0.5) & ((0.8,,0.1,0.1),0.6) & ((0.6,0.2,0.1),0.8) \\ ((0.5,0.2,0.3),0.6) & ((0.3,0.1,0.2),0.7) & ((0.6,0.1,0.2),0.9) \end{bmatrix}$$

3.6 | Union and Intersection of Possibility Plithogenic Soft Sets

Let (F_{PP}^{z}, H) and (G_{PP}^{z}, H) be two possibility plithogenic soft sets the union of two:

Possibility Plithogenic soft sets is defined as $(F_{PP}^{z}, \mathrm{H}) \vee_P^z (G_{PP}^{z}, \mathrm{H})$ and the intersection of two:

Possibility Plithogenic soft sets is defined as $(F_{PP}^{z}, \mathrm{H}) \wedge_P^z (G_{PP}^{z}, \mathrm{H})$. The similar procedure of union and intersection of plithogenic soft sets discussed by Alkhazaleh [6] shall be considered with the t-norm and t-conorm defined respectively as $\mathrm{a} \wedge_{\mathrm{P}} \mathrm{b} = \mathrm{ab}$ and $\mathrm{a} \vee_{\mathrm{P}} \mathrm{b} = \mathrm{a} + \mathrm{b} - \mathrm{ab}$

In addition to it, the possibility degree in case of union is determined using max operator and in case of intersection the min operator is used.

Let us understand the union and intersection of Possibility Plithogenic soft sets with a simple example in the case of fuzzy environment.

$$\text{Let } (F_{PP}^{z}, \mathrm{H}) = \left\{\begin{array}{l}\left(h_1,\left\{\frac{(s_1,(0.3,0,0.6)_d)}{(0.6,0.4,0.7)},0.6,\frac{(s_2,c(0.3,0,0.6)_d)}{(0.5,0.7,0.2)},0.7,\frac{(s_3,(0.3,0,0.6)_d)}{(0.4,0.5,0.7)},0.5\right\}\right),\\ \left(h_2,\left\{\frac{(s_1,(0,0.6,0)_d)}{(0.2,0.4,0.7)},0.3,\frac{(s_2,c(0,0.6,0)_d)}{(0.7,0.5,0.8)},0.6,\frac{(s_3,(0,0.6,0)_d)}{(0.7,0.5,0.6)},0.8\right\}\right),\\ \left(h_3,\left\{\frac{(s_1,(0,0.3,0)_d)}{(0.7,0.9,0.5)},0.6,\frac{(s_2,c(0,0.3,0)_d)}{(0.7,0.5,0.3)},0.8,\frac{(s_3,(0,0.3,0)_d)}{(0.2,0.7,0.6)},0.7\right\}\right)\end{array}\right\}$$

And

$$(G_{PP}^{z}, \mathrm{H}) = \left\{\begin{array}{l}\left(h_1,\left\{\frac{(s_1,(0.3,0,0.6)_d)}{(0.5,0.4,0.3)},0.5,\frac{(s_2,c(0.3,0,0.6)_d)}{(0.7,0.3,0.4)},0.6,\frac{(s_3,(0.3,0,0.6)_d)}{(0.6,0.7,0.3)},0.4\right\}\right),\\ \left(h_2,\left\{\frac{(s_1,(0,0.6,0)_d)}{(0.2,0.4,0.7)},0.7,\frac{(s_2,c(0,0.6,0)_d)}{(0.6,0.6,0.8)},0.6,\frac{(s_3,(0,0.6,0)_d)}{(0.8,0.7,0.6)},0.7\right\}\right),\\ \left(h_3,\left\{\frac{(s_1,(0,0.3,0)_d)}{(0.5,0.7,0.5)},0.5,\frac{(s_2,c(0,0.3,0)_d)}{(0.6,0.5,0.4)},0.6,\frac{(s_3,(0,0.3,0)_d)}{(0.5,0.7,0.4)},0.8\right\}\right)\end{array}\right\}$$

The union of two possible plithogenic soft sets of (F_{PP}^{z}, H) and (G_{PP}^{z}, H) is (M_{PP}^{z}, H) and it is obtained as:

$$\text{Let } (M_{PP}^{Z}, \mathrm{H}) = \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(s_1,(0.3,0,0.6)_d)}{(0.65,0.64,0.44)}, 0.6, \frac{(s_2,c(0.3,0,0.6)_d)}{(0.7,0.79,0.26)}, 0.7, \frac{(s_3,(0.3,0,0.6)_d)}{(0.6,0.85,0.44)}, 0.5\right\}\right), \\ \left(h_2, \left\{\frac{(s_1,(0,0.6,0)_d)}{(0.36,0.35,0.81)}, 0.7, \frac{(s_2,c(0,0.6,0)_d)}{(0.88,0.5,0.96)}, 0.6, \frac{(s_3,(0,0.6,0)_d)}{(0.94,0.55,0.84)}, 0.8\right\}\right), \\ \left(h_3, \left\{\frac{(s_1,(0,0.3,0)_d)}{(0.85,0.78,0.76)}, 0.6, \frac{(s_2,c(0,0.3,0)_d)}{(0.88,0.6,0.58)}, 0.8, \frac{(s_3,(0,0.3,0)_d)}{(0.6,0.78,0.76)}, 0.8\right\}\right) \end{array} \right\}$$

To describe the result in the Example above let's compute $\left(\frac{(s_1,(0.3,0,0.6)_d)}{(0.6,0.4,0.7)}, 0.6\right) \vee_P^F \left(\frac{(s_1,(0.3,0,0.6)_d)}{(0.5,0.4,0.3)}, 0.5\right)$

(1-0.3)*(0.6+0.5-0.6*0.5)+0.3*(0.6*0.5) = 0.65,(1-0)*(0.4+0.4-0.4*0.4) +0*(0.4*0.4) = 0.64, (1-0.6)*(0.7+0.3-0.7*0.3)+0.6*(0.7*0.3) = 0.44

The intersection of two possible plithogenic soft sets of (F_{PP}^{Z}, H) and (G_{PP}^{Z}, H) is (N_{PP}^{Z}, H) and it is obtained as:

$$\text{Let } (N_{PP}^{Z}, \mathrm{H}) = \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(s_1,(0.3,0,0.6)_d)}{(0.45,0.16,0.56)}, 0.5, \frac{(s_2,c(0.3,0,0.6)_d)}{(0.5,0.21,0.34)}, 0.6, \frac{(s_3,(0.3,0,0.6)_d)}{(0.39,0.35,0.56)}, 0.4\right\}\right), \\ \left(h_2, \left\{\frac{(s_1,(0,0.6,0)_d)}{(0.04,0.45,0.49)}, 0.3, \frac{(s_2,c(0,0.6,0)_d)}{(0.42,0.6,0.64)}, 0.6, \frac{(s_3,(0,0.6,0)_d)}{(0.56,0.65,0.36)}, 0.7\right\}\right), \\ \left(h_3, \left\{\frac{(s_1,(0,0.3,0)_d)}{(0.35,0.73,0.25)}, 0.5, \frac{(s_2,c(0,0.3,0)_d)}{(0.42,0.4,0.12)}, 0.6, \frac{(s_3,(0,0.3,0)_d)}{(0.1,0.62,0.24)}, 0.7\right\}\right) \end{array} \right\}$$

To describe the result in the Example above let's compute $\left(\frac{(s_1,(0.3,0,0.6)_d)}{(0.6,0.4,0.7)}, 0.6\right) \wedge_P^F \left(\frac{(s_1,(0.3,0,0.6)_d)}{(0.5,0.4,0.3)}, 0.5\right)$

(1-0.3)*(0.6*0.5)+0.3*(0.6+0.5-0.6*0.5) = 0.45,(1-0)*(0.4*0.4) +0*(0.4+0.4-0.4*0.4) = 0.16, (1-0.6)*(0.7*0.3)+0.6*(0.7+0.3-0.7*0.3) = 0.56.

In a similar fashion, the union and intersections of the possibility of plithogenic intuitionistic and neutrosophic soft sets shall be determined based on the Plithogenic operations discussed in [].

3.7 | Similarity Measures of Possibility Plithogenic Soft Sets

Let (F_{PP}^{Z}, H) and (G_{PP}^{Z}, H) be two possibility Plithogenic soft sets. The similarity measure between these two sets is defined as S_P (F_{PP}^{Z}, G_{PP}^{Z}) = M$(F_{PP}^{Z}(e)\ , G_{PP}^{Z}(e))$ *M$(\mu_{F_{pp}}(e), \mu_{G_{pp}}(e))$

Where M$(F_{PP}^{Z}(e)\ , G_{PP}^{Z}(e)) = \max_k M_k(F_{PP}^{Z}(e)\ , G_{PP}^{Z}(e))$

M$(\mu_{F_{pp}}(e), \mu_{G_{pp}}(e))$= $\max_k M_k\left(\mu_{F_{pp}}(e), \mu_{G_{pp}}(e)\right)$

$$M_k(F_{PP}^{Z}(e)\ , G_{PP}^{Z}(e)) = 1 - \frac{\sum_{j=1}^{|U|}\sum_{i=1}^{|e|}\left|F_j(e_{ik}) - G_j(e_{ik})\right|}{\sum_{j=1}^{|U|}\sum_{i=1}^{|e|}\left|F_j(e_{ik}) + G_j(e_{ik})\right|}$$

$$M_k(\mu_{F_{pp}}(e), \mu_{G_{pp}}(e)) = 1 - \frac{\sum_{j=1}^{|U|}\sum_{i=1}^{|e|}\left|\mu_{F_{pp}}(e_{ik}) - \mu_{G_{pp}}(e_{ik})\right|}{\sum_{j=1}^{|U|}\sum_{i=1}^{|e|}\left|\mu_{F_{pp}}(e_{ik}) + \mu_{G_{pp}}(e_{ik})\right|}$$

Let us consider an example

$$\text{Let } (F_{PP}^{F}, \mathrm{H}) = \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(s_1,(0.3,0,0.6)_d)}{(0.6,0.4,0.7)}, 0.6, \frac{(s_2,c(0.3,0,0.6)_d)}{(0.5,0.7,0.2)}, 0.7, \frac{(s_3,(0.3,0,0.6)_d)}{(0.4,0.5,0.7)}, 0.5\right\}\right), \\ \left(h_2, \left\{\frac{(s_1,(0,0.6,0)_d)}{(0.2,0.4,0.7)}, 0.3, \frac{(s_2,c(0,0.6,0)_d)}{(0.7,0.5,0.8)}, 0.6, \frac{(s_3,(0,0.6,0)_d)}{(0.7,0.5,0.6)}, 0.8\right\}\right), \\ \left(h_3, \left\{\frac{(s_1,(0,0.3,0)_d)}{(0.7,0.9,0.5)}, 0.6, \frac{(s_2,c(0,0.3,0)_d)}{(0.7,0.5,0.3)}, 0.8, \frac{(s_3,(0,0.3,0)_d)}{(0.2,0.7,0.6)}, 0.7\right\}\right) \end{array} \right\}$$

And

$$(G_{PP}^{F},\mathrm{H})=\left\{\begin{array}{c}\left(h_1,\left\{\frac{(s_1,(0.3,0,0.6)_d)}{(0.5,0.4,0.3)},0.5,\frac{(s_2,c(0.3,0,0.6)_d)}{(0.7,0.3,0.4)},0.6,\frac{(s_3,(0.3,0,0.6)_d)}{(0.6,0.7,0.3)},0.4\right\}\right),\\ \left(h_2,\left\{\frac{(s_1,(0,0.6,0)_d)}{(0.2,0.4,0.7)},0.7,\frac{(s_2,c(0,0.6,0)_d)}{(0.6,0.6,0.8)},0.6,\frac{(s_3,(0,0.6,0)_d)}{(0.8,0.7,0.6)},0.7\right\}\right),\\ \left(h_3,\left\{\frac{(s_1,(0,0.3,0)_d)}{(0.5,0.7,0.5)},0.5,\frac{(s_2,c(0,0.3,0)_d)}{(0.6,0.5,0.4)},0.6,\frac{(s_3,(0,0.3,0)_d)}{(0.5,0.7,0.4)},0.8\right\}\right)\end{array}\right\}$$

The similarity between these two possibility plithogenic fuzzy soft sets is computed as follows:

$$M_1(F_{PP}^{F}(e)\,,G_{PP}^{F}(e))=1-\frac{\sum_{j=1}^{3}\sum_{i=1}^{3}|F_j(e_{i1})-G_j(e_{i1})|}{\sum_{j=1}^{3}\sum_{i=1}^{3}|F_j(e_{i1})+G_j(e_{i1})|}=1-\frac{0.5+0.8+0.8}{2.9+2.8+3.2}=0.76$$

$M_2(F_{PP}^{F}(e)\,,G_{PP}^{F}(e))=0.95$, $M_3(F_{PP}^{F}(e)\,,G_{PP}^{F}(e))=0.89$

$M_1\left(\mu_{F_{pp}}(e),\mu_{G_{pp}}(e)\right)=0.91$, $M_2\left(\mu_{F_{pp}}(e),\mu_{G_{pp}}(e)\right)=0.86$, $M_3\left(\mu_{F_{pp}}(e),\mu_{G_{pp}}(e)\right)=0.9$

$\mathrm{M}(F_{PP}^{F}(e)\,,G_{PP}^{F}(e))=0.95$ $\mathrm{M}(\mu_{F_{pp}}(e),\mu_{G_{pp}}(e))=0.91$

$S_P\,(F_{PP}^{F},G_{PP}^{F})=0.95*0.91=0.8645$

4 | Application of Possibility Plithogenic Soft Sets in Decision-Making

Let us consider a decision-making situation where the manufacturers have to make the selection of the materials for their production. The company has decided to purchase five different materials with the following attributes and attribute values as listed below.

a_1 Durability A_1 = { long-standing, average, poor}, cost a_2, A_2 = {cheap, expensive}, a_3 Environmental impact A_3 = { high, moderate, low}. In general, the company expects the materials to possess the attribute values of long-standing durability at cheap cost with low environmental impacts. In this case let us consider these attribute values as dominant, i.e. d = {long-standing, cheap, low}. However the company compromises by considering the set H of attribute values. H= {h_1 = (long-standing, expensive, low), h_2 = (average, cheap, moderate)}. The expected standards of the materials is assumed with respect to the set of attribute values and are expressed as possibility plithogenic fuzzy sets of the below-given form.

Let

$$(F_{PP}^{F},\mathrm{H})=\left\{\begin{array}{c}\left(h_1,\left\{\frac{(m_1,(0,0.5,0)_d)}{(0.6,0.4,0.7)},0.8,\frac{(m_2,c(0,0.5,0)_d)}{(0.5,0.7,0.2)},0.7,\frac{(m_3,(0,0.5,0)_d)}{(0.7,0.6,0.8)},0.8,\frac{(m_3,(0,0.5,0)_d)}{(0.4,0.5,0.7)},0.8,\frac{(m_3,(0,0.5,0)_d)}{(0.6,0.7,0.8)},0.9\right\}\right),\\ \left(h_2,\left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.7,0.6,0.8)},0.6,\frac{(m_2,c(0.3,0,0.3)_d)}{(0.6,0.8,0.7)},0.9,\frac{(m_3,(0.3,0,0.3)_d)}{(0.5,0.7,0.7)},0.7,\frac{(m_3,(0.3,0,0.3)_d)}{(0.8,0.5,0.6)},0.9,\frac{(m_3,(0.3,0,0.3)_d)}{(0.7,0.8,0.8)},0.8\right\}\right)\end{array}\right\}$$

The company receives orders from different supplier sources and they are also represented as Possibility Plithogenic Soft sets as given below.

$$(S1_{PP}^{F},\mathrm{H})=\left\{\begin{array}{c}\left(h_1,\left\{\frac{(m_1,(0,0.5,0)_d)}{(0.7,0.5,0.3)},0.8,\frac{(m_2,c(0,0.5,0)_d)}{(0.2,0.5,0.3)},0.6,\frac{(m_3,(0,0.5,0)_d)}{(0.3,0.2,0.6)},0.6,\frac{(m_3,(0,0.5,0)_d)}{(0.4,0.5,0.7)},0.3,\frac{(m_3,(0,0.5,0)_d)}{(0.3,0.6,0.7)},0.5\right\}\right),\\ \left(h_2,\left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.5,0.4,0.6)},0.6,\frac{(m_2,c(0.3,0,0.3)_d)}{(0.3,0.6,0.2)},0.7,\frac{(m_3,(0.3,0,0.3)_d)}{(0.2,0.5,0.8)},0.5,\frac{(m_3,(0.3,0,0.3)_d)}{(0.4,0.5,0.5)},0.5,\frac{(m_3,(0.3,0,0.3)_d)}{(0.6,0.5,0.7)},0.5\right\}\right)\end{array}\right\}$$

$$(S2_{PP}^{F},\mathrm{H})=\left\{\begin{array}{c}\left(h_1,\left\{\frac{(m_1,(0,0.5,0)_d)}{(0.6,0.6,0.3)},0.7,\frac{(m_2,c(0,0.5,0)_d)}{(0.4,0.5,0.5)},0.3,\frac{(m_3,(0,0.5,0)_d)}{(0.2,0.2,0.3)},0.5,\frac{(m_3,(0,0.5,0)_d)}{(0.4,0.6,0.6)},0.3,\frac{(m_3,(0,0.5,0)_d)}{(0.2,0.6,0.8)},0.8\right\}\right),\\ \left(h_2,\left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.4,0.4,0.5)},0.5,\frac{(m_2,c(0.3,0,0.3)_d)}{(0.5,0.6,0.3)},0.4,\frac{(m_3,(0.3,0,0.3)_d)}{(0.3,0.5,0.7)},0.2,\frac{(m_3,(0.3,0,0.3)_d)}{(0.3,0.6,0.7)},0.6,\frac{(m_3,(0.3,0,0.3)_d)}{(0.5,0.4,0.6)},0.8\right\}\right)\end{array}\right\}$$

$$(S3_{PP}^F, \mathrm{H}) = \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(m_1,(0,0.5,0)_d)}{(0.6,0.5,0.2)}, 0.6, \frac{(m_2,c(0,0.5,0)_d)}{(0.3,0.5,0.4)}, 0.5, \frac{(m_3,(0,0.5,0)_d)}{(0.5,0.2,0.6)}, 0.4, \frac{(m_3,(0,0.5,0)_d)}{(0.4,0.6,0.7)}, 0.4, \frac{(m_3,(0,0.5,0)_d)}{(0.2,0.6,0.7)}, 0.6\right\}\right), \\ \left(h_2, \left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.7,0.4,0.6)}, 0.7, \frac{(m_2,c(0.3,0,0.3)_d)}{(0.3,0.7,0.2)}, 0.8, \frac{(m_3,(0.3,0,0.3)_d)}{(0.2,0.6,0.8)}, 0.6, \frac{(m_3,(0.3,0,0.3)_d)}{(0.4,0.7,0.5)}, 0.6, \frac{(m_3,(0.3,0,0.3)_d)}{(0.8,0.5,0.7)}, 0.7\right\}\right) \end{array} \right\}$$

$$(S4_{PP}^F, \mathrm{H}) = \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(m_1,(0,0.5,0)_d)}{(0.7,0.4,0.3)}, 0.3, \frac{(m_2,c(0,0.5,0)_d)}{(0.2,0.5,0.3)}, 0.5, \frac{(m_3,(0,0.5,0)_d)}{(0.3,0.1,0.5)}, 0.6, \frac{(m_3,(0,0.5,0)_d)}{(0.4,0.3,0.6)}, 0.7, \frac{(m_3,(0,0.5,0)_d)}{(0.2,0.6,0.7)}, 0.8\right\}\right), \\ \left(h_2, \left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.6,0.2,0.6)}, 0.6, \frac{(m_2,c(0.3,0,0.3)_d)}{(0.4,0.6,0.8)}, 0.5, \frac{(m_3,(0.3,0,0.3)_d)}{(0.3,0.1,0.8)}, 0.3, \frac{(m_3,(0.3,0,0.3)_d)}{(0.4,0.2,0.4)}, 0.9, \frac{(m_3,(0.3,0,0.3)_d)}{(0.2,0.5,0.9)}, 0.5\right\}\right) \end{array} \right\}$$

$$(S5_{PP}^F, \mathrm{H}) = \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(m_1,(0,0.5,0)_d)}{(0.1,0.6,0.3)}, 0.7, \frac{(m_2,c(0,0.5,0)_d)}{(0.2,0.5,0.1)}, 0.6, \frac{(m_3,(0,0.5,0)_d)}{(0.3,0.2,0.5)}, 0.7, \frac{(m_3,(0,0.5,0)_d)}{(0.4,0.5,0.2)}, 0.6, \frac{(m_3,(0,0.5,0)_d)}{(0.3,0.6,0.2)}, 0.2\right\}\right), \\ \left(h_2, \left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.5,0.1,0.2)}, 0.3, \frac{(m_2,c(0.3,0,0.3)_d)}{(0.3,0.1,0.1)}, 0.6, \frac{(m_3,(0.3,0,0.3)_d)}{(0.2,0.5,0.7)}, 0.5, \frac{(m_3,(0.3,0,0.3)_d)}{(0.4,0.2,0.5)}, 0.4, \frac{(m_3,(0.3,0,0.3)_d)}{(0.3,0.5,0.2)}, 0.3\right\}\right) \end{array} \right\}$$

The similarity measures between the company's expected standards of the materials and the supplier's quality of the materials are determined. The highest similarity measures help to identify the optimal supplier that matches the expected standards of the company.

S_P $(F_{PP}^F, S1_{PP}^F) = 0.6921$, S_P $(F_{PP}^F, S2_{PP}^F) = 0.6357$, S_P $(F_{PP}^F, S3_{PP}^F) = 0.7567$, S_P $(F_{PP}^F, S4_{PP}^F) = 0.6557$, S_P $(F_{PP}^F, S5_{PP}^F) = 0.5688$

From the similarity measures the suppliers S1, S2, S3, S4, and S5 are ranked accordingly. S3 > S1>S4>S2>S5.

Thus, the suppliers are ranked and the company shall make use of the ranking results in choosing the optimal suppliers. In a similar fashion, the Possibility of Plithogenic soft sets shall be applied in making optimal decisions in various environments.

5 | Comparative Analysis

The efficacy of Possibility Plithogenic soft sets shall be determined by comparing the ranking results of Plithogenic soft sets with that of the Possibility-oriented Plithogenic soft sets. The ranking results obtained using both Plithogenic and Possibility Plithogenic Soft sets are presented in Table 2.

To obtain the similarity measures of the Plithogenic soft sets, the possibility degrees are excluded and then considered for calculation as given below.

$$\text{Let } (F_P^F, \mathrm{H}) = \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(m_1,(0,0.5,0)_d)}{(0.6,0.4,0.7)}, \frac{(m_2,c(0,0.5,0)_d)}{(0.5,0.7,0.2)}, \frac{(m_3,(0,0.5,0)_d)}{(0.7,0.6,0.8)}, \frac{(m_3,(0,0.5,0)_d)}{(0.4,0.5,0.7)}, \frac{(m_3,(0,0.5,0)_d)}{(0.6,0.7,0.8)}\right\}\right), \\ \left(h_2, \left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.7,0.6,0.8)}, \frac{(m_2,c(0.3,0,0.3)_d)}{(0.6,0.8,0.7)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.5,0.7,0.7)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.8,0.5,0.6)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.7,0.8,0.8)}\right\}\right) \end{array} \right\}$$

The company receives orders from different supplier sources and they are also represented as Possibility Plithogenic Soft sets as given below.

$$(S1_P^F, \mathrm{H}) = \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(m_1,(0,0.5,0)_d)}{(0.7,0.5,0.3)}, \frac{(m_2,c(0,0.5,0)_d)}{(0.2,0.5,0.3)}, \frac{(m_3,(0,0.5,0)_d)}{(0.3,0.2,0.6)}, \frac{(m_3,(0,0.5,0)_d)}{(0.4,0.5,0.7)}, \frac{(m_3,(0,0.5,0)_d)}{(0.3,0.6,0.7)}\right\}\right), \\ \left(h_2, \left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.5,0.4,0.6)}, \frac{(m_2,c(0.3,0,0.3)_d)}{(0.3,0.6,0.2)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.2,0.5,0.8)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.4,0.5,0.5)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.6,0.5,0.7)}\right\}\right) \end{array} \right\}$$

$(S2_P^F, \mathrm{H})$

$$= \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(m_1,(0,0.5,0)_d)}{(0.6,0.6,0.3)}, \frac{(m_2,c(0,0.5,0)_d)}{(0.4,0.5,0.5)}, \frac{(m_3,(0,0.5,0)_d)}{(0.2,0.2,0.3)}, \frac{(m_3,(0,0.5,0)_d)}{(0.4,0.6,0.6)}, \frac{(m_3,(0,0.5,0)_d)}{(0.2,0.6,0.8)}\right\}\right), \\ \left(\left(h_2, \left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.4,0.4,0.5)}, \frac{(m_2,c(0.3,0,0.3)_d)}{(0.5,0.6,0.3)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.3,0.5,0.7)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.3,0.6,0.7)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.5,0.4,0.6)}\right\}\right)\right) \end{array} \right\}$$

$(S3_P^F, \mathrm{H})$

$$= \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(m_1,(0,0.5,0)_d)}{(0.6,0.5,0.2)}, \frac{(m_2,c(0,0.5,0)_d)}{(0.3,0.5,0.4)}, \frac{(m_3,(0,0.5,0)_d)}{(0.5,0.2,0.6)}, \frac{(m_3,(0,0.5,0)_d)}{(0.4,0.6,0.7)}, \frac{(m_3,(0,0.5,0)_d)}{(0.2,0.6,0.7)}\right\}\right), \\ \left(\left(h_2, \left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.7,0.4,0.6)}, \frac{(m_2,c(0.3,0,0.3)_d)}{(0.3,0.7,0.2)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.2,0.6,0.8)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.4,0.7,0.5)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.8,0.5,0.7)}\right\}\right)\right) \end{array} \right\}$$

$(S4_P^F, \mathrm{H})$

$$= \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(m_1,(0,0.5,0)_d)}{(0.7,0.4,0.3)}, \frac{(m_2,c(0,0.5,0)_d)}{(0.2,0.5,0.3)}, \frac{(m_3,(0,0.5,0)_d)}{(0.3,0.1,0.5)}, \frac{(m_3,(0,0.5,0)_d)}{(0.4,0.3,0.6)}, \frac{(m_3,(0,0.5,0)_d)}{(0.2,0.6,0.7)}\right\}\right), \\ \left(\left(h_2, \left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.6,0.2,0.6)}, \frac{(m_2,c(0.3,0,0.3)_d)}{(0.4,0.6,0.8)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.3,0.1,0.8)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.4,0.2,0.4)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.2,0.5,0.9)}\right\}\right)\right) \end{array} \right\}$$

$(S5_P^F, \mathrm{H})$

$$= \left\{ \begin{array}{c} \left(h_1, \left\{\frac{(m_1,(0,0.5,0)_d)}{(0.1,0.6,0.3)}, \frac{(m_2,c(0,0.5,0)_d)}{(0.2,0.5,0.1)}, \frac{(m_3,(0,0.5,0)_d)}{(0.3,0.2,0.5)}, \frac{(m_3,(0,0.5,0)_d)}{(0.4,0.5,0.2)}, \frac{(m_3,(0,0.5,0)_d)}{(0.3,0.6,0.2)}\right\}\right), \\ \left(\left(h_2, \left\{\frac{(m_1,(0.3,0,0.3)_d)}{(0.5,0.1,0.2)}, \frac{(m_2,c(0.3,0,0.3)_d)}{(0.3,0.1,0.1)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.2,0.5,0.7)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.4,0.2,0.5)}, \frac{(m_3,(0.3,0,0.3)_d)}{(0.3,0.5,0.2)}\right\}\right)\right) \end{array} \right\}$$

The similarity measures are computed using the formula given in [6], in that case the similarity measures thus obtained are

S $(F_P^F, S1_P^F)$ =0.816734, S $(F_P^F, S2_P^F)$ = 0.79249, S $(F_P^F, S3_P^F)$ = 0.832677, S $(F_P^F, S4_P^F)$ = 0.779961, S $(F_P^F, S5_P^F)$ = 0.663205. Hence S3>S1>S2>S4>S5.

Table 2. Ranking results.

Suppliers	Plithogenic Fuzzy Soft Sets (PFS)	Possibility Plithogenic Fuzzy Soft Sets (PPFS)
S1	2	2
S2	3	4
S3	1	1
S4	4	3
S5	5	5

It is inferred that the ranking results are closer and not much deviated, however, the inclusion of the possibility degree has added to the concreteness of the results. In the case of involving experts in decision-making, the inclusion of a possibility degree will strengthen the optimality of the results.

6 | Conclusion

This research work introduces the Possibility of Plithogenic soft sets as both an extension and generalization of Plithogenic soft sets. The operations and similarity measures discussed in this paper play a significant role in decision-making. The application of Possibility Plithogenic soft sets in supplier selection signifies its viability in ranking-based decision problems. The comparison between PSS and PPSS under a fuzzy environment explicates the consistency and efficiency of PPSS over PSS. The decision-making models based on PPSS shall be developed and further the theoretical framework of PPSS shall be discussed with other kinds of representations such as Fermatean sets, Pythagorean sets, vague and cube sets.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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