1 | Introduction

A. Csaszar [1] in 2002, introduced Generalized topological space which generalization the concept of “topological space” and the concept of “supra topological space” which was introduced by A.S. Mashhour et al. [2]. Since then many concepts in “topology” and “supra topology” have been developed to “generalized topological space”.

Recently, F.Smarandache introduced “the theory of neutrosophic sets” [3, 4]. The theory of “neutrosophic topology” and neutrosophic topological space was introduced in 2012 [5].

In [6], G.Jayaparthisarathy, et al. studied the concept of “neutrosophic supra topological space” in 2019. Moreover, a new method is proposed to solve medical diagnosis problems by using a “single valued neutrosophic score function”. Also, neutrosophic supra-topological space is more generalization than neutrosophic topological space. Finally, they investigate new weak open and closed sets in this new space, called neutrosophic supra open (closed) sets. After P. G.Jayaparthisarathy, et al. found out “neutrosophic supra topological space”. This encouraged us to think of presenting neutrosophic generalized topological space. Moreover, this new generalization of neutrosophic space enabled researchers to define and study many concepts in “neutrosophic topology”. On the other hand, the “neutrosophic sets” and the new “neutrosophic sets” in this space may have applications in “the medical field” such as “diagnosis of bipolar disorder diseases” [7], “Intelligent Medical Decision Support Model” Based on “Soft Computing” and “IoT” [8], “evaluation...
In this work, we are defining a more generalized concept in “neutrosophic topology”, as we define the “neutrosophic generalized topological space”, which is more general than “neutrosophic topological space” and “neutrosophic supra topological space”. Also, we define many kinds of open and closed sets in this new space. Moreover, we study the properties of these sets. Finally, We introduce interior and closure operators in “neutrosophic generalized topological space”, and we study their properties.

NSO(\(\mathcal{N}\)), NPO(\(\mathcal{N}\)), and NatO(\(\mathcal{N}\)) mean the family of all “neutrosophic semi-open sets” [11], “neutrosophic pre-open sets” [11], and “neutrosophic \(\alpha\) open sets” [11] respectively. For a neutrosophic set \(\hat{A}\), \(\hat{A}^c\) denotes the “neutrosophic complement” of \(\hat{A}\).

More for “neutrosophic bi-topological space” in [12, 13]. And for “neutrosophic topological space” [14, 15].

2 | Preliminaries

We need to recall some necessary definitions which are important to complement this paper.

Definition 2.1. [3] Let \(\mathbb{N}\neq\emptyset\) be a set. A neutrosophic set (NS) \(\hat{A}\) is a subset having the form \(U=\{<x, U_1, U_2, U_3>: x\in\mathbb{N}\}\); \(U_1, U_2\) and \(U_3\) are the “degree of membership”, the “degree of indeterminacy” and the “degree of nonmembership” of all \(x\in\mathbb{N}\).

Definition 2.2. [6] A neutrosophic supra topology (NST) on \(\mathbb{N}\neq\emptyset\) is a family \(\hat{W}\) of neutrosophic subsets in \(\mathbb{N}\), satisfying the following axioms:

i. \(1_\mathbb{N}\in\hat{W}\) and \(0_\mathbb{N}\in\hat{W}\).

ii. \(\hat{W}\) is closed under “arbitrary union”.

\((\mathbb{N}, \hat{W}\) is called a “neutrosophic supra topological space” (NSTS) in X. Also, every element \(\theta\in\hat{W}\) is said to be a “neutrosophic open set” (NOS), A neutrosophic set \(\hat{F}\) is a closed set (NCS) iff \(\hat{F}\in\hat{W}\).

3 | Neutrosophic Generalized Topological Space

In this part, we present the neutrosophic generalized topological space and introduce the “neutrosophic generalized open set” and “neutrosophic generalized closed set” in this new space.

Definition 3.1. Let \(\mathbb{N}\neq\emptyset\) be a set, \(g_\mathbb{N}\) is a subcollection of neutrosophic sets on \(\mathbb{N}\), then:

\(g_\mathbb{N}\) is called “neutrosophic generalized topology” (NGT) on X if \(0_\mathbb{N}\in g_\mathbb{N}\) and \(g_\mathbb{N}\) are closed under “arbitrary union”.

Definition 3.2. Let \(g_\mathbb{N}\) be a NGT on \(\mathbb{N}\), then:

\((\mathbb{N}, g_\mathbb{N}\) is called a “neutrosophic generalized topological space” (NGTS). The “neutrosophic set” \(\hat{A}\) in \(\mathbb{N}\) is said to be “neutrosophic generalized open set” (NGOS) if \(\hat{A}\in g_\mathbb{N}\), a neutrosophic set \(\hat{F}\) in \(\mathbb{N}\) is said to be “neutrosophic generalized closed set” (NGCS) if \(\hat{F}\in g_\mathbb{N}\).

- NGOS(\(\mathbb{N}\)) are a family of “neutrosophic generalized open sets”.
- NGCS(\(\mathbb{N}\)) are a family of “neutrosophic generalized closed sets”.

Remark 3.3. The following table illustrates the comparison of “fuzzy supra topological space”, “neutrosophic supra topological space”, “neutrosophic generalized topological space” and “neutrosophic topological space”.

Hospital medical care systems” [9] and “A novel group decision-making model” for “heart disease diagnosis” [10].
Example 3.4. Let \( \mathcal{N} = \{ p, q, r \} \) \( g_N = \{ 0_N, D, C, B \} \); \( E = \{ 0_N, E, G, H \} \); \( D = \{ < p, 0.3, 0.2, 0.5 >, < q, 0.6, 0.5, 0.3 >, < r, 0.7, 0.1, 0.9 > \} \); \( C = \{ < p, 0.4, 0.1, 0.3 >, < q, 0.2, 0.6, 0.7 >, < r, 0.1, 0.3, 0.4 > \} \); \( B = \{ < p, 0.4, 0.2, 0.3 >, < q, 0.6, 0.6, 0.3 >, < r, 0.7, 0.3, 0.4 > \} \).

\( (\mathcal{N}, g_N) \) is (NGTS).

0\( _N \), D, C, B are NGOS.

Example 3.5. Let \((\mathcal{N}, g_N)\) be a (NGTS), then:

0\( _N \) is a NGOS, 1\( _N \) is a NGCS.

Remark 3.6.

(i) Every NTS is NGTS. But, the converse is not true. see the following example.

(ii) Every NSTS is NGTS. But, the converse is not true. see the following example.

(iii) \( g_N \) is not closed under arbitrary intersection, see the following example.

Example 3.7. In example 3.4, \((\mathcal{N}, g_N)\) is NGTS, but \((\mathcal{N}, g_N)\) is not NCTS nor NSTS.

In example 3.4, D, C are neutrosophic generalized open sets, but \( D \cap C = \{ < p, 0.3, 0.1, 0.5 >, < q, 0.2, 0.5, 0.7 >, < r, 0.1, 0.1, 0.9 > \} \) is not a “neutrosophic generalized open set”.

Example 3.8. Let \((\mathcal{N}, T)\) be a NCTS:

Let \( g_N = NSO(\mathcal{N}) \) or \( g_N = NPO(\mathcal{N}) \) or \( g_N = N^{\alpha}O(\mathcal{N}) \) or \( g_N = N^{\beta}O(\mathcal{N}) \) then \( g_N \) is NGT on \( \mathcal{N} \).

Remark 3.9. 1\( _N \) is a “neutrosophic open set” and 1\( _N \) is a “neutrosophic supra open set”, but 1\( _N \) is not a “neutrosophic generalized open set”.

Remark 3.10. Let \((\mathcal{N}, g_N)\) be a NGTS, then:

Since \( g_N \) are closed under arbitrary union. Then the arbitrary intersection of any NGCS is an NGCS.

Remark 3.11. In any NSTS (or NTS) \((\mathcal{N}, S)\), the union of any two “neutrosophic supra open sets”, maybe 1\( _N \), see the following example

Example 3.12. Let \( \mathcal{N} = \{ p \} \) \( S = \{ 0_N, 1_N, D, C, B \} \); \( D = \{ < p, 0.3, 0.2, 0.5 > \} \); \( C = \{ < p, 0.4, 0.1, 0.3 > \} \);
\( \mathcal{B} = \{ < \mathcal{P}, 0.4, 0.2, 0.3 > \} \).

Then \( (\mathcal{N}, \mathcal{S}) \) is a NSTS.

0\(_N\), 1\(_N\), \( \mathcal{D}, \mathcal{C}, \mathcal{B} \) are “neutrosophic supra open sets”. 1\(_N\) \( \cup \mathcal{D} = 1\_N \).

- The following theorem shows that this is not true in an NGTS.

**Theorem 3.13.** Let \( (\mathcal{N}, \mathcal{G}_N) \) be a NGTS. But it is not NSTS then, the union of any two “neutrosophic generalized open sets”, fails to be 1\(_N\).

**Proof:**

Let \( \mathcal{A}, \mathcal{B} \) two NGOS, that satisfied \( \mathcal{A} \cup \mathcal{B} = 1\_N \).

Since \( \mathcal{A}, \mathcal{B} \) two NGOS, we get \( \mathcal{A} \cup \mathcal{B} \) is NGOS, therefore 1\(_N\) is an NGOS, which contradicts with assumption.

### 4 | The Interior and Closure Operations

In this part, we define the “closure” and “interior” via a neutrosophic generalized open set.

**Definition 4.1.** Let \( (\mathcal{N}, \mathcal{G}_N) \) be an NGTS, and \( \mathcal{A} \) is a neutrosophic set then:

The union of any NGOS, contained in \( \mathcal{A} \) is called the “neutrosophic generalized interior” of \( \mathcal{A} \)

\( (N_g-i(\mathcal{A})) \).

\( N_g-i(\mathcal{A}) = \cup \{ \mathcal{B} ; \mathcal{B} \subseteq \mathcal{A} ; \mathcal{B} \in \text{NGOS}(\mathcal{N}) \} \).

**Theorem 4.2.** Let \( (\mathcal{N}, \mathcal{G}_N) \) be a NGTS, \( \mathcal{A}, \mathcal{B} \) are “neutrosophic sets” then:

i. \( N_g-i(\mathcal{A}) \subseteq \mathcal{A} \).

ii. \( N_g-i(\mathcal{A}) \) is NGOS.

iii. \( \mathcal{A} \subseteq \mathcal{B} \Rightarrow N_g-i(\mathcal{A}) \subseteq N_g-i(\mathcal{B}) \).

**Proof:**

i. Follow from the definition of \( N_g-i(\mathcal{A}) \) as a union of any “neutrosophic generalized open sets”, contained in \( \mathcal{A} \).

ii. since union of any NGOS, is NGOS, then \( N_g-i(\mathcal{A}) = \cup \{ \mathcal{B} ; \mathcal{B} \subseteq \mathcal{A} ; \mathcal{B} \in \text{NGOS}(\mathcal{N}) \} \) is NGOS.

iii. Prof is Obvious.

**Definition 4.3.** Let \( (\mathcal{N}, \mathcal{G}_N) \) be an NGTS, \( \mathcal{A} \) is neutrosophic set then:

The intersection of any NGCS, including \( \mathcal{A} \), is called “neutrosophic generalized closure” of \( \mathcal{A} \)

\( (N_g-c(\mathcal{A})) \).

\( N_g-c(\mathcal{A}) = \cap \{ \mathcal{B} ; \mathcal{B} \supseteq \mathcal{A} ; \mathcal{B} \in \text{NGCS}(\mathcal{N}) \} \).

**Theorem 4.4.** Let \( (\mathcal{N}, \mathcal{G}_N) \) be a NGTS, \( \mathcal{A} \) is “NS” then:

i. \( \mathcal{A} \subseteq N_g-c(\mathcal{A}) \).

ii. \( N_g-c(\mathcal{A}) \) is NGCS.

**Proof:**

i. Since \( N_g-c(\mathcal{A}) \) is an intersection of any NGCS contained in \( \mathcal{A} \).

ii. Follow from remark 3.9.
Theorem 4.5. Let \( (\mathcal{R}, g_{\mathcal{R}}) \) be a NGTS, \( \hat{A} \) is neutrosophic set then:

i. \( N_{g-c}(\mathcal{R} - \hat{A}) = \mathcal{R} - N_{g-i}(\hat{A}) \).

ii. \( N_{g-i}(\mathcal{R} - \hat{A}) = \mathcal{R} - N_{g-c} (\hat{A}) \).

iii. \( N_{g-i}(\hat{A}) = \mathcal{R} - N_{g-c}(\mathcal{R} - \hat{A}) \).

iv. \( N_{g-c}(\hat{A}) = \mathcal{R} - N_{g-i}(\mathcal{R} - \hat{A}) \).

Proof:

i. \( \mathcal{R} - N_{g-i}(\hat{A}) = \mathcal{R} - \{ \beta ; \beta \subseteq \hat{A} ; \beta \in \text{NGOS}(\mathcal{R}) \} \)

\[ = \cap \{ \mathcal{R} - \beta ; \mathcal{R} - \beta \subseteq \mathcal{R} - \hat{A} ; \mathcal{R} - \beta \in \text{NGOS}(X) \} = N_{g-c}(\mathcal{R} - \hat{A}) \]

\[ = \cup \{ \mathcal{R} - \beta ; \mathcal{R} - \beta \subseteq \mathcal{R} - \hat{A} ; \mathcal{R} - \beta \in \text{NGCS}(\mathcal{R}) \} = N_{g-i}(\hat{A}) \).

ii. \( \mathcal{R} - N_{g-c}(\hat{A}) = \mathcal{R} - \{ \beta ; \beta \supseteq \hat{A} ; \beta \in \text{NGCS}(\mathcal{R}) \} \)

\[ = \cup \{ \mathcal{R} - \beta ; \mathcal{R} - \beta \subseteq \mathcal{R} - \hat{A} ; \mathcal{R} - \beta \in \text{NGOS}(\mathcal{R}) \} \)

\[ = N_{g-i}(\mathcal{R} - \hat{A}) \]

iii. Follows from (2) by putting \( \mathcal{R} - \hat{A} \) in place of \( \hat{A} \).

iv. Follows from (1) by putting \( \mathcal{R} - \hat{A} \) in place of \( \hat{A} \).

Theorem 4.6. Let \( (\mathcal{R}, g_{\mathcal{R}}) \) be a NGTS, and \( \hat{A} \subseteq \mathcal{R} \), then:

i. \( \hat{A} \) is NGCS, then \( N_{g-c}(\hat{A}) = \hat{A} \).

ii. \( \hat{A} \) is NGOS, then \( N_{g-i}(\hat{A}) = \hat{A} \).

Proof:

i. Follow the definition of \( N_{g-c}(\hat{A}) \) and Theorem 4.4.

ii. Follow the definition of \( N_{g-i}(\hat{A}) \) and Theorem 4.2.

Theorem 4.7. Let \( (\mathcal{R}, g_{\mathcal{R}}) \) be a NGTS, \( \hat{A} \) is neutrosophic set then:

i. \( N_{g-c} [N_{g-c}(\hat{A})] = N_{g-c}(\hat{A}) \).

ii. \( N_{g-i} [N_{g-i}(\hat{A})] = N_{g-i}(\hat{A}) \).

Proof:

The proof is Obvious.

Remark 4.8. Let \( (\mathcal{R}, g_{\mathcal{R}}) \) be a NGTS, Let \( \hat{A}, \beta \subseteq \mathcal{R} \), then:

i. \( N_{g-i} (\hat{A} \cap \beta) \subseteq N_{g-i} (\hat{A}) \cap N_{g-i} (\beta) \).

ii. \( N_{g-c} (\hat{A} \cap \beta) \subseteq N_{g-c} (\hat{A}) \cap N_{g-c} (\beta) \).

iii. \( N_{g-i} (\hat{A} \cup \beta) \supseteq N_{g-i} (\hat{A}) \cup N_{g-i} (\beta) \).

iv. \( N_{g-c} (\hat{A} \cup \beta) \supseteq N_{g-c} (\hat{A}) \cup N_{g-c} (\beta) \).

Proof:

i. Since \( \hat{A} \cap \beta \subseteq \hat{A} \), \( \hat{A} \cap \beta \subseteq \beta \) then \( N_{g-i} (\hat{A} \cap \beta) \subseteq N_{g-i} (\hat{A}) \) and \( N_{g-i} (\hat{A} \cap \beta) \subseteq N_{g-i} (\beta) \), hence \( N_{g-i} (\hat{A} \cap \beta) \subseteq N_{g-i} (\hat{A}) \cap N_{g-i} (\beta) \).

ii. Since \( \hat{A} \cap \beta \subseteq \hat{A} \), \( \hat{A} \cap \beta \subseteq \beta \) then \( N_{g-c} (\hat{A} \cap \beta) \subseteq N_{g-c} (\hat{A}) \) and \( N_{g-c} (\hat{A} \cap \beta) \subseteq N_{g-c} (\beta) \), hence \( N_{g-c} (\hat{A} \cap \beta) \subseteq N_{g-c} (\hat{A}) \cap N_{g-c} (\beta) \).
iii. Since $\hat{A} \subseteq \hat{A} \cup \hat{B}$, $\beta \subseteq \hat{A} \cup \hat{B}$ then $N_{e^{-1}}(\hat{A}) \subseteq N_{e^{-1}}(\hat{A} \cup \hat{B})$ and $N_{e^{-1}}(\beta) \subseteq N_{e^{-1}}(\hat{A} \cup \hat{B})$, hence $N_{e^{-1}}(\hat{A}) \cup N_{e^{-1}}(\beta) \subseteq N_{e^{-1}}(\hat{A} \cup \hat{B})$.

iv. Since $\hat{A} \subseteq \hat{A} \cup \hat{B}$, $\beta \subseteq \hat{A} \cup \hat{B}$ then $N_{e^{-1}}(\hat{A}) \subseteq N_{e^{-1}}(\hat{A} \cup \hat{B})$ and $N_{e^{-1}}(\beta) \subseteq N_{e^{-1}}(\hat{A} \cup \hat{B})$, hence $N_{e^{-1}}(\hat{A}) \cup N_{e^{-1}}(\beta) \subseteq N_{e^{-1}}(\hat{A} \cup \hat{B})$.

**Remark 4.9.**

i. $N_{e^{-1}}(\hat{A} \cap \hat{B}) = N_{e^{-1}}(\hat{A}) \cap N_{e^{-1}}(\beta)$.

ii. $N_{e^{-1}}(\hat{A} \cup \hat{B}) = N_{e^{-1}}(\hat{A}) \cup N_{e^{-1}}(\beta)$.

It is easy to show that remark 4.9 is true in neutrosophic topological space (in this case, $A$, $B$ are neutrosophic sets in neutrosophic topological space). However, this remark 4.9 is not true in NGTS (in this case, $A$, $B$ are neutrosophic sets in NGTS), as shown in the following example.

**Example 4.10.** Let $\mathbb{N} = \{1\}$, $S = \{0, 1, \infty, E, G, H\}$; $E = \{0.5, 1, 0\}$, $G = \{0.25, 0, 1\}$, $H = \{0.5, 1, 1\}$, $C = \{0.5, 0.5, 0\}$, $D = \{0.5, 0, 0.5\}$.

Then

$N_{e}(C) = \{0.75, 1, 0\}$, $N_{e}(D) = \{0.5, 0, 1\}$

$C \cup D = \{0.5, 0.5, 0.5\}$, $N_{e}(C \cup D) = \{1, 1, 1\}$.

$N_{e}(C \cup D) \neq N_{e}(C) \cup N_{e}(D) = \{0.75, 1, 1\}$,

$N = \{0.5, 1, 0.25\}$, $M = \{0.5, 0.5, 1\}$.

Then

$N_{d}(N) = \{0.5, 1, 0\}$, $N_{d}(D) = \{0.25, 0, 1\}$

$C \cap D = \{0.5, 0.5, 0.25\}$, $N_{d}(C \cap D) = \{0, 0, 0\}$.

$N_{d}(C \cap D) \neq N_{d}(C) \cap N_{d}(D) = \{0.25, 0, 0\}$.

5 | New Neutrosophic Generalized Sets

**Definition 5.1.** Let $(\mathbb{N}, g_{\alpha})$ be a NGTS, then:

i. A “NS” $U$ is said to be a “neutrosophic generalized pre-open set” (NGPOS) if $U \subseteq\text{Ng-i}[\text{Ng-e}(U)]$.

ii. A neutrosophic set $U$ is said to be a “neutrosophic generalized semi-open set” (NGSOS) if $U \subseteq\text{Ng-e}[\text{Ng-i}(U)]$.

iii. A neutrosophic set $U$ is said to be a “neutrosophic generalized $\alpha$-open set” (NG$\alpha$OS) if $U \subseteq\text{Ng-e}[\text{Ng-i}(U)]$.

**Definition 5.2.** Let $(\mathbb{N}, g_{\alpha})$ be a NGTS, then:

i. A Neutrosophic set $F$ is said to be a “neutrosophic generalized pre-closed set” (NGPCS) if $U^{c}$ is NGPOS. The family of NGPOS(NGPCS) set in $\mathbb{N}$ is denoted by NGPOS($\mathbb{N}, g_{\alpha}$) (NGPCS($\mathbb{N}, g_{\alpha}$)).

ii. A neutrosophic set $F$ is said to be a “neutrosophic generalized semi-closed set” if $U^{c}$ is NGSOS. The family of NGSOS(NGSCS) set in $\mathbb{N}$ is denoted by NGSOS($\mathbb{N}, g_{\alpha}$) (NGSCS($\mathbb{N}, g_{\alpha}$)).

iii. A neutrosophic set $F$ is said to be a “neutrosophic generalized $\alpha$-closed set” (NG$\alpha$CS) if $U^{c}$ is NG$\alpha$OS. The family of NG$\alpha$OS (NG$\alpha$CS) in $\mathbb{N}$ is denoted by NG$\alpha$OS($\mathbb{N}, g_{\alpha}$) (NG$\alpha$CS($\mathbb{N}, g_{\alpha}$)).
Theorem 5.3. Let \((\mathcal{K}, g\mathbb{N})\) be an NGTS then, Every NGOS is NGSOS.

Proof.

Let U be a NGOS in \((\mathcal{K}, g\mathbb{N})\), We know that \(U \subseteq N_{g}^{-i}(\mathcal{A})\), and since U is a NGOS, we get \(U = N_{g}^{-i}(\mathcal{A})\), therefore \(U \subseteq N_{g}^{-c}(N_{g}^{-i}(\mathcal{A}))\).

Therefore U is NGSOS.

Remark 5.4. The converse of the above theorem 5.3 is not necessarily true, see the following example.

Example 5.5. Let \(\mathcal{K} = \{p, q, r\} \ g\mathbb{N} = \{\ 0_{N}, E, G\} \ ; \ E = \{< 0.4, 0.5, 0.2 >, < 0.3, 0.2, 0.1 >, < 0.9, 0.6, 0.8 >\}, \ G = \{< 0.2, 0.4, 0.5 >, < 0.1, 0.1, 0.2 >, < 0.6, 0.5, 0.8 >\}, \ \mathcal{A} = \{< 0.5, 0.6, 0.1 >, < 0.4, 0.3, 0.1 >, < 0.9, 0.8, 0.5 >\}.

Then \((\mathcal{K}, g\mathbb{N})\) is NGTS.

\(\mathcal{A}\) is NGSOS, but \(\mathcal{A}\) is not NGOS.

Theorem 5.6. Let \((\mathcal{K}, g\mathbb{N})\) be an NGTS then, Every NG\(\alpha\)OS is NGSOS.

Proof.

Let \(\mathcal{A}\) be a NG\(\alpha\)OS in \((\mathcal{K}, g\mathbb{N})\), then \(\mathcal{A} \subseteq Ng^{-i}(Ng^{-c}[Ng^{-i}(\mathcal{A})])\)…(1), since \(Ng^{-i}(Ng^{-c}[Ng^{-i}(\mathcal{A})]) \subseteq Ng^{-c}[Ng^{-i}(\mathcal{A})]\)…(2).

(1),(2) \(\Rightarrow \ U \subseteq Ng^{-c}[Ng^{-i}(\mathcal{A})]\), Therefore \(\mathcal{A}\) is NGSOS.

Remark 5.7. The Converse of above theorem 5.6 is not necessarily true, see the following example.

Example 5.8. Let \(\mathcal{K} = \{p\} \ g\mathbb{N} = \{\ 0_{N}, E, G\} \ ; \ E = \{< 0.4, 0.5, 0.4 >\}, \ G = \{< 0.3, 0.4, 0.4 >\}, \ H = \{< 0.5, 0.5, 0.5 >\}.

Then \((\mathcal{K}, g\mathbb{N})\) is NGTS.

\(H = \{< 0.5, 0.5, 0.5 >\} \) is NGSO, but H is not NG\(\alpha\)OS.

Theorem 5.9. Let \((\mathcal{K}, g\mathbb{N})\) be NGTS then, Every NGOS is NG\(\alpha\)OS.

Proof.

Let U be a NGOS in \((\mathcal{K}, g\mathbb{N})\), then \(U = N_{g}^{-i}(\mathcal{A})\), since \(U \subseteq N_{g}^{-c}(U)\), then \(N_{g}^{-i}(U) \subseteq N_{g}^{-i}(N_{g}^{-c}(\mathcal{A}))\).

therefore \(U \subseteq N_{g}^{-i}(N_{g}^{-c}(\mathcal{A}))\), hence U is NG\(\alpha\)OS.

Remark 5.10. The Converse of theorem 5.9 is not true, see example 3.11.

Example 5.11. Let \(\mathcal{K} = \{p, q, r\} \ g\mathbb{N} = \{\ 0_{N}, E, G\} \ ; \ E = \{< 0.4, 0.5, 0.4 >, < 0.5, 0.5, 0.5 >, < 0.4, 0.5, 0.4 >\}, \ G = \{< 0.7, 0.6, 0.5 >, < 0.2, 0.3, 0.4 >, < 0.9, 0.4, 0.4 >\}, \ H = \{< 0.5, 0.5, 0.5 >, < 0.5, 0.5, 0.5 >, < 0.5, 0.5, 0.5 >\}.

Then \((\mathcal{K}, g\mathbb{N})\) is NGTS.

\(H\) is NG\(\alpha\)OS, but \(H\) is not NGOS.

Theorem 5.12. Let \((\mathcal{K}, g\mathbb{N})\) be an NGTS then, every NGOS is NG\(\alpha\)OS.

Proof.

Let U be a NGOS in \((\mathcal{K}, g\mathbb{N})\), We now \(U \subseteq N_{g}^{-c}(\mathcal{A})\), and since U is a NGOS, we get \(U = N_{g}^{-i}(\mathcal{A})\). Then \(U \subseteq N_{g}^{-c}(N_{g}^{-i}(\mathcal{A}))\). Hence \(N_{g}^{-i}(U) \subseteq N_{g}^{-i}(N_{g}^{-c}(N_{g}^{-i}(\mathcal{A})))\), then \(U \subseteq N_{g}^{-i}(N_{g}^{-c}(N_{g}^{-i}(\mathcal{A})))\).
Remark 5.13. The Converse of theorem 5.12 is not true, see example 4.14.

Example 4.14. Let $\mathcal{N} = \{p, q\}$ $g\in \{0, E, G\}; E = \{<0.3, 0.4>, <0.4, 0.5>, <0.2, 0.3>, <0.6, 0.4>\}$, $G = \{<0.4, 0.5>, <0.3, 0.4>, <0.3, 0.4>, <0.6, 0.4>\}$.

Then $(\mathcal{N}, g\in \mathcal{N})$ is NGTS.

H is NG$\alpha$OS, but H is not NGOS.

Theorem 5.15. Let $(\mathcal{N}, g\in \mathcal{N})$ be an NGTS then, every NG$\alpha$OS is NGPOS.

Proof.

Let U be a NG$\alpha$OS in $(\mathcal{N}, g\in \mathcal{N})$, then $U \subseteq \overline{\text{Ng-}i(\text{Ng-}c(\text{Ng-}i(U)))}$...(1), since $\text{Ng-}i(\text{Ng-}c(\text{Ng-}i(U))) \subseteq \text{Ng-}i(\text{Ng-}c(U))$...(2).

$(1),(2) \Rightarrow U \subseteq \text{Ng-}i(\text{Ng-}c(U))$, Therefore U is NGPOS.

Remark 5.16. The converse of the above theorem 4.18 is not necessarily true, see the following example.

Example 5.17. Let $\mathcal{N} = \{p, q\}$ $g\in \{0, E, G\}; E = \{<0.3, 0.4>, <0.5, 0.6>, <0.3, 0.4>, <0.6, 0.4>\}$, $G = \{<0.4, 0.5>, <0.3, 0.4>, <0.3, 0.4>, <0.6, 0.4>\}$. Then $(\mathcal{N}, g\in \mathcal{N})$ is NGTS.

$H = \{<0.4, 0.5>, <0.5, 0.6>, <0.3, 0.4>, <0.6, 0.4>\}$ is NGPOS, but H is not NG$\alpha$OS.

Remark 5.18. Relations among the new types of generalized neutrosophic open sets that were studied in this paper in the following Figure 1:

![Diagram showing relations among NGPOS, NGOS, NG$\alpha$OS, and NGSOS]

Figure 1. Relations among the new types of generalized neutrosophic open sets.

6 Conclusion

The neutrosophic generalized topological structure, which is a more general structure than neutrosophic supra topological spaces is built on neutrosophic sets. Also, we study a new type of neutrosophic open and closed in this space. These new neutrosophic sets can be used to find new research about, neutrosophic connectedness, neutrosophic compactness, neutrosophic continuity, and neutrosophic separation axioms in NGTS.
Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References


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