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Study for General Plithogenic Soft Expert Graphs

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Abstract

Graph theory, a branch of mathematics, focuses on studying graphs that model relationships between objects through vertices and edges [29]. The Turiyam Neutrosophic Graph handles uncertainty by assigning four values to vertices and edges: truth, indeterminacy, falsity, and liberal state. In contrast, the Plithogenic Graph is a general structure where vertices and edges are defined by degrees of appurtenance and contradiction across multiple attributes [45]. The General Plithogenic Graph extends the Plithogenic Graph, offering a broader framework [45]. This study presents new concepts of the General Plithogenic Soft Expert Graph and the Turiyam Neutrosophic Soft Expert Graph, with the former expanding the existing Soft Expert Graph model into a more generalized structure.

Keywords: Neutrosophic Graph; Fuzzy Graph; Plithogenic Graph; Soft Expert Graph.

1 | Introduction

1.1 | Uncertain Graph

Graph theory, a branch of mathematics, explores the study of graphs that model relationships between objects using vertices and edges [29]. It has found extensive applications in both mathematical and real-world contexts [38, 64, 74, 76, 94]. Recently, graph theory has also been widely utilized in artificial intelligence research [23 96, 114, 119].

To address real-world uncertainty, concepts like Fuzzy Sets [116] were introduced, later extended to Neutrosophic Sets [98] and other frameworks, leading to various applications across multiple fields. This paper examines several models of uncertain graphs-namely, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic Graphs-designed to handle uncertainty in diverse contexts. These models, collectively referred to as uncertain graphs, extend classical graph theory by integrating various levels of uncertainty. Due to their significance, numerous related graph classes and applications have been developed [37, 39, 41–44, 48, 50, 52–56]. In addition to graph models, foundational concepts like Fuzzy Sets and Neutrosophic Sets have been extensively explored and are widely recognized in the literature [10–14, 26, 30–32, 34, 73, 75, 83, 98, 116, 117].

This paper specifically focuses on the Turiyam Neutrosophic Graph and the General Plithogenic Graph. The Turiyam Neutrosophic Graph represents uncertainty using four values for vertices and edges: truth, indeterminacy, falsity, and liberal state. Conversely, the Plithogenic Graph is a general structure in which

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vertices and edges are characterized by degrees of appurtenance and contradiction across multiple attributes [45]. The General Plithogenic Graph is an extension of the Plithogenic Graph, providing a broader framework [45]. Note that the Turiyam Neutrosophic Set is actually a particular case of the quadripartitioned Neutrosophic Set, by replacing "Contradiction" with "Liberal" [97]. The corresponding graph concept known as quadripartitioned neutrosophic graphs is well-documented [67,68].

1.2 | Soft Expert Graph

A Soft Expert Graph combines graph theory with expert evaluations, where vertices and edges are associated with fuzzy sets, and experts' inputs define uncertainty and relationships within the graph. Related concepts include the Fuzzy Soft Expert Graph [92], Intuitionistic Fuzzy Soft Expert Graph [109], and Neutrosophic Soft Expert Graph [111]. These models have been studied for applications such as multi-criteria decision-making (e.g., [15, 33, 88, 106, 108]). Additionally, related concepts such as Soft Expert Sets are also well-known [6, 9, 84, 91].

1.3 | Contributions

Building upon the research of Uncertain Graphs and Soft Expert Graphs, this study introduces and analyzes new concepts of the General Plithogenic Soft Expert Graph and the Turiyam Neutrosophic Soft Expert Graph. The General Plithogenic Soft Expert Graph extends the existing Soft Expert Graph model into a more generalized framework.

2 | Preliminaries and Definitions

In this section, we present a brief overview of the definitions and notations used throughout this paper.

2.1 | Basic Graph Concepts

Here are a few basic graph concepts listed below. For more foundational graph concepts and notations, please refer to [27, 28-29, 64, 113].

Definition 1 (Graph). [29] A graph G is a mathematical structure consisting of a set of vertices $V(G)$ and a set of edges $E(G)$ that connect pairs of vertices, representing relationships or connections between them. Formally, a graph is defined as $G = (V, E)$, where V is the vertex set and E is the edge set.

Definition 2 (Degree). [29] Let $G = (V, E)$ be a graph. The degree of a vertex $v \in V$, denoted $\deg(v)$, is the number of edges incident to v. Formally, for undirected graphs:

$$\deg(v) = |\{e \in E \mid v \in e\}|$$

In the case of directed graphs, the in-degree $\deg^-(v)$ is the number of edges directed into v, and the out-degree $\deg^+(v)$ is the number of edges directed out of v.

2.2 | Uncertain Graph

This paper addresses Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic concepts within the framework of Unified Uncertain Graphs. Note that Turiyam Neutrosophic Set is actually a particular case of the Quadruple Neutrosophic Set, by replacing "Contradiction" with "Liberal" 97].

Definition 3 (Unified Uncertain Graphs Framework). (cf.[51]) Let $G = (V, E)$ be a classical graph with a set of vertices V and a set of edges E. Depending on the type of graph, each vertex $v \in V$ and edge $e \in E$ is assigned membership values to represent various degrees of truth, indeterminacy, falsity, and other nuanced measures of uncertainty.

1. Fuzzy Graph [18,40,60,61,72,77,80,89,90,104,112]:
   - Each vertex $v \in V$ is assigned a membership degree $\sigma(v) \in [0,1]$.

- Each edge $e = (u, v) \in E$ is assigned a membership degree $\mu(u, v) \in [0,1]$.

2. Intuitionistic Fuzzy Graph (IFG) 1 1, 17, 24, 70,78,107,110,118:
   - Each vertex $v \in V$ is assigned two values: $\mu_A(v) \in [0,1]$ (degree of membership) and $v_A(v) \in [0,1]$ (degree of non-membership), such that $\mu_A(v) + v_A(v) \leq 1$.
   - Each edge $e = (u, v) \in E$ is assigned two values: $\mu_B(u, v) \in [0,1]$ and $v_B(u, v) \in [0,1]$, with $\mu_B(u, v) + v_B(u, v) \leq 1$.
3. Neutrosophic Graph [4,5,22,35,36,45,47,49,53,56,65,66,71,93,101,102]:
   - Each vertex $v \in V$ is assigned a triplet $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, where $\sigma_T(v), \sigma_I(v), \sigma_F(v) \in [0,1]$ and $\sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3$.
   - Each edge $e = (u, v) \in E$ is assigned a triplet $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$.
4. Turiyam Neutrosophic Graph [57-59]:
   - Each vertex $v \in V$ is assigned a quadruple $\sigma(v) = (t(v), iv(v), fv(v), lv(v))$, where each component is in $[0,1]$ and $t(v) + iv(v) + fv(v) + lv(v) \leq 4$.
   - Each edge $e = (u, v) \in E$ is similarly assigned a quadruple.
5. Vague Graph [2, 3, 19, 21, 86, 87, 95]:
   - Each vertex $v \in V$ is assigned a pair $(\tau(v), \phi(v))$, where $\tau(v) \in [0,1]$ is the degree of truthmembership and $\phi(v) \in [0,1]$ is the degree of false-membership, with $\tau(v) + \phi(v) \leq 1$.
   - The grade of membership is characterized by the interval $[\tau(v), 1 - \phi(v)]$.
   - Each edge $e = (u, v) \in E$ is assigned a pair $(\tau(e), \phi(e))$, satisfying:
   $$\tau(e) \leq \min\{\tau(u), \tau(v)\},\ \phi(e) \geq \max\{\phi(u), \phi(v)\}$$
6. Hesitant Fuzzy Graph [16, 63, 81, 82,115]:
   - Each vertex $v \in V$ is assigned a hesitant fuzzy set $\sigma(v)$, represented by a finite subset of $[0,1]$, denoted $\sigma(v) \subseteq [0,1]$.
   - Each edge $e = (u, v) \in E$ is assigned a hesitant fuzzy set $\mu(e) \subseteq [0,1]$.
   - Operations on hesitant fuzzy sets (e.g., intersection, union) are defined to handle the hesitation in membership degrees.
7. Single-Valued Pentapartitioned Neutrosophic Graph [25, 67, 69, 85]:
   - Each vertex $v \in V$ is assigned a quintuple $\sigma(v) = (T(v), C(v), R(v), U(v), F(v))$, where:
     - $T(v) \in [0,1]$ is the truth-membership degree.
     - $C(v) \in [0,1]$ is the contradiction-membership degree.
     - $R(v) \in [0,1]$ is the ignorance-membership degree.
     - $U(v) \in [0,1]$ is the unknown-membership degree.
     - $F(v) \in [0,1]$ is the false-membership degree.
     - $T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$.

- Each edge $e = (u, v) \in E$ is assigned a quintuple $\mu(e) = (T(e), C(e), R(e), U(e), F(e))$, satisfying:

$$\begin{cases} T(e) \leq \min\{T(u), T(v)\} \\ C(e) \leq \min\{C(u), C(v)\} \\ R(e) \geq \max\{R(u), R(v)\} \\ U(e) \geq \max\{U(u), U(v)\} \\ F(e) \geq \max\{F(u), F(v)\} \end{cases}$$

Definition 4. $[62, 99, 100, 103, 105]$ Let $G = (V, E)$ be a crisp graph where V is the set of vertices and $E \subseteq V \times V$ is the set of edges. A Plithogenic Graph PG is defined as:

$$PG = (PM, PN)$$

where:

1. Plithogenic Vertex Set $PM = (M, l, Ml, adf, aCf)$:
   - $M \subseteq V$ is the set of vertices.
   - l is an attribute associated with the vertices.
   - Ml is the range of possible attribute values.
   - adf : $M \times Ml \rightarrow [0,1]^s$ is the Degree of Appurtenance Function (DAF) for vertices.
   - $aCf: Ml \times Ml \rightarrow [0,1]^t$ is the Degree of Contradiction Function (DCF) for vertices.
2. Plithogenic Edge Set $PN = (N, m, Nm, bdf, bCf)$:
   - $N \subseteq E$ is the set of edges.
   - m is an attribute associated with the edges.
   - Nm is the range of possible attribute values.
   - bdf: $N \times Nm \rightarrow [0,1]^s$ is the Degree of Appurtenance Function (DAF) for edges.
   - bCf: Nm $\times Nm \rightarrow [0,1]^t$ is the Degree of Contradiction Function (DCF) for edges.

The Plithogenic Graph PG must satisfy the following conditions:

1. Edge Appurtenance Constraint: For all $(x, a), (y, b) \in M \times Ml$:

$$bdf((xy), (a, b)) \leq \min\{adf(x, a), adf(y, b)\}$$

where $xy \in N$ is an edge between vertices x and y, and $(a, b) \in Nm \times Nm$ are the corresponding attribute values.

2. Contradiction Function Constraint: For all $(a, b), (c, d) \in Nm \times Nm$:

$$bCf((a, b), (c, d)) \leq \min\{aCf(a, c), aCf(b, d)\}$$

3. Reflexivity and Symmetry of Contradiction Functions:

$$\begin{aligned} aCf(a, a) &= 0, & \forall a &\in Ml \\ aCf(a, b) &= aCf(b, a), & \forall a, b &\in Ml \\ bCf(a, a) &= 0, & \forall a &\in Nm \\ bCf(a, b) &= bCf(b, a), & \forall a, b &\in Nm \end{aligned}$$

Example 5. (cf. 45 51]) The following examples are provided.

- When $s = t = 1, PG$ is called a Plithogenic Fuzzy Graph.
- When $s = 2, t = 1, PG$ is called a Plithogenic Intuitionistic Fuzzy Graph.
- When $s = 3, t = 1, PG$ is called a Plithogenic Neutrosophic Graph.
- When $s = 4, t = 1, PG$ is called a Plithogenic Turiyam Neutrosophic Graph.

The General Plithogenic Graph is a generalization of the Plithogenic Graph (cf.[35, 45, 79]).

Definition 6 (General Plithogenic Graph). [45] Let $G = (V, E)$ be a classical graph, where V is a finite set of vertices, and $E \subseteq V \times V$ is a set of edges.

A General Plithogenic Graph $G^{GP} = (PM, PN)$ consists of:

1. General Plithogenic Vertex Set PM:

$$PM = (M, l, Ml, adf, aCf)$$

Where:

- $M \subseteq V$: Set of vertices.
- l : Attribute associated with the vertices.
- Ml : Range of possible attribute values.
- $adf: M \times Ml \rightarrow [0,1]^s$: Degree of Appurtenance Function (DAF) for vertices.
- $aCf: Ml \times Ml \rightarrow [0,1]^t$: Degree of Contradiction Function (DCF) for vertices.

2. General Plithogenic Edge Set PN:

$$PN = (N, m, Nm, bdf, bCf)$$

Where:

- $N \subseteq E$: Set of edges.
- m : Attribute associated with the edges.
- Nm: Range of possible attribute values.
- $bdf: N \times Nm \rightarrow [0,1]^s$: Degree of Appurtenance Function (DAF) for edges.
- $bCf: Nm \times Nm \rightarrow [0,1]^t$: Degree of Contradiction Function (DCF) for edges.

The General Plithogenic Graph G^{GP} Only needs to satisfy the following Reflexivity and Symmetry properties of the Contradiction Functions:

- Reflexivity and Symmetry of Contradiction Functions:

$$\begin{aligned} aCf(a,a) &= 0, && \forall a \in Ml \\ aCf(a,b) &= aCf(b,a), && \forall a,b \in Ml \\ bCf(a,a) &= 0, && \forall a \in Nm \\ bCf(a,b) &= bCf(b,a), && \forall a,b \in Nm \end{aligned}$$

2.3 | Soft Expert Graph

The definitions of the Intuitionistic Fuzzy Soft Expert Graph and the Neutrosophic Soft Expert Graph are provided below.

Definition 7. [109] An Intuitionistic Fuzzy Soft Expert Graph (IFSEG) is defined over a simple graph $G^* = (\mathcal{V}, \mathcal{E})$, where:

- $\mathcal{V}$ is a set of vertices,
- $\mathcal{E}$ is a set of edges,
- y is a set of parameters,
- X is a set of experts,
- $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ is a set of opinions, and
- $Z = \mathcal{Y} \times \mathcal{X} \times O$ is the Cartesian product of the sets.

Let $A \subseteq Z$ and let $\text{IFSE}(\mathcal{V})$ denote the set of all intuitionistic fuzzy sets in $\mathcal{V}$. The IFSEG is represented as a 4-tuple:

$$G = (G^*, A, f, g)$$

where:

- $f: A \to \text{IFSE}(\mathcal{V})$ is a function mapping each parameter in A to an intuitionistic fuzzy set of vertices,
- $g: A \to \text{IFSE}(\mathcal{V} \times \mathcal{V})$ is a function mapping each parameter in A to an intuitionistic fuzzy set of edges.

  The mappings f and g are defined as:

$$f(\alpha) = f_\alpha = \{\langle x, \mu_{f_\alpha}(x), v_{f_\alpha}(x)\rangle : x \in \mathcal{V}\}$$
$$g(\alpha) = g_\alpha = \{\langle (x, y), \mu_{g_\alpha}(x, y), v_{g_\alpha}(x, y)\rangle : (x, y) \in \mathcal{V} \times \mathcal{V}\}$$

where:

- $\mu_{f_\alpha}(x)$ and $v_{f_\alpha}(x)$ represent the membership and non-membership degrees of vertex x under parameter α, respectively.
- $\mu_{g_\alpha}(x, y)$ and $v_{g_\alpha}(x, y)$ represent the membership and non-membership degrees of the edge (x, y) under parameter α, respectively.

  These mappings satisfy the following conditions for all $(x, y) \in \mathcal{V} \times \mathcal{V}$ and $\alpha \in A$:

$$\mu_{g_\alpha}(x, y) \leq \min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\}$$
$$v_{g_\alpha}(x, y) \leq \min\{v_{f_\alpha}(x), v_{f_\alpha}(y)\}$$

  The IFSEG can also be denoted as:

$$G = (G^*, A, f, g) = \{\text{IFSE}(\alpha) : \alpha \in A\}$$

where $\text{IFSE}(\alpha)$ represents a family of parameterized intuitionistic fuzzy soft expert graphs.

Definition 8. [111] A Neutrosophic Soft Expert Graph (NSEG) is defined over a simple graph $G^* = (V, E)$, where V is the set of vertices, E is the set of edges, A is a set of parameters, and X is a set of experts. Let $N(V)$ denote the set of all neutrosophic sets in V. The NSEG is represented as a 4-tuple:

$$G = (G^*, A, f, g)$$

Where:

- $f: A \to N(V)$ is a function mapping each parameter in A to a neutrosophic set of vertices,
- $g: A \to N(V \times V)$ is a function mapping each parameter in A to a neutrosophic set of edges.

The mappings f and g are defined as:

$$f(\alpha) = \{x, \mu_f(x), v_f(x), \pi_f(x): x \in V\}$$
$$g(\alpha) = \{(x, y), \mu_f(x, y), v_f(x, y), \pi_f(x, y): (x, y) \in V \times V\}$$

Where:

- $\mu_f(x), v_f(x)$, and $\pi_f(x)$ represent the truth, indeterminacy, and falsity membership degrees of vertex x, respectively.
- $\mu_f(x, y), v_f(x, y)$, and $\pi_f(x, y)$ represent the truth, indeterminacy, and falsity membership degrees of the edge (x, y), respectively.

These mappings satisfy the following conditions for all $(x, y) \in V \times V$ and $\alpha \in A$:

$$\mu_g(x, y) \leq \min\{\mu_f(x), \mu_f(y)\}$$
$$v_g(x, y) \leq \min\{v_f(x), v_f(y)\}$$
$$\pi_g(x, y) \geq \max\{\pi_f(x), \pi_f(y)\}$$

The NSEG can also be denoted as:

$$G = (G^*, A, f, g) = \{N(\alpha): \alpha \in A\}$$

Where $N(\alpha)$ represents a family of parameterized neutrosophic graphs.

3 | Result in this Paper

In this section, we present the results of this paper.

3.1 | Turiyam Neutrosophic Soft Expert Graph

Definition 9. A Turiyam Neutrosophic Soft Expert Graph (TSEG) is defined over a simple graph $G^* = (V, E)$, where:

- V is a set of vertices,
- E is a set of edges,
- Y is a set of parameters,
- X is a set of experts,
- $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ is a set of opinions,
- $Z = Y \times X \times O$ is the Cartesian product of the sets.

Let $A \subseteq Z$ and let $T(V)$ denote the set of all Turiyam Neutrosophic sets in V. The TSEG is represented as a 4-tuple:

$$G = (G^*, A, f, g)$$

where:

- $f: A \to T(V)$ is a function mapping each parameter in A to a Turiyam Neutrosophic set of vertices,
- $g: A \to T(V \times V)$ is a function mapping each parameter in A to a Turiyam Neutrosophic set of edges.

The mappings f and g are defined as:

$$f(\alpha) = \{x, t_f(x), iv_f(x), fv_f(x), lv_f(x) \mid x \in V\}$$
$$g(\alpha) = \{(x,y), t_f(x,y), iv_f(x,y), fv_f(x,y), lv_f(x,y) \mid (x,y) \in V \times V\}.$$

where:

- $t_f(x), iv_f(x), fv_f(x)$, and $lv_f(x)$ represent the truth, indeterminacy, falsity, and liberal state membership degrees of vertex x, respectively.
- $t_f(x,y), iv_f(x,y), fv_f(x,y)$, and $lv_f(x,y)$ represent the corresponding membership degrees of the edge (x,y).

  These mappings satisfy the following conditions for all $(x,y) \in V \times V$ and $\alpha \in A$:

$$t_g(x,y) \leq \min\{t_f(x), t_f(y)\}$$
$$iv_g(x,y) \leq \min\{iv_f(x), iv_f(y)\}$$
$$fv_g(x,y) \geq \max\{fv_f(x), fv_f(y)\}$$
$$lv_g(x,y) \leq \min\{lv_f(x), lv_f(y)\}$$

The TSEG can also be denoted as:

$$G = (G^*, A, f, g) = \{T(\alpha) \mid \alpha \in A\}$$

Where $T(\alpha)$ represents a family of parameterized Turiyam Neutrosophic soft graphs.

Theorem 10. The Turiyam Neutrosophic Soft Expert Graph (TSEG) can be transformed into the following graphs under appropriate parameter settings:

i). Turiyam Neutrosophic Graph (TG)

ii). Neutrosophic Soft Expert Graph (NSEG)

iii). Neutrosophic Graph (NG)

iv). Intuitionistic Fuzzy Soft Expert Graph (IFSEG)

Proof. We will demonstrate that by adjusting the parameters of a TSEG, it can be reduced to each of the specified graph types.

We consider Transformation to Turiyam Neutrosophic Graph (TG). By considering a single parameter and a single expert, and ignoring the soft expert structure, the TSEG reduces to a Turiyam Neutrosophic Graph.

- Let A be a singleton set: $A = \{\alpha\}$.
- Let X be a singleton set: $X = \{x_0\}$.
- The opinion set $O = \{1\}$, indicating agreement only.

Under these settings, the set $Z = Y \times X \times O$ simplifies, and the TSEG becomes:

$$G = (G^*, A, f, g)$$

Where:

- $f: A \to T(V)$ assigns Turiyam Neutrosophic membership degrees to vertices.
- $g: A \to T(V \times V)$ assigns Turiyam Neutrosophic membership degrees to edges.

Since there's only one parameter and one expert, the soft expert aspect is eliminated, resulting in a standard Turiyam Neutrosophic Graph.

We consider Transformation to Neutrosophic Soft Expert Graph (NSEG). By setting the liberal state membership degree $lv_f(x) = 0$ for all vertices and edges, the TSEG reduces to an NSEG.

- For all $x \in V$ and $(x, y) \in E$, set $lv_f(x) = 0$ and $lv_g(x, y) = 0$.
- The Turiyam Neutrosophic membership degrees become $t_f(x), iv_f(x)$, and $fv_f(x)$.
- These correspond to the truth-membership, indeterminacy-membership, and falsity-membership degrees in a neutrosophic set.

Therefore, the TSEG simplifies to a Neutrosophic Soft Expert Graph with mappings:

$$f(\alpha) = \{x, \mu_f(x), v_f(x), \pi_f(x) \mid x \in V\}$$
$$g(\alpha) = \{(x, y), \mu_g(x, y), v_g(x, y), \pi_g(x, y) \mid (x, y) \in E\}$$

Where:

- $\mu_f(x) = t_f(x)$,
- $v_f(x) = iv_f(x)$,
- $\pi_f(x) = fv_f(x)$.

We consider Transformation to a Neutrosophic Graph (NG). By considering a single parameter and a single expert in an NSEG, it reduces to an NG.

- Let $A = \{\alpha\}$ and $X = \{x_0\}$.

  The NSEG becomes:

$$G = (G^*, A, f, g),$$

With f and g mapping to neutrosophic sets over V and E, respectively. Without the soft expert framework, this structure aligns with the definition of a Neutrosophic Graph.

We consider transformation to Intuitionistic Fuzzy Soft Expert Graph (IFSEG). By setting $iv_f(x) = 0$ and $lv_f(x) = 0$ for all vertices and edges, the TSEG reduces to an IFSEG.

- For all $x \in V$ and $(x, y) \in E$, set $iv_f(x) = 0, lv_f(x) = 0, iv_g(x, y) = 0$, and $lv_g(x, y) = 0$.
- The remaining membership degrees are $t_f(x)$ and $fv_f(x)$.
- Define:

$$\mu_f(x) = t_f(x)$$
$$v_f(x) = fv_f(x)$$

  satisfying $\mu_f(x) + v_f(x) \leq 1$.

- Similarly for edges.

  This conforms to the definition of an Intuitionistic Fuzzy Soft Expert Graph.

3.2 | General Plithogenic Soft Expert Graph

Definition 11. A General Plithogenic Soft Expert Graph (GPSEG) is defined over a simple graph $G^* = (V, E)$, where:

- V is a set of vertices,
- E is a set of edges,

- Y is a set of parameters,
- X is a set of experts,
- $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ is a set of opinions,
- $Z = Y \times X \times O$ is the Cartesian product of the sets.

Let $A \subseteq Z$ and let $P(V)$ denote the set of all Plithogenic sets in V. The GPSEG is represented as a 4-tuple:

$$G = (G^*, A, f, g),$$

Where:

- $f: A \rightarrow P(V)$ maps each parameter to a Plithogenic set of vertices,
- $g: A \rightarrow P(V \times V)$ maps each parameter to a Plithogenic set of edges.

The mappings f and g are defined as:

$$f(\alpha) = \{x, \text{DAF}(x), \text{DCF}(x) \mid x \in V\},$$
$$g(\alpha) = \{(x, y), \text{DAF}(x, y), \text{DCF}(x, y) \mid (x, y) \in V \times V\}.$$

The GPSEG can also be denoted as:

$$G = (G^*, A, f, g) = \{P(\alpha) \mid \alpha \in A\},$$

Where $P(\alpha)$ represents a family of parameterized Plithogenic soft graphs.

Theorem 12. The General Plithogenic Soft Expert Graph (GPSEG) can be transformed into the following graphs under appropriate parameter settings:

i). General Plithogenic Graph (GPG)

ii). Turiyam Neutrosophic Soft Expert Graph (TSEG)

iii). Turiyam Neutrosophic Graph (TG)

iv). Neutrosophic Soft Expert Graph (NSEG)

v). Intuitionistic Fuzzy Soft Expert Graph (IFSEG)

Proof. We will demonstrate that by adjusting the parameters and settings of a GPSEG, it can be reduced to each of the specified graph types.

We consider Transformation to be a General Plithogenic Graph (GPG). By considering a single parameter and a single expert, and eliminating the soft expert structure, the GPSEG reduces to a General Plithogenic Graph.

- Let A be a singleton set: $A = \{\alpha\}$.
- Let X be a singleton set: $X = \{x_0\}$.
- The opinion set $O = \{1\}$, indicating agreement only.

Under these settings, the set $Z = Y \times X \times O$ simplifies, and the GPSEG becomes:

$$G = (G^*, A, f, g),$$

Where:

- $f: A \rightarrow P(V)$ assigns Plithogenic membership degrees to vertices.
- $g: A \rightarrow P(V \times V)$ assigns Plithogenic membership degrees to edges.

Since there's only one parameter and one expert, and opinions are fixed, the soft expert aspect is eliminated, resulting in a standard General Plithogenic Graph.

We consider Transformation to Turiyam Neutrosophic Soft Expert Graph (TSEG). By setting the Degree of Appurtenance Function (DAF) to correspond to Turiyam Neutrosophic membership degrees and adjusting the Degree of Contradiction Function (DCF) accordingly, the GPSEG reduces to a TSEG.

- Set $s = 4$ and $t = 1$ in the GPSEG, where $[0,1]^s$ Corresponds to the Turiyam Neutrosophic membership degrees.
- The DAF for vertices and edges becomes:

$$\text{DAF} : M \times Ml \rightarrow [0,1]^4$$

which aligns with the quadruple membership degrees $(t(v), iv(v), fv(v), lv(v))$ in the Turiyam Neutrosophic set.

- The DCF is adjusted to match the Turiyam Neutrosophic logic.

Therefore, the GPSEG becomes a Turiyam Neutrosophic Soft Expert Graph.

We consider Transformation to Turiyam Neutrosophic Graph (TG). By considering a single parameter and a single expert, and eliminating the soft expert structure from the TSEG obtained in step 2, we get a Turiyam Neutrosophic Graph.

- Let $A = \{\alpha\}$ and $X = \{x_0\}$.

The TSEG reduces to a Turiyam Neutrosophic Graph with Turiyam Neutrosophic membership degrees assigned directly to vertices and edges.

We consider Transformation to a Neutrosophic Soft Expert Graph (NSEG). By setting the liberal state membership degree to zero in the GPSEG configured as a TSEG, the GPSEG reduces to an NSEG.

- In the GPSEG, set $s = 4, t = 1$, and for all vertices and edges, set the fourth component of the DAF to zero:

$$lv_f(x) = 0,\ lv_g(e) = 0$$

- The remaining components correspond to the truth, indeterminacy, and falsity membership degrees of the Neutrosophic set.

Adjust the DCF accordingly.

Thus, the GPSEG reduces to a Neutrosophic Soft Expert Graph.

We consider Transformation to Intuitionistic Fuzzy Soft Expert Graph (IFSEG). By setting the indeterminacy and liberal state membership degrees to zero in the GPSEG, we can reduce it to an IFSEG.

- In the GPSEG, set $s = 4, t = 1$, and for all vertices and edges, set:

$$iv_f(x) = 0,\ lv_f(x) = 0,\ iv_g(e) = 0,\ lv_g(e) = 0$$

- The remaining components $t_f(x)$ and $fv_f(x)$ correspond to the membership and non-membership degrees in an intuitionistic fuzzy set.
- Adjust the DCF accordingly.

  Therefore, the GPSEG reduces to an Intuitionistic Fuzzy Soft Expert Graph.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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