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On K-idempotent Neutrosophic Fuzzy Matrices

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Abstract

The main intention of this paper is to introduce and study the concepts of idempotent neutrosophic fuzzy matrices and k-idempotent neutrosophic fuzzy matrices. Here the concepts of idempotent neutrosophic fuzzy matrices and k-idempotent neutrosophic fuzzy matrices are discussed in details as a generalization of idempotent fuzzy matrix and k-idempotent fuzzy matrix.

Keywords: Fuzzy Matrix; Idempotent Fuzzy Matrix; Neutrosophic Fuzzy Matrix; Idempotent Neutrosophic.

1 | Introduction

In matrix theory, there are many special types of matrices and one of these matrices is called the idempotent matrix which plays a very important role in functional analysis especially spectral theory of transformation and projections. Idempotent matrices are associated with the theory of generalized inverses. The term idempotent is a matrix that when multiplied by itself does not change. Generalizing the concept of idempotent matrices via permutations, k-idempotent matrices were introduced.

The neutrosophic set theory introduced by Smarandache [1] in which each element has associated defining functions, namely the membership function (T), the non-membership function (F), and the indeterminacy function (I) defined on the universe of discourse X and the three functions are completely independent. Neutrosophic sets provide a more reasonable mathematical framework to deal with indeterminate and inconsistent information. Many uncertain situations can be handled easily by the theory of neutrosophic sets. Dhar al el. [3] defined neutrosophic fuzzy matrices and defined some operations and properties. Further study was made by Das et al. [4] as an extension of [3]. The theory has been applied successfully in various fields by many researchers as can be seen from the references [2-7].

Lee et al. [9] have discussed the concept of idempotent fuzzy matrices. The k-idempotent matrix was the concept of Krishnamoorthy et al. [10] as a generalization of idempotent matrices.

In this article, the idempotent neutrosophic matrix and k-idempotent neutrosophic fuzzy matrix are defined [11, 12]. Moreover, some properties of newly introduced k-idempotent neutrosophic fuzzy matrices are discussed and some numerical examples are provided to make the concept clear and it can be found that the characterizations obtained are analogous to those of idempotent matrices.

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2 | Materials and Methods

This section mainly focuses on some definitions of fuzzy matrix, neutrosophic set, neutrosophic fuzzy matrix etc which are necessary for the subsequent sections.

Definition 2.1: Permutation matrix

A square matrix is said to be a permutation matrix if every row and every column contains exactly one 1 and all other entries are zero.

Definition 2.2: Idempotent fuzzy matrix

A fuzzy matrix $A=\left[a_{ij}\right]$ is said to be idempotent fuzzy matrix when $A^2=A$

Definition 2.3: Fuzzy matrix

Let A be an mXn matrix defined by $A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ : & : & : & : \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$ is said to be a fuzzy matrix if and only if $a_{ij}\in[0,1]$ for $1\le i\le m, 1\le j\le n$.

Definition 2.4: k-Idempotent fuzzy matrix(Gilchrist[8])

A fuzzy matrix $A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ : & : & : & : \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$ is said to be k-idempotent if with respect to fuzzy operative, $KA^2K=A$, where $K=\begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & 0 & 1 & 0 \\ : & : & : & : \\ 1 & 0 & \dots & 0 \end{bmatrix}$

Definition 2.5: Neutrosophic set

Let the universe of discourse be X. Then the neutrosophic set A is an object having the form $A=\{<x:T_A(x),I_A(x),F_A(x)>,x\in X\}$, where functions $T,I,F:X\to\left]^-0,1^+\right[$, define respectively the degree of membership (T), the degree of indeterminacy (I), and the degree of nonmembership (F) of the element $x\in X$ to the set A with the condition $0\le T_A(x)+I_A(x)+F_A(x)\le 3^+$.

Philosophically, the neutrosophic set takes the value from a real standard or non-standard subset of $\left]0,1^+\right[$. For technical applications, instead of $\left]0,1^+\right[$ the is to be considered for better applications interval [0,1].

Definition 2.6: Neutrosophic fuzzy matrix

The entries of the neutrosophic fuzzy matrix will take the form a+Ib(fuzzy number), where a, b are elements on [0,1]and I is an indeterminant such that $I^n=I$,n is a positive integer. For example

$$A=\begin{bmatrix} a_{11}+Ib_{11} & a_{12}+Ib_{12} \\ a_{21}+Ib_{21} & a_{22}+Ib_{22} \end{bmatrix}$$

Is a neutrosophic fuzzy matrix.

Definition 2.7: Square Neutrosophic Fuzzy Matrix

A neutrosophic fuzzy matrix is said to be square neutrosophic fuzzy matrix when the number of rows is equal to the number of columns.

Definition 2.8: Neutrosophic Identity matrix

A square neutrosophic fuzzy matrix is said to be an identity neutrosophic fuzzy matrix when the diagonal elements are of the form 1+I1 and others are of the form 0+I0.

For example

$I=\begin{bmatrix} 1+I1 & 0+I0 \\ 0+I0 & 1+I1 \end{bmatrix}$, $\begin{bmatrix} 1+I1 & 0+I0 & 0+I0 \\ 0+I0 & 1+I1 & 0+I0 \\ 0+I0 & 0+I0 & 1+I1 \end{bmatrix}$ are neutrosophic identity matrices of order 2 and 3 respectively.

3 | Operations on Neutrosophic Matrices

This section deals with some operations on neutrosophic fuzzy matrices.

3.1 | Sum of Two Neutrosophic Fuzzy Matrices

Let

$$A=\begin{bmatrix} a_{11}+Ib_{11} & a_{12}+Ib_{12} \\ a_{21}+Ib_{21} & a_{22}+Ib_{22} \end{bmatrix} \& B=\begin{bmatrix} c_{11}+Id_{11} & c_{12}+Id_{12} \\ c_{21}+Id_{21} & c_{22}+Id_{22} \end{bmatrix}$$

be two neutrosophic fuzzy matrices.

The sum of two neutrosophic fuzzy matrices is defined when the number of rows and the number of columns of both matrices are the same.

$$A+B=\left[C_{ij}\right]$$

Where

$$C_{11}=\max(a_{11},c_{11})+\mathrm{Im}ax(b_{11},d_{11})$$

$$C_{12}=\max(a_{12},c_{12})+\mathrm{Im}ax(b_{12},d_{12})$$

$$C_{21}=\max(a_{21},c_{21})+\mathrm{Im}ax(b_{21},d_{21})$$

$$C_{22}=\max(a_{22},c_{22})+\mathrm{Im}ax(b_{22},d_{22})$$

3.2 | Multiplication of Two Neutrosophic Matrices

The multiplication operations of two neutrosophic fuzzy matrices are defined when the number of columns of the first matrix is equal to the number of rows of the second matrix. Let us consider the two neutrosophic fuzzy matrices $A=\left[a_{ij}+Ib_{ij}\right]$, $B=\left[c_{ij}+Id_{ij}\right]$

Then

$$AB=[\max\min(a_{ij},c_{ji})+\mathrm{Im}ax\min(b_{ij},d_{ji})]$$

Let

$$AB=[D_{ij}]$$

Where

$$D_{11}=[\max\{\min(a_{11},c_{11}),\min(a_{12},c_{21})\}+\mathrm{Im}ax\{\min(b_{11},d_{11}),\min(b_{12},d_{21})\}]$$

$$D_{12}=[\max\{\min(a_{11},c_{12}),\min(a_{12},c_{22})\}+\mathrm{Im}ax\{\min(b_{11},d_{11}),\min(b_{12},d_{22})\}]$$

$D_{21} = [\max\{\min(a_{21}, c_{11}), \min(a_{22}, c_{21})\} + \text{I}\max\{\min(b_{21}, d_{11}), \min(b_{22}, d_{21})\}]$

$D_{22} = [\max\{\min(a_{21}, c_{12}), \min(a_{12}, c_{22})\} + \text{I}\max\{\min(b_{21}, d_{12}), \min(b_{22}, d_{22})\}]$

4 | Results

In this section, some new concepts of neutrosophic fuzzy matrices are introduced.

Definition 4.1: Idempotent neutrosophic fuzzy matrix.

A square neutrosophic fuzzy matrix $A = \left[a_{ij} + Ib_{ij}\right]$ is said to be an idempotent neutrosophic fuzzy matrix when $A^2 = A$.

Proposition: If a neutrosophic fuzzy matrix A is idempotent then $A^n = A$.

Proof: Since A is an idempotent neutrosophic fuzzy matrix then

$A^2 = A$

Pre-multiplying by A, it is seen that

$A^2 = A$

$A.A^2 = A.A$

$\Rightarrow A^3 = A^2 = A$

Similarly, it can be shown that $A^n = A$.

Definition 4.2: Permutation neutrosophic matrix.

A permutation matrix is obtained by repeated interchanging the rows and columns of an identity matrix. Proceeding in the same way, permutation neutrosophic matrix can also be defined. A nXn neutrosophic matrix K is a permutation neutrosophic matrix if and only if it can be obtained from the nXn identity neutrosophic matrix I by performing one or more interchanges of the rows and columns of I.

Some examples

$$\begin{bmatrix} 0+I0 & 1+I1 \\ 1+I1 & 0+I0 \end{bmatrix}, \begin{bmatrix} 0+I0 & 0+I0 & 1+I1 \\ 0+I0 & 1+I1 & 0+I0 \\ 1+I1 & 0+I0 & 0+I0 \end{bmatrix}$$

are neutrosophic matrices of order 2 and order 3 respectively.

Remark 1: The product of permutation matrix k is a neutrosophic identity matrix.

Proposition: If A is a k-idempotent neutrosophic matrix and K is a permutation neutrosophic matrix which is conformable for multiplication then AK=KA.

Example: Let us consider a 2X2 idempotent neutrosophic matrix.

$$A = \begin{bmatrix} 0.4+I0.5 & 0.2+I0.3 \\ 0.2+I0.3 & 0.4+I0.5 \end{bmatrix}$$

And

$$K = \begin{bmatrix} 0+I0 & 1+I1 \\ 1+I1 & 0+I0 \end{bmatrix}$$

Then

$$AK = \begin{bmatrix} 0.4+I0.5 & 0.2+I0.3 \\ 0.2+I0.3 & 0.4+I0.5 \end{bmatrix} \begin{bmatrix} 0+I0 & 1+I1 \\ 1+I1 & 0+I0 \end{bmatrix}$$

$$= \begin{bmatrix} \max(0,0.2)+\mathrm{Im}\,ax(0,0.3) & \max(0,0.4)+\mathrm{Im}\,ax(0,0.5) \\ \max(0.4,0)+\mathrm{Im}\,ax(0.5,0) & \max(0.2,0)+I\max(0.3,0) \end{bmatrix}$$

$$= \begin{bmatrix} 0.2+I0.3 & 0.4+I0.5 \\ 0.4+I0.5 & 0.2+I0.3 \end{bmatrix}$$

Similarly

$$KA = \begin{bmatrix} 0.2+I0.3 & 0.4+I0.5 \\ 0.4+I0.5 & 0.2+I0.3 \end{bmatrix}$$

Hence AK=KA.

That is

$$k^2 = \begin{bmatrix} 0+I0 & 0+I0 & 1+I1 \\ 0+I0 & 1+I1 & 0+I0 \\ 1+I1 & 0+I0 & 0+I0 \end{bmatrix} \begin{bmatrix} 0+I0 & 0+I0 & 1+I1 \\ 0+I0 & 1+I1 & 0+I0 \\ 1+I1 & 0+I0 & 0+I0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+I0 & 0+I0 & 1+I1 \\ 0+I0 & 1+I1 & 0+I0 \\ 1+I1 & 0+I0 & 0+I0 \end{bmatrix} = I$$

The identity matrix.

Let us consider a 3X3 idempotent matrix A of the form

$$A = \begin{bmatrix} 0.4+I0.5 & 0.2+I0.3 & 0.2+I0.4 \\ 0.2+I0.3 & 0.4+I0.3 & 0.2+I0.3 \\ 0.2+I0.4 & 0.2+I0.3 & 0.4+I0.5 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0.4+I0.5 & 0.2+I0.3 & 0.2+I0.4 \\ 0.2+I0.3 & 0.4+I0.3 & 0.2+I0.3 \\ 0.2+I0.4 & 0.2+I0.3 & 0.4+I0.5 \end{bmatrix}$$

$$A^2k = \begin{bmatrix} 0.4+I0.5 & 0.2+I0.3 & 0.2+I0.4 \\ 0.2+I0.3 & 0.4+I0.3 & 0.2+I0.3 \\ 0.2+I0.4 & 0.2+I0.3 & 0.4+I0.5 \end{bmatrix} \begin{bmatrix} 0+I0 & 0+I0 & 1+I1 \\ 0+I0 & 1+I1 & 0+I0 \\ 1+I1 & 0+I0 & 0+I0 \end{bmatrix}$$

$$A^2k = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

Where

$$x_{11} = \max(0,0,0.2)+\mathrm{Im}\,ax(0,0,0.4)$$

=0.2+I0.4

$x_{12} = \max(0, 0.2, 0) + \mathrm{Im}ax(0, 0.3, 0)$

=0.2+I0.3

$x_{13} = \max(0.4, 0, 0) + \mathrm{Im}ax(0.5, 0, 0)$

=0.4+I0.5

$x_{21} = \max(0, 0, 0.2) + \mathrm{Im}ax(0, 0, 0.3)$

=0.2+I0.3

$x_{22} = \max(0, 0.4, 0) + \mathrm{Im}ax(0, 0.3, 0)$

=0.4+I0.3

$x_{23} = \max(0.2, 0, 0) + \mathrm{Im}ax(0.3, 0, 0)$

=0.2+I0.3

$x_{31} = \max(0, 0, 0.4) + \mathrm{Im}ax(0, 0, 0.5)$

=0.4+I0.5

$x_{32} = \max(0, 0.2, 0) + \mathrm{Im}ax(0, 0.3, 0)$

=0.2+I0.3

$x_{33} = \max(0.2, 0, 0) + \mathrm{Im}ax(0.4, 0, 0)$

=0.2+I0.4

Therefore

$$A^2k = \begin{bmatrix} 0.2+I0.4 & 0.2+I0.3 & 0.4+I0.5 \\ 0.2+I0.3 & 0.4+I0.3 & 0.2+I0.3 \\ 0.4+I0.5 & 0.2+I0.3 & 0.2+I0.4 \end{bmatrix}$$

Then

$$kA^2k = \begin{bmatrix} 0+I0 & 0+I0 & 1+I1 \\ 0+I0 & 1+I1 & 0+I0 \\ 1+I1 & 0+I0 & 0+I0 \end{bmatrix} \begin{bmatrix} 0.2+I0.4 & 0.2+I0.3 & 0.4+I0.5 \\ 0.2+I0.3 & 0.4+I0.3 & 0.2+I0.3 \\ 0.4+I0.5 & 0.2+I0.3 & 0.2+I0.4 \end{bmatrix}$$

Hence it can be said that matrix A is ak-idempotent matrix.

Theorem: Characterization of a 2X2 K-idempotent neutrosophic matrix).

Let $$A = \begin{bmatrix} a_{11}+Ib_{11} & a_{12}+Ib_{12} \\ a_{21}+Ib_{21} & a_{22}+Ib_{22} \end{bmatrix}$$

Be a 2X2 K-idempotent neutrosophic matrix. Then

$a_{11} = a_{22}$
$a_{21} = a_{12}$
$b_{11} = b_{22}$
$b_{12} = b_{21}$

Proof: Let

$$A = \begin{bmatrix} a_{11} + Ib_{11} & a_{12} + Ib_{12} \\ a_{21} + Ib_{21} & a_{22} + Ib_{22} \end{bmatrix}$$

Then

$$A^2 = \begin{bmatrix} a_{11} + Ib_{11} & a_{12} + Ib_{12} \\ a_{21} + Ib_{21} & a_{22} + Ib_{22} \end{bmatrix} \begin{bmatrix} a_{11} + Ib_{11} & a_{12} + Ib_{12} \\ a_{21} + Ib_{21} & a_{22} + Ib_{22} \end{bmatrix}$$

$$= \begin{bmatrix} x_{11} + Iy_{11} & x_{12} + Iy_{12} \\ x_{21} + Iy_{21} & x_{22} + Iy_{22} \end{bmatrix}$$

Where

$$x_{11} = \max\{\min(a_{11}, a_{11}), \min(a_{12}, a_{21})\} + I\max\{\min(b_{11}, b_{11}), \min(b_{12}, b_{21})\}$$

$$x_{12} = \max\{\min(a_{11}, a_{12}), \min(a_{12}, a_{22})\} + I\max\{\min(b_{11}, b_{12}), \min(b_{12}, b_{22})\}$$

$$x_{21} = \max\{\min(a_{21}, a_{11}), \min(a_{22}, a_{21})\} + I\max\{\min(b_{21}, b_{11}), \min(b_{22}, b_{21})\}$$

$$x_{22} = \max\{\min(a_{21}, a_{12}), \min(a_{22}, a_{22})\} + I\max\{\min(b_{21}, b_{12}), \min(b_{22}, b_{22})\}$$

$$KA^2 = \begin{bmatrix} 0 + I0 & 1 + I1 \\ 1 + I1 & 0 + I0 \end{bmatrix} \begin{bmatrix} x_{11} + Iy_{11} & x_{12} + Iy_{12} \\ x_{21} + Iy_{21} & x_{22} + Iy_{22} \end{bmatrix}$$

$$= \begin{bmatrix} C_{11} + ID_{11} & C_{12} + ID_{12} \\ C_{21} + ID_{21} & C_{22} + ID_{22} \end{bmatrix}$$

$$C_{11} = \max\{\min(0, x_{11}), \min(1, x_{21})\} + I\max\{\min(0, y_{11}), \min(1, y_{21})\}$$

$$C_{12} = \max\{\min(0, x_{12}), \min(1, x_{22})\} + I\max\{\min(0, y_{12}), \min(1, y_{22})\}$$

$$C_{21} = \max\{\min(1, x_{11}), \min(0, x_{21})\} + I\max\{\min(1, y_{11}), \min(0, y_{21})\}$$

$$C_{22} = \max\{\min(1, x_{12}), \min(0, x_{22})\} + I\max\{\min(1, y_{12}), \min(0, y_{22})\}$$

$$KA^2K = \begin{bmatrix} C_{11} + ID_{11} & C_{12} + ID_{12} \\ C_{21} + ID_{21} & C_{22} + ID_{22} \end{bmatrix} \begin{bmatrix} 0 + I0 & 1 + I1 \\ 1 + I1 & 0 + I0 \end{bmatrix}$$

$$= \begin{bmatrix} E_{11} + IF_{11} & E_{12} + IF_{12} \\ E_{21} + IF_{21} & E_{22} + IF_{22} \end{bmatrix}$$

Where

$$E_{11} = \max\{\min(0, C_{11}), \min(1, C_{12})\} + I\max\{\min(0, D_{11}), \min(1, D_{12})\}$$

$$= \max(0, C_{12}) + I\max(0, D_{12})$$
$$= C_{12} + ID_{12}$$

$$= \max\{\min(0, x_{12}), \min(1, x_{22})\} + I\max\{\min(0, y_{12}), \min(1, y_{22})\}$$

$$= \max(0, x_{22}) + I\max(0, y_{22})$$

$$= x_{22} + Iy_{22}$$

$$= \max\{\min(a_{21}, a_{12}), \min(a_{22}, a_{22})\} + I\max\{\min(b_{21}, b_{12}), \min(b_{22}, b_{22})\}$$

$$= \max\{\min(a_{21}, a_{12}), a_{22})\} + I\max\{\min(b_{21}, b_{12}), b_{22})\}$$

Since A is k-idempotent then

$$KA^2K = A$$

and hence the following

$E_{11} + IF_{11} = a_{11} + Ib_{11}$, $E_{12} + IF_{12} = a_{12} + Ib_{12}$, $E_{21} + IF_{21} = a_{21} + Ib_{21}$ and $E_{22} + IF_{22} = a_{22} + Ib_{22}$

From the above results

$$a_{11} + Ib_{11} = \max\{\min(a_{21}, a_{12}), a_{22})\} + I\max\{\min(b_{21}, b_{12}), b_{22})\}$$

Then

$a_{11} \geq a_{22}$, $b_{11} \geq b_{22}$

$$E_{22} = \max\{\min(1, C_{21}), \min(0, C_{22})\} + I\max\{\min(1, D_{21}), \min(0, D_{22})\}$$
$$= C_{21} + ID_{21}$$
$$= x_{11} + Iy_{11}$$

$$= \max\{\min(a_{11}, a_{11}), \min(a_{12}, a_{21})\} + I\max\{\min(b_{11}, b_{11}), \min(b_{12}, b_{21})\}$$
$$= \max\{(a_{11}, \min(a_{12}, a_{21}))\} + I\max\{(b_{11}, \min(b_{12}, b_{21}))\}$$

Since

$$E_{22} + IF_{22} = a_{22} + Ib_{22}$$

Then

$$\max\{(a_{11}, \min(a_{12}, a_{21}))\} + I\max\{(b_{11}, \min(b_{12}, b_{21}))\} = a_{22} + Ib_{22}$$
$$\Rightarrow \max\{(a_{11}, \min(a_{12}, a_{21}))\} = a_{22} \;\&\; \max\{(b_{11}, \min(b_{12}, b_{21}))\} = b_{22}$$
$$\Rightarrow a_{22} \geq a_{11} \;\&\; b_{22} \geq b_{11}$$

From the above two results it is obtained that

$a_{11} = a_{22}$, $b_{11} = b_{22}$

Similarly

$$E_{12} = \max\{\min(1, C_{11}), \min(0, C_{12})\} + I\max\{\min(1, D_{11}), \min(0, D_{12})\}$$

$$= C_{11} + ID_{11}$$
$$= x_{21} + Iy_{21}$$

$$= \max\{\min(a_{21}, a_{11}), \min(a_{22}, a_{21})\} + I\max\{\min(b_{21}, b_{11}), \min(b_{22}, b_{21})\}$$

Since

$a_{11} = a_{22}$, $b_{11} = b_{22}$

$$\Rightarrow \max\{\min(a_{21}, a_{11}), \min(a_{22}, a_{21})\} + I \max\{\min(b_{21}, b_{11}), \min(b_{22}, b_{21})\}$$
$$= \max\{\min(a_{21}, a_{22}), \min(a_{22}, a_{21})\} + I \max\{\min(b_{21}, b_{22}), \min(b_{22}, b_{21})\}$$
$$= \min(a_{21}, a_{22}) + I \min(b_{21}, b_{22})$$

Again since

$E_{12} + IF_{12} = a_{12} + Ib_{12}$

It is seen that

$\min(a_{21}, a_{22}) + I \min(b_{21}, b_{22}) = a_{12} + Ib_{12}$
$\Rightarrow a_{12} \le a_{21} \ \& \ b_{12} \le b_{21}$

$$E_{21} = \max\{\min(0, C_{21}), \min(1, C_{22})\} + I \max\{\min(0, D_{21}), \min(1, D_{22})\}$$
$$= C_{22} + ID_{22}$$
$$= x_{12} + Iy_{12}$$

$$= \max\{\min(a_{11}, a_{12}), \min(a_{12}, a_{22})\} + I \max\{\min(b_{11}, b_{12}), \min(b_{12}, b_{22})\}$$

Since

$a_{11} = a_{22}$, $b_{11} = b_{22}$

Then

$$E_{21} + ID_{21} = \max\{\min(a_{11}, a_{12}), \min(a_{12}, a_{22})\} + I \max\{\min(b_{11}, b_{12}), \min(b_{12}, b_{22})\}$$
$$\Rightarrow E_{21} + ID_{21} = \max\{\min(a_{11}, a_{12}), \min(a_{12}, a_{11})\} + I \max\{\min(b_{11}, b_{12}), \min(b_{12}, b_{11})\}$$
$$\Rightarrow \min(a_{11}, a_{12}) + I \min(b_{11}, b_{12}) = a_{21} + Ib_{21}$$
$$\Rightarrow a_{21} = \min(a_{11}, a_{12}) \ \& \ b_{21} = \min(b_{11}, b_{12})$$
$$\Rightarrow a_{21} \le a_{12} \ \& \ b_{21} \le b_{12}$$

From the above two results, it can concluded that

$a_{21} = a_{12} \ \& \ b_{21} = b_{12}$

Hence the proof.

Proposition: The sum of two k-idempotent matrices is also k-idempotent.

Let us consider two k-idempotent neutrosophic matrices A and B as

$$A = \begin{bmatrix} 0.4 + I0.5 & 0.2 + I0.3 \\ 0.2 + I0.3 & 0.4 + I0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 + I0.6 & 0.4 + I0.3 \\ 0.4 + I0.3 & 0.5 + I0.6 \end{bmatrix}$$

Then the sum of the above matrices

$$A + B = \begin{bmatrix} 0.5 + I0.6 & 0.4 + I0.3 \\ 0.4 + I0.3 & 0.5 + I0.6 \end{bmatrix}$$

This is again k-idempotent.

Proposition: The subtraction of two k-idempotent matrices is also k-idempotent.

Let us consider two k-idempotent matrices A and B as

$$A=\begin{bmatrix}0.4+I0.5 & 0.2+I0.3\\ 0.2+I0.3 & 0.4+I0.5\end{bmatrix},\quad B=\begin{bmatrix}0.5+I0.6 & 0.4+I0.3\\ 0.4+I0.3 & 0.5+I0.6\end{bmatrix}$$

Then the sum of the above matrices

$$A-B=\begin{bmatrix}0.4+I0.5 & 0.2+I0.3\\ 0.2+I0.3 & 0.4+I0.5\end{bmatrix}$$

This is again k-idempotent.

Proposition : The transpose of A k-idempotent matrix is k-idempotent.

Let us consider a k-idempotent matrix A as

$$A=\begin{bmatrix}0.4+I0.5 & 0.2+I0.3 & 0.2+I0.4\\ 0.2+I0.3 & 0.4+I0.3 & 0.2+I0.3\\ 0.2+I0.4 & 0.2+I0.3 & 0.4+I0.5\end{bmatrix}$$

The transpose of the above matrix is

$$A^T=\begin{bmatrix}0.4+I0.5 & 0.2+I0.3 & 0.2+I0.4\\ 0.2+I0.3 & 0.4+I0.3 & 0.2+I0.3\\ 0.2+I0.4 & 0.2+I0.3 & 0.4+I0.5\end{bmatrix}$$

Here

$$A=A^T$$

Since A is k-idempotent then it is obvious that A^T is also k-idempotent.

Proposition: Let A and B be two k-idempotent fuzzy matrices. If AB=BA, then AB is also a k-idempotent fuzzy matrix.

If AB=BA then $(AB)^2=A^2B^2$

$$k(AB)^2k=kA^2B^2k$$

$$=kA^2k.kB^2k$$
$$=AB$$

Theorem: If $A_1, A_2, A_3, \ldots\ldots\ldots, A_3$ are k-idempotent then $A_1A_2A_3\ldots\ldots\ldots A_3$ is also k-idempotent.

$$k(A_1A_2A_3\ldots\ldots A_n)^2k=k(A_1A_2A_3\ldots.A_n.A_1A_2A_3\ldots.A_n)k$$

$$=k(A_1^2A_2^2A_3^2\ldots.A_n^2)k$$

$$=kA_1^2k.kA_2^2k.kA_3^2k\ldots.kA_n^2k$$

$$=A_1A_2A_3\ldots A_n$$

Hence $A_1A_2A_3\ldots.A_n$ is also k-idempotent.

Proposition: An idempotent fuzzy matrix commutes with the associated permutation.

That is if A is an idempotent fuzzy matrix and k is the associated permutation then AK=KA.

Proposition: The complement of a k-idempotent matrix is also k-idempotent.

Let us consider a k-idempotent matrix A as

$$A=\begin{bmatrix} 0.4+I0.5 & 0.2+I0.3 & 0.2+I0.4 \\ 0.2+I0.3 & 0.4+I0.3 & 0.2+I0.3 \\ 0.2+I0.4 & 0.2+I0.3 & 0.4+I0.5 \end{bmatrix}$$

Then the complement of the above matrix is

$$A^c=\begin{bmatrix} 0.5+I0.4 & 0.3+I0.2 & 0.4+I0.2 \\ 0.3+I0.2 & 0.3+I0.4 & 0.3+I0.2 \\ 0.4+I0.2 & 0.3+I0.2 & 0.5+I0.4 \end{bmatrix}$$

Proposition: If A and B are two k-idempotent fuzzy matrices then A(A+B)B commutes with the permutation matrix K.

Proof:

$$A(A+B)B=A^2B+AB^2$$

$$=KA^2KB+AKB^2K$$

$$=KA^2BK+KAB^2K$$

$$=K(A(A+B)B)K$$

$$=K(A(A+B)B)$$

$$=(A(A+B)B)K$$

5 | Conclusion

The main content of the article is to discuss about idempotent and k-idempotent neutrosophic matrices. Some properties associated with these introduced concepts are discussed and supported by some numerical examples. Most of the properties are found to be analogous to the existing properties of idempotent and k-idempotent matrices.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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