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Neutrosophic Systems of Linear Equations by Symbolic Calculus

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Abstract

This article explores a key challenge in mathematics: how to solve systems of neutrosophic linear equations, where uncertainty and indeterminacy complicate traditional approaches. The research focuses on determining how symbolic calculus can offer an effective solution to these systems, which incorporate non-absolutely defined values, typical of the neutrosophic framework. Although numerous previous studies have addressed classical linear systems, few have considered the limitations imposed by the inherent ambiguity of neutrosophic contexts, leaving a gap that this work seeks to fill. To address this challenge, we employ symbolic calculus, a computational technique that manipulates mathematical expressions exactly, integrating it with neutrosophic principles to systematically process uncertain and contradictory data. The relevance of this topic lies in its potential to transform areas such as artificial intelligence, optimization, and the modeling of complex phenomena, where uncertainty is a constant. Using specialized software and specifically designed algorithms, the study successfully solves these systems, revealing that the symbolic approach is not only viable but also overcomes the limitations of conventional numerical methods when handling indeterminacies. The findings show accurate and generalizable solutions, highlighting the method's ability to adapt to varying degrees of uncertainty. In terms of contributions, this research enriches theory by proposing an innovative tool for neutrosophic problems and offers practical applications in fields requiring decision-making under ambiguous conditions. Thus, in addition to broadening the mathematical horizon, the work provides a valuable resource for professionals and academics who deal with systems with undefined elements, consolidating a significant advance in the intersection of neutrosophics and symbolic computation.

Keywords: Neutrosophic Systems; Linear Equations; Symbolic Calculus; Uncertainty; Resolution; Neutrosophy; Computation; Indeterminacy; Mathematics; Algorithms.

1 | Introduction

The word Neutrosophic means knowledge of neutral thought, and this third/neutral represents the main distinction, ie, the neutral/indeterminate/unknown part (in addition to "truth" / "belonging" and "falsehood" Components of "non-belonging" that appears in the fuzzy logic/set). Neutrosophic logic (NL) is a generalization of Zadeh's fuzzy logic (FL), and especially of Atanassov's intuitionistic fuzzy logic (IFL), and other multi-valued logics, see Figure 1 and [1].

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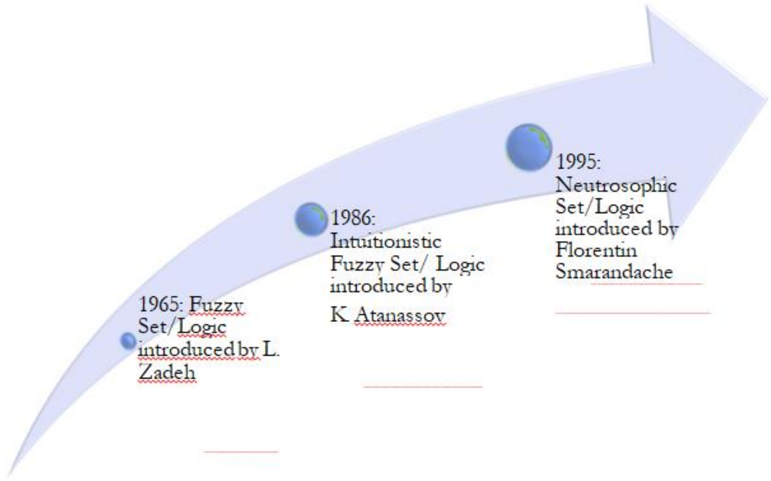

Figure 1. Neutrosophic and its fundamental background [1].

Let U be to universe of discourse, and M to set included in U. An element x of OR which is denoted concerning M as x (T, I, F) and belongs to M according to the Neutrosophic Set as follows: it is t % true in the set, i % indeterminate (unknowns) in the set, and f% false, where t varies in T, i varies in I and f varies in F. Statically T, I, F are subsets, but dynamically T, YO, F are functions/operators that it depends on many known or unknown parameters [2, 3].

Neutrosophic sets generalize the fuzzy sets (especially the fuzzy and fuzzy intuitionist sets), the paraconsistent sets. It allows dealing with a more significant number of situations that occur in real life [4].

2 | Preliminary

A statistical neutrosophic number is a number given in the following form [5]:

N= d+i (1)

Where d is the determined part and i is the indeterminate part, see [6].

For example:

a=8.6+i if i ∈ [3, 3.4] the number is equivalent to i ∈ [11.6, 12].

Additionally, to neutrosophic matrix es to a matrix such that the elements to = (aij) have been replaced by elements in <R ∪ I>, where <R ∪ I> is a neutrosophic ring, see [7].

TO neutrosophic graph is a kind of graph in which at least one arc is a neutrosophic arc, see [8]. The weights associated with the arcs have the following meanings: 0 = "no connection between nodes", 1 = "connection between nodes", I = "indeterminate connection (unknown if it is or not)". The neutrosophic adjacency matrix is an adjacency matrix with at least one neutrosophic arc. Such notions are not used in fuzzy theory, an example of which is shown below:

$$\begin{matrix} 0 & 0 & I \\ I & 0 & 1 \\ 1 & 0 & 0 \end{matrix}$$

Further, we shall describe practical implementations of this approach. Google Colaboratory is a web application that allows us to create and share documents containing source codes, equations, visualizations, and illustrative texts, as shown in Figure 2.

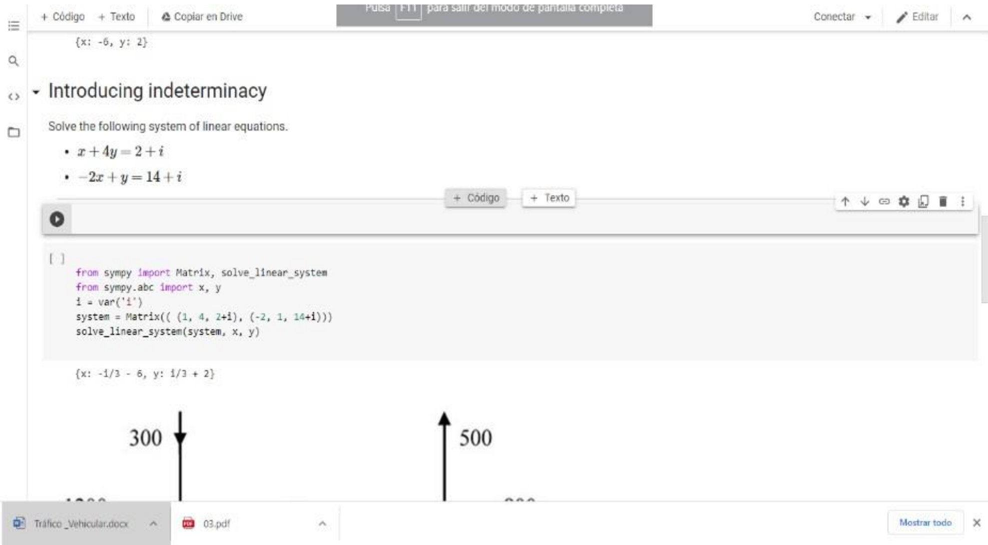

Figure 2. Google Colaboratory.

Jupyter allows us to interact with several programming languages; in this case, Python is used. Python is a simple and powerful programming language with access to a great variety of useful libraries.

3 | Neutrosophic Computing and Sympy

SymPy can be used for computational works with neutrosophic numbers in Python language. It is a library written in Python language to bring together all the features of a computer algebra system, which is easily extensible and maintains the code as simple as possible [9].

Here we propose a calculus that is based on interval-valued arithmetic and accordingly, a de -neutralsification process is required [10] to obtain a representative numerical value. Thus, I ∈ [0,1] is replaced by its maximum and minimum values.

In this case, we employ the MP math library and the MPI type [11]. The mpi type deals with intervals of a pair of mpf values. Interval arithmetic uses conservative rounding, SW that, if an interval es interpreted to ace the numerical uncertainty concerning a single-valued number, any sequence of interval operations will produce intervals that contain the result of applying the same sequence of operations to a precise number, see Figure 3.

Figure 3. Working with Neutrosophic Numbers.

In this case, systems of neutrosophic linear equations dog be solved, see [12]. For example, given the following system of equations:

3x+8y = 1+i (2)

4x + 7y = 8+i (3)

Item es solved as follows, see Figure 4:

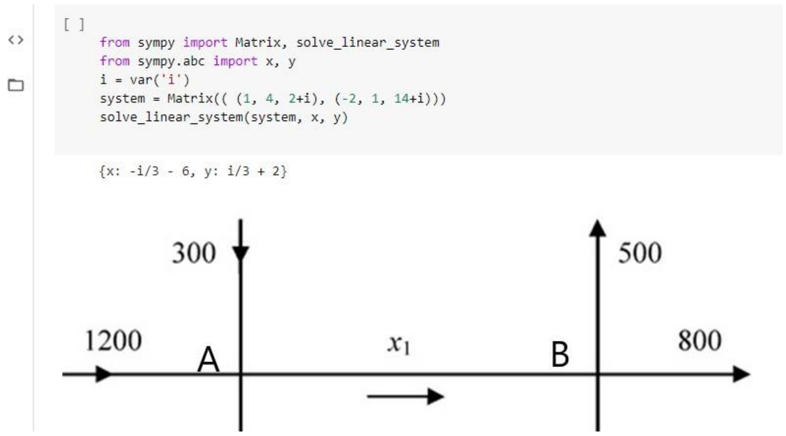

Figure 4. Solution to to system of linear equations with Neutrosophic Numbers, by using Colaboratory.

An example of application in a real-life problem is a system of linear equations that determines the flow of traffic at different intercepts, see Figure 5.

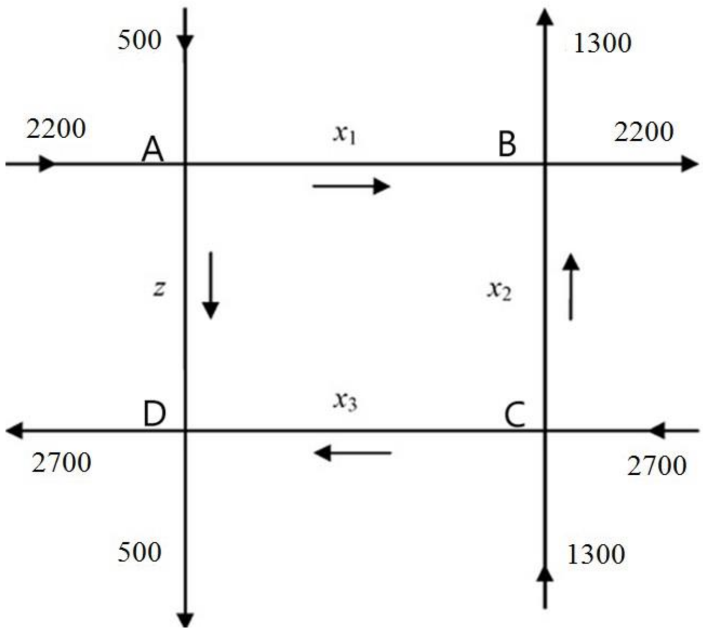

Figure 5. Schematic representation of the Vehicle Flow [12]. At each intercept, the outflow must be equal to the inflow.

Intercept A: 2700=x1+z

Intercept B: 3500=x1+x2

Intercept C: 4000=x2+x3

Interception D: 3200=x3+z

If z=500.

Then the system of equations is ace follows:

x1 = 2200

x1 + x2 = 3500

x2 +2x3=6700

The solution for this system is the following:

x1 = 2200

x2 = 1300

x3 = 2700

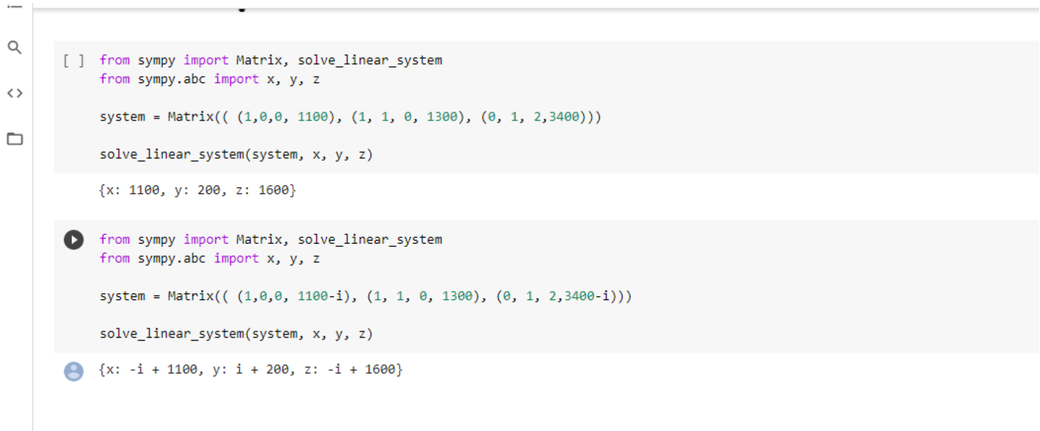

Figure 6. Solution of the vehicular flow with indetermination.

In the case of z = 500 + i.

Then the system of equations is shown in Figure 6:

x1 = 2200-i

x1 + x2 = 3500

x1 + 2x3 = 6700 - i

The solution for this system is:

x1 = 2200-i

x2 = 1300 + i

x3 = 2250

This kind of computation allows opening new ways to compute using indeterminacy in different real-world problems [13-19].

4 | Conclusions

The conclusions obtained in this article demonstrate that the use of symbolic calculus, integrated with neutrosophic principles, constitutes a powerful methodology for solving systems of linear equations in contexts of uncertainty and indeterminacy. This approach has successfully addressed problems where conventional numerical methods become ineffective or insufficient due to the presence of ambiguous or undefined values, characteristic of the neutrosophic framework.

The use of symbolic computation, specifically through Python on the Google Colaboratory platform, has shown significant advantages in precision and flexibility. This approach makes it possible to manipulate complex mathematical expressions accurately, effectively integrating uncertainty into the calculation process and providing more robust and adaptable solutions for varying levels of ambiguity.

Additionally, the incorporation of specialized libraries such as SymPy and interval arithmetic provided effective tools to manage and solve neutrosophic linear systems. The results obtained in the vehicular flow case study exemplify how this methodology can be successfully applied in real-world contexts, generating intuitive and accurate solutions even in situations of indeterminacy.

Theoretically, this research significantly contributes to the development of neutrosophic logic by integrating concepts of symbolic calculus into equation solving, providing a conceptual advance that facilitates new ways of addressing problems where uncertainty is inherent. From a practical perspective, the presented results suggest potential applications in areas such as vehicular traffic, engineering, logistics, and other fields frequently dealing with decisions under conditions of uncertainty or partial information.

Finally, this work opens multiple avenues for future research, such as deepening the development of more advanced neutrosophic computational techniques, integrating other neutrosophic operators into symbolic analysis, and exploring specific applications in neutrosophic statistics, multicriteria models, optimization, artificial intelligence, and machine learning. This research trajectory promises not only to expand theoretical knowledge of the neutrosophic approach but also to consolidate its practical usefulness in complex real-world scenarios.

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Author Contribution

All authors contributed equally to this work.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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