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A VIKOR Method Based on the Relative Closeness on Interval Linguistic Neutrosophic Uncertain Linguistic Numbers

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Abstract

In actual multi-attribute group decision-making problems, due to the complexity and uncertainty of objective things and the ambiguity of human thinking, decision-makers find it hard to give accurate evaluation information by crisp numbers. Even the weights of attribute values and decision-makers are ambiguous. At this time, we are more inclined to adopt the intuitive form of linguistic variables such as "excellent", "good" or "bad" to describe attribute values and weights. So in this paper, based on a neutrosophic set (NS), we further propose interval linguistic neutrosophic uncertain linguistic number (ILNULN). ILNULN combines interval linguistic neutrosophic and uncertain linguistic numbers, and it has the advantages of both. At the same time, due to the weighted arithmetic Bonferroni mean operator considering the interrelationship between aggregation parameters, therefore we combine the ILNULN and weighted arithmetic Bonferroni mean operator to propose the interval linguistic neutrosophic uncertain linguistic weighted arithmetic Bonferroni mean (ILNULWABM) operator. Finally, under the environment of interval linguistic neutrosophic and uncertain linguistic numbers, this article uses the linguistic weights and ILNULWABM operator to make VIKOR decision based on the relative closeness, and gives a practical example.

Keywords: Multi-Attribute Group Decision-Making, Interval Linguistic Neutrosophic Uncertain Linguistic Number, Weighted Arithmetic Bonferroni Mean Operator, VIKOR.

1 | Introduction

Since its birth, the theory of multi-attribute group decision-making has been a research hotspot in academia. With the development of society and economy, the complexity, uncertainty, and ambiguity of human thinking are increasing. In the actual decision-making process, decision information is often expressed as fuzzy information. In 1965 Zadeh [1] put forward the concept of fuzzy set (FS). Fuzzy Set represents the uncertainty of decision information by membership degree, which refers to the degree that which something belongs to a certain judgment. However, in the process of cognition, people tend to hesitate to different degrees or show a certain degree of lack of knowledge, so the cognitive results are shown as positive, negative, or intermediate between positive and negative hesitation. Therefore, in 1986, Atanassov [2, 3] extended the theory of fuzzy sets and proposed the concept of intuitionistic fuzzy sets (IFS). IFS considers both membership and non-membership information at the same time, so it provides more choices in the description of the attributes

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and has a stronger performance in dealing with uncertain information. In addition, in some practical situations, the membership, non-membership, and hesitation of elements may not be specific values, so IFS extended to interval-value intuitionistic fuzzy set (IVIFS) by Atanassov and Gargov [4]. Although the FS theory has been widely developed and popularized, it still can not handle all types of uncertain problems in real life, such as uncertain information and inconsistent information. To this end, Smarandache [5] proposed the concept of a neutrosophic set (NS). NS includes membership degree $T(x)$, uncertainty degree $I(x)$, and non-membership degree $F(x)$ of elements. Wang and Zhang [6] further proposed the concept of an interval neutrosophic set (INS), where the representation of the $T(x)$, $I(x)$ and $F(x)$ extended from a single value to an interval number. Later Wang and Smarandache et al. [7] proposed the single-valued neutrosophic set (SVNS) theory. In addition, Ye [8] combined the uncertain linguistic set with INS to define the interval neutrosophic uncertain linguistic set (INULS), and he also defined the score function, accuracy function, and operational laws of INULS. The first part of the interval neutrosophic uncertain linguistic variable represents the subjective evaluation value of the thing being evaluated, and the second part indicates membership degree, uncertainty degree, and non-membership degree. In 2017, Ye and Fang [9] proposed the concept of linguistic neutrosophic number (LNN), which was characterized independently by the truth, indeterminacy, and falsity of linguistic variables.

VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [10] is a method of multi-attribute decision-making based on the ideal point proposed by Opricuvic in 1998. This method gives the ranking index with the ideal closest to the ideal solution, which maximizes the group utility and minimizes individual regret when selecting a solution. At present, many scholars have studied the VIKOR method and its application. Bayakzkan and Ruan [11] extended the VIKOR method to the fuzzy environment to solve the software evaluation problem. Sayadi et al. [12] discussed the VIKOR method with the attribute values as interval numbers and the weights as real numbers. Sanayei et al. [13] researched the VIKOR method using fuzzy sets and linguistic values, and they applied it to supplier selection problems. In 2011, Park et al. [14] considered the VIKOR method with attribute values of intuitionistic interval fuzzy numbers and weights of real numbers. Zhang and Wei [15] extended the VIKOR method to the hesitating fuzzy set. Due to the traditional VIKOR method only considering the closeness between the alternatives and the positive ideal solution, Liu [16] proposed the VIKOR method based on the relative closeness coefficient. This method takes the closeness coefficient between alternatives and positive ideal solution as well as the closeness coefficient between alternatives and negative ideal solution into account.

Information integration is a common activity in our daily life. The Bonferroni mean (BM) operator is one of the aggregation methods proposed by Bonferroni [17]. BM operator has a desirable characteristic that it can capture the interrelationship of input arguments. Then Yager [18] further extended the BM operator and proposed some more efficient integration operators. Xu and Yager [19] introduced a new BM operator to solve the multi-attribute decision-making (MADM) problems under fuzzy conditions. To solve the reducibility of the weighted BM operator, Xia et al. [20] proposed a modified generalized weighted BM operator and applied it to the intuitionistic fuzzy environment. Since the arithmetic average only considers group decisions and ignores individual decisions, Zhou et al. [21] proposed the standardized weighted BM operator and fully considered the correlation between attribute values. Later the BM operator extended to a neutrosophic environment. Wei et al. [22] developed an uncertain linguistic Bonferroni mean (ULBM) operator and an uncertain linguistic geometric Bonferroni mean (ULGBM) operator to aggregate the uncertain linguistic information. Liu [23] and Wang introduced a single-valued neutrosophic normalized weighted Bonferroni mean (SVNNWBM) operator. Wang et al. [24] developed a simplified neutrosophic linguistic Bonferroni mean (SNLBM) operator and a simplified neutrosophic linguistic normalized weighted Bonferroni mean (SNLNWBM) operator.

Although the combination of the neutrosophic set and the linguistic set has been further developed, there are few studies on the combination of the interval linguistic neutrosophic and uncertain linguistic number.

Therefore, this paper proposes the concept of interval linguistic neutrosophic uncertain linguistic number, combining the WABM operator and linguistic weights to make the VIKOR decision.

2 | Preliminaries

This section is the theoretical foundation of this thesis. Some basic concepts about FS, ILNULS, BM operator, and VIKOR are reviewed to provide mathematical support and theoretical guarantee for the following research.

Definition 1 [25]: Let $a \triangleq [a^L, a^U] = \{x | a^L \le x \le a^U\}$, then a is an interval fuzzy number. When $0 \le a^L \le a^U$, a is a positive interval fuzzy number.

Definition 2 [26]: Let $S = \{s_i | i = 0, 1, 2, ... t-1\}$, then S is a linguistic set (LS) and t is an odd number. S satisfies the following conditions:

(1) If $i \ge j$, than $s_i \ge s_j$

(2) There is the inverse operator $reg(s_i) = s_j$, and $i + j = t$-1

(3) If $s_i \ge s_j$, then $\max(s_i, s_j) = s_i$

(4) If $s_i \le s_j$, then $\min(s_i, s_j) = s_i$

Definition 3 [26]: If a linguistic variable $s = [s_\theta, s_\rho]$, $s \in$ linguistic set S and θ, $\lambda \in [0, t-1]$, then s is an uncertain linguistic variable (ULV). s_θ, s_ρ are the upper and lower limits respectively.

Definition 4 [26]: For any three uncertain linguistic variables $s = [s_\theta, s_\rho]$, $s_1 = [s_{\theta_1}, s_{\rho_1}]$ and $s_2 = [s_{\theta_2}, s_{\rho_2}]$, then the algorithms for uncertain linguistic variables are as follows:

(1) $s_1 \oplus s_2 = [s_{\theta_1}, s_{\rho_1}] + [s_{\theta_2}, s_{\rho_2}] = [s_{\theta_1+\theta_2}, s_{\rho_1+\rho_2}]$

(2) $s_1 \otimes s_2 = [s_{\theta_1}, s_{\rho_1}] \otimes [s_{\theta_2}, s_{\rho_2}] = [s_{\theta_1\theta_2}, s_{\rho_1\rho_2}]$

(3) $s^\lambda = [s_\theta, s_\rho]^\lambda = [(s_\theta)^\lambda, (s_\rho)^\lambda] = [s_{\theta^\lambda}, s_{\rho^\lambda}]$ $(\lambda \ge 0)$

(4) $\lambda s = \lambda[s_\theta, s_\rho] = [\lambda s_\theta, \lambda s_\rho] = [s_{\lambda\theta}, s_{\lambda\rho}]$ $(\lambda \ge 0)$

Definition 5 [26]: For any three uncertain linguistic variables $s = [s_\theta, s_\rho]$, $s_1 = [s_{\theta_1}, s_{\rho_1}]$ and $s_2 = [s_{\theta_2}, s_{\rho_2}]$, then the operational properties are as follows:

(1) $s_1 \oplus s_2 = s_2 \oplus s_1$

(2) $s_1 \otimes s_2 = s_2 \otimes s_1$

(3) $\lambda(s_1 \oplus s_2) = \lambda s_1 \oplus \lambda s_2$

(4) $\lambda_1 s \oplus \lambda_2 s (\lambda_1 + \lambda_2) s$

(5) $s_1^\lambda \otimes s_2^\lambda = (s_1 \otimes s_2)^\lambda$

(6) $s^{\lambda_1} \otimes s^{\lambda_2} = s^{\lambda_1+\lambda_2}$

Definition 6 [27]: Let $s_\theta, s_\rho \in$ linguistic set S and $\gamma=(s_\theta, s_\rho)$, if $\theta+\rho \le t$-1, then we call γ the linguistic intuitionistic fuzzy number (LIFN) defined on S. If $s_\theta, s_\rho \in S$, then we call γ the original linguistic intuitionistic fuzzy number; otherwise, we call γ the virtual linguistic intuitionistic fuzzy number.

Definition 7 [27]: For any three linguistic intuitionistic fuzzy numbers $\gamma=(s_\theta, s_\rho)$, $\gamma_1=(s_{\theta_1}, s_{\rho_1})$ and $\gamma_2=(s_{\theta_2}, s_{\rho_2})$, the following operations of linguistic intuitionistic fuzzy numbers have been defined:

(1) $\gamma_1 \oplus \gamma_2 = [s_{\theta_1}, s_{\rho_1}] + [s_{\theta_2}, s_{\rho_2}] = [s_{\theta_1+\theta_2-\theta_1\theta_2}, s_{\rho_1\rho_2}]$

(2) $\gamma_1 \otimes \gamma_2 = [s_{\theta_1}, s_{\rho_1}] \otimes [s_{\theta_2}, s_{\rho_2}] = [s_{\theta_1\theta_2}, s_{\rho_1+\rho_2-\rho_1\rho_2}]$

(3) $\gamma^\lambda = [s_\theta, s_\rho]^\lambda = [(s_\theta)^\lambda, (s_\rho)^\lambda] = [s_{\theta^\lambda}, s_{1-(1-\rho)^\lambda}] (\lambda \ge 0)$

(4) $\lambda\gamma = \lambda[s_\theta, s_\rho] = [\lambda s_\theta, \lambda s_\rho] = [s_{1-(1-\theta)^\lambda}, s_{\rho^\lambda}] (\lambda \ge 0)$

Definition 8 [5]: Let X be a set of objects and x be the element in X. The neutrosophic set (NS) A in X consists of $T_A(x)$ - membership degree, $I_A(x)$ - uncertainty degree, and $F_A(x)$ - non-membership degree, and it is defined as $A=\{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are non-standard subsets in $]0^-, 1^+[$, i.e. $T_A(x): X \to]0^-, 1^+[$, $I_A(x): X \to]0^-, 1^+[$, and $F_A(x): X \to]0^-, 1^+[$. Due to the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ is unlimited, so $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 9 [7]: Let X be a set of objects and x be the element in X. When $T_A(x)$, $I_A(x)$ and $F_A(x)$ respectively degenerate to an exact number, then A is a single neutrosophic set (SVNS).

Definition 10 [6]: Let X be a set of objects and x be the element in X. The neutrosophic set A X consists of $T_A(x)$ a membership degree, $I_A(x)$ -an uncertainty degree, and $F_A(x)$ a non-membership degree. When $T_A(x)$, $I_A(x)$ and $F_A(x)$ belong to a closed interval [0,1], i.e. $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$, and $F_A(x): X \to [0,1]$, then A is an interval neutrosophic set (INS) which can be expressed as follows:

$A=\{x, [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] | x \in X\}$. Similarly, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies: $0^- \le T_A^U(x) + I_A^U(x) + F_A^U(x) \le 3^+$.

Definition 11 [28]: Let U be a space of objects. S is a linguistic set where $S=\{s_i | i=0,1,...t-1\}$ and t is odd. Then an interval neutrosophic linguistic set (INLS) A X can be defined as : $A=\{x, \langle s_{\theta(x)}, ([T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)])\rangle | x \in X\}$.

The membership degree, uncertainty degree and non-membership degree of x in X to the linguistic term $s_{\theta(x)}$ satisfies: $[T_A^L(x), T_A^U(x)] \subseteq [0,1]$, $[I_A^L(x), I_A^U(x)] \subseteq [0,1]$, and $[F_A^L(x), F_A^U(x)] \subseteq [0,1]$.

Besides, the interval neutrosophic linguistic number (INLN), which is an element of A, can be expressed as $\langle s_{\theta(x)}, ([T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)])\rangle$.

Definition 12 [29]: An interval neutrosophic uncertain linguistic set A on X can be defined as $A=\{x, \langle [s_{\theta(x)}, s_{\rho(x)}], ([T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)])\rangle | x \in X\}$, where $s_{\theta(x)}$ and $s_{\rho(x)} \in S$,

$\left[T_A^L(x),T_A^U(x)\right]\subseteq[0,1]$, $\left[I_A^L(x),I_A^U(x)\right]\subseteq[0,1]$ and $\left[F_A^L(x),F_A^U(x)\right]\subseteq[0,1]$ with the condition $0\le T_A^U(x)+I_A^U(x)+F_A^U(x)\le 3$ for any $x\in X$. The function $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent the membership degree, uncertainty degree, and non-membership degree respectively with interval values of the element x in X to the uncertain linguistic variable $\left[s_{\theta(x)},s_{\rho(x)}\right]$.

Definition 13 [29]: For any three-interval neutrosophic uncertain linguistic variables

$a=\left\langle\left[s_{\theta(a)},s_{\rho(a)}\right],\left(\left[T_A^L(a),T_A^U(a)\right],\left[I_A^L(a),I_A^U(a)\right],\left[F_A^L(a),F_A^U(a)\right]\right)\right\rangle$,

$a_1=\left\langle\left[s_{\theta(a_1)},s_{\rho(a_1)}\right],\left(\left[T_A^L(a_1),T_A^U(a_1)\right],\left[I_A^L(a_1),I_A^U(a_1)\right],\left[F_A^L(a_1),F_A^U(a_1)\right]\right)\right\rangle$,

$a_2=\left\langle\left[s_{\theta(a_2)},s_{\rho(a_2)}\right],\left(\left[T_A^L(a_2),T_A^U(a_2)\right],\left[I_A^L(a_2),I_A^U(a_2)\right],\left[F_A^L(a_2),F_A^U(a_2)\right]\right)\right\rangle$ and $\lambda\ge 0$, then the operational laws for interval neutrosophic uncertain linguistic variables are as follows:

(1) $a_1\oplus a_2=\left\langle\left[s_{\theta(a_1)+\theta(a_2)},s_{\rho(a_1)+\rho(a_2)}\right],\left(\left[T_A^L(a_1)+T_A^L(a_2)-T_A^L(a_1)T_A^L(a_2),T_A^U(a_1)+T_A^U(a_2)-T_A^U(a_1)T_A^U(a_2)\right],\left[I_A^L(a_1)I_A^L(a_2),\right.\right.$ $\left.\left.I_A^U(a_1)I_A^U(a_2)\right],\left[F_A^L(a_1)F_A^L(a_2),F_A^U(a_1)F_A^U(a_2)\right]\right)\right\rangle$

(2) $a_1\otimes a_2=\left\langle\left[s_{\theta(a_1)\theta(a_2)},s_{\rho(a_1)\rho(a_2)}\right],\left(\left[T_A^L(a_1)T_A^L(a_2),T_A^U(a_1)T_A^U(a_2)\right],\left[I_A^L(a_1)+I_A^L(a_2)-I_A^L(a_1)I_A^L(a_2),\ I_A^U(a_1)+I_A^U(a_2)-\right.\right.$ $\left.\left.I_A^U(a_1)I_A^U(a_2)\right],\left[F_A^L(a_1)+F_A^L(a_2)-F_A^L(a_1)F_A^L(a_2),F_A^U(a_1)+F_A^U(a_2)\text{-}F_A^U(a_1)F_A^U(a_2)\right]\right)\right\rangle$.

(3) $\lambda\otimes a=\left\langle\left[s_{\lambda\theta(a)},s_{\lambda\rho(a)}\right],\left(\left[1-\left(1-T_A^L(a)\right)^{\lambda},1-\left(1-T_A^U(a)\right)^{\lambda}\right],\left[\left(I_A^L(a)\right)^{\lambda},\left(I_A^U(a)\right)^{\lambda}\right],\left[\left(F_A^L(a)\right)^{\lambda},\left(F_A^U(a)\right)^{\lambda}\right]\right)\right\rangle$

(4) $a^{\lambda}=\left\langle\left[s_{\theta^{\lambda}(a)},s_{\rho^{\lambda}(a)}\right],\left(\left[\left(T_A^L(a)\right)^{\lambda},\left(T_A^U(a)\right)^{\lambda}\right],\left[1-\left(1-I_A^L(a)\right)^{\lambda},1-\left(1-I_A^U(a)\right)^{\lambda}\right],\left[1-\left(1-F_A^L(a)\right)^{\lambda},1-\left(1-F_A^U(a)\right)^{\lambda}\right]\right)\right\rangle$

Definition 14 [29]: For any three-interval neutrosophic uncertain linguistic variables

$a=\left\langle\left[s_{\theta(a)},s_{\rho(a)}\right],\left(\left[T_A^L(a),T_A^U(a)\right],\left[I_A^L(a),I_A^U(a)\right],\left[F_A^L(a),F_A^U(a)\right]\right)\right\rangle$,

$a_1=\left\langle\left[s_{\theta(a_1)},s_{\rho(a_1)}\right],\left(\left[T_A^L(a_1),T_A^U(a_1)\right],\left[I_A^L(a_1),I_A^U(a_1)\right],\left[F_A^L(a_1),F_A^U(a_1)\right]\right)\right\rangle$,

$a_2=\left\langle\left[s_{\theta(a_2)},s_{\rho(a_2)}\right],\left(\left[T_A^L(a_2),T_A^U(a_2)\right],\left[I_A^L(a_2),I_A^U(a_2)\right],\left[F_A^L(a_2),F_A^U(a_2)\right]\right)\right\rangle$ and $\lambda\ge 0$, then the operational properties for interval neutrosophic uncertain linguistic variables are as follows:

(1) $a_1\oplus a_2=a_2\oplus a_1$

(2) $a_1\otimes a_2=a_2\otimes a_1$

(3) $\lambda(a_1\oplus a_2)=\lambda a_1\oplus\lambda a_2$

(4) $\lambda_1 a\oplus\lambda_2 a=(\lambda_1+\lambda_2)\ a$

(5) $a_1^{\lambda}\otimes a_2^{\lambda}=(a_1\otimes a_2)^{\lambda}$

(6) $a^{\lambda 1}\otimes a^{\lambda 2}=a^{(\lambda 1+\lambda 2)}$

Definition 15 [8]: For any two interval neutrosophic uncertain linguistic variables $a_1=\left\langle\left[s_{\theta(a_1)},s_{\rho(a_1)}\right],\left(\left[T_A^L(a_1),T_A^U(a_1)\right],\left[I_A^L(a_1),I_A^U(a_1)\right],\left[F_A^L(a_1),F_A^U(a_1)\right]\right)\right\rangle$,

$a_2=\left\langle\left[s_{\theta(a_2)},s_{\rho(a_2)}\right],\left(\left[T_A^L(a_2),T_A^U(a_2)\right],\left[I_A^L(a_2),I_A^U(a_2)\right],\left[F_A^L(a_2),F_A^U(a_2)\right]\right)\right\rangle$ be any two interval neutrosophic uncertain linguistic variables, then the Hamming distance between a_1 and a_2 can be defined as:

$$d(a_1,a_2)=\frac{1}{12(\mathrm{t}-1)}\Big(\left|\theta(a_1)T_A^L(a_1)-\theta(a_2)T_A^L(a_2)\right|+\left|\theta(a_1)T_A^U(a_1)-\theta(a_2)T_A^U(a_2)\right|+\left|\theta(a_1)I_A^L(a_1)-\theta(a_2)I_A^L(a_2)\right|+\left|\theta(a_1)I_A^U(a_1)-\theta(a_2)I_A^U(a_2)\right|+$$
$$\left|\theta(a_1)F_A^L(a_1)-\theta(a_2)F_A^L(a_2)\right|+\left|\theta(a_1)F_A^U(a_1)-\theta(a_2)F_A^U(a_2)\right|+\left|\rho(a_1)T_A^L(a_1)-\rho(a_2)T_A^L(a_2)\right|+\left|\rho(a_1)T_A^U(a_1)-\rho(a_2)T_A^U(a_2)\right|+$$
$$\left|\rho(a_1)I_A^L(a_1)-\rho(a_2)I_A^L(a_2)\right|+\left|\rho(a_1)I_A^U(a_1)-\rho(a_2)I_A^U(a_2)\right|+\left|\rho(a_1)F_A^L(a_1)-\rho(a_2)F_A^L(a_2)\right|+\left|\rho(a_1)F_A^U(a_1)-\rho(a_2)F_A^U(a_2)\right|\Big)$$

Definition 16 [30]: For an interval neutrosophic uncertain linguistic variable $a=\left\langle [s_{\theta(a)},s_{\rho(a)}],\left(\left[T_A^L(a),T_A^U(a)\right],\left[I_A^L(a),I_A^U(a)\right],\left[F_A^L(a),F_A^U(a)\right]\right)\right\rangle$, then the score function of a can be expressed by the equation below: $S(a)=\frac{1}{12}\left(\theta(a)+\rho(a)\right)\left(4+T_A^L(a)-I_A^L(a)-F_A^L(a)+T_A^U(a)-I_A^U(a)-F_A^U(a)\right).$

Definition 17 [31]: Assume that $S=\{s_i \mid i=0,1,2,...t-1\}$, then S is a linguistic set (LS) and t is an odd number. If $a=\langle s_T,s_I,s_F\rangle$ is defined for $s_T,s_I,s_F\in S$ and $T,I,F\in[0,t-1]$, where s_T, s_I and s_F expresses independently the membership degree, uncertainty degree, and non-membership degree by linguistic numbers, then a is called a LNN.

Definition 18 [17]: Let p, $q\geq 0$ and $a_i\ (i=1,2,...n)$ be a collection of nonnegative real numbers. If

$$BM^{p,q}\left(a_1,a_2,...a_n\right)=\left(\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}a_i^p a_j^q\right)^{\frac{1}{p+q}}$$

, then $BM^{p,q}$ is called the Bonferroni mean (BM) operator.

Definition 19 [32]: Let $s_i=\left[s_{\theta_i},s_{\rho_i}\right],(i=1,2,...n)$ be a set of uncertain linguistic numbers and p, q ≥ 0. $\omega=\left(\omega_1,\omega_2,...\omega_n\right)^T$ is the weight vector of s, $\omega_i\in[0,1]$ and $\sum_{i=1}^{n}\omega_i=1$. If

$$IULWABM_{\omega}^{p,q}\left(s_1,s_2,...s_n\right)=\left(\frac{1}{n(n\text{-}1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\left(\omega_i s_i\right)^p\left(\omega_j s_j\right)^q\right)^{\frac{1}{p+q}}=\left[\left(\frac{1}{n(n\text{-}1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\left(\omega_i s_{\theta_i}\right)^p\left(\omega_j s_{\theta_j}\right)^q\right)^{\frac{1}{p+q}},\left(\frac{1}{n(n\text{-}1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\left(\omega_i s_{\rho_i}\right)^p\left(\omega_j s_{\rho_j}\right)^q\right)^{\frac{1}{p+q}}\right],$$

Then $IULWABM_{\omega}^{p,q}$ is called the intuitionistic uncertain linguistic weighted arithmetic Bonferroni mean operator.

Definition 20 [10]: VIKOR is a method of multi-attribute decision-making based on the ideal point proposed by Opricuvic in 1998. It is regarded as a pragmatic approach to searching for a compromise solution appearing in a set that includes conflicting criteria. The multi-criterion measurement of compromise order is developed from the L_P measure and it is an aggregate function of distance functions. L_1 is the sum of all individual regrets, and L_∞ is the maximum of individual regrets. The assembly function of the VIKOR method is as follows:

$$L_{p,j}=\left\{\sum_{j=1}^{n}\left[\frac{\omega_j\left(f_{ij}^{+}-f_{ij}\right)}{\left(f_{ij}^{+}-f_{ij}^{-}\right)}\right]^p\right\}^{\frac{1}{p}},1\leq p\leq+\infty,\ j=1,2,...n$$

Where $\omega_j\ (j=1,2,...n)$ is the relevant weight of the criteria, L_{Pj} represents the distance of each scheme from the positive ideal solution, $f_{ij}^{+}=\max_j f_{ij}$ represents the positive ideal solution, and $f_{ij}^{-}=\min_j f_{ij}$ represents the negative ideal solution. Therefore, the main advantage of this method is that it produces a solution by maximizing group utility and minimizing the opponent's individual regret.

2.1 | Calculation Steps of VIKOR Method

$A=\{A_1, A_2, \ldots A_m\}$ is a set of alternatives; $C=\{C_1, C_2, \ldots C_n\}$ represent n criteria; and $\omega=(\omega_1, \omega_2, \ldots \omega_n)^T$ denotes a weight vector of criteria with $\omega_j \geq 0$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} \omega_j = 1$. The evaluation value of A_i on attribute C_j is represented by the decision matrix $Y = \left(y_{ij}\right)_{m \times n}$

Step 1: Normalize the decision matrix $Y = \left(y_{ij}\right)_{m \times n}$

Step 2: Calculate the positive ideal alternative y_j^+ and the negative ideal alternative y_j^- by score function

$$y_j^+ = \left\{\max y_{ij}\right\} = \left\{\max S\left(y_{ij}\right)\right\},$$
$$y_j^- = \left\{\min y_{ij}\right\} = \left\{\min S\left(y_{ij}\right)\right\}$$

Step 3: Compute the group utility values S_i and the individual regret values R_i $(i = 1, 2, \cdots, m)$.

$$S_i = \sum_{j=1}^{n} \omega_j \frac{d\left(y_{ij}^+ - y_{ij}\right)}{d\left(y_{ij}^- - y_{ij}^+\right)}, \quad R_i = \max_j \omega_j \frac{d\left(y_{ij}^+ - y_{ij}\right)}{d\left(y_{ij}^- - y_{ij}^+\right)}$$

Step 4: Calculate the values Q_i

$$Q_i = \varepsilon \frac{S_i - S^-}{S^+ - S^-} + (1-\varepsilon) \frac{R_i - R^-}{R^+ - R^-}$$

Where $S^+ = \max_i S_i$, $S^- = \min_i S_i$, $R^+ = \max_i R_i$, $R^- = \min_i R_i$ and ε represents the weight of the strategy of "the majority of criteria". In the comprehensive evaluation, the value of ε is determined according to the subjective tendency of the decision-maker. If the decision-maker pays more attention to group benefits, then $\varepsilon > 0.5$; if the decision-maker is focused more on individual regret minimization, then $\varepsilon < 0.5$; otherwise if the decision-maker pursues both the group benefit and the individual regret value minimum, then $\varepsilon = 0.5$.

Step 5: Sort the Q_i in ascending order

Step 6: Test the compromise solution

Condition 1: Acceptable advantage: $Q\left(A^2\right)\text{-}Q\left(A^1\right) \geq \frac{1}{m\text{-}1}$, where A^2 ranks second in the ordered list by Q;

Condition 2: Acceptable stability in the process of decision making

A^1 must be the best sorted by S or/and R. This compromise solution holds steady during the whole decision-making process.

A set of compromise solutions is obtained if it does not satisfy one of the following conditions:

(1) A^1 and A^2 are compromise solutions if only condition 2 is not satisfied; or

(2) $A^1, A^2, \ldots A^M$ are compromise solutions if condition 1 is not satisfied; and A^M is decided by the constraint $Q\left(A^M\right)\text{-}Q\left(A^1\right) \leq \frac{1}{m-1}$ for maximum M.

Definition 21 [16]: Liu thought that the traditional VIKOR method was not reasonable to consider only the closeness of the scheme to the positive ideal solution. So she proposed the VIKOR method based on the relative closeness coefficient. This method takes the closeness coefficient between alternatives and positive ideal solution as well as the closeness coefficient between alternatives and negative ideal solution into account and aims to obtain a relative optimal compromise solution through relative group utility and relative individual regret.

Different from the traditional VIKOR method, she computed the utility values S_i and the regret values $R_i\ (i = 1, 2, \cdots, m)$ by following formulations:

$$\Delta S_i = \sum_{j=1}^{n} \omega_j \frac{d\left(y_j^- - y_{ij}\right) - d\left(y_j^+ - y_{ij}\right)}{d\left(y_j^- - y_j^+\right)}$$

$$\Delta R_i = \max_j \omega_j \frac{d\left(y_j^- - y_{ij}\right) - d\left(y_j^+ - y_{ij}\right)}{d\left(y_j^- - y_j^+\right)}$$

Obviously, the bigger the ΔS_i and R_i, the bigger the Q_i, and the better alternative i.

3 | Interval Linguistic Neutrosophic Uncertain Linguistic Number and Interval Linguistic Neutrosophic Uncertain Linguistic Weighted Arithmetic Bonferroni Mean Operator

Definition 22: An interval linguistic neutrosophic uncertain linguistic set A in X can be defined as $A = \left\langle \left[s_{\theta(x)}, s_{\rho(x)} \right], \left(\left[s_{T_A^L(x)}, s_{T_A^U(x)} \right], \left[s_{I_A^L(x)}, s_{I_A^U(x)} \right], \left[s_{F_A^L(x)}, s_{F_A^U(x)} \right] \right) \right\rangle$, Where $s \in S$. The function $\left[s_{T_A^L(x)}, s_{T_A^U(x)} \right], \left[s_{I_A^L(x)}, s_{I_A^U(x)} \right]$ and $\left[s_{F_A^L(x)}, s_{F_A^U(x)} \right]$ are interval linguistic numbers and represent the membership degree, uncertainty degree, and non-membership degree respectively with interval values of the element x in X to the uncertain linguistic number $\left[s_{\theta(x)}, s_{\rho(x)} \right]$.

Definition 23: For any three interval linguistic neutrosophic uncertain linguistic numbers $a = \left\langle \left[s_{\theta(a)}, s_{\rho(a)} \right], \left(\left[s_{T_A^L(a)}, s_{T_A^U(a)} \right], \left[s_{I_A^L(a)}, s_{I_A^U(a)} \right], \left[s_{F_A^L(a)}, s_{F_A^U(a)} \right] \right) \right\rangle$, $a_1 = \left\langle \left[s_{\theta(a_1)}, s_{\rho(a_1)} \right], \left(\left[s_{T_A^L(a_1)}, s_{T_A^U(a_1)} \right], \left[s_{I_A^L(a_1)}, s_{I_A^U(a_1)} \right], \left[s_{F_A^L(a_1)}, s_{F_A^U(a_1)} \right] \right) \right\rangle$, $a_2 = \left\langle \left[s_{\theta(a_2)}, s_{\rho(a_2)} \right], \left(\left[s_{T_A^L(a_2)}, s_{T_A^U(a_2)} \right], \left[s_{I_A^L(a_2)}, s_{I_A^U(a_2)} \right], \left[s_{F_A^L(a_2)}, s_{F_A^U(a_2)} \right] \right) \right\rangle$ and $\lambda \geq C$, then the operational laws for interval linguistic neutrosophic uncertain linguistic numbers are as follows:

(1) $$a_1 \oplus a_2 = \left\langle \left[s_{\theta(a_1)+\theta(a_2)}, s_{\rho(a_1)+\rho(a_2)} \right], \left(\left[s_{T_A^L(a_1)+T_A^L(a_2)-T_A^L(a_1)T_A^L(a_2)}, s_{T_A^U(a_1)+T_A^U(a_2)-T_A^U(a_1)T_A^U(a_2)} \right], \left[s_{I_A^L(a_1)I_A^L(a_2)}, s_{I_A^U(a_1)I_A^U(a_2)} \right], \left[s_{F_A^L(a_1)F_A^L(a_2)}, s_{F_A^U(a_1)F_A^U(a_2)} \right] \right) \right\rangle$$

(2) $$a_1 \otimes a_2 = \left\langle \left[s_{\theta(a_1)\theta(a_2)}, s_{\rho(a_1)\rho(a_2)} \right], \left(\left[s_{T_A^L(a_1)T_A^L(a_2)}, s_{T_A^U(a_1)T_A^U(a_2)} \right], \left[s_{I_A^L(a_1)+I_A^L(a_2)-I_A^L(a_1)I_A^L(a_2)}, s_{I_A^U(a_1)+I_A^U(a_2)-I_A^U(a_1)I_A^U(a_2)} \right] \left[s_{F_A^L(a_1)+F_A^L(a_2)-F_A^L(a_1)F_A^L(a_2)}, s_{F_A^U(a_1)+F_A^U(a_2)-F_A^U(a_1)F_A^U(a_2)} \right] \right) \right\rangle$$

(3) $$\lambda \otimes a = \left\langle \left[s_{\lambda\theta(a)}, s_{\lambda\rho(a)} \right], \left(\left[s_{1-(1-T_A^L(a))^\lambda}, s_{1-(1-T_A^U(a))^\lambda} \right], \left[s_{\left(I_A^L(a)\right)^\lambda}, s_{\left(I_A^U(a)\right)^\lambda} \right], \left[s_{\left(F_A^L(a)\right)^\lambda}, s_{\left(F_A^U(a)\right)^\lambda} \right] \right) \right\rangle$$

(4) $$a^\lambda = \left\langle \left[s_{\theta^\lambda(a)}, s_{\rho^\lambda(a)} \right], \left(\left[s_{(T_A^L(a))^\lambda}, s_{(T_A^U(a))^\lambda} \right], \left[s_{1-(1-I_A^L(a))^\lambda}, s_{1-(1-I_A^U(a))^\lambda} \right], \left[s_{(1-(1-F_A^L(a))^\lambda}, s_{1-(1-F_A^U(a))^\lambda} \right] \right) \right\rangle$$

Definition 24: For any three interval linguistic neutrosophic uncertain linguistic numbers $a=\left\langle\left[s_{\theta(a)},s_{\rho(a)}\right],\left(\left[s_{T_A^L(a)},s_{T_A^U(a)}\right],\left[s_{I_A^L(a)},s_{I_A^U(a)}\right],\left[s_{F_A^L(a)},s_{F_A^U(a)}\right]\right)\right\rangle$,
$a_1=\left\langle\left[s_{\theta(a_1)},s_{\rho(a_1)}\right],\left(\left[s_{T_A^L(a_1)},s_{T_A^U(a_1)}\right],\left[s_{I_A^L(a_1)},s_{I_A^U(a_1)}\right],\left[s_{F_A^L(a_1)},s_{F_A^U(a_1)}\right]\right)\right\rangle$,
$a_2=\left\langle\left[s_{\theta(a_2)},s_{\rho(a_2)}\right],\left(\left[s_{T_A^L(a_2)},s_{T_A^U(a_2)}\right],\left[s_{I_A^L(a_2)},s_{I_A^U(a_2)}\right],\left[s_{F_A^L(a_2)},s_{F_A^U(a_2)}\right]\right)\right\rangle$ and $\lambda\geq 0$, then the operational properties for interval linguistic neutrosophic uncertain linguistic numbers are as follows:

(1) $a_1\oplus a_2=a_2\oplus a_1$

(2) $a_1\otimes a_2=a_2\otimes a_1$

(3) $\lambda(a_1\oplus a_2)=\lambda a_1\oplus\lambda a_2$

(4) $\lambda_1 a\oplus\lambda_2 a=(\lambda_1+\lambda_2)\,a$

(5) $a_1^{\lambda}\otimes a_2^{\lambda}=(a_1\otimes a_2)^{\lambda}$

(6) $a^{\lambda_1}\otimes a^{\lambda_2}=a^{(\lambda_1+\lambda_2)}$

Definition 25: For any two-interval linguistic neutrosophic uncertain linguistic numbers

$a_1=\left\langle\left[s_{\theta(a_1)},s_{\rho(a_1)}\right],\left(\left[s_{T_A^L(a_1)},s_{T_A^U(a_1)}\right],\left[s_{I_A^L(a_1)},s_{I_A^U(a_1)}\right],\left[s_{F_A^L(a_1)},s_{F_A^U(a_1)}\right]\right)\right\rangle$,
$a_2=\left\langle\left[s_{\theta(a_2)},s_{\rho(a_2)}\right],\left(\left[s_{T_A^L(a_2)},s_{T_A^U(a_2)}\right],\left[s_{I_A^L(a_2)},s_{I_A^U(a_2)}\right],\left[s_{F_A^L(a_2)},s_{F_A^U(a_2)}\right]\right)\right\rangle$, the Hamming distance between a_1 and a_2 can be defined as:

$$
\begin{aligned}
d(a_1,a_2)=\frac{1}{12(t-1)}\Big(&\left|\theta(a_1)T_A^L(a_1)-\theta(a_2)T_A^L(a_2)\right|+\left|\theta(a_1)T_A^U(a_1)-\theta(a_2)T_A^U(a_2)\right|+\left|\theta(a_1)I_A^L(a_1)-\theta(a_2)I_A^L(a_2)\right|+\left|\theta(a_1)I_A^U(a_1)-\theta(a_2)I_A^U(a_2)\right|+\\
&\left|\theta(a_1)F_A^L(a_1)-\theta(a_2)F_A^L(a_2)\right|+\left|\theta(a_1)F_A^U(a_1)-\theta(a_2)F_A^U(a_2)\right|+\left|\rho(a_1)T_A^L(a_1)-\rho(a_2)T_A^L(a_2)\right|+\left|\rho(a_1)T_A^U(a_1)-\rho(a_2)T_A^U(a_2)\right|+\\
&\left|\rho(a_1)I_A^L(a_1)-\rho(a_2)I_A^L(a_2)\right|+\left|\rho(a_1)I_A^U(a_1)-\rho(a_2)I_A^U(a_2)\right|+\left|\rho(a_1)F_A^L(a_1)-\rho(a_2)F_A^L(a_2)\right|+\left|\rho(a_1)F_A^U(a_1)-\rho(a_2)F_A^U(a_2)\right|\Big)
\end{aligned}
$$

Definition 26: For an interval linguistic neutrosophic uncertain linguistic number

$a=\left\langle\left[s_{\theta(a)},s_{\rho(a)}\right],\left(\left[s_{T_A^L(a)},s_{T_A^U(a)}\right],\left[s_{I_A^L(a)},s_{I_A^U(a)}\right],\left[s_{F_A^L(a)},s_{F_A^U(a)}\right]\right)\right\rangle$, then the score function a can be expressed by the equation below:

$$
S(a)=\frac{1}{12}\left(\theta(a)+\rho(a)\right)\left(4+T_A^L(a)-I_A^L(a)-F_A^L(a)+T_A^U(a)-I_A^U(a)-F_A^U(a)\right)
$$

Definition 27: Let $p,q\geq 0$, $a_i=\left\langle\left[s_{\theta(a_i)},s_{\rho(a_i)}\right],\left(\left[s_{T_A^L(a_i)},s_{T_A^U(a_i)}\right],\left[s_{I_A^L(a_i)},s_{I_A^U(a_i)}\right],\left[s_{F_A^L(a_i)},s_{F_A^U(a_i)}\right]\right)\right\rangle$ be a set of interval linguistic neutrosophic uncertain linguistic numbers, and $\omega=(\omega_1,\omega_2,...\omega_n)^T$ be the weight vector of a_i, $\omega_i\geq 0$ (j = 1, 2,⋯, n) and $\sum_{j=1}^{n}\omega_i=1$. Then the aggregated result by interval linguistic neutrosophic uncertain linguistic weighted arithmetic Bonferroni mean (ILNULWABM) operator can be expressed as:

$$ILNULWABM_{s_\omega}^{p,q}\left(a_1,a_2,\ldots a_n\right)=\left(\frac{1}{n(n\text{-}1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\left(\omega_i a_i\right)^p\left(\omega_j a_j\right)^q\right)^{\frac{1}{p+q}}$$

$$=\left\langle\left[s_{\left(\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(\omega_i\theta_i)^p(\omega_j\theta_j)^q\right)^{\frac{1}{p+q}}},s_{\left(\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(\omega_i\rho_i)^p(\omega_j\rho_j)^q\right)^{\frac{1}{p+q}}}\right],\right.$$

$$\left(\left[s_{\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\left(1-\left(\omega_i T_A^L(a_i)\right)^p\left(\omega_j T_A^L(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}},s_{\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\left(1-\left(\omega_i T_A^U(a_i)\right)^p\left(\omega_j T_A^U(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}}\right],\right.$$

$$\left[s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-I_A^L(a_i)+\omega_i I_A^L(a_i)\right)^p\left(1-\omega_j-I_A^L(a_j)+\omega_j I_A^L(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}},s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-I_A^U(a_i)+\omega_i I_A^U(a_i)\right)^P\left(1-\omega_j-I_A^U(a_j)+\omega_j I_A^U(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}}\right],$$

$$\left.\left.\left[s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-F_A^L(a_i)+\omega_i F_A^L(a_i)\right)^p\left(1-\omega_j-F_A^L(a_j)+\omega_j F_A^L(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}},s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-F_A^U(a_i)+\omega_i F_A^U(a_i)\right)^P\left(1-\omega_j-F_A^U(a_j)+\omega_j F_A^U(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}}\right]\right)\right\rangle$$

Proof.

Since

$$s_{\omega_i}a_i=\left\langle\left[s_{\omega_i\theta_i},s_{\omega_i\rho_i}\right],\left(\left[s_{\omega_i T_A^L(a_i)},s_{\omega_i T_A^U(a_i)}\right]\left[s_{\omega_i+I_A^L(a_i)-\omega_i I_A^L(a_i)},s_{\omega_i+I_A^U(a_i)-\omega_i I_A^U(a_i)}\right],\right.\right.$$
$$\left.\left.\left[s_{\omega_i+F_A^L(a_i)-\omega_i F_A^L(a_i)},s_{\omega_i+F_A^U(a_i)-\omega_i F_A^U(a_i)}\right]\right)\right\rangle,$$

$$s_{\omega_j}a_j=\left\langle\left[s_{\omega_j\theta_j},s_{\omega_j\rho_j}\right],\left(\left[s_{\omega_j T_A^L(a_j)},s_{\omega_j T_A^U(a_j)}\right]\left[s_{\omega_j+I_A^L(a_j)-\omega_j I_A^L(a_j)},s_{\omega_j+I_A^U(a_j)-\omega_j I_A^U(a_j)}\right],\right.\right.$$
$$\left.\left.\left[s_{\omega_j+F_A^L(a_j)-\omega_j F_A^L(a_j)},s_{\omega_j+F_A^U(a_j)-\omega_j F_A^U(a_j)}\right]\right)\right\rangle$$

and

$$\left(s_{\omega_i}a_i\right)^p=\left\langle\left[s_{(\omega_i\theta_i)^p},s_{(\omega_i\rho_i)^p}\right],\left(\left[s_{\left(\omega_i T_A^L(a_i)\right)^p},s_{\left(\omega_i T_A^U(a_i)\right)^p}\right],\left[s_{1-\left(1-\omega_i-I_A^L(a_i)+\omega_i I_A^L(a_i)\right)^p},s_{1-\left(1-\omega_i-I_A^U(a_i)+\omega_i I_A^U(a_i)\right)^P}\right],\right.\right.$$
$$\left.\left.\left[s_{1-\left(1-\omega_i-F_A^L(a_i)+\omega_i F_A^L(a_i)\right)^p},s_{1-\left(1-\omega_i-F_A^U(a_i)+\omega_i F_A^U(a_i)\right)^P}\right]\right)\right\rangle,$$

$$\left(s_{\omega_j}a_j\right)^q=\left\langle\left[s_{(\omega_j\theta_j)^q},s_{(\omega_j\rho_j)^q}\right],\left(\left[s_{\left(\omega_j T_A^L(a_j)\right)^q},s_{\left(\omega_j T_A^U(a_j)\right)^q}\right],\left[s_{1-\left(1-\omega_j-I_A^L(a_j)+\omega_j I_A^L(a_j)\right)^q},s_{1-\left(1-\omega_j-I_A^U(a_j)+\omega_j I_A^U(a_j)\right)^q}\right]\right.\right.$$
$$\left.\left.\left[s_{1-\left(1-\omega_j-F_A^L(a_j)+\omega_j F_A^L(a_j)\right)^q},s_{1-\left(1-\omega_j-F_A^U(a_j)+\omega_j F_A^U(a_j)\right)^q}\right]\right)\right\rangle,$$

Then

$$\left(s_{\omega_i}a_i\right)^p \times \left(s_{\omega_j}a_j\right)^q = \left\langle \left[s_{(\omega_i\theta_i)^p(\omega_j\theta_j)^q}, s_{(\omega_i\rho_i)^p(\omega_j\rho_j)^q} \right], \left(\left[s_{\left(\omega_i T_A^L(a_i)\right)^p\left(\omega_j T_A^L(a_j)\right)^q}, \left[s_{\left(\omega_i T_A^U(a_i)\right)^p\left(\omega_j T_A^U(a_j)\right)^q} \right], \right.\right.$$

$$\left[s_{1-\left(1-\omega_i - I_A^L(a_i)+\omega_i I_A^L(a_i)\right)^p\left(1-\omega_j - I_A^L(a_j)+\omega_j I_A^L(a_j)\right)^q}, s_{1-\left(1-\omega_i - I_A^U(a_i)+\omega_i I_A^U(a_i)\right)^P\left(1-\omega_j - I_A^U(a_j)+\omega_j I_A^U(a_j)\right)^q} \right],$$

$$\left.\left. \left[s_{1-\left(1-\omega_i - F_A^L(a_i)+\omega_i F_A^L(a_i)\right)^p\left(1-\omega_j - F_A^L(a_j)+\omega_j F_A^L(a_j)\right)^q}, s_{1-\left(1-\omega_i - F_A^U(a_i)+\omega_i F_A^U(a_i)\right)^P\left(1-\omega_j - F_A^U(a_j)+\omega_j F_A^U(a_j)\right)^q} \right] \right) \right\rangle$$

So

$$\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\left(s_{\omega_i}a_i\right)^p\left(s_{\omega_j}a_j\right)^q = \left\langle \left[s_{\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(\omega_i\theta_i)^p(\omega_j\theta_j)^q}, s_{\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(\omega_i\rho_i)^p(\omega_j\rho_j)^q} \right], \right.$$

$$\left(\left[s_{1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\left(1-\left(\omega_i T_A^L(a_i)\right)^p\left(\omega_j T_A^L(a_j)\right)^q\right)}, s_{1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\left(1-\left(\omega_i T_A^U(a_i)\right)^p\left(\omega_j T_A^U(a_j)\right)^q\right)} \right], \left[s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i - I_A^L(a_i)+\omega_i I_A^L(a_i)\right)^p\left(1-\omega_j - I_A^L(a_j)+\omega_j I_A^L(a_j)\right)^q\right)}, \right.\right.$$

$$\left. s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i - I_A^U(a_i)+\omega_i I_A^U(a_i)\right)^P\left(1-\omega_j - I_A^U(a_j)+\omega_j I_A^U(a_j)\right)^q\right)} \right], \left[s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i - F_A^L(a_i)+\omega_i F_A^L(a_i)\right)^p\left(1-\omega_j - F_A^L(a_j)+\omega_j F_A^L(a_j)\right)^q\right)}, \right.$$

$$\left.\left.\left. s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i - F_A^U(a_i)+\omega_i F_A^U(a_i)\right)^P\left(1-\omega_j - F_A^U(a_j)+\omega_j F_A^U(a_j)\right)^q\right)} \right] \right) \right\rangle$$

Finally, we can get

$$ILNULWABM_{s_\omega}^{p,q}(a_1,a_2,\ldots a_n) = \left(\frac{1}{n(n\text{-}1)} \sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n} (\omega_i a_i)^p (\omega_j a_j)^q \right)^{\frac{1}{p+q}}$$

$$= \left\langle \left[s_{\left(\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(\omega_i\theta_i)^p(\omega_j\theta_j)^q\right)^{\frac{1}{p+q}}}, s_{\left(\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(\omega_i\rho_i)^p(\omega_j\rho_j)^q\right)^{\frac{1}{p+q}}} \right], \right.$$

$$\left(\left[s_{\left(1-\prod_{i=1}^{n}\prod_{\substack{i=1\\ i\neq j}}^{n}\left(1-\left(\omega_i T_A^L(a_i)\right)^P\left(\omega_j T_A^L(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}}, s_{\left(1-\prod_{i=1}^{n}\prod_{\substack{i=1\\ i\neq j}}^{n}\left(1-\left(\omega_i T_A^L(a_i)\right)^P\left(\omega_j T_A^L(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}} \right], \right.$$

$$\left[s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i - I_A^L(a_i)+\omega_i I_A^L(a_i)\right)^p\left(1-\omega_j - I_A^L(a_j)+\omega_j I_A^L(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}}, s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i - I_A^U(a_i)+\omega_i I_A^U(a_i)\right)^P\left(1-\omega_j - I_A^U(a_j)+\omega_j I_A^U(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}} \right],$$

$$\left.\left. \left[s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i - F_A^L(a_i)+\omega_i F_A^L(a_i)\right)^P\left(1-\omega_j - F_A^L(a_j)+\omega_j F_A^L(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}}, s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i - F_A^U(a_i)+\omega_i F_A^U(a_i)\right)^P\left(1-\omega_j - F_A^U(a_j)+\omega_j F_A^U(a_j)\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}} \right] \right) \right\rangle$$

Next, some special cases of the ILNULWABM operator concerning the parameters p and q will be demonstrated respectively.

(1) When p=0 and q=0, then

$$ILNULWABM_{s_\omega}^{p=1,q=0}\left(s_1,s_2,\dots s_n\right)$$

$$=\frac{1}{n(n\text{-}1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\omega_i a_i=\left\langle\left[s_{\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\omega_i\theta_i},s_{\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\omega_i\rho_i}\right],\right.$$

$$\left(\left[s_{1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\left(1-\omega_i T_A^L(a_i)\right)^{\frac{1}{n(n-1)}}},s_{1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\left(1-\omega_i T_A^U(a_i)\right)^{\frac{1}{n(n-1)}}}\right],\left[s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(\omega_i+I_A^L(a_i)-\omega_i I_A^L(a_i)\right)^{\frac{1}{n(n-1)}}},\right.\right.$$

$$\left.\left.\left.s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(\omega_i+I_A^U(a_i)\text{-}\omega_i I_A^U(a_i)\right)^{\frac{1}{n(n-1)}}}\right],\left[s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(\omega_i+F_A^L(a_i)-\omega_i F_A^L(a_i)\right)^{\frac{1}{n(n-1)}}},s_{\left(\omega_i+F_A^U(a_i)-\omega_i F_A^U(a_i)\right)^{\frac{1}{n(n-1)}}}\right]\right)\right\rangle$$

(2) When p=1 and q=1, then

$$ILNULWABM_{s_\omega}^{p=q=1}\left(s_1,s_2,\dots s_n\right)$$

$$=\left(\frac{1}{n(n\text{-}1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\omega_i\omega_j a_i a_j\right)^{\frac{1}{2}}=\left\langle\left[s_{\left(\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\omega_i\omega_j\theta_i\theta_j\right)^{\frac{1}{2}}},s_{\left(\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\omega_i\omega_j\rho_i\rho_j\right)^{\frac{1}{2}}}\right],\right.$$

$$\left(\left[s_{\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\left(1-\omega_i\omega_j T_A^L(a_i)T_A^L(a_j)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}},s_{\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\left(1-\omega_i\omega_j T_A^U(a_i)T_A^U(a_j)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}}\right],\right.$$

$$\left[s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-I_A^L(a_i)+\omega_i I_A^L(a_i)\right)\left(1-\omega_j-I_A^L(a_j)+\omega_j I_A^L(a_j)\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}},\right.$$

$$\left.s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-I_A^U(a_i)+\omega_i I_A^U(a_i)\right)\left(1-\omega_j-I_A^U(a_j)+\omega_j I_A^U(a_j)\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}}\right],\left[s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-F_A^L(a_i)+\omega_i F_A^L(a_i)\right)\left(1-\omega_j-F_A^L(a_j)+\omega_j F_A^L(a_j)\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}},\right.$$

$$\left.\left.\left.s_{1-\left(1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-F_A^U(a_i)+\omega_i F_A^U(a_i)\right)\left(1-\omega_j-F_A^U(a_j)+\omega_j F_A^U(a_j)\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}}\right]\right)\right\rangle$$

(3) When p=0.5 and q=0.5, then

$$ILNULWABM_{s_{\omega}}^{p=q=0.5}\left(s_1,s_2,\ldots s_n\right)$$

$$=\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\left(\omega_i a_i\right)^{0.5}\left(\omega_j a_j\right)^{0.5}=\left\langle\left[s_{\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(\omega_i\theta_i)^{0.5}(\omega_j\theta_j)^{0.5}},s_{\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(\omega_i\rho_i)^{0.5}(\omega_j\rho_j)^{0.5}}\right],\right.$$

$$\left(\left[s_{1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\left(1-\left(\omega_i T_A^L(a_i)\right)^{0.5}\left(\omega_j T_A^L(a_j)\right)^{0.5}\right)^{\frac{1}{n(n-1)}}},s_{1-\prod_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\left(1-\left(\omega_i T_A^U(a_i)\right)^{0.5}\left(\omega_j T_A^U(a_j)\right)^{0.5}\right)^{\frac{1}{n(n-1)}}}\right],\right.$$

$$\left[s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-I_A^L(a_i)+\omega_i I_A^L(a_i)\right)^{0.5}\left(1-\omega_j-I_A^L(a_j)+\omega_j I_A^L(a_j)\right)^{0.5}\right)^{\frac{1}{n(n-1)}}},s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-I_A^U(a_i)+\omega_i I_A^U(a_i)\right)^{0.5}\left(1-\omega_j-I_A^U(a_j)+\omega_j I_A^U(a_j)\right)^{0.5}\right)^{\frac{1}{n(n-1)}}}\right],$$

$$\left.\left.\left[s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-F_A^L(a_i)+\omega_i F_A^L(a_i)\right)^{0.5}\left(1-\omega_j-F_A^L(a_j)+\omega_j F_A^L(a_j)\right)^{0.5}\right)^{\frac{1}{n(n-1)}}},s_{\prod_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\left(1-\left(1-\omega_i-F_A^U(a_i)+\omega_i F_A^U(a_i)\right)^{0.5}\left(1-\omega_j-F_A^U(a_j)+\omega_j F_A^U(a_j)\right)^{0.5}\right)^{\frac{1}{n(n-1)}}}\right]\right)\right\rangle$$

4 | The VIKOR Method Based on Relative Closeness Coefficient under ILNULN and ILNULWABM Operator

For a group decision-making problem, there is a discrete set of alternatives $A=\{A_1,A_2,...,A_m\}$ and n attributes $C=\{C_1,C_2,...C_n\}$, and the attribute weight vector of C is $\omega=(\omega_1,\omega_2,...\omega_n)^T$. There are λ decision-makers $D=\{D_1,D_2,...D_\lambda\}$ assess this problem and the relative importance vector is $W=(W_1,W_2,...W_\lambda)^T$.

For the decision maker D^k, the evaluation value of A_i on attribute C_j is represented by the decision matrix $R^k=(r_{ij}^k)_{m\times n}$, where $r_{ij}^k=\left\langle\left[s_{\theta(r_{ij}^k)},s_{\rho(r_{ij}^k)}\right],\left(\left[s_{T^L(r_{ij}^k)},s_{T^U(r_{ij}^k)}\right],\left[s_{I^L(r_{ij}^k)},s_{I^U(r_{ij}^k)}\right],\left[s_{F^L(r_{ij}^k)},s_{F^U(r_{ij}^k)}\right]\right)\right\rangle$.

The steps of the VIKOR method based on the relative closeness coefficient under interval linguistic neutrosophic uncertain linguistic numbers and ILNULWABM operators are shown as follows:

Step 1: Normalize the decision matrix $R^k=(r_{ij}^k)_{m\times n}$.

First, the decision-making information r_{ij}^k in the matrix $R^k=(r_{ij}^k)_{m\times n}$ must be normalized. The normalized matrix F of the decision matrix can be expressed as:

$$F^k=\left(f_{ij}^k\right)_{m\times n},\ f_{ij}^k=\frac{r_{ij}^k}{\sum_{i=1}^{m}\left(r_{ij}^k\right)^2}\left(i=1,2,\ldots m;\ j=1,2,\ldots n\right)$$

Step 2: Aggregate information from each decision-maker.

Different decision-makers give different evaluation information. To aggregate the evaluation values of experts, we can use the ILNULWABM operator to aggregate the evaluation information matrix F^k given by the decision maker D^k to obtain the integration matrix F.

$$F=\left(f_{ij}\right)_{m\times n},\ f_{ij}=ILNULWABM\left(f_{ij}^1,f_{ij}^2,\ldots f_{ij}^\lambda\right)$$

Step 3: Calculate the positive ideal alternative f_{ij}^+ and the negative ideal alternative f_{ij}^-.

We can use the score function to obtain the positive ideal alternative and the negative ideal alternative:

$$f_j^+=\{\max f_{ij}\}=\{\max S(f_{ij})\},\ f_j^-=\{\min f_{ij}\}=\{\min S(f_{ij})\}$$

$$S(f_{ij})=\frac{1}{12}\left(\theta(f_{ij})+\rho(f_{ij})\right)\left(4+T_A^L(f_{ij})-I_A^L(f_{ij})-F_A^L(f_{ij})+T_A^U(f_{ij})-I_A^U(f_{ij})-F_A^U(f_{ij})\right)$$

Step 4: Compute the group utility values ΔS_i and individual regret values ΔR_i .

$$\Delta S_i=\sum_{j=1}^{n}\omega_j\frac{d(f_j^- - f_{ij})-d(f_j^+ - f_{ij})}{d(f_j^+ - f_j^-)}$$

$$\Delta R_i=\max_j \omega_j\frac{d(f_j^- - f_{ij})-d(f_j^+ - f_{ij})}{d(f_j^+ - f_j^-)}$$

Step 5: Compute the values Q_i .

$$\Delta Q_i=\varepsilon\frac{\Delta S_i-\Delta S^-}{\Delta S^+-\Delta S^-}+(1-\varepsilon)\frac{\Delta R_i-\Delta R^-}{\Delta R^+-\Delta R^-}$$

Where $\Delta S^+=\max_i \Delta S_i$, $\Delta S^-=\min_i \Delta S_i$, $\Delta R^+=\max_i \Delta R_i$, $\Delta R^-=\min_i \Delta R_i$ and ε represents the weight of the strategy of the "the majority of criteria".

Step 6: Sort the ΔQ_i in descending order.

Step 7: Test the compromise solution.

5 | A Numerical Example

Now we consider a group decision-making problem. Suppose there are four alternatives labeled A_1, A_2, A_3, A_4 and three attributes labeled C_1, C_2, C_3 whose weight vector is $\omega=(s_3, s_4, s_7)^T$. Three decision-makers assess this problem and the relative importance vector $W=(S_4, S_6, S_2)^T$. Here, we let $S=\{s_i \mid i=0,1,2,...8\}$ where s_i represent a possible value for a linguistic number, and

$$\begin{aligned}S=\{s_0,s_1,s_2,s_3,s_4,s_5,s_6,s_7,s_8\}=\{&s_0=\text{extremely poor}, s_1=\text{very poor}, s_2=\text{poor},\\ &s_3=\text{a little poor}, s_4=\text{medium}, s_5=\text{a little good},\\ &s_6=\text{good}, s_7=\text{very good}, s_8=\text{excellent}\}\end{aligned}$$

The decision-makers assign values to the alternatives through the interval linguistic neutrosophic uncertain linguistic numbers to form three decision matrices, as shown in Tables 1-3.

Step 1: Normalize the decision matrix R^k . Firstly, we should normalize the original data utilized. The normalized decision matrices are shown in Tables 4-6.

Step 2: Aggregate information from each expert. For easy calculation, we should make the weights normalized by the formula

$$\omega_j^{'}=\frac{\omega_j}{\sum_{j=1}^{3}(\omega_j)}, W_k^{'}=\frac{W^k}{\sum_{k=1}^{3}(W^k)}.$$

Through calculation, we can get the normalized weight vectors: $\omega^{'}=(s_{0.21}, s_{0.29}, s_{0.5})^T$, $W^{'}=(s_{0.33}, s_{0.5}, s_{0.17})^T$.

Next, we use the ILNULWABM operators to gather decision information from all decision-makers. The specific calculation method is shown in Definition 27. Here we let p=1 and q=1. The group decision matrix we obtained is shown in Table 7.

Table 1. Decision matrix R1 of the decision-maker D^1 .

	C$_1$	**C$_2$**	**C$_3$**
A$_1$	$<(S_3,S_4),([S_6,S_7][S_2,S_4][S_0,S_1])>$	$<(S_5,S_6),([S_5,S_6][S_2,S_3][S_1,S_2])>$	$<(S_4,S_5),([S_4,S_5][S_2,S_3][S_3,S_4])>$
A$_2$	$<(S_4,S_5),([S_5,S_6][S_4,S_5][S_2,S_3])>$	$<(S_4,S_5),([S_4,S_5][S_3,S_4][S_2,S_3])>$	$<(S_7,S_8),([S_5,S_6][S_1,S_2][S_2,S_3])>$
A$_3$	$<(S_7,S_8),([S_4,S_5][S_2,S_3][S_3,S_4])>$	$<(S_3,S_4),([S_6,S_7][S_2,S_4][S_1,S_2])>$	$<(S_3,S_4),([S_4,S_5][S_3,S_4][S_3,S_4])>$
A$_4$	$<(S_5,S_6),([S_5,S_6][S_2,S_3][S_2,S_3])>$	$<(S_6,S_7),([S_3,S_4][S_2,S_4][S_4,S_5])>$	$<(S_7,S_8),([S_7,S_8][S_2,S_3][S_0,S_1])>$

Table 2. Decision matrix R2 of the decision-maker D^2 .

	C$_1$	**C$_2$**	**C$_3$**
A$_1$	$<(S_3,S_4),([S_7,S_8][S_3,S_4][S_1,S_2])>$	$<(S_4,S_5),([S_5,S_6][S_2,S_3][S_2,S_3])>$	$<(S_2,S_3),([S_5,S_6][S_3,S_4][S_2,S_3])>$
A$_2$	$<(S_6,S_7),([S_4,S_5][S_3,S_4][S_2,S_3])>$	$<(S_7,S_8),([S_5,S_7][S_3,S_4][S_1,S_2])>$	$<(S_7,S_8),([S_4,S_5][S_2,S_3][S_3,S_4])>$
A$_3$	$<(S_2,S_3),([S_6,S_7][S_1,S_2][S_0,S_1])>$	$<(S_3,S_4),([S_4,S_5][S_2,S_3][S_3,S_4])>$	$<(S_5,S_6),([S_6,S_7][S_3,S_4][S_0,S_1])>$
A$_4$	$<(S_7,S_8),([S_5,S_6][S_2,S_3][S_2,S_3])>$	$<(S_5,S_6),([S_6,S_7][S_3,S_4][S_1,S_2])>$	$<(S_4,S_5),([S_5,S_6][S_2,S_3][S_2,S_3])>$

Table 3. Decision matrix R3 of the decision-maker D^3 .

	C$_1$	**C$_2$**	**C$_3$**
A$_1$	$<(S_4,S_5),([S_6,S_7][S_2,S_3][S_1,S_2])>$	$<(S_4,S_5),([S_5,S_6][S_2,S_3][S_1,S_2])>$	$<(S_2,S_3),([S_6,S_7][S_2,S_4][S_1,S_2])>$
A$_2$	$<(S_3,S_4),([S_5,S_6][S_1,S_2][S_2,S_3])>$	$<(S_6,S_7),([S_7,S_8][S_1,S_2][S_0,S_1])>$	$<(S_5,S_6),([S_5,S_6][S_2,S_3][S_3,S_4])>$
A$_3$	$<(S_2,S_3),([S_4,S_5][S_2,S_3][S_3,S_4])>$	$<(S_2,S_3),([S_6,S_7][S_2,S_3][S_1,S_2])>$	$<(S_3,S_4),([S_5,S_6][S_3,S_4][S_2,S_3])>$
A$_4$	$<(S_6,S_7),([S_7,S_8][S_2,S_4][S_0,S_1])>$	$<(S_5,S_6),([S_5,S_6][S_2,S_4][S_2,S_3])>$	$<(S_7,S_8),([S_4,S_5][S_2,S_3][S_3,S_4])>$

Table 4. Normalized decision matrix F1 of the decision-maker D^1 .

	C$_1$	**C$_2$**
A$_1$	$<(S_{0.3015},S_{0.3369}),([S_{0.5941},S_{0.5793}][S_{0.3780},S_{0.5208}][S_{0.0000},S_{0.1690}])>$	$<(S_{0.5392},S_{0.5345}),([S_{0.5392},S_{0.5345}][S_{0.4364},S_{0.3974}][S_{0.2132},S_{0.3086}])>$
A$_2$	$<(S_{0.4020},S_{0.4211}),([S_{0.4951},S_{0.4966}][S_{0.7559},S_{0.6509}][S_{0.4851},S_{0.5071}])>$	$<(S_{0.4313},S_{0.4454}),([S_{0.4313},S_{0.4454}][S_{0.6547},S_{0.5298}][S_{0.4264},S_{0.4629}])>$
A$_3$	$<(S_{0.7035},S_{0.6737}),([S_{0.3961},S_{0.4138}][S_{0.3780},S_{0.3906}][S_{0.7276},S_{0.6761}])>$	$<(S_{0.3235},S_{0.3563}),([S_{0.6470},S_{0.6236}][S_{0.4364},S_{0.5298}][S_{0.2132},S_{0.3086}])>$
A$_4$	$<(S_{0.5025},S_{0.5253}),([S_{0.4951},S_{0.4966}][S_{0.3780},S_{0.3906}][S_{0.4851},S_{0.5071}])>$	$<(S_{0.6470},S_{0.6236}),([S_{0.3235},S_{0.3563}][S_{0.4364},S_{0.5298}][S_{0.8528},S_{0.7715}])>$
	C$_3$	
A$_1$	$<(S_{0.3607},S_{0.3846}),([S_{0.3885},S_{0.4082}][S_{0.4714},S_{0.4867}][S_{0.6396},S_{0.6172}])>$	
A$_2$	$<(S_{0.6312},S_{0.6154}),([S_{0.4856},S_{0.4899}][S_{0.2357},S_{0.3244}][S_{0.4264},S_{0.4629}])>$	
A$_3$	$<(S_{0.2705},S_{0.3077}),([S_{0.3885},S_{0.4082}][S_{0.7071},S_{0.6489}][S_{0.6396},S_{0.6172}])>$	
A$_4$	$<(S_{0.6312},S_{0.6154}),([S_{0.6799},S_{0.6532}][S_{0.4714},S_{0.4867}][S_{0.0000},S_{0.1543}])>$	

Table 5. Normalized decision matrix F2 of the decision-maker D^2

	C$_1$	**C$_2$**
A$_1$	$<(S_{0.3030},S_{0.3405}),([S_{0.6236},S_{0.6065}][S_{0.6255},S_{0.5963}][S_{0.3333},S_{0.4170}])>$	$<(S_{0.4020},S_{0.4211}),([S_{0.4951},S_{0.4758}][S_{0.3922},S_{0.4243}][S_{0.5164},S_{0.5222}])>$
A$_2$	$<(S_{0.6061},S_{0.5959}),([S_{0.3563},S_{0.3790}][S_{0.6255},S_{0.5963}][S_{0.6667},S_{0.6255}])>$	$<(S_{0.7035},S_{0.6737}),([S_{0.4951},S_{0.5551}][S_{0.5883},S_{0.5657}][S_{0.2582},S_{0.3482}])>$
A$_3$	$<(S_{0.2020},S_{0.2554}),([S_{0.5345},S_{0.5307}][S_{0.2085},S_{0.2981}][S_{0.0000},S_{0.2085}])>$	$<(S_{0.3015},S_{0.3369}),([S_{0.3961},S_{0.3965}][S_{0.3922},S_{0.4243}][S_{0.7746},S_{0.6963}])>$
A$_4$	$<(S_{0.7071},S_{0.6810}),([S_{0.4454},S_{0.4549}][S_{0.4170},S_{0.4472}][S_{0.6667},S_{0.6255}])>$	$<(S_{0.5025},S_{0.5053}),([S_{0.5941},S_{0.5551}][S_{0.5883},S_{0.5657}][S_{0.2582},S_{0.3482}])>$
	C$_3$	
A$_1$	$<(S_{0.2063},S_{0.2592}),([S_{0.4951},S_{0.4966}][S_{0.5883},S_{0.5657}][S_{0.4851},S_{0.5071}])>$	
A$_2$	$<(S_{0.7220},S_{0.6911}),([S_{0.3961},S_{0.4138}][S_{0.3922},S_{0.4243}][S_{0.7276},S_{0.6761}])>$	
A$_3$	$<(S_{0.5157},S_{0.5183}),([S_{0.5941},S_{0.5793}][S_{0.5883},S_{0.5657}][S_{0.0000},S_{0.1690}])>$	
A$_4$	$<(S_{0.4126},S_{0.4319}),([S_{0.4951},S_{0.4966}][S_{0.3922},S_{0.4243}][S_{0.4851},S_{0.5071}])>$	

Table 6. Normalized decision matrix F3 of the decision- maker D^3.

	C1	**C2**
A1	$<(S_{0.4961},S_{0.5025}),([S_{0.5345},S_{0.5307}][S_{0.5547},S_{0.4867}][S_{0.2673},S_{0.3651}])>$	$<(S_{0.4444},S_{0.4583}),([S_{0.4303},S_{0.4411}][S_{0.5547},S_{0.4867}][S_{0.4082},S_{0.4714}])>$
A2	$<(S_{0.3721},S_{0.4020}),([S_{0.4454},S_{0.4549}][S_{0.2774},S_{0.3244}][S_{0.5345},S_{0.5477}])>$	$<(S_{0.6667},S_{0.6417}),([S_{0.6025},S_{0.5882}][S_{0.2774},S_{0.3244}][S_{0.0000},S_{0.2357}])>$
A3	$<(S_{0.2481},S_{0.3015}),([S_{0.3563},S_{0.3790}][S_{0.5547},S_{0.4867}][S_{0.8018},S_{0.7303}])>$	$<(S_{0.2222},S_{0.2750}),([S_{0.5164},S_{0.5147}][S_{0.5547},S_{0.4867}][S_{0.4082},S_{0.4714}])>$
A4	$<(S_{0.7442},S_{0.7035}),([S_{0.6236},S_{0.6065}][S_{0.5547},S_{0.6489}][S_{0.0000},S_{0.1826}])>$	$<(S_{0.5556},S_{0.5500}),([S_{0.4303},S_{0.4411}][S_{0.5547},S_{0.6489}][S_{0.8165},S_{0.7071}])>$
	C3	
A1	$<(S_{0.2144},S_{0.2683}),([S_{0.5941},S_{0.5793}][S_{0.4364},S_{0.5657}][S_{0.2085},S_{0.2981}])>$	
A2	$<(S_{0.5361},S_{0.5367}),([S_{0.4951},S_{0.4966}][S_{0.4364},S_{0.4243}][S_{0.6255},S_{0.5963}])>$	
A3	$<(S_{0.3216},S_{0.3578}),([S_{0.4951},S_{0.4966}][S_{0.6547},S_{0.5657}][S_{0.4170},S_{0.4472}])>$	
A4	$<(S_{0.7505},S_{0.7155}),([S_{0.3961},S_{0.4138}][S_{0.4364},S_{0.4243}][S_{0.6255},S_{0.5963}])>$	

Table 7. Group decision matrix F.

	C1	**C2**
A1	$<(S_{0.1099},S_{0.1197}),([S_{0.1896},S_{0.1853}],[S_{0.6818},S_{0.6896}],[S_{0.4670},S_{0.5464}])>$	$<(S_{0.1465},S_{0.1498}),([S_{0.1586},S_{0.1563}],[S_{0.6503},S_{0.6294}],[S_{0.5865},S_{0.6248}])>$
A2	$<(S_{0.1503},S_{0.1541}),([S_{0.1355},S_{0.1396}],[S_{0.7154},S_{0.6871}],[S_{0.7072},S_{0.7071}])>$	$<(S_{0.1880},S_{0.1847}),([S_{0.1584},S_{0.1668}],[S_{0.6773},S_{0.6487}],[S_{0.4895},S_{0.5693}])>$
A3	$<(S_{0.1118},S_{0.1243}),([S_{0.1405},S_{0.1441}],[S_{0.6062},S_{0.6054}],[S_{0.7388},S_{0.7270}])>$	$<(S_{0.0928},S_{0.1052}),([S_{0.1619},S_{0.1595}],[S_{0.6503},S_{0.6594}],[S_{0.6394},S_{0.6598}])>$
A4	$<(S_{0.2036},S_{0.2004}),([S_{0.1606},S_{0.1610}],[S_{0.6423},S_{0.6771}],[S_{0.5923},S_{0.6255}])>$	$<(S_{0.1803},S_{0.1778}),([S_{0.1431},S_{0.1442}],[S_{0.6865},S_{0.7269}],[S_{0.8098},S_{0.7642}])>$
	C3	
A1	$<(S_{0.0824},S_{0.0969}),([S_{0.1525},S_{0.1541}],[S_{0.6646},S_{0.6957}],[S_{0.6377},S_{0.6542}])>$	
A2	$<(S_{0.2058},S_{0.2002}),([S_{0.1437},S_{0.1468}],[S_{0.5754},S_{0.5971}],[S_{0.7316},S_{0.7203}])>$	
A3	$<(S_{0.1183},S_{0.1268}),([S_{0.1571},S_{0.1582}],[S_{0.7722},S_{0.7326}],[S_{0.5976},S_{0.6270}])>$	
A4	$<(S_{0.1788},S_{0.1778}),([S_{0.1711},S_{0.1699}],[S_{0.6274},S_{0.6339}],[S_{0.6059},S_{0.6279}])>$	

Step 3: Calculate the positive ideal alternative f_{ij}^{+} and the negative ideal alternative f_{ij}^{-}.

We can use the score function to obtain the positive ideal alternative and the negative ideal alternative. According to the score function $S(f_{ij})=\frac{1}{12}\left(\theta(f_{ij})+\rho(f_{ij})\right)\left(4+T_A^L(f_{ij})-I_A^L(f_{ij})-F_A^L(f_{ij})+T_A^U(f_{ij})-I_A^U(f_{ij})-F_A^U(f_{ij})\right)$, we can get the scores as follows:

$$S(f_{11})=0.0381,\ S(f_{12})=0.045,\ S(f_{13})=0.0247$$
$$S(f_{21})=0.037,\ S(f_{22})=0.0603,\ S(f_{23})=0.0564$$
$$S(f_{31})=0.0316,\ S(f_{32})=0.0283,\ S(f_{33})=0.0324$$
$$S(f_{41})=0.0601,\ S(f_{42})=0.0388,\ S(f_{43})=0.0549$$

According to the formula $f_j^{+}=\{\max f_{ij}\}=\{\max S(f_{ij})\}$, $f_j^{-}=\{\min f_{ij}\}=\{\min S(f_{ij})\}$, the positive ideal alternatives f_{ij}^{+} and the negative ideal alternatives f_{ij}^{-} are shown as follows:

$$f_1^{+}=\{\max f_{i1}\}=\{\max S(f_{i1})\}=f_{41}=\left\langle (S_{0.2036},S_{0.2004}),([S_{0.1606},S_{0.1610}],[S_{0.6423},S_{0.6771}],[S_{0.5923},S_{0.6255}])\right\rangle,$$
$$f_1^{-}=\{\min f_{i1}\}=\{\min S(f_{i1})\}=f_{31}=\left\langle (S_{0.1118},S_{0.1243}),([S_{0.1405},S_{0.1441}],[S_{0.6062},S_{0.6054}],[S_{0.7388},S_{0.7270}])\right\rangle,$$
$$f_2^{+}=\{\max f_{i2}\}=\{\max S(f_{i2})\}=f_{22}=\left\langle (S_{0.1880},S_{0.1847}),([S_{0.1584},S_{0.1668}],[S_{0.6773},S_{0.6487}],[S_{0.4895},S_{0.5693}])\right\rangle;$$
$$f_2^{-}=\{\min f_{i2}\}=\{\min S(f_{i2})\}=f_{32}=\left\langle (S_{0.0928},S_{0.1052}),([S_{0.1619},S_{0.1595}],[S_{0.6503},S_{0.6594}],[S_{0.6394},S_{0.6598}])\right\rangle,$$
$$f_3^{+}=\{\max f_{i3}\}=\{\max S(f_{i3})\}=f_{23}=\left\langle (S_{0.2058},S_{0.2002}),([S_{0.1437},S_{0.1468}],[S_{0.5754},S_{0.5971}],[S_{0.7316},S_{0.7203}])\right\rangle,$$
$$f_3^{-}=\{\min f_{i3}\}=\{\min S(f_{i3})\}=f_{13}=\left\langle (S_{0.0824},S_{0.0969}),([S_{0.1525},S_{0.1541}],[S_{0.6646},S_{0.6957}],[S_{0.6377},S_{0.6542}])\right\rangle.$$

Step 4: Compute the group utility values ΔS_i and individual regret values ΔR_i

$$\Delta S_1 = \sum_{j=1}^{n} \omega_j \frac{d\left(f_j^- - f_{1j}\right) - d\left(f_j^+ - f_{1j}\right)}{d\left(f_j^- - f_j^+\right)} = -0.6202, \Delta S_2 = \sum_{j=1}^{n} \omega_j \frac{d\left(f_j^- - f_{2j}\right) - d\left(f_j^+ - f_{2j}\right)}{d\left(f_j^- - f_j^+\right)} = 0.8034,$$

$$\Delta S_3 = \sum_{j=1}^{n} \omega_j \frac{d\left(f_j^- - f_{3j}\right) - d\left(f_j^+ - f_{3j}\right)}{d\left(f_j^- - f_j^+\right)} = -0.6765, \Delta S_4 = \sum_{j=1}^{n} \omega_j \frac{d\left(f_j^- - f_{3j}\right) - d\left(f_j^+ - f_{4j}\right)}{d\left(f_j^- - f_j^+\right)} = 0.6957,$$

$$\Delta R_1 = \max_j \omega_j \frac{d\left(f_j^- - f_{1j}\right) - d\left(f_j^+ - f_{1j}\right)}{d\left(f_j^- - f_j^+\right)} = 0.0451, \Delta R_2 = \max_j \omega_j \frac{d\left(f_j^- - f_{2j}\right) - d\left(f_j^+ - f_{2j}\right)}{d\left(f_j^- - f_j^+\right)} = 0.5,$$

$$\Delta R_3 = \max_j \omega_j \frac{d\left(f_j^- - f_{3j}\right) - d\left(f_j^+ - f_{3j}\right)}{d\left(f_j^- - f_j^+\right)} = -0.1765, \Delta R_4 = \max_j \omega_j \frac{d\left(f_j^- - f_{4j}\right) - d\left(f_j^+ - f_{4j}\right)}{d\left(f_j^- - f_j^+\right)} = 0.2578.$$

Step 5: Compute the values ΔQ_i

Here we make ε = 0.5. The VIKOR values ΔQ_i for each alternative can be calculated as follows:

$$\Delta Q_1 = \varepsilon \frac{\Delta S_1 - \Delta S^-}{\Delta S^+ - \Delta S^-} + (1-\varepsilon)\frac{\Delta R_1 - \Delta R^-}{\Delta R^+ - \Delta R^-} = 0.5 \times \frac{-0.6202+0.6765}{0.8034+0.6765} + 0.5 \times \frac{0.0451+0.1765}{0.5+0.1765} = 0.1828$$

$$\Delta Q_2 = \varepsilon \frac{\Delta S_2 - \Delta S^-}{\Delta S^+ - \Delta S^-} + (1-\varepsilon)\frac{\Delta R_2 - \Delta R^-}{\Delta R^+ - \Delta R^-} = 0.5 \times \frac{0.8034+0.6765}{0.8034+0.6765} + 0.5 \times \frac{0.5+0.1765}{0.5+0.1765} = 1$$

$$\Delta Q_3 = \varepsilon \frac{\Delta S_3 - \Delta S^-}{\Delta S^+ - \Delta S^-} + (1-\varepsilon)\frac{\Delta R_3 - \Delta R^-}{\Delta R^+ - \Delta R^-} = 0.5 \times \frac{-0.6765+0.6765}{0.8034+0.6765} + 0.5 \times \frac{-0.1765+0.1765}{0.5+0.1765} = 0$$

$$\Delta Q_4 = \varepsilon \frac{\Delta S_4 - \Delta S^-}{\Delta S^+ - \Delta S^-} + (1-\varepsilon)\frac{\Delta R_4 - \Delta R^-}{\Delta R^+ - \Delta R^-} = 0.5 \times \frac{0.6957+0.6765}{0.8034+0.6765} + 0.5 \times \frac{0.2578+0.1765}{0.5+0.1765} = 0.7846$$

Step 6: Sort the ΔQ_i in descending order.

We can sort the alternatives according to the values of ΔS_i , ΔR_i and ΔQ_i . The larger the value, the better the alternative. Then, according to the ranking process, three ordered lists can be obtained as displayed in Table 8.

Table 8. Group utility value, individual regret value, and compromise evaluation value.

	A_1	A_2	A_3	A_4	Ranking results	compromise solution
ΔS_i	-0.6202	0.8034	-0.6765	0.6957	$A_2 > A_4 > A_1 > A_3$	A_2, A_4
ΔR_i	0.0451	0.5000	-0.1765	0.2578	$A_2 > A_4 > A_1 > A_3$	A_2, A_4
ΔQ_i	0.1828	1.0000	0.0000	0.7846	$A_2 > A_4 > A_1 > A_3$	A_2, A_4

Step 7: Test the compromise solutions.

The alternatives are ranked by ΔQ : $\Delta Q_2 > \Delta Q_4 > \Delta Q_1 > \Delta Q_3$. The best alternative is A_2 with $\Delta Q_2 = 1$, and the alternative A_4 is the second with $\Delta Q(A_4) = 0.7846$. Due to this $\Delta Q_2 - \Delta Q_4 = 0.2154 \leq \frac{1}{4-1} = 0.3333$, it doesn't satisfy condition 1- acceptable advantage. However, alternative A_2 is also the best sorted by ΔS and ΔR and satisfies condition 2. In that $\Delta Q_2 - \Delta Q_1 = 0.8172 > \frac{1}{4-1} = 0.3333$, so A_2 and A_4 are both compromise solutions. These results indicate that A_2 the best choice among the four alternatives, at the same time, A_4 could be the compromise solution that holds steady during the whole decision-making process.

Due to the decision results being related to the parameters p, and q on the ILNULWABM operator, it is necessary to make an analysis and discussion.

Similarly, in the VIKOR method, the compromise evaluation value of each alternative is affected by the group utility weight ε. To consider the impact of different values of ε on the evaluation results, the analysis is performed by setting different ε to observe their impact. The impact of the sorting result is shown in Table 9.

It can be seen from Table 9 that under the same value of ε, whatever the p, q is, the optimal scheme remains unchanged. Similarly, keeping the p, q fixed, the optimal solution remains the same based on different ε. So p, q, and ε have a limited impact on the ranking results.

Table 9. Group utility value, individual regret value, and compromise evaluation value in different p, q and ε

			A_1	A_2	A_3	A_4	Ranking results	compromise solution
p=1, q=1	ΔS		-0.6202	0.8034	-0.6765	0.6957	A2> A4> A1> A3	A_2,A_4
	ΔR		0.0451	0.5000	-0.1765	0.2578	A2> A4> A1> A3	
	ΔQ	ε=0.4	0.2117	1.0000	0.0000	0.7561	A2> A4> A1> A3	
		ε=0.5	0.1828	1.0000	0.0000	0.7846	A2> A4> A1> A3	
		ε=0.6	0.1538	1.0000	0.0000	0.8131	A2> A4> A1> A3	
p=1, q=0	ΔS		-0.6700	0.8068	-0.6047	0.6544	A2> A4> A3> A1	A_2,A_4
	ΔR		0.0400	0.5000	-0.1047	0.2700	A2> A4> A3> A1	
	ΔQ	ε=0.4	0.1435	1.0000	0.0177	0.7305	A2> A4> A1> A3	
		ε=0.5	0.1196	1.0000	0.0221	0.7582	A2> A4> A1> A3	
		ε=0.6	0.0957	1.0000	0.0266	0.7859	A2> A4> A1> A3	
p=0.5, q=0.5	ΔS		-0.6442	0.5800	-0.1683	0.5702	A2> A4> A3> A1	A_2,A_4
	ΔR		0.0378	0.5000	0.2100	0.5000	A2= A4> A3> A1	
	ΔQ	ε=0.4	0.0000	1.0000	0.3790	0.9968	A2> A4> A3> A1	
		ε=0.5	0.0000	1.0000	0.3807	0.9960	A2> A4> A3> A1	
		ε=0.6	0.0000	1.0000	0.3823	0.9952	A2> A4> A3> A1	

6 | Conclusion

This article proposes the concept of interval linguistic neutrosophic uncertain linguistic numbers. ILNULN consists of two parts: interval linguistic neutrosophic and uncertain linguistic number. The interval linguistic neutrosophic reflects the subjective linguistic judgment of the decision maker on the given uncertain linguistic number, and the uncertain linguistic number reflects the attitude of the decision maker towards the evaluation object. Based on interval linguistic neutrosophic uncertain linguistic number, this paper studies its basic properties, algorithms, scores function, and Hamming distance between two numbers, and proposes an interval linguistic neutrosophic uncertain linguistic weighted arithmetic Bonferroni Mean (ILNULWABM) operator. In addition, this paper applies ILNULN and ILNULWABM operators to the VIKOR method based on the relative closeness coefficient and discusses the impact of different parameters p, q, and ε on the MAGDM. Finally, we give an example to illustrate our theory, which proves the practicability and feasibility of the method proposed in this paper, and it improves and enriches the theory of MAGDM.

This article discusses and studies the VIKOR problem with ILNULN, and it has achieved certain results. But this research still needs to be further improved:

1) This article only considers the MAGDM problem in which the attribute weights and expert weights are single-valued linguistic numbers. The attribute weights and expert weights are not yet considered in the interval linguistic value. However, this situation is common in practical decision-making. Therefore, we can conduct further expansion research in the future.

2) In future research, it will be necessary and meaningful to apply the proposed interval linguistic neutrosophic uncertain linguistic MAGDM method to solve some practical problems in other areas, such as personnel evaluation, medical artificial intelligence, and pattern recognition.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] L. A. Zadeh, "Fuzzy sets," Information Control , vol. 8, no. 3, pp. 338-353, 1965.
[2] K. T. Atanassov, "Intuitionistic fuzzy sets," Fuzzy Sets Systems, vol. 20, no. 1, pp. 87-96, Aug. 1986.
[3] K. T. Atanassov, "More on intuitionistic fuzzy sets," Fuzzy Sets Systems, vol. 33, no. 1, pp. 33-37, Oct. 1989.
[4] K. T. Atanassov and G. Gargov, "Interval-valued intuitionistic fuzzy sets," Fuzzy Sets Systems, vol. 31, no. 3, pp. 343-349, Jul. 1989.
[5] F. Smarandache, "A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic," Rehoboth: American Research Press, pp. 1-141, 1999.
[6] H. B. Wang, P. Madiraju, Y. Q. Zhang and R. Sunderraman, "Interval Neutrosophic Sets," International Journal of Applied Mathematics and Statistics, vol. 3, no. 5, pp. 1-18, 2005.
[7] H. B. Wang, F. Smarandache, Y. Q. Zhang and R. Sunderraman, "Single Valued Neutrosophic Sets," Review of the Air Force Academy, vol. 1, pp 10-14, 2010.
[8] S. Broumi, J. Ye and F. Smarandache, "An Extended TOPSIS Method for Multiple Attribute Decision Making based on Interval Neutrosophic Uncertain Linguistic Variables," Neutrosophic Sets and Systems, vol. 8, pp. 22-31, Mar. 2015.
[9] Z. B. Fang and J. Ye, "Multiple attribute group decision-making method based on linguistic neutrosophic numbers," Symmetry, vol. 9, no. 7, pp. 1-12, Jul. 2017.
[10] S. Opricovic, "Multi-criteria optimization of civil engineering systems," Faculty of Civil Engineering, vol. 2, no. 1, pp. 5-21, 1998.
[11] G. Büyüközkan, D. Ruan, "Evaluation of software development projects using a fuzzy multi-criteria decision approach," Mathematics and Computers in Simulation, vol. 77, no. 5, pp. 464-475, Nov. 2008.
[12] M. K. Sayadi, M. Heydari, and K. Shahanaghi. "Extension of VIKOR method for decision making problem with interval numbers," Applied Mathematical Modelling, vol. 33, no. 5, pp. 2257-2262, May. 2009.
[13] A. Sanayei, S. FaridMousavi and A. Yazdankhah. "Group decision making process for supplier selection with VIKOR under fuzzy environment," Expert Systems with Applications, vol. 37, no. 1, pp. 24-30, Jan. 2010.
[14] J. H. Park, H. J. Cho and Y. C. Kwun. "Extension of the VIKOR method for group decision-making with interval-valued intuitionistic fuzzy information," Fuzzy Optimization and Decision Making. vol. 10, no. 3, pp. 233-253, Sep. 2011.
[15] N. Zhang and G. W. Wei, "Extension of VIKOR method for decision-making problem based on hesitant fuzzy set," Applied Mathematical Modelling, vol. 37, no. 7, pp. 4938-4947, Apr. 2013.

[16] Sh. N. Liu, "Multi-attribute decision-making method and its application based on neutrosophic sets," M.S. thesis, Shanxi Univ., Taiyuan, China, 2018.

[17] C. Bonferroni, "Sulle medie multiple di potenze," Bollettino dell'Unione Matematica Italiana, vol. 5, no. 3-4, pp. 267-270, 1950.

[18] R. R. Yager, "On generalized Bonferroni mean operators for multi-criteria aggregation," International Journal of Approximate Reasoning, vol. 50, no. 8, pp. 1279-1286, Sep. 2009.

[19] Z. S. Xu and R. R. Yager, "Intuitionistic Fuzzy Bonferroni Means," IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, vol. 41, no. 2, pp. 568-578, Apr. 2011.

[20] M. M. Xia, Z. S. Xu and B. Zhu, "Generalized intuitionistic fuzzy Bonferroni means," International Journal of Intelligent Systems, vol. 27, no. 1, pp. 23-47, Jan. 2012.

[21] W. Zhou and J. M. He, "Intuitionistic fuzzy normalized weighted Bonferroni mean and its application in multicriteria decision making," Journal of Applied Mathematics, vol. 2012, pp. 1-22, Jul. 2012.

[22] G. W. Wei, X. F. Zhang, R. Lin and H. J. Wang, "Uncertain linguistic Bonferroni mean operators and their application to multiple attribute decision making," Applied Mathematical Modelling, vol. 37, no. 7, pp. 5277-5285, Apr. 2013.

[23] P. D. Liu and Y. M. Wang, "Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean," Neural Computing and Applications, vol. 25, no. 7-8, pp. 2001–2010, Dec. 2014.

[24] Z. P. Tian, J. Wang, H. Y. Zhang, X. H. Chen and J. Q. Wang, "Simplified neutrosophic linguistic normalized weighted Bonferroni mean operator and its application to multi-criteria decision-making problems," Faculty of Sciences and Mathematics, vol. 30, no. 12, pp. 3339-3360, Jan. 2016.

[25] N. Chen, Z. S. Xui and M. M. Xia. "Interval-valued hesitant preference relations and their applications to group decision making," Knowledge-Based Systems, vol. 37, pp. 528-540. Jan. 2013.

[26] Z. S. Xu. "Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment," Information Sciences, vol. 168, no. 1-4, pp. 171-184. Dec. 2004.

[27] Z. Ch. Chen, P. H. Liu and Z. Pei, "An approach to multiple attribute group decision making based on linguistic intuitionistic fuzzy numbers," International Journal of Computational Intelligence Systems, vol. 8, no. 4, pp. 747-760, Jul. 2015.

[28] Y. Sh. Geng, X. G. Wang, X. M Li, K. Yu and P. D. Liu, "Some Interval Neutrosophic Linguistic Maclaurin Symmetric Mean Operators and Their Application in Multiple Attribute Decision Making," Symmetry, vol.10, no. 4, pp. 1-23, Apr. 2018.

[29] J. Ye, "Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making," Journal of Intelligent and Fuzzy Systems, vol. 27, no. 5, pp. 2231-2241, 2014.

[30] J. Ye. "Multiple attribute group decision-making based on interval neutrosophic uncertain linguistic variables," International Journal of Machine Learning and Cybernetics. vol. 8, no. 3, pp. 837-848, Jun. 2017.

[31] Ch. X. Fan and J. Ye, "Heronian Mean Operator of Linguistic Neutrosophic Cubic Numbers and Their Multiple Attribute Decision-Making Methods," Mathematical Problems in Engineering, vol. 2018, pp. 1-14, Jul. 2018.

[32] P. D. Liu and X. H. Zhang. "Some intuitionistic uncertain linguistic Bonferroni mean operators and their application to group decision making," Soft Computing, vol. 23, no. 11, pp. 3869–3886, Jun. 2019.