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Basic Introduction of Neutrosophic Set Theory

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Abstract

This paper is devoted to introducing the basic introduction to neutrosophic set theory. Neutrosophic laws defined by the partial algebra are totally (100%) true. We study three types of neutrosophic sets $H_1^t[I]$, $H_2^t[I]$, and $H_3^t[I]$, and defined the notions of empty neutrosophic set, universal neutrosophic set, equality of neutrosophic set, neutrosophic subsets, complement of neutrosophic set, and their properties.

Keywords: Neutrosophic Set of Type-1, Neutrosophic Set of Type-2, Neutrosophic Set of Type-3.

1 | Introduction

Concepts have the property of development and growth, through the scientific activity carried out by humans searching for solutions to problems, imposed by the urgent need of the physical world, to solve some new issues. Mathematical concepts (like others) are subject to the principle of development and growth, including the concept of set, which is considered the cornerstone of the philosophy of mathematics. We can claim that the following proposition is true (without a doubt): If there is no set, then there is no mathematical object. Anyway, whoever tries to create a mathematical object, without the concept of a set, is like someone searching for a black cat in a dark room. If he raises his hands, he can hardly see it. Anyway, the neutrosophic set comes from the school of Neutrosophy, it is a branch of philosophy related to the concept of neutrality, it is means the tendency to not take a side in a conflict (physical or ideological or in war). The principle of neutrality as expressed by Smarandache states the following: suppose that $\langle A \rangle$ represents any: proposition, event, theorem, idea, or concept; $\langle Non - A \rangle$ means that the negation of $\langle A \rangle$; $\langle Anti - A \rangle$ means that the opposite of $\langle A \rangle$, and $\langle Neut - A \rangle$ meant that neither $\langle A \rangle$ nor $\langle Anti - A \rangle$, Smarandache said, " Between an idea and its opposite, there is a continuum-power spectrum of neutralities [1], ". Furthermore, the neutrosophic set can be viewed from two different directions, one depends on the degree of membership functions like [2, 3]. The other depends on the neutrosophic number $a + bI$, neutrosophic number term used in many structures of neutrosophic Algebra such as: neutrosophic linear algebra, neutrosophic groups, neutrosophic rings, neutrosophic number theory and so on, such as [4, 5]. The target of this article is to present an introduction to neutrosophic set theory, according to the generalization of the classical set, depending on the neutrosophic set generated by indeterminacy I which proposed by Smarandache, where $I^2 = I$ and $oI = 0$. This work enhances the neutrosophic number in the previous works of works [6-10].

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2 | Short Historical Review about Neutrosophic Sets

In this section, we present a short review to see how the particular object develops and grows from classical set theory, fuzzy set theory, intuitionistic fuzzy set theory, and neutrosophic set theory respectively.

Definition 2.1. A Set (or class) is any well-defined collection of objects. An object in a set is called an element or member of that set. This concept is due to G. Cantor (1845-1918), who was named the father of set theory. The relation between the domain of the sub-set of a universal set and the co-domain of set {0,1} is illustrated by the following definition.

Definition 2.2. Let U be a universal set and $A \subseteq U$. The characteristics function (or degree of membership function) of A is defined by: $\mu_A(x): X \longmapsto \{0,1\}$ such that:

$$\mu_A(x) = \begin{cases} 1, if\ x \in A \\ 0, if\ x \notin A \end{cases}$$, the set of order paired

$\mu_A(x) = \{(x, \mu_A(x)): \forall\, x \in U\} \subseteq U \times \{0,1\}$ is called a crisp set. This class (or set) made the cornerstone of new mathematical systems. Of course, it is a particular class in set theory or a conceptualization of the concept of a set within the framework of set theory. In 1965, a new era of fuzzy mathematics was introduced by Lotfi Zadeh, he extended the co-domain from set $\{0,1\}$ into the interval [0,1] by the next definition.

Definition 2.3. [11] Let X be a non-empty set and $A \subseteq X$. Define $T_A(x): X \longmapsto [0.1]$ such that $0 \leq T_A(x) \leq 1$, for all $x \in X$. The set (or class) $A_{FS} = \{(x, T_A(x)): x \in X\}$ is called a fuzzy set (FS), and he created the Fuzzy Set theory that is associated with Fuzzy Logic. Where, $T_A(x)$ is called the membership function or grade of membership (also the degree of compatibility or degree of truth) of x in A. later on, Krassimir Atanassov added a new extension by adding the degree of falsity (or degree of non-membership function) as the following:

Definition 2.4. [12] Let X be a non-empty set and $A \subseteq X$. Define $T_A(x): X \longmapsto [0.1]$, and $F_A(x): X \longmapsto [0.1]$, such that $0 \leq T_A(x), F_A(x) \leq 1$, for all $x \in X$, $T_A(x) \leq F_A(x), \forall x \in X$, and $0 \leq \mu_T(x) + F_A(x) \leq 2$. The set $A_{IFS} = \{(x, T_A(x), F_A(x)\ ): x \in X\}$ is called is called a Intutionstic Fuzzy Set(IFS), and he created the Intutionstic Fuzzy Set theory that is associated with Intuitionistic Fuzzy Logic. Where, $T_A(x)$ A has the previous description, and $F_A(x)$ is called the non-membership function or grade of non-membership (also a degree of incompatibility or degree of falsehood) of x in A. We see that every A_{FS} has the form:

$A_{FS} = \{(x, T_A(x), 1 - F_A(x)\ ): x \in X\}$. The process of growth and development of the particular concept continues, but this time, through Smarandache with a new extension when the idea of indeterminacy is third-degree according to the following definition:

Definition 2.5. [3, 13] Let X be a non-empty set and $A \subseteq X$. Define $T_A(x): X \longmapsto [0.1]$, and $I_A(x): X \longmapsto [0.1]$, and $F_A(x): X \longmapsto [0.1]$, such that $0 \leq T_A(x), I_A(x), F_A(x) \leq 1$, for all $x \in X$ and $0 \leq \mu_A(x) + \mu_A(x) \leq 3$. The set $A_{SVNS} = \{(x, T_A(x), I_A(x), F_A(x)\ ): x \in X\}$ is called a single-valued neutrosophic set(SVNS), and he created the neutrosophic Set theory that is associated with neutrosophic Logic. Where, $T_A(x)$and $F_A(x)$ have the previous description, and $I_A(x)$ is called the indeterminacy for $T_A(x)$or $F_A(x)$. The original neutrosophic set is defined by Smarandache by taking the co-domain as a non-standard interval such as $]^{0-}, 1^{+}[$. The previous concepts give us a neutrosophic set according to degree functions.

3 | Neutrosphic-Sets of Type-1, Type-2, and Type-3

In this section, we will begin our development of the axiomatic neutrosophic set theory that corresponds to the axiomatic set theory. In the literature philosophy of mathematical axiomatic systems, it consists of a set of undefined terms and axioms, axioms mean that a declarative sentence (or proposition) is assumed to be true. We will present three types of neutrosophic sets type-1, type-2, and type-3.

Definition 3.1 Let U be a universal set, then:

1. $U_1^t[I] = \{u_1 + u_2 I : u_1, u_2 \in U\}$ is a unversial neutrosophic-set of type-1, where I is an indeterminacy.
2. $U_2^t[I] = \{uI \cup \{u\} : u \in U\}$ is a universal neutrosophic set of type-2, where I is an indeterminacy.
3. $U_3^t[I] = \{(u_1 + u_2 I) \cup \{u_1\} : u_1, u_2 \in U\}$ is a universal neutrosophic-set of type-3, where I is an indeterminacy

Definition 3.2 Let $\emptyset$ be the empty-set, then:

1. $\emptyset_1^t[I] = \{u_1 + u_2 I : u_1, u_2 \in \emptyset\} = \emptyset$ is an empty neutrosophic set of type-1, where I is an indeterminacy.
2. $\emptyset_2^t[I] = \{uI \cup \{u\} : u \in \emptyset\} = \emptyset$ is an empty neutrosophic set of type-2, where I is an indeterminacy.
3. $\emptyset_3^t[I] = \{(u_1 + u_2 I) \cup \{u_1\} : u_1, u_2 \in \emptyset\} = \emptyset$ is a empy neutrosophic-set of type-3, where I is an indeterminacy

Definition 3.3 Let $H \neq \emptyset \subset U$ be a non-empty-set, then $H_1^t[I] = \{h_1 + h_2 I : h_1, h_2 \in H\}$ is a neutrosophic-set of type-1, where I is an indeterminacy.

Definition 3.4 Let $H \neq \emptyset \subset U$ be a non-empty-set, and $H_1^t[I] = \{h_1 + h_2 I : h_1, h_2 \in H\}$ is a neutrosophic-set of type-1, Every classical-element $h \in H$ is a neutrosophic-element of $H_1^t[I]$, if $0 \in H$, because h can be representative as $h = h + 0I$, and consequently, $H \subset H_1^t[I]$. Otherwise, $H \not\subseteq H_1^t[I]$.

Definition 5.5 Let $H \neq \emptyset \subset U$ be a non-empty-set, then $H_2^t[I] = \{aI \cup \{a\} : a \in H\}$ is a neutrosophic set of type-2, where I is an indeterminacy. It is clear that $H \subset H_2^t[I]$.

Definition 3.6 Let $H \neq \emptyset \subset U$ be a non-empty-set, then $H_3^t[I] = \{(h_1 + h_2 I) \cup \{h_1\} : h_1, h_2 \in H\}$ is a neutrosophic-set of type-3, where I is an indeterminacy, and $H \subset H_3^t[I]$.

Example 3.1 Let H be a set, given by $H = \{1,2,3\}$, Then:

i). Then the neutrosophic set of type-1 as the form:

$$H_1^t[I] = \{h_1 + h_2 I : h_1, h_2 \in H\} =$$
$$\{1 + I, 1 + 2I, 1 + 3I, 2 + I, 2 + 2I, 2 + 3I, 3 + I, 3 + 2I, 3 + 3I\}$$

ii). The neutrosophic set of type-2 looks like this:

$$H_2^t[I] = \{aI \cup \{a\} : a \in H\} = \{1,2,3, I, 2I, 3I\}$$

iii). The neutrosophic-set of type-2 becomes like:

$$H_3^t[I] = \{(h_1 + h_2 I) \cup \{h_1\} : h_1, h_2 \in H\} =$$
$$\{1,2,3,1 + I, 1 + 2I, 1 + 3I, 2 + I, 2 + 2I, 2 + 3I, 3 + I, 3 + 2I, 3 + 3I\}$$

Example 3.2 Let $\mathbb{N} = \{0,1,2, \dots\}$ be the set of natural numbers. Then:

1) Then neutrosophic-natural numbers of type-1 is given by:

$$\mathbb{N}_1^t[I] = \begin{Bmatrix} 0, & 0 + I, & 0 + 2I, & 0 + 3I, & \cdots \\ 1, & 1 + I, & 1 + 2I, & 1 + 3I, & \cdots \\ 2, & 2 + I & 2 + 2I & 2 + 3I, & \cdots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{Bmatrix}$$

2) Then neutrosophic-natural numbers of type-2 like:

$$\mathbb{N}_2^t[I] = \begin{Bmatrix} 0, & 0I, \\ 1, & 1I, \\ 2, & 2I \\ \vdots & \vdots \end{Bmatrix}$$

Observation, since $0 \in \mathbb{N}$, then $\mathbb{N}_1^t[I] = \mathbb{N}_3^t[I]$.

Example 3.3 Let $\mathbb{Z} = \{0, \pm 1, \pm 2, \dots\}$ be the set of integers numbers. Then:

1) Then neutrosophic-integer numbers of type-1 is given by:

$$\mathbb{Z}_1^t[I] = \begin{Bmatrix} 0, & 0 \pm I, & 0 \pm 2I, & 0 \pm 3I, & \cdots \\ \pm 1, & \pm 1 \pm I, & \pm 1 \pm 2I, & \pm 1 \pm 3I, & \cdots \\ \pm 2, & \pm 2 \pm I & \pm 2 \pm 2I & \pm 2 \pm 3I, & \cdots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{Bmatrix}$$

2) Then neutrosophic-natural numbers of type-2 like:

$$\mathbb{Z}_2^t[I] = \begin{Bmatrix} 0, & 0I, \\ \pm 1, & \pm 1I, \\ \pm 2, & \pm 2I \\ \vdots & \vdots \end{Bmatrix}$$

Observation, since Since $0 \in \mathbb{Z}$, then $\mathbb{Z}_1^t[I] = \mathbb{Z}_3^t[I]$.

Theorem 3.1 Let $H_1^t[I\,] = \{h_1 + h_2 I : h_1, h_2 \in H\}$ be a neutrosophic-set of type-1,if $N \subset H$, then $N_1^t[I] \subset H_1^t[I\,]$.

Proof. Suppose that $N \subset H$. Let $n \in N_1^t[I] \Rightarrow \exists\, n_1, n_2 \in N$, and indeterminacy I such that $n = n_1 + n_2 I$

$\Rightarrow$ $n_1, n_2 \in H$, and indeterminacy I such that $n = n_1 + n_2 I$

$\Rightarrow$ $n \in H_1^t[I\,]$, hence $N_1^t[I] \subset H_1^t[I\,]$.

Example 3.4 Let $N = \{1,2\}$ and $H = \{1,2,3\}$ be two classical sets, clearly, $N \subset H$. Then the neutrosophic set of type-1 of $N_1^t[I] = \{n_1 + n_2 I : n_1, n_2 \in N\} = \{1 + 1I, 1 + 2I, 2 + 1I, 2 + 2I\}$, and the

neutrosophic set of type-1 of $H_1^t[I\,]$is given by:

$$H_1^t[I\,] = \{h_1 + h_2 I : h_1, h_2 \in H\}$$
$$= \begin{Bmatrix} 1 + 1I, 1 + 2I, 1 + 3I, \\ 2 + 1I, 2 + 2I, 2 + 3I \\ 3 + 1I, 3 + 2I, 3 + 3I \end{Bmatrix}$$

We see that, $N_1^t[I] \subset H_1^t[I\,]$.

Theorem 3.2 Let $H_2^t[I] = \{aI \cup \{a\} : a \in H\}$ be a neutrosophic-set of type-2, if $N \subset H$, then: $N_2^t[I] \subset H_2^t[I\,]$.

Proof. Let $n \in N_2^t[I] \Rightarrow \exists\, n' \in N$, and indeterminacy I such that $n = n'I \cup \{n'\}$

$\Rightarrow$ $n' \in H$, and indeterminacy I such that $n = n'I \cup \{n'\}$

$\Rightarrow$ $n \in H_2^t[I\,]$, hence $N_2^t[I] \subset H_2^t[I\,]$.

Example 3.5 Let $N = \{2,4\}$ and $H = \{2,4,6\}$ be two classical sets, obviously, $N \subset H$. Then the neutrosophic set of type-2 of $N_2^t[I] = \{nI \cup \{n\} : n \in N\} = \{2, 2I, 4, 4I\}$, and the

neutrosophic set of type-2 of $H_2^t[I\,]$ is given by: $H_2^t[I] = \{aI \cup \{a\} : a \in H\}$

$$= \begin{Bmatrix} 2,2I \\ 4,4I \\ 6,6I \end{Bmatrix}$$

We see that, $N_2^t[I] \subset H_2^t[I\,]$.

Theorem 3.3 Let $H_3^t[I\,] = \{(h_1 + h_2I) \cup \{h_1\}: h_1, h_2 \in H\}$ be a neutrosophic-set of type-3, if $\subset H$ **,** then $N_3^t[I] \subset H_3^t[I\,]$.

Proof. Suppose that $N \subset H$. Let $n \in N_3^t[I] \Rightarrow \exists\, n_1, n_2 \in N$, and indeterminacy I such that $n = (n_1 + n_2I) \cup \{n_1\}$

$\Rightarrow\ n_1, n_2 \in H$, and indeterminacy I such that $n = (n_1 + n_2I) \cup \{n_1\}$

$\Rightarrow\ n \in N_3^t[I]$, hence $N_3^t[I] \subset H_3^t[I\,]$.

Example 3.6 Let $N = \{a, b\}$ and $H = \{a, b, c\}$ be two classical sets, evidently, $N \subset H$. Then the neutrosophic set of type-3 of $N_3^t[I] = \{(n_1 + n_2I) \cup \{n_1\}: n_1, n_2 \in N\} = \begin{Bmatrix} a, & a + aI & a + bI \\ b, & b + aI & b + bI \end{Bmatrix}$ and the neutrosophic set of type-3 of $N_3^t[I]$ is given by: $N_3^t[I] = \{(h_1 + h_2I) \cup \{h_1\}: h_1, h_2 \in H\}$

$$= \begin{Bmatrix} a. & a + aI, & a + bI, & a + cI, \\ b, & b + aI, & b + bI, & b + cI, \\ c, & c + aI, & a + aI, & a + aI, \end{Bmatrix}$$

We see that, $N_3^t[I] \subset H_3^t[I\,]$.

Definition 3.7 Let $H_1^t[I\,]$, $N_1^t[I\,]$, $H_2^t[I\,]$, $N_2^t[I\,]$, $H_3^t[I\,]$, and $N_3^t[I\,]$ be six neutrosophic-sets of type-1, type-2, and type-3, respectively. Then :

i. $H_1^t[I\,] = N_1^t[I\,]$, if $H = N$,
ii. $H_2^t[I\,] = N_2^t[I\,]$, if $H = N$, and
iii. $H_3^t[I\,] = N_3^t[I\,]$, if $H = N$.

Theorem 3.4 Let $H_1^t[I\,]$, $N_1^t[I\,]$, $H_2^t[I\,]$, $N_2^t[I\,]$, $H_3^t[I\,]$, and $N_3^t[I\,]$ be six neutrosophic-sets of type-1, type-2, and type-3, respectively. Then :

i. $H_1^t[I\,] = N_1^t[I\,] \Leftrightarrow H_1^t[I\,] \subset N_1^t[I\,] \wedge N_1^t[I\,] \subset H_1^t[I\,]$.
ii. $H_2^t[I\,] = N_2^t[I\,] \Leftrightarrow H_2^t[I\,] \subset N_2^t[I\,] \wedge N_2^t[I\,] \subset H_2^t[I\,]$.
iii. $H_3^t[I\,] = N_3^t[I\,] \Leftrightarrow H_3^t[I\,] \subset N_3^t[I\,] \wedge N_3^t[I\,] \subset H_3^t[I\,]$.

Proof. (i). Suppose that $H_1^t[I\,] = N_1^t[I\,] \Leftrightarrow H = N$

$\Leftrightarrow (H \subset N) \wedge (N \subset H)$

$\Leftrightarrow (H_1^t[I\,] \subset N_1^t[I\,]) \wedge (N_1^t[I\,] \subset H_1^t[I\,])$.

(ii) and (iii) by the same argument.

Theorem 3.5 Let $\emptyset_1^t[I\,], \emptyset_2^t[I\,]$, and $\emptyset_3^t[I\,]$ be three emptyl neutrosophic-sets of type-1, type-2, and type-3 respectevely, then :

1. $\emptyset_1^t[I\,] \subset H_1^t[I\,]$,
2. $\emptyset_2^t[I\,] \subset H_2^t[I\,]$,
3. $\emptyset_3^t[I\,] \subset H_3^t[I\,]$, where H is any arbitrary classical set.
4. The empty neutrosophic sets of type-1, type-2, and type-3 are unique.

Proof. (1). Suppose that $\emptyset_1^t[I\,] \not\subset H_1^t[I\,] \Rightarrow \exists\, x \in \emptyset_1^t[I\,] \wedge x \notin H_1^t[I\,]$

$\Rightarrow \exists\, x_1, x_2 \in \emptyset$, and indeterminacy I, this led to a contradiction, since $\emptyset$ is an empty set. Proof (2) and (3) by the same argument.

(4). To prove that uniqueness. Suppose that $\underbrace{\emptyset_1^t[I\,]}_{1}$ and $\underbrace{\emptyset_1^t[I\,]}_{2}$ are two empty neutrosophic sets of type-1, then by part (1) in theorem 2.5, we have $\underbrace{\emptyset_1^t[I\,]}_{1} \subset \underbrace{\emptyset_1^t[I\,]}_{2}$ and $\underbrace{\emptyset_1^t[I\,]}_{2} \subset \underbrace{\emptyset_1^t[I\,]}_{1}$, therefore $\underbrace{\emptyset_1^t[I\,]}_{1} = \underbrace{\emptyset_1^t[I\,]}_{2}$ by theorem 3.4.

Definition 3.8 Let $H_1^t[I\,]$, $H_2^t[I\,]$, and $H_3^t[I\,]$, be three neutrosophic-sets of type-1, type-2, and type-3, respectively. The neutrosophic complment-sets of of type-1, type-2, and type-3 denoted by $\overbrace{H_1^t[I\,]}^{c}$, $\overbrace{H_2^t[I\,]}^{c}$, and $\overbrace{H_3^t[I\,]}^{c}$, and defined by:

1. $\overbrace{H_1^t[I\,]}^{c} = \{x: x \notin H_1^t[I\,] \wedge x \in U_1^t[I\,]\} = \{x: x \notin H \wedge x \in U\}$,
2. $\overbrace{H_2^t[I\,]}^{c} = \{x: x \notin H_2^t[I\,] \wedge x \in U_2^t[I\,]\} = \{x: x \notin H \wedge x \in U\}$, and
3. $\overbrace{H_3^t[I\,]}^{c} = \{x: x \notin H_3^t[I\,] \wedge x \in U_3^t[I\,]\} = \{x: x \notin H \wedge x \in U\}$.

Example 3.7 Let $U = \{a, b, c, d, e\}$ and $H = \{a, b, d\}$ be two classical sets, then, $x \notin H$, implies that $x \in H^c = \{c, e\}$, we have,

1. $\overbrace{H_1^t[I\,]}^{c} = \begin{Bmatrix} c + cI, & c + eI \\ e + cI, & e + eI \end{Bmatrix}$, where $U_1^t[I\,] = \begin{Bmatrix} a + aI, & a + bI, & a + dI, \\ b + aI, & b + bI & b + dI \\ d + aI, & d + bI & d + dI, \end{Bmatrix}$,
2. $\overbrace{H_2^t[I\,]}^{c} = \begin{Bmatrix} c, & cI \\ e & eI \end{Bmatrix}$, where $H_2^t[I\,] = \begin{Bmatrix} a, & aI \\ b, & bI \\ c & c, \end{Bmatrix}$, and
3. $\overbrace{H_3^t[I\,]}^{c} = \begin{Bmatrix} c, & c + cI & c + eI \\ e, & e + cI, & e + eI \end{Bmatrix}$, where $U_3^t[I\,] = \begin{Bmatrix} a, & a + aI, & a + bI, & a + dI, \\ b, & b + aI, & b + bI & b + dI \\ d, & d + aI, & d + bI & d + dI, \end{Bmatrix}$.

Theorem 3.6 Let $H_1^t[I\,], H_2^t[I\,]$, and $H_3^t[I\,]$ be three neutrosophic-sets of type-1, type-2, and type-3 respectevely, then :

1. $\overbrace{H_1^t[I\,]}^{c^c} = H_1^t[I\,]$,
2. $\overbrace{H_2^t[I\,]}^{c^c} = H_2^t[I\,]$,
3. $\overbrace{H_3^t[I\,]}^{c^c} = H_3^t[I\,]$,
4. $\overbrace{\emptyset_1^t[I\,]}^{c} = U_1^t[I\,]$,
5. $\overbrace{\emptyset_2^t[I\,]}^{c} = U_2^t[I\,]$,
6. $\overbrace{\emptyset_3^t[I\,]}^{c} = U_3^t[I\,]$,
7. $\overbrace{U_1^t[I\,]}^{c} = \emptyset_1^t[I\,]$,
8. $\overbrace{U_2^t[I\,]}^{c} = \emptyset_2^t[I\,]$, and
9. $\overbrace{U_3^t[I\,]}^{c} = \emptyset_3^t[I\,]$, where H is any arbitrary classical set and U is any arbitrary universal classical set.

Proof. Suppose that $U_1^t[I\,]$ is any arbitrary universal neutrosophic set, where U is any arbitrary universal classical set such that $H_1^t[I\,] \subset U_1^t[I\,]$, when $H \subset U$. To show that $\overbrace{H_1^t[I\,]}^{c^c} = H_1^t[I\,]$

Suppose that $x \in \overbrace{H_1^t[I\,]}^{c^c} \Rightarrow x \notin \overbrace{H_1^t[I\,]}^{c} \Rightarrow x \in H_1^t[I\,] \Rightarrow \overbrace{H_1^t[I\,]}^{c^c} \subset H_1^t[I\,]$. Conversely, assume that $x \in H_1^t[I\,] \Rightarrow x \notin \overbrace{H_1^t[I\,]}^{c} \Rightarrow x \in \overbrace{H_1^t[I\,]}^{c^c} \Rightarrow H_1^t[I\,] \subset \overbrace{H_1^t[I\,]}^{c^c}$, hence $\overbrace{H_1^t[I\,]}^{c^c} = H_1^t[I\,]$, by theorem 3.4 part-1, for (2) and (3) by similar method and theorem 3.4 part-2 and part-3, respectively.

(4). Suppose that $\overbrace{\emptyset_1^t[I\,]}^{c} \neq U_1^t[I\,] \Rightarrow \exists\, x \notin \overbrace{\emptyset_1^t[I\,]}^{c} \wedge x \in U_1^t[I\,]$

$\Rightarrow \exists\, x \in \emptyset_1^t[I\,] \wedge x \in U_1^t[I\,]$

$\Rightarrow \exists\, x \in \emptyset \wedge x \in U.$

$\Rightarrow x \in \emptyset$, this contradiction, hence $\overbrace{\emptyset_1^t[I\,]}^{c} = U_1^t[I\,]$.

Prove (5),(6),(7),(8), and (9) by a similar argument.

Definition 3.9 Let $H_1^t[I\,]$, $H_2^t[I\,]$, and $H_3^t[I\,]$, be three neutrosophic sets of type-1, type-2, and type-3, respectively, where H is any arbitrary classical set, either H is a finite set, then $H_1^t[I\,]$, $H_2^t[I\,]$, and $H_3^t[I\,]$ are finite neutrosophic sets, the number of all neutrosophic elements is called neutrosophic-order, and denoted by $\psi(H_1^t[I\,])$, $\psi(H_2^t[I\,])$, and $\psi(H_3^t[I\,])$ respectively or H is an infinite set, and consequently, $H_1^t[I\,]$, $H_2^t[I\,]$, and $H_3^t[I\,]$ have an infinite neutrosophic order.

Definition 3.10 Let $H_1^t[I\,]$, $H_2^t[I\,]$, and $H_3^t[I\,]$, be three neutrosophic-sets of type-1, type-2, and type-3, respectively. The neutrosophic power-sets of of type-1, type-2, and type-3 , then:

1. $\Im\,(H_1^t[I\,]) = \{N_1^t[I\,] : \subseteq H_1^t[I\,]\}$ is a neutrosophic power set of type-1,
2. $\Im\,(H_2^t[I\,]) = \{N_2^t[I\,] : \subseteq H_2^t[I\,]\}$ is a neutrosophic power set of type-2, and
3. $\Im\,(H_3^t[I\,]) = \{N_3^t[I\,] : \subseteq H_3^t[I\,]\}$ is a neutrosophic power set of type-3.

Example 2.8 Let $H = \{a, b, c\}$ be two classical-set, the neutrosophic-sets of type-1, type-2, and type-3, with neutrosophic-order, and the number of all neutrosophic-subsets of $H_1^t[I\,]$, $H_2^t[I\,]$, and $H_3^t[I\,]$ such as:

$$H_1^t[I\,] = \begin{Bmatrix} a+aI, & a+bI, & a+cI, \\ b+aI, & b+bI, & b+cI \\ c+aI, & c+bI, & c+cI \end{Bmatrix}, \psi(H_1^t[I\,]) = 9 \text{ and } \psi\big(\Im\,(H_2^t[I\,])\big) = 512,$$

$$H_2^t[I\,] = \begin{Bmatrix} a, & aI \\ b, & bI \\ c, & cI \end{Bmatrix}, \psi(H_2^t[I\,]) = 6 \text{ and } \psi\big(\Im\,(H_1^t[I\,])\big) = 64,$$

$$H_3^t[I\,] = \begin{Bmatrix} a, & a+aI, & a+bI, & a+cI, \\ b, & b+aI, & b+bI, & b+cI \\ c, & c+aI, & c+bI, & c+cI \end{Bmatrix}, \psi(H_2^t[I\,]) = 12 \text{ and } \psi\big(\Im\,(H_2^t[I\,])\big) = 4096,$$

Observations. The neutrosophic order of $\psi\big(H_\mathrm{i}^t[I\,]\big),\ i = 1,2,3$ does not dvide the $\psi\left(\Im\left(H_\mathrm{i}^t[I\,]\right)\right),\ i = 1,2,3.$

Theorem 3.7 Let $H_1^t[I\,]$, $N_1^t[I\,]$, $H_2^t[I\,]$, $N_2^t[I\,]$, $H_3^t[I\,]$, and $N_3^t[I\,]$ be six neutrosophic-sets of type-1, type-2, and type-3, respectively. Then :

i. $H_1^t[I\,] = H_1^t[I\,]$,
ii. $H_2^t[I\,] = H_2^t[I\,]$,
iii. $H_3^t[I\,] = H_3^t[I\,]$,
iv. $H_1^t[I\,] = N_1^t[I\,] \Rightarrow N_1^t[I\,] = H_1^t[I\,]$,
v. $H_2^t[I\,] = N_2^t[I\,] \Rightarrow N_2^t[I\,] = H_2^t[I\,]$, and
vi. $H_3^t[I\,] = N_3^t[I\,] \Rightarrow N_3^t[I\,] = H_3^t[I\,]$.

Proof. (i). Suppose that $H_1^t[I\,] = H_1^t[I\,] \Leftrightarrow H = H$

$\Leftrightarrow (H \subset H) \wedge (H \subset H)$

$\Leftrightarrow (H_1^t[I\,] \subset N_1^t[I\,]) \wedge (N_1^t[I\,] \subset H_1^t[I\,])$.

(ii). and (iii) by similar method.

(iv). Suppose that $H_1^t[I\,] = N_1^t[I\,] \Rightarrow H = N$

$$\Rightarrow (N \subset H) \wedge (H \subset N)$$
$$\Rightarrow (N_1^t[I\,] \subset H_1^t[I\,]) \wedge (H_1^t[I\,] \subset N_1^t[I\,])$$
$$\Rightarrow N_1^t[I\,] = H_1^t[I\,].$$

(v). and (iv) by similar method.

Theorem 3.8 Let $H_1^t[I\,]$, $N_1^t[I\,]$, and $M_1^t[I\,]$ be three neutrosophic sets of type-1. Then

$$H_1^t[I\,] = N_1^t[I\,] \wedge N_1^t[I\,] = M_1^t[I\,] \Rightarrow H_1^t[I\,] = M_1^t[I\,].$$

Proof. Assume that $H_1^t[I\,] = N_1^t[I\,] \wedge N_1^t[I\,] = M_1^t[I\,]$

$\because H_1^t[I\,] = N_1^t[I\,] \Rightarrow H = N$

$\because N_1^t[I\,] = M_1^t[I\,] \Rightarrow N = M$

$$\Rightarrow H = M$$
$$\Rightarrow H_1^t[I\,] = M_1^t[I\,].$$

Theorem 3.9 Let $H_1^t[I\,]$, $N_1^t[I\,]$, and $M_1^t[I\,]$ be three neutrosophic sets of type-2. Then

$$H_1^t[I\,] = N_1^t[I\,] \wedge N_1^t[I\,] = M_1^t[I\,] \Rightarrow H_1^t[I\,] = M_1^t[I\,].$$

Theorem 3.10 Let $H_3^t[I\,]$, $N_3^t[I\,]$, and $M_3^t[I\,]$ be three neutrosophic sets of type-3. Then

$$H_3^t[I\,] = N_3^t[I\,] \wedge N_3^t[I\,] = M_3^t[I\,] \Rightarrow H_3^t[I\,] = M_3^t[I\,].$$

The proof of theorems 2.7 and 2.8 is like theorem 3.6, and we note that the relation of equality for three types of neutrosophic sets is a transitive relation.

4 | Conclusion

This paper is devoted to introducing the basic introduction to neutrosophic set theory. Neutrosophic laws defined by the partial algebra are totally (100%) true. We study three types of neutrosophic sets $H_1^t[I\,]$, $H_2^t[I\,]$, and $H_3^t[I\,]$, and defined the notions of empty neutrosophic set, universal neutrosophic set, equality of neutrosophic set, neutrosophic subsets, complement of neutrosophic set, and their properties.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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