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# Neutrosophic Generalized Topological Space

**Riad K. Al-Hamido** [1],*

[1] Faculty of Science, AlFurat University, Deir-ez-Zor, Syrian Arab Republic.
[1] Cordoba private University, Alqamishli branch, Syrian Arab Republic; riad-hamido1983@hotmail.com.

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## Abstract

In classical topology, the generalized topological space develops the topological space and supra topological space. In this work, we define and study neutrosophic generalized topological space which is more generalization than the neutrosophic supra topological space, this space may be opening a new horizon and new direction in neutrosophic topology. Also, we study new "neutrosophic open (closed) sets" in this new space such as "neutrosophic generalized open (closed) sets", "neutrosophic generalized semi open (closed) sets", "neutrosophic generalized pre open (closed) sets", "neutrosophic generalized $\alpha$-open (closed) sets", and we obtain its basic Properties and study the relations between them. Finally, we introduced "the associated interior and closure operators" in neutrosophic generalized topological space.

**Keywords:** Neutrosophic Generalized Topological Space, Neutrosophic Generalized Open Set, Neutrosophic Generalized Semi Open Set, Neutrosophic Generalized Pre Open Set, Neutrosophic Generalized $\alpha$-open Set.

# 1 | Introduction

A. Csaszar [1] in 2002, introduced Generalized topological space which generalization the concept of "topological space" and the concept of "supra topological space" which was introduced by A.S. Mashhour et al. [2]. Since then many concepts in "topology" and "supra topology" have been developed to "generalized topological space".

Recently, F.Smarandache introduced "the theory of neutrosophic sets" [3, 4]. The theory of "neutrosophic topology" and neutrosophic topological space was introduced in 2012 [5].

In [6], G.Jayaparthasarathy, et al. studied the concept of "neutrosophic supra topological space" in 2019. Moreover, a new method is proposed to solve medical diagnosis problems by using a "single valued neutrosophic score function". Also, neutrosophic supra-topological space is more generalization than neutrosophic topological space. Finally, they investigate new weak open and closed sets in this new space, called neutrosophic supra open (closed) sets. After P. G.Jayaparthasarathy, et al. found out "neutrosophic supra topological space". this encouraged us to think of presenting neutrosophic generalized topological space. Moreover, this new generalization of neutrosophic space enabled researchers to define and study many concepts in "neutrosophic topology". On the other hand, the "neutrosophic sets" and the new "neutrosophic sets" in this space may have applications in "the medical field" such as "diagnosis of bipolar disorder diseases" [7], "Intelligent Medical Decision Support Model" Based on "Soft Computing" and "IoT" [8], "evaluation

Corresponding Author: riad-hamido1983@alfuratuniv.edu.sy
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Hospital medical care systems" [9] and "A novel group decision-making model" for "heart disease diagnosis" [10].

In this work, we are defining a more generalized concept in "neutrosophic topology", as we define the "neutrosophic generalized topological space", which is more general than "neutrosophic topological space" and "neutrosophic supra topological space". Also, we define many kinds of open and closed sets in this new space. Moreover, we study the properties of these sets. Finally, We introduce interior and closure operators in "neutrosophic generalized topological space", and we study their properties.

NSO(χ), NPO(χ), and NαO(χ) mean the family of all "neutrosophic semi-open sets" [11], "neutrosophic pre-open sets" [11], and "neutrosophic α open sets" [11]respectively. For a neutrosophic set Â, $Â^c$ denotes the "neutrosophic complement" of Â.

More for "neutrosophic bi-topological space" in [12, 13]. And for "neutrosophic topological space" [14, 15].

## 2 | Preliminaries

We need to recall some necessary definitions which are important to complement this paper.

**Definition 2.1.** [3] Let ℵ≠ϕ be a set. A neutrosophic set (NS) Â is a subset having the form U={< x, $U_1$, $U_2$, $U_3$ > : x∈ℵ}; $U_1$, $U_2$, and $U_3$ are the "degree of membership", the "degree of indeterminacy" and the "degree of nonmembership" of all x∈ℵ.

**Definition 2.2.** [6] A neutrosophic supra topology (NST) on ℵ≠∅ is a family Ŵ of neutrosophic subsets in ℵ, satisfying the following axioms:

i. $1_N$ ∈Ŵ and $0_N$ ∈Ŵ.

ii. Ŵ is closed under "arbitrary union".

(ℵ, Ŵ) is called a "neutrosophic supra topological space" (NSTS) in X. Also, every element $\vartheta$∈ Ŵ is said to be a "neutrosophic open set" (NOS), A neutrosophic set fi is a closed set (NCS) iff $fi^c$∈ Ŵ.

## 3 | Neutrosophic Generalized Topological Space

In this part, we present the neutrosophic generalized topological space and introduce the "neutrosophic generalized open set" and "neutrosophic generalized closed set" in this new space.

**Definition 3.1.** let ℵ ≠ ∅ be a set, $g_N$ is a subcollection of neutrosophic sets on ℵ, then:

$g_N$ is called "neutrosophic generalized topology" (NGT) on X if $0_N$∈$g_N$ and $g_N$ are closed under "arbitrary union".

**Definition 3.2.** Let $g_N$ be a NGT on ℵ, then:

(ℵ, $g_N$) is called a "neutrosophic generalized topological space" (NGTS). The "neutrosophic set" Â in ℵ is said to be "neutrosophic generalized open set" (NGOS) if Â∈ $g_N$, a neutrosophic set F in ℵ is said to be "neutrosophic generalized closed set" (NGCS) if $Â^c$∈ $g_N$.

- NGOS(ℵ) are a family of "neutrosophic generalized open sets".
- NGCS(ℵ) are a family of "neutrosophic generalized closed sets".

**Remark 3.3.** The following table illustrates the comparison of "fuzzy supra topological space", "neutrosophic supra topological space", "neutrosophic generalized topological space" and "neutrosophic topological space".

| S.No | Fuzzy supra topological space | Neutrosophic supra topological space | neutrosophic generalized topological space | neutrosophic topological space |
|---|---|---|---|---|
| 1 | It deals with fuzzy sets | It deals with neutrosophic sets | It deals with neutrosophic sets | **It deals with neutrosophic sets** |
| 2 | It is a generalization of classical supra-topological space | It is a generalization of Fuzzy supra-topological space | It is a generalization of neutrosophic supra-topological space | **It is a generalization of neutrosophic generalized topological space** |
| 3 | Every fuzzy topological space is fuzzy supra topological | Every neutrosophic topological space is a neutrosophic supra topological space | Every neutrosophic supra topological space is a neutrosophic generalized topological space | |

**Example 3.4.** Let $\aleph=\{p, q, r\}$ $g_N = \{0_N, \mathfrak{D}, \mathfrak{C}, \mathfrak{B}\}$; E ={ $0_N$, E , G , H }; $\mathfrak{D}$ ={< $p$, 0.3, 0.2 , 0.5 >, < $q$, 0.6, 0.5 , 0.3 >,< $r$, 0.7, 0.1 , 0.9 >}, $\mathfrak{C}$ ={< $p$, 0.4, 0.1 , 0.3 >, < $q$, 0.2, 0.6 , 0.7 >,< $r$, 0.1, 0.3 , 0.4 >},

$\mathfrak{B}$ ={< $p$, 0.4, 0.2 , 0.3 >, < $q$, 0.6, 0.6 , 0.3 >,< $r$, 0.7, 0.3 , 0.4 >}.

($\aleph$ , $g_N$) is (NGTS).

$0_N$, $\mathfrak{D}, \mathfrak{C}, \mathfrak{B}$ are NGOS.

**Example 3.5.** Let ($\aleph$ , $g_N$) be a (NGTS), then:

$0_N$ is a NGOS, $1_N$ is a NGCS.

**Remark 3.6.**

(i) Every NTS is NGTS. But, the converse is not true. see the following example.

(ii) Every NSTS is NGTS. But, the converse is not true. see the following example.

(iii) $g_N$ is not closed under arbitrary intersection, see the following example.

**Example 3.7.** In example 3.4, ($\aleph$, $g_N$) is NGTS, but ($\aleph$, $g_N$) is not NCTS nor NSTS.

In example 3.4, $\mathfrak{D}, \mathfrak{C}$ are neutrosophic generalized open sets, but $\mathfrak{D} \cap \mathfrak{C}$ ={< $p$, 0.3, 0.1 , 0.5 >, < $q$, 0.2, 0.5 , 0.7 >,< $r$, 0.1, 0.1 , 0.9 >} is not a "neutrosophic generalized open set".

**Example 3.8.** Let ($\aleph$, T) be a NCTS:

Let $g_N$ =NSO($\aleph$) or $g_N$ =NPO($\aleph$) or $g_N$ =N$\alpha$O($\aleph$) or $g_N$ =N$\beta$O($\aleph$) then $g_N$ is NGT on $\aleph$.

**Remark 3.9.** $1_N$ is a "neutrosophic open set" and $1_N$ is a "neutrosophic supra open set", but $1_N$ is not a "neutrosophic generalized open set".

**Remark 3.10.** Let ($\aleph$ , $g_N$) be a NGTS, then:

Since $g_N$ are closed under arbitrary union. Then the arbitrary intersection of any NGCS is an NGCS.

**Remark 3.11.** In any NSTS (or NTS) (X, S), the union of any two "neutrosophic supra open sets", maybe $1_N$, see the following example

**Example 3.12.** Let $\aleph$={ $p$ } S ={ $0_N$, $1_N$, $\mathfrak{D}, \mathfrak{C}, \mathfrak{B}$}; $\mathfrak{D}$ ={< $p$, 0.3, 0.2 , 0.5 >}, $\mathfrak{C}$ ={< $p$, 0.4, 0.1 , 0.3>,

$\mathfrak{B}$ ={< $\wp$, 0.4, 0.2, 0.3 >}.

Then (ℵ ,S) is a NSTS.

$0_N$, $1_N$, $\mathfrak{D}$, $\mathfrak{C}$, $\mathfrak{B}$ are "neutrosophic supra open sets". $1_N \cup \mathfrak{D} = 1_N$ .

- The following theorem shows that this is not true in an NGTS.

**Theorem 3.13.** Let (ℵ, $g_N$) be a NGTS. But it is not NSTS then, the union of any two "neutrosophic generalized open sets", fails to be $1_N$.

**Proof:**

Let Â, ß two NGOS, that satisfied Â∪ß $=1_N$.

Since Â, ß two NGOS, we get Â∪ß is NGOS, therefore $1_N$ is an NGOS, which contradicts with assumption.

## 4 | The Interior and Closure Operations

In this part, we define the "closure" and "interior" via a neutrosophic generalized open set.

**Definition 4.1**. Let (ℵ, $g_N$) be an NGTS, and Â is a neutrosophic set then:

The union of any NGOS, contained in Â is called the "neutrosophic generalized interior" of Â ($N_g$-i(Â)).

$N_g$-i(Â)=∪{ ß ; ß⊆Â ; ß∈ NGOS(ℵ) }.

**Theorem 4.2**. Let (ℵ , $g_N$) be a NGTS, Â, ß are "neutrosophic sets" then:

i. $N_g$-i (Â) ⊆ Â.

ii. $N_g$-i (Â) is NGOS.

iii. Â ⊆ ß ⇒ $N_g$-i (Â) ⊆ $N_g$-i (ß).

**Proof :**

i. Follow from the definition of Ng-i (Â) as a union of any "neutrosophic generalized open sets", contained in Â.

ii. since union of any NGOS, is NGOS, then Ng-i(Â)=∪{ ß ; ß⊆Â ; ß∈ NGOS(X)} is NGOS.

iii. Prof is Obvious.

**Definition 4.3**. Let (ℵ, $g_N$) be an NGTS, Â is neutrosophic set then:

The intersection of any NGCS, including Â, is called "neutrosophic generalized closure" of Â ( $N_g$-c(Â) ).

$N_g$-c(Â) =∩{ß ; ß⊇Â ; ß∈ NGCS(ℵ) }

**Theorem 4.4.** Let (ℵ , $g_N$) be a NGTS, Â is "NS" then:

i. Â ⊆ $N_g$-c (Â).

ii. $N_g$-c (Â) is NGCS.

**Proof :**

i. Since $N_g$-c(Â)is an intersection of any NGCS contained in Â.

ii. Follow from remark 3.9.

**Theorem 4.5.** Let (ℵ , $g_N$) be a NGTS, Â is neutrosophic set then:

i. $N_g$-c (ℵ-Â)= ℵ- $N_g$-i (Â).

ii. $N_g$-i (ℵ-Â)= ℵ- $N_g$-c (Â).

iii. $N_g$-i (Â)= ℵ- $N_g$-c (ℵ-Â).

iv. $N_g$-c (Â)= ℵ- $N_g$-i (ℵ-Â).

**Proof :**

i. ℵ- $N_g$-i(Â)= ℵ-[∪{ß ; ß ⊆ Â ; ß∈ NGOS(ℵ) }]

=∩{ℵ-ß ; ℵ-ß ⊇ ℵ-Â ; X-ß∈ NGOS(X) }= $N_g$-c(X-Â)

=∪{ℵ-ß ; ℵ-ß ⊆ Â ; ℵ-ß∈ NGOS(ℵ) }=ℵ- $N_g$-i(Â).

ii. ℵ- $N_g$-c (Â)= ℵ-[∩{ß ; ß⊇Â ; ß∈ NGCS(ℵ) }]

=∪{ℵ-ß ; ℵ-ß ⊆ ℵ-Â ; ℵ-ß∈ NGCS(ℵ) }= $N_g$-i (ℵ-Â)

iii. Follows from (2) by putting ℵ-Â in place of Â.

iv. Follows from (1) by putting ℵ-Â in place of Â.

**Theorem 4.6.** Let (ℵ , $g_N$) be a NGTS, and Â ⊆ℵ, then:

i. Â is NGCS, then $N_g$-c (Â) = Â.

ii. Â is NGOS, then $N_g$-i (Â)= Â.

**Proof :**

i. Follow the definition of Ng-c (Â) and Theorem 4.4.

ii. Follow the definition of Ng-i (Â) and Theorem 4.2.

**Theorem 4.7.** Let (ℵ, $g_N$) be an NGTS, Â is neutrosophic set then :

i. $N_g$-c [$N_g$-c(Â)]= $N_g$-c (Â).

ii. $N_g$-i [$N_g$-i (Â)]= $N_g$-i (Â).

**Proof :**

The proof is Obvious.

**Remark 4.8**. Let (ℵ , $g_N$) be a NGTS, Let Â, ß ⊆X, then:

i. $N_g$-i (Â∩ß)⊆ $N_g$-i (Â)∩$N_g$-i (ß).

ii. $N_g$-c (Â∩ß) ⊆ $N_g$-c (Â)∩$N_g$-c (ß).

iii. $N_g$-i (Â∪ß) ⊇ $N_g$-i (Â)∪$N_g$-i (ß).

iv. $N_g$-c (Â∪ß) ⊇ $N_g$-c (Â)∪$N_g$-c (ß).

**Proof:**

i. Since Â∩ß ⊆ Â, Â∩ß ⊆ ß then $N_g$-i (Â∩ß)⊆ $N_g$-i (Â) and $N_g$-i (Â∩ß)⊆ $N_g$-i (ß), hence $N_g$-i (Â∩ß)⊆ $N_g$-i (Â)∩$N_g$-i (Â).

ii. Since Â∩ß ⊆ Â, Â∩ß ⊆ ß then $N_g$-c (Â∩ß)⊆ $N_g$-c (Â) and $N_g$-c (A∩ß)⊆ $N_g$-c (ß), hence $N_g$-c (Â∩ß)⊆ $N_g$-c (Â)∩$N_g$-c (Â).

iii. Since $\hat{A}\subseteq\hat{A}\cup ß$, $ß\subseteq\hat{A}\cup ß$ then $N_g$-i $(\hat{A})\subseteq N_g$-i $(\hat{A}\cup ß)$ and $N_g$-i $(ß)\subseteq N_g$-i $(\hat{A}\cup ß)$, hence

$N_g$-i $(\hat{A})\cup N_g$-i $(ß)\subseteq N_g$-i $(\hat{A}\cap ß)$.

iv. Since $\hat{A}\subseteq\hat{A}\cup ß$, $ß\subseteq\hat{A}\cup ß$ then $N_g$-c $(\hat{A})\subseteq N_g$-c $(\hat{A}\cup ß)$ and $N_g$-c $(ß)\subseteq N_g$-c $(\hat{A}\cup ß)$, hence $N_g$-c $(\hat{A})\cup N_g$-c $(ß)\subseteq N_g$-c $(\hat{A}\cap ß)$.

**Remark 4.9.**

i. $N_g$-i $(\hat{A}\cap ß)= N_g$-i $(\hat{A})\cap N_g$-i $(ß)$.

ii. $N_g$-c $(\hat{A}\cup ß)= N_g$-c $(\hat{A})\cup N_g$-c $(ß)$.

It is easy to show that remark 4.9 is true in neutrosophic topological space (in this case, A, ß are neutrosophic sets in neutrosophic topological space). However, this remark 4.9 is not true in NGTS (in this case, A, ß are neutrosophic sets in NGTS), as shown in the following example.

**Example 4.10.** Let ℵ={p} S ={ $0_N$, $1_N$, E,G,H}; E ={< 0.5, 1 , 0 >}, G ={< 0.25, 0 , 1>,

H ={< 0.5, 1 , 1 >}, C ={< 0.5, 0.5 , 0>, D ={< 0.5, 0 , 0.5 >}.

then

$N_g c(C)$ = {< 0.75, 1 , 0 >}, $N_g c(D)$= {< 0.5, 0 , 1 >}

$C\cup D$={< 0.5, 0.5 , 0.5 >}, $N_g c(C\cup D)$= {< 1, 1 , 1 >}.

$N_g c(C\cup D)\neq N_g c(C)\cup N_g c(D)$ = {< 0.75, 1 , 1 >},

N={< 0.5, 1 , 0.25 >}, M= {< 0.5, 0.5 , 1 >}.

then

$N_g i(N)$ ={< 0.5, 1 , 0 >} $N_g i(D)$={< 0.25, 0 , 1 >}

$C\cap D$={< 0.5, 0.5 , 0.25 >}, $N_g i(C\cap D)$={< 0, 0 , 0 > }.

$N_g i\ (C\cap D)\neq N_g i\ (C)\cap N_g i\ (D)$ ={< 0.25, 0 , 0 > }.

## 5 | New Neutrosophic Generalized Sets

**Definition 5.1.** Let (ℵ , $g_N$) be a NGTS, then:

i. Â "NS" U is said to be a "neutrosophic generalized pre-open set" (NGPOS) if U ⊆ Ng-i [Ng-c (U)].

ii. Â neutrosophic set U is said to be a" neutrosophic generalized semi-open set" (NGSOS) if U⊆Ng-c [Ng-I (U)].

iii. Â neutrosophic set U is said to be a "neutrosophic generalized α-open set" (NGαOS) if U⊆ Ng-i (Ng-c [Ng-i (U)]).

**Definition 5.2.** Let (ℵ , $g_N$) be a NGTS, then:

i. Â Neutrosophic set F is said to be a "neutrosophic generalized pre-closed set" (NGPCS) if $U^c$ is NGPOS. The family of NGPOS(NGPCS) set in ℵ is denoted by NGPOS(ℵ, $g_N$) ( NGPCS(ℵ, $g_N$)).

ii. Â neutrosophic set F is said to be a "neutrosophic generalized semi-closed set" if $U^c$ is NGSOS. The family of NGSOS(NGSCS) in ℵ is denoted by NGSOS(ℵ, $g_N$) (NGSCS(ℵ, $g_N$)).

iii. Â neutrosophic set F is said to be Â "neutrosophic generalized α-closed set" (NGαCS) if $U^c$ is NGαOS. The family of NGαOS (NGαCS) in ℵ is denoted by NGαOS(ℵ, $g_N$) (NGαCS(ℵ, $g_N$)).

**Theorem 5.3.** Let (ℵ, gN) be an NGTS then, Every NGOS is NGSOS.

**Proof.**

Let U be a NGOS in (ℵ, gN), We know that U⊆$N_g$-c(Â), and since U is an NGOS, we get U=$N_g$-i(Â), therefore U⊆$N_g$-c($N_g$-i(Â)).

Therefore U is NGSOS.

**Remark 5.4.** The converse of the above theorem 5.3 is not necessarily true, see the following example.

**Example 5.5.** Let ℵ={𝓅, 𝓆, 𝓇} $g_N$ ={ $0_N$, 𝔇, 𝔊}; 𝔇 ={< 0.4, 0.5 , 0.2 >, < 0.3, 0.2 , 0.1 >,< 0.9, 0.6 , 0.8 }, 𝔊 ={< 0.2, 0.4 , 0.5 >, < 0.1, 0.1 , 0.2 >,< 0.6, 0.5 , 0.8 >},

𝓐 ={< 0.5, 0.6 , 0.1 >, < 0.4, 0.3 , 0.1 >,< 0.9, 0.8 , 0.5 >}.

Then (ℵ ,$g_N$) is NGTS.

𝓐 is NGSOS, but 𝓐 is not NGOS.

**Theorem 5.6.** Let (ℵ, gN) be an NGTS then, Every NGαOS is NGSOS.

**Proof.**

Let 𝓐 be a NGαOS in (X , gN), then 𝓐 ⊆ Ng-i(Ng-c[Ng-i(𝓐)])…(1), since Ng-i(Ng-c[Ng-i(𝓐)])⊆ Ng-c[Ng-i(𝓐)]….(2) .

(1),(2) ⇒ U⊆ Ng-c[Ng-i(𝓐)], Therefore 𝓐 is NGSOS.

**Remark 5.7.** The converse of above theorem 5.6 is not necessarily true, see the following example.

**Example 5.8.** Let ℵ={p}, $g_N$ ={ $0_N$, E,G}; E ={< 0.4, 0.5 , 0.5 >}, G ={< 0.3, 0.4 , 0.4 >},

Then (ℵ ,$g_N$) is NGTS.

H ={< 0.5, 0.5 , 0.5 >} is NGSO, but H is not NGαOS.

**Theorem 5.9.** Let (ℵ, gN) be NGTS then, Every NGOS is NGPOS.

**Proof.**

Let U be a NGOS in (ℵ, gN), then U=$N_g$-i(Â), since

U⊆$N_g$-c(U), then $N_g$-i(U)⊆$N_g$-i($N_g$-c(Â)).

therefore U ⊆ $N_g$-i($N_g$-c(Â))]. hence U is NGPOS.

**Remark 5.10.** The Converse of theorem 5.9 is not true, see example 3.11.

**Example 5.11.** Let ℵ={p, q, r} $g_N$ ={ $0_N$, E, G}; E ={< 0.4, 0.5 , 0.4 >, < 0.5, 0.5 , 0.5 >,< 0.4, 0.5 , 0.4 }, G ={< 0.7, 0.6 , 0.5 >, < 0.3, 0.4 , 0.5 >,< 0.3, 0.4 , 0.4 >}, H ={< 0.5, 0.5 , 0.5 >, < 0.5, 0.5 , 0.5 >,< 0.5, 0.5 , 0.5 >}.

Then (ℵ ,$g_N$) is NGTS.

H is NGPOS, but H is not NGOS.

**Theorem 5.12.** Let (ℵ, gN) be an NGTS then, every NGOS is NGαOS.

**Proof.**

Let U be a NGOS in (ℵ, gN), We now U⊆$N_g$-c(Â), and since U is a NGOS, we get U=$N_g$-i(Â). Then U⊆$N_g$-c($N_g$-i(Â)). Hence $N_g$-i (U)⊆ $N_g$-i($N_g$-c($N_g$-i(Â))), then U⊆ $N_g$-i($N_g$-c($N_g$-i(Â))).

**Remark 5.13.** The Converse of theorem 5.12 is not true, see example 4.14.

**Example 4.14.** Let ℵ={p, q} $g_N$ ={ $0_N$, E, G}; E ={< 0.3, 0.3 , 0.4 >, < 0.4, 0.4 , 0.5 >}, G ={< 0.4, 0.4, 0.5 >, < 0.2, 0.2, 0.3 >}, H ={< 0.4, 0.4, 0.3 >, < 0.6, 0.6, 0.4 >}.

Then (ℵ ,$g_N$) is NGTS.

H is NGαOS, but H is not NGOS.

**Theorem 5.15.** Let (ℵ, gN) be an NGTS then, every NGαOS is NGPOS.

**Proof.**

Let U be a NGαOS in (ℵ , gN), then U⊆ Ng-i(Ng-c[Ng-i(U)])…(1), since Ng-i(Ng-c[Ng-i(U)])⊆ Ng-i[Ng-c(U)]….(2) .

(1),(2) ⇒ U⊆ Ng-i[Ng-c(U)], Therefore U is NGPOS.

**Remark 5.16.** The converse of the above theorem 4.18 is not necessarily true, see the following example.

**Example 5.17.** Let ℵ={p, q} $g_N$ ={ $0_N$, E, G}; E ={< 0.3, 0.3 , 0.4 >, < 0.5, 0.5 , 0.5 >}, G ={< 0.4, 0.4 , 0.5 >, < 0.3, 0.3 , 0.4 >}.Then (ℵ ,$g_N$) is NGTS.

H ={< 0.4, 0.4 , 0.5 >, < 0.5, 0.5 , 0.4 >} is NGPOS, but H is not NGαOS.

**Remark 5.18.** Relations among the new types of generalized neutrosophic open sets that were studied in this paper in the following Figure 1:

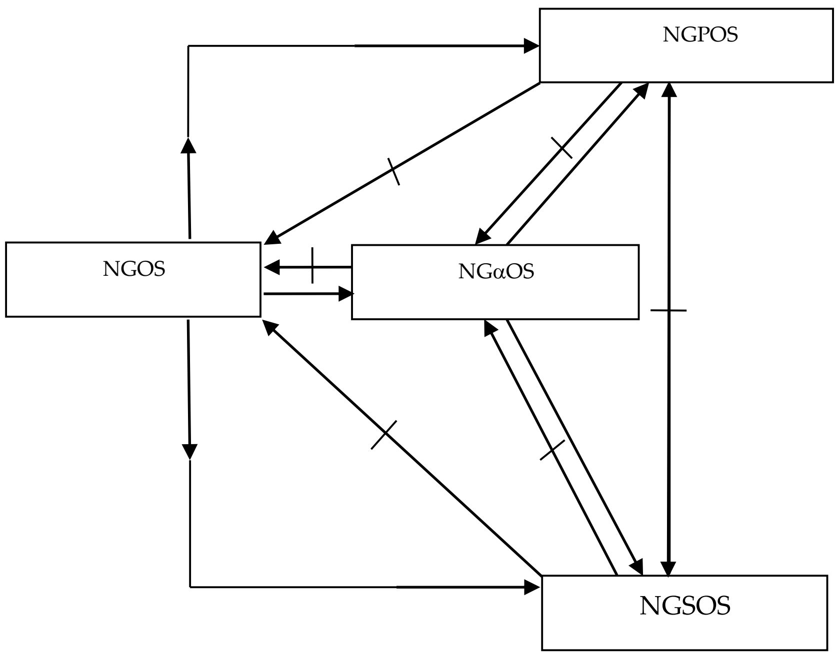


**Figure 1.** Relations among the new types of generalized neutrosophic open sets.

## 6 | Conclusion

The neutrosophic generalized topological structure, which is a more general structure than neutrosophic supra topological spaces is built on neutrosophic sets. Also, we study a new type of neutrosophic open and closed in this space. These new neutrosophic sets can be used to find new research about, neutrosophic connectedness, neutrosophic compactness, neutrosophic continuity, and neutrosophic separation axioms in NGTS.

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## Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

## Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

## Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

# References


[1] Csaszar, A. (2002). Generalized topology, generalized continuity, Acta Math. Hungar.,96,351-357.

[2] Mashhour, S. Allam, A. A. Mahmoud, F.S. & Khedr, F.H. (1983). On Supra topological spaces, Indian Jr.Pure and Appl.Math.No.4(14), 502-510.

[3] Smarandache, F. (1999). A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM.

[4] Smarandache, F. (2002). Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA.

[5] Salma, A. A., & Alblowi, S.A. (2012). Neutrosophic set and neutrosophic topological spaces. IOSR J. Math., 3(4), 31–35.

[6] Jayaparthasarathy, G. Little Flower, V. F., & Arockia Dasan,. M. (2019).Neutrosophic Supra Topological Applications in Data Mining Process, Neutrosophic Sets and Systems, ,vol. 27, pp. 80-97.

[7] Abdel-Basset, M., Mai, M., Mohamed, E., Francisco, C. & Abd El-Nasser, H. Z. (2019).Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases, Artificial Intelligence in Medicine 101, 101735.

[8] Abdel-Basset, M., Gunasekaran, M., Abduallah, G., & Victor, C. (2019). A Novel Intelligent Medical Decision Support Model Based on Soft Computing and IoT, IEEE Internet of Things Journal.

[9] Abdel-Basset, M., Mohamed, E., Abduallah G., & Florentin, (2019).F, A novel model for evaluation Hospital medical care systems based on plithogenic sets, Artificial intelligence in medicine 100, 101710.

[10] Abdel-Basset, M., Abduallah, G. , Gunasekaran, M., & Hoang, L. (2019). A novel group decision making model based on neutrosophic sets for heart disease diagnosis, Multimedia Tools and Applications, 1-26.

[11] Arokiarani, I., Dhavaseelan, R., Jafari, S., & Parimala, M. (2016). On Some New Notions and Functions in Neutrosophic Topological Spaces, Neutrosophic Sets and Systems, vol. 13, 96–104.

[12] Al-Hamido, R. K. (2020).Neutrosophic Crisp Supra Bi-Topological Spaces, International Journal of Neutrosophic Science, Vol 4, Issue 1, 36-46.

[13] Al-Hamido, R. K. (2019). A study of multi-Topological Spaces, PhD Theses, AlBaath university, Syria.

[14] Al-Hamido, R. K. (2018). Neutrosophic Crisp Bi-Topological Spaces, Neutrosophic Sets and Systems, vol. 21, 66-73.

[15] Al- Omeri, W. F. (2016). Neutrosophic crisp sets via neutrosophic crisp topological spaces, Neutrosophic Sets and Systems, Vol. 13, 96–104.