Plithogenic Logic and Computation

Journal Homepage: sciencesforce.com/plc

SCIENCES FORCE
International Academic Publisher

Plithogenic Log. Comp. Vol. 2 (2024) 37–48

Paper Type: Original Article

The Extended Study of the 2-symbolic Plithogenic Numbers

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Received: 15 Apr 2024 **Revised:** 12 Jul 2024 **Accepted:** 13 Aug 2024 **Published:** 17 Aug 2024

Abstract

In this paper, we presented the extended study of the 2-symbolic plithogenic numbers, where we studied the square root of a 2-symbolic plithogenic real number and 2-symbolic plithogenic real polynomial. In addition, this paper is considered a generalization of the result that was presented in On the Refined AH-Isometry, which it presented one solution for the square root of the 2-symbolic plithogenic real numbers but we discussed all possible solutions when solving 2-symbolic plithogenic equation.

Keywords: Refined Neutrosophic; Square Root; Complex Polynomial; Real Polynomial.

1 | Introduction

The genesis, origination, formation, development, and evolution of new entities through dynamics of contradictory and/or neutral and/or noncontradictory multiple old entities is known as plithogenic. Plithogeny advocates for the integration of theories from several fields.

We use numerous "knowledge" from domains like soft sciences, hard sciences, arts, and literature theories, etc. as "entities" in this study, this is what Smarandache introduced, as he presented a study on Plithogeny, plithogenic set, logic, probability, and statistics [2], in addition to presenting introduction to the symbolic plithogenic algebraic structures (revisited), through which he discussed several ideas, including mathematical operations on plithogenic numbers [1]. Also, an overview of plithogenic set and symbolic plithogenic algebraic structures is discussed by him [3]. It is thought that the symbolic n-plithogenic sets are a good place to start when developing algebraic extensions for other classical structures including rings, vector spaces, modules, and equations [4-7]. Also, many other extensions and algorithms in cryptography were presented in [11-12].

Alhasan also presented several papers on calculus, in which he discussed neutrosophic definite and indefinite integrals. He also presented the most important applications of definite integrals in neutrosophic logic [8-10].

Many papers [13] were published about neutrosophic and symbolic plithogenic.

2 | Main Discussion

2.1 | The Square Root of the Symbolic Plithogenic Number

First, let's compute the square root: $\sqrt{a_0 + a_1P_1 + a_2P_2}$; $a_0 \geq 0, a_0 + a_2 \geq 0, a_0 + a_1 + a_2 \geq 0$

$$\sqrt{a_0 + a_1P_1 + a_2P_2} = x_0 + x_1P_1 + x_2P_2$$

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https://doi.org/10.61356/j.plc.2024.2360

Where x_0, x_1, x_2 are real unknowns that we need to find out.

By raising both sides to the second power, we get:

$$a_0 + a_1P_1 + a_2P_2 = (x_0 + x_1P_1 + x_2P_2)^2$$

$$a_0 + a_1P_1 + a_2P_2 = (x_0 + x_1P_1)^2 + 2(x_0 + x_1P_1)(x_2P_2) + (x_2P_2)^2$$

$$a_0 + a_1P_1 + a_2P_2 = {x_0}^2 + 2x_0x_1P_1 + (x_1P_1)^2 + 2(x_0 + x_1P_1)(x_2P_2) + (x_2P_2)^2$$

$$a_0 + a_1P_1 + a_2P_2 = {x_0}^2 + 2x_0x_1P_1 + {x_1}^2P_1 + 2x_0x_2P_2 + 2x_1x_2P_2 + {x_2}^2P_2$$

$$a_0 + a_1P_1 + a_2P_2 = {x_0}^2 + ({x_1}^2 + 2x_0x_1)P_1 + ({x_2}^2 + 2x_0x_2 + 2x_1x_2)P_2$$

Whence:

$$\begin{cases} {x_0}^2 = a_0 \\ {x_1}^2 + 2x_0x_1 = a_1 \\ {x_2}^2 + 2x_0x_2 + 2x_1x_2 = a_2 \end{cases}$$

$$\Rightarrow \begin{cases} x_0 = \pm\sqrt{a_0} \\ {x_1}^2 + 2x_0x_1 = a_1 \quad (1) \\ {x_2}^2 + 2x_0x_2 + 2x_1x_2 = a_2 \quad (2) \end{cases}$$

Case 1: $x_0 = \sqrt{a_0}$ by substituting in (1)

$${x_1}^2 + 2\sqrt{a_0}x_1 - a_1 = 0$$

$$\Delta = 4a_0 + 4a_1$$

Then:

$$\begin{cases} x_1 = \dfrac{-2\sqrt{a_0} + 2\sqrt{a_0 + a_1}}{2} = -\sqrt{a_0} + \sqrt{a_0 + a_1} \\ x_1 = \dfrac{-2\sqrt{a_0} - 2\sqrt{a_0 + a_1}}{2} = -\sqrt{a_0} - \sqrt{a_0 + a_1} \end{cases}$$

- For $x_1 = -\sqrt{a_0} + \sqrt{a_0 + a_1}$ we substitute in (2)

$${x_2}^2 + 2\sqrt{a_0}x_2 + 2\left(-\sqrt{a_0} + \sqrt{a_0 + a_1}\right)x_2 - a_2 = 0$$

$${x_2}^2 + 2\sqrt{a_0 + a_1}\, x_2 - a_2 = 0$$

$$\Delta = 4(a_0 + a_1) + 4a_2 = 4(a_0 + a_1 + a_2)$$

Then:

$$\begin{cases} x_2 = \dfrac{-2\sqrt{a_0 + a_1} + 2\sqrt{a_0 + a_1 + a_2}}{2} = -\sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2} \\ x_2 = \dfrac{-2\sqrt{a_0 + a_1} - 2\sqrt{a_0 + a_1 + a_2}}{2} = -\sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2} \end{cases}$$

Hence:

$$\sqrt{a_0 + a_1P_1 + a_2P_2} = \sqrt{a_0} + \left(-\sqrt{a_0} + \sqrt{a_0 + a_1}\right)P_1 + \left(-\sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2}\right)P_2$$

Or:

$$= \sqrt{a_0} + \left(-\sqrt{a_0} + \sqrt{a_0 + a_1}\right)P_1 + \left(-\sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2}\right)P_2$$

- For $x_1 = -\sqrt{a_0} - \sqrt{a_0 + a_1}$ we substitute in (2)

$$x_2{}^2 + 2\sqrt{a_0}x_2 + 2\left(-\sqrt{a_0} - \sqrt{a_0 + a_1}\right)x_2 - a_2 = 0$$

$$x_2{}^2 - 2\sqrt{a_0 + a_1}\, x_2 - a_2 = 0$$

$$\Delta = 4(a_0 + a_1) + 4a_2 = 4(a_0 + a_1 + a_2)$$

Then:

$$\begin{cases} x_2 = \dfrac{2\sqrt{a_0 + a_1} + 2\sqrt{a_0 + a_1 + a_2}}{2} = \sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2} \\ x_2 = \dfrac{2\sqrt{a_0 + a_1} - 2\sqrt{a_0 + a_1 + a_2}}{2} = \sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2} \end{cases}$$

Hence:

$$\sqrt{a_0 + a_1P_1 + a_2P_2} = \sqrt{a_0} + \left(-\sqrt{a_0} - \sqrt{a_0 + a_1}\right)P_1 + \left(\sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2}\right)P_2$$

Or:

$$= \sqrt{a_0} + \left(-\sqrt{a_0} - \sqrt{a_0 + a_1}\right)P_1 + \left(\sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2}\right)P_2$$

Case 2: $x_0 = -\sqrt{a_0}$ by substitution in (1)

$$x_1{}^2 - 2\sqrt{a_0}x_1 - a_1 = 0$$

$$\Delta = 4a_0 + 4a_1$$

Then:

$$\begin{cases} x_1 = \dfrac{2\sqrt{a_0} + 2\sqrt{a_0 + a_1}}{2} = \sqrt{a_0} + \sqrt{a_0 + a_1} \\ x_1 = \dfrac{2\sqrt{a_0} - 2\sqrt{a_0 + a_1}}{2} = \sqrt{a_0} - \sqrt{a_0 + a_1} \end{cases}$$

- For $x_1 = \sqrt{a_0} + \sqrt{a_0 + a_1}$ we substitute in (2)

$$x_2{}^2 - 2\sqrt{a_0}x_2 + 2\left(\sqrt{a_0} + \sqrt{a_0 + a_1}\right)x_2 - a_2 = 0$$

$$x_2{}^2 + 2\sqrt{a_0 + a_1}\, x_2 - a_2 = 0$$

$$\Delta = 4(a_0 + a_1) + 4a_2 = 4(a_0 + a_1 + a_2)$$

Then:

$$\begin{cases} x_2 = \dfrac{-2\sqrt{a_0 + a_1} + 2\sqrt{a_0 + a_1 + a_2}}{2} = -\sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2} \\ x_2 = \dfrac{-2\sqrt{a_0 + a_1} - 2\sqrt{a_0 + a_1 + a_2}}{2} = -\sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2} \end{cases}$$

Hence:

$$\sqrt{a_0 + a_1P_1 + a_2P_2} = -\sqrt{a_0} + \left(\sqrt{a_0} + \sqrt{a_0 + a_1}\right)P_1 + \left(-\sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2}\right)P_2$$

Or:

$$= -\sqrt{a_0} + \left(\sqrt{a_0} + \sqrt{a_0 + a_1}\right)P_1 + \left(-\sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2}\right)P_2$$

- For $x_1 = \sqrt{a_0} - \sqrt{a_0 + a_1}$ we substitute in (2)

$$x_2{}^2 - 2\sqrt{a_0}x_2 + 2(\sqrt{a_0} - \sqrt{a_0 + a_1})x_2 - a_2 = 0$$

$$x_2{}^2 - 2\sqrt{a_0 + a_1}\,x_2 - a_2 = 0$$

$$\Delta = 4(a_0 + a_1) + 4a_2 = 4(a_0 + a_1 + a_2)$$

Then:

$$\begin{cases} x_2 = \dfrac{2\sqrt{a_0 + a_1} + 2\sqrt{a_0 + a_1 + a_2}}{2} = \sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2} \\ x_2 = \dfrac{2\sqrt{a_0 + a_1} - 2\sqrt{a_0 + a_1 + a_2}}{2} = \sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2} \end{cases}$$

Hence:

$$\sqrt{a_0 + a_1P_1 + a_2P_2} = -\sqrt{a_0} + (\sqrt{a_0} - \sqrt{a_0 + a_1})P_1 + (\sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2})P_2$$

Or:

$$= -\sqrt{a_0} + (\sqrt{a_0} - \sqrt{a_0 + a_1})P_1 + (\sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2})P_2$$

The eight results are:

$$(x_0, x_1, x_2) = (\sqrt{a_0}, -\sqrt{a_0} + \sqrt{a_0 + a_1}, -\sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2}), (\sqrt{a_0}, -\sqrt{a_0} + \sqrt{a_0 + a_1}, -\sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2})$$

$$(\sqrt{a_0}, -\sqrt{a_0} - \sqrt{a_0 + a_1}, \sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2}), (\sqrt{a_0}, -\sqrt{a_0} - \sqrt{a_0 + a_1}, \sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2})$$

$$(-\sqrt{a_0}, \sqrt{a_0} + \sqrt{a_0 + a_1}, -\sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2}), (-\sqrt{a_0}, \sqrt{a_0} + \sqrt{a_0 + a_1}, -\sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2})$$

$$(-\sqrt{a_0}, \sqrt{a_0} - \sqrt{a_0 + a_1}, \sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2}), (-\sqrt{a_0}, \sqrt{a_0} - \sqrt{a_0 + a_1}, \sqrt{a_0 + a_1} - \sqrt{a_0 + a_1 + a_2})$$

Because we are now calculating the square root of a symbolic plithogenic number (according to classical analysis), we only take the result with a positive value, hence:

$$\sqrt{a_0 + a_1P_1 + a_2P_2} = \sqrt{a_0} + (-\sqrt{a_0} + \sqrt{a_0 + a_1})P_1 + (-\sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2})P_2$$

Clearly: $\sqrt{a_0} \geq 0, -\sqrt{a_0} + \sqrt{a_0 + a_1} \geq 0 \; and \; -\sqrt{a_0 + a_1} + \sqrt{a_0 + a_1 + a_2} \geq 0$

However, when we solve the symbolic plithogenic number equation, we take all eight results.

Example 1

Let's find: $\sqrt{1 + 3P_1 + 12P_2}$

$$\sqrt{1 + 3P_1 + 12P_2} = x_0 + x_1P_1 + x_2P_2$$

By raising both sides to the second power, we get:

$$1 + 3P_1 + 12P_2 = (x_0 + x_1P_1 + x_2P_2)^2$$

$$1 + 3P_1 + 12P_2 = (x_0 + x_1P_1)^2 + 2(x_0 + x_1P_1)(x_2P_2) + (x_2P_2)^2$$

$$1 + 3P_1 + 12P_2 = x_0{}^2 + 2x_0x_1P_1 + (x_1P_1)^2 + 2(x_0 + x_1P_1)(x_2P_2) + (x_2P_2)^2$$

$$1 + 3P_1 + 12P_2 = x_0{}^2 + 2x_0x_1P_1 + x_1{}^2P_1 + 2x_0x_2P_2 + 2x_1x_2P_2 + x_2{}^2P_2$$

$$1 + 3P_1 + 12P_2 = {x_0}^2 + ({x_1}^2 + 2x_0x_1)P_1 + ({x_2}^2 + 2x_0x_2 + 2x_1x_2)P_2$$

Whence:

$$\begin{cases} {x_0}^2 = 1 \\ {x_1}^2 + 2x_0x_1 = 3 \\ {x_2}^2 + 2x_0x_2 + 2x_1x_2 = 12 \end{cases}$$

$$\Rightarrow \begin{cases} x_0 = \pm 1 \\ {x_1}^2 + 2x_0x_1 = 3 \quad (1) \\ {x_2}^2 + 2x_0x_2 + 2x_1x_2 = 12 \quad (2) \end{cases}$$

Case 1: $x_0 = 1$ by substituting in (1)

$$ {x_1}^2 + 2x_1 - 3 = 0 $$

$$ (x_1 - 1)(x_1 + 3) $$

Then:

$$\begin{cases} x_1 = 1 \\ x_1 = -3 \end{cases}$$

- For $x_1 = 1$ we substitute in (2)

$$ {x_2}^2 + 4x_2 - 12 = 0 $$

$$ (x_1 - 2)(x_1 + 6) = 0 $$

Then:

$$\begin{cases} x_2 = 2 \\ x_2 = -6 \end{cases}$$

Hence:

$\sqrt{1 + 3P_1 + 12P_2} = 1 + P_1 + 2P_2$ (Accepted)

Or:

$= 1 + P_1 - 6P_2$ (Rejected)

- For $x_1 = -3$ we substitute in (2)

$$ {x_2}^2 - 4x_2 - 12 = 0 $$

$$ (x_1 + 2)(x_1 - 6) = 0 $$

Then:

$$\begin{cases} x_2 = -2 \\ x_2 = 6 \end{cases}$$

Hence:

$\sqrt{1 + 3P_1 + 12P_2} = 1 - 3P_1 - 2P_2$ (Rejected)

Or:

$= 1 - 3P_1 + 6P_2$ (Rejected)

Case 2: $x_0 = -1$ by substitution in (1)

$$x_1{}^2 - 2x_1 - 3 = 0$$

$$(x_1 - 3)(x_1 + 1) = 0$$

Then:

$$\begin{cases} x_1 = 3 \\ x_1 = -1 \end{cases}$$

For $x_1 = 3$ we substitute in (2)

$$x_2{}^2 + 4x_2 - 12 = 0$$

$$(x_1 - 2)(x_1 + 6) = 0$$

Then:

$$\begin{cases} x_2 = 2 \\ x_2 = -6 \end{cases}$$

Hence:

$\sqrt{1 + 3P_1 + 12P_2} = -1 + 3P_1 + 2P_2$ (Rejected)

Or:

$= -1 + 3P_1 - 6P_2$ (Rejected)

- For $x_1 = -1$ we substitute in (2)

$$x_2{}^2 - 4\,x_2 - 12 = 0$$

$$(x_1 - 6)(x_1 + 2) = 0$$

Then:

$$\begin{cases} x_2 = 6 \\ x_2 = -2 \end{cases}$$

Hence:

$\sqrt{1 + 3P_1 + 12P_2} = -1 - P_1 + 6P_2$ (Rejected)

Or:

$= -1 - P_1 - 2P_2$ (Rejected)

Remarks

As a particular case we can compute $\sqrt{P_1 + P_2}$

$$\sqrt{P_1 + P_2} = x_0 + x_1P_1 + x_2P_2$$

By raise both sides to the second power, we get:

$$P_1 + P_2 = (x_0 + x_1P_1 + x_2P_2)^2$$

$$P_1 + P_2 = (x_0 + x_1P_1)^2 + 2(x_0 + x_1P_1)(x_2P_2) + (x_2P_2)^2$$

$$P_1 + P_2 = x_0{}^2 + 2x_0x_1P_1 + (x_1P_1)^2 + 2(x_0 + x_1P_1)(x_2P_2) + (x_2P_2)^2$$

$$P_1 + P_2 = x_0{}^2 + 2x_0x_1P_1 + x_1{}^2P_1 + 2x_0x_2P_2 + 2x_1x_2P_2 + x_2{}^2P_2$$

$$P_1 + P_2 = x_0{}^2 + (x_1{}^2 + 2x_0x_1)P_1 + (x_2{}^2 + 2x_0x_2 + 2x_1x_2)P_2$$

Whence:

$$\begin{cases} {x_0}^2 = 0 \\ {x_1}^2 + 2x_0x_1 = 1 \\ {x_2}^2 + 2x_0x_2 + 2x_1x_2 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} x_0 = 0 \\ {x_1}^2 + 2x_0x_1 = 1 \\ {x_2}^2 + 2x_0x_2 + 2x_1x_2 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} x_0 = 0 \\ x_1 = \pm 1 \\ {x_2}^2 + 2x_1x_2 = 1 \;\; (1) \end{cases}$$

Case1: $x_1 = 1$ by substituting in (1)

$${x_2}^2 + 2x_2 - 1 = 0$$

$$\Delta = 8$$

Then:

$$\begin{cases} x_2 = \frac{-2 + 2\sqrt{2}}{2} = -1 + \sqrt{2} \\ x_2 = \frac{-2 - 2\sqrt{2}}{2} = -1 - \sqrt{2} \end{cases}$$

Hence:

$$\sqrt{P_1 + P_2} = P_1 + (-1 + \sqrt{2})P_2$$

Or:

$$= P_1 + (-1 - \sqrt{2})P_2$$

- Case 1: $x_1 = -1$ by substituting in (1)

$${x_2}^2 - 2x_2 - 1 = 0$$

$$\Delta = 8$$

Then:

$$\begin{cases} x_2 = \frac{2 + 2\sqrt{2}}{2} = 1 + \sqrt{2} \\ x_2 = \frac{2 - 2\sqrt{2}}{2} = 1 - \sqrt{2} \end{cases}$$

Hence:

$$\sqrt{P_1 + P_2} = -P_1 + (1 + \sqrt{2})P_2$$

Or:

$$= -P_1 + (1 - \sqrt{2})P_2$$

The four results are:

$$(x_0, x_1, x_2) = (0\,, 1\,, -1 + \sqrt{2}), (0\,, 1\,, -1 - \sqrt{2})$$

$$\left(0\,,-1\,,1+\sqrt{2}\right),\left(0\,,-1\,,1-\sqrt{2}\right)$$

Because we are now calculating the square root of a symbolic plithogenic number (according to classical analysis), we only take the result with a positive value, hence:

$$\sqrt{P_1+P_2}=P_1+\left(-1+\sqrt{2}\right)P_2$$

2.2 | The Symbolic Plithogenic Real Polynomial

Definition: A polynomial whose coefficients (at least one of them containing P_1 or P_2 or both P_1, P_2) are symbolic plithogenic numbers called symbolic plithogenic polynomials.

Similarly, we may have symbolic plithogenic real polynomials if their coefficients are symbolic plithogenic real numbers.

Example 2:

- $P(x)=2x^2+(1+4P_1-P_2)x-1+P_1-18P_2$
- $Q(x)=x^3+(-11-9P_1+15P_2)x^2+6I_2x^2-P_2x-14+15P_2$

Example 3:

Solve the equation: $x^2-36-45P_1-19P_2=0$

Solution:

$$x^2=36+45P_1+19P_2$$

Let's find: $\sqrt{36+45P_1+19P_2}$

$$\sqrt{36+45P_1+19P_2}=x_0+x_1P_1+x_2P_2$$

By raising both sides to the second power, we get:

$$36+45P_1+19P_2=(x_0+x_1P_1+x_2P_2)^2$$

$$36+45P_1+19P_2=(x_0+x_1P_1)^2+2(x_0+x_1P_1)(x_2P_2)+(x_2P_2)^2$$

$$36+45P_1+19P_2={x_0}^2+2x_0x_1P_1+(x_1P_1)^2+2(x_0+x_1P_1)(x_2P_2)+(x_2P_2)^2$$

$$36+45P_1+19P_2={x_0}^2+2x_0x_1P_1+{x_1}^2P_1+2x_0x_2P_2+2x_1x_2P_2+{x_2}^2P_2$$

$$36+45P_1+19P_2={x_0}^2+({x_1}^2+2x_0x_1)P_1+({x_2}^2+2x_0x_2+2x_1x_2)P_2$$

Whence:

$$\begin{cases} {x_0}^2=36 \\ {x_1}^2+2x_0x_1=45 \\ {x_2}^2+2x_0x_2+2x_1x_2=19 \end{cases}$$

$$\Rightarrow\begin{cases} x_0=\pm 6 \\ {x_1}^2+2x_0x_1=45 \quad (1) \\ {x_2}^2+2x_0x_2+2x_1x_2=19 \quad (2) \end{cases}$$

Case1: $x_0=6$ by substituting in (1)

$${x_1}^2+12x_1-45=0$$

$$(x_1-3)(x_1+15)$$

Then:

$$\begin{cases} x_1 = 3 \\ x_1 = -15 \end{cases}$$

- For $x_1 = 3$ we substitute in (2)

$$x_2{}^2 + 18x_2 - 19 = 0$$

$$(x_1 - 1)(x_1 + 19) = 0$$

Then:

$$\begin{cases} x_2 = 1 \\ x_2 = -19 \end{cases}$$

Hence:

$$\sqrt{36 + 45P_1 + 19P_2} = 6 + 3P_1 + P_2$$

Or:

$$= 6 + 3P_1 - 19P_2$$

- For $x_1 = -15$ we substitute in (2)

$$x_2{}^2 - 18x_2 - 19 = 0$$

$$(x_1 - 19)(x_1 + 1) = 0$$

Then:

$$\begin{cases} x_2 = 19 \\ x_2 = -1 \end{cases}$$

Hence:

$$\sqrt{36 + 45P_1 + 19P_2} = 6 - 15P_1 + 19P_2$$

Or:

$$= 6 - 15P_1 - P_2$$

Case2: $x_0 = -6$ by substitution in (1)

$$x_1{}^2 - 12x_1 - 45 = 0$$

$$(x_1 - 15)(x_1 + 3) = 0$$

Then:

$$\begin{cases} x_1 = 15 \\ x_1 = -3 \end{cases}$$

- For $x_1 = 15$ we substitute in (2)

$$x_2{}^2 + 18x_2 - 19 = 0$$

$$(x_1 - 1)(x_1 + 19) = 0$$

Then:

$$\begin{cases} x_2 = 1 \\ x_2 = -19 \end{cases}$$

Hence:

$$\sqrt{36 + 45P_1 + 19P_2} = -6 + 15P_1 + P_2$$

Or:

$= -6 + 15P_1 - 19P_2$

- For $x_1 = -3$ we substitute in (2)

$$x_2{}^2 - 18x_2 - 19 = 0$$

$$(x_1 - 19)(x_1 + 1) = 0$$

Then:

$$\begin{cases} x_2 = 19 \\ x_2 = -1 \end{cases}$$

Hence:

$\sqrt{36 + 45P_1 + 19P_2} = -6 - 3P_1 + 19P_2$

Or:

$= -6 - 3P_1 - P_2$

Hence, we got eight solutions:

$$x = \sqrt{1 + 5I_1 + 3I_2} = 6 + 3P_1 + P_2$$

Or:

$= 6 + 3P_1 - 19P_2$

Or:

$= 6 - 15P_1 + 19P_2$

Or:

$= 6 - 15P_1 - P_2$

Or:

$= -6 + 15P_1 + P_2$

Or:

$= -6 + 15P_1 - 19P_2$

Or:

$= -6 - 3P_1 + 19P_2$

Or:

$= -6 - 3P_1 - P_2$

First symbolic plithogenic factoring:

$$P(x) = x^2 - 36 - 45P_1 - 19P_2 = (x - 6 - 3P_1 - P_2)(x + 6 + 3P_1 + P_2)$$

Second symbolic plithogenic factoring:

$$P(x) = x^2 - 36 - 45P_1 - 19P_2 = (x - 6 - 3P_1 + 19P_2)(x + 6 + 3P_1 - 19P_2)$$

Third symbolic plithogenic factoring:

$$P(x) = x^2 - 36 - 45P_1 - 19P_2 = (x - 6 + 15P_1 - 19P_2)(x + 6 - 15P_1 + 19P_2)$$

Fourth symbolic plithogenic factoring:

$$P(x) = x^2 - 36 - 45P_1 - 19P_2 = (x - 6 + 15P_1 + P_2)(x + 6 - 15P_1 - P_2)$$

Differently from the classical polynomial with real coefficients, the symbolic plithogenic polynomials do not have unique factoring! If we check each solution, we get:

$$P(x_1) = P(x_2) = P(x_3) = P(x_4) = P(x_5) = P(x_6) = P(x_7) = P(x_8) = 0$$

Let's compute:

$$P(x_2) = P(6 + 3P_1 - 19P_2) = (6 + 3P_1 - 19P_2)^2 - 36 - 45P_1 - 19P_2$$

$$= 36 + 36P_1 + 9P_1 - 228P_2 - 114P_2 + 361P_2 - 36 - 45P_1 - 19P_2 = 0 \text{ (True)}$$

$$P(x_7) = P(-6 + 15P_1 - 19P_2) = (-6 + 15P_1 - 19P_2)^2 - 36 - 45P_1 - 19P_2$$

$$= 36 - 180P_1 + 225P_1 + 228P_2 - 570P_2 + 361P_2 - 36 - 45P_1 - 19P_2 = 0 \text{ (True)}$$

And so on for the rest of the results.

3 | Conclusions

This paper presented an extensive study on 2-symbolic plithogenic numbers, and its most important results were taking all solutions when solving the 2-symbolic plithogenic equation not just one solution as in the case of finding the square root of the 2-symbolic plithogenic number.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Funding

"This study is supported via funding from Prince Sattam bin Abdulaziz University project number (PSAU/2024/R/1445)".

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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