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The Cardinal of the m-powerset of a Set of n Elements used in the SuperHyperStructures and Neutrosophic SuperHyperStructures

Florentin Smarandache [1,*]

[1] University of New Mexico, Mathematics, Physics and Natural Science Division, Gallup, NM 87301, USA; smarand@unm.edu.

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Abstract

In this paper, we find a formula for computing the cardinal of the m-th powerset of a set of n elements, which is needed in the SuperHyperStructures.

Keywords: m-powerset of a Set; SuperHyperStructure; SuperHyperAlegebra; SuperHyperAxiom; SuperHyperOperation.

1 | Introduction

The SuperHyperStructure and Neutrosophic SuperHyperStructure [2, 3], together with their particular cases such as:

SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra (endowed with superHyperOperations and SuperHyperAxioms) [2016, 2022], **SuperHyperGraph** (including **SuperHyperTree**) and Neutrosophic SuperHyperGraph (including Neutrosophic SuperHyperTree) [2019-2022], **SuperHyperSoft Set**, **SuperHyperFunction** and Neutrosophic SuperHyperFunction [2022], **SuperHyperTopology** (that is a topology built on the *powersets of* $P(H)$, *or* $P^n(H)$, for $n \geq 1$) and Neutrosophic SuperHyperTopology [2022] were founded by Smarandache [2, 3] and developed between 2016 - 2024.

2 | Definition of the SuperHyperStructure

A **SuperHyperStructure** is a structure built on the ***n-th PowerSet of a Set H,*** for n ≥ 1, as in our real world {because a set (or system) H (that may be a set of items, an organization, country, etc.) is composed by *sub-sets* that are parts of $P(H)$, which in their turn are organized in *sub-sub-sets* that are parts of $P(P(H)) = P^2(H)$, then in *sub-sub-sub-sets* that are parts of $P^3(H)$, and so on, $P^{n+1}(H) = P(P^n(H))$ }.

The powerset $P(H)$ means all non-empty and empty subsets of H, including the empty set (ф), which represents the indeterminacy that occurs into the set H (as in our real world where we deal with unclear/indeterminate information in any space/set); and similarly for $P^n(H)$.

While $P_*(H)$ means all non-empty subsets of H, or $P_*(H) = P(H)$ - ф. And similarly for $P_*^n(H)$.

A structure built on $P_*^n(H)$ is called a **SuperHyperStructure** (has no indeterminacy), while built on $P^n(H)$ it is called **Neutrosophic SuperHyperStructure** (does have indeterminacy).

Corresponding Author: smarand@unm.edu

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In a SuperHyperStructure we deal with **SuperHyperAxioms**, **SuperHyperOperators**, etc.

3 | "Hyper" and "Super" Prefixes

The prefix "Hyper" {Marty [1], 1934} stand for the codomain of the functions and operations to be *P(H)*, or the powerset of the set *H*. While the prefix "Super" (Smarandache [2, 3], 2016) stands for using the $P^n(H)$, $n \geq 2$, or the *n-th* PowerSet of the Set *H* {because a *set* (or *system*) *H* (that may be a set of items, a company, institution, country, region, etc.) is organized in *sub-sets* that are part *of P(H)*, which in their turn are organized in *sub-sub-sets*, that are part of $P(P(H)) = P^2(H)$ and so on} in the domain and/or codomain of the functions and operations and axioms.

4 | SuperHyperStructure

A SuperHyperStructure [2, in 2016] is a structure built on the *n-th powerset* $P_*^n(H)$ of a non-empty set *H*, for integer $n \geq 1$, whose SuperHyperOperators are defined as follows:

$$\#_{SHS} : \left(P_*^r(H)\right)^m \longrightarrow P_*^n(H)$$

where $P_*^r(H)$ is the *r-powerset of H,* for integer $r \geq 1$, while similarly $P_*^n(H)$ is the *n-th powerset of H*, and the **SuperHyperAxioms** act on it.

Indeterminacy is not allowed on this structure.

5 | Neutrosophic SuperHyperStructure

A Neutrosophic SuperHyperStructure is a structure built on the *n-th powerset* $P^n(H)$ of a non-empty set *H*, for integer $n \geq 1$, whose Neutrosophic SuperHyperOperators are defined as follows:

$\#_{SHS}: (P^r(H))^m \longrightarrow P^n(H)$,

where $P^r(H)$ is the *r-powerset of H,* for integer $r \geq 1$, while similarly

$P^n(H)$ is the *n-th powerset of H*, and the **SuperHyperAxioms** act on it.

Indeterminacy is allowed on this structure and represented by the empty-set (φ).

6 | Theorem 1 (with indeterminacy)

Let $H = \{a_1, a_2, \ldots, a_n\}$, $n \geq 1$, a set.

$Card(H) = n$, where $Card$ means cardinal of H, or the number of elements of H.

$\mathcal{P}(H)$ is the powerset of H, including the empty set $\emptyset$ (which represents indeterminacy):

$$\mathcal{P}(H) = \{\emptyset;\ a_1, a_2, \ldots, a_n;\ \{a_1, a_2\}, \ldots; \{a_1, a_2, a_3\}, \ldots; \ldots\}$$

The cardinal of $\mathcal{P}(H)$ is:

$$Card(\mathcal{P}(H)) = C_n^0 + C_n^1 + C_n^2 + \cdots + C_n^n = (1+1)^n = 2^n.$$

Let us rename the elements of $\mathcal{P}(H)$ to easily compute the cardinal of $\mathcal{P}^2(H) = \mathcal{P}(\mathcal{P}(H))$, as $\mathcal{P}(H) = \{\alpha_1, \alpha_2, \ldots, \alpha_{2^n}\}$

$$Card\left(\mathcal{P}(\mathcal{P}(H))\right) = C_{2^n}^0 + C_{2^n}^1 + C_{2^n}^2 + \cdots + C_{2^n}^{2^n} = (1+1)^{2^n} = 2^{2^n}\Big\} \text{ two 2's}$$

or $Card(\mathcal{P}^2(H)) = 2^{2^n}$.

Similarly,

$Card(\mathcal{P}^3(H)) = 2^{2^{2^n}}\}$ three 2's

By induction, it is easily proved that:

$Card(\mathcal{P}^m(H)) = 2^{2^{\cdots 2^n}}\} m$ 2's

or the cardinal of the m-powerset of a set of n elements, for integers $m \geq 1$ and $n \geq 1$ is equal to: $2^{2^{\cdots 2^n}}\}$ m *of* 2's.

7 | Example of Cardinality (with indeterminacy) of a 2-powerset of a Set of 3 Elements

H = {a_1, a_2, a_3}.

For m = 3 and n = 3, in the above formula, one gets:

$Card(\mathcal{P}^2(H)) = 2^{2^3} = 2^{(2^3)} = 2^8 = 256$ elements.

8 | Theorem 2 (without indeterminacy)

Let $H = \{a_1, a_2, \dots, a_n\}$, $n \geq 1$, a set.

$Card(H) = n.$

$P_*(H)$ is the powerset of H, *excluding* the empty set $\emptyset$ (which represents indeterminacy):

$$P_*(H) = \{a_1, a_2, \dots, a_n;\ \{a_1, a_2\}, \dots; \{a_1, a_2, a_3\}, \dots; \dots\}$$

The cardinal of $P_*(H)$ is:

$$Card\left(P_*(H)\right) = C_n^1 + C_n^2 + \cdots + C_n^n = (1+1)^n = 2^n - 1.$$

Let us rename the elements of $P_*(H)$, to easily compute the cardinal of $P_*^2(H) = P_*(P_*(H))$, as $P_*(H) = \{\beta_1, \beta_2, \dots, \beta_{2^n-1}\}$, then:

$Card(P_*^2(H)) = Card(P_*(\{\beta_1, \beta_2, \dots, \beta_{2^n-1}\})) =$

$$= C_{2^n-1}^1 + C_{2^n-1}^2 + \dots + C_{2^n-1}^{2^n-1} = (1+1)^{2^n-1} - 1 = 2^{2^n-1} - 1.$$

Similarly, let's rename $P_*^2(H) = \{\gamma_1, \gamma_2, \dots, \gamma_{2^{2^n-1}-1}\}$

$$Card(P_*^3(H)) = Card(P_*(P_*^2(H))) = Card(P_*(\{\gamma_1, \gamma_2, \dots, \gamma_{2^{2^n-1}-1}\})) = C_{2^{2^n-1}-1}^1 + C_{2^{2^n-1}-1}^2 + \dots + C_{2^{2^n-1}-1}^{2^{2^n-1}-1} =$$

$$= (1+1)^{2^{2^n-1}-1} - 1 = 2^{2^{2^n-1}-1} - 1$$

Then by mathematical induction we find the general formula for the cardinal of $P_*^m(H)$:

$$Card(P_*^m(H)) = 2^{2^{\cdot^{\cdot^{2^n-1}}-1}}-1$$

Where, in the above formula, one has m of 2's, and m of -1's.

9 | Example 2

Same set H = $\{a_1, a_2, a_3\}$, with n = 3.

For m = 2, one has $Card(P_*^2(H)) = 2^{2^3-1} - 1 = 2^{8-1} - 1 = 2^7 - 1 = 127$.

10 | Conclusion

In this paper we calculated the cardinals of the m-powerset of a set of n elements, when indeterminacy (empty-set) was included, but also for the case when the indeterminacy (empty-set) was not included.

These m-powersets $P^m(H)$ are used in the Neutrosophic SuperHyperStructures (when the indeterminacy does exist), while $P_*^m(H)$ are used in the SuperHyperStructures (when indeterminacy does not exist).

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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