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# Multi-Criteria Group Decision-Making with ELECTRE-III Method for Selection of Female Spouse in Pythagorean Neutrosophic Environment

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## Abstract

Selecting a female partner in the 21st century is more challenging as compared to the 20th century. The globalized world, changing societal norms, and technology have increased the pool of potential partners, making it harder to find a compatible match. The prevalence of social media and online dating has also changed how people interact, leading to new challenges in establishing genuine connections. This paper presents a novel Multiple-Criteria Group Decision Making (MCGDM) model for selecting a female spouse using the ELECTRE-III method in a Pythagorean neutrosophic environment. The proposed model takes into account multiple criteria and addresses the challenges of handling conflicting and uncertain information. The Pythagorean neutrosophic set theory (PNST) is used to handle the vagueness and uncertainty of the decision-making process. The proposed model is validated using a case study of selecting a female spouse based on education, family background, beauty, and skill capability. The case study results demonstrate the proposed model's effectiveness in addressing the complexity and uncertainty of the decision-making process and providing a useful tool for decision-makers in selecting a spouse.

**Keywords:** Female Spouse, Multi-Criteria Group Decision-Making, Pythagorean Neutrosophic Sets, ELECTRE-III Method.

## 1 | Introduction

The process of selecting a spouse [1, 2] is a complex and challenging decision that individual's face in their lives. It is not only a personal decision but also has social, cultural, and economic implications [3]. Therefore, the importance of making an informed decision is paramount. With advancements in technology and the availability of information, individuals have access to a wide range of options for selecting a spouse [4, 5]. However, the abundance of choices often leads to confusion and indecision, making it difficult to make the right choice. To assist individuals in making the best decision, Multi-Criteria Decision Making [MCDM] methods [6, 7] have been used extensively in recent years. These methods help individuals to evaluate the alternatives based on multiple criteria and select the best one. The existing MCDM models, however, do not take into account the uncertainties present in the decision-making process. These uncertainties arise due to

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the imprecise nature of the criteria, the preferences of the decision-maker, and the available information. However, to choose the appropriate alternative is very difficult because of vague information in some cases.

To overwhelmed such circumstances, Zadeh established the notion of fuzzy sets [FSs] [8] to solve those problems which comprise uncertainty and vagueness. As a result of the observation that some situations cannot be resolved by fuzzy sets, Turksen [9-11] proposed the concept of interval-valued fuzzy sets [IVFS]. In some circumstances, we must consider membership unbiased as well as non-membership values for the appropriate representation of an object under ambiguous and indefinite circumstances that neither FSs nor IVFSs could manage. Atanassov [12] proposed the idea of intuitionistic fuzzy sets [IFSs] to get around these issues. Atanassov's theory only deals with incomplete data when both membership and non-membership values are taken into account, but intuitionistic fuzzy set theory is unable to cope with contradictory and imprecise data. To deal with such incompatible and imprecise data Smarandache [13] expanded on Atanassov's [12] work with IFSs and introduced Neutrosophic sets [NSs] as a potent tool for dealing with ambiguous, insufficient, and inconsistent information that is encountered in real-world issues. NSs are comparable to FSs and IFSs. So, using NSs directly for ELECTRE-III is not always easy. Wang et al. [14] in created a subclass of NSs called as single-valued Neutrosophic sets [SVNSs] to be used with the NSs. The author of suggested a geometric explanation based on NSs. In the Neutrosophic context, Gulfam et al. [15] created new methodologies and a new distance formula for SVNSs. the idea of a single-valued Neutrosophic soft expert set that was first out in by fusing soft expert sets with SVNSs. To address this gap, this research paper proposes a new MCGDM model for the selection of a female spouse using the ELECTRE-III method in a Pythagorean neutrosophic environment. Pythagorean neutrosophic sets [PNSs] are an extension of NSs, which provide a more comprehensive and flexible framework for dealing with uncertainties. The proposed model incorporates the concept of PNSs to handle the uncertainties present in the decision-making process.

The proposed model is designed to consider the preferences of the decision-maker, the imprecise nature of the criteria, and the uncertainties present in the decision-making process. It is expected to provide a more comprehensive and accurate assessment of the alternatives, thereby assisting the decision-maker in making the best possible decision. To demonstrate the effectiveness of the proposed model, it is applied to a real-life case study. The case study involves selecting a female spouse based on multiple criteria, such as education, income, family background, and personal qualities. The results of the case study are analysed and compared with the existing MCGDM models to show the superiority of the proposed model.

The introduction should briefly place the study in a broad context and highlight why it is important. It should define the purpose of the work and its significance. The current state of the research field should be reviewed carefully and key publications cited. Please highlight controversial and diverging hypotheses when necessary. Finally, briefly mention the main aim of the work and highlight the principal conclusions. As far as possible, please keep the introduction comprehensible to scientists outside your particular field of research. References should be numbered in order of appearance and indicated by a numeral or numerals in square brackets, e.g., [1] or [2,3], or [4–6]. See the end of the document for further details on references.

## 1.1 | Motivation of the Research Work

The process of selecting a spouse [16, 17] is a complex and important decision that has a significant impact on an individual's personal and social life. The existing MCDM models, however, do not take into account the uncertainties present in the decision-making process. These uncertainties arise due to the imprecise nature of the criteria, the preferences of the decision-maker, and the available information. Therefore, there is a need for a new MCGDM model that can address these uncertainties and assist individuals in making better-informed decisions when selecting a spouse.

## 1.2 | Objective of the Investigation

The objective of this research paper is to propose a new MCGDM model for the selection of a female spouse using the ELECTRE-III method in a Pythagorean neutrosophic environment.(see [18-22]). The proposed model incorporates the concept of PNSs to handle the uncertainties present in the decision-making process. The ELECTRE-III method is used to rank the alternatives based on the preferences of the DM. The proposed model is expected to provide a more comprehensive and accurate assessment of the alternatives, thereby assisting the decision-maker in making the best possible decision.

### 1.3 | Related Work of the Study

Several studies have been conducted in the field of MCDM to assist individuals in selecting a spouse. The Analytic Hierarchy Process [AHP] and the Technique for Order of Preference by Similarity to Ideal Solution [TOPSIS] ([11], [23], [24]) are two widely used methods for selecting a spouse. However, these methods do not take into account the uncertainties present in the decision-making process.

PNSs are an extension of NS, which provide a more comprehensive and flexible framework for dealing with uncertainties. Several studies have been conducted on the application of PNSs in MCDM. However, the use of PNSs in the selection of a spouse has not been explored.

The ELECTRE-III method is a widely used MCDM method that takes into account the preferences of the decision-maker and the criteria weights. Several studies have been conducted on the application of the ELECTRE-III method in various fields. However, its application in the selection of a spouse using PNSs has not been explored.

Therefore, this research paper proposes a new MCGDM model that combines the use of PNSs and the ELECTRE-III method to address the uncertainties present in the decision-making process when selecting a spouse. The proposed model is expected to provide a more comprehensive and accurate assessment of the alternatives, thereby assisting the DM in making the best possible decision.

## 2 | Preliminaries

In this section, some essentials introductory concept of NSs, SVNSs, PNSs are briefly presented which will enable the conversation in following sections.

**Definition 2.1.** [25]: Let $Z$ be a non-empty set. A NS $A$ on $Z$, comprising $\alpha_A(z)$ as membership, $\beta_A(z)$ as indeterminacy and $\gamma_A(z)$ as non-membership functions, defined as

$$A = \left\{ \left\langle z, \alpha_A(z), \beta_A(z), \gamma_A(z) \right\rangle : z \in Z \right\},$$

where $\alpha_A(z), \beta_A(z), \gamma_A(z) \in \left]^{-}0, 1^{+}\right[$ such that $^{-}0 \le \alpha_A(z) + \beta_A(z) + \gamma_A(z) \le 3^{+}$ for all $z \in Z$.

**Definition 2.2.** [15]: A SVNS $A$ on a non-empty universal set $Z$ is defined as

$$A = \left\{ \left\langle z, \alpha_A(z), \beta_A(z), \gamma_A(z) \right\rangle : z \in Z \right\},$$

where $\alpha_A(z), \beta_A(z), \gamma_A(z) \in [0,1]$ such that $0 \le \alpha_A(z) + \beta_A(z) + \gamma_A(z) \le 3$ for all $z \in Z$. And $\alpha_A(z)$ as membership, $\beta_A(z)$ as indeterminacy and $\gamma_A(z)$ as non-membership functions.

**Definition 2.3.** [26]: Let $Z$ be a discourse. A PNS $A$ on $Z$ is defined as

$$A = \left\{ \left\langle z, \alpha_A(z), \beta_A(z), \gamma_A(z) \right\rangle : z \in Z \right\}$$

where $\alpha_A(z), \beta_A(z), \gamma_A(z) \in [$ such that $0 \le \left(\alpha_A(z)\right)^2 + \left(\beta_A(z)\right)^2 + \left(\gamma_A(z)\right)^2 \le 2$ for all $z \in Z$. $\alpha_A(z)$ as membership, $\beta_A(z)$ as indeterminacy and $\gamma_A(z)$ as non-membership. Here truth $(\alpha_A(z))$ and falsity $(\gamma_A(z))$ are dependent components and indeterminacy $(\beta_A(z))$ is an independent component. The triplet

$A=(\alpha_A(z),\beta_A(z),\gamma_A(z))$ is called a Pythagorean neutrosophic number [PNN]. For suitability, we denote a PNN $A=(\alpha_A(z),\beta_A(z),\gamma_A(z))$ as $A=(\alpha_A,\beta_A,\gamma_A)$, throughout in this article.

**Definition 2.4.** ([20],[21],[27]): Let three PNN $A=(\alpha_A,\beta_A,\gamma_A)$, $A_1=(\alpha_{A_1},\beta_{A_1},\gamma_{A_1})$ and $A_2=(\alpha_{A_2},\beta_{A_2},\gamma_{A_2})$ then the elementary mathematical operations over these PNNs are defined as:

(i). Complement: $A^c=(\gamma_A,\beta_A,\alpha_A)$

(ii). Union: $A_1\bigcup A_2=\left(\max\left\{\alpha_{A_1},\alpha_{A_2}\right\},\min\left\{\beta_{A_1},\beta_{A_2}\right\},\min\left\{\gamma_{A_1},\gamma_{A_2}\right\}\right)$

(iii). Intersection: $A_1\bigcap A_2=\left(\min\left\{\alpha_{A_1},\alpha_{A_2}\right\},\max\left\{\beta_{A_1},\beta_{A_2}\right\},\max\left\{\gamma_{A_1},\gamma_{A_2}\right\}\right)$

(iv). Addition: $A_1\oplus A_2=\left(\sqrt{\alpha_{A_1}^2+\alpha_{A_2}^2-\alpha_{A_1}^2\cdot\alpha_{A_2}^2},\beta_{A_1}\cdot\beta_{A_2},\gamma_{A_1}\cdot\gamma_{A_2}\right)$ (1)

(v). Multiplication: $A_1\otimes A_2=\left(\alpha_{A_1}\cdot\alpha_{A_2},\sqrt{\beta_{A_1}^2+\beta_{A_2}^2-\beta_{A_1}^2\cdot\beta_{A_2}^2},\sqrt{\gamma_{A_1}^2+\gamma_{A_2}^2-\gamma_{A_1}^2\cdot\gamma_{A_2}^2}\right)$ (2)

(vi). Scalar Multiplication: $r.A=\left(\sqrt{1-\left(1-\alpha_A^2\right)^r},\left(\beta_A\right)^r,\left(\gamma_A\right)^r\right);\ r>0.$ (3)

(vii). Exponentiation: $A^r=\left(\left(\alpha_A\right)^r,\sqrt{1-\left(1-\beta_A^2\right)^r},\sqrt{1-\left(1-\gamma_A^2\right)^r}\right);r>0.$ (4)

**Definition 2.5.** [De-Neutrosophication]

(i). Score Function: $s(A)=\alpha_A^2-\beta_A^2-\gamma_A^2$ , provided $-1\le s(A)\le 1$ (5)

(ii). Accuracy Function: $a(A)=\alpha_A^2+\beta_A^2+\gamma_A^2$, provided $0\le a(A)\le 2$ (6)

(iii). Normalized Euclidean Distance:

$$d(A_1,A_2)=\sqrt{\left(\alpha_{A_1}{}^2-\alpha_{A_2}{}^2\right)^2+\left(\beta_{A_1}{}^2-\beta_{A_2}{}^2\right)^2+\left(\gamma_{A_1}{}^2-\gamma_{A_2}{}^2\right)^2} \tag{7}$$

**Definition 2.6.** [Comparison]

(i). If $s(A_1)>s(A_2)$, then $A_1\succ A_2$ ($A_1$ is superior to $A_2$)

(ii). If $s(A_1)=s(A_2)$, then

a. If $a(A_1)>a(A_2)$, then $A_1\succ A_2$ ($A_1$ is superior to $A_2$)

b. If $a(A_1)=a(A_2)$, then $A_1 \approx A_2$ ($A_1$ is equivalent to $A_2$)

**Definition 2.7.** [Aggregation]: Let some PNN are $A_i=(\alpha_{A_i},\beta_{A_i},\gamma_{A_i})$ and their respective weights are $w=(w_1,w_2,w_3,...,w_n)$ such that $\sum_{i=1}^n w_i=1$. then

(i). Pythagorean Neutrosophic Weighted Aggregation [PNWA] Operator $\text{PNWA}_w(A_1,A_2,A_3,...,A_n)=w_1A_1\oplus w_2A_2\oplus w_3A_3\oplus...\oplus w_nA_n$

$$=\left(\sqrt{1-\prod_{i=1}^n\left(1-\alpha_{A_i}^2\right)^{w_i}},\prod_{i=1}^n\left(\beta_{A_i}\right)^{w_i},\prod_{i=1}^n\left(\gamma_{A_i}\right)^{w_i}\right) \tag{8}$$

(ii). Pythagorean Neutrosophic Weighted Geometric [PNWG] Operator $\text{PNWG}_w\left(A_1, A_2, A_3, ..., A_n\right) = w_1 A_1 \otimes w_2 A_2 \otimes w_3 A_3 \otimes ... \otimes w_n A_n$

$$= \left( \prod_{i=1}^{n} \left(\alpha_{A_i}\right)^{w_i}, \sqrt{1 - \prod_{i=1}^{n} \left(1 - \beta_{A_i}^2\right)^{w_i}}, \sqrt{1 - \prod_{i=1}^{n} \left(1 - \gamma_{A_i}^2\right)^{w_i}} \right) \quad (9)$$

# 3 | Two-Phase Pythagorean Neutrosophic ELECTRE III Method (Algorithm)

In this section, the PNSs and the conventional ELECTRE III technique [7] are combined to form the two-phase Pythagorean neutrosophic ELECTRE-III (PN-ELECTRE-III) group decision support system. In Pythagorean neutrosophic environment, let for a multi-criteria group decision making [MCGDM] problem, $Y = \left\{y_1, y_2, y_3, ..., y_a, ..., y_f\right\}$ be a set of accessible $f$ alternatives and $C = \left\{c_1, c_2, c_3, ..., c_b, ..., c_g\right\}$ be a set of $g$ criteria assigning to individually alternative. A group $V = \left\{v_1, v_2, v_3, ... v_t, ... v_h\right\}$ of $h$ decision-maker [DM] allocates the feasibility data of alternative $y_a \in Y$ with respect to criterion $c_b \in C$ as; $c_b(y_a)$. As all the criteria can have their own and unequal importance considering objective of the MCGDM problem thus criteria weight vector will be denoted by $w = \left\{w_1, w_2, w_3, ..., w_b, ..., w_g\right\}$ such that $\sum_{b=1}^{g} w_b = 1$. Likewise, the importance expert weight vector will be $\ell = \left\{\ell_1, \ell_2, \ell_3, ..., \ell_t, ..., \ell_h\right\}$ such that $\sum_{t=1}^{h} \ell_t = 1$. Let the subscript set of criterions i.e. $\tau = \left\{1, 2, 3, ..., g\right\}$.

## 3.1 | Phase I: (Pythagorean Neutrosophic Evaluation Phase)

**Step-1.** For the evaluation of, feasibility ratings of alternatives, importance weights of criteria, important weights of experts, and establishment of threshold functions, we first build certain linguistic variables/terms in the form of PNN.

**Step-2.** Methodically measuring each alternative $y_a$ w.r.t. each criterion $c_b$. Expert $v_t$ delivers his/her assessment data in the form of Pythagorean neutrosophic decision matrix [PNDM] $P^{(t)} = [P_{ab}^{(t)}]_{f \times g}$, as in table 1, where $P_{ab}^{(t)} = \left(\alpha_{P_a}^{(t)}(c_b), \beta_{P_a}^{(t)}(c_b), \gamma_{P_a}^{(t)}(c_b)\right)$ is the PNN assigned by the DM $v_t$.

**Table 1.** PNDM by DM $v_t$

| $P^{(t)}$ | $c_1$ | $c_2$ | $\cdots$ | $c_g$ |
|---|---|---|---|---|
| $y_1$ | $\left(\alpha_{P_1}^{(t)}(c_1), \beta_{P_1}^{(t)}(c_1), \gamma_{P_1}^{(t)}(c_1)\right)$ | $\left(\alpha_{P_1}^{(t)}(c_2), \beta_{P_1}^{(t)}(c_2), \gamma_{P_1}^{(t)}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P_1}^{(t)}(c_g), \beta_{P_1}^{(t)}(c_g), \gamma_{P_1}^{(t)}(c_g)\right)$ |
| $y_2$ | $\left(\alpha_{P_2}^{(t)}(c_1), \beta_{P_2}^{(t)}(c_1), \gamma_{P_2}^{(t)}(c_1)\right)$ | $\left(\alpha_{P_2}^{(t)}(c_2), \beta_{P_2}^{(t)}(c_2), \gamma_{P_2}^{(t)}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P_2}^{(t)}(c_g), \beta_{P_2}^{(t)}(c_g), \gamma_{P_2}^{(t)}(c_g)\right)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $y_f$ | $\left(\alpha_{P_f}^{(t)}(c_1), \beta_{P_f}^{(t)}(c_1), \gamma_{P_f}^{(t)}(c_1)\right)$ | $\left(\alpha_{P_f}^{(t)}(c_2), \beta_{P_f}^{(t)}(c_2), \gamma_{P_f}^{(t)}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P_f}^{(t)}(c_g), \beta_{P_f}^{(t)}(c_g), \gamma_{P_f}^{(t)}(c_g)\right)$ |

**Step-3.** Weight of $t^{th}$ expert can be strong minded by the subsequent equation

$$\ell_t = \alpha_t + \gamma_t \left( \frac{\alpha_t}{\alpha_t + \beta_t} \right) \Bigg/ \sum_{t=1}^{h} \left( \alpha_t + \gamma_t \left( \frac{\alpha_t}{\alpha_t + \beta_t} \right) \right) \quad (10)$$

where the $\ell_t$ satisfy the normalized condition $\sum_{t=1}^{h} \ell_t = 1$.

**Step-4.** The separate opinions of DM or experts essential to be combined into a collective opinion to construct aggregated Pythagorean neutrosophic decision matrix (PNDM) $P' = \left(P'_{ab}\right)_{f\times g}$ by using PNWA operator, see equation**Error! Reference source not found.**,

$$P'_{ab} = PNWA_{\ell}\left(P_{ab}^{(1)}, P_{ab}^{(2)}, P_{ab}^{(3)}, \ldots, P_{ab}^{(h)}\right) = \ell_1 P_{ab}^{(1)} \oplus \ell_2 P_{ab}^{(2)} \oplus \ell_3 P_{ab}^{(3)} \oplus \ldots \oplus \ell_h P_{ab}^{(h)}$$

$$= \left(\sqrt{1-\prod_{t=1}^{h}\left(1-\left(\alpha_{ab}^{(t)}\right)^2\right)^{\ell_t}}, \prod_{t=1}^{h}\left(\beta_{ab}^{(t)}\right)^{\ell_t}, \prod_{t=1}^{h}\left(\gamma_{ab}^{(t)}\right)^{\ell_t}\right) \tag{11}$$

where $P'_{ab} = \left(\alpha_{P'_a}(c_b), \beta_{P'_a}(c_b), \gamma_{P'_a}(c_b)\right)$ shown in table 2 as follows:

**Table 2.** Aggregated Pythagorean Neutrosophic Decision Matrix $\left(\text{A - PNDM}\right)$

| $P'$ | $c_1$ | $c_2$ | $\cdots$ | $c_g$ |
|---|---|---|---|---|
| $y_1$ | $\left(\alpha_{P'_1}(c_1), \beta_{P'_1}(c_1), \gamma_{P'_1}(c_1)\right)$ | $\left(\alpha_{P'_1}(c_2), \beta_{P'_1}(c_2), \gamma_{P'_1}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P'_1}(c_g), \beta_{P'_1}(c_g), \gamma_{P'_1}(c_g)\right)$ |
| $y_2$ | $\left(\alpha_{P'_2}(c_1), \beta_{P'_2}(c_1), \gamma_{P'_2}(c_1)\right)$ | $\left(\alpha_{P'_2}(c_2), \beta_{P'_2}(c_2), \gamma_{P'_2}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P'_2}(c_g), \beta_{P'_2}(c_g), \gamma_{P'_2}(c_g)\right)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $y_f$ | $\left(\alpha_{P'_f}(c_1), \beta_{P'_f}(c_1), \gamma_{P'_f}(c_1)\right)$ | $\left(\alpha_{P'_f}(c_2), \beta_{P'_f}(c_2), \gamma_{P'_f}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P'_f}(c_g), \beta_{P'_f}(c_g), \gamma_{P'_f}(c_g)\right)$ |

**Step-5.** Let $C_B$ and $C_N$ represent the corresponding collections of criteria that are of the benefit-type and cost-types. The combined PNDM can be transformed into the normalized aggregated PNDM, $P = \left(P_{ab}\right)_{f\times g}$ which displays the evaluation information of each alternative w.r.t. each benefit or cost criterion, in standard form, for supplementary calculations, table 3 establishes how the matrix $P$ is erected. You can define the PNN for $P_{ab}$ as trails:

$$P_{ab} = \left(\alpha_{P_a}(c_b), \beta_{P_a}(c_b), \gamma_{P_a}(c_b)\right) = \begin{cases} P'_{ab} = \left(\alpha_{P'_a}(c_b), \beta_{P'_a}(c_b), \gamma_{P'_a}(c_b)\right), & \text{if } c_b \in Y_B \\ \left(P'_{ab}\right)^C = \left(\gamma_{P'_a}(c_b), \beta_{P'_a}(c_b), \alpha_{P'_a}(c_b)\right), & \text{if } c_b \in Y_N \end{cases} \tag{12}$$

**Table 3**. Normalized Aggregated Pythagorean Neutrosophic Decision Matrix $\left(\text{NA-PNDM}\right)$

| $P$ | $c_1$ | $c_2$ | $\cdots$ | $c_g$ |
|---|---|---|---|---|
| $y_1$ | $\left(\alpha_{P_1}(c_1), \beta_{P_1}(c_1), \gamma_{P_1}(c_1)\right)$ | $\left(\alpha_{P_1}(c_2), \beta_{P_1}(c_2), \gamma_{P_1}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P_1}(c_g), \beta_{P_1}(c_g), \gamma_{P_1}(c_g)\right)$ |
| $y_2$ | $\left(\alpha_{P_2}(c_1), \beta_{P_2}(c_1), \gamma_{P_2}(c_1)\right)$ | $\left(\alpha_{P_2}(c_2), \beta_{P_2}(c_2), \gamma_{P_2}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P_2}(c_g), \beta_{P_2}(c_g), \gamma_{P_2}(c_g)\right)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $y_f$ | $\left(\alpha_{P_f}(c_1), \beta_{P_f}(c_1), \gamma_{P_f}(c_1)\right)$ | $\left(\alpha_{P_f}(c_2), \beta_{P_f}(c_2), \gamma_{P_f}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P_f}(c_g), \beta_{P_f}(c_g), \gamma_{P_f}(c_g)\right)$ |

**Step-6.** Not all criteria might have likewise important. Let $w_b^{(t)} = \left(\alpha_w^{(t)}(c_b), \beta_w^{(t)}(c_b), \gamma_w^{(t)}(c_b)\right)$ denote the PNN that the expert $v_t$ assigned for the comparative weight of criterion $c_b$. By aggregating the opinions of experts on $c_b$, determine the PN weight $w_b = \left(\alpha_w(c_b), \beta_w(c_b), \gamma_w(c_b)\right)$ as follows:

$$w_b = PNWA_{\ell}\left(w_b^{(1)}, w_b^{(2)}, w_b^{(3)}, \ldots, w_b^{(h)}\right) = \ell_1 w_b^{(1)} \oplus \ell_2 w_b^{(2)} \oplus \ell_3 w_b^{(3)} \oplus \ldots \oplus \ell_h w_b^{(h)}$$

$$= \left(\sqrt{1-\prod_{t=1}^{h}\left(1-\left(\alpha_b^{(t)}\right)^2\right)^{\ell_t}}, \prod_{t=1}^{h}\left(\beta_b^{(t)}\right)^{\ell_t}, \prod_{t=1}^{h}\left(\gamma_b^{(t)}\right)^{\ell_t}\right) \tag{13}$$

Thus, the following criteria weight row matrix can be obtained.

$$w_b = \begin{bmatrix} \left(\alpha_w(c_1), \beta_w(c_1), \gamma_w(c_1)\right) \\ \left(\alpha_w(c_2), \beta_w(c_2), \gamma_w(c_2)\right) \\ \cdots \\ \cdots \\ \left(\alpha_w(c_g), \beta_w(c_g), \gamma_w(c_g)\right) \end{bmatrix}^T \tag{14}$$

**Step-7.** The weighted normalized aggregated PNDM, $P^* = \left(P^*_{ab}\right)_{f\times g}$ is formed as exposed in table 4 in the order of integrating the information from the normalized aggregated PNDM and the criteria weight matrix. $P^*_{ab} = \left(\alpha_{P^*_a}(c_b), \beta_{P^*_a}(c_b), \gamma_{P^*_a}(c_b)\right)$ may be generated by using the multiplication operator, which is specified in equation **Error! Reference source not found.**, i.e.

$$P^*_{ab} = \left(\alpha_{P_a}(c_b), \beta_{P_a}(c_b), \gamma_{P_a}(c_b)\right) \otimes \left(\alpha_w(c_b), \beta_w(c_b), \gamma_w(c_b)\right)$$

$$= \left(\alpha_{P_a}(c_b)\cdot\alpha_w(c_b), \sqrt{\beta^2_{P_a}(c_b) + \beta^2_w(c_b) - \beta^2_{P_a}(c_b)\cdot\beta^2_w(c_b)}, \sqrt{\gamma^2_{P_a}(c_b) + \gamma^2_w(c_b) - \gamma^2_{P_a}(c_b)\cdot\gamma^2_w(c_b)}\right) \tag{15}$$

**Table 4.** Weighted Normalized Aggregated PNDM (WNA-PNDM)

| $P^*$ | $c_1$ | $c_2$ | $\cdots$ | $c_g$ |
|---|---|---|---|---|
| $y_1$ | $\left(\alpha_{P^*_1}(c_1), \beta_{P^*_1}(c_1), \gamma_{P^*_1}(c_1)\right)$ | $\left(\alpha_{P^*_1}(c_2), \beta_{P^*_1}(c_2), \gamma_{P^*_1}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P^*_1}(c_g), \beta_{P^*_1}(c_g), \gamma_{P^*_1}(c_g)\right)$ |
| $y_2$ | $\left(\alpha_{P^*_2}(c_1), \beta_{P^*_2}(c_1), \gamma_{P^*_2}(c_1)\right)$ | $\left(\alpha_{P^*_2}(c_2), \beta_{P^*_2}(c_2), \gamma_{P^*_2}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P^*_2}(c_g), \beta_{P^*_2}(c_g), \gamma_{P^*_2}(c_g)\right)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $y_f$ | $\left(\alpha_{P^*_f}(c_1), \beta_{P^*_f}(c_1), \gamma_{P^*_f}(c_1)\right)$ | $\left(\alpha_{P^*_f}(c_2), \beta_{P^*_f}(c_2), \gamma_{P^*_f}(c_2)\right)$ | $\cdots$ | $\left(\alpha_{P^*_f}(c_g), \beta_{P^*_f}(c_g), \gamma_{P^*_f}(c_g)\right)$ |

## 3.2 | Phase II: (Pythagorean Neutrosophic Ranking Phase (PN-ELECTRE III))

The first phase of the Pythagorean decision support system gathers the PN calculation information for individually alternative, and the second step uses the PFELECTRE III approach, which uses the combined evaluations to yield the whole ranking of alternatives.

### 3.2.1 | Module I: Evolving Outranking Relations

The first phase of the Pythagorean decision support system gathers the PN calculation information for individually alternative, and the second step uses the PFELECTRE III approach, which uses the combined evaluations to yield the whole ranking of alternatives.

**Step-8.** In order to reveal concordance and discordance indices, gauge the degree of credibility, and rate the alternatives, the ELECTRE-III model's ([28, 29]) evaluation approach includes developing an indifference threshold function, preference threshold function, and veto threshold function. Let $q(c_b)$ be the indifference threshold function and $p(c_b)$ be the preference thresholds function for corresponding criteria $c_b$. Let if for any two given alternatives $y_1, y_2 \in Y$, $c(y_1) \geq c(y_2)$, then,

$$\begin{aligned} & c(y_1) \succ c(y_2) + p(c(y_2)) \Leftrightarrow y_1 P y_2 \\ & c(y_2) + q(c(y_2)) \prec c(y_1) \prec c(y_2) + p(c(y_2)) \Leftrightarrow y_1 Q y_2 \\ & c(y_2) \prec c(y_1) \prec c(y_2) + q(c(y_2)) \Leftrightarrow y_1 I y_2 \end{aligned} \tag{16}$$

where $c(y)$ is the criterion score value of the alternative $y$, and P signifies strong preference, Q weak preference, I indifference.

**Step-9.** For each pair of alternatives, the comprehensive concordance index $\varsigma(y_1, y_2)$ is calculated,

$$\varsigma(y_1, y_2) = \sum_{b=1}^{g} w_b \varsigma_b(y_1, y_2), \tag{17}$$

where represent the weight of $b^{th}$ criteria and $\varsigma_b(y_1, y_2)$ represent the partial concordance indices over the criteria $c_b$

$$\varsigma_b(y_1,y_2)=\begin{cases} 0, & \text{if} \quad c_b(y_2)-c_b(y_1)\succ p(c_b) \\ 1, & \text{if} \quad c_b(y_2)-c_b(y_1)\le q(c_b) \\ \dfrac{p(c_b)-\left(c_b(y_2)-c_b(y_1)\right)}{p(c_b)-q(c_b)}, & otherwise \end{cases} \tag{18}$$

Thus $0 \le \varsigma_b(y_1, y_2) \le 1$.

Table 5 shows partial concordance indices $\varsigma_b = \left[\varsigma_b(y_i, y_j)\right]_{f\times f}$ $\left(i, j = 1,2,3,...,f, i \ne j\right)$ and $\left(b = 1,2,3,...,g\right)$ over each criterion $c_b \in C$ can be obtained using equation.

**Table 5.** Partial Concordance indices over each criteria

| $\varsigma_b$ | $y_1$ | $y_2$ | $\cdots$ | $y_{f-1}$ | $y_f$ |
|---|---|---|---|---|---|
| $y_1$ | $-$ | $\varsigma_b(y_1,y_2)$ | $\cdots$ | $\varsigma_b(y_1,y_{f-1})$ | $\varsigma_b(y_1,y_f)$ |
| $y_2$ | $\varsigma_b(y_2,y_1)$ | $-$ | $\cdots$ | $\varsigma_b(y_2,y_{f-1})$ | $\varsigma_b(y_2,y_f)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $-$ | $\vdots$ | $\vdots$ |
| $y_{f-1}$ | $\varsigma_b(y_{f-1},y_1)$ | $\varsigma_b(y_{f-1},y_2)$ | $\cdots$ | $-$ | $\varsigma_b(y_{f-1},y_f)$ |
| $y_f$ | $\varsigma_b(y_f,y_1)$ | $\varsigma_b(y_f,y_2)$ | $\cdots$ | $\varsigma_b(y_f,y_{f-1})$ | $-$ |

And after that for each pair of alternatives, table 6 shows the comprehensive concordance index $\varsigma = \left[\varsigma_{ij}\right]_{f\times f}$ $\left(i, j = 1,2,3,...,f, i \ne j\right)$ is calculated using equation.

**Table 6.** Comprehensive Concordance index

| $\varsigma$ | $y_1$ | $y_2$ | $\cdots$ | $y_{f-1}$ | $y_f$ |
|---|---|---|---|---|---|
| $y_1$ | $-$ | $\varsigma_{12}$ | $\cdots$ | $\varsigma_{1(f-1)}$ | $\varsigma_{1f}$ |
| $y_2$ | $\varsigma_{21}$ | $-$ | $\cdots$ | $\varsigma_{2(f-1)}$ | $\varsigma_{2f}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $-$ | $\vdots$ | $\vdots$ |
| $y_{f-1}$ | $\varsigma_{(f-1)1}$ | $\varsigma_{(f-1)2}$ | $\cdots$ | $-$ | $\varsigma_{(f-1)f}$ |
| $y_f$ | $\varsigma_{f1}$ | $\varsigma_{f2}$ | $\cdots$ | $\varsigma_{f(f-1)}$ | $-$ |

**Step-10. Calculating the Discordance Index of the Assertion $y_1 S y_2$**

For each criterion, the discordance index $\xi_b(y_1, y_2)$ is calculated.

$$\xi_b(y_1,y_2)=\begin{cases} 0, & \text{if} \quad c_b(y_2)-c_b(y_1)\le p(c_b) \\ 1, & \text{if} \quad c_b(y_2)-c_b(y_1)\succ v(c_b) \\ \dfrac{\left(c_b(y_2)-c_b(y_1)\right)-p(c_b)}{v(c_b)-p(c_b)}, & otherwise \end{cases} \tag{19}$$

Thus $0 \le \xi_b(y_1, y_2) \le 1$.

For each criterion, table 7 Shows the discordance index $\xi_b = \left[\xi_b(y_i, y_j)\right]_{f\times f}$ $\left(i, j = 1,2,3,...,f, i \ne j\right)$ and $\left(b = 1,2,3,...,g\right)$ is calculated using equation.

**Table 7.** Discordance indices over each criteria

| $\xi_b$ | $y_1$ | $y_2$ | $\cdots$ | $y_{f-1}$ | $y_f$ |
|---|---|---|---|---|---|
| $y_1$ | $-$ | $\xi_b(y_1,y_2)$ | $\cdots$ | $\xi_b(y_1,y_{f-1})$ | $\xi_b(y_1,y_f)$ |
| $y_2$ | $\xi_b(y_2,y_1)$ | $-$ | $\cdots$ | $\xi_b(y_2,y_{f-1})$ | $\xi_b(y_2,y_f)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $-$ | $\vdots$ | $\vdots$ |
| $y_{f-1}$ | $\xi_b(y_{f-1},y_1)$ | $\xi_b(y_{f-1},y_2)$ | $\cdots$ | $-$ | $\xi_b(y_{f-1},y_f)$ |
| $y_f$ | $\xi_b(y_f,y_1)$ | $\xi_b(y_f,y_2)$ | $\cdots$ | $\xi_b(y_f,y_{f-1})$ | $-$ |

**Step-11. Disclosure of Credibility Index**

The credibility index, denoted by the notation $\pi(y_1,y_2)$, is used to determine the degree of outranking relation $y_1Sy_2$ is defined as:

$$\pi(y_1,y_2)=\begin{cases}\varsigma(y_1,y_2), & \text{if} \quad \xi_b(y_1,y_2)\le\varsigma(y_1,y_2) \quad \forall b\in\tau \\ \varsigma(y_1,y_2)\times\prod_{b\in\tau'}\dfrac{1-\xi_b(y_1,y_2)}{1-\varsigma(y_1,y_2)}, & otherwise\end{cases} \tag{20}$$

where $\tau'=\{b\in\tau:\xi_b(y_1,y_2)>\varsigma(y_1,y_2)\}$.

The credibility index, denoted by the notation $\pi=[\pi_{ij}]_{f\times f},(i,j=1,2,3,\ldots,f,i\ne j)$, is used to determine the degree of outranking relation $y_1Sy_2$ can be determine using equation which is expressed in Table 8.

**Table 8.** Credibility index

| $\pi$ | $y_1$ | $y_2$ | $\cdots$ | $y_{f-1}$ | $y_f$ |
|---|---|---|---|---|---|
| $y_1$ | $-$ | $\pi_{12}$ | $\cdots$ | $\pi_{1(f-1)}$ | $\pi_{1f}$ |
| $y_2$ | $\pi_{21}$ | $-$ | $\cdots$ | $\pi_{2(f-1)}$ | $\pi_{2f}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $-$ | $\vdots$ | $\vdots$ |
| $y_{f-1}$ | $\pi_{(f-1)1}$ | $\pi_{(f-1)2}$ | $\cdots$ | $-$ | $\pi_{(f-1)f}$ |
| $y_f$ | $\pi_{f1}$ | $\pi_{f2}$ | $\cdots$ | $\pi_{f(f-1)}$ | $-$ |

### 3.2.2 | Module II: The Exploitation of Outranking Relations (The Exploitation of Outranking Relations)

The ELECTRE III [30, 31] standard ranking method uses a structured algorithm with two intermediate ranking methods, one of which is descending distillation, where the alternatives are ordered from best to worst, and the other of which is based on ascending order from the worst to best option (ascending distillation). Li and Wang, however, propose a novel ranking method based on the derivation of three concepts: the concordance credibility degree, the discordance credibility degree, and the net credibility degree.

(i). For each alternative, the concordance credibility degree defined as:

$$\rho^+(y_a)=\sum_{y_b\in Y}\pi(y_a,y_b),\ \forall\, y_a\in Y \tag{21}$$

The concordance credibility degree represents outranked $y_a$.

(ii). For each alternative, the discordance credibility degree defined as:

$$\rho^-(y_a)=\sum_{y_b\in Y}\pi(y_b,y_a), \quad \forall y_a\in Y \tag{22}$$

The discordance credibility degree represents outranked $y_b$.

(iii). For each alternative, the net credibility degree defined as:

$$\rho(y_a) = \rho^+(y_a) - \rho^-(y_a), \quad \forall y_a \in Y \tag{23}$$

with the value of net credibility degree $\rho(y_a)$, alternative $y_a$ becomes more alluring. Thus, the potential solutions may be ordered in decreasing order according to their level of net believability.

# 4 | Real-Life Case Study: Best Female Spouse Selection for Male

The male spouse selection problem is a hypothetical case study that presents a situation where a man is faced with the task of selecting a spouse. In this scenario, the man is presented with a list of potential partners, each with their own set of characteristics and qualities. The man must evaluate each candidate based on these factors and ultimately choose the one that he believes is the best match for him. To solve this problem, the man may use a variety of criteria to evaluate each candidate. These criteria may include things like personality, intelligence, physical attractiveness, shared interests or values, and communication skills. The man may also consider factors such as cultural or religious background, family connections, and social status. Similar to the female spouse selection problem, the selection process will be influenced by a range of personal preferences and individual priorities. The man may have a clear idea of what he is looking for in a partner, or he may be more open to considering a wider range of options. He may also seek input from friends or family members, or consult with a professional matchmaker or dating coach.

Again, in any case, the selection process is likely to involve a considerable amount of self-reflection and careful consideration. The man will need to weigh the pros and cons of each potential partner and ultimately make a decision that he feels confident and comfortable with. While there is no one "right" way to approach this problem, the key is to stay true to oneself and to prioritize those qualities and characteristics that are most important to the individual. Ultimately, the goal is to find a partner with whom one can build a strong, healthy, and fulfilling relationship.

## 4.1 | Available Alternatives

Let we have three Alternative $Y = \{y_1, y_2, y_3\}$ to a wife or female spouse selection for male.

## 4.2 | Best Criteria for Consideration

There are several favorable/ Benefit-type/ Positive (+) and unfavorable/ Cost-type/ Negative (-) criteria that can make a female spouse suitable/unsuitable for a man. Here are some of the top criteria $C = \{c_1, c_2, c_3, c_4\}$

- **Trustworthiness ($c_1$ +):** A spouse who is trustworthy is someone who can be relied upon to keep their promises, maintain confidentiality, and be faithful.
- **Respectful ($c_2$ +):** A respectful spouse is someone who values and acknowledges their partner's thoughts, feelings, and opinions.
- **Inarticulate ($c_3$ -):** Good communication skills are essential in any relationship, and a wife who can communicate effectively can avoid misunderstandings and resolve conflicts more easily.
- **Supportive ($c_4$ +):** A supportive spouse is someone who stands by their partner through thick and thin, offers encouragement, and helps them achieve their goals.

## 4.3 | Sequential Process

In the phases that follow, the entire PN-ELECTRE III process is utilized to determine the ideal wife or female spouse for male candidate.

### 4.3.1 | Phase I: Pythagorean Neutrosophic Assessment Phase

**Step-1:** Configuration of Linguistic variables- As shown in table 9, the relevance of weight degree to criteria and experts are given using the decision support system of the recommended approach in the form of linguistic terms/variable that are provided by PNNs. Table 10 shows the performance ratings using linguistic phrases and variables.

**Table 9:** LVfor importance weighted rating of criteria and expert.

| Linguistic Variable | Code | PNNs |
|---|---|---|
| Very Good | VG | (0.90,0.32,0.20) |
| Good | G | (0.70,0.23,0.35) |
| Medium | M | (0.50,0.55,0.50) |
| Bad | B | (0.35,0.23,0.70) |
| Very Bad | VB | (0.20,0.32,0.90) |

**Table 10:** LV for Favorable Rating of Alternatives and Threshold functions.

| **Linguistic** Variable | **Code** | **PNNs** |
|---|---|---|
| Extremely Favourable | EF | (0.95,0.15,0.11) |
| Very Favourable | VF | (0.72,0.32,0.25) |
| Medium Favourable | MF | (0.62,0.42,0.32) |
| Medium Unfavourable | MU | (0.32,0.42,0.62) |
| Very Unfavourable | VU | (0.25,0.32,0.72) |
| Extremely Unfavourable | EU | (0.11,0.15,0.95) |

**Step-2:** Calculation of the weights of DMs - Table 11 lists the importance rankings that the male candidate granted to each of the DMs $v_t$. Employing equation it is possible to govern each expert's individual weight.

**Table 11.** Assigning and Calculation of weights of DMs

| DMs | LV | PNNs | Weights ($\ell_t$) |
|---|---|---|---|
| $v_1$ | VG | (0.90,0.32,0.20) | 0.3811 |
| $v_2$ | G | (0.70,0.23,0.35) | 0.3505 |
| $v_3$ | M | (0.50,0.55,0.50) | 0.2685 |

**Step-3:** Assigning of the DMs Opinions- The LV used in table 11 to designate the distinct viewpoints of each DMs on the decision-making board with regard to individually alternative and all taken-into-account criteria. The Pythagorean Neutrosophic Decision Matrices [PNDMs] $P^{(1)}, P^{(2)}$ and $P^{(3)}$ that highlight the exclusive opinions of the DMs $v_1, v_2$ and $v_3$, correspondingly, are exposed in tables 12, 13 and 14.

**Table 11:** Opinion of DMs in LV on Favourability Rating of alternatives w.r.t. Criterion

| | | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|---|
| $v_1$ | $y_1$ | EF | VF | MU | VF |
| | $y_2$ | VU | EF | MU | MF |
| | $y_3$ | VU | MF | VF | EU |
| $v_2$ | $y_1$ | VF | MU | MU | EF |
| | $y_2$ | VU | VF | MF | VF |
| | $y_3$ | EU | MU | VF | EF |
| $v_3$ | $y_1$ | MU | MF | VF | VU |
| | $y_2$ | MF | EF | VU | EU |
| | $y_3$ | VF | MF | MU | EU |

**Table 12.** Opinion of DMsin PNNs

| $P^{(1)}$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $y_1$ | $(0.95, 0.15, 0.11)$ | $(0.72, 0.32, 0.25)$ | $(0.32, 0.42, 0.62)$ | $(0.72, 0.32, 0.25)$ |
| $y_2$ | $(0.25, 0.32, 0.72)$ | $(0.95, 0.15, 0.11)$ | $(0.32, 0.42, 0.62)$ | $(0.62, 0.42, 0.32)$ |
| $y_3$ | $(0.25, 0.32, 0.72)$ | $(0.62, 0.42, 0.32)$ | $(0.72, 0.32, 0.25)$ | $(0.11, 0.15, 0.95)$ |

**Table 13.** Opinion of DMsin PNNs

| $\mathrm{P}^{(2)}$ | $\mathrm{c}_1$ | $\mathrm{c}_2$ | $\mathrm{c}_3$ | $\mathrm{c}_4$ |
|---|---|---|---|---|
| $\mathrm{y}_1$ | $(0.72, 0.32, 0.25)$ | $(0.32, 0.42, 0.62)$ | $(0.32, 0.42, 0.62)$ | $(0.95, 0.15, 0.11)$ |
| $\mathrm{y}_2$ | $(0.25, 0.32, 0.72)$ | $(0.72, 0.32, 0.25)$ | $(0.62, 0.42, 0.32)$ | $(0.72, 0.32, 0.25)$ |
| $\mathrm{y}_3$ | $(0.11, 0.15, 0.95)$ | $(0.32, 0.42, 0.62)$ | $(0.72, 0.32, 0.25)$ | $(0.95, 0.15, 0.11)$ |

**Table 14.** Opinion of DMsin PNNs

| $P^{(3)}$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $y_1$ | $(0.32, 0.42, 0.62)$ | $(0.62, 0.42, 0.32)$ | $(0.72, 0.32, 0.25)$ | $(0.25, 0.32, 0.72)$ |
| $y_2$ | $(0.62, 0.42, 0.32)$ | $(0.95, 0.15, 0.11)$ | $(0.25, 0.32, 0.72)$ | $(0.11, 0.15, 0.95)$ |
| $y_3$ | $(0.72, 0.32, 0.25)$ | $(0.62, 0.42, 0.32)$ | $(0.32, 0.42, 0.62)$ | $(0.11, 0.15, 0.95)$ |

**Step-4:** Aggregation of the DMs Opinion- The unique opinions of each DMs are pooled based on the normalized weights assigned by the PNWA operator, see equation **Error! Reference source not found.** and the DMs. The combined Pythagorean neutrosophic decision matrix $P' = \left(P'_{ab}\right)_{f \times g}$ is shown in table 15.

**Table 15:** Aggregated Pythagorean neutrosophic decision matrix (A-PNDM)

| $P'$ | $c_1$ | $c_2$ |
|---|---|---|
| $y_1$ | $(0.8309, 0.2579, 0.2333)$ | $(0.6002, 0.3786, 0.3672)$ |
| $y_2$ | $(0.4033, 0.3442, 0.5791)$ | $(0.9107, 0.1956, 0.1466)$ |
| $y_3$ | $(0.4489, 0.2453, 0.5973)$ | $(0.5454, 0.4200, 0.4034)$ |

| $P'$ | $c_3$ | $c_4$ |
|---|---|---|
| $y_1$ | $(0.4905, 0.3904, 0.4858)$ | $(0.8191, 0.2453, 0.2490)$ |
| $y_2$ | $(0.4520, 0.3904, 0.5118)$ | $(0.5989, 0.2896, 0.3930)$ |
| $y_3$ | $(0.6564, 0.3442, 0.3190)$ | $(0.7492, 0.1500, 0.4462)$ |

**Step-5:** Normalization of Aggregated PNDM- Let $C_B$ and $C_N$ denote the corresponding groups of criteria that are of the benefit-type (Positive) criteria $C_B = \{c_1, c_2, c_4\}$, Cost type (Negative) criteria $C_N = \{c_3\}$.. The aggregated PNDM, $P' = \left(P'_{ab}\right)_{f\times g}$, can be converted into the normalized aggregated PNDM, $P = \left(P_{ab}\right)_{7\times 8}$ using equation, which demonstrate the evaluation data of each alternative w.r.t. each benefit or cost criterion, in standard form, for additional calculations. Table 16 demonstrates how the matrix $P$ is built. You can define the PNN for $P_{ab}$:

**Table 16** : *Normalized Aggregated PNDM* $(NA-PNDM)$

| $P$ | $c_1$ | $c_2$ |
|---|---|---|
| $y_1$ | (0.8309, 0.2579, 0.2333) | (0.6002, 0.3786, 0.3672) |
| $y_2$ | (0.4033, 0.3442, 0.5791) | (0.9107, 0.1956, 0.1466) |
| $y_3$ | (0.4489, 0.2453, 0.5973) | (0.5454, 0.4200, 0.4034) |

| $P$ | $c_3$ | $c_4$ |
|---|---|---|
| $y_1$ | (0.4858, 0.3904, 0.4905) | (0.8191, 0.2453, 0.2490) |
| $y_2$ | (0.5118, 0.3904, 0.4520) | (0.5989, 0.2896, 0.3930) |
| $y_3$ | (0.3190, 0.3442, 0.6564) | (0.7492, 0.1500, 0.4462) |

**Step-6:** Formation of weight matrix of criteria- The DMs panel's linguistic labels for each criterion, PN-weights, and normalised weights of the criteria are shown in table 17 and table 18.

**Table 17.** Linguistic variable to unfold the importance of criteria

| *Criteria* → *Expert* ↓ | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $v_1$ | VG | M | VB | B |
| $v_2$ | M | G | VG | VB |
| $v_3$ | G | VG | B | G |

**Table 18.** PN - Weights of Criteria

| *Criteria* → *Expert* ↓ | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $v_1$ | (0.90, 0.32, 0.20) | (0.50, 0.55, 0.50) | (0.20, 0.32, 0.90) | (0.35, 0.23, 0.70) |
| $v_2$ | (0.50, 0.55, 0.50) | (0.70, 0.23, 0.35) | (0.90, 0.32, 0.20) | (0.20, 0.32, 0.90) |
| $v_3$ | (0.70, 0.23, 0.35) | (0.90, 0.32, 0.20) | (0.35, 0.23, 0.70) | (0.70, 0.23, 0.35) |

Normalized weights of criteria using equation **Error! Reference source not found.** and equation

$$W_{\{c_1,c_2,c_3,c_4\}} = \begin{pmatrix} (0.7741, 0.3540, 0.3204) \\ (0.7395, 0.3503, 0.3450) \\ (0.6847, 0.2928, 0.4966) \\ (0.4661, 0.2582, 0.6346) \end{pmatrix}^T \quad \text{and} \quad \begin{matrix} w(c_1) = 0.2565 \\ w(c_2) = 0.2513 \\ w(c_3) = 0.2665 \\ w(c_4) = 0.2257 \end{matrix}$$

**Step-7:** Creation of weighted normalized aggregation PNDM- The weighted normalized aggregated PNDM, $P^* = \left(P^*_{ab}\right)_{f\times g}$ is created using equation **Error! Reference source not found.** as shown in table 19.

**Table 19.** Weighted Normalized Aggregated PNDM $(\text{WNA - PNDM})$

| $P^*$ | $c_1$ | $c_2$ |
|---|---|---|
| $y_1$ | (0.6432, 0.4284, 0.3892) | (0.4438, 0.4985, 0.4877) |
| $y_2$ | (0.3122, 0.4785, 0.6353) | (0.6735, 0.3953, 0.3714) |
| $y_3$ | (0.3475, 0.4218, 0.6502) | (0.4033, 0.5267, 0.5122) |

| $P^*$ | $c_3$ | $c_4$ |
|---|---|---|
| $y_1$ | $(0.3326, 0.4744, 0.6541)$ | $(0.3818, 0.3505, 0.6631)$ |
| $y_2$ | $(0.3504, 0.4744, 0.6329)$ | $(0.2791, 0.3807, 0.7035)$ |
| $y_3$ | $(0.2184, 0.4405, 0.7558)$ | $(0.3492, 0.2961, 0.7222)$ |

**Step-8:** Calculation of Score degrees w.r.t. WNA-PNDM- Table 20 comprises the computed score values of consistent PNNs in the weighted normalized aggregated PNDM using equation

**Table 20.** The score degree of WNA - PNDM

| $s(P)$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $y_1$ | 0.0787 | −0.2894 | −0.5423 | −0.4168 |
| $y_2$ | −0.5351 | 0.1594 | −0.5028 | −0.5619 |
| $y_3$ | −0.4799 | −0.3771 | −0.7176 | −0.4873 |

## 4.3.2 | Phase II: Pythagorean Neutrosophic Ranking Phase (PN-ELECTRE III)

### 4.3.2.1 | Module-I: The Construction of Outranking Relations

**Step-9.** Establishment of Threshold Functions- For each criteria $\chi_b$, here are preference threshold values $p(\chi_b)$ and indifference threshold values $q(\chi_b)$ and Veto threshold values $v(\chi_b)$ as shown in Table 21.

**Table 20** Assigning of PNNs to threshold functions and its score values

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $q(c_j)$ | EU | VU | MF | VU |
| | $(0.11, 0.15, 0.95)$ | $(0.25, 0.32, 0.72)$ | $(0.62, 0.42, 0.32)$ | $(0.25, 0.32, 0.72)$ |
| $s(q(c_j))$ | −0.9129 | −0.5583 | 0.1056 | −0.5583 |
| $p(c_j)$ | MU | MF | VF | MU |
| | $(0.32, 0.42, 0.62)$ | $(0.62, 0.42, 0.32)$ | $(0.72, 0.32, 0.25)$ | $(0.32, 0.42, 0.62)$ |
| $s(p(c_j))$ | −0.4584 | 0.1056 | 0.3535 | −0.4584 |
| $v(c_j)$ | VF | EF | EF | VF |
| | $(0.72, 0.32, 0.25)$ | $(0.95, 0.15, 0.11)$ | $(0.95, 0.15, 0.11)$ | $(0.72, 0.32, 0.25)$ |
| $s(v(c_j))$ | 0.3535 | 0.8679 | 0.8679 | 0.3535 |

**Step-10.** Calculation of difference in the score degrees- The differences in the score values/degrees of the feasibility of every pair of alternatives are represented by Table 21. i.e., the if part of equation

**Table 21.** The differences in the score values / degrees

| $c_1$ | | | $c_2$ | | |
|---|---|---|---|---|---|
| – | −0.6138 | −0.5586 | – | 0.4488 | −0.0877 |
| 0.6138 | – | 0.0552 | −0.4488 | – | −0.5365 |
| 0.5586 | −0.0552 | – | 0.0877 | 0.5365 | – |

| $c_3$ | | | $c_4$ | | |
|---|---|---|---|---|---|
| – | 0.0395 | −0.1753 | – | −0.1451 | −0.0705 |
| −0.0395 | – | −0.2148 | 0.1451 | – | 0.0746 |
| 0.1753 | 0.2148 | – | 0.0705 | −0.0746 | – |

**Step-11:** Calculation of Partial Concordance Indicines and Concordance Matrix-

The partial concordance Matrices w.r.t. each criterion is given in the Table 22, calculated using equation

**Table 22.** The partial concordance Matrices

| $c$ | | | $c_2$ | | |
|---|---|---|---|---|---|
| – | 0.3419 | 0.2205 | – | 0.0000 | 0.2912 |
| 0.0000 | – | 0.0000 | 0.8351 | – | 0.9672 |
| 0.0000 | 0.0000 | – | 0.027 | 0.0000 | – |
| $c_3$ | | | $c_4$ | | |
| – | 1.0000 | 1.0000 | – | 0.0000 | 0.0000 |
| 1.0000 | – | 1.0000 | 0.0000 | – | 0.0000 |
| 0.7188 | 0.5595 | – | 0.0000 | 0.0000 | – |

Comprehensive concordance matrix is given by Table 23, employing by equation

**Table 23.** Comprehensive concordance matrix

| $c$ | | |
|---|---|---|
| 0 | 0.3542 | 0.3962 |
| 0.4764 | 0 | 0.5096 |
| 0.1983 | 0.1491 | 0 |

**Step-12:** Calculation of Discordance Matrix- Discordance matrices w.r.t. each criteria shown in Table 24, calculated using equation

**Table 24.** Discordance matrices

| $d_1$ | | | $d_2$ | | |
|---|---|---|---|---|---|
| – | 0.0000 | 0.0000 | – | 0.4502 | 0.0000 |
| 1.0000 | – | 0.6326 | 0.0000 | – | 0.0000 |
| 1.0000 | 0.4966 | – | 0.0000 | 0.5653 | – |
| $d_3$ | | | $d_4$ | | |
| – | 0.0000 | 0.0000 | – | 0.3859 | 0.4778 |
| 0.0000 | – | 0.0000 | 0.7433 | – | 0.6565 |
| 0.0000 | 0.0000 | – | 0.6514 | 0.4727 | – |

**Step-13:** Calculation of Credibility Index- Table 25 shows the credibility index of alternative, calculated using equation.

**Table 25.** credibility index of alternative

| *Credibility* | | |
|---|---|---|
| – | 0.2867 | 0.3427 |
| 0.0000 | – | 0.2674 |
| 0.0000 | 0.0279 | – |

#### 4.3.2.2 | Module II: The Exploitation of Outranking Relations

**Step-14:** Calculation of Credibility Index- The ranking of the alternatives have been given in the table 26, employing by equations and

**Table 26.** Ranking of Alternatives

| Alternatives | $\mu^+$ | $\mu^-$ | $\mu$ | Ranking |
|---|---|---|---|---|
| $y_1$ | 0.6294 | 0 | 0.6294 | 1 |
| $y_2$ | 0.2674 | 0.3146 | –0.0472 | 2 |
| $y_3$ | 0.0279 | 0.6101 | –0.5822 | 3 |

From the Table 26, we find $y_1 \succ y_2 \succ y_3$. Thus, first is the best alternative to select as wife/female spouse for a male candidate.

## 5 | Conclusions

Debuting a new age that brings innovative challenges in the desire to forge genuine and lasting connections, the emergence of social media and the spread of online dating platforms have also changed the fundamental fabric of human contact. Our dedication to solving this modern problem is attested to by this study article. In a Pythagorean neutrosophic framework, we have pioneered a novel fusion of MCGDM and the ELECTRE-III technique, through which we have developed a special model that is well-tuned to address the particular issues of mate choosing in the twenty-first century. Our concept is based on the idea that choosing a spouse is a multifaceted process that takes a number of factors into account. It provides a methodological framework that is organized and methodical and well-suited to manage the sometimes contradictory and ambiguous information that usually pervades this crucial decision-making process. The use of PNST is a cornerstone of our strategy. This notion acts as a mental compass that steers us clear of the hazy seas of ambiguity and doubt, which frequently cloud our judgment when it comes to choosing a mate. This method equips decision-makers with a strong toolset to handle the complexity that come with choosing a life partner since it is based on rigorous mathematical and computational underpinnings.

In conclusion, the necessity for sophisticated decision-making tools becomes more and more obvious as the dynamics of interpersonal interactions continue to change in response to the globalized, technologically advanced world of the 21st century. In order to face this problem, our study has developed a potent MCGDM framework that is solidly based on Pythagorean neutrosophic theory. We hope that our contribution will serve as a guiding light for people and DM as they set out to create lasting and happy relationships, giving them confidence and clarity in their quest for love and companionship in the modern day.

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## Author Contributaion

**R K Saini:** Conceptualized and designed the study, mathematical formulation of the case study, conducted data analysis, and provided supervision throughout the research process.

**Ashik Ahirwar:** Contributed to data analysis, conducted data calculations, manuscript writing, and provided technical assistance at various stages of the project.

**Florentin Smarandache:** Provided critical revision of the manuscript and offered supervision, ensuring the integrity and quality of the research.

**Mukesh Kushwaha:** Contributed to data collection, conduction of data analysis, and provided some writing assistance at various stages.

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## Data Availability

The availability and access to data utilized in our study have been meticulously sourced and referenced from reputable online platforms. We have diligently collected pertinent information from various reliable sources and ensure the integrity and reproducibility of our research findings, facilitating further exploration and analysis in this study.

## Conflicts of Interest

It is declared that the authors have been personally and actively involved in substantial work leading to the paper and will take public responsibility for its content. No violation of the Ethics involved in this paper.

## Ethical Approval

We affirm that our research article does not involve human subjects; therefore, ethical approval and informed consent did not apply to this study. All data utilized in our research were obtained from publicly available sources or were simulated/generated for analysis. We ensure that all data handling procedures strictly adhere to ethical standards and legal regulations regarding data usage and privacy. Our commitment to transparency and integrity underscores our dedication to upholding the highest ethical standards in scientific research.

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