Plithogenic Logic and Computation

Journal Homepage: sciencesforce.com/plc

Plithogenic Log. Comp. Vol. 2 (2024) 10–19

Paper Type: Original Article

Functions of $\mathbf{Neu}_{g\zeta^*H}$ in NT Spaces

Myvizhi Muthuswamy [1,*]

[1] Department of Mathematics, KPR Institute of Engineering and Technology, Coimbatore, 641407, Tamilnadu, India; myvizhi.m@kpriet.ac.in.

Received: 16 Feb 2024 **Revised:** 25 May 2024 **Accepted:** 24 Jun 2024 **Published:** 26 Jun 2024

Abstract

In this paper, the concept of $g\zeta^*$-Neutrosophic Continuous Functions and $g\zeta^*$-Neutrosophic Irresolute Functions were introduced and their characterizations were analyzed. Furthermore, the work is extended to $g\zeta^*$-Homeomorphism and some of its properties are explored in Neutrosophic topological spaces.

Keywords: Neutrosophic Continuous Functions, Neutrosophic Irresolute Functions, Homeomorphism, Neutrosophic Topological Spaces.

1 | Introduction

Topology is traditionally defined as the mathematical study of shapes and topological spaces. Topology is an area of mathematics, which deals with the properties of space that is preserved under continuous deformations including stretching and bending. The term topology was introduced by Johann Benedict Listing in the 19th century. The theory of fuzzy topological spaces was introduced and developed by C.L Chang (1968) [9]. Lowen innovatively proposed fuzzy topology in 1976. Later topological structures in fuzzy topological spaces were generalized to intuitionistic fuzzy topological spaces by Coker (1997) [10], A.A.Salama (2012) [52], and S.A.Albowi (2012) [53] devised the concept of Neutrosophic topological spaces. They extended the concept to generalized Neutrosophic topological spaces, and Neutrosophic Crisp topological concepts and studied various properties.

General topology or point-set topology is one of the most basic and traditional divisions within topology which studies the topological properties along with its structure. It is the foundation for several areas of research in topology such as Nano topology, digital topology, fuzzy topology, supra topology, Bi topology, and so on. Many authors like Abd El-Monsef M.E[1-3], Balachandaran K[11-13], Dontchev J[18-26], Hatir E[29-31], Jafari S[4, 14-17, 32-34], Jankovic D[35-37], Lellis Thivagar M[38-44], Levine N[45-48], Noiri T[49,50], Sundaram P[55-59], Tong J[60,61] and Veerakumar M.K.R.S[62] have contributed in the field of general topology.

A fuzzy set is a class of objects with a continuum of grades of membership and is characterized by the membership (characteristic) function, which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established.

Corresponding Author: myvizhi.m@kpriet.ac.in

https://doi.org/10.61356/j.plc.2024.2313

The traditional fuzzy set was introduced by Lot Aliasker Zadeh (1965) [63] which was characterized by the grade of the membership value. A fuzzy subset A of a universal set U is a function $A: U \rightarrow I$, where I is the interval [0, 1] and is called a membership function.

The intuitionistic fuzzy set is an extension of the fuzzy set introduced by Atanassov (1983) [5-7] and is found to be more efficient in dealing with vagueness and ambiguity. It is characterized by a membership function $\mu A(x)$ and a non-membership function $\gamma A(x)$ with their sum being less than or equal to one $\mu A(x)+\gamma A(x) \leq 1$. This relaxes the enforced duality $\gamma A(x) \leq 1-\mu A(x)$ from fuzzy set theory. An intuitionistic fuzzy set allows one to address the positive and negative sides of an imprecise concept separately. An intuitionistic fuzzy set is beneficial in decision-making problems, particularly in the case of medical diagnosis, sales analysis, new product marketing, financial services, etc. In recent times various applications of intuitionistic fuzzy sets have been used in artificial intelligence like intuitionistic fuzzy expert systems, intuitionistic fuzzy neural networks, intuitionistic fuzzy decision making, intuitionistic fuzzy machine learning, and intuitionistic fuzzy semantic representations. Using the notion of intuitionistic fuzzy sets, Dogan Coker (1997) [10] introduced the notion of intuitionistic fuzzy topological spaces. Exploring the fundamental definitions with appropriate examples, he acquainted the definitions of intuitionistic fuzzy continuity, intuitionistic fuzzy compactness, and intuitionistic fuzzy connectedness and obtained several preservation properties and some characterizations concerning intuitionistic fuzzy connectedness.

The neutrosophic set was introduced by Smarandache [54] and the details of the neutrosophic sets are a generalization of intuitionistic fuzzy set. Salama A. A and Alblowi [53] introduced Neutrosophic topological space by using Neutrosophic sets in the year of 2012 and also Neutrosophic closed set and neutrosophic continuous function were introduced by the same author Salama A. A [52]. Arokiarani et al. [1] introduced the neutrosophic α-closed set in the year of 2017. Neutrosophic Homeomorphism plays an important role in Neutrosphic topology. Parimala M et al [51] presented by Neutrosophic Homeomorphism.

In this work, we introduced the $Neu_{g\zeta^*}$-CF and $Neu_{g\zeta^*}$-IF and also investigate the characteristics of $Neu_{g\zeta^*H}$-Functions.

2 | Preliminary Results

Definition 2.1. [53] Let $\mathbb{J}$ be a non-empty fixed set. A Neu set E is an object having the form $E = \{\langle j, \mu_p(E(j)), \sigma_q(E(j)), \upsilon_r(E(j))\rangle \, for\ every\ j \in \mathbb{J}\}$, where $\mu_p(E(j))$ represents the membership, $\sigma_q(E(j))$ represents indeterminacy and $\upsilon_r(E(j))$ represents non-membership functions of each element $j \in \mathbb{J}$ to the set E.

Remark 2.1. [53] A Neu set $E = \{\langle j, \mu_p(E(j)), \sigma_q(E(j)), \upsilon_r(E(j))\rangle \, for\ every\ j \in \mathbb{J}\}$ and it can be denoted as an ordered triple $\{\langle \mu_p(E(j)), \sigma_q(E(j)), \upsilon_r(E(j))\rangle\}$ on $\mathbb{J}$.

Definition 2.2. [51] In Neu Topological Spaces,

for every $j \in \mathbb{J}$, $0_{(N)}$ defined as

$0_{(N)} = \langle j, 0, 0, 1\rangle$

$0_{(N)} = \langle j, 0, 1, 1\rangle$

$0_{(N)} = \langle j, 0, 1, 0\rangle$

$0_{(N)} = \langle j, 0, 0, 0\rangle$

for every $j \in \mathbb{J}$, $1_{(N)}$ defined as

$1_{(N)} = \langle j, 1, 0, 0\rangle$

$1_{(N)} = \langle j, 1, 0, 1\rangle$

$1_{(N)} = \langle j, 1, 1, 0\rangle$

$1_{(N)} = \langle j, 1, 1, 1\rangle$

Definition 2.3. [51] Let E be the neu set of the form $E = \{\langle j, \mu_p(E(j)), \sigma_q(E(j)), v_r(E(j))\rangle \; for\; every\; j \in \mathbb{J}\}$and then$[E^c]$ defined as: $E^c = \{\langle j, 1 - \mu_p(E(j)), 1 - \sigma_q(E(j)), 1 - v_r(E(j))\rangle \; for\; every\; j \in \mathbb{J}\}$.

Definition 2.4. [52] Let E and F be two neu sets of the form, $E = \{\langle j, \mu_p(E(j)), \sigma_q(E(j)), v_r(E(j))\rangle \; for\; every\; j \in \mathbb{J}\}$

and $F = \{\langle j, \mu_p(F(j)), \sigma_q(F(j)), v_r(F(j))\rangle \; for\; every\; j \in \mathbb{J}\}$. Then,

i). The Subsets of $E \; and \; F$ defined as $E \subseteq F$ if and only if $\mu_p(E(j)) \leq \mu_p(F(j)), \sigma_q(E(j)) \geq \sigma_q(F(j)), v_r(E(j)) \geq v_r(F(j))$

ii). The Subsets of $E = F$ if and only if $E \subseteq F$ and $F \subseteq E$

iii). The Union of subsets $E \; and \; F$ defined as

$E \cup F = \{\mathrm{j}, \; \max[\mu_p(E(j)), \mu_p(F(j))], \max\, [\sigma_q(E(j)), \sigma_q(F(j))], \min[v_r(E(j)), v_r(F(j))]$ $for\; every\; j \in \mathbb{J}\}$.

iv). The Intersection of subsets $E \; and \; F$ defined as

$E \cap F = \{\mathrm{j}, \min[\mu_p(E(j)), \mu_p(F(j))], \min\, [\sigma_q(E(j)), \sigma_q(F(j))], \max[v_r(E(j)), v_r(F(j))]$ $for\; every\; j \in \mathbb{J}\}$.

Definition 2.5. [52] A Neu topological space $(\mathbb{J}, \tau)$ satisfies the following conditions:

i). $0_{(N)}, 1_{(N)} \in \tau$

ii). $K_1 \cap K_2 \in \tau \; for\; any\; K_1, K_2 \in \tau$

iii). $\cup K_i \in \tau \; for\; every\; \{K_i : i \in I\} \subseteq \tau$.

Then $(\mathbb{J}, \tau)$ is called a Neu topological space.

Definition 2.6. [54] Let E be a Neu set in $(\mathbb{J}, \tau)$. Then

i). $Neu_{int(E)} = \cup\{F \;/\; F$ is a $Neu\text{-}O$ set in $(\mathbb{J}, \tau)$ and $F \subseteq E\}$;

ii). $Neu_{cl(E)} = \cap\{F \;/\; F$ is a $Neu\text{-}C$ set in $(\mathbb{J}, \tau)$ and $F \supseteq E\}$.

3 | $\mathbf{Neu}_{g\zeta^*}$-CF in NTS's

Definition 3.1. A function $\mathcal{D}: (\mathcal{R}, \vartheta) \to (S, \omega)$ is said to be $Neu_{g\zeta^*}$-Continuous Functions (briefly $Neu_{g\zeta^*}$-CF) if $\mathcal{D}^{-1}(\mathcal{P})$ is $Neu_{g\zeta^*}$-CS in $(\mathcal{R}, \vartheta)$ for each Neu-CS in (S, ω).

Theorem 3.1. Each $Neu_{g\zeta^*}$-CF is Neu_{gs}-CF (resp. $Neu_{\alpha g}$-CF, Neu_{gp}-CF, Neu_{gsp}-CF). Converse is not true as shown in the following example.

Proof: Let $\mathcal{P}$ be a Neu-CS in (S, ω). Since $\mathcal{D}$ is $Neu_{g\zeta^*}$-CF. $\mathcal{D}^{-1}(\mathcal{P})$ is $Neu_{g\zeta^*}$-CS in$(\mathcal{R}, \vartheta)$. Since each $Neu_{g\zeta^*}$-CS is Neu_{gs}-CS (resp. $Neu_{\alpha g}$-CS, Neu_{gp}-CS, Neu_{gsp}-CS), therefore $\mathcal{D}^{-1}(\mathcal{P})$ is Neu_{gs}-CS (resp. $Neu_{\alpha g}$-CS, Neu_{gp}-CS, Neu_{gsp}-CS) in $(\mathcal{R}, \vartheta)$. Hence $\mathcal{D}$ is Neu_{gs}-CF (resp. $Neu_{\alpha g}$-CF, Neu_{gp}-CF, Neu_{gsp}-CF).

Example 3.1. Assume $\mathbb{I} = \{u, v, w\}$ and then the Neu sets D_1, D_2, D_3, D_4 and H_1, H_2, H_3, H_4 are defined as

$D_1 = \{(0.3,0.3,0.2), (0.3,0.2,0.2), (0.4,0.6,0.6)\}$

$D_2 = \{(0.4,0.6,0.6), (0.5,0.4,0.4), (0.3,0.2,0.2)\}$

$D_3 = \{(0.4,0.6,0.6), (0.3,0.2,0.2), (0.3,0.3,0.2)\}$

$D_4 = \{(0.3,0.3,0.2), (0.5,0.4,0.4), (0.4,0.6,0.6)\}$ and

$H_1 = \{(0.4,0.4,0.4), (0.5,0.4,0.4), (0.3,0.2,0.2)\}$

$H_2 = \{(0.3,0.2,0.2), (0.3,0.2,0.2), (0.4,0.6,0.6)\}$

$H_3 = \{(0.4,0.4,0.4), (0.3,0.2,0.2), (0.3,0.2,0.2)\}$

$H_4 = \{(0.3,0.2,0.2), (0.5,0.4,0.4), (0.4,0.6,0.6)\}$

$Neu_{g\zeta^*}\text{-}CS = \{(0.2,0.2,0.2), (0.5,0.4,0.4), (0.3,0.3,0.2)\}$ and

$Neu_{gs}\text{-}CS = \{(0.3,0.2,0.2), (0.3,0.2,0.2), (0.4,0.4,0.4)\}$. Here $\mathcal{D}^{-1}({H_3}^c)$ is $Neu_{gs}\text{-}CS$ not $Neu_{g\zeta^*}\text{-}CS$.

Example 3.2. Assume $\mathbb{I} = \{u, v, w\}$ and then the Neu sets D_1, D_2, D_3, D_4 and I_1, I_2, I_3, I_4 are defined as

$D_1 = \{(0.3,0.3,0.2), (0.3,0.2,0.2), (0.4,0.6,0.6)\}$

$D_2 = \{(0.4,0.6,0.6), (0.5,0.4,0.4), (0.3,0.2,0.2)\}$

$D_3 = \{(0.4,0.6,0.6), (0.3,0.2,0.2), (0.3,0.3,0.2)\}$

$D_4 = \{(0.3,0.3,0.2), (0.5,0.4,0.4), (0.4,0.6,0.6)\}$ and

$I_1 = \{(0.3,0.3,0.4), (0.4,0.3,0.3), (0.3,0.4,0.4)\}$

$I_2 = \{(0.3,0.3,0.2), (0.4,0.3,0.4), (0.3,0.5,0.5)\}$

$I_3 = \{(0.3,0.3,0.2), (0.4,0.3,0.4), (0.3,0.5,0.5)\}$

$I_4 = \{(0.3,0.3,0.4), (0.4,0.3,0.3), (0.3,0.4,0.4)\}$

$Neu_{g\zeta^*}\text{-}CS = \{(0.2,0.2,0.2), (0.5,0.4,0.4), (0.3,0.3,0.2)\}$ and

$Neu_{\alpha g}\text{-}CS = \{(0.3,0.5,0.5), (0.4,0.3,0.3), (0.3,0.3,0.2)\}$. Here $\mathcal{D}^{-1}({H_4}^c)$ is $Neu_{\alpha g}\text{-}CS$ not $Neu_{g\zeta^*}\text{-}CS$.

Example 3.3 Assume $\mathbb{I} = \{u, v, w\}$ and then the Neu sets D_1, D_2, D_3, D_4 and F_1, F_2, F_3, F_4 are defined as

$D_1 = \{(0.3,0.3,0.2), (0.3,0.2,0.2), (0.4,0.6,0.6)\}$

$D_2 = \{(0.4,0.6,0.6), (0.5,0.4,0.4), (0.3,0.2,0.2)\}$

$D_3 = \{(0.4,0.6,0.6), (0.3,0.2,0.2), (0.3,0.3,0.2)\}$

$D_4 = \{(0.3,0.3,0.2), (0.5,0.4,0.4), (0.4,0.6,0.6)\}$ and

$F_1 = \{(0.4,0.5,0.6), (0.5,0.6,0.6), (0.4,0.5,0.3)\}$

$F_2 = \{(0.4,0.5,0.5), (0.5,0.4,0.4), (0.3,0.4,0.5)\}$

$F_3 = \{(0.4,0.5,0.6), (0.5,0.4,0.4), (0.3,0.4,0.5)\}$

$F_4 = \{(0.4,0.5,0.5), (0.5,0.6,0.6), (0.4,0.5,0.3)\}$

$Neu_{g\zeta^*}\text{-}CS = \{(0.2,0.2,0.2), (0.5,0.4,0.4), (0.3,0.3,0.2)\}$ and

$Neu_{gp}\text{-}CS = \{(0.3,0.4,0.5), (0.5,0.4,0.4), (0.3,0.4,0.5)\}$. Here $\mathcal{D}^{-1}({F_3}^c)$ is $Neu_{gp}\text{-}CS$ not $Neu_{g\zeta^*}\text{-}CS$.

Example 3.4. Assume $\mathbb{I} = \{u, v, w\}$ and then the Neu sets D_1, D_2, D_3, D_4 and J_1, J_2, J_3, J_4 are defined as

$D_1 = \{(0.3,0.3,0.2), (0.3,0.2,0.2), (0.4,0.6,0.6)\}$

$D_2 = \{(0.4,0.6,0.6), (0.5,0.4,0.4), (0.3,0.2,0.2)\}$

$D_3 = \{(0.4,0.6,0.6), (0.3,0.2,0.2), (0.3,0.3,0.2)\}$

$D_4 = \{(0.3,0.3,0.2), (0.5,0.4,0.4), (0.4,0.6,0.6)\}$ and

$J_1 = \{(0.4,0.6,0.4), (0.5,0.4,0.3), (0.4,0.6,0.5)\}$

$J_2 = \{(0.3,0.6,0.5), (0.3,0.2,0.2), (0.4,0.4,0.4)\}$

$J_3 = \{(0.4,0.6,0.4), (0.3,0.2,0.2), (0.4,0.4,0.4)\}$

$J_4 = \{(0.3,0.6,0.5), (0.5,0.4,0.3), (0.4,0.6,0.5)\}$

$Neu_{g\zeta^*}$-$CS = \{(0.2,0.2,0.2), (0.5,0.4,0.4), (0.3,0.3,0.2)\}$ and

Neu_{gsp}-$CS = \{(0.4,0.6,0.5), (0.5,0.4,0.3), (0.3,0.6,0.5)\}$. Here $\mathcal{D}^{-1}({J_4}^c)$ is Neu_{gsp}-CS not $Neu_{g\zeta^*}$-CS.

Theorem 3.2. The composition of two $Neu_{g\zeta^*}$-CF's is also a $Neu_{g\zeta^*}$-CF.

Proof: Let $\mathcal{D}: (\mathcal{R}, \vartheta) \to (S, \omega)$ and $\ell: (S, \omega) \to (\mathcal{W}, \rho)$ are two $Neu_{g\zeta^*}$-CF's.

Let $\mathfrak{M}$ be a Neu-CS in $(\mathcal{W}, \rho)$, then $l^{-1}(\mathfrak{M})$ is Neu-CS in (S, ω), since l is Neu-continuous, $Neu_{g\zeta^*}$-Continuity of $\mathcal{D}$ implies that $\mathcal{D}^{-1}\big(l^{-1}(\mathfrak{M})\big) = (l \circ \mathcal{D})(\mathfrak{M})$ is $Neu_{g\zeta^*}$-CS in $(\mathcal{R}, \vartheta)$. Hence $l \circ \mathcal{D}$ is $Neu_{g\zeta^*}$-CF.

4 | $\mathbf{Neu_{g\zeta^*}}$-IF in NTS's

Definition 4.1. A function $\mathcal{D}: (\mathcal{R}, \vartheta) \to (S, \omega)$ is called $Neu_{g\zeta^*}$-Irresolute Functions (briefly $Neu_{g\zeta^*}$-IF) if $\mathcal{D}^{-1}(\mathfrak{M})$ is a $Neu_{g\zeta^*}$-CS of $(\mathcal{R}, \vartheta)$ for every $Neu_{g\zeta^*}$-CS $\mathfrak{M}$ of (S, ω).

Theorem 4.1. Let $\mathcal{D}: (\mathcal{R}, \vartheta) \to (S, \omega)$ and $\ell: (S, \omega) \to (\mathcal{W}, \rho)$ are any two functions, then

i). $l \circ \mathcal{D} : (\mathcal{R}, \vartheta) \to (\mathcal{W}, \rho)$ is $Neu_{g\zeta^*}$-CF if l is Neu-CF and $\mathcal{D}$ is $Neu_{g\zeta^*}$-CF.

ii). $l \circ \mathcal{D} : (\mathcal{R}, \vartheta) \to (\mathcal{W}, \rho)$ is $Neu_{g\zeta^*}$-IF if both l and $\mathcal{D}$ is $Neu_{g\zeta^*}$-IF.

iii). $l \circ \mathcal{D} : (\mathcal{R}, \vartheta) \to (\mathcal{W}, \rho)$ is $Neu_{g\zeta^*}$-CF if l is $Neu_{g\zeta^*}$-CF and $\mathcal{D}$ is $Neu_{g\zeta^*}$-IF.

Proof:

i). Let us assume that $\mathcal{P}$ is a Neu-CS in $(\mathcal{W}, \rho)$. Since l is Neu-CF, $l^{-1}(\mathcal{P})$ is Neu-CS in (S, ω). Since $\mathcal{D}$ is $Neu_{g\zeta^*}$-CF, $\mathcal{D}^{-1}\big(l^{-1}(\mathcal{P})\big) = (l \circ \mathcal{D})^{-1}(\mathcal{P})$ is $Neu_{g\zeta^*}$-CS in $(\mathcal{R}, \vartheta)$, Therefore $l \circ \mathcal{D}$ is $Neu_{g\zeta^*}$-CF.

ii). Let us assume that $\mathcal{P}$ is a $Neu_{g\zeta^*}$-CS in $(\mathcal{W}, \rho)$. Since l is $Neu_{g\zeta^*}$-IF, $l^{-1}(\mathcal{P})$ is $Neu_{g\zeta^*}$-CS in (S, ω). Since $\mathcal{D}$ is $Neu_{g\zeta^*}$-IF, $\mathcal{D}^{-1}\big(l^{-1}(\mathcal{P})\big) = (l \circ \mathcal{D})^{-1}(\mathcal{P})$ is $Neu_{g\zeta^*}$-CS in $(\mathcal{R}, \vartheta)$, Therefore $l \circ \mathcal{D}$ is $Neu_{g\zeta^*}$-IF.

iii). Let us assume that $\mathcal{P}$ is a Neu-CS in $(\mathcal{W}, \rho)$. Since l is $Neu_{g\zeta^*}$-CF , $l^{-1}(\mathcal{P})$ is $Neu_{g\zeta^*}$-CS in (S, ω). Since $\mathcal{D}$ is $Neu_{g\zeta^*}$-IF, $\mathcal{D}^{-1}\big(l^{-1}(\mathcal{P})\big) = (l \circ \mathcal{D})^{-1}(\mathcal{P})$ is $Neu_{g\zeta^*}$-CS in $(\mathcal{R}, \vartheta)$, Therefore $l \circ \mathcal{D}$ is $Neu_{g\zeta^*}$-CF.

5 | $\mathbf{Neu_{g\zeta^* H}}$-Functions in NTS's

Definition 5.1. A function $\mathcal{D}: (\mathcal{R}, \vartheta) \to (S, \omega)$ is said to be $Neu_{g\zeta^* H}$-Functions if $\mathcal{D}$ is bijective, $\mathcal{D}$ and $\mathcal{D}^{-1}$ are $Neu_{g\zeta^*}$-CF.

Theorem 5.1. Each $Neu_{g\zeta^*H}$-Functions is Neu_{gsH}-Functions.

Proof: Let $\mathcal{D}: (\mathcal{R}, \vartheta) \rightarrow (S, \omega)$ be $Neu_{g\zeta^*H}$-Functions, then $\mathcal{D}$ is bijective, $Neu_{g\zeta^*}$-CF and $Neu_{g\zeta^*}$-OF. Let $\mathfrak{M}$ be Neu-CS in (S, ω), then $\mathcal{D}^{-1}(\mathfrak{M})$ is $Neu_{g\zeta^*}$-CS in $(\mathcal{R}, \vartheta)$. Since each $Neu_{g\zeta^*}$-CS is Neu_{gs}-CS, then $\mathcal{D}^{-1}(\mathfrak{M})$ is Neu_{gs}-CS in $(\mathcal{R}, \vartheta)$, Therefore $\mathcal{D}$ is Neu_{gs}-CF. Let $\mathfrak{J}$ be Neu-OS in $(\mathcal{R}, \vartheta)$, then $\mathcal{D}(\mathfrak{J})$ is $Neu_{g\zeta^*}$-OS in (S, ω). Since each $Neu_{g\zeta^*}$-OS is Neu_{gs}-OS, then $\mathcal{D}(\mathfrak{J})$ is Neu_{gs}-OS in (S, ω), therefore $\mathcal{D}$ is Neu_{gs}-OF. Hence $\mathcal{D}$ is Neu_{gsH}-Functions.

Example 5.1. Assume $\mathbb{I} = \{u, v, w\}$ and then the Neu sets D_1, D_2, D_3, D_4 and H_1, H_2, H_3, H_4 are defined as:

$D_1 = \{(0.3,0.3,0.2), (0.3,0.2,0.2), (0.4,0.6,0.6)\}$

$D_2 = \{(0.4,0.6,0.6), (0.5,0.4,0.4), (0.3,0.2,0.2)\}$

$D_3 = \{(0.4,0.6,0.6), (0.3,0.2,0.2), (0.3,0.3,0.2)\}$

$D_4 = \{(0.3,0.3,0.2), (0.5,0.4,0.4), (0.4,0.6,0.6)\}$ and

$H_1 = \{(0.4,0.4,0.4), (0.5,0.4,0.4), (0.3,0.2,0.2)\}$

$H_2 = \{(0.3,0.2,0.2), (0.3,0.2,0.2), (0.4,0.6,0.6)\}$

$H_3 = \{(0.4,0.4,0.4), (0.3,0.2,0.2), (0.3,0.2,0.2)\}$

$H_4 = \{(0.3,0.2,0.2), (0.5,0.4,0.4), (0.4,0.6,0.6)\}$

Then the families $\gamma = \{0_{(N)}, 1_{(N)}, D_1, D_2, D_3, D_4\}$ and $\eta = \{0_{(N)}, 1_{(N)}, H_1, H_2, H_3, H_4\}$ are Neu Topologies on $\mathbb{I}$. Thus, $(\mathbb{I}, \gamma)$ and $(\mathbb{I}, \eta)$ are Neu Topological Spaces. Define $\mathcal{D}: (\mathbb{I}, \gamma) \rightarrow (\mathbb{I}, \eta)$ as $\mathcal{D}(u) = u, \mathcal{D}(v) = v, \mathcal{D}(w) = w$. Then $\mathcal{D}$ is Neu_{gsH}-Function but not $Neu_{g\zeta^*H}$-Function. Hence in $(\mathbb{I}, \gamma)$,

$Neu_{g\zeta^*}$-$CS = \{(0.2,0.2,0.2), (0.5,0.4,0.4), (0.3,0.3,0.2)\}$ and

Neu_{gs}-$CS = \{(0.3,0.2,0.2), (0.3,0.2,0.2), (0.4,0.4,0.4)\}$. Here $\mathcal{D}^{-1}({H_3}^c)$ is Neu_{gs}-CS but not

$Neu_{g\zeta^*}$-CS.

$Neu_{g\zeta^*}$-$OS = \{(0.3,0.3,0.2), (0.5,0.4,0.4), (0.2,0.2,0.2)\}$ and

Neu_{gs}-$OS = \{(0.4,0.4,0.4), (0.3,0.2,0.2), (0.3,0.2,0.2)\}$ is Neu_{gs}-OS but not $Neu_{g\zeta^*}$-OS.

Theorem 5.2. For any bijective function $\mathcal{D}: (\mathcal{R}, \vartheta) \rightarrow (S, \omega)$ then the following statement is equivalent.

i). $\mathcal{D}^{-1}: (S, \omega) \rightarrow (\mathcal{R}, \vartheta)$ is $Neu_{g\zeta^*}$-CF.

ii). $\mathcal{D}$ is a $Neu_{g\zeta^*}$-OF.

iii). $\mathcal{D}$ is a $Neu_{g\zeta^*}$-CF.

Proof:

i). $\Rightarrow (ii)$ Let $\mathfrak{B}$ is an Neu-OS in $(\mathcal{R}, \vartheta)$, then $\mathcal{R} - \mathfrak{B}$ is Neu-CS in $(\mathcal{R}, \vartheta)$. Since $\mathcal{D}^{-1}$ is $Neu_{g\zeta^*}$-CF, then $(\mathcal{D}^{-1})^{-1}(\mathfrak{B})$ is $Neu_{g\zeta^*}$-CS in (S, ω). That is $\mathcal{D}(\mathcal{R} - \mathfrak{B})$ is $Neu_{g\zeta^*}$-CS in (S, ω), that is $S - \mathcal{D}(\mathfrak{B})$ is $Neu_{g\zeta^*}$-CS in (S, ω). Therefore $\mathcal{D}(\mathfrak{B})$ is $Neu_{g\zeta^*}$-OS in (S, ω). Thus $\mathcal{D}$ is $Neu_{g\zeta^*}$-OF.

ii). $\Rightarrow (iii)$ Let $\mathfrak{T}$ is an Neu-CS in $(\mathcal{R}, \vartheta)$, then $\mathcal{R} - \mathfrak{T}$ is Neu-OS in $(\mathcal{R}, \vartheta)$. Since $\mathcal{D}$ is $Neu_{g\zeta^*}$-OF, then $\mathcal{D}(\mathcal{R} - \mathfrak{T})$ is $Neu_{g\zeta^*}$-OS in (S, ω). That is $S - \mathcal{D}(\mathfrak{T})$ is $Neu_{g\zeta^*}$-OS in (S, ω). Therefore $\mathcal{D}(\mathfrak{T})$ is $Neu_{g\zeta^*}$-CS in (S, ω). Thus $\mathcal{D}$ is $Neu_{g\zeta^*}$-CF.

iii). $\Rightarrow (ii)$ Let $\mathfrak{C}$ is an Neu-CS in $(\mathcal{R},\vartheta)$, then $\mathcal{D}$ is $Neu_{g\zeta^*}$-CF, then $\mathcal{D}(\mathfrak{C})$ is $Neu_{g\zeta^*}$-CS in (S,ω). That is $(\mathcal{D}^{-1})^{-1}(\mathfrak{C})$ is $Neu_{g\zeta^*}$-CS in (S,ω), hence $\mathcal{D}^{-1}$ is $Neu_{g\zeta^*}$-CF.

Theorem 5.3. Let $\mathcal{D}:(\mathcal{R},\vartheta)\to(S,\omega)$ is bijective and $Neu_{g\zeta^*}$-CF, then the following statements are equivalent.

i). $\mathcal{D}$ is a $Neu_{g\zeta^*}$-OF.

ii). $\mathcal{D}$ is a $Neu_{g\zeta^*H}$-Function.

iii). $\mathcal{D}$ is a $Neu_{g\zeta^*}$-CF.

Proof:

i). $\Rightarrow (ii)$ Let us assume that $\mathcal{D}$ is a $Neu_{g\zeta^*}$-OF. Since $\mathcal{D}$ is bijective and $Neu_{g\zeta^*}$-CF, $\mathcal{D}$ is $Neu_{g\zeta^*H}$-Function.

ii). $\Rightarrow (iii)$ Let us assume that $\mathcal{D}$ is a $Neu_{g\zeta^*H}$-Function. Then $\mathcal{D}$ is $Neu_{g\zeta^*}$-OF. If $\mathfrak{T}$ is Neu-CS in $\mathcal{R}$, then $\mathcal{D}(\mathcal{R}-\mathfrak{T})$ is $Neu_{g\zeta^*}$-OS in (S,ω). That is $S-\mathcal{D}(\mathfrak{T})$ is $Neu_{g\zeta^*}$-OS in (S,ω). Therefore $\mathcal{D}(\mathfrak{T})$ is $Neu_{g\zeta^*}$-CS in (S,ω). Hence $\mathcal{D}$ is $Neu_{g\zeta^*}$-CF.

iii). $\Rightarrow (i)$ Let us assume that $\mathfrak{B}$ is Neu-OS in $(\mathcal{R},\vartheta)$. Then $\mathcal{R}-\mathfrak{B}$ is Neu-CS in $(\mathcal{R},\vartheta)$. Since $\mathcal{D}$ is $Neu_{g\zeta^*}$-CS, then $\mathcal{D}(\mathcal{R}-\mathfrak{B})$ is $Neu_{g\zeta^*}$-CS in (S,ω). That is $S-\mathcal{D}(\mathfrak{B})$ is $Neu_{g\zeta^*}$-CS in (S,ω). Hence $\mathcal{D}(\mathfrak{B})$ is $Neu_{g\zeta^*}$-OS in (S,ω).

Theorem 5.4. The composition of two $Neu_{g\zeta^*H}$-Functions is also a $Neu_{g\zeta^*H}$-Function.

Proof:

Assume $\mathcal{D}:(\mathcal{R},\vartheta)\to(S,\omega)$ and $\ell:(S,\omega)\to(\mathcal{W},\rho)$ are two $Neu_{g\zeta^*}$-CF. Assume $\mathfrak{B}$ is a Neu-CS in $(\mathcal{W},\rho)$. Since ℓ is a $Neu_{g\zeta^*}$-CF, $l^{-1}(\mathfrak{B})$ is $Neu_{g\zeta^*}$-CS in (S,ω). Since any $Neu_{g\zeta^*}$-CS is Neu-CS, $l^{-1}(\mathfrak{B})$ is Neu-CS in (S,ω). Since $\mathcal{D}$ is a $Neu_{g\zeta^*}$-CF, $(\mathcal{D}^{-1}(l^{-1}(\mathfrak{B}) = l\circ\mathcal{D}(\mathfrak{B})$ is $Neu_{g\zeta^*}$-CS in $(\mathcal{R},\vartheta)$, therefore $l\circ\mathcal{D}$ is also $Neu_{g\zeta^*}$-CF.

Assume $\mathcal{G}$ is a Neu-CS in $(\mathcal{R},\vartheta)$ then $\mathcal{R}-\mathcal{G}$ is a Neu-OS in $(\mathcal{R},\vartheta)$. Since $\mathcal{D}$ is $Neu_{g\zeta^*H}$-Functions, then $\mathcal{D}(\mathcal{R}-\mathcal{G})$ is a $Neu_{g\zeta^*}$-OS in (S,ω), implies $\mathcal{D}(\mathcal{G})$ is $Neu_{g\zeta^*}$-CS in (S,ω). Since any $Neu_{g\zeta^*}$-CS is Neu-CS, then $\mathcal{D}(\mathcal{G})$ is Neu-CS in (S,ω), then $S-\mathcal{D}(\mathcal{G})$ is Neu-OS in (S,ω). Since ℓ is $Neu_{g\zeta^*H}$-Functions. $\ell(S-\mathcal{D}(\mathcal{G}))$ is $Neu_{g\zeta^*}$-OS in $(\mathcal{W},\rho)$, implies $\ell(\mathcal{D}(\mathcal{G}))=l\circ\mathcal{D}(\mathcal{G})$ is $Neu_{g\zeta^*}$-CS in $(\mathcal{W},\rho)$ therefore $l\circ\mathcal{D}$ is $Neu_{g\zeta^*}$-CF and $Neu_{g\zeta^*}$-OF, implies $l\circ\mathcal{D}$ is $Neu_{g\zeta^*H}$-Functions.

6 | Conclusion

In this paper, we defined the notion of $Neu_{g\zeta^*}$-CF and $Neu_{g\zeta^*}$-IF in neutrosophic topological spaces and its relation with details. Along with that some of their properties were discussed. Also, introduce the new class of $Neu_{g\zeta^*H}$-functions and studied some of their properties in Neutrosophic Topological Spaces. In future work, we will use the neutrosophic complex sets and their characterizations.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] Abd El-Monsef M.E, El-Deeb S.N and Mahmoud R.A., -open sets and -continuous Mappings, Bull. Fac. Sci, (12)(1983), 77 - 90.
[2] Abd El-Monsef M.E, Lashien E.F, and Nasef A.A., On I-open sets and I-continuous Functions, Kyungpook Mathematical Journal., (32)(1)(1992), 21 - 30.
[3] Abd El-Monsef M.E., Mahmoud R.A, and Nasef A.A., Al-most I-openness and almost I-continuity, J.Egyptian Math.Soc., (7)(2)(1993),191 - 200.
[4] Arokiarani I, Dhavaeelan R, Jafari S and Parimala M., On Some New Notions and Functions in Neutrosophic Topological Spaces, Neutro-sophic Sets and Systems,(16)(2017), 16 - 19.
[5] Atanassov K.T., Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, (20)(1986), 87 - 96.
[6] Atanassov K.T., Intuitionistic Fuzzy Sets, Physica - Verlag, Heidel-berg, N.Y (1999).
[7] Atanassov K.T., Review and New Result on Intuitionistic Fuzzy Sets, preprint IMMFAIS, (1988), 1 - 88.
[8] Balachandaran K, Sundaram P and Maki H., On generalized continuous functions in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser.A Math, (12)(1991), 5 - 13.
[9] Chang, C.L., Fuzzy Topological Spaces, J. Math. Anal. Appl. (24)(1968), 182 - 190.
[10] Coker D., An Introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy Sets and Systems, (88)(1997), 81 - 89.
[11] Devi R, Balachandaran K and Maki H., Semi-generalized closed maps and generalized semi-closed maps, Mem. Fac. Sci. Kochi Univ. Ser.A.Math., (14)(1993), 41 - 54.
[12] Devi R, Balachandaran K, and Maki H., Generalized -closed maps and -generalized closed maps, Indian J.Pure.Appl.Math., (29)(1998), 37 - 49.
[13] Devi R, Balachandaran K, and Maki H., On generalized –continuous functions and generalized continuous functions, Far East J. Math. Sci.,Special, (1)(1997), 1 - 15.
[14] Dhavaseelan R, Jafari S and Hanifpage .M.D, Neutrosophic generalized a-contra-continuity, creat.math. inform. (27)(2)(2018), 133 - 139.
[15] Dhavaseelan R, Jafari S, Generalized Neutrosophic closed sets, New trends in Neutrosophic theory and applications, (2)(2018), 261 – 273.
[16] Dhavaseelan R, Jafari S, OzelC and Al-Shumrani.M.A., Generalized Neutrosophic Contra-Continuity, New Trends in Neutrosophic Theory and Applications - (2)(2017).
[17] Dhavaseelan R, Narmada Devi R, Jafari S, QaysHatem Imran., On Neutrosophic m-continuity Neutrosophic, Sets and Systems, (27)(2009).
[18] Dontchev J, Ganster M and Noiri T., Unied operation approach of generalized closed sets via topological ideal, Math. Japanica., (49)(1999),395 - 401.
[19] Dontchev J, Noiri T., Contra-semi continuous functions, Math.Pannonica., (10)(1999), 159 |168.
[20] Dontchev J, Noiri T., Quasi-Normal Spaces and g-Closed Sets, ActaMathematica Hungarica (89)(2000), 211 - 219.
[21] Dontchev J., Contra-Continuous Functions And Strongly S-Closed Spaces, Internat. J. Math. and Math. Sci. (19)(2)(1996), 303 - 310.
[22] Dontchev J., On Contra-continuous functions and strongly s-closed spaces, Internat. J. Math. and Math. Sci., (19)(1996),303 - 310.
[23] Dontchev J., On generalizing semi-pre open sets, Mem. Fac. Sci. Kochi Univ. Ser.A. Math., (16)(1995), 35 - 48.

[24] Dontchev J., On pre I open sets and decomposition of I continuity, Banyan Mathematical Journal.,(2)(1996).
[25] Dontchev J., On some separation axioms associated with the -topology, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., (18)(1997), 31 - 35.
[26] Dontchev J., The characterization of spaces and maps via semi pre open sets, Indian J. Pure. Appl. Math., (25)(1994), 939 - 947.
[27] Ekici E, Noiri T., On a generalization of g-closed sets and g-open sets, Italian Journal of Pure and Applied Mathematics, in press.
[28] Florentin Smaradache., Neutrosophic Set: - A Generalization of Intuitionistic Fuzzy Set, Journal of Defense Resources Management, (10)(2010), 107 - 116.
[29] Hatir E and Jafari S., On some new classes of sets and a new decomposition of continuity via grills, J. Adv. Math. Studies, (3)(1)(2010), 33 - 40.
[30] Hatir E, and Jafari S., On decompositions of continuity via idealization, Acta. Math.Hungar, (96)(4)(2002), 341 - 349.
[31] Hatir E, Jafari S., On semi I open sets and semi continuous functions, Acta. Math.Hungar,(107)(4) (2005), 345 - 354.
[32] Jafari S, Lellis Thivagar M, and Athisayaponmani S., -open sets based on bi-topological separation axioms , Soochow Journal of Mathematics, (33)(3)(2007),375 - 381.
[33] Jafari S, Noiri T, Rajesh N, and LellisThivagar M., Another generalization of closed sets, Kochi J. Math (Japan)., (3)(2008), 25 - 38.
[34] Jafari S, On a weak separation axioms, Far East J. Math. Sci., 3(5)(2001), 779 - 787.
[35] Jankovic D, Hamlett T.R., Compatible extensions of ideals, Union e Mathematica Italiana Bollettino. B. Series (6)(3)(1992), 453 - 465.
[36] Jankovic D, Hamlett T.R., Ideals in topological spaces and the set operator, Boll. Univ. Math. Ital., (7)(1990), 863 - 874.
[37] Jankovic D, Hamlett T.R., New topologies from old via ideals, Amer.Math. Monthly., (97)(1990), 295 - 310.
[38] Lellis Thivagar M, Ekici E and Ravi O., On -sets and bitopological decompositions of -continuous mappings, Kochi Journal of Mathematics (Japan)., (3)(2008), 181 - 189.
[39] Lellis Thivagar M, Nirmala Mariappan., On weak separation axioms associated with (1; 2) sg closed sets in bi-topological spaces, Int.Jour.of. Math.Analysis.,(4)(13) (2010), 631 - 644.
[40] Lellis Thivagar M, Raja Rajeswar R, and Athisaya Ponmani S., Characterizations of ultra-separation axioms via -kernel, Lobachevski Journal of Mathematics, (25) (2007), 217 - 229.
[41] Lellis Thivagar M, Ravi O., A bi-topological - semi generalized continuous mappings, Bull. Malayasian Math. Soc., (29)(1) (2006), 79 - 88.
[42] Lellis Thivagar M, Ravi O., On stronger forms of -quotient mappings in bi-topological spaces, Int. J. Math. Game theory and Algebra., (6)(14)(2004), 481 - 492.
[43] Lellis Thivagar M., A note on quotient mappings, Bull. Malayasian Math. Soc., (14)(1991), 21 - 30.
[44] Lellis Thivagar M., Generalization of pairwise -continuous functions, Pure and Applied Mathematicka Sciences., (28)(1991), 55 - 63.
[45] Levine N., Generalized Closed Sets in Topology, Rend. Cir. Mat. palermo,(2)(1970) 89 - 96.
[46] Levine N., Generalized closed sets in Topology, Rend. Circ. Math. Palermo., (19)(2)(1970), 89 - 96.
[47] Levine N., On Strong continuity in topological spaces, Amer. Math. Monthly, (67)(1960).
[48] Levine N., Semi-open sets and semi-continuity in topological spaces, Amer.Math. Monthly.,(70) (1963), 36 - 41.
[49] Noiri T, Keskin A., I-sets and some weak separation axioms, Int. J.Math. Analysis., (5)(11)(2011), 539 - 548.
[50] Noiri T., Almost g-closed functions and separation axioms, Acta Math. Hungar, (82)(3)(1999), 193 - 205.
[51] Parimala M, Jeevitha R, Jafari S, Smaramdache F and Udhayakumar R., Neutrosophic-Homeomorphisms in Neutrosophic Topological space, Journal of Information, (9)(187)(2018), 1 - 10.
[52] Salama A.A, Alblowi S.A., Generalized Neutrosophic Set and Generalized Neutrousophic Topological Spaces, Journal computer Sci. Engineering, (2)(7)(2012).
[53] Salama A.A, Alblowi S.A., Neutrosophic Set and Neutrosophic Topological Space, ISOR Journal of Mathematics, (3)(4)(2012), 31 - 35.
[54] Smarandache F, A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic set, Neutrosophic Probability; American Research press: Rehoboth, NM, USA, 1999.
[55] Sundaram P, Maki H and Balachandran K., Semi-generalized continuous maps and semi-T1/2 spaces, Bull. Fukuoka Univ. Edu. (40)(1991), 33-40.
[56] Sundaram P, Maki H and Balachandran K., Semi-generalized continuous maps and semi-T1/2 spaces, Bull. Fukuoka Univ. Edu. (40) (1991), 33-40.
[57] Sundaram P, Rajamani M., Some decompositions of -continuity in topological spaces, Acta Ciencia Indica Mathematics, (27)(2)(2007),185 - 190.
[58] Sundaram P, Sheik John M,, Weakly closed sets and weak continuous maps in topological spaces, Proc. 82nd Sci. Cong. Calcutta, (1995), 49 - 54.
[59] Sundaram P., Studies on generalizations of continuous maps in topological spaces, Ph.D. Thesis, Bharathiar University, (1991).
[60] Tong J., A decomposition of continuity in topological spaces, Acta. Math.Hungar, (54)(1989), 51 - 55.
[61] Tong J., A separation axioms between T0 and T1, Ann. Soc. Sci. Bruxelles, (96)(2)(1982), 85 - 90.

[62] Veerakumar M.K.R.S., # g-semi closed sets in topological spaces, Antartica J.Math, (2)(2) (2005), 201 - 222.
[63] Zadeh L. A., Fuzzy Sets, Inform and Control, (8)(3)(1965), 338 - 353.