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Reconsideration of Neutrosophic Social Science and Neutrosophic Phenomenology with Non-classical logic

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Abstract

Body-Mind-Soul-Spirit Fluidity is a concept rooted in psychology and phenomenology, offering significant insights into human decision-making and well-being. Similarly, in social analysis and social sciences, frameworks such as PDCA, DMAIC, SWOT, and OODA have been established to enable structured evaluation and effective problem-solving. Furthermore, in phenomenology and social sciences, various logical systems have been developed to address specific objectives and practical applications.
This paper extends these concepts using the Neutrosophic theory, revisiting their mathematical definitions and exploring their properties. The Neutrosophic Set, an extension of the Fuzzy Set, is a highly flexible framework that has been widely studied in fields such as social sciences. By incorporating Neutrosophic Sets, we aim to improve their suitability for programming and mathematical analysis, providing advanced methods to tackle complex, multi-dimensional problems.
We hope that this research will inspire further studies and foster the development of practical applications across various related disciplines.

Keywords: Neutrosophic Set, Plithogenic set, Fuzzy set, Phenomenology, Social analysis

1 | Short Introduction

1.1 | Phenomenology: Body-Mind-Soul-Spirit Fluidity

Phenomenology is a philosophical approach that investigates conscious experiences as they are perceived, focusing on intentionality, subjective interpretation, and the suspension of preconceived notions to reveal the essence of phenomena and lived experiences [345, 244, 139, 398, 411, 110]. Its relevance spans disciplines such as psychology,

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sociology [87, 107], education [98], and healthcare [297], highlighting the importance of continued research in phenomenological studies.

Body-Mind-Soul-Spirit Fluidity is a concept originating from psychology and phenomenology (cf. [183, 112, 360, 195, 105]). It reflects the interconnected dimensions of human existence: the physical body, mental processes, emotional soul, and spiritual awareness. Recently, this concept has been extended through the framework of Neutrosophic Sets, giving rise to *Neutrosophic Body-Mind-Soul-Spirit Fluidity*, a more flexible and robust model for understanding the dynamics of these interconnected dimensions [337].

1.2 | Social Analysis: PDCA, DMAIC, SWOT, OODA, and Five Forces Analysis

Social Science studies human behavior, societies, and cultures using systematic research and interdisciplinary approaches[142, 393]. Social Analysis examines societal structures, relationships, and processes to understand social dynamics and address challenges[34, 138]. In the field of Social Analysis and Social Sciences, various frameworks have been established to facilitate structured evaluation and problem-solving[138]. Notable examples include the following frameworks, which are widely recognized for their practical applications. In this paper, these concepts will be extended using the Neutrosophic Set framework discussed later.

- *PDCA (Plan-Do-Check-Act)*: A cyclical framework designed for continuous improvement. It involves planning strategies, executing actions, evaluating results, and refining processes to achieve better outcomes [133, 291, 173, 256].
- *DMAIC (Define-Measure-Analyze-Improve-Control)*: A methodology derived from Six Sigma that emphasizes defining problems, collecting and measuring data, analyzing root causes, implementing improvements, and controlling processes to maintain quality [242, 285, 300, 286, 356, 224].
- *SWOT (Strengths-Weaknesses-Opportunities-Threats)*: A strategic planning tool used to assess internal strengths and weaknesses, as well as external opportunities and threats, for effective organizational analysis [311, 140, 391, 237, 305, 93].
- *OODA (Observe-Orient-Decide-Act)*: A decision-making process that focuses on observing situations, orienting oneself to the context, making informed decisions, and acting promptly, particularly in dynamic or competitive environments [236, 282, 298, 131, 415, 198].
- *Porter's Five Forces Analysis*: A framework for analyzing industry competition. It examines five key forces: industry rivalry, buyer power, supplier power, the threat of substitutes, and the threat of new entrants[94, 150, 278].

1.3 | Neutrosophic Set and Related Set Theories

Psychology, Phenomenology, and Social Analysis are inherently intertwined with uncertainty. The Neutrosophic Set provides a comprehensive framework for effectively addressing and managing these uncertainties. This subsection explains the Neutrosophic Set and its related concepts.

Set theory is a foundational branch of mathematics that focuses on the study of "sets," which are collections of objects [90, 385, 382, 180]. Over time, extensions of classical set theory have been developed to better handle the complexities and uncertainties encountered in real-world scenarios. These include Fuzzy Sets [403, 88, 358, 417, 405, 406, 407, 408], Vague Sets [9, 58, 63, 165, 412], Soft Sets [222, 15, 120, 14, 241, 400], Hypersoft Sets [333, 332], Rough Sets [266, 272, 270, 267, 271, 268, 269], Hyperfuzzy Sets [182, 349, 136, 119], and Neutrosophic Sets [319, 320, 11, 100, 390, 251, 340, 115, 324, 323, 54].

Each of these frameworks addresses specific forms of ambiguity or uncertainty. For example, *Fuzzy Sets* assign to each element a membership degree within the interval $[0, 1]$, representing partial rather than binary membership [403]. *Neutrosophic Sets* extend this concept by assigning three independent degrees—truth, indeterminacy, and falsity—to each element, making them particularly suitable for managing complex uncertainties [319, 320].

1.4 | Our Contribution in This Paper

In this paper, we extend the concepts of Body-Mind-Soul-Spirit Fluidity, PDCA, DMAIC, SWOT, OODA, and Five Forces Analysis within the framework of Neutrosophic theory and provide a brief exploration of their properties. Furthermore, we investigate various types of logic in the contexts of Neutrosophic Phenomenology and Neutrosophic Social Science. It is important to note that the term "logic" here refers specifically to non-classical logic. While some of these concepts are already established, we revisit their mathematical definitions to facilitate programming and mathematical analysis using Neutrosophic Sets.

We hope that this research will inspire further studies and encourage the development of practical applications in this emerging field.

2 | Preliminaries and Definitions

This section introduces essential concepts from set theory that are used throughout this work. For a deeper exploration of these concepts and their applications, readers are encouraged to consult the cited references as necessary [159, 113, 207, 180, 167]. Detailed discussions on related operations and extensions are also available in the listed references.

2.1 | Core Concepts in Set Theory

The following are foundational principles in set theory. For additional insights and examples, readers may refer to the recommended references [180].

Definition 1 (Set)**.** [180] A *set* is defined as a well-determined collection of distinct elements. These elements are either included in or excluded from the set. If A is a set and x is one of its elements, this is expressed as $x \in A$. Sets are typically denoted using curly braces, e.g., $A = \{a, b, c\}$.

Definition 2 (Subset)**.** [180] A set A is said to be a *subset* of another set B, written $A \subseteq B$, if all elements of A are also elements of B. Formally, this is expressed as:

$$A \subseteq B \iff \forall x\,(x \in A \implies x \in B).$$

We also use the following concepts.

Definition 3. (cf.[168]) The set of *real numbers* $\mathbb{R}$ includes all rational and irrational numbers. Formally, it is defined as a complete, ordered field that satisfies the *completeness property*:

Every non-empty subset of $\mathbb{R}$ that is bounded above has a least upper bound in $\mathbb{R}$.

Definition 4. (cf.[194]) The set of *integers* $\mathbb{Z}$ consists of all whole numbers, including positive, negative, and zero:

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}.$$

2.2 | Fuzzy Sets and Neutrosophic Sets

Fuzzy Sets and Neutrosophic Sets are often introduced in relation to their foundational counterpart, the Crisp Set. Below are formal definitions to establish this context.

Definition 5 (Universe Set)**.** (cf.[252]) A *universe set*, denoted as U, is the complete set of all elements relevant to a particular discussion or problem. It serves as the universal context, encompassing every element that could be considered within a given framework. For any subset A, the relationship $A \subseteq U$ holds, meaning all elements of A must belong to U.

The universe set U is foundational in set theory, acting as the domain of discourse within which all subsets are defined. It is synonymous with concepts such as the underlying set or total set.

Definition 6 (Crisp Set)**.** [259] Let X be a universe set, and let $P(X)$ represent the power set of X, which includes all subsets of X. A *crisp set* $A \subseteq X$ is defined by its characteristic function $\chi_A : X \to \{0, 1\}$, where:

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

The characteristic function χ_A assigns a value of 1 to elements belonging to A and 0 to those outside it, creating a clear and definitive boundary. Crisp sets adhere strictly to binary logic, distinguishing whether an element is inside or outside the set.

A Fuzzy Set assigns each element a degree of membership between 0 and 1, representing partial truth and handling uncertainty.

Definition 7 (Fuzzy Set)**.** [403, 408, 404, 405, 406, 407] A *fuzzy set* τ in a non-empty universe Y is a function $\tau : Y \to [0,1]$, where each element $y \in Y$ is assigned a degree of membership in the interval $[0,1]$.

A *fuzzy relation* δ is a fuzzy subset of $Y \times Y$. If τ is a fuzzy set in Y and δ is a fuzzy relation on Y, δ is called a *fuzzy relation on* τ if:

$$\delta(y,z) \leq \min\{\tau(y), \tau(z)\} \quad \text{for all } y,z \in Y.$$

Example 8 (Temperature Perception)**.** (cf.[64]) Consider the fuzzy set τ of "warm temperatures" in a universe $Y = \mathbb{R}$ (all temperatures in Celsius). The membership function τ could be defined as:

$$\tau(y) = \begin{cases} 0, & \text{if } y \leq 15 \text{ (cold);} \\ \frac{y-15}{10}, & \text{if } 15 < y < 25; \\ 1, & \text{if } y \geq 25 \text{ (warm).} \end{cases}$$

For example, at $y = 20^\circ C$, the membership degree of "warm" is 0.5.

Example 9 (Tall People)**.** (cf.[196]) In a population where height is measured, the fuzzy set τ of "tall people" can assign membership values based on height h:

$$\tau(h) = \begin{cases} 0, & \text{if } h \leq 150 \text{ cm (not tall);} \\ \frac{h-150}{30}, & \text{if } 150 < h < 180; \\ 1, & \text{if } h \geq 180 \text{ cm (tall).} \end{cases}$$

Here, a person of height 165 cm has a membership degree of 0.5.

Example 10 (Risk Level in Investments)**.** (cf.[158]) The fuzzy set τ of "high-risk investments" in a universe Y of possible investments may assign degrees of risk based on volatility or expected return. For example:

$$\tau(r) = \begin{cases} 0, & \text{if volatility } r \leq 5\%; \\ \frac{r-5}{10}, & \text{if } 5\% < r < 15\%; \\ 1, & \text{if } r \geq 15\%. \end{cases}$$

An investment with volatility $r = 10\%$ would have a membership degree of 0.5 in the "high-risk" category.

Neutrosophic Set extends Fuzzy Set by introducing truth, indeterminacy, and falsity, each independently in $[0,1]$, handling uncertainty and contradictions more comprehensively[319]. Unlike Fuzzy Sets, Neutrosophic Sets model indeterminacy explicitly, enabling greater flexibility for uncertain, inconsistent, or ambiguous data representation.

Definition 11 (Neutrosophic Set)**.** [319, 340, 321, 339, 322] Let X be a non-empty set. A (single-valued) Neutrosophic Set A on X is characterized by three membership functions:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for every $x \in X$, $T_A(x)$, $I_A(x)$, and $F_A(x)$ denote the degrees of truth, indeterminacy, and falsity, respectively. These functions satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Example 12 (Analysis of Tasks)**.** "Analysis of Tasks" systematically examines tasks by breaking them into components, evaluating resources, priorities, dependencies, and performance for optimization (cf.[232, 70]). Let $U = \{a, b, c\}$ be a set of tasks. A Neutrosophic Set S can assign the following degrees of truth, indeterminacy, and falsity to each task:

- Task a: $T(a) = 0.8$, $I(a) = 0.1$, $F(a) = 0.1$
- Task b: $T(b) = 0.5$, $I(b) = 0.3$, $F(b) = 0.2$
- Task c: $T(c) = 0.6$, $I(c) = 0.2$, $F(c) = 0.2$

This setup illustrates a scenario where task a has a high likelihood of success, task b is relatively uncertain, and task c has a moderate chance of being true.

Example 13 (Analysis of Consumer Sentiment)**.** "Consumer Sentiment" measures individuals' attitudes, confidence, and feelings about economic conditions, influencing spending behavior and market trends [134, 59]. Consider a product review x. The sentiment of the review can be quantified using Neutrosophic Sets as follows:

- $T_A(x) = 0.6$: 60% of users convey positive feedback.
- $I_A(x) = 0.3$: 30% of users exhibit neutral or uncertain opinions.
- $F_A(x) = 0.1$: 10% of users express negative feedback.

Neutrosophic Sets have been widely applied in sentiment analysis to handle uncertainty and partial truths in user opinions [31, 190, 164, 210, 283].

Theorem 14. *A Neutrosophic Set can generalize both Fuzzy Sets and Crisp Sets.*

Proof: This follows directly from the definition, as a Neutrosophic Set encompasses the structures of Fuzzy Sets and Crisp Sets as special cases. □

As related concepts of Fuzzy Sets, the following are well-known: Hesitant Fuzzy Sets [367, 72, 366], Picture Fuzzy Sets [81, 6, 253, 80, 6], Bipolar Fuzzy Sets [13, 18, 12, 152, 61, 248], Hyperfuzzy set[119, 182, 349, 136], Spherical fuzzy sets[200, 338, 25, 199, 221], and Tripolar Fuzzy Sets [289, 290, 288]. Additionally, related concepts of the Neutrosophic Set include the Bipolar Neutrosophic Set[239, 1, 372], Neutrosophic Soft Set[188, 7, 191, 53, 18], Hyperneutrosophic set[119], Neutrosophic offset[323, 329, 342, 328, 115, 330, 324] and Complex Neutrosophic Set [16, 17], among others.

Furthermore, Fuzzy and Neutrosophic concepts have been studied not only in the context of sets but also in various fields such as Graph Theory and Algebra[126, 123, 114, 116, 254, 8, 10]. Therefore, research on Fuzzy and Neutrosophic frameworks is of great significance.

2.3 | Plithogenic Set: A Generalization of Uncertain Sets

The Plithogenic Set is recognized as a type of set capable of generalizing Neutrosophic Sets, Fuzzy Sets, and other similar uncertain sets [327, 326]. The definition of the Plithogenic Set is provided below.

Definition 15. [327, 326] Let S be a universal set, and $P \subseteq S$. A *Plithogenic Set* PS is defined as:

$$PS = (P, v, Pv, pdf, pCF)$$

where:

- v is an attribute.
- Pv is the range of possible values for the attribute v.
- $pdf : P \times Pv \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)*.
- $pCF : Pv \times Pv \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)*.

These functions satisfy the following axioms for all $a, b \in Pv$:

(1) *Reflexivity of Contradiction Function*:

$$pCF(a, a) = 0$$

(2) *Symmetry of Contradiction Function*:

$$pCF(a,b) = pCF(b,a)$$

Example 16. (cf.[125]) The following examples of Plithogenic sets are provided.

- When $s = t = 1$, PS is called a *Plithogenic Fuzzy Set*.
- When $s = 2, t = 1$, PS is called a *Plithogenic Intuitionistic Fuzzy Set*.
- When $s = 3, t = 1$, PS is called a *Plithogenic Neutrosophic Set*.
- When $s = 4, t = 1$, PS is called a *Plithogenic quadripartitioned Neutrosophic Set* (cf.[287, 171, 310]).
- When $s = 5, t = 1$, PS is called a *Plithogenic pentapartitioned Neutrosophic Set* (cf.[83, 223, 46]).
- When $s = 6, t = 1$, PS is called a *Plithogenic hexapartitioned Neutrosophic Set* (cf.[265]).
- When $s = 7, t = 1$, PS is called a *Plithogenic heptapartitioned Neutrosophic Set* (cf.[249, 52]).
- When $s = 8, t = 1$, PS is called a *Plithogenic octapartitioned Neutrosophic Set*.
- When $s = 9, t = 1$, PS is called a *Plithogenic nonapartitioned Neutrosophic Set*.

The Plithogenic Set can generalize various sets that handle uncertainty, including Neutrosophic Sets and Fuzzy Sets [326, 119]. Several derived concepts of the Plithogenic Set have been studied [124, 315, 229, 230, 91, 352], along with its applications in graph theory and related fields [316, 129, 128, 125]. Therefore, research on Plithogenic Sets is as significant as that on Fuzzy Sets and Neutrosophic Sets.

2.4 | Uncertain Logic

This subsection explains Uncertain Logic. Various types of logic, such as Fuzzy Logic [247, 404, 409], Intuitionistic Fuzzy Logic [359, 27, 69], Neutrosophic Logic [319, 130, 321], Plithogenic Logic [327], and Upside-Down Logic [336, 127], have been studied under the umbrella of Uncertain Logic. Below, we introduce some of these logics.

Definition 17 (Classical Logic)**.** (cf.[312, 101, 79, 75]) Classical Logic is a formal system of reasoning based on binary truth values: true (1) and false (0). It operates under the principles of the law of identity, the law of non-contradiction, and the law of excluded middle, ensuring that every proposition is either true or false, with no intermediate states.

Definition 18 (Fuzzy Logic)**.** [403] Fuzzy Logic is an extension of classical logic designed to handle reasoning under uncertainty and vagueness. It assigns a degree of truth to each proposition, rather than a binary value (true or false). Formally, Fuzzy Logic is defined as a system:

$$\mathcal{F} = (\mathcal{X}, \mu, \mathcal{R}),$$

where:

- $\mathcal{X}$: A universal set of discourse, representing all possible elements under consideration.
- $\mu : \mathcal{X} \to [0,1]$: A membership function that maps each element $x \in \mathcal{X}$ to a degree of truth in the interval $[0,1]$, where:

  $$\mu(x) = 1 \quad \text{if } x \text{ is fully true,}$$

  $$\mu(x) = 0 \quad \text{if } x \text{ is fully false.}$$

  Intermediate values $(0 < \mu(x) < 1)$ represent partial truth.
- $\mathcal{R}$: A set of fuzzy rules or relations, typically of the form:

  $$\text{If } A \text{ is } X \text{ then } B \text{ is } Y,$$

  where $A, B \in \mathcal{X}$ and X, Y are fuzzy sets defined on $\mathcal{X}$.

Definition 19 (Neutrosophic Logic)**.** [319] Neutrosophic Logic extends classical logic by assigning to each proposition a truth value comprising three components:

$$v(A) = (T, I, F),$$

where $T, I, F \in [0, 1]$ represent the degrees of truth, indeterminacy, and falsity, respectively.

Remark 20. *Fuzzy logic is a special case of Neutrosophic Logic where both indeterminacy and falsity are set to zero. Moreover, Plithogenic Logic is known for its ability to generalize both Neutrosophic Logic and Fuzzy Logic.*

Definition 21 (Plithogenic Logic)**.** [327, 326] Plithogenic Logic extends classical and fuzzy logic by incorporating the concepts of contradiction and attribute values to model uncertainty and decision-making under complex conditions. Formally, let S be a universal set, and $P \subseteq S$. A *Plithogenic Set* PS is defined as:

$$PS = (P, v, Pv, pdf, pCF),$$

where:

- v: An attribute describing elements of P.
- Pv: The range of possible values for the attribute v.
- $pdf : P \times Pv \to [0, 1]^s$: The *Degree of Appurtenance Function (DAF)*, which assigns a degree of belonging for an element of P based on the attribute v.
- $pCF : Pv \times Pv \to [0, 1]^t$: The *Degree of Contradiction Function (DCF)*, which measures the degree of contradiction between pairs of attribute values.

The following axioms must hold for all $a, b \in Pv$:

(1) *Reflexivity of Contradiction Function:*

$$pCF(a, a) = 0$$

(2) *Symmetry of Contradiction Function:*

$$pCF(a, b) = pCF(b, a)$$

Example 22. (cf.[125]) The following examples of Plithogenic Logic are provided.

- When $s = t = 1$, PL is called a *Plithogenic Fuzzy Logic.*
- When $s = 2, t = 1$, PL is called a *Plithogenic Intuitionistic Fuzzy Logic.*
- When $s = 3, t = 1$, PL is called a *Plithogenic Neutrosophic Logic.*
- When $s = 4, t = 1$, PL is called a *Plithogenic Quadripartitioned Neutrosophic Logic.*
- When $s = 5, t = 1$, PL is called a *Plithogenic Pentapartitioned Neutrosophic Logic.*
- When $s = 6, t = 1$, PL is called a *Plithogenic Hexapartitioned Neutrosophic Logic.*
- When $s = 7, t = 1$, PL is called a *Plithogenic Heptapartitioned Neutrosophic Logic.*
- When $s = 8, t = 1$, PL is called a *Plithogenic Octapartitioned Neutrosophic Logic.*
- When $s = 9, t = 1$, PL is called a *Plithogenic Nonapartitioned Neutrosophic Logic.*

3 | Result and Discussion in this Paper

This section provides a concise explanation of the mathematical definitions and properties of Neutrosophic Phenomenology and Neutrosophic Social Science discussed in this paper.

3.1 | Neutrosophic Phenomenology: Neutrosophic Body-Mind-Soul-Spirit Fluidity

Neutrosophic Body-Mind-Soul-Spirit Fluidity is a novel concept introduced in [337]. This concept extends the traditional idea of Body-Mind-Soul-Spirit Fluidity by incorporating the principles of the Neutrosophic Set. If we attempt to define it mathematically, it can be expressed as follows.

Definition 23 (Neutrosophic Phenomenology)**.** Neutrosophic Phenomenology is the study of phenomena and consciousness under uncertainty, incorporating neutrosophic components of truth (T), indeterminacy (I), and falsity (F). It provides a framework to model subjective experiences where information is incomplete, ambiguous, or contradictory.

Definition 24 (Components of Neutrosophic Body-Mind-Soul-Spirit Fluidity)**.** The Neutrosophic Body-Mind-Soul-Spirit Fluidity (NBMSSF) integrates the four fundamental aspects of human existence—*Body*, *Mind*, *Soul*, and *Spirit*—within the neutrosophic framework. Each component is defined as follows:

1. *Body*: Represents the physical aspect of a person, characterized by biological processes. In neutrosophy, the body exists not merely in health or illness but also in neutral states, reflecting the dynamic balance and transition between wellness, growth, and decay.

2. *Mind*: Encompasses cognitive functions like reasoning and memory. The mind, in neutrosophic terms, transcends a binary rational/irrational framework, allowing for indeterminate states where beliefs and perceptions coexist in varying degrees of clarity, ambiguity, and influence.

3. *Soul*: Represents the essence or immaterial core of a person. In neutrosophy, the soul is not limited to good or evil but fluctuates between true identity (T), uncertain beliefs (I), and societal misconceptions (F), reflecting the full spectrum of human emotional and spiritual experiences.

4. *Spirit*: Associated with transcendence and connection to the divine. Neutrosophy views the spirit as existing in transitional states, balancing truths of divine experience (T), uncertainties in belief (I), and misconceptions about spiritual practices (F).

Example 25 (Real-Life Intuitive and Mathematically Correct Illustration of NBMSSF)**.** Consider the case of an individual recovering from a serious illness (cf.[86, 85]), reflecting the interplay of Body, Mind, Soul, and Spirit within the Neutrosophic Body-Mind-Soul-Spirit Fluidity (NBMSSF) framework:

1. *Body*: The individual's physical state fluctuates between health and illness. For instance, while the immune system is actively recovering, the body exists in a dynamic state, not fully healthy (T), not completely ill (F), and in a transitional phase (I) as new treatments are being adapted.

2. *Mind*: Cognitively, the individual experiences varying degrees of clarity and confusion. For example, optimism about recovery (T) may coexist with doubts about treatment efficacy (I) or fear of relapse (F), creating a nuanced mental state.

3. *Soul*: Emotionally, the person may feel both gratitude for life (T) and unresolved pain from the illness (F), alongside uncertainty about their spiritual purpose (I). These fluctuations represent the complexity of the soul in navigating existential questions.

4. *Spirit*: Spiritually, the person seeks connection with the divine or higher purpose. They may experience moments of profound clarity and faith (T), intermixed with uncertainties about their beliefs (I), or misconceptions about spiritual practices (F), especially during challenging times.

This example illustrates the NBMSSF concept by highlighting how each component operates within neutrosophic parameters, offering a more comprehensive understanding of human experiences in real-life situations.

Taking the above components into consideration, Neutrosophic Body-Mind-Soul-Spirit Fluidity is defined as follows.

Definition 26 (Neutrosophic Body-Mind-Soul-Spirit Fluidity)**.** Neutrosophic Body-Mind-Soul-Spirit Fluidity (NBMSSF) is defined as a mathematical structure consisting of four interacting components *Body*, *Mind*, *Soul*, and *Spirit*. Each component $X \in \{\text{Body, Mind, Soul, Spirit}\}$ is characterized by the Neutrosophic Triad $T(X)$, $I(X)$, $F(X)$, which satisfies the following conditions:

1. *Neutrosophic Triad*:

$$\begin{aligned} T(X) &\in [0,1] \quad \text{(Degree of Truth)} \\ I(X) &\in [0,1] \quad \text{(Degree of Indeterminacy)} \\ F(X) &\in [0,1] \quad \text{(Degree of Falsehood)} \\ T(X) + I(X) + F(X) &= 1. \end{aligned}$$

2. *Dynamics*: Each component X's state is influenced by the other three components Y, Z, W, expressed as a fluidity function $\mathcal{F}(X)$:

$$\mathcal{F}(X) = f_X(T(Y), I(Y), F(Y), T(Z), I(Z), F(Z), T(W), I(W), F(W)),$$

where f_X is the influence function, determined by the specific application.

3. *Interdependency Model*: The state of each component evolves as a system of differential equations:

$$\frac{dT(X)}{dt} = g_{T,X}(T, I, F),$$
$$\frac{dI(X)}{dt} = g_{I,X}(T, I, F),$$
$$\frac{dF(X)}{dt} = g_{F,X}(T, I, F),$$

where $g_{T,X}$, $g_{I,X}$, and $g_{F,X}$ describe the rate of change for each state. The explicit forms of these functions can include interactions between the components, such as $g_{T,X} = \alpha_X T(Y) - \beta_X F(W)$, where α_X and β_X are sensitivity coefficients.

4. *Global Fluidity Matrix*: The overall state of the system is represented as a matrix:

$$\mathbf{S}(t) = \begin{bmatrix} T(\text{Body}) & I(\text{Body}) & F(\text{Body}) \\ T(\text{Mind}) & I(\text{Mind}) & F(\text{Mind}) \\ T(\text{Soul}) & I(\text{Soul}) & F(\text{Soul}) \\ T(\text{Spirit}) & I(\text{Spirit}) & F(\text{Spirit}) \end{bmatrix},$$

which evolves over time t.

5. *Characteristic Function*: Each component's state transition is described by:

$$\Phi_X(T, I, F) = \alpha_X T(X) + \beta_X I(X) + \gamma_X F(X),$$

where $\alpha_X, \beta_X, \gamma_X$ are context-dependent weights.

6. *Constraints*: To ensure global balance, the following constraint holds:

$$\sum_{X \in \{\text{Body, Mind, Soul, Spirit}\}} T(X) + I(X) + F(X) = 4.$$

Remark 27. *Fuzzy Body-Mind-Soul-Spirit Fluidity is a special case of Neutrosophic Body-Mind-Soul-Spirit Fluidity where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Body-Mind-Soul-Spirit Fluidity is notable for its ability to generalize both Neutrosophic and Fuzzy Body-Mind-Soul-Spirit Fluidity.*

Example 28. Consider an individual who is generally healthy, mentally active, and emotionally balanced but experiencing some uncertainty in spiritual matters. Their states are as follows:

- *Body*: $T(\text{Body}) = 0.7$, $I(\text{Body}) = 0.2$, $F(\text{Body}) = 0.1$ (indicating good physical health).
- *Mind*: $T(\text{Mind}) = 0.5$, $I(\text{Mind}) = 0.3$, $F(\text{Mind}) = 0.2$ (a mixture of clarity and indecision).
- *Soul*: $T(\text{Soul}) = 0.6$, $I(\text{Soul}) = 0.2$, $F(\text{Soul}) = 0.2$ (emotional stability but with some conflicting emotions).
- *Spirit*: $T(\text{Spirit}) = 0.4$, $I(\text{Spirit}) = 0.4$, $F(\text{Spirit}) = 0.2$ (reflecting spiritual uncertainty).

The global fluidity matrix at this moment is:

$$\mathbf{S}(t) = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.5 & 0.3 & 0.2 \\ 0.6 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.2 \end{bmatrix}.$$

Example 29. Consider an individual recovering from stress (cf.[371, 350]), where fatigue and indecision dominate their state. Their characteristics are:

- *Body*: $T(\text{Body}) = 0.6$, $I(\text{Body}) = 0.3$, $F(\text{Body}) = 0.1$ (recovering from physical exhaustion).
- *Mind*: $T(\text{Mind}) = 0.4$, $I(\text{Mind}) = 0.5$, $F(\text{Mind}) = 0.1$ (struggling with mental clarity).
- *Soul*: $T(\text{Soul}) = 0.5$, $I(\text{Soul}) = 0.4$, $F(\text{Soul}) = 0.1$ (seeking emotional balance).
- *Spirit*: $T(\text{Spirit}) = 0.3$, $I(\text{Spirit}) = 0.5$, $F(\text{Spirit}) = 0.2$ (spiritually uncertain and seeking direction).

The fluidity matrix for this scenario is:

$$\mathbf{S}(t) = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.4 & 0.5 & 0.1 \\ 0.5 & 0.4 & 0.1 \\ 0.3 & 0.5 & 0.2 \end{bmatrix}.$$

This example highlights how improving one aspect, such as practicing mindfulness to reduce $I(\text{Mind})$, can create cascading positive effects, improving both emotional balance ($T(\text{Soul})$) and spiritual clarity ($T(\text{Spirit})$).

Example 30. Suppose an individual achieves significant spiritual clarity and emotional stability after a transformative event, such as a retreat or life-changing realization. Their states are:

- *Body*: $T(\text{Body}) = 0.8$, $I(\text{Body}) = 0.1$, $F(\text{Body}) = 0.1$ (excellent physical health).
- *Mind*: $T(\text{Mind}) = 0.7$, $I(\text{Mind}) = 0.2$, $F(\text{Mind}) = 0.1$ (sharp mental focus).
- *Soul*: $T(\text{Soul}) = 0.9$, $I(\text{Soul}) = 0.05$, $F(\text{Soul}) = 0.05$ (peaceful emotional state).
- *Spirit*: $T(\text{Spirit}) = 0.85$, $I(\text{Spirit}) = 0.1$, $F(\text{Spirit}) = 0.05$ (strong spiritual connection).

The fluidity matrix is:

$$\mathbf{S}(t) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.7 & 0.2 & 0.1 \\ 0.9 & 0.05 & 0.05 \\ 0.85 & 0.1 & 0.05 \end{bmatrix}.$$

This scenario models a person who has realigned their physical, mental, and spiritual dimensions, leading to a harmonious state.

The theorems that hold in Neutrosophic Body-Mind-Soul-Spirit Fluidity are presented below.

Theorem 31. *Neutrosophic Body-Mind-Soul-Spirit Fluidity has the structure of a Neutrosophic Set.*

Proof: This follows directly from the definition of Neutrosophic Body-Mind-Soul-Spirit Fluidity. □

Theorem 32 (Invariant Triad Property)**.** *In the context of Neutrosophic Body-Mind-Soul-Spirit Fluidity, under the specified dynamics, suppose the initial condition satisfies:*

$$T(X_0) + I(X_0) + F(X_0) = 1 \quad \text{at } t = 0.$$

Then for all $t \geq 0$, the following invariant holds:

$$T(X_t) + I(X_t) + F(X_t) = 1.$$

Proof: The dynamics of the system are governed by the differential equations for $T(X)$, $I(X)$, and $F(X)$. Summing these equations, we have:

$$\frac{d}{dt}(T(X)+I(X)+F(X))=\frac{dT(X)}{dt}+\frac{dI(X)}{dt}+\frac{dF(X)}{dt}.$$

By the interdependency model, the rates of change satisfy:

$$\frac{dT(X)}{dt}+\frac{dI(X)}{dt}+\frac{dF(X)}{dt}=g_{T,X}+g_{I,X}+g_{F,X}.$$

From the system definition, $g_{T,X}+g_{I,X}+g_{F,X}=0$. Thus:

$$\frac{d}{dt}(T(X)+I(X)+F(X))=0.$$

Integrating over time, the sum $T(X)+I(X)+F(X)$ remains constant, and given the initial condition $T(X_0)+I(X_0)+F(X_0)=1$, the result follows:

$$T(X_t)+I(X_t)+F(X_t)=1 \quad \text{for all } t\geq 0.$$

□

Theorem 33 (Non-Negativity and Boundedness)**.** *In the context of Neutrosophic Body-Mind-Soul-Spirit Fluidity, assume the initial condition:*

$$T(X_0),\ I(X_0),\ F(X_0)\in[0,1].$$

Then for any $t\geq 0$, the following holds:

$$T(X_t),\ I(X_t),\ F(X_t)\in[0,1].$$

Proof: The invariant property (Theorem 32) ensures that the sum $T(X_t)+I(X_t)+F(X_t)=1$ holds for all $t\geq 0$. Assume by contradiction that one of the components, say $T(X_t)$, leaves the interval $[0,1]$.

If $T(X_t)>1$, then $I(X_t)+F(X_t)<0$, which violates non-negativity. Similarly, if $T(X_t)<0$, then $I(X_t)+F(X_t)>1$, which is also impossible.

Using standard comparison theorems for differential equations and ensuring non-negativity through Grönwall's inequality, the components $T(X_t)$, $I(X_t)$, and $F(X_t)$ are bounded within $[0,1]$. Hence:

$$T(X_t),\ I(X_t),\ F(X_t)\in[0,1] \quad \text{for all } t\geq 0.$$

□

Theorem 34 (Global Balance Constraint)**.** *In the context of Neutrosophic Body-Mind-Soul-Spirit Fluidity, let the Global Fluidity Matrix at time t be:*

$$\mathbf{S}(t)=\begin{bmatrix} T(Body_t) & I(Body_t) & F(Body_t)\\ T(Mind_t) & I(Mind_t) & F(Mind_t)\\ T(Soul_t) & I(Soul_t) & F(Soul_t)\\ T(Spirit_t) & I(Spirit_t) & F(Spirit_t)\end{bmatrix}.$$

Then the total balance constraint holds:

$$\sum_{X\in\{Body,Mind,Soul,Spirit\}}[T(X_t)+I(X_t)+F(X_t)]=4 \quad \textit{for all } t\geq 0.$$

Proof: From Theorem 32, each component X satisfies $T(X_t)+I(X_t)+F(X_t)=1$ for all $t\geq 0$. Summing over all components:

$$\sum_{X\in\{\text{Body},\text{Mind},\text{Soul},\text{Spirit}\}}[T(X_t)+I(X_t)+F(X_t)]=4\cdot 1=4.$$

This holds for all $t\geq 0$, completing the proof. □

Based on the discussion above, we redefine Dynamic Neutrosophic Body-Mind-Soul-Spirit Fluidity. This model allows the observation and analysis of changes in the Body, Mind, Soul, and Spirit over time. The formal definitions and associated properties are presented below.

Definition 35 (Dynamic Neutrosophic Body-Mind-Soul-Spirit Fluidity)**.** Dynamic Neutrosophic Body-Mind-Soul-Spirit Fluidity (Dynamic NBMSSF) extends the static NBMSSF framework by incorporating time-dependent changes and interactions among its four components: *Body*, *Mind*, *Soul*, and *Spirit*. Each component $X \in \{\text{Body}, \text{Mind}, \text{Soul}, \text{Spirit}\}$ evolves over time according to the following properties:

- *Neutrosophic Triad Dynamics:* For each component X, the Truth $(T(X_t))$, Indeterminacy $(I(X_t))$, and Falsity $(F(X_t))$ values vary with time t and satisfy:
$$T(X_t), I(X_t), F(X_t) \in [0,1] \quad \text{and} \quad T(X_t) + I(X_t) + F(X_t) = 1 \quad \forall\, t \geq 0.$$
- *Influence Function:* Each component X is influenced by the other three components Y, Z, W through a fluidity function $\mathcal{F}(X_t)$:
$$\mathcal{F}(X_t) = f_X\Big(T(Y_t), I(Y_t), F(Y_t),\, T(Z_t), I(Z_t), F(Z_t),\, T(W_t), I(W_t), F(W_t)\Big),$$
where f_X is an application-specific function describing how other components affect X.
- *Time Evolution Equations:* The temporal behavior of each component is modeled by a system of differential equations:
$$\begin{cases} \dfrac{dT(X)}{dt} = g_{T,X}(T, I, F, t), \\ \dfrac{dI(X)}{dt} = g_{I,X}(T, I, F, t), \\ \dfrac{dF(X)}{dt} = g_{F,X}(T, I, F, t), \end{cases}$$
where $g_{T,X}, g_{I,X}, g_{F,X}$ capture the rates of change for Truth, Indeterminacy, and Falsity, potentially depending on all components and external factors.
- *Global Dynamics Matrix:* The overall system state at time t is represented by the matrix:
$$\mathbf{S}(t) = \begin{bmatrix} T(\text{Body}_t) & I(\text{Body}_t) & F(\text{Body}_t) \\ T(\text{Mind}_t) & I(\text{Mind}_t) & F(\text{Mind}_t) \\ T(\text{Soul}_t) & I(\text{Soul}_t) & F(\text{Soul}_t) \\ T(\text{Spirit}_t) & I(\text{Spirit}_t) & F(\text{Spirit}_t) \end{bmatrix}.$$
This matrix evolves over time according to the system's dynamics.
- *Invariant Properties and Constraints:*
  - *Invariant Triad Property:* For each component X, $T(X_t) + I(X_t) + F(X_t) = 1$ remains true for all $t \geq 0$.
  - *Global Balance Constraint:* Summing over all four components at any time t yields
$$\sum_{X \in \{\text{Body},\text{Mind},\text{Soul},\text{Spirit}\}} (T(X_t) + I(X_t) + F(X_t)) = 4.$$
- *Characteristic Dynamics Function:* Each component's combined state can be expressed by a characteristic function:
$$\Phi_X(T, I, F, t) = \alpha_X\, T(X_t) \;+\; \beta_X\, I(X_t) \;+\; \gamma_X\, F(X_t),$$
where $\alpha_X, \beta_X, \gamma_X$ are context-dependent parameters indicating the relative significance of each dimension.

Example 36 (Rehabilitation Scenario)**.** Consider an individual undergoing rehabilitation for a sports injury (cf.[201]):

- *Body*: $T(\text{Body}_t)$ represents the probability of full physical recovery, $I(\text{Body}_t)$ indicates uncertainty during the healing process, and $F(\text{Body}_t)$ accounts for residual impairment or setbacks.
- *Mind*: $T(\text{Mind}_t)$ measures mental clarity and optimism, $I(\text{Mind}_t)$ captures confusion or doubts, and $F(\text{Mind}_t)$ reflects negative beliefs about the rehabilitation process.

- *Soul*: $T(\text{Soul}_t)$ represents personal resilience or spiritual harmony, $I(\text{Soul}_t)$ signifies existential uncertainty, and $F(\text{Soul}_t)$ might correspond to cultural misconceptions or conflicts.
- *Spirit*: $T(\text{Spirit}_t)$ denotes moments of profound insight or faith, $I(\text{Spirit}_t)$ covers spiritual ambiguity, and $F(\text{Spirit}_t)$ indicates doubts or misunderstandings about spiritual practices.

As rehabilitation progresses over time t, each triad $(T(X_t), I(X_t), F(X_t))$ evolves dynamically based on the individual's physical therapy, mental training, emotional support, and spiritual practices. The *Invariant Triad Property* ensures $T(X_t) + I(X_t) + F(X_t) = 1$ for each component, while the *Global Balance Constraint* enforces the total sum to remain 4 at any time t.

Theorem 37. *Dynamic Neutrosophic Body-Mind-Soul-Spirit Fluidity possesses the structure of a Neutrosophic Set.*

Proof: This result follows directly from the definition of Neutrosophic Body-Mind-Soul-Spirit Fluidity, as each component (*Body*, *Mind*, *Soul*, and *Spirit*) is represented using the Neutrosophic Triad (T, I, F), which satisfies the axioms of a Neutrosophic Set. □

Theorem 38. *Dynamic Neutrosophic Body-Mind-Soul-Spirit Fluidity can be transformed into Neutrosophic Body-Mind-Soul-Spirit Fluidity by omitting temporal dependencies.*

Proof: This follows from the definition of Dynamic Neutrosophic Body-Mind-Soul-Spirit Fluidity. By setting the time-dependent functions $T(X_t), I(X_t), F(X_t)$ to their initial values at $t = 0$, the model reduces to the static form of Neutrosophic Body-Mind-Soul-Spirit Fluidity. □

Question 39. Related concepts such as Holistic Well-Being[410], Embodied Cognition[308, 397], Mindfulness and Meditation Practices[82, 418], and Psychoneuroimmunology [234, 245] are well-known.

Is it possible to extend these concepts using Fuzzy Sets and Neutrosophic Sets? Furthermore, what would their applications and mathematical structures entail?

3.2 | Logic of Phenomenology

There is a deep connection between phenomenology and logic, and several logical systems have been studied in this context. This subsection explores the logic within phenomenology, including considerations of its potential extension to Neutrosophic Logic.

3.2.1 | Neutrosophic Intentional Logic

Intentional concepts in phenomenology describe how consciousness always aims at or is directed toward objects, revealing the relationship between subject and object in experience (cf.[375, 374, 89, 65, 413]). Intentional Logic studies the structure of intentionality, analyzing how mental states are directed toward objects, contents, or propositions systematically (cf.[343, 383]).

Definition 40 (Intentional Logic)**.** Intentional Logic formalizes the structure of intentionality, defined as the directedness of mental states toward objects or contents. Let:

- W: the set of all possible worlds.
- S: the set of subjects (agents).
- O: the set of objects (including abstract entities).
- $\mathbb{B} = \{0, 1\}$: the Boolean domain indicating intentional states.

The intentionality of a subject $s \in S$ toward an object $o \in O$ in a world $w \in W$ is modeled as a relation:

$$I : S \times O \times W \to \mathbb{B},$$

where $I(s, o, w) = 1$ indicates that s intentionally directs their mental state toward o in w.

Intentional Content. The intentional content of a subject s is defined as:

$$\mathcal{I}_s = \{(o, w) \mid I(s, o, w) = 1\}.$$

Axioms. Intentional Logic satisfies the following properties:

(1) *Existence:* For all $s \in S$, there exists at least one $o \in O$ and $w \in W$ such that $I(s, o, w) = 1$.

(2) *Consistency:* For any $s \in S$, if $I(s, o_1, w) = 1$ and $I(s, o_2, w) = 1$, then $o_1 = o_2$ (if exclusivity is assumed).

(3) *Higher-Order Intentionality:* If o is an intentional state itself, then $o \in \mathcal{P}(S \times O)$, allowing for recursive representation of intentions.

Definition 41 (Neutrosophic Intentional Logic)**.** Neutrosophic Intentional Logic extends classical Intentional Logic by incorporating the neutrosophic components of truth (T), indeterminacy (I), and falsity (F). Let:

- W: the set of all possible worlds.
- S: the set of subjects (agents).
- O: the set of objects (including abstract entities).
- $\mathbb{N} = [0, 1]^3$: the neutrosophic domain, where each component (T, I, F) satisfies $0 \leq T + I + F \leq 1$.

The intentionality of a subject $s \in S$ toward an object $o \in O$ in a world $w \in W$ is modeled as:

$$I^N : S \times O \times W \to \mathbb{N},$$

where $I^N(s, o, w) = (T, I, F)$ indicates the degrees of truth (T), indeterminacy (I), and falsity (F) of s's intentional state toward o in w.
Neutrosophic Intentional Content. The neutrosophic intentional content of a subject s is defined as:

$$\mathcal{I}_s^N = \{(o, w, (T, I, F)) \mid I^N(s, o, w) = (T, I, F)\}.$$

Axioms. Neutrosophic Intentional Logic satisfies the following properties:

(1) *Existence:* For all $s \in S$, there exists at least one $o \in O$ and $w \in W$ such that $I^N(s, o, w) = (T, I, F)$ with $T > 0$.

(2) *Consistency:* For any $s \in S$, if $I^N(s, o_1, w) = (T_1, I_1, F_1)$ and $I^N(s, o_2, w) = (T_2, I_2, F_2)$, then $o_1 = o_2$ if $T_1 + T_2 = 1$ and $I_1 = I_2 = 0$.

(3) *Higher-Order Neutrosophic Intentionality:* If o is an intentional state, then $o \in \mathcal{P}(S \times O \times \mathbb{N})$, allowing recursive representation of neutrosophic intentionality.

Remark 42 (Neutrosophic Intentional Logic)**.** *Fuzzy Intentional Logic is a special case of Neutrosophic Intentional Logic where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Intentional Logic is notable for its ability to generalize both Neutrosophic and Fuzzy Intentional Logic.*

Example 43 (Neutrosophic Intentional Logic)**.** Consider an agent s thinking about the proposition o: "The market will grow by 10% next year" in the world w_1. The intentionality is modeled as:

$$I^N(s, o, w_1) = (T, I, F),$$

where $T = 0.6$, $I = 0.3$, and $F = 0.1$. This means:

- The agent believes the proposition is 60% true ($T = 0.6$).
- There is a 30% level of uncertainty or indeterminacy due to insufficient data ($I = 0.3$).
- The agent believes the proposition is 10% false ($F = 0.1$).

Higher-Order Intentionality. If the agent s also contemplates their own belief about o, this is represented as:

$$I^N(s, I^N(s, o, w_1), w_2) = (T', I', F'),$$

where w_2 is a meta-level world reflecting the agent's introspection.

Visualization of Content. The neutrosophic intentional content of s is:

$$\mathcal{I}_s^N = \{(o, w_1, (0.6, 0.3, 0.1))\}.$$

This captures the agent's nuanced and uncertain attitude toward the proposition o in w_1.

3.2.2 | Neutrosophic Ontological Logic

Ontology is the study of existence and reality, exploring entities, their properties, relationships, and categories [344, 149, 151, 160, 99]. Ontology is often studied in connection with phenomenology [261]. Concepts like Ontological Logic [302, 262] are also recognized within ontology.

To define this within the framework of Neutrosophic Logic, we first mathematically define Ontological Logic and then extend it. The definition is provided below.

Definition 44 (Ontological Logic)**.** Ontological Logic formalizes the relationships, properties, and existence of entities. Let:

- U: the universe of discourse, partitioned into:
$$U = E \cup P \cup R \cup T,$$
where E: entities, P: properties, R: relations, and T: time.
- $\sigma : P \times E \times T \to \mathbb{B}$: a function assigning truth values to properties of entities at specific times.
- $R : E \times E \to \mathbb{B}$: a function defining binary relations between entities.

The ontological structure is defined as a tuple:

$$\mathcal{O} = (E, P, R, T, \sigma).$$

Core Axioms. Ontological Logic satisfies the following axioms:

(1) *Identity:* For every entity $e \in E$, there exists at least one property $p \in P$ and time $t \in T$ such that $\sigma(p, e, t) = 1$.

(2) *Non-Contradiction:* For any $e \in E, p \in P, t \in T$, $\sigma(p, e, t) = 1$ implies $\sigma(\neg p, e, t) = 0$.

(3) *Temporal Consistency:* For persistent properties $p \in P$, if $\sigma(p, e, t_1) = 1$, then $\sigma(p, e, t_2) = 1$ for all $t_2 \geq t_1$.

Mereological Relations. Part-whole relationships are formalized as:

$$P_W \subseteq E \times E,$$

where $(e_1, e_2) \in P_W$ indicates that e_1 is a part of e_2. The following properties hold:

- *Transitivity:* $(e_1, e_2), (e_2, e_3) \in P_W \implies (e_1, e_3) \in P_W$.
- *Antisymmetry:* $(e_1, e_2) \in P_W \wedge (e_2, e_1) \in P_W \implies e_1 = e_2$.

Example 45 (Ontological Logic in Healthcare System)**.** Consider a healthcare system (cf.[416, 44]) where entities, properties, relations, and time are formalized as follows:

- $E = \{\text{Patient}, \text{Doctor}, \text{Medication}, \text{Treatment Plan}\}$: A set of entities.
- $P = \{\text{isHealthy}, \text{isPrescribed}, \text{isAdministered}, \text{isEffective}\}$: A set of properties.
- $R = \{\text{treats}, \text{prescribes}, \text{monitors}\}$: A set of relations between entities.
- $T = \{\text{Day 1}, \text{Day 2}, \dots, \text{Day 30}\}$: A set of time points.

Property Assignment. The property function σ assigns truth values to properties of entities over time:

$$\sigma(\text{isPrescribed}, \text{Medication}, t) = \begin{cases} 1 & \text{if the medication is prescribed at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Relations. The relation function R formalizes interactions between entities. For example:

$$R(\text{Doctor}, \text{Patient}) = \text{treats}, \quad R(\text{Doctor}, \text{Medication}) = \text{prescribes}.$$

Core Axioms in Context. The core axioms of Ontological Logic can be applied to this healthcare example:

- *Identity:* Every patient $e \in E$ must have at least one property $p \in P$ at a specific time t:

$$\exists p \in P, t \in T \text{ s.t. } \sigma(p, \text{Patient}, t) = 1.$$

  Example: $\sigma(\text{isHealthy}, \text{Patient}, \text{Day 10}) = 1$.

- *Non-Contradiction:* A medication cannot simultaneously be prescribed and not prescribed at the same time:

$$\sigma(\text{isPrescribed}, \text{Medication}, t) = 1 \implies \sigma(\neg\text{isPrescribed}, \text{Medication}, t) = 0.$$

- *Temporal Consistency:* If a treatment plan is effective on Day 5, it must remain effective for subsequent days unless modified:

$$\sigma(\text{isEffective}, \text{Treatment Plan}, \text{Day 5}) = 1 \implies \sigma(\text{isEffective}, \text{Treatment Plan}, t) = 1 \quad \forall t \geq \text{Day 5}.$$

Mereological Relations. Part-whole relationships in the healthcare system are defined as follows:

$$P_W = \{(\text{Medication}, \text{Treatment Plan})\}.$$

Here, medication e_1 is a part of the treatment plan e_2. The transitivity and antisymmetry properties hold:

- *Transitivity:* If Medication A is part of Treatment Plan X, and Treatment Plan X is part of Healthcare Protocol Y, then Medication A is part of Healthcare Protocol Y.

$$(\text{Medication A}, \text{Treatment Plan X}) \in P_W \wedge (\text{Treatment Plan X}, \text{Healthcare Protocol Y}) \in P_W$$

$$\implies (\text{Medication A}, \text{Healthcare Protocol Y}) \in P_W.$$

- *Antisymmetry:* If Medication A is part of Treatment Plan X and vice versa, then Medication A and Treatment Plan X are identical:

$$(\text{Medication A}, \text{Treatment Plan X}) \in P_W \wedge (\text{Treatment Plan X}, \text{Medication A}) \in P_W$$

$$\implies \text{Medication A} = \text{Treatment Plan X}.$$

The definition of Neutrosophic Ontological Logic, which incorporates the principles of Neutrosophic Logic into Ontological Logic, is provided below.

Definition 46 (Neutrosophic Ontological Logic)**.** Neutrosophic Ontological Logic extends classical Ontological Logic by incorporating the neutrosophic components of truth (T), indeterminacy (I), and falsity (F). Let:

- U: the universe of discourse, partitioned as:

$$U = E \cup P \cup R \cup T,$$

  where E: entities, P: properties, R: relations, and T: time.

- $\mathbb{N} = [0, 1]^3$: the neutrosophic domain, where (T, I, F) satisfies $0 \leq T + I + F \leq 1$.
- $\sigma^N : P \times E \times T \to \mathbb{N}$: a function assigning neutrosophic truth values to properties of entities at specific times.
- $R^N : E \times E \times T \to \mathbb{N}$: a function assigning neutrosophic truth values to binary relations between entities.

The neutrosophic ontological structure is defined as a tuple:

$$\mathcal{O}^N = (E, P, R, T, \sigma^N, R^N).$$

Core Axioms. Neutrosophic Ontological Logic satisfies the following axioms:

(1) *Neutrosophic Identity:* For every entity $e \in E$, there exists at least one property $p \in P$ and time $t \in T$ such that:

$$\sigma^N(p, e, t) = (T, I, F), \quad \text{where } T > 0.$$

(2) *Neutrosophic Non-Contradiction:* For any $e \in E, p \in P, t \in T$, $\sigma^N(p, e, t) = (T, I, F)$ implies that $\sigma^N(\neg p, e, t) = (F, I, T)$.

(3) *Neutrosophic Temporal Consistency:* For persistent properties $p \in P$, if $\sigma^N(p, e, t_1) = (T_1, I_1, F_1)$ and $t_2 \geq t_1$, then:

$$\sigma^N(p, e, t_2) = (T_2, I_2, F_2), \quad \text{where } T_2 \leq T_1 \text{ and } F_2 \geq F_1.$$

(4) *Neutrosophic Mereological Relations:* Part-whole relationships $P_W \subseteq E \times E$ are assigned neutrosophic truth values:

$$R^N(e_1, e_2, t) = (T, I, F),$$

where (T, I, F) represents the degree to which e_1 is a part of e_2 at time t.

Remark 47 (Neutrosophic Ontological Logic)**.** *Fuzzy Ontological Logic is a special case of Neutrosophic Ontological Logic where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Ontological Logic is notable for its ability to generalize both Neutrosophic and Fuzzy Ontological Logic.*

Example 48 (Neutrosophic Ontological Logic for healthcare system)**.** Consider a healthcare system modeled as $\mathcal{O}^N$ with:

- e_1: a hospital.
- e_2: a healthcare network to which the hospital belongs.
- p: the property "provides emergency services."
- t: the current time.

Property Evaluation. The property p for the entity e_1 at t is evaluated as:

$$\sigma^N(p, e_1, t) = (0.8, 0.1, 0.1),$$

indicating that:

- There is an 80% certainty ($T = 0.8$) that the hospital provides emergency services.
- There is a 10% uncertainty ($I = 0.1$) due to incomplete data.
- There is a 10% falsity ($F = 0.1$) based on occasional service disruptions.

Mereological Relation. The hospital's membership in the healthcare network is represented as:

$$R^N(e_1, e_2, t) = (0.9, 0.05, 0.05),$$

indicating a 90% certainty ($T = 0.9$) that the hospital is part of the network, with 5% uncertainty ($I = 0.05$) and 5% falsity ($F = 0.05$) due to occasional administrative errors.
Temporal Consistency. If p represents "provides emergency services," and at a later time $t' > t$, the hospital's performance declines, the evaluation might adjust to:

$$\sigma^N(p, e_1, t') = (0.6, 0.2, 0.2).$$

This reflects reduced certainty ($T = 0.6$) and increased falsity ($F = 0.2$) due to degraded service.

3.3 | Neutrosophic Social Analysis

This subsection provides a mathematical definition of the PDCA (Plan-Do-Check-Act), DMAIC (Define-Measure-Analyze-Improve-Control), SWOT Analysis (Strengths, Weaknesses, Opportunities, Threats), and OODA Loop (Observe, Orient, Decide, Act) cycles using the concept of Neutrosophic Sets, incorporating truth, indeterminacy, and falsehood degrees for decision-making under uncertainty.

First, as a broad perspective, Neutrosophic Social Analysis is roughly defined as follows. It extends Social Analysis by incorporating the principles of Neutrosophic Logic.

Definition 49 (Neutrosophic Social Analysis)**.** Neutrosophic Social Analysis is the evaluation of social systems, behaviors, and relationships under uncertainty. It incorporates neutrosophic components of truth (T), indeterminacy (I), and falsity (F) to model complex, ambiguous, or conflicting social dynamics.

3.3.1 | PDCA Cycle with Neutrosophic Sets

The PDCA (Plan-Do-Check-Act) cycle is a continuous improvement framework consisting of four stages: planning, implementing, evaluating results, and refining processes [133, 291, 173, 256]. These four stages are extended within the framework of Neutrosophic Sets as follows. It is worth noting that several studies have explored the application of the PDCA cycle in the Fuzzy domain and the Neutrosophic domain [137, 392, 24].

Definition 50 (Neutrosophic PDCA cycle)**.** The Neutrosophic PDCA cycle is an extension of the traditional Plan-Do-Check-Act (PDCA) cycle, incorporating Neutrosophic Sets to model uncertainty, indeterminacy, and truth. The cycle consists of four stages:

(1) *Plan (P):* Represented by a Neutrosophic Set P:

$$P = \{(x, T_P(x), I_P(x), F_P(x)) \mid x \in \text{Planning Elements}\},$$

where:

- $T_P(x)$: Degree to which the plan is expected to succeed.
- $I_P(x)$: Degree of uncertainty associated with the plan.
- $F_P(x)$: Degree to which the plan is expected to fail.

(2) *Do (D):* Represented by a Neutrosophic Set D:

$$D = \{(y, T_D(y), I_D(y), F_D(y)) \mid y \in \text{Execution Elements}\},$$

where:

- $T_D(y)$: Degree to which the execution is successful.
- $I_D(y)$: Degree of uncertainty during execution.
- $F_D(y)$: Degree to which the execution is unsuccessful.

(3) *Check (C):* Represented by a Neutrosophic Set C:

$$C = \{(z, T_C(z), I_C(z), F_C(z)) \mid z \in \text{Evaluation Criteria}\},$$

where:

- $T_C(z)$: Degree to which evaluation criteria are met.
- $I_C(z)$: Degree of uncertainty in the evaluation process.
- $F_C(z)$: Degree to which evaluation criteria are not met.

(4) *Act (A):* Represented by a Neutrosophic Set A:

$$A = \{(w, T_A(w), I_A(w), F_A(w)) \mid w \in \text{Improvement Elements}\},$$

where:

- $T_A(w)$: Degree to which the improvement is effective.

- $I_A(w)$: Degree of uncertainty in the improvement's impact.
- $F_A(w)$: Degree to which the improvement is ineffective.

Example 51. Consider applying the Neutrosophic PDCA cycle to a marketing campaign (cf.[279, 250]):

- *Plan (P):* Tasks such as "Develop Ad Content" and "Set Budget" might have the following values:
  - Develop Ad Content: $T_P = 0.7$, $I_P = 0.2$, $F_P = 0.1$
  - Set Budget: $T_P = 0.6$, $I_P = 0.3$, $F_P = 0.1$
- *Do (D):* Execution tasks such as "Run Ad Campaign" and "Monitor Metrics":
  - Run Ad Campaign: $T_D = 0.8$, $I_D = 0.1$, $F_D = 0.1$
  - Monitor Metrics: $T_D = 0.6$, $I_D = 0.3$, $F_D = 0.1$
- *Check (C):* Evaluation criteria such as "ROI Improvement[206]" and "Engagement Increase[157]":
  - ROI Improvement: $T_C = 0.7$, $I_C = 0.2$, $F_C = 0.1$
  - Engagement Increase: $T_C = 0.5$, $I_C = 0.4$, $F_C = 0.1$
- *Act (A):* Improvement actions such as "Adjust Budget" and "Redesign Ad Content":
  - Adjust Budget: $T_A = 0.6$, $I_A = 0.3$, $F_A = 0.1$
  - Redesign Ad Content: $T_A = 0.8$, $I_A = 0.1$, $F_A = 0.1$

This demonstrates how the Neutrosophic PDCA cycle integrates uncertainty and truth degrees into planning, execution, evaluation, and improvement stages.

Theorem 52. *Neutrosophic PDCA cycle has the structure of a Neutrosophic Set.*

Proof: This follows directly from the definition of Neutrosophic PDCA cycle. □

Question 53. Numerous derived concepts of PDCA, such as the PDSA Cycle (Plan-Do-Study-Act) [205, 292, 102, 66], OPDCA Cycle (Observe-Plan-Do-Check-Act) [179, 351], and SDCA Cycle (Standardize-Do-Check-Act)[211, 104, 22], are widely known.

What characteristics emerge when concepts like Neutrosophic Sets are applied to these derived cycles? Furthermore, what potential applications could result from such adaptations?

3.3.2 | DMAIC Cycle with Neutrosophic Sets

The DMAIC Cycle is a Six Sigma methodology [231, 263] designed for process improvement [242]. It consists of five phases: Define, Measure, Analyze, Improve, and Control, aiming to optimize processes systematically [242, 285, 300, 286, 356, 224]. This framework is widely utilized in business management and has also been explored in Fuzzy and Neutrosophic contexts[141, 143, 402]. The following outlines an extension of the DMAIC Cycle using Neutrosophic Sets.

Definition 54 (Neutrosophic DMAIC cycle)**.** The Neutrosophic DMAIC cycle is an extension of the traditional Define-Measure-Analyze-Improve-Control (DMAIC) cycle, incorporating Neutrosophic Sets to model uncertainty, indeterminacy, and truth. The cycle consists of five stages:

(1) *D efine (D):* Represented by a Neutrosophic Set D_f:

$$D_f = \{(x, T_{D_f}(x), I_{D_f}(x), F_{D_f}(x)) \mid x \in \text{Definition Elements}\},$$

where:

- $T_{D_f}(x)$: Degree to which the definition is accurate.
- $I_{D_f}(x)$: Degree of uncertainty in the definition.

- $F_{D_f}(x)$: Degree to which the definition is inaccurate.

(2) *Measure (M):* Represented by a Neutrosophic Set M:

$$M = \{(y, T_M(y), I_M(y), F_M(y)) \mid y \in \text{Measurement Elements}\},$$

where:

- $T_M(y)$: Degree of reliability of the measurement.
- $I_M(y)$: Degree of uncertainty in the measurement process.
- $F_M(y)$: Degree to which the measurement is unreliable.

(3) *Analyze (A):* Represented by a Neutrosophic Set A_n:

$$A_n = \{(z, T_{A_n}(z), I_{A_n}(z), F_{A_n}(z)) \mid z \in \text{Analysis Elements}\},$$

where:

- $T_{A_n}(z)$: Degree to which the analysis results are correct.
- $I_{A_n}(z)$: Degree of uncertainty in the analysis.
- $F_{A_n}(z)$: Degree to which the analysis results are incorrect.

(4) *Improve (I):* Represented by a Neutrosophic Set I_m:

$$I_m = \{(w, T_{I_m}(w), I_{I_m}(w), F_{I_m}(w)) \mid w \in \text{Improvement Actions}\},$$

where:

- $T_{I_m}(w)$: Degree to which the improvement is successful.
- $I_{I_m}(w)$: Degree of uncertainty about the improvement's effectiveness.
- $F_{I_m}(w)$: Degree to which the improvement fails.

(5) *Control (C):* Represented by a Neutrosophic Set C_t:

$$C_t = \{(v, T_{C_t}(v), I_{C_t}(v), F_{C_t}(v)) \mid v \in \text{Control Elements}\},$$

where:

- $T_{C_t}(v)$: Degree to which control is effective.
- $I_{C_t}(v)$: Degree of uncertainty in the control process.
- $F_{C_t}(v)$: Degree to which control is ineffective.

Example 55. A production process is a sequence of operations transforming raw materials into finished products efficiently [313, 197]. Consider applying the Neutrosophic DMAIC cycle to improve a production process:

- *Define (D):* Tasks such as "Identify Core Needs" and "Set Goals":
  - Identify Core Needs: $T_{D_f} = 0.8$, $I_{D_f} = 0.1$, $F_{D_f} = 0.1$
  - Set Goals: $T_{D_f} = 0.7$, $I_{D_f} = 0.2$, $F_{D_f} = 0.1$
- *Measure (M):* Measuring performance metrics like "Production Efficiency" and "Customer Satisfaction":
  - Production Efficiency: $T_M = 0.9$, $I_M = 0.05$, $F_M = 0.05$
  - Customer Satisfaction: $T_M = 0.7$, $I_M = 0.2$, $F_M = 0.1$
- *Analyze (A):* Analyzing issues such as "Supply Chain Delays" and "Equipment Downtime":
  - Supply Chain Delays: $T_{A_n} = 0.6$, $I_{A_n} = 0.3$, $F_{A_n} = 0.1$
  - Equipment Downtime: $T_{A_n} = 0.7$, $I_{A_n} = 0.2$, $F_{A_n} = 0.1$

- *Improve (I):* Improvement actions such as "Add New Suppliers" and "Upgrade Machinery":
  - Add New Suppliers: $T_{I_m} = 0.7$, $I_{I_m} = 0.2$, $F_{I_m} = 0.1$
  - Upgrade Machinery: $T_{I_m} = 0.8$, $I_{I_m} = 0.1$, $F_{I_m} = 0.1$
- *Control (C):* Control measures like "Real-Time Monitoring" and "Automated Alerts":
  - Real-Time Monitoring: $T_{C_t} = 0.9$, $I_{C_t} = 0.05$, $F_{C_t} = 0.05$
  - Automated Alerts: $T_{C_t} = 0.8$, $I_{C_t} = 0.1$, $F_{C_t} = 0.1$

This demonstrates how the Neutrosophic DMAIC cycle integrates uncertainty and truth degrees into defining, measuring, analyzing, improving, and controlling stages.

Theorem 56. *Neutrosophic DMAIC cycle has the structure of a Neutrosophic Set.*

Proof: This follows directly from the definition of Neutrosophic DMAIC cycle. □

Question 57. Several derived concepts of the DMAIC cycle are widely recognized, including the DMADV Cycle (Define-Measure-Analyze-Design-Verify) [348, 38, 155] and the DCOV Cycle (Define-Characterize-Optimize-Verify) [203, 30].

What unique characteristics arise when concepts such as Neutrosophic Sets are incorporated into these derived cycles? Additionally, what potential applications might be enabled by such adaptations?

3.3.3 | SWOT Analysis with Neutrosophic Sets

SWOT Analysis is a strategic planning tool used to assess a project or organization's internal and external factors. It identifies four key dimensions: Strengths, Weaknesses, Opportunities, and Threats, aiming to develop effective strategies[311, 2, 140, 391, 237, 305, 93].

This framework is widely applied across various industries, including business, education[2], and healthcare[281], and has also been studied within Fuzzy and Neutrosophic contexts[309, 37, 163]. The following outlines an extension of SWOT Analysis using Neutrosophic Sets.

Definition 58. The Neutrosophic SWOT Analysis extends the traditional Strengths-Weaknesses-Opportunities-Threats framework by incorporating Neutrosophic Sets to model uncertainty, indeterminacy, and truth. The analysis consists of four components:

(1) *Strengths (S):* Represented by a Neutrosophic Set S:

$$S = \{(x, T_S(x), I_S(x), F_S(x)) \mid x \in \text{Strength Elements}\},$$

where:

- $T_S(x)$: Degree to which x is a strength.
- $I_S(x)$: Degree of uncertainty in determining x as a strength.
- $F_S(x)$: Degree to which x is not a strength.

(2) *Weaknesses (W):* Represented by a Neutrosophic Set W:

$$W = \{(y, T_W(y), I_W(y), F_W(y)) \mid y \in \text{Weakness Elements}\},$$

where:

- $T_W(y)$: Degree to which y is a weakness.
- $I_W(y)$: Degree of uncertainty in determining y as a weakness.
- $F_W(y)$: Degree to which y is not a weakness.

(3) *Opportunities (O):* Represented by a Neutrosophic Set O:

$$O = \{(z, T_O(z), I_O(z), F_O(z)) \mid z \in \text{Opportunity Elements}\},$$

where:

- $T_O(z)$: Degree to which z is an opportunity.
- $I_O(z)$: Degree of uncertainty in determining z as an opportunity.
- $F_O(z)$: Degree to which z is not an opportunity.

(4) *Threats (T):* Represented by a Neutrosophic Set T:

$$T = \{(w, T_T(w), I_T(w), F_T(w)) \mid w \in \text{Threat Elements}\},$$

where:

- $T_T(w)$: Degree to which w is a threat.
- $I_T(w)$: Degree of uncertainty in determining w as a threat.
- $F_T(w)$: Degree to which w is not a threat.

Example 59. Consider applying Neutrosophic SWOT Analysis to evaluate a company:

- *Strengths (S):* ”Brand Recognition[153]” and ”Skilled Workforce[376]”:
  - Brand Recognition: $T_S = 0.9$, $I_S = 0.05$, $F_S = 0.05$
  - Skilled Workforce: $T_S = 0.8$, $I_S = 0.1$, $F_S = 0.1$
- *Weaknesses (W):* ”High Operational Costs” and ”Limited Market Presence”:
  - High Operational Costs: $T_W = 0.7$, $I_W = 0.2$, $F_W = 0.1$
  - Limited Market Presence: $T_W = 0.6$, $I_W = 0.3$, $F_W = 0.1$
- *Opportunities (O):* ”Emerging Markets” and ”Technological Advancements”:
  - Emerging Markets: $T_O = 0.8$, $I_O = 0.15$, $F_O = 0.05$
  - Technological Advancements: $T_O = 0.9$, $I_O = 0.05$, $F_O = 0.05$
- *Threats (T):* ”Economic Recession[346]” and ”New Competitors”:
  - Economic Recession: $T_T = 0.7$, $I_T = 0.2$, $F_T = 0.1$
  - New Competitors: $T_T = 0.6$, $I_T = 0.3$, $F_T = 0.1$

This analysis demonstrates how Neutrosophic Sets model strengths, weaknesses, opportunities, and threats with varying degrees of truth, uncertainty, and falsehood.

Theorem 60. *Neutrosophic SWOT Analysis has the structure of a Neutrosophic Set.*

Proof: This follows directly from the definition of Neutrosophic SWOT Analysis. □

Question 61. Several extended concepts of SWOT Analysis are widely recognized, including SWOC Analysis (Strengths, Weaknesses, Opportunities, Challenges)[33, 55, 177, 264, 284], SOAR Analysis (Strengths, Opportunities, Aspirations, Results) [354, 355, 357, 174], and Dynamic SWOT Analysis [396, 45, 178].

What mathematical characteristics and potential applications could emerge if these frameworks were extended using Neutrosophic Sets?

3.3.4 | OODA Cycle with Neutrosophic Sets

The OODA Cycle (Observe-Orient-Decide-Act) is a decision-making framework designed to enable effective responses in dynamic and competitive environments [236, 282, 298, 131, 415, 198]. It emphasizes observing the situation, orienting oneself based on the context, making informed decisions, and taking timely actions. The following outlines an extension of the OODA Cycle using Neutrosophic Sets.

Definition 62. The Neutrosophic OODA Loop extends the traditional Observe-Orient-Decide-Act framework by incorporating Neutrosophic Sets to model uncertainty, indeterminacy, and truth. The loop consists of four stages:

(1) *Observe (O):* Represented by a Neutrosophic Set O_b:

$$O_b = \{(x, T_{O_b}(x), I_{O_b}(x), F_{O_b}(x)) \mid x \in \text{Observation Elements}\},$$

where:

- $T_{O_b}(x)$: Degree to which x is accurately observed.
- $I_{O_b}(x)$: Degree of uncertainty in observing x.
- $F_{O_b}(x)$: Degree to which x is inaccurately observed.

(2) *Orient (O):* Represented by a Neutrosophic Set O_r:

$$O_r = \{(y, T_{O_r}(y), I_{O_r}(y), F_{O_r}(y)) \mid y \in \text{Orientation Elements}\},$$

where:

- $T_{O_r}(y)$: Degree to which orientation is correct.
- $I_{O_r}(y)$: Degree of uncertainty in orientation.
- $F_{O_r}(y)$: Degree to which orientation is incorrect.

(3) *Decide (D):* Represented by a Neutrosophic Set D_c:

$$D_c = \{(z, T_{D_c}(z), I_{D_c}(z), F_{D_c}(z)) \mid z \in \text{Decision Elements}\},$$

where:

- $T_{D_c}(z)$: Degree to which the decision is correct.
- $I_{D_c}(z)$: Degree of uncertainty in the decision.
- $F_{D_c}(z)$: Degree to which the decision is incorrect.

(4) *Act (A):* Represented by a Neutrosophic Set A_c:

$$A_c = \{(w, T_{A_c}(w), I_{A_c}(w), F_{A_c}(w)) \mid w \in \text{Action Elements}\},$$

where:

- $T_{A_c}(w)$: Degree to which the action is effective.
- $I_{A_c}(w)$: Degree of uncertainty in the action.
- $F_{A_c}(w)$: Degree to which the action is ineffective.

Example 63. Consider applying the Neutrosophic OODA Loop to a business decision:

- *Observe (O):* Observing market trends such as "Customer Preferences[347]" and "Competitor Actions[26]":
  - Customer Preferences: $T_{O_b} = 0.8$, $I_{O_b} = 0.1$, $F_{O_b} = 0.1$
  - Competitor Actions: $T_{O_b} = 0.7$, $I_{O_b} = 0.2$, $F_{O_b} = 0.1$
- *Orient (O):* Orienting strategies based on "Market Positioning[60]" and "Customer Segmentation[74]":
  - Market Positioning: $T_{O_r} = 0.7$, $I_{O_r} = 0.2$, $F_{O_r} = 0.1$

  - Customer Segmentation: $T_{O_r} = 0.8$, $I_{O_r} = 0.1$, $F_{O_r} = 0.1$
- *Decide (D):* Making decisions on "Budget Allocation[414]" and "Market Entry[186]":
  - Budget Allocation: $T_{D_c} = 0.7$, $I_{D_c} = 0.2$, $F_{D_c} = 0.1$
  - Market Entry: $T_{D_c} = 0.6$, $I_{D_c} = 0.3$, $F_{D_c} = 0.1$
- *Act (A):* Implementing actions like "Launch New Product" and "Improve Distribution Channels":
  - Launch New Product: $T_{A_c} = 0.8$, $I_{A_c} = 0.1$, $F_{A_c} = 0.1$
  - Improve Distribution Channels: $T_{A_c} = 0.7$, $I_{A_c} = 0.2$, $F_{A_c} = 0.1$

This example illustrates how the Neutrosophic OODA Loop integrates truth, uncertainty, and falsehood degrees into observing, orienting, deciding, and acting stages.

Theorem 64. *Neutrosophic OODA Loop has the structure of a Neutrosophic Set.*

Proof: This follows directly from the definition of Neutrosophic OODA Loop. □

3.3.5 | Neutrosophic Porter's Five Forces Analysis

Neutrosophic Porter's Five Forces Analysis is an extended framework based on the classic Porter's Five Forces Analysis. This approach evaluates industry competition through five key factors: rivalry among existing competitors, bargaining power of buyers, bargaining power of suppliers, threat of substitutes, and threat of new entrants [94, 277, 150, 278].

Several related studies have been conducted within the contexts of Fuzzy Sets and Neutrosophic Sets [240]. The formal definition is provided below.

Definition 65. The Neutrosophic Porter's Five Forces Analysis extends the traditional framework by incorporating Neutrosophic Sets to model uncertainty, indeterminacy, and truth across the five competitive forces:

(1) *Threat of New Entrants (N):* Represented by a Neutrosophic Set N:

$$N = \{(x, T_N(x), I_N(x), F_N(x)) \mid x \in \text{New Entrant Factors}\},$$

where:

- $T_N(x)$: Degree to which x increases the threat of new entrants.
- $I_N(x)$: Degree of uncertainty regarding the influence of x.
- $F_N(x)$: Degree to which x does not influence the threat of new entrants.

(2) *Bargaining Power of Suppliers (S):* Represented by a Neutrosophic Set S:

$$S = \{(y, T_S(y), I_S(y), F_S(y)) \mid y \in \text{Supplier Factors}\},$$

where:

- $T_S(y)$: Degree to which y increases supplier bargaining power.
- $I_S(y)$: Degree of uncertainty regarding the influence of y.
- $F_S(y)$: Degree to which y does not influence supplier bargaining power.

(3) *Bargaining Power of Buyers (B):* Represented by a Neutrosophic Set B:

$$B = \{(z, T_B(z), I_B(z), F_B(z)) \mid z \in \text{Buyer Factors}\},$$

where:

- $T_B(z)$: Degree to which z increases buyer bargaining power.
- $I_B(z)$: Degree of uncertainty regarding the influence of z.

- $F_B(z)$: Degree to which z does not influence buyer bargaining power.

(4) *Threat of Substitutes (U):* Represented by a Neutrosophic Set U:

$$U = \{(w, T_U(w), I_U(w), F_U(w)) \mid w \in \text{Substitute Factors}\},$$

where:

- $T_U(w)$: Degree to which w increases the threat of substitutes.
- $I_U(w)$: Degree of uncertainty regarding the influence of w.
- $F_U(w)$: Degree to which w does not influence the threat of substitutes.

(5) *Industry Rivalry (R):* Represented by a Neutrosophic Set R:

$$R = \{(v, T_R(v), I_R(v), F_R(v)) \mid v \in \text{Rivalry Factors}\},$$

where:

- $T_R(v)$: Degree to which v intensifies industry rivalry.
- $I_R(v)$: Degree of uncertainty regarding the influence of v.
- $F_R(v)$: Degree to which v does not influence industry rivalry.

Example 66. Consider applying Neutrosophic Porter's Five Forces Analysis to a retail business (cf.[208]):

- *Threat of New Entrants (N):* Factors such as ”Low Capital Requirements” and ”Lack of Brand Loyalty”:
  - Low Capital Requirements: $T_N = 0.8$, $I_N = 0.15$, $F_N = 0.05$
  - Lack of Brand Loyalty: $T_N = 0.7$, $I_N = 0.2$, $F_N = 0.1$
- *Bargaining Power of Suppliers (S):* Factors such as ”Few Suppliers” and ”High Switching Costs”:
  - Few Suppliers: $T_S = 0.9$, $I_S = 0.05$, $F_S = 0.05$
  - High Switching Costs: $T_S = 0.8$, $I_S = 0.1$, $F_S = 0.1$
- *Bargaining Power of Buyers (B):* Factors such as ”Availability of Alternatives” and ”Price Sensitivity”:
  - Availability of Alternatives: $T_B = 0.7$, $I_B = 0.2$, $F_B = 0.1$
  - Price Sensitivity: $T_B = 0.8$, $I_B = 0.1$, $F_B = 0.1$
- *Threat of Substitutes (U):* Factors such as ”Ease of Switching” and ”Low Cost of Substitutes”:
  - Ease of Switching: $T_U = 0.8$, $I_U = 0.1$, $F_U = 0.1$
  - Low Cost of Substitutes: $T_U = 0.7$, $I_U = 0.2$, $F_U = 0.1$
- *Industry Rivalry (R):* Factors such as ”High Number of Competitors” and ”Slow Market Growth”:
  - High Number of Competitors: $T_R = 0.9$, $I_R = 0.05$, $F_R = 0.05$
  - Slow Market Growth: $T_R = 0.8$, $I_R = 0.1$, $F_R = 0.1$

This example illustrates how Neutrosophic Sets can quantify and model the dynamics of Porter's Five Forces in the context of a competitive market.

Theorem 67. *Neutrosophic Porter's Five Forces has the structure of a Neutrosophic Set.*

Proof: This follows directly from the definition of Neutrosophic Porter's Five Forces. □

Question 68. As a related concept, frameworks such as six-forces analysis have been studied [189, 19, 48, 49]. Can the principles of Neutrosophic Logic be applied to these frameworks, and what potential applications might emerge?

3.4 | Some Neutrosophic (Social or Business) Logic

In the field of Social Science, various logics have been studied (e.g., [257, 21, 176]). This paper aims to explore potential extensions of these logics, including their expansion into Neutrosophic Logic.

3.4.1 | Neutrosophic Institutional Logics

Institutional Logics are frameworks guiding behavior within societal institutions, integrating material practices and symbolic systems to shape actions and norms[364, 43, 362, 363].

Definition 69 (Institutional Logics)**.** [363] Institutional logics are formalized as a structure $\mathcal{L} = (\mathcal{I}, \mathcal{S}, \mathcal{R}, \mathcal{C})$, where:

(1) $\mathcal{I} = \{I_1, I_2, \ldots, I_k\}$ is a finite set of *institutional orders*. Each $I_i \in \mathcal{I}$ corresponds to a domain such as markets, states, families, or religions.

(2) $\mathcal{S}$ is the set of *structural-symbolic systems*, defined as:

$$\mathcal{S} = \{S_i = (M_i, C_i) \mid i \in \{1, 2, \ldots, k\}\},$$

where:

- M_i is a set of *material practices*, formalized as a function $M_i : X \to Y$, where X represents resource inputs and Y represents outputs.
- C_i is a *symbolic system*, defined as a tuple $C_i = (\Sigma, \mathcal{G})$, where Σ is a set of cultural symbols and $\mathcal{G} : \Sigma \to [0, 1]$ is a probability distribution encoding the salience of each symbol.

(3) $\mathcal{R} \subseteq \mathcal{I} \times \mathcal{S}$ is a relation mapping institutional orders $I_i \in \mathcal{I}$ to their corresponding structural-symbolic systems $S_i \in \mathcal{S}$.

(4) $\mathcal{C}$ is a set of *constraints*, where $\mathcal{C} : A \times \mathcal{I} \to \mathbb{B}$ maps actions A and institutional orders $\mathcal{I}$ to a boolean domain $\mathbb{B} = \{0, 1\}$, enforcing domain-specific norms and rules.

Definition 70 (Behavior under Institutional Logics)**.** The behavior of an actor $a \in A$ within an institutional logic $\mathcal{L}$ is defined as a function:

$$B_{\mathcal{L}}(a) = \arg\max_{b \in B} U(b \mid \mathcal{L}),$$

where B is the set of all possible behaviors, and $U : B \times \mathcal{L} \to \mathbb{R}$ is a utility function defined as:

$$U(b \mid \mathcal{L}) = \sum_{i=1}^{k} \Big(\omega_i \cdot (f_M(b, M_i) + f_C(b, C_i))\Big),$$

with:

- $\omega_i \in [0, 1]$ representing the weight of the i-th institutional order.
- $f_M(b, M_i)$ quantifying the compatibility of behavior b with material practices M_i.
- $f_C(b, C_i)$ quantifying the alignment of behavior b with symbolic systems C_i.

Definition 71 (Institutional Change)**.** Institutional change occurs when the relation $\mathcal{R}$ or constraints $\mathcal{C}$ are updated due to exogenous events or endogenous contradictions. Formally, institutional change is a process:

$$\Phi : \mathcal{L}_t \to \mathcal{L}_{t+1},$$

where $\mathcal{L}_t$ and $\mathcal{L}_{t+1}$ represent institutional logics at time t and $t + 1$, respectively, and Φ satisfies:

$$\Phi(\mathcal{L}_t) = \Big(\mathcal{I}, \mathcal{S}', \mathcal{R}', \mathcal{C}'\Big),$$

with $\mathcal{S}', \mathcal{R}', \mathcal{C}'$ reflecting updated material practices, symbolic systems, or constraints.

Remark 72 (Neutrosophic Institutional Logic)**.** *Fuzzy Institutional Logic is a special case of Neutrosophic Institutional Logic where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Institutional Logic is notable for its ability to generalize both Neutrosophic and Fuzzy Institutional Logic.*

Example 73 (Market Logic)**.** Consider a market logic $\mathcal{L}_{\text{market}} = (I_{\text{market}}, S_{\text{market}}, \mathcal{R}_{\text{market}}, \mathcal{C}_{\text{market}})$, where:

- I_{market} represents the institutional order of markets.
- $S_{\text{market}} = (M_{\text{exchange}}, C_{\text{profit}})$, with:
  - M_{exchange} formalized as a function $M_{\text{exchange}}(p, q) = p \cdot q$, where p is price and q is quantity.
  - C_{profit} representing the cultural schema of profit maximization, encoded as $\mathcal{G}(\text{profit}) = 1$.
- $\mathcal{C}_{\text{market}}(a, I_{\text{market}}) = 1$ if a adheres to legal and competitive norms, otherwise 0.

The following describes Institutional Neutrosophic Logics, which extend this concept using Neutrosophic Logic.

Definition 74 (Institutional Neutrosophic Logics)**.** Institutional Neutrosophic Logics extend classical Institutional Logics by incorporating uncertainty, represented by the neutrosophic components of truth (T), indeterminacy (I), and falsity (F). Formally, an Institutional Neutrosophic Logic is defined as:

$$\mathcal{L}^N = (\mathcal{I}, \mathcal{S}, \mathcal{R}, \mathcal{C}, \mathcal{N}),$$

where $\mathcal{N}$ maps each proposition P about an institutional action or state to a neutrosophic value:

$$\mathcal{N}(P) = (T, I, F),$$

with $T, I, F \in [0, 1]$ satisfying $0 \leq T + I + F \leq 1$.

- T: Degree to which P is true within the institutional logic.
- I: Degree to which P is indeterminate due to conflicting or insufficient evidence.
- F: Degree to which P is false.

The behavior under Institutional Neutrosophic Logics is defined by a neutrosophic utility function:

$$U^N(b \mid \mathcal{L}^N) = \sum_{i=1}^{k} \Big(\omega_i \cdot (f_M^N(b, M_i) + f_C^N(b, C_i)) \Big),$$

where f_M^N and f_C^N incorporate neutrosophic evaluations of material practices and symbolic systems.

Remark 75. *Institutional Fuzzy Logic is a special case of Institutional Neutrosophic Logic where both indeterminacy and falsity are set to zero. Furthermore, Institutional Plithogenic Logic can also be defined using Plithogenic Logic.*

Example 76 (Neutrosophic Market Logic)**.** Consider a neutrosophic market logic

$$\mathcal{L}^N_{\text{market}} = (I_{\text{market}}, S_{\text{market}}, \mathcal{R}_{\text{market}}, \mathcal{C}_{\text{market}}, \mathcal{N})$$

, where:

- $\mathcal{N}(P) = (T, I, F)$ evaluates propositions such as "The market will grow by 10% next year" with $T = 0.6$, $I = 0.3$, and $F = 0.1$. This reflects a moderately confident prediction with some uncertainty and minimal falsity.
- U^N incorporates these neutrosophic values into decision-making. For example, an investor uses T, I, F to decide whether to allocate resources, balancing the confidence (T) against the uncertainty (I) and risk (F).
- Material practices M_{market} include pricing strategies modeled as $M_{\text{market}}(p, q) = p \cdot q$, where p is the price per unit and q is the quantity sold.
- Symbolic systems C_{profit} prioritize profit maximization, encoded as $\mathcal{G}(\text{profit}) = 1$.

Theorem 77. *Institutional Neutrosophic Logics naturally incorporate the structure of Neutrosophic Logic.*

Proof: This follows directly from the definition of Institutional Neutrosophic Logics, as they extend the principles and framework of Neutrosophic Logic to institutional contexts. □

Theorem 78. *Institutional Neutrosophic Logics naturally incorporate the structure of Institutional Logics.*

Proof: This follows directly from the definition of Institutional Neutrosophic Logics, as they integrate the fundamental aspects of traditional Institutional Logics into a neutrosophic framework. □

Theorem 79. *Every Institutional Neutrosophic Logic $\mathcal{L}^N$ is a superset of Institutional Fuzzy Logic $\mathcal{L}^F$.*

Proof: By definition, an Institutional Neutrosophic Logic $\mathcal{L}^N = (\mathcal{I}, \mathcal{S}, \mathcal{R}, \mathcal{C}, \mathcal{N})$ includes a neutrosophic mapping:

$$\mathcal{N}(P) = (T, I, F),$$

where $T, I, F \in [0, 1]$ and $0 \leq T + I + F \leq 1$. In Institutional Fuzzy Logic $\mathcal{L}^F = (\mathcal{I}, \mathcal{S}, \mathcal{R}, \mathcal{C}, \mathcal{F})$, the mapping:

$$\mathcal{F}(P) = T,$$

can be viewed as a special case of $\mathcal{N}(P)$ where $I = 0$ and $F = 0$. Since $\mathcal{L}^F$ is defined within the constraints of $\mathcal{L}^N$, every Institutional Fuzzy Logic is inherently embedded within an Institutional Neutrosophic Logic. Thus, $\mathcal{L}^N$ is a superset of $\mathcal{L}^F$. □

Theorem 80. *Institutional Neutrosophic Logic can model multiple institutional orders simultaneously, preserving independence and interdependence of $\mathcal{I}_i$.*

Proof: Let $\mathcal{L}^N = (\mathcal{I}, \mathcal{S}, \mathcal{R}, \mathcal{C}, \mathcal{N})$, where $\mathcal{I} = \{I_1, I_2, \ldots, I_k\}$ is the set of institutional orders. The neutrosophic mapping $\mathcal{N}(P) = (T, I, F)$ applies independently to propositions P_i within each institutional order I_i. Additionally, interdependencies between institutional orders are encoded in the relation $\mathcal{R} \subseteq \mathcal{I} \times \mathcal{S}$. The independence of $\mathcal{I}_i$ is preserved by maintaining separate evaluations for each I_i, while interdependencies are modeled via shared structural-symbolic systems $\mathcal{S}$ and constraints $\mathcal{C}$. Thus, $\mathcal{L}^N$ accommodates both independence and interdependence among multiple institutional orders. □

3.4.2 | Dominant Neutrosophic Logic

Dominant Logic refers to the mindset or cognitive framework organizations use to make decisions, allocate resources, and interpret information, shaping strategy and performance [378, 280, 162, 295, 193, 204].

Definition 81 (Dominant Logic)**.** (cf.[378]) Let F be a firm operating a portfolio of businesses $\mathcal{B} = \{B_1, B_2, \ldots, B_n\}$. The *Dominant Logic* $\mathcal{L}$ of the firm is a cognitive and operational framework defined as:

$$\mathcal{L} = (\mathcal{S}, \mathcal{D}, \mathcal{K}, \mathcal{P}),$$

where:

- $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$: A set of schemas, where each schema S_i is a mapping:

$$S_i : \mathcal{E} \to \mathcal{A},$$

  that transforms environmental inputs $\mathcal{E}$ (e.g., market trends) into actionable decisions $\mathcal{A}$.

- $\mathcal{D}$: A decision-making function defined as:

$$\mathcal{D} : \mathcal{V} \times \mathcal{C} \to \mathbb{R}^+,$$

  where $\mathcal{V}$ represents strategic variables (e.g., product pricing, market share), $\mathcal{C}$ represents organizational capabilities, and $\mathcal{D}(\mathbf{v}, \mathbf{c})$ is the resource allocation decision.

- $\mathcal{K}$: A knowledge structure represented as a directed graph $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of knowledge nodes and $\mathcal{E}$ are directed edges encoding the relationships among knowledge components.

- $\mathcal{P} = \{P_1, P_2, \ldots, P_k\}$: A set of performance metrics, where each $P_j : \mathcal{O} \to \mathbb{R}$ maps observable outcomes $\mathcal{O}$ (e.g., revenue, market share) to a real-valued evaluation.

Remark 82 (Neutrosophic Dominant Logic)**.** *Fuzzy Dominant Logic is a special case of Neutrosophic Dominant Logic where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Dominant Logic is notable for its ability to generalize both Neutrosophic and Fuzzy Dominant Logic.*

Example 83 (Application of Dominant Logic)**.** Consider a firm F with two businesses:

$$\mathcal{B} = \{B_1 : \text{Consumer Electronics}, \ B_2 : \text{Healthcare Products}\}.$$

The firm's Dominant Logic $\mathcal{L}$ is described as follows:

- *Schemas ($\mathcal{S}$):* S_1 is a schema that responds to market trends by adjusting product pricing. For instance:

  $$S_1(\text{Increase in demand}) = \text{Increase price by } 10\%.$$

- *Decision-making ($\mathcal{D}$):* The firm allocates R&D resources to maximize revenue. For example:

  $$\mathcal{D}(\text{Budget Share: } 0.6, \text{Capabilities: Advanced R\&D}) = 0.8,$$

  indicating 80% of the R&D budget is allocated to Consumer Electronics.

- *Knowledge Structure ($\mathcal{K}$):* Nodes represent domain expertise such as "Electronics Design" and "Healthcare Regulation," with directed edges denoting knowledge dependencies.

- *Performance Metrics ($\mathcal{P}$):* Metrics include P_1 = Revenue Growth and P_2 = Customer Retention, measured as:

  $$P_1 = \frac{\text{Revenue}_{\text{current}} - \text{Revenue}_{\text{previous}}}{\text{Revenue}_{\text{previous}}}.$$

Through this Dominant Logic, the firm evaluates whether R&D investments optimize the metrics P_1 and P_2, adapting to feedback from market performance.

Definition 84 (Strategic Fit)**.** A Dominant Logic $\mathcal{L}$ achieves *strategic fit* if, for each business $B_i \in \mathcal{B}$, there exists a schema $S_i \in \mathcal{S}$ and a decision $\mathcal{D}(\mathbf{v}, \mathbf{c})$ such that:

$$\mathcal{P}_j(B_i) \text{ is maximized for all } P_j \in \mathcal{P}.$$

Example 85 (Strategic Fit)**.** In the earlier example, the firm aligns $\mathcal{D}$ with $\mathcal{P}$ by prioritizing R&D spending in Consumer Electronics, where revenue growth (P_1) shows the highest marginal return per unit investment. If the healthcare business (B_2) exhibits diminishing returns, resources are reallocated to B_1 to maximize overall firm performance.

Next, the following describes Dominant Neutrosophic Logic, which extends Dominant Logic using Neutrosophic Logic.

Definition 86 (Dominant Neutrosophic Logic)**.** Dominant Neutrosophic Logic is an extension of Dominant Logic that incorporates neutrosophic components of truth (T), indeterminacy (I), and falsity (F) to handle uncertainty and incomplete information in decision-making processes. It is formally defined as a tuple:

$$\mathcal{L}^N = (\mathcal{S}^N, \mathcal{D}^N, \mathcal{K}^N, \mathcal{P}^N),$$

where:

(1) $\mathcal{S}^N = \{S_1^N, S_2^N, \dots, S_m^N\}$ is a set of neutrosophic schemas. Each schema S_i^N is a mapping:

$$S_i^N : \mathcal{E} \to \mathcal{A}^N,$$

where $\mathcal{E}$ is the space of environmental inputs, and $\mathcal{A}^N$ is the space of neutrosophic-valued actions defined as:

$$\mathcal{A}^N = \{(T, I, F) \mid T, I, F \in [0, 1], \ T + I + F \leq 1\}.$$

Here, T represents the degree of truth, I represents the degree of indeterminacy, and F represents the degree of falsity.

(2) $\mathcal{D}^N$ is a neutrosophic decision-making function:

$$\mathcal{D}^N : \mathcal{V} \times \mathcal{C} \to \mathcal{A}^N,$$

where $\mathcal{V}$ is the space of strategic variables, $\mathcal{C}$ is the space of organizational capabilities, and $\mathcal{D}^N(\mathbf{v}, \mathbf{c})$ assigns a neutrosophic value to each decision.

(3) $\mathcal{K}^N$ is a neutrosophic knowledge structure, represented as a directed graph $(\mathcal{N}, \mathcal{E})$, where:

$$\mathcal{N} = \{K_1^N, K_2^N, \dots, K_p^N\},$$

is a set of knowledge nodes, and each K_i^N is associated with a neutrosophic value (T_i, I_i, F_i). The edges in $\mathcal{E}$ represent knowledge dependencies, each assigned a neutrosophic weight.

(4) $\mathcal{P}^N = \{P_1^N, P_2^N, \dots, P_k^N\}$ is a set of neutrosophic performance metrics. Each metric P_j^N is a function:

$$P_j^N : \mathcal{O} \to \mathcal{A}^N,$$

where $\mathcal{O}$ is the space of observable outcomes, and $P_j^N(o) = (T_j, I_j, F_j)$ evaluates the outcome o in terms of truth, indeterminacy, and falsity.

Remark 87. *Dominant Neutrosophic Logic generalizes Dominant Logic by explicitly modeling uncertainty and conflict through the neutrosophic components (T, I, F). Fuzzy Dominant Logic is a special case where indeterminacy (I) and falsity (F) are zero, i.e., $(T, I, F) = (T, 0, 0)$.*

Example 88 (Application of Dominant Neutrosophic Logic)**.** Consider a firm F managing two business domains:

$$\mathcal{B} = \{B_1 : \text{Artificial Intelligence},\ B_2 : \text{Healthcare Devices}\}.$$

The Dominant Neutrosophic Logic $\mathcal{L}^N$ for F can be described as follows:

(1) *Neutrosophic Schema ($\mathcal{S}^N$):* A schema S_1^N evaluates the proposition "Invest in AI R&D" based on market trends:

$$S_1^N(\text{Positive market trend}) = (T = 0.8, I = 0.15, F = 0.05).$$

(2) *Neutrosophic Decision-making ($\mathcal{D}^N$):* Allocates resources with uncertainty in mind. For example:

$$\mathcal{D}^N(\text{R\&D Budget: } 60\%, \text{Capabilities: AI Research}) = (T = 0.7, I = 0.2, F = 0.1).$$

(3) *Neutrosophic Knowledge Structure ($\mathcal{K}^N$):* Nodes include "Market Trends" and "Technology Readiness," with neutrosophic weights:

$$(\text{Market Trends}) \to (\text{AI Research}) = (T = 0.9, I = 0.05, F = 0.05).$$

(4) *Neutrosophic Performance Metrics ($\mathcal{P}^N$):* Metrics include revenue growth, evaluated as:

$$P_1^N(\text{Revenue Growth}) = (T = 0.75, I = 0.2, F = 0.05).$$

The neutrosophic framework helps the firm balance confidence (T), uncertainty (I), and risk (F).

Theorem 89. *Dominant Neutrosophic Logic naturally incorporates the structure of Neutrosophic Logic.*

Proof: This follows directly from the definition of Dominant Neutrosophic Logic, as it extends the principles and framework of Neutrosophic Logic to dominant logical structures and reasoning processes. □

Theorem 90. *Dominant Neutrosophic Logic naturally incorporates the structure of Dominant Logic.*

Proof: This follows directly from the definition of Dominant Neutrosophic Logic, as it integrates the fundamental aspects of traditional Dominant Logic into a neutrosophic framework. □

Theorem 91. *Dominant Neutrosophic Logic is a superset of Fuzzy Dominant Logic.*

Proof: Fuzzy Dominant Logic is a special case of Dominant Neutrosophic Logic where $I = 0$ and $F = 0$, reducing the neutrosophic value (T, I, F) to $(T, 0, 0)$. Since Dominant Neutrosophic Logic allows T, I, F to independently range within $[0, 1]$ under the constraint $T + I + F \leq 1$, Fuzzy Dominant Logic is fully embedded within this broader framework. Thus, Dominant Neutrosophic Logic generalizes Fuzzy Dominant Logic. □

Theorem 92. *Dominant Neutrosophic Logic accommodates multiple schemas, preserving independence and interdependence among decision components.*

Proof: Let $\mathcal{L}^N = (\mathcal{S}^N, \mathcal{D}^N, \mathcal{K}^N, \mathcal{P}^N)$, where $\mathcal{S}^N = \{S_1^N, S_2^N, \dots, S_m^N\}$ represents neutrosophic schemas. Each schema S_i^N operates independently as a mapping $S_i^N : \mathcal{E} \to \mathcal{A}^N$, where $\mathcal{A}^N = \{(T, I, F)\}$. Interdependence is introduced through shared resources or dependencies represented in $\mathcal{K}^N$, a directed graph linking knowledge nodes. This structure preserves independence at the schema level while modeling interdependencies through relationships in $\mathcal{K}^N$. Thus, Dominant Neutrosophic Logic effectively manages independent and interdependent components. □

Theorem 93. *Dominant Neutrosophic Logic enhances decision-making by explicitly modeling uncertainty and conflict through* (T, I, F)*.*

Proof: In traditional Dominant Logic, decisions rely on deterministic or probabilistic values, lacking explicit representation of indeterminacy or falsity. Dominant Neutrosophic Logic extends this framework by incorporating neutrosophic values (T, I, F), allowing decisions to account for truth, uncertainty, and conflict simultaneously. This enriched representation improves decision-making in complex scenarios with incomplete or conflicting information, as each decision component $\mathcal{D}^N$ evaluates strategic variables and organizational capabilities under neutrosophic uncertainty. □

3.4.3 | Service-Dominant Neutrosophic Logic

Service-Dominant Logic emphasizes value co-creation through service exchange, viewing goods as service delivery mechanisms, focusing on relationships, collaboration, and customer-centricity in value creation [379, 380, 217, 216, 219, 215, 381, 218, 317, 147, 260].

Definition 94 (Service-Dominant Logic)**.** (cf.[379, 380]) Service-Dominant Logic (S-D Logic) is a theoretical framework that conceptualizes value creation as a collaborative process among multiple actors within a service ecosystem. Formally, it is defined as a tuple:

$$\mathcal{L}_{SD} = (\mathcal{A}, \mathcal{R}, \mathcal{I}, \mathcal{V}),$$

where:

(1) $\mathcal{A} = \{A_1, A_2, \dots, A_n\}$: A set of actors in the service ecosystem, where each actor A_i is a resource integrator.

(2) $\mathcal{R} = \{\rho_{ij}\}$: A set of resource exchanges between actors A_i and A_j, where:

$$\rho_{ij} = (R_{ij}, E_{ij}),$$

with R_{ij} being the resource provided by A_i to A_j, and E_{ij} being the corresponding value exchange.

(3) $\mathcal{I} = \{I_1, I_2, \dots, I_m\}$: A set of institutional arrangements, where each I_k defines the rules, norms, and practices governing resource exchanges within the ecosystem.

(4) $\mathcal{V} = \{V_1, V_2, \dots, V_p\}$: A set of value cocreation processes, where each V_l is a mapping:

$$V_l : \mathcal{A} \times \mathcal{R} \to \mathbb{R},$$

assigning a value $v \in \mathbb{R}$ to each interaction based on the integration of resources by the actors.

Example 95 (Service Ecosystem)**.** Consider a healthcare service ecosystem:

$$\mathcal{A} = \{\text{Patients}, \text{Doctors}, \text{Pharmacies}, \text{Insurers}\}.$$

Here:

- Resource exchanges ($\mathcal{R}$) include the transfer of medical knowledge (R_{ij}) from doctors to patients and financial resources (R_{ji}) from insurers to healthcare providers.
- Institutional arrangements ($\mathcal{I}$) include healthcare regulations and insurance policies.

- Value cocreation processes ($\mathcal{V}$) evaluate outcomes such as patient health improvement or cost-effectiveness.

Through this framework, the ecosystem collectively cocreates value.

Definition 96 (Service-Dominant Neutrosophic Logic)**.** Service-Dominant Neutrosophic Logic (SDN Logic) extends Service-Dominant Logic by incorporating neutrosophic components of truth (T), indeterminacy (I), and falsity (F) to address uncertainty and incomplete information within a service ecosystem. Formally, SDN Logic is defined as:

$$\mathcal{L}_{SDN} = (\mathcal{A}, \mathcal{R}^N, \mathcal{I}, \mathcal{V}^N),$$

where:

(1) $\mathcal{A} = \{A_1, A_2, \ldots, A_n\}$: A set of actors in the service ecosystem. Each actor A_i is a resource integrator and decision-maker.

(2) $\mathcal{R}^N = \{\rho_{ij}^N\}$: A set of neutrosophic resource exchanges between actors A_i and A_j, where:

$$\rho_{ij}^N = (R_{ij}, (T_{ij}, I_{ij}, F_{ij})),$$

with R_{ij} being the resource provided by A_i to A_j, and (T_{ij}, I_{ij}, F_{ij}) representing the neutrosophic truth, indeterminacy, and falsity values of the resource exchange.

(3) $\mathcal{I} = \{I_1, I_2, \ldots, I_m\}$: A set of institutional arrangements defining the rules, norms, and practices governing interactions and exchanges within the ecosystem.

(4) $\mathcal{V}^N = \{V_1^N, V_2^N, \ldots, V_p^N\}$: A set of neutrosophic value cocreation processes, where each V_l^N is a mapping:

$$V_l^N : \mathcal{A} \times \mathcal{R}^N \to \mathcal{A}^N,$$

assigning a neutrosophic value (T, I, F) to each interaction based on the integration of resources by the actors.

The neutrosophic constraints require that:

$$T, I, F \in [0, 1], \quad T + I + F \leq 1.$$

Remark 97 (Neutrosophic Service-Dominant Logic)**.** *Fuzzy Service-Dominant Logic is a special case of Neutrosophic Service-Dominant Logic where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Service-Dominant Logic is notable for its ability to generalize both Neutrosophic and Fuzzy Service-Dominant Logic.*

Example 98 (Healthcare Service Ecosystem)**.** A Healthcare Service Ecosystem is a dynamic network of interconnected stakeholders collaboratively co-creating value through services, resources, and relationships to improve health outcomes (cf.[67, 399, 4]). Consider a healthcare service ecosystem:

$$\mathcal{A} = \{\text{Patients}, \text{Doctors}, \text{Pharmacies}, \text{Insurers}\}.$$

Here:

- Neutrosophic resource exchanges ($\mathcal{R}^N$) include the transfer of medical advice (R_{ij}) from doctors to patients with:

$$\rho_{\text{Doctor, Patient}}^N = (\text{Medical Advice}, (T = 0.9, I = 0.05, F = 0.05)).$$

- Institutional arrangements ($\mathcal{I}$) include healthcare regulations and insurance policies.
- Neutrosophic value cocreation processes ($\mathcal{V}^N$) evaluate outcomes such as patient health improvement. For instance:

$$V_{\text{Health Improvement}}^N(\text{Doctor}, \text{Patient}) = (T = 0.85, I = 0.1, F = 0.05).$$

Through this framework, the ecosystem balances confidence (T), uncertainty (I), and risk (F) in resource exchanges and value creation.

Theorem 99. *Service-Dominant Neutrosophic Logic generalizes Service-Dominant Logic by incorporating neutrosophic components* (T, I, F).

Proof: Service-Dominant Logic $\mathcal{L}_{SD}$ is defined as $\mathcal{L}_{SD} = (\mathcal{A}, \mathcal{R}, \mathcal{I}, \mathcal{V})$, where $\mathcal{R}$ represents deterministic resource exchanges and $\mathcal{V}$ deterministic value cocreation processes. Service-Dominant Neutrosophic Logic $\mathcal{L}_{SDN} = (\mathcal{A}, \mathcal{R}^N, \mathcal{I}, \mathcal{V}^N)$ extends $\mathcal{L}_{SD}$ by introducing $\mathcal{R}^N$ and $\mathcal{V}^N$, where resource exchanges and value cocreation processes are represented with neutrosophic components (T, I, F). These components allow $\mathcal{L}_{SDN}$ to explicitly model uncertainty (I) and conflict (F), which are absent in $\mathcal{L}_{SD}$. Thus, Service-Dominant Neutrosophic Logic generalizes Service-Dominant Logic. □

Theorem 100. *Service-Dominant Neutrosophic Logic inherently possesses the structure of Neutrosophic Logic.*

Proof: Service-Dominant Neutrosophic Logic $\mathcal{L}_{SDN}$ incorporates neutrosophic resource exchanges $\mathcal{R}^N$ and neutrosophic value cocreation processes $\mathcal{V}^N$, which map interactions and outcomes to neutrosophic values (T, I, F). These mappings align directly with the principles of Neutrosophic Logic, where T, I, F represent truth, indeterminacy, and falsity, respectively. As $T + I + F \leq 1$ is a fundamental constraint in both frameworks, $\mathcal{L}_{SDN}$ naturally inherits the structure of Neutrosophic Logic. □

Theorem 101. *Service-Dominant Neutrosophic Logic balances resource exchanges and value cocreation under uncertainty, enabling robust decision-making.*

Proof: In Service-Dominant Neutrosophic Logic $\mathcal{L}_{SDN}$, resource exchanges $\rho_{ij}^N = (R_{ij}, (T_{ij}, I_{ij}, F_{ij}))$ explicitly account for uncertainty (I) and falsity (F) in interactions. Neutrosophic value cocreation processes $V_l^N : \mathcal{A} \times \mathcal{R}^N \to \mathcal{A}^N$ integrate these components to evaluate outcomes with confidence (T) while accommodating uncertainty and conflict. This balanced approach ensures that decision-making within the service ecosystem is robust, adapting to incomplete or conflicting information. □

Theorem 102. *Service-Dominant Neutrosophic Logic enables dynamic optimization of resource exchanges and value cocreation processes in complex ecosystems.*

Proof: The neutrosophic components (T, I, F) in Service-Dominant Neutrosophic Logic allow dynamic assessment of resource exchanges ρ_{ij}^N and value processes V_l^N. By continuously updating (T, I, F) based on new information, the framework adapts to changes in the service ecosystem, optimizing interactions and outcomes. This flexibility supports decision-making in complex and evolving environments, where uncertainty and conflicting information are prevalent. □

3.4.4 | Neutrosophic Critical Thinking (Neutrosophic Critical Logic)

Critical Thinking is the objective analysis and evaluation of information to form reasoned judgments, emphasizing logic, and evidence [202, 170, 243, 35, 103, 275].

Definition 103 (Critical Thinking)**.** Critical thinking is the systematic, recursive, and logical process of analyzing, evaluating, and synthesizing information to derive coherent conclusions and self-reflectively improve reasoning. Mathematically, it can be represented as:

$$\mathcal{C}(X) = \mathcal{R} \circ \mathcal{F} \circ \mathcal{E} \circ \mathcal{A} \circ \mathcal{I}(X),$$

where:

(1) $\mathcal{I} : \mathcal{X} \to \mathcal{R}$ (*Interpretation Function*): A function that maps raw data $X \in \mathcal{X}$ into a structured representation $R \in \mathcal{R}$, capturing its semantic meaning. Formally:

$$\mathcal{I}(X) = R, \quad \text{where } R \text{ is a structured framework.}$$

(2) $\mathcal{A} : \mathcal{R} \to \mathcal{P}(\mathcal{E})$ (*Analysis Operator*): A function that decomposes R into its atomic elements or subcomponents $\{e_1, e_2, \dots, e_n\} \subseteq \mathcal{E}$, where $\mathcal{P}(\mathcal{E})$ denotes the power set of $\mathcal{E}$. Formally:

$$\mathcal{A}(R) = \{e_i \mid e_i \text{ represents an atomic element of } R\}.$$

(3) $\mathcal{E} : \mathcal{E} \to [0,1]$ (*Evaluation Metric*): A function that assigns a weight $w(e_i)$ to each element e_i, quantifying its credibility or logical strength. Formally:

$$\mathcal{E}(e_i) = w(e_i), \quad w(e_i) \text{ indicates the reliability of } e_i.$$

(4) $\mathcal{F} : \mathcal{P}(\mathcal{E}) \to \mathcal{C}$ (*Inference Function*): A function that aggregates weighted elements $\{(e_i, w(e_i))\}$ into a conclusion $C \in \mathcal{C}$ based on logical or probabilistic rules. Formally:

$$\mathcal{F}(\{(e_i, w(e_i))\}) = C, \quad \text{where } C \text{ is logically consistent.}$$

(5) $\mathcal{R} : \mathcal{C} \to \mathcal{C}'$ (*Self-Regulation Operator*): A recursive function that reassesses and refines all preceding steps, resulting in an improved critical thinking process $\mathcal{C}'$. Formally:

$$\mathcal{R}(\mathcal{C}) = \mathcal{C}', \quad \mathcal{C}' \text{ is an updated process.}$$

Remark 104. *The critical thinking process $\mathcal{C}$ is inherently recursive, as the self-regulation operator $\mathcal{R}$ allows iterative improvement. This ensures both logical rigor and adaptability to new information.*

Example 105. Consider X as a dataset of experimental observations supporting a scientific hypothesis. The process proceeds as follows:

(1) *Interpretation ($\mathcal{I}$)*: Organize X into a structured hypothesis R.

(2) *Analysis ($\mathcal{A}$)*: Decompose R into key premises $\{e_1, e_2, \dots, e_n\}$.

(3) *Evaluation ($\mathcal{E}$)*: Assign weights $w(e_i)$ to each premise based on empirical evidence.

(4) *Inference ($\mathcal{F}$)*: Derive a conclusion C by combining weighted premises.

(5) *Regulation ($\mathcal{R}$)*: Reassess $\mathcal{I}, \mathcal{A}, \mathcal{E}, \mathcal{F}$ and refine the conclusion C.

Definition 106 (Neutrosophic Critical Thinking)**.** Neutrosophic Critical Thinking (NCT) is an extension of classical critical thinking that operates under the framework of neutrosophic logic, incorporating degrees of truth (T), indeterminacy (I), and falsity (F). This enables reasoning and decision-making in the presence of uncertainty and contradictions. Formally, NCT is a structured process defined as:

$$\mathcal{NCT}(X) = \mathcal{R}^N \circ \mathcal{F}^N \circ \mathcal{E}^N \circ \mathcal{A}^N \circ \mathcal{I}^N(X),$$

where $X \in \mathcal{X}$ is the input data or information, and the components are defined as follows:

(1) $\mathcal{I}^N : \mathcal{X} \to \mathcal{R}^N$ (*Neutrosophic Interpretation*): A mapping that converts raw data X into a neutrosophic representation $R^N \in \mathcal{R}^N$. For each proposition P in R^N, a neutrosophic truth value is assigned:

$$\mathcal{N}(P) = (T, I, F), \quad T, I, F \in [0,1], \quad 0 \le T + I + F \le 1,$$

where:

- T: Degree to which P is true.
- I: Degree to which P is indeterminate (uncertain or conflicting).
- F: Degree to which P is false.

(2) $\mathcal{A}^N : \mathcal{R}^N \to \mathcal{P}(\mathcal{E}^N)$ (*Neutrosophic Analysis*): Decomposes R^N into atomic components $\{e_1, e_2, \dots, e_n\} \subseteq \mathcal{E}^N$, where each component e_i represents a fundamental unit of R^N. Each e_i is associated with a neutrosophic evaluation:

$$\mathcal{A}^N(R^N) = \{(e_i, \mathcal{N}(e_i)) \mid e_i \text{ is an atomic component of } R^N\}.$$

(3) $\mathcal{E}^N : \mathcal{E}^N \to [0,1]^3$ (*Neutrosophic Evaluation*): Assigns a neutrosophic truth value $\mathcal{N}(e_i) = (T_{e_i}, I_{e_i}, F_{e_i})$ to each atomic component e_i, quantifying its truth, indeterminacy, and falsity. Formally:

$$\mathcal{E}^N(e_i) = (T_{e_i}, I_{e_i}, F_{e_i}), \quad T_{e_i}, I_{e_i}, F_{e_i} \in [0,1], \quad T_{e_i} + I_{e_i} + F_{e_i} \le 1.$$

(4) $\mathcal{F}^N : \mathcal{P}(\mathcal{E}^N) \to \mathcal{C}^N$ (*Neutrosophic Inference*): Synthesizes the neutrosophic evaluations $\{(e_i, \mathcal{N}(e_i))\}$ into a conclusion C^N, represented as:

$$\mathcal{N}(C^N) = (T_C, I_C, F_C),$$

where:

$$T_C = \sum_{i=1}^{n} w_i T_{e_i}, \quad I_C = \sum_{i=1}^{n} w_i I_{e_i}, \quad F_C = \sum_{i=1}^{n} w_i F_{e_i},$$

and w_i are weights such that $\sum_{i=1}^{n} w_i = 1$.

(5) $\mathcal{R}^N : \mathcal{C}^N \to \mathcal{C}^N$ (*Neutrosophic Self-Regulation*): A recursive operator that re-evaluates and refines $\mathcal{C}^N$ by iteratively applying the process to updated information or revised assumptions. Formally:

$$\mathcal{R}^N(\mathcal{C}^N) = \mathcal{C}^N_{\text{updated}},$$

where $\mathcal{C}^N_{\text{updated}}$ incorporates new evaluations or corrections.

Remark 107 (Neutrosophic Critical Thinking)**.** *Fuzzy Critical Thinking is a special case of Neutrosophic Critical Thinking where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Critical Thinking is notable for its ability to generalize both Neutrosophic and Fuzzy Critical Thinking.*

Example 108 (Neutrosophic Decision-Making in Scientific Hypotheses)**.** Let X represent experimental data supporting a scientific hypothesis. The process unfolds as follows:

(1) *Interpretation ($\mathcal{I}^N$)*: Convert X into R^N, where each proposition P (e.g., "The hypothesis holds under condition A") is assigned $\mathcal{N}(P) = (T, I, F)$.

(2) *Analysis ($\mathcal{A}^N$)*: Decompose R^N into atomic premises $\{e_1, e_2, \dots, e_n\}$, with $\mathcal{N}(e_i) = (T_{e_i}, I_{e_i}, F_{e_i})$.

(3) *Evaluation ($\mathcal{E}^N$)*: Assign $T_{e_i}, I_{e_i}, F_{e_i}$ values to each e_i based on empirical evidence and logical consistency.

(4) *Inference ($\mathcal{F}^N$)*: Compute $\mathcal{N}(C^N) = (T_C, I_C, F_C)$ as the weighted aggregate of $\mathcal{N}(e_i)$.

(5) *Self-Regulation ($\mathcal{R}^N$)*: Reassess $\mathcal{N}(C^N)$ and update components based on additional data or new hypotheses.

For instance, a hypothesis with $\mathcal{N}(C^N) = (0.7, 0.2, 0.1)$ indicates 70% confidence, 20% uncertainty, and 10% falsity.

Theorem 109. *Neutrosophic Critical Thinking inherently extends classical critical thinking by modeling uncertainty and contradiction through (T, I, F).*

Proof: In classical critical thinking, each proposition P is evaluated as either true or false, lacking an explicit representation of uncertainty or contradiction. Neutrosophic Critical Thinking extends this framework by assigning to each proposition $\mathcal{N}(P) = (T, I, F)$, where:

$$T, I, F \in [0, 1], \quad T + I + F \leq 1.$$

This representation allows propositions to simultaneously have degrees of truth (T), indeterminacy (I), and falsity (F). By incorporating I and F, Neutrosophic Critical Thinking explicitly accounts for uncertainty and contradictions, providing a more comprehensive framework for reasoning in ambiguous or complex scenarios. □

Theorem 110. *Neutrosophic Critical Thinking improves decision-making by balancing confidence, uncertainty, and falsity in evaluations.*

Proof: In Neutrosophic Critical Thinking, the inference process $\mathcal{F}^N$ aggregates neutrosophic evaluations:

$$\mathcal{N}(C^N) = (T_C, I_C, F_C),$$

where:

$$T_C = \sum_{i=1}^{n} w_i T_{e_i}, \quad I_C = \sum_{i=1}^{n} w_i I_{e_i}, \quad F_C = \sum_{i=1}^{n} w_i F_{e_i}.$$

The weights w_i are adjusted based on the importance or reliability of atomic components e_i. This balanced approach enables decision-making that considers confidence (T_C), uncertainty (I_C), and falsity (F_C), allowing for nuanced conclusions that classical frameworks cannot achieve. □

Theorem 111. *Neutrosophic Critical Thinking provides a self-regulating mechanism for iterative reasoning and decision-making.*

Proof: The self-regulation operator $\mathcal{R}^N$ in Neutrosophic Critical Thinking re-evaluates and refines conclusions $\mathcal{C}^N$ by incorporating new data or updated assumptions:

$$\mathcal{R}^N(\mathcal{C}^N) = \mathcal{C}^N_{\text{updated}}.$$

This recursive process ensures that decisions and conclusions remain adaptive to evolving information, improving robustness and accuracy over time. Such iterative refinement is absent in classical critical thinking, highlighting the advanced capabilities of the neutrosophic approach. □

Theorem 112. *Neutrosophic Critical Thinking is applicable to systems with incomplete or conflicting data, where classical critical thinking fails.*

Proof: In systems with incomplete or conflicting data, propositions P cannot be fully classified as true or false. Neutrosophic Critical Thinking assigns $\mathcal{N}(P) = (T, I, F)$, where indeterminacy ($I$) captures the ambiguity or conflict. By explicitly modeling I alongside T and F, the framework accommodates incomplete or contradictory information, enabling reasoning and decision-making where classical approaches are inadequate. □

3.4.5 | Neutrosophic Climate Change Logic

In social science, Climate Change Logic models the interplay between human behavior, policies, and environmental impacts, analyzing strategies to mitigate climate change while accounting for societal, economic, and regulatory factors [23, 42, 373, 166, 77].

Definition 113 (Climate Change Logic)**.** *Climate Change Logic* is a formal mathematical system for modeling, evaluating, and optimizing the dynamic interactions between environmental states, human activities, and their impacts on climate systems under uncertainty. It is formally expressed as:

$$\mathcal{L}_{CC} = (S, A, T, F, P, V, C, R, \mathcal{U}),$$

where:

- $S = \{s_1, s_2, \dots, s_n\}$: A finite or infinite set of environmental states representing measurable climate-related variables such as CO_2 concentration, global temperature rise, or sea-level rise.
- $A = \{a_1, a_2, \dots, a_m\}$: A finite set of human activities or interventions influencing the state transitions, such as emissions, deforestation, industrial output, or renewable energy adoption.
- $T = \{t_0, t_1, \dots, t_p\}$: A discrete or continuous time horizon over which environmental dynamics and human interventions are observed.
- $F : S \times A \times T \to \Delta(S)$: The *state transition function*, where $F(s, a, t)$ gives the probability distribution over S at time $t + 1$, conditioned on the current state $s \in S$ and activity $a \in A$. Here, $\Delta(S)$ is the set of probability distributions on S.
- $P : S \to [0, 1]$: The *risk function*, assigning the probability of adverse events (e.g., natural disasters, economic damage) occurring at state s.
- $V : S \times T \to \mathbb{R}^+$: The *valuation function*, quantifying the severity of impacts or costs (e.g., economic losses, biodiversity loss, or health damage) associated with state s at time t.
- $C : A \times T \to \mathbb{R}^+$: The *activity cost function*, representing the cost associated with implementing activity a at time t.

- $R : S \times A \to \mathbb{R}^+$: The *regulation function*, defining the regulatory or mitigation costs required to control the state transition induced by activity a from state s.
- $\mathcal{U} : \mathcal{P}(S \times A) \to \mathbb{R}$: The *utility function*, capturing the decision-maker's preferences over states and actions, accounting for both immediate and future impacts.

Climate Impact Evaluation. The cumulative climate impact I over a time horizon T is expressed as:

$$I = \int_{t_0}^{t_p} \sum_{s\in S} \sum_{a\in A} P(s) \cdot F(s,a,t) \cdot V(s,t)\, dt.$$

Optimal Climate Policy. The optimal climate policy π^* is a strategy that minimizes the cumulative impact I and total costs C, while accounting for regulatory constraints:

$$\pi^* = \arg\min_{\pi\in\Pi} \left[\int_{t_0}^{t_p} \left(I + \sum_{a\in A} C(a,t) + \sum_{s\in S} R(s,a) \right) dt \right],$$

subject to:

$$F(s,a,t) \in \Delta(S), \quad \forall t \in T,\ s \in S,\ a \in A.$$

Here, Π is the set of all feasible policies mapping states S to actions A.

Uncertainty in Climate Change Logic. If uncertainty in state transitions or valuations is represented by a neutrosophic framework, the state transition function F^N and valuation function V^N are extended as follows:

$$F^N(s,a,t) = (T_F, I_F, F_F), \quad V^N(s,t) = (T_V, I_V, F_V),$$

where T, I, and F denote the truth, indeterminacy, and falsity components, respectively.

The cumulative neutrosophic impact I^N becomes:

$$I^N = \int_{t_0}^{t_p} \sum_{s\in S} \sum_{a\in A} (T_F - F_F) \cdot P(s) \cdot V^N(s,t)\, dt.$$

Example 114 (Climate Change Logic: Renewable Energy vs. Forest Regeneration)**.** Consider a climate policy scenario where policymakers aim to reduce greenhouse gas (GHG) emissions [258] over a time horizon $T = \{t_0, t_1, t_2\}$. The components of the Climate Change Logic are as follows:

- $S = \{s_1, s_2, s_3\}$: Environmental states.
  - s_1: Low GHG emissions.
  - s_2: Moderate GHG emissions.
  - s_3: High GHG emissions.
- $A = \{a_1, a_2, a_3\}$: Climate mitigation activities.
  - a_1: Adoption of renewable energy (solar, wind).
  - a_2: Forest regeneration programs.
  - a_3: No intervention.
- $F : S \times A \times T \to \Delta(S)$: State transition probabilities under mitigation activities.

  $$F(s_3, a_1, t_1) = 0.7, \quad F(s_3, a_2, t_1) = 0.6.$$

  Interpretation: At t_1, adopting a_1 reduces s_3 to lower states with 70% probability, while a_2 achieves a 60% reduction probability.
- $V : S \times T \to \mathbb{R}^+$: Climate impact valuation.

  $$V(s_3, t_2) = 100, \quad V(s_2, t_2) = 50, \quad V(s_1, t_2) = 10.$$

  Interpretation: The cost of high emissions (s_3) at t_2 is 100, while moderate (s_2) and low emissions (s_1) cost 50 and 10, respectively.

- $C : A \times T \to \mathbb{R}^+$: Activity cost function.

$$C(a_1, t_1) = 30, \quad C(a_2, t_1) = 20, \quad C(a_3, t_1) = 0.$$

Interpretation: The implementation costs of a_1 (renewable energy) and a_2 (forest regeneration) at t_1 are 30 and 20, respectively. a_3 (no intervention) incurs no cost.

- $R : S \times A \to \mathbb{R}^+$: Regulatory compliance cost.

$$R(s_3, a_3) = 50.$$

Interpretation: Maintaining high emissions (s_3) under no intervention (a_3) incurs regulatory penalties of 50.

Cumulative Climate Impact. The total climate impact I over T is calculated as:

$$I = \int_{t_0}^{t_2} \sum_{s \in S} \sum_{a \in A} F(s, a, t) \cdot V(s, t)\, dt.$$

Cost-Benefit Comparison. The total costs C_{total} for each policy (activity) include implementation costs and regulatory penalties:

$$C_{\text{total}}(a_1) = 30, \quad C_{\text{total}}(a_2) = 20, \quad C_{\text{total}}(a_3) = 50.$$

Optimal Policy. The optimal activity a^* minimizes the sum of cumulative climate impact and total costs:

$$a^* = \arg\min_{a \in A} \left[I + C_{\text{total}} \right].$$

Results.

- a_1 (renewable energy) achieves the largest emission reduction probability (70%), reducing I significantly, but incurs higher upfront costs.
- a_2 (forest regeneration) provides a lower reduction probability (60%) but is more cost-effective.
- a_3 (no intervention) results in the highest regulatory penalties and climate impact, making it the least optimal choice.

Thus, policymakers must evaluate trade-offs between emission reductions and associated costs to determine the optimal climate mitigation policy.

Definition 115 (Neutrosophic Climate Change Logic)**.** *Neutrosophic Climate Change Logic* is a formalized framework that models climate systems, human activities, and their interactions under uncertainty, indeterminacy, and falsity. It is defined as:

$$\mathcal{L}_{CC}^N = (S, A, T, F^N, P^N, V^N, C^N, R^N, \mathcal{U}^N),$$

where:

- $S = \{s_1, s_2, \ldots, s_n\}$: A finite or infinite set of environmental states (e.g., temperature rise, CO_2 concentration, sea-level change).
- $A = \{a_1, a_2, \ldots, a_m\}$: A finite set of human activities or mitigation strategies that influence state transitions, such as energy consumption, reforestation, or carbon capture.
- $T = \{t_0, t_1, \ldots, t_p\}$: A time domain (discrete or continuous) representing the temporal evolution of climate states.
- $F^N : S \times A \times T \to [0, 1]^3$: The *neutrosophic state transition function*, where:

$$F^N(s, a, t) = (T_F, I_F, F_F),$$

assigns the degrees of truth (T_F), indeterminacy (I_F), and falsity (F_F) for the probability of transitioning to a new state $s \in S$ under activity a at time t.

- $P^N : S \to [0, 1]^3$: The *neutrosophic risk function*, where:

$$P^N(s) = (T_P, I_P, F_P),$$

represents the neutrosophic probabilities of risks (e.g., disasters or adverse effects) occurring at state s.

- $V^N : S \times T \to \mathbb{R}^3$: The *neutrosophic valuation function*, where:
$$V^N(s,t) = (T_V, I_V, F_V),$$
gives the truth (T_V), indeterminacy (I_V), and falsity (F_V) components of the impacts or costs associated with state s at time t (e.g., economic loss, biodiversity decline).
- $C^N : A \times T \to \mathbb{R}^3$: The *neutrosophic cost function*, where:
$$C^N(a,t) = (T_C, I_C, F_C),$$
quantifies the truth, indeterminacy, and falsity of the costs incurred by implementing activity a at time t.
- $R^N : S \times A \to \mathbb{R}^3$: The *neutrosophic regulation function*, where:
$$R^N(s,a) = (T_R, I_R, F_R),$$
represents the costs or regulatory constraints (with uncertainty) for controlling state s under activity a.
- $\mathcal{U}^N : \mathcal{P}(S \times A) \to \mathbb{R}^3$: The *neutrosophic utility function*, evaluating the decision-maker's preferences over states and actions, incorporating truth, indeterminacy, and falsity.

Neutrosophic Climate Impact. The cumulative neutrosophic climate impact I^N over a time horizon T is defined as:
$$I^N = \int_{t_0}^{t_p} \sum_{s\in S} \sum_{a \in A} (T_F - F_F) \cdot P^N(s) \cdot V^N(s,t)\, dt,$$
where T_F and F_F are the truth and falsity degrees from F^N.
Optimal Neutrosophic Climate Policy. The optimal policy π_N^* minimizes the cumulative neutrosophic impact and costs while respecting regulatory constraints:
$$\pi_N^* = \arg\min_{\pi\in\Pi} \left[\int_{t_0}^{t_p} \left(I^N + \sum_{a\in A} C^N(a,t) + \sum_{s\in S} R^N(s,a) \right) dt \right],$$
subject to:
$$F^N(s,a,t) \in [0,1]^3, \quad \forall t \in T,\ s \in S,\ a \in A.$$
Neutrosophic Uncertainty Representation. In this framework, uncertainty is explicitly represented through neutrosophic triplets:
$$(T, I, F),$$
where:

- T: Degree of truth, reflecting known and verified information.
- I: Degree of indeterminacy, accounting for ambiguity or incomplete information.
- F: Degree of falsity, representing contradictory or false information.

The neutrosophic extension allows for a comprehensive evaluation of climate change processes, enabling decision-makers to handle uncertain, incomplete, and conflicting data effectively.

Remark 116 (Neutrosophic Climate Change Logic)**.** *Fuzzy Climate Change Logic is a special case of Neutrosophic Climate Change Logic where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Climate Change Logic is notable for its ability to generalize both Neutrosophic and Fuzzy Climate Change Logic.*

Example 117 (Neutrosophic Climate Change Logic: GHG Emission Reduction)**.** Consider a scenario where policymakers aim to reduce greenhouse gas (GHG) emissions to mitigate climate change. Let the components of Neutrosophic Climate Change Logic be defined as follows:

- $S = \{s_1, s_2, s_3\}$: Set of environmental states.
  - s_1: Low GHG emission level (below target threshold).
  - s_2: Moderate GHG emission level.
  - s_3: High GHG emission level.

- $A = \{a_1, a_2, a_3\}$: Set of mitigation actions.
  - a_1: Implementation of renewable energy (solar, wind).
  - a_2: Industrial carbon capture and storage (CCS).
  - a_3: No intervention (business as usual).
- $T = \{t_0, t_1, t_2\}$: Discrete time steps t_0 (initial), t_1 (mid-term), t_2 (long-term).
- $F^N : S \times A \times T \to [0,1]^3$: Neutrosophic state transition function.

$$F^N(s_2, a_1, t_1) = (0.8, 0.1, 0.1), \quad F^N(s_3, a_2, t_2) = (0.6, 0.3, 0.1).$$

  Interpretation: Implementing a_1 at t_1 reduces emissions to s_2 with 80% certainty ($T = 0.8$), 10% indeterminacy ($I = 0.1$), and 10% falsity ($F = 0.1$).
- $P^N : S \to [0,1]^3$: Neutrosophic risk function.

$$P^N(s_3) = (0.9, 0.05, 0.05),$$

  indicating a 90% chance of severe climate risks under high emissions (s_3), with 5% indeterminacy and 5% falsity.
- $V^N : S \times T \to \mathbb{R}^3$: Neutrosophic valuation function for environmental impact.

$$V^N(s_3, t_2) = (-100, 0.2, -5),$$

  meaning the environmental cost of state s_3 at t_2 is highly negative ($T = -100$), with 20% uncertainty and -5 representing an overestimated loss.
- $C^N : A \times T \to \mathbb{R}^3$: Neutrosophic cost function for mitigation actions.

$$C^N(a_1, t_1) = (30, 0.1, 2), \quad C^N(a_2, t_2) = (50, 0.2, 5),$$

  where implementing a_1 at t_1 incurs a cost of 30 with 10% indeterminacy, while a_2 at t_2 incurs a higher cost of 50 with 20% uncertainty.
- $R^N : S \times A \to \mathbb{R}^3$: Neutrosophic regulation function for compliance costs.

$$R^N(s_3, a_3) = (0, 0.1, 0),$$

  indicating no additional regulation cost under s_3 with no intervention (a_3).

Neutrosophic Climate Impact. The cumulative neutrosophic climate impact I^N over T is calculated as:

$$I^N = \sum_{t \in T} \sum_{s \in S} \sum_{a \in A} (T_F - F_F) \cdot P^N(s) \cdot V^N(s,t),$$

where T_F and F_F are the truth and falsity degrees, respectively.
Optimal Policy. The optimal mitigation policy π_N^* minimizes the total neutrosophic impact and associated costs:

$$\pi_N^* = \arg\min_{\pi \in \Pi} \left[I^N + \sum_{t \in T} \sum_{a \in A} C^N(a,t) + R^N(s,a) \right].$$

Interpretation. Based on the neutrosophic values:

- Implementing a_1 (renewable energy) reduces emissions to s_2 with high certainty and low indeterminacy, making it a cost-effective option.
- a_2 (carbon capture) achieves results with moderate certainty but incurs higher costs.
- a_3 (no intervention) results in severe climate risks (s_3) with high probability.

The decision-maker uses the neutrosophic framework to weigh uncertainties, evaluate trade-offs, and determine the most effective policy.

3.4.6 | Neutrosophic Social Media Logic

Social Media Logic refers to the principles driving social media platforms, focusing on programmability, popularity, connectivity, and datafication to shape user interactions and content dynamics [377, 368, 76, 187, 296]. This is extended using Neutrosophic Logic. The definition is provided below.

Definition 118 (Social Media Logic)**.** (cf.[377, 368, 76, 187, 296]) Social Media Logic (SML) is a mathematical framework that models the underlying principles governing social media platforms. It is defined as:

$$\text{SML} = (\mathcal{P}, \mathcal{L}, \mathcal{C}, \mathcal{D}),$$

where:

- *Programmability* ($\mathcal{P}$): A bidirectional function $\mathcal{P} : (U \times A) \to (R \times A')$, where:
  - U: Set of users,
  - A: Set of algorithms,
  - R: Set of platform responses,
  - A': Updated state of algorithms based on user interactions.
- *Popularity* ($\mathcal{L}$): A scalar function $\mathcal{L} : C \to \mathbb{R}^+$, where C is the set of content items, and $\mathcal{L}(c)$ quantifies the popularity of content c using a weighted sum of metrics.
- *Connectivity* ($\mathcal{C}$): A dynamic graph $G = (V, E)$, where:
  - $V = U \cup C$: Set of users and content,
  - $E \subseteq V \times V$: Set of directed edges representing relationships or interactions.
- *Datafication* ($\mathcal{D}$): A function $\mathcal{D} : E \to \mathbb{R}^n$, mapping each edge $e \in E$ to a vector of numerical features describing interaction attributes.

Example 119 (Components of Social Media Logic)**.** Consider a simplified social media scenario:

- *Programmability ($\mathcal{P}$):* User u_1 interacts with algorithm a_1, resulting in a response r_1 (e.g., recommended content), and updates the algorithm to state a_1':

$$\mathcal{P}(u_1, a_1) = (r_1, a_1').$$

- *Popularity ($\mathcal{L}$):* The popularity of a post c_1 is calculated as:

$$\mathcal{L}(c_1) = w_1 \cdot \text{Likes} + w_2 \cdot \text{Shares} + w_3 \cdot \text{Comments},$$

  where w_1, w_2, w_3 are weights assigned to each metric.
- *Connectivity ($\mathcal{C}$):* The platform is represented as a graph $G = (V, E)$, where:

$$V = \{u_1, u_2, c_1, c_2\}, \quad E = \{(u_1, c_1), (c_1, u_2)\}.$$

- *Datafication ($\mathcal{D}$):* An edge $e = (u_1, c_1)$ is mapped to a vector representing interaction attributes:

$$\mathcal{D}(e) = [\text{time\_spent}, \text{clicks}, \text{likes}].$$

Definition 120 (Social Media Neutrosophic Logic)**.** Social Media Neutrosophic Logic (SMNL) is a framework for analyzing the uncertainty, indeterminacy, and truthfulness of propositions on social media. It extends classical Social Media Logic by incorporating neutrosophic components. Formally, SMNL is defined as:

$$\text{SMNL} = (\mathcal{P}, \mathcal{L}, \mathcal{C}, \mathcal{D}, \mathcal{N}),$$

where:

- *Programmability* ($\mathcal{P}$): A bidirectional function $\mathcal{P} : (U \times A) \to (R \times A')$, where:
  - U: Set of users,

  - A: Set of algorithms,
  - R: Set of platform responses,
  - A': Updated state of algorithms influenced by user interactions.

- *Popularity* ($\mathcal{L}$): A neutrosophic scalar function $\mathcal{L} : C \to \mathbb{R}^3$, where C is the set of content items, and:
$$\mathcal{L}(c) = (T_c, I_c, F_c),$$
where $T_c, I_c, F_c \in [0, 1]$ represent the truth, indeterminacy, and falsity of content c, satisfying:
$$0 \leq T_c + I_c + F_c \leq 1.$$

- *Connectivity* ($\mathcal{C}$): A dynamic graph $G = (V, E)$, where:
  - $V = U \cup C$: Set of users and content,
  - $E \subseteq V \times V$: Set of directed edges representing relationships or interactions,
  - Each edge $e \in E$ is assigned a neutrosophic value $\mathcal{N}(e) = (T_e, I_e, F_e)$.

- *Datafication* ($\mathcal{D}$): A function $\mathcal{D} : E \to \mathbb{R}^n$, mapping edges $e \in E$ to feature vectors of quantified interaction data.

- *Neutrosophic Evaluation* ($\mathcal{N}$): A mapping $\mathcal{N} : P \to \mathbb{R}^3$, where P represents propositions about user interactions, platform algorithms, or content. For any P, we have:
$$\mathcal{N}(P) = (T, I, F),$$
where $T, I, F \in [0, 1]$ denote the degrees of truth, indeterminacy, and falsity, satisfying $0 \leq T + I + F \leq 1$.

Remark 121 (Neutrosophic Social Media Logic)**.** *Fuzzy Social Media Logic is a special case of Neutrosophic Social Media Logic where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Social Media Logic is notable for its ability to generalize both Neutrosophic and Fuzzy Social Media Logic.*

Example 122 (Application of SMNL Components)**.** A social media platform is a digital environment enabling users to create, share, and interact with content, fostering communication, networking, and engagement (cf.[68, 62]).

Consider a social media platform evaluating a post c_1:

- *Programmability ($\mathcal{P}$):* User u_1 interacts with the platform's algorithm a_1, which generates a response r_1 (e.g., content recommendation) and updates itself to state a_1':
$$\mathcal{P}(u_1, a_1) = (r_1, a_1').$$

- *Popularity ($\mathcal{L}$):* The post c_1 is evaluated as:
$$\mathcal{L}(c_1) = (T_{c_1}, I_{c_1}, F_{c_1}) = (0.7, 0.2, 0.1),$$
indicating high truthfulness, moderate indeterminacy, and low falsity.

- *Connectivity ($\mathcal{C}$):* The platform graph $G = (V, E)$ includes nodes $V = \{u_1, u_2, c_1, c_2\}$ and edges $E = \{(u_1, c_1), (c_1, u_2)\}$. The edge (u_1, c_1) has a neutrosophic value:
$$\mathcal{N}((u_1, c_1)) = (T_e, I_e, F_e) = (0.8, 0.1, 0.1).$$

- *Datafication ($\mathcal{D}$):* The edge (u_1, c_1) is mapped to a feature vector:
$$\mathcal{D}((u_1, c_1)) = [\text{time\_spent}, \text{clicks}, \text{likes}] = [300, 5, 10].$$

- *Neutrosophic Evaluation ($\mathcal{N}$):* A proposition P: "Post c_1 is reliable" is evaluated as:
$$\mathcal{N}(P) = (T, I, F) = (0.7, 0.2, 0.1).$$

Theorem 123. *Social Media Neutrosophic Logic inherently possesses the structure of a Neutrosophic Logic.*

Proof: This result follows directly from the definition of Social Media Neutrosophic Logic, as it extends the principles and components of Neutrosophic Logic to the domain of social media. □

Theorem 124. *Social Media Neutrosophic Logic inherently possesses the structure of a Social Media Logic.*

Proof: This result follows directly from the definition of Social Media Neutrosophic Logic, as it adapts the principles and mechanisms of Social Media Logic within a neutrosophic framework. □

Theorem 125. *The neutrosophic popularity function $\mathcal{L}(c)$ in SMNL balances truth, indeterminacy, and falsity to model content evaluation.*

Proof: The popularity function $\mathcal{L}(c)$ in SMNL maps each content item $c \in C$ to a neutrosophic value:

$$\mathcal{L}(c) = (T_c, I_c, F_c), \quad T_c, I_c, F_c \in [0,1], \quad T_c + I_c + F_c \leq 1.$$

This representation balances:

- T_c: The degree to which the content is truthful or reliable.
- I_c: The degree of uncertainty or ambiguity in evaluating the content.
- F_c: The degree to which the content is false or unreliable.

The constraint $T_c + I_c + F_c \leq 1$ ensures that the evaluation is consistent and accounts for all available information. By incorporating I_c, SMNL captures ambiguity that deterministic or probabilistic models cannot, providing a nuanced evaluation of content. □

Theorem 126. *SMNL explicitly models uncertainty and conflict in social media interactions through neutrosophic connectivity $\mathcal{C}$.*

Proof: The connectivity component $\mathcal{C}$ in SMNL is a dynamic graph $G = (V, E)$, where:

$$\mathcal{N}(e) = (T_e, I_e, F_e), \quad T_e, I_e, F_e \in [0,1], \quad T_e + I_e + F_e \leq 1.$$

For each edge $e \in E$, the neutrosophic value $\mathcal{N}(e)$ represents:

- T_e: The degree to which the interaction is meaningful or reliable.
- I_e: The degree of uncertainty or ambiguity in the interaction.
- F_e: The degree to which the interaction is misleading or false.

By modeling interactions with $\mathcal{N}(e)$, SMNL captures the uncertainty and conflict inherent in social media interactions, enabling a more accurate analysis of network dynamics. □

Theorem 127. *SMNL enhances decision-making by integrating neutrosophic evaluations into the programmability component $\mathcal{P}$.*

Proof: In SMNL, the programmability function $\mathcal{P}$ is defined as:

$$\mathcal{P} : (U \times A) \to (R \times A'),$$

where U is the set of users, A is the set of algorithms, R is the set of platform responses, and A' is the updated state of algorithms. By incorporating neutrosophic evaluations $\mathcal{N}(P) = (T, I, F)$ for propositions P about user interactions or algorithm behavior, $\mathcal{P}$ enables algorithms to:

- Prioritize responses with high T (truthfulness).
- Mitigate decisions with high I (uncertainty).
- Avoid actions with high F (falsity).

This integration ensures that platform decisions are robust and adaptive to uncertainty and conflicting information. □

3.4.7 | Neutrosophic Critical Service Logic

In the field of social science, Service Logic is well recognized. As a related concept, Critical Service Logic is also known. Critical Service Logic focuses on understanding value creation through interactions, emphasizing customer experiences, resources, and context within service ecosystems [148]. This framework is extended using Neutrosophic Logic to incorporate uncertainty, indeterminacy, and falsity into the analysis of value co-creation processes. Definitions and formalizations are provided below.

Definition 128 (Neutrosophic Critical Service Logic)**.** Neutrosophic Critical Service Logic (NCSL) is a mathematical framework for value creation and co-creation under uncertainty, ambiguity, and conflict, using neutrosophic components of truth (T), indeterminacy (I), and falsity (F). Formally, NCSL is defined as:

$$\mathcal{NCSL} = (\mathcal{A}, \mathcal{R}, \mathcal{V}^N, \mathcal{E}^N, \mathcal{D}^N, \mathcal{N}),$$

where:

(1) $\mathcal{A} = \{A_1, A_2, \dots, A_n\}$: A set of actors in the service system, where each actor A_i integrates resources for value creation.

$$A_i = (\text{role}, \text{capabilities}, \mathcal{N}).$$

(2) $\mathcal{R} = \{R_1, R_2, \dots, R_p\}$: A set of resources, where each resource R_k includes:

$$R_k = (\text{financial}, \text{human}, \text{technological}, \mathcal{N}),$$

and each resource effectiveness is evaluated as:

$$\mathcal{N}(R_k) = (T_{R_k}, I_{R_k}, F_{R_k}), \quad T_{R_k} + I_{R_k} + F_{R_k} \leq 1.$$

(3) $\mathcal{V}^N = \{V_1^N, V_2^N, \dots, V_m^N\}$: A set of neutrosophic value functions. Each value function V_j^N maps time horizons to neutrosophic evaluations:

$$V_j^N : \mathcal{T} \to \mathbb{R}^3, \quad V_j^N(t) = (T_j(t), I_j(t), F_j(t)).$$

(4) $\mathcal{E}^N = \{E_1^N, E_2^N, \dots, E_q^N\}$: A set of neutrosophic environmental states affecting value co-creation, where:

$$E_h^N : \mathcal{T} \to \mathbb{R}^3, \quad E_h^N(t) = (T_{E_h}(t), I_{E_h}(t), F_{E_h}(t)).$$

(5) $\mathcal{D}^N = \{D_1^N, D_2^N, \dots, D_r^N\}$: A set of neutrosophic decisions, where each decision D_l^N is defined as:

$$D_l^N : (\mathcal{R}, \mathcal{E}^N) \to \mathcal{V}^N.$$

(6) $\mathcal{N}$: A neutrosophic evaluation function assigning a truth value to propositions P about actors, resources, or environmental states:

$$\mathcal{N}(P) = (T_P, I_P, F_P), \quad T_P, I_P, F_P \in [0, 1], \quad T_P + I_P + F_P \leq 1.$$

Remark 129 (Neutrosophic Critical Service Logic)**.** *Fuzzy Critical Service Logic is a special case of Neutrosophic Critical Service Logic where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Critical Service Logic is notable for its ability to generalize both Neutrosophic and Fuzzy Critical Service Logic.*

Example 130 (Neutrosophic Critical Service Logic in Renewable Energy)**.** Renewable energy is energy derived from naturally replenishing sources like sunlight, wind, water, and biomass, providing sustainable, eco-friendly power [56, 92, 214, 369]. Consider a renewable energy service ecosystem where stakeholders collaborate to create sustainable energy solutions under uncertainty.

- *Actors ($\mathcal{A}$)*:

  $$\mathcal{A} = \{\text{Energy Providers}, \text{Governments}, \text{Investors}, \text{Consumers}\}.$$

  Each actor A_i integrates resources for value creation. For instance:

  $$A_{\text{Investors}} = (\text{Financial support}, \text{Capital allocation}, \mathcal{N} = (T = 0.8, I = 0.15, F = 0.05)).$$

- *Resources ($\mathcal{R}$)*: Resources include financial investments, technological infrastructure, and human expertise:
$$R_1 = (\$10\,\text{Million}, 20\,\text{Engineers}, \text{Solar Panels}, (T = 0.9, I = 0.05, F = 0.05)).$$

- *Neutrosophic Value Functions ($\mathcal{V}^N$)*: The value "energy production efficiency" is measured over a year as:
$$V^N_{\text{efficiency}}(t) = (T_{\text{efficiency}}, I_{\text{efficiency}}, F_{\text{efficiency}}),$$
where:
$$V^N_{\text{efficiency}}(12\,\text{months}) = (0.75, 0.2, 0.05).$$

- *Neutrosophic Environmental States ($\mathcal{E}^N$)*: External factors such as government subsidies and climate conditions influence outcomes:
$$E^N_{\text{subsidy}}(t) = (0.7, 0.2, 0.1).$$

- *Neutrosophic Decisions ($\mathcal{D}^N$)*: A decision to invest in solar energy is evaluated based on resources and environmental states:
$$D^N_{\text{solar}}(R_1, E^N_{\text{subsidy}}) = V^N_{\text{efficiency}}.$$

In this example, NCSL quantifies the uncertainties (I) and risks (F) involved in renewable energy investments, allowing stakeholders to make informed and balanced decisions.

Theorem 131. *The Neutrosophic Critical Service Logic exhibits the structure of a Neutrosophic Set.*

Proof: The result follows directly from the definition. □

Theorem 132. *The Neutrosophic Critical Service Logic exhibits the structure of a Classic Critical Service Logic.*

Proof: The result follows directly from the definition. □

Theorem 133 (Non-negativity of Neutrosophic Components)**.** *For any neutrosophic evaluation $\mathcal{N}(P) = (T_P, I_P, F_P)$ in NCSL, the components T_P, I_P, and F_P are non-negative:*
$$T_P \geq 0, \quad I_P \geq 0, \quad F_P \geq 0.$$

Proof: By the definition of the neutrosophic evaluation function:
$$\mathcal{N}(P) = (T_P, I_P, F_P), \quad \text{where } T_P, I_P, F_P \in [0, 1].$$
The interval $[0, 1]$ imposes the lower bound 0 for T_P, I_P, and F_P. Hence, the components are non-negative:
$$T_P \geq 0, \quad I_P \geq 0, \quad F_P \geq 0.$$
□

Theorem 134 (Bounded Sum of Neutrosophic Components)**.** *For any neutrosophic evaluation $\mathcal{N}(P) = (T_P, I_P, F_P)$ in NCSL, the sum of components is bounded:*
$$T_P + I_P + F_P \leq 1.$$

Proof: By the definition of the neutrosophic evaluation function, we have:
$$\mathcal{N}(P) = (T_P, I_P, F_P), \quad T_P, I_P, F_P \in [0, 1].$$
The condition $T_P + I_P + F_P \leq 1$ ensures that the total evaluation remains within the valid range. If any of the components T_P, I_P, or F_P increase, the other components must decrease to satisfy this bound. Thus:
$$T_P + I_P + F_P \leq 1.$$
□

Theorem 135 (Optimal Neutrosophic Decision-Making)**.** *Given a set of resources $\mathcal{R}$ and environmental states $\mathcal{E}^N$, a neutrosophic decision D^N is optimal if it maximizes the truth component T while minimizing indeterminacy I and falsity F:*

$$D^N_{optimal} = \arg\max_{D^N_l \in \mathcal{D}^N} \left(T_{D_l} - I_{D_l} - F_{D_l} \right).$$

Proof: Let D^N_l be a neutrosophic decision such that:

$$D^N_l : (\mathcal{R}, \mathcal{E}^N) \to \mathcal{V}^N, \quad D^N_l = (T_{D_l}, I_{D_l}, F_{D_l}).$$

The optimal decision D^N_{optimal} seeks to balance the neutrosophic components by maximizing the truth T_{D_l} and simultaneously minimizing the indeterminacy I_{D_l} and falsity F_{D_l}. Formally:

$$D^N_{\text{optimal}} = \arg\max_{D^N_l \in \mathcal{D}^N} \left(T_{D_l} - I_{D_l} - F_{D_l} \right),$$

subject to the constraint:

$$T_{D_l} + I_{D_l} + F_{D_l} \leq 1.$$

This ensures that the decision D^N_{optimal} satisfies the neutrosophic bounds while optimizing the value for the decision-maker. □

4 | Future Tasks: Various Extensions

This section provides a brief overview of the future prospects of this research.

It is important to note that the concepts defined in this Future Tasks section are merely examples and hold significant potential for improvement depending on the objectives and perspectives involved. However, by engaging in such mathematical modeling, we believe that these concepts can be analyzed using various existing mathematical frameworks and logics.

Further exploration of these definitions, their applications, and related research developments are expected to progress in the future.

4.1 | Real-World Applications within a New Social Framework

In this subsection, we discuss potential real-world applications within an uncertain social framework.

4.1.1 | Plithogenic Social Framework

As previously mentioned, the plithogenic set is widely recognized for its flexibility and its ability to generalize Fuzzy Sets and Neutrosophic Sets [314, 352, 20, 327, 304, 325, 326, 341]. Owing to its versatile structure, the plithogenic set holds significant potential for real-world applications. In this study, we propose extending the plithogenic set framework to established methodologies such as Neutrosophic Psychology, PDCA, DMAIC, SWOT, and OODA. By integrating plithogenic sets into these frameworks, our aim is to explore their interconnections and enhance their capability to address complex, multi-dimensional, and contradictory scenarios effectively.

Below, we outline conceptual definitions for applying plithogenic sets to these systems:

- *Plithogenic Body-Mind-Soul-Spirit Fluidity*: A framework capturing multi-attribute dynamics in psychological and spiritual contexts, enabling nuanced assessments of human decision-making and well-being.
- *Plithogenic PDCA*: An extension of the Plan-Do-Check-Act cycle that incorporates multi-criteria and contradictory attributes for more effective quality improvement and problem-solving.
- *Plithogenic DMAIC*: A generalized approach to Define-Measure-Analyze-Improve-Control, leveraging plithogenic attributes to address complex operational challenges in Six Sigma processes.
- *Plithogenic SWOT*: An enriched version of Strengths-Weaknesses-Opportunities-Threats analysis, integrating multi-dimensional perspectives and contradictions for strategic decision-making.
- *Plithogenic OODA*: A plithogenic adaptation of the Observe-Orient-Decide-Act loop, enabling flexible and adaptive responses in dynamic and uncertain environments.

- *Plithogenic Five Forces*: An extension of Porter's Five Forces framework, incorporating multi-attribute and contradictory factors to analyze industry competition with greater flexibility and precision.

Theorem 136 (Generalization of Fuzzy and Neutrosophic Concepts in Plithogenic Frameworks)**.** *The frameworks of Plithogenic Body-Mind-Soul-Spirit Fluidity, Plithogenic PDCA, Plithogenic DMAIC, Plithogenic SWOT, Plithogenic OODA, and Plithogenic Five Forces extend the Fuzzy and Neutrosophic concepts by integrating multi-attribute, multi-criteria, and contradictory characteristics. These generalizations facilitate the modeling and analysis of complex, multi-dimensional, and dynamic systems.*

Proof: The claim is evident from the definitions of the Plithogenic frameworks. Similar proofs have been provided in the literature [125, 119, 129]. Readers may refer to these works for detailed justifications if needed. □

By applying plithogenic sets to these widely used frameworks, we hope to provide more robust tools for decision-making, strategic planning, and continuous improvement in diverse real-world contexts.

4.1.2 | Hyperanalysis and Hypercycle

We also hope that concepts such as Hyperanalysis/Hypercycle and Superhyperanalysis/SuperHypercycle, which hierarchically represent the ideas presented in this paper, will be explored as needed. These approaches are envisioned as applications of hyperstructure [41, 51] and superhyperstructure [117, 121, 334, 118, 122, 154, 331] principles to the concepts introduced in this study.

First, we provide the definitions related to hyperstructure and superhyperstructure below. In set theory, hyperstructure and superhyperstructure can be viewed as the power set and nth-superhyperset, respectively. Intuitively, they represent iterative structures. For detailed definitions of Hyperstructure and Superhyperstructure, readers are encouraged to refer to relevant works such as [335, 318, 122] as needed.

Definition 137 (Powerset)**.** [118] The *powerset* of a set S, denoted by $\mathcal{P}(S)$, is the set of all subsets of S, including both the empty set and S itself. Formally:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 138 (Hyperoperation)**.** (cf.[386, 387, 294, 388]) A *hyperoperation* is an extension of a traditional binary operation where the result of applying the operation to two elements is a subset of the base set rather than a single element. Formally, given a set S, a hyperoperation $\circ$ is defined as:

$$\circ : S \times S \to \mathcal{P}(S),$$

where $\mathcal{P}(S)$ is the powerset of S.

Definition 139 (Hyperstructure)**.** (cf.[335, 118, 318]) A *Hyperstructure* is a mathematical construct that generalizes operations on a set using its powerset. Formally, it is defined as:

$$\mathcal{H} = (\mathcal{P}(S), \circ),$$

where:

- S is the underlying base set.
- $\mathcal{P}(S)$ denotes the powerset of S, which includes all subsets of S.
- $\circ$ is an operation acting on the elements of $\mathcal{P}(S)$.

Definition 140 (n-th Powerset)**.** (cf.[335, 118, 318]) The *n-th powerset* of a set H, denoted as $\mathcal{P}_n(H)$, is constructed recursively through successive powerset operations. Specifically:

$$\mathcal{P}_1(H) = \mathcal{P}(H), \quad \mathcal{P}_{n+1}(H) = \mathcal{P}(\mathcal{P}_n(H)) \quad \text{for } n \geq 1.$$

Similarly, the *n-th non-empty powerset*, denoted as $\mathcal{P}^*_n(H)$, excludes the empty set at each level and is defined as:

$$\mathcal{P}^*_1(H) = \mathcal{P}^*(H), \quad \mathcal{P}^*_{n+1}(H) = \mathcal{P}^*(\mathcal{P}^*_n(H)),$$

where $\mathcal{P}^*(H)$ represents the standard powerset $\mathcal{P}(H)$ with the empty set removed.

Definition 141 (SuperHyperOperations)**.** (cf.[335]) Let H be a non-empty set, and let $\mathcal{P}(H)$ represent the powerset of H. The n-th powerset, denoted as $\mathcal{P}^n(H)$, is recursively defined as:

$$\mathcal{P}^0(H) = H, \quad \mathcal{P}^{k+1}(H) = \mathcal{P}(\mathcal{P}^k(H)), \quad \forall k \geq 0.$$

A *SuperHyperOperation* of order (m, n) is an m-ary operation expressed as:

$$\circ^{(m,n)} : H^m \to \mathcal{P}^n_*(H),$$

where $\mathcal{P}^n_*(H)$ denotes the n-th powerset of H, with two variations depending on inclusion or exclusion of the empty set:

- If the codomain excludes the empty set, the operation is referred to as a *classical-type* (m, n)-*SuperHyperOperation*.
- If the codomain includes the empty set, it is termed a *Neutrosophic* (m, n)-*SuperHyperOperation*.

These SuperHyperOperations generalize hyperoperations to higher-order structures, accommodating multi-layered relationships through iterative powerset constructions.

Definition 142 (n-Superhyperstructure)**.** (cf.[335, 318]) An n-*Superhyperstructure* is an advanced extension of a hyperstructure that incorporates n-fold iterations of the powerset operation. It is defined as:

$$\mathcal{SH}_n = (\mathcal{P}_n(S), \circ),$$

where:

- S is the base set.
- $\mathcal{P}_n(S)$ represents the n-th powerset of S, obtained through recursive applications of the powerset operation.
- $\circ$ is an operation defined on elements of $\mathcal{P}_n(S)$.

The aforementioned concepts of hyperstructure and superhyperstructure can be applied not only to various mathematical frameworks but also to concepts beyond pure mathematics. Consequently, it is natural to consider their applicability to the ideas presented in this paper. For instance, the definitions of the PDCA Hypercycle and PDCA n-SuperhyperCycle are provided above. We anticipate further exploration of these frameworks and their potential applications to other models.

Definition 143 (PDCA Hypercycle)**.** A *PDCA Hypercycle* is defined as:

$$\mathcal{H}_{PDCA} = (\mathcal{P}(S), \circ),$$

where S is a set of system states, and $\circ$ maps:

$$\mathcal{H}_{PDCA}(X) = A(C(D(P(X)))), \quad X \subseteq S.$$

Example 144 (PDCA Hypercycle in Quality Management)**.** Consider a manufacturing process aimed at improving product quality using the PDCA (Plan-Do-Check-Act) Hypercycle framework. The process can be described as follows:

- $S = \{s_1, s_2, \dots, s_5\}$: A set of system states, where each s_i represents a different stage of product quality, such as:

$$s_1 = \text{Initial state},\ s_2 = \text{Design stage},\ s_3 = \text{Production stage},\ s_4 = \text{Quality inspection},\ s_5 = \text{Defect correction}.$$

- $\mathcal{P}(S)$: The powerset of S, capturing all subsets of system states $X \subseteq S$, such as:

$$X = \{s_2, s_3\},\ \mathcal{P}(X) = \{\{s_2\}, \{s_3\}, \{s_2, s_3\}\}.$$

- The PDCA Hypercycle operates through the following steps:

(1) $P(X)$: **Plan** phase – Define quality objectives and prepare production plans for the subset of states $X = \{s_2, s_3\}$. For example, improving the defect rate by optimizing production parameters.

(2) $D(X)$: **Do** phase – Implement the plans, such as testing new production methods or upgrading machinery in states s_2 and s_3.

(3) $C(X)$: **Check** phase – Evaluate the outcomes of the Do phase by inspecting the quality results and collecting metrics, such as:

$$\text{Defect rate reduced from 5\% to 3\%.}$$

(4) $A(X)$: **Act** phase – Adjust processes based on the Check phase results. For instance, fine-tune the machine settings further or update training protocols for workers.

The PDCA Hypercycle iteratively refines X, evolving system states through higher-order feedback loops. The process can be expressed mathematically as:

$$\mathcal{H}_{PDCA}(X) = A(C(D(P(X)))).$$

Outcome. After multiple iterations of the PDCA Hypercycle, the system achieves an improved state with a defect rate of 1%, meeting the quality target.

Definition 145 (PDCA n-SuperhyperCycle)**.** A *PDCA n-SuperhyperCycle* is defined as:

$$\mathcal{SH}^n_{PDCA} = (\mathcal{P}^n(S), \circ^{(4,n)}),$$

where $\mathcal{P}^n(S)$ is the n-th powerset of S, and:

$$\mathcal{SH}^n_{PDCA}(X) = A^n \circ C^n \circ D^n \circ P^n(X), \quad X \in \mathcal{P}^n(S).$$

Example 146 (PDCA n-SuperhyperCycle in Project Management)**.** In a complex project management scenario:

- S: Tasks $\{T_1, T_2, \dots, T_5\}$.
- $\mathcal{P}^2(S)$: Powerset of subsets of tasks, capturing interdependent subtasks and their groupings.

The PDCA 2-SuperhyperCycle proceeds as follows:

(1) $P^2(X)$: Generates plans across grouped subtasks. For example:

$$P^2(X) = \{\{T_1, T_2\}, \{T_3, T_4\}\}.$$

(2) $D^2(X)$: Executes actions on these subsets, producing partial results.

(3) $C^2(X)$: Evaluates subset outcomes, such as task completion percentages.

(4) $A^2(X)$: Adjusts task groupings and priorities based on evaluation.

The process evolves X iteratively through multi-level refinement, achieving higher-order optimization.

For clarification, as with the PDCA Hypercycle and n-SuperhyperCycle, the following concepts are defined. We look forward to further research and advancements in these areas.

- *DMAIC Hypercycle*: A generalized approach to Define-Measure-Analyze-Improve-Control, leveraging hyperstructure attributes to address complex operational challenges within Six Sigma processes.
- *SWOT Hyperanalysis*: An enhanced version of the Strengths-Weaknesses-Opportunities-Threats analysis, integrating multi-dimensional perspectives and interdependencies to improve strategic decision-making.
- *OODA Hypercycle*: A hyperstructure-based adaptation of the Observe-Orient-Decide-Act loop, enabling flexible and adaptive responses in dynamic and uncertain environments.
- *Five Forces Hyperanalysis*: An extended version of Porter's Five Forces framework, incorporating multi-attribute and interdependent factors to analyze industry competition with greater precision and adaptability.

- *DMAIC n-Superhypercycle*: A higher-order extension of the Define-Measure-Analyze-Improve-Control process, addressing n-fold complexities through multi-level operational analysis.
- *SWOT n-Superhyperanalysis*: A multi-level enhancement of SWOT analysis, incorporating n-fold dimensions and contradictions to enable comprehensive strategic planning and decision-making.
- *OODA n-Superhypercycle*: A higher-order adaptation of the Observe-Orient-Decide-Act loop, capturing n-fold interdependencies to support adaptive and resilient decision-making in uncertain environments.
- *Five Forces n-Superhyperanalysis*: An advanced extension of Porter's Five Forces model, integrating n-fold multi-attribute and hierarchical structures to analyze industry competition with greater depth and flexibility.

4.1.3 | Other Frameworks

In addition to the frameworks discussed in this paper, numerous others are developed daily across various fields. For example:

- **COBIT** (Control Objectives for Information and Related Technologies) [273, 172],
- **BADIR** (Business Question, Analysis Plan, Data Collection, Insights Derivation, Recommendations) [175],
- **ITIL** (Information Technology Infrastructure Library) [227, 135],
- **Five Whys** [303, 39, 353],
- **Kanban** [5, 365],
- **VRIO** (Value, Rarity, Imitability, Organization) [235, 192],
- **OGSM** (Objectives, Goals, Strategies, and Measures) [274, 246, 213],
- **PEST Analysis** [78, 220, 106].

We hope to explore the potential for extending these frameworks using concepts such as Neutrosophic Structures, Uncertain Structures, and Superhyperstructures. Future research may focus on examining the mathematical structures of these extended frameworks and exploring their applications in fields such as social sciences.

4.2 | New Strategic Leadership

4.2.1 | Neutrosophic Strategic Leadership

In addition to the concepts discussed in this paper, the neutrosophic framework can be applied to a variety of other fields and ideas. As an example, we introduce the concept of *Neutrosophic Strategic Leadership*.

Leadership refers to the ability to influence, guide, and inspire individuals or groups to achieve objectives through effective communication, motivation, and vision [57, 40]. Strategic Leadership, in particular, focuses on balancing short-term goals with long-term vision, emphasizing resource allocation, adaptability, and organizational alignment to ensure sustained success [108, 109, 384].

The related definitions and formalizations are presented below. It is important to note that leadership itself is a multifaceted concept that can be defined and studied from various perspectives, depending on the context or scope of analysis. The definitions provided here represent only one example among many.

We hope that future research will further explore concepts like Neutrosophic Strategic Leadership and its applications. Additionally, many related leadership frameworks have been studied extensively in existing literature, including examples such as *servant leadership* [299], *meta-leadership* [226, 225, 95], *e-leadership* [84, 29, 184], *Agile leadership*[28, 181], and *followership* [370, 36], among others.

Definition 147 (Classic Leadership)**.** Classic Leadership is a structured decision-making framework that formalizes the process of directing, influencing, and coordinating individuals or groups to achieve organizational goals. It is mathematically defined as a tuple:

$$\mathcal{L}_{CL} = (\mathcal{A}, \mathcal{T}, \mathcal{R}, \mathcal{S}, \mathcal{P}),$$

where:

(1) $\mathcal{A} = \{A_1, A_2, \dots, A_n\}$: A set of agents (leaders and followers), where each agent A_i has attributes:

$$A_i = (\text{role}, \text{capabilities}, \text{preferences}).$$

Here:

- role $\in \{\text{Leader}, \text{Follower}\}$ defines the agent's position.
- capabilities $\in \mathbb{R}^d$ represents the skillset or competence vector in d-dimensional space.
- preferences $\in \mathbb{R}^k$ indicates the agent's goals or utility preferences.

(2) $\mathcal{T} = \{T_1, T_2, \dots, T_m\}$: A set of tasks to be accomplished, where each task T_j is defined as:

$$T_j = (\mathcal{R}_j, \mathcal{O}_j, \mathcal{C}_j),$$

with:

- $\mathcal{R}_j$: Resource requirements for T_j.
- $\mathcal{O}_j$: The output or measurable outcome of T_j.
- $\mathcal{C}_j$: Constraints, such as deadlines or quality thresholds.

(3) $\mathcal{R} = \{R_1, R_2, \dots, R_p\}$: A set of resources required to execute tasks, where each resource R_k has a finite capacity:

$$R_k = (\text{type}, \text{capacity}), \quad \text{capacity} \in \mathbb{R}^+.$$

(4) $\mathcal{S} = \{S_1, S_2, \dots, S_q\}$: A set of strategies for resource allocation and task assignment, where each strategy S_l maps agents and resources to tasks:

$$S_l : \mathcal{A} \times \mathcal{R} \to \mathcal{T}.$$

(5) $\mathcal{P} = \{P_1, P_2, \dots, P_r\}$: A set of performance metrics to evaluate leadership effectiveness. Each performance metric P_h is a mapping:

$$P_h : \mathcal{T} \to \mathbb{R},$$

where $P_h(T_j)$ measures the success or efficiency of completing task T_j.

Remark 148 (Components of Classic Leadership)**.** *Classic Leadership focuses on task execution and organizational performance by:*

- *Aligning agents ($\mathcal{A}$) with appropriate tasks ($\mathcal{T}$) using their capabilities.*
- *Optimizing resource allocation ($\mathcal{R}$) under constraints.*
- *Selecting strategies ($\mathcal{S}$) to achieve goals efficiently.*
- *Evaluating performance ($\mathcal{P}$) based on measurable outcomes.*

Example 149 (Classic Leadership in a Project Management Scenario)**.** Consider a project with three agents, two tasks, and finite resources:

$$\mathcal{A} = \{A_1 : \text{Leader}, A_2 : \text{Follower}, A_3 : \text{Follower}\},$$

$$\mathcal{T} = \{T_1 : \text{Design Phase}, T_2 : \text{Implementation Phase}\},$$

$$\mathcal{R} = \{R_1 : \text{Budget} = \$10{,}000, R_2 : \text{Human Resources} = 5 \text{ engineers}\}.$$

The leader A_1 assigns resources and strategies:

$$S_1(A_2, R_1) \to T_1, \quad S_1(A_3, R_2) \to T_2.$$

The performance metrics P_1 evaluate success:

$$P_1(T_1) = 90\% \text{ completion}, \quad P_1(T_2) = 80\% \text{ completion}.$$

This example demonstrates how Classic Leadership optimizes task execution and resource utilization.

The concept of *Neutrosophic Leadership*, which integrates the principles of Neutrosophic Logic into the above definition of leadership, is presented below.

Definition 150 (Neutrosophic Leadership)**.** Neutrosophic Leadership is a mathematical framework that models leadership under uncertainty, ambiguity, and contradiction by extending classical leadership principles with neutrosophic logic. It incorporates truth (T), indeterminacy (I), and falsity (F) to evaluate decisions, strategies, and resource allocations. Formally, it is defined as:

$$\mathcal{L}_{NL} = (\mathcal{A}, \mathcal{T}, \mathcal{R}^N, \mathcal{S}^N, \mathcal{P}^N),$$

where:

(1) $\mathcal{A} = \{A_1, A_2, \dots, A_n\}$: A set of agents (leaders and followers), where each agent A_i is defined as:

$$A_i = (\text{role}, \text{capabilities}, \mathcal{N}^A),$$

with:

- role $\in$ {Leader, Follower}: The position of the agent.
- capabilities $\in \mathbb{R}^d$: The agent's skillset or competence vector in d-dimensional space.
- $\mathcal{N}^A = (T_{A_i}, I_{A_i}, F_{A_i})$: A neutrosophic evaluation of the agent's effectiveness, where:

$$T_{A_i} \text{ (truth)}, I_{A_i} \text{ (indeterminacy)}, F_{A_i} \text{ (falsity)} \in [0,1], \quad T_{A_i} + I_{A_i} + F_{A_i} \leq 1.$$

(2) $\mathcal{T} = \{T_1, T_2, \dots, T_m\}$: A set of tasks, where each task T_j is described as:

$$T_j = (\mathcal{R}_j^N, \mathcal{O}_j, \mathcal{C}_j),$$

with:

- $\mathcal{R}_j^N$: Neutrosophic resource requirements evaluated as:

$$\mathcal{N}(R_j) = (T_{R_j}, I_{R_j}, F_{R_j}).$$

- $\mathcal{O}_j$: The outcome of task T_j.
- $\mathcal{C}_j$: Task constraints such as deadlines or priorities.

(3) $\mathcal{R}^N = \{R_1^N, R_2^N, \dots, R_p^N\}$: A set of neutrosophic resources, where each resource R_k^N includes:

$$R_k^N = (\text{type}, \text{capacity}, \mathcal{N}^R), \quad \mathcal{N}^R = (T_{R_k}, I_{R_k}, F_{R_k}).$$

(4) $\mathcal{S}^N = \{S_1^N, S_2^N, \dots, S_q^N\}$: A set of neutrosophic strategies that allocate agents and resources to tasks under uncertainty:

$$S_l^N : \mathcal{A} \times \mathcal{R}^N \to \mathcal{T}.$$

Each strategy S_l^N is evaluated as:

$$\mathcal{N}(S_l^N) = (T_{S_l}, I_{S_l}, F_{S_l}).$$

(5) $\mathcal{P}^N = \{P_1^N, P_2^N, \dots, P_r^N\}$: A set of neutrosophic performance metrics to evaluate leadership effectiveness. Each performance metric P_h^N maps tasks to neutrosophic evaluations:

$$P_h^N : \mathcal{T} \to \mathbb{R}^3, \quad P_h^N(T_j) = (T_{P_h}, I_{P_h}, F_{P_h}).$$

Remark 151 (Characteristics of Neutrosophic Leadership)**.** *Neutrosophic Leadership extends classical leadership by:*

- *Incorporating truth, indeterminacy, and falsity components into agents, resources, tasks, and strategies.*

- *Managing ambiguity and uncertainty in decision-making.*
- *Balancing resource allocation and performance evaluations under incomplete information.*

Remark 152 (Neutrosophic Leadership)**.** *Fuzzy Leadership is a special case of Neutrosophic Leadership where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Leadership is notable for its ability to generalize both Neutrosophic and Fuzzy Leadership.*

Example 153 (Neutrosophic Leadership in a Construction Project)**.** Consider a construction project with three agents, two tasks, and limited resources:

$$\mathcal{A} = \{A_1 : \text{Leader}, A_2 : \text{Engineer}, A_3 : \text{Worker}\}.$$

The resources and tasks are as follows:

$$R_1^N = (\text{Budget} = \$50{,}000, \mathcal{N} = (T = 0.8, I = 0.15, F = 0.05)), \quad T_1 = (\mathcal{R}_1^N, \text{Foundation work}, \mathcal{C}_1).$$

The leader A_1 evaluates the strategy S_1^N as:

$$S_1^N(A_2, R_1^N) \to T_1, \quad \mathcal{N}(S_1^N) = (T_{S_1} = 0.85, I_{S_1} = 0.1, F_{S_1} = 0.05).$$

The performance metric P_1^N for task T_1 is evaluated as:

$$P_1^N(T_1) = (T_{P_1} = 0.8, I_{P_1} = 0.15, F_{P_1} = 0.05).$$

This example demonstrates how neutrosophic leadership handles uncertainty and evaluates performance with truth, indeterminacy, and falsity components.

Theorem 154. *Neutrosophic Leadership generalizes Classical Leadership by incorporating uncertainty, indeterminacy, and falsity into all components of leadership.*

Proof: By definition, Classical Leadership uses precise values for agents, resources, and tasks. In Neutrosophic Leadership, these components are extended to include neutrosophic evaluations (T, I, F). Since $T + I + F \leq 1$, Neutrosophic Leadership preserves the classical framework while accommodating uncertainty and contradiction. Hence, Classical Leadership is a special case of Neutrosophic Leadership when $I = 0$ and $F = 0$. □

Theorem 155. *Neutrosophic Leadership improves decision-making under uncertainty compared to Classical Leadership.*

Proof: In Classical Leadership, decisions are based solely on precise values. In Neutrosophic Leadership, decisions incorporate uncertainty (I) and falsity (F) to provide a more robust evaluation. For any strategy S^N, the neutrosophic evaluation:

$$\mathcal{N}(S^N) = (T_S, I_S, F_S),$$

allows leaders to account for ambiguity and risk. By assigning weights to I and F, decisions reflect a realistic assessment of uncertain environments, which improves outcomes. □

Next, the definition of Strategic Leadership is provided below.

Definition 156 (Strategic Leadership)**.** Strategic Leadership is a mathematical framework for decision-making and organizational guidance, balancing short-term and long-term goals through resource allocation, environmental analysis, and stakeholder alignment. Formally, it is defined as a tuple:

$$\mathcal{L}_{SL} = (\mathcal{V}, \mathcal{O}, \mathcal{R}, \mathcal{D}, \mathcal{E}),$$

where:

(1) $\mathcal{V} = \{V_1, V_2, \dots, V_n\}$: A set of organizational visions or objectives, where each V_i is a function:

$$V_i : \mathcal{T} \to \mathbb{R},$$

mapping time horizons $\mathcal{T}$ to a measurable outcome, such as profit, market share, or sustainability.

(2) $\mathcal{O} = \{O_1, O_2, \dots, O_m\}$: A set of operational strategies, where each O_j is defined as:

$$O_j : \mathcal{R} \to \mathcal{V},$$

mapping resources $\mathcal{R}$ to organizational objectives.

(3) $\mathcal{R} = \{R_1, R_2, \dots, R_p\}$: A set of resources, where each R_k is a tuple:

$$R_k = (\text{financial}, \text{human}, \text{technological}),$$

representing resource allocations across critical categories.

(4) $\mathcal{D} = \{D_1, D_2, \dots, D_q\}$: A set of strategic decisions, where each D_l is defined as:

$$D_l : (\mathcal{O}, \mathcal{E}) \to \mathcal{V},$$

mapping operational strategies and environmental states to organizational objectives.

(5) $\mathcal{E} = \{E_1, E_2, \dots, E_r\}$: A set of environmental states, where each E_h represents external conditions, modeled as:

$$E_h : \mathcal{T} \to \mathcal{S},$$

with $\mathcal{S}$ being a set of state variables, such as market trends, regulatory changes, or competitive dynamics.

Remark 157 (Components and Relationships)**.** *The framework integrates the following key components:*

- *Vision Alignment: Leaders optimize:*

$$\max_{O_j \in \mathcal{O}} \sum_{i=1}^{n} \alpha_i V_i(T),$$

where α_i represents the weight assigned to each objective V_i at time T.

- *Resource Allocation: Resources $\mathcal{R}$ are allocated by solving:*

$$\min_{R_k \in \mathcal{R}} \left(\sum_{j=1}^{m} \beta_j O_j(R_k) - \gamma \mathcal{C}(R_k) \right),$$

where β_j is the importance of strategy O_j, γ is a penalty factor, and $\mathcal{C}(R_k)$ is the cost function of resource R_k.

- *Adaptability: Strategic decisions $\mathcal{D}$ adapt to environmental states by satisfying:*

$$D_l(O_j, E_h) = \arg\max_{O_j} \sum_{i=1}^{n} \delta_i V_i(E_h(T)),$$

where δ_i represents the sensitivity of V_i to $E_h(T)$.

Remark 158 (The differences between *Strategic Leadership* and *Classical Leadership*)**.** *The differences between Strategic Leadership and Classical Leadership are summarized as follows:*

(1) *Focus and Goals:*

- *Classical Leadership: Task-oriented, focusing on short-term objectives and immediate resource utilization.*
- *Strategic Leadership: Balances short-term goals and long-term visions by aligning resources and strategies for sustainability.*

(2) *Decision-Making Framework:*

- *Classical Leadership: Uses predefined roles and strategies for decision-making.*
- *Strategic Leadership: Incorporates flexibility by dynamically adapting to external conditions.*

(3) *Resource Management:*

- *Classical Leadership: Focuses on resource allocation for immediate task execution.*

- *Strategic Leadership: Dynamically allocates resources to achieve broader, long-term organizational goals.*

(4) *Environmental Adaptability:*

- *Classical Leadership: Assumes a static environment with limited external influence.*
- *Strategic Leadership: Explicitly models external conditions ($\mathcal{E}$) and adapts to changing environments.*

(5) *Evaluation:*

- *Classical Leadership: Evaluates performance using task-specific metrics ($\mathcal{P}$).*
- *Strategic Leadership: Measures success using broader, vision-oriented metrics ($\mathcal{V}$).*

Example 159 (Strategic Leadership in Renewable Energy Development)**.** Consider a renewable energy company aiming to expand its operations by balancing short-term profitability with long-term sustainability. The components of Strategic Leadership $\mathcal{L}_{SL}$ are instantiated as follows:

(1) $\mathcal{V} = \{V_1, V_2, V_3\}$: The organizational visions are defined as:

- $V_1(T)$: Short-term profitability, measured in millions of dollars over time T.
- $V_2(T)$: Long-term sustainability, quantified as the percentage of energy sourced from renewable resources over time T.
- $V_3(T)$: Market share in the renewable energy sector, measured as a percentage over time T.

(2) $\mathcal{O} = \{O_1, O_2, O_3\}$: The operational strategies are:

- $O_1(R)$: Investing in wind energy infrastructure.
- $O_2(R)$: Developing solar energy projects.
- $O_3(R)$: Marketing campaigns to promote renewable energy solutions.

Each strategy O_j maps resource allocations R to organizational objectives $\mathcal{V}$.

(3) $\mathcal{R} = \{R_1, R_2, R_3\}$: The resource allocations are:

$$R_1 = (\$50\,\text{M},\ 200\,\text{employees, wind turbines}),$$
$$R_2 = (\$30\,\text{M},\ 150\,\text{employees, solar panels}),$$
$$R_3 = (\$20\,\text{M},\ 50\,\text{employees, marketing tools}).$$

(4) $\mathcal{D} = \{D_1, D_2\}$: The strategic decisions are:

- $D_1(O, E)$: Allocating 60% of resources to O_1 and 40% to O_2, based on favorable environmental conditions E.
- $D_2(O, E)$: Shifting resources to O_3 during periods of high public demand for renewable energy awareness.

(5) $\mathcal{E} = \{E_1, E_2\}$: The environmental states are:

- $E_1(T)$: Government incentives for renewable energy projects.
- $E_2(T)$: Fluctuations in fossil fuel prices affecting market dynamics.

These states are modeled as functions of time, influencing operational strategies and resource allocations.

Optimization: The company optimizes its strategies by solving:

$$\max_{O_j \in \mathcal{O}} \left(\alpha_1 V_1(T) + \alpha_2 V_2(T) + \alpha_3 V_3(T)\right),$$

where $\alpha_1 = 0.4$, $\alpha_2 = 0.4$, and $\alpha_3 = 0.2$ reflect the relative importance of each objective.

Adaptability: Strategic decisions are adjusted dynamically based on environmental changes. For instance, when $E_1(T)$ increases government subsidies, the company prioritizes O_1 and O_2, maximizing long-term sustainability.

Resource Allocation: Resources R_k are allocated to minimize costs:

$$\min_{R_k \in \mathcal{R}} \left(\sum_{j=1}^{3} \beta_j O_j(R_k) - \gamma \mathcal{C}(R_k) \right),$$

where β_j is the importance of each strategy, and $\mathcal{C}(R_k)$ represents resource costs.

This framework ensures the company achieves its objectives while remaining responsive to market and environmental dynamics.

We extend the above framework using Neutrosophic Sets to introduce Neutrosophic Strategic Leadership. The following outlines this concept. We anticipate that further research and validation of this approach will progress in the future.

Definition 160 (Neutrosophic Strategic Leadership)**.** Neutrosophic Strategic Leadership (NSL) extends classical Strategic Leadership by incorporating uncertainty, indeterminacy, and falsity into the decision-making process. It is defined as:

$$\mathcal{L}_{NSL} = (\mathcal{V}^N, \mathcal{O}^N, \mathcal{R}^N, \mathcal{D}^N, \mathcal{E}^N),$$

where:

(1) $\mathcal{V}^N = \{V_1^N, V_2^N, \dots, V_n^N\}$: A set of neutrosophic organizational visions or objectives, where each V_i^N is a mapping:

$$V_i^N : \mathcal{T} \to \mathbb{R}^3,$$

such that:

$$V_i^N(T) = (T_{V_i}, I_{V_i}, F_{V_i}),$$

where $T_{V_i}, I_{V_i}, F_{V_i} \in [0,1]$ represent the truth, indeterminacy, and falsity of achieving V_i over the time horizon T, satisfying $T_{V_i} + I_{V_i} + F_{V_i} \leq 1$.

(2) $\mathcal{O}^N = \{O_1^N, O_2^N, \dots, O_m^N\}$: A set of neutrosophic operational strategies, where each O_j^N is defined as:

$$O_j^N : \mathcal{R}^N \to \mathcal{V}^N,$$

mapping neutrosophic resource allocations $\mathcal{R}^N$ to neutrosophic organizational objectives $\mathcal{V}^N$.

(3) $\mathcal{R}^N = \{R_1^N, R_2^N, \dots, R_p^N\}$: A set of neutrosophic resources, where each R_k^N is a tuple:

$$R_k^N = (\text{financial}, \text{human}, \text{technological}, \mathcal{N}),$$

and $\mathcal{N}$ assigns a neutrosophic value:

$$\mathcal{N}(R_k^N) = (T_{R_k}, I_{R_k}, F_{R_k}),$$

representing the truth, indeterminacy, and falsity of the effectiveness of resource R_k^N.

(4) $\mathcal{D}^N = \{D_1^N, D_2^N, \dots, D_q^N\}$: A set of neutrosophic strategic decisions, where each D_l^N is defined as:

$$D_l^N : (\mathcal{O}^N, \mathcal{E}^N) \to \mathcal{V}^N,$$

mapping neutrosophic operational strategies and neutrosophic environmental states to neutrosophic organizational objectives.

(5) $\mathcal{E}^N = \{E_1^N, E_2^N, \dots, E_r^N\}$: A set of neutrosophic environmental states, where each E_h^N represents external conditions, modeled as:

$$E_h^N : \mathcal{T} \to \mathbb{R}^3,$$

such that:

$$E_h^N(T) = (T_{E_h}, I_{E_h}, F_{E_h}),$$

where $T_{E_h}, I_{E_h}, F_{E_h} \in [0,1]$ denote the truth, indeterminacy, and falsity of the state variables at time T, satisfying $T_{E_h} + I_{E_h} + F_{E_h} \leq 1$.

Remark 161 (Neutrosophic Strategic Leadership)**.** *Fuzzy Strategic Leadership is a special case of Neutrosophic Strategic Leadership where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Strategic Leadership is notable for its ability to generalize both Neutrosophic and Fuzzy Strategic Leadership.*

Example 162 (Application in Corporate Sustainability)**.** A company evaluates its sustainability strategy (cf.[161, 132, 389]) under uncertain environmental regulations:

- $\mathcal{V}^N$: The objective "achieve 50% renewable energy usage by 2030" is represented as:
$$V_1^N(T) = (T_{V_1}, I_{V_1}, F_{V_1}) = (0.6, 0.3, 0.1).$$
- $\mathcal{O}^N$: Operational strategies include investments in solar and wind energy, each evaluated with neutrosophic values:
$$O^N_{\text{solar}}(R) = (T = 0.7, I = 0.2, F = 0.1).$$
- $\mathcal{R}^N$: Resources for solar investments have a neutrosophic effectiveness:
$$R^N_{\text{solar}} = (\$100M, 500\,\text{employees}, \text{solar panels}, (T = 0.8, I = 0.1, F = 0.1)).$$
- $\mathcal{E}^N$: Environmental state "government incentives for renewables" is represented as:
$$E^N_{\text{incentives}}(T) = (T_E, I_E, F_E) = (0.7, 0.2, 0.1).$$

The framework ensures robust decision-making by balancing T, I, and F across all components.

4.2.2 | HyperLeadership

Furthermore, we anticipate future advancements in the research on the applications and validity of HyperLeadership and n-SuperhyperLeadership, which extend the principles of hyperstructure and superhyperstructure to leadership. Although these ideas remain at the conceptual stage, the definitions are outlined below. As previously mentioned, for detailed definitions of Hyperstructure and Superhyperstructure, readers are encouraged to consult relevant works such as [119, 335, 318, 122] as needed.

Definition 163 (HyperLeadership)**.** HyperLeadership is an extended leadership framework that operates on the powerset of agents, tasks, and resources, capturing hierarchical, multi-level, and interdependent leadership dynamics. It is formally defined as a tuple:
$$\mathcal{H}_{HL} = (\mathcal{P}(\mathcal{A}), \mathcal{P}(\mathcal{T}), \mathcal{P}(\mathcal{R}), \mathcal{P}(\mathcal{S}), \mathcal{P}(\mathcal{P})),$$
where:

(1) $\mathcal{P}(\mathcal{A})$: The powerset of agents, including individual agents and their groupings:
$$\mathcal{P}(\mathcal{A}) = \{A, A' \subseteq \mathcal{A} \mid A \neq \emptyset\}.$$

(2) $\mathcal{P}(\mathcal{T})$: The powerset of tasks, capturing interdependencies between tasks:
$$\mathcal{P}(\mathcal{T}) = \{T, T' \subseteq \mathcal{T} \mid T \neq \emptyset\}.$$

(3) $\mathcal{P}(\mathcal{R})$: The powerset of resources, representing combinations and allocations:
$$\mathcal{P}(\mathcal{R}) = \{R, R' \subseteq \mathcal{R} \mid R \neq \emptyset\}.$$

(4) $\mathcal{P}(\mathcal{S})$: The powerset of strategies, where each strategy subset assigns resources and agents to task subsets:
$$S : \mathcal{P}(\mathcal{A}) \times \mathcal{P}(\mathcal{R}) \to \mathcal{P}(\mathcal{T}).$$

(5) $\mathcal{P}(\mathcal{P})$: The powerset of performance metrics, evaluating leadership effectiveness at various levels:
$$P_h : \mathcal{P}(\mathcal{T}) \to \mathbb{R}.$$

Remark 164. *HyperLeadership extends Classic Leadership by incorporating higher-order interactions among agents, tasks, and resources. It enables hierarchical grouping and complex interrelations across organizational levels.*

Definition 165 (n-SuperhyperLeadership)**.** n-SuperhyperLeadership is a higher-order generalization of HyperLeadership achieved through n-fold applications of the powerset operation. It is formally defined as:

$$\mathcal{SHL}_n = (\mathcal{P}^n(\mathcal{A}), \mathcal{P}^n(\mathcal{T}), \mathcal{P}^n(\mathcal{R}), \mathcal{P}^n(\mathcal{S}), \mathcal{P}^n(\mathcal{P})),$$

where:

(1) $\mathcal{P}^n(\mathcal{A})$: The n-th powerset of agents, recursively defined as:

$$\mathcal{P}^0(\mathcal{A}) = \mathcal{A}, \quad \mathcal{P}^{k+1}(\mathcal{A}) = \mathcal{P}(\mathcal{P}^k(\mathcal{A})),\ k \geq 0.$$

(2) $\mathcal{P}^n(\mathcal{T})$: The n-th powerset of tasks, capturing multi-layered task hierarchies:

$$\mathcal{P}^n(\mathcal{T}) = \mathcal{P}(\mathcal{P}^{n-1}(\mathcal{T})).$$

(3) $\mathcal{P}^n(\mathcal{R})$: The n-th powerset of resources, describing higher-order combinations and allocations:

$$\mathcal{P}^n(\mathcal{R}) = \mathcal{P}(\mathcal{P}^{n-1}(\mathcal{R})).$$

(4) $\mathcal{P}^n(\mathcal{S})$: The n-th powerset of strategies, mapping higher-order subsets of agents and resources to task hierarchies:

$$S_n : \mathcal{P}^n(\mathcal{A}) \times \mathcal{P}^n(\mathcal{R}) \to \mathcal{P}^n(\mathcal{T}).$$

(5) $\mathcal{P}^n(\mathcal{P})$: The n-th powerset of performance metrics, evaluating leadership effectiveness at multi-level task structures:

$$P_n : \mathcal{P}^n(\mathcal{T}) \to \mathbb{R}.$$

Remark 166. *n-SuperhyperLeadership provides a comprehensive framework for analyzing and managing leadership dynamics across multiple organizational layers, accounting for interdependencies, feedback loops, and iterative refinements.*

Example 167 (HyperLeadership in Multi-Team Project Management)**.** Consider a project with three teams of agents ($\mathcal{A}$), six tasks ($\mathcal{T}$), and three types of resources ($\mathcal{R}$):

$$\mathcal{A} = \{\text{Team 1}, \text{Team 2}, \text{Team 3}\}, \quad \mathcal{T} = \{T_1, T_2, \dots, T_6\}, \quad \mathcal{R} = \{R_1, R_2, R_3\}.$$

- HyperLeadership generates subsets of agents, tasks, and resources:

$$\mathcal{P}(\mathcal{A}) = \{\{\text{Team 1}\}, \{\text{Team 2, Team 3}\}, \dots\}.$$

- A strategy S maps teams and resources to tasks:

$$S(\{\text{Team 1, Team 2}\}, \{R_1, R_2\}) \to \{T_1, T_3, T_4\}.$$

- Performance metrics evaluate task outcomes:

$$P(T_1, T_3, T_4) = 85\% \text{ completion}.$$

This example illustrates the role of HyperLeadership in managing interdependent teams, tasks, and resources.

4.3 | New Negotiation Theory

4.3.1 | Neutrosophic Negotiation Theory

Negotiation Theory is the study of strategies and processes that parties use to reach agreements, focusing on balancing interests, alternatives, and outcomes [361, 3, 394, 96, 395]. In Negotiation Theory, the frameworks of BATNA and ZOPA are well known. BATNA refers to the best outcome a party can achieve if negotiations fail, serving as their most advantageous alternative or fallback option [50, 301, 228, 293, 73]. ZOPA is the range of possible agreements where both parties' outcomes overlap, enabling a mutually beneficial deal; outside this range, no rational agreement can be reached[401, 185, 238, 212].

Definition 168 (Best Alternative to a Negotiated Agreement (BATNA))**.** Let N represent a negotiation between two parties, A (Agent 1) and B (Agent 2), where the set of all possible deals is $\mathcal{D} \subseteq \mathbb{R}^2$.

The *Best Alternative to a Negotiated Agreement (BATNA)* for each party is the utility associated with their best achievable outcome if no agreement is reached. Formally:

$$\text{BATNA}_i = \max_{\alpha \in \mathcal{A}_i} U_i(\alpha), \quad i \in \{A, B\},$$

where:

- $\mathcal{A}_i$: The set of alternatives available to party i outside the current negotiation N (e.g., other partners, fallback options).
- $U_i : \mathcal{A}_i \to \mathbb{R}$: The utility function of party i, representing their valuation for each alternative outcome.
- BATNA_i: The maximum utility value party i can achieve independently of the current negotiation.

Interpretation. The BATNA represents the threshold utility for each party to accept any negotiated deal $d \in \mathcal{D}$. Specifically, party i will accept a deal d only if:

$$U_i(d) \geq \text{BATNA}_i.$$

Definition 169 (Zone of Possible Agreement (ZOPA))**.** The *Zone of Possible Agreement (ZOPA)* is the set of feasible deals where both parties' utilities meet or exceed their respective BATNAs.

Let $U_A : \mathcal{D} \to \mathbb{R}$ and $U_B : \mathcal{D} \to \mathbb{R}$ represent the utility functions of parties A and B, respectively. Then the ZOPA is defined as:

$$\text{ZOPA} = \{d \in \mathcal{D} \mid U_A(d) \geq \text{BATNA}_A \text{ and } U_B(d) \geq \text{BATNA}_B\},$$

where:

- $\mathcal{D} \subseteq \mathbb{R}^2$: The set of all possible deals $d = (d_A, d_B)$, where d_A and d_B represent the utilities for parties A and B, respectively.
- $U_A(d)$ and $U_B(d)$: The utilities for parties A and B when deal d is agreed upon.
- BATNA_A and BATNA_B: The BATNAs for parties A and B, as defined earlier.

Conditions for ZOPA Existence. The ZOPA exists if and only if there exists a deal $d \in \mathcal{D}$ such that:

$$U_A(d) \geq \text{BATNA}_A \quad \text{and} \quad U_B(d) \geq \text{BATNA}_B.$$

The conditions for the existence of a ZOPA can be expressed as:

$$\max_{d \in \mathcal{D}} U_A(d) \geq \text{BATNA}_A \quad \text{and} \quad \max_{d \in \mathcal{D}} U_B(d) \geq \text{BATNA}_B.$$

Negative Bargaining Zone. If no such $d \in \mathcal{D}$ exists where both conditions hold, then the ZOPA does not exist, and the negotiation is said to have a *Negative Bargaining Zone* (NBZ).

Example 170 (ZOPA in Practice)**.** Suppose two parties A and B negotiate over the price of a car. Let:

$$\text{BATNA}_A = 5,000 \quad \text{and} \quad \text{BATNA}_B = 4,500.$$

The possible deals d (prices) are represented by $d \in \mathcal{D} = [4,000, 6,000]$, where:

$$U_A(d) = 6,000 - d \quad \text{and} \quad U_B(d) = d - 4,000.$$

The ZOPA is the set of prices d where both utilities exceed their BATNAs:

$$6,000 - d \geq 5,000 \quad \text{and} \quad d - 4,000 \geq 4,500.$$

Simplifying these conditions gives:

$$d \leq 5,000 \quad \text{and} \quad d \geq 4,500.$$

Therefore, the ZOPA is:

$$\text{ZOPA} = [4,500, 5,000].$$

The above concepts are extended by incorporating the conditions of the Neutrosophic Set.

Definition 171 (Neutrosophic Best Alternative to a Negotiated Agreement (Neutrosophic BATNA))**.** Let N represent a negotiation between two parties A (Agent 1) and B (Agent 2), where the set of all possible deals is $\mathcal{D} \subseteq \mathbb{R}^2$. The *Neutrosophic Best Alternative to a Negotiated Agreement (Neutrosophic BATNA)* incorporates the degrees of truth (T), indeterminacy (I), and falsity (F) into the evaluation of alternatives.

Formally, the Neutrosophic BATNA for each party $i \in \{A, B\}$ is defined as:

$$\text{NBATNA}_i = \max_{\alpha \in \mathcal{A}_i} \mathcal{N}_i(\alpha), \quad \mathcal{N}_i(\alpha) = (T_i(\alpha), I_i(\alpha), F_i(\alpha)),$$

where:

- $\mathcal{A}_i$: The set of alternatives available to party i outside the current negotiation N (e.g., fallback options, external agreements).
- $\mathcal{N}_i : \mathcal{A}_i \to [0,1]^3$: The neutrosophic utility function of party i, mapping each alternative α to a tuple:

  $$\mathcal{N}_i(\alpha) = (T_i(\alpha), I_i(\alpha), F_i(\alpha)),$$

  where:

  $$T_i(\alpha) + I_i(\alpha) + F_i(\alpha) \leq 1, \quad T_i, I_i, F_i \in [0,1].$$

- NBATNA_i: The maximum neutrosophic utility for party i, which quantifies the best outcome they can achieve independently.

Acceptance Condition. For any negotiated deal $d \in \mathcal{D}$, party i will only accept d if:

$$\mathcal{N}_i(d) \succeq \text{NBATNA}_i,$$

where $\succeq$ denotes a partial order such that:

$$(T_i(d), I_i(d), F_i(d)) \succeq (T_i, I_i, F_i) \iff T_i(d) \geq T_i,\ I_i(d) \leq I_i,\ \text{and } F_i(d) \leq F_i.$$

Remark 172 (Neutrosophic BATNA)**.** *Fuzzy BATNA is a special case of Neutrosophic BATNA where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic BATNA is notable for its ability to generalize both Neutrosophic and Fuzzy BATNA.*

Theorem 173. *The Neutrosophic BATNA exhibits the structure of a Neutrosophic Set.*

Proof: The result follows directly from the definition. □

Theorem 174. *The Neutrosophic BATNA exhibits the structure of a Classic BATNA.*

Proof: The result follows directly from the definition. □

Definition 175 (Neutrosophic Zone of Possible Agreement (Neutrosophic ZOPA))**.** The *Neutrosophic Zone of Possible Agreement (Neutrosophic ZOPA)* is the set of feasible deals where the neutrosophic utility of both parties meets or exceeds their respective Neutrosophic BATNAs.

Let $\mathcal{N}_A : \mathcal{D} \to [0,1]^3$ and $\mathcal{N}_B : \mathcal{D} \to [0,1]^3$ represent the neutrosophic utility functions of parties A and B, respectively. Then the Neutrosophic ZOPA is defined as:

$$\text{NZOPA} = \{d \in \mathcal{D} \mid \mathcal{N}_A(d) \succeq \text{NBATNA}_A \text{ and } \mathcal{N}_B(d) \succeq \text{NBATNA}_B\}.$$

Existence Condition. The Neutrosophic ZOPA exists if and only if there exists a deal $d \in \mathcal{D}$ such that:

$$\mathcal{N}_A(d) \succeq \text{NBATNA}_A \quad \text{and} \quad \mathcal{N}_B(d) \succeq \text{NBATNA}_B.$$

If no such deal d exists, the negotiation is said to have a *Neutrosophic Negative Bargaining Zone (NNBZ).*

Remark 176 (Neutrosophic ZOPA)**.** *Fuzzy ZOPA is a special case of Neutrosophic ZOPA where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic ZOPA is notable for its ability to generalize both Neutrosophic and Fuzzy ZOPA.*

Example 177 (Neutrosophic ZOPA in Practice)**.** Suppose two parties A and B negotiate over a service fee. Their Neutrosophic BATNAs are:

$$\mathrm{NBATNA}_A = (0.8, 0.1, 0.1), \quad \mathrm{NBATNA}_B = (0.7, 0.2, 0.1).$$

The possible deals $d \in \mathcal{D}$ are represented by their Neutrosophic utility values:

$$\mathcal{N}_A(d) = (T_A(d), I_A(d), F_A(d)), \quad \mathcal{N}_B(d) = (T_B(d), I_B(d), F_B(d)).$$

For a deal d to belong to the Neutrosophic ZOPA, the following conditions must hold:

$$\mathcal{N}_A(d) \succeq (0.8, 0.1, 0.1) \quad \text{and} \quad \mathcal{N}_B(d) \succeq (0.7, 0.2, 0.1).$$

Assume a deal d_1 has the following utilities:

$$\mathcal{N}_A(d_1) = (0.85, 0.08, 0.07), \quad \mathcal{N}_B(d_1) = (0.75, 0.15, 0.1).$$

Since both conditions are satisfied:

$$0.85 \geq 0.8,\ 0.08 \leq 0.1,\ 0.07 \leq 0.1, \quad \text{and} \quad 0.75 \geq 0.7,\ 0.15 \leq 0.2,\ 0.1 \leq 0.1,$$

we conclude that $d_1 \in \mathrm{NZOPA}$.

Theorem 178. *The Neutrosophic ZOPA exhibits the structure of a Neutrosophic Set.*

Proof: The result follows directly from the definition. □

Theorem 179. *The Neutrosophic ZOPA exhibits the structure of a Classic ZOPA.*

Proof: The result follows directly from the definition. □

4.4 | New Framing

4.4.1 | Neutrosophic Framing

Framing is the presentation of identical information in different ways, influencing decision-making behavior by altering perception of outcomes and choices.

Definition 180 (Framing)**.** *Framing* is a representation of a decision problem where the same problem is presented in different ways, influencing decision-making behavior and preferences. Mathematically, a frame F is defined as:

$$F = (A, O, P, V, U),$$

where:

- $A = \{a_1, a_2, \dots, a_n\}$: The set of available *actions* or *choices*.
- $O = \{o_1, o_2, \dots, o_m\}$: The set of possible *outcomes*.
- $P : A \times O \to [0, 1]$: The *probability function*, assigning a probability $P(o_j|a_i)$ to each outcome $o_j \in O$ for a given action $a_i \in A$, satisfying:
$$\forall a_i \in A, \sum_{o_j \in O} P(o_j|a_i) = 1.$$
- $V : O \to \mathbb{R}$: The *valuation function*, assigning a numerical value $V(o_j)$ (e.g., gain or loss) to each outcome $o_j \in O$.
- $U : A \to \mathbb{R}$: The *utility function*, defined as:
$$U(a_i) = \sum_{o_j \in O} P(o_j|a_i) \cdot V(o_j).$$

The decision-maker selects the action $a^* \in A$ that maximizes their perceived utility:

$$a^* = \arg\max_{a_i \in A} U(a_i).$$

Remark 181 (Impact of Framing)**.** *Framing influences V, the valuation of outcomes, depending on how the outcomes are presented. Specifically:*

- *A* positive frame *presents outcomes as* gains*, leading to risk-averse behavior.*
- *A* negative frame *presents outcomes as* losses*, leading to risk-seeking behavior.*

Thus, the same A, O, and P may yield different decisions due to changes in V.

Example 182 (Framing Effect: Risk Preferences)**.** Consider two equivalent frames for a medical treatment decision:

- **Positive Frame:** "200 lives will be saved."
- **Negative Frame:** "400 people will die."

The outcomes $O = \{o_1, o_2\}$ are identical, with:

$$V(o_1) = 200 \text{ lives saved}, \quad V(o_2) = 400 \text{ lives lost}.$$

Given the same probabilities P, a decision-maker under the positive frame tends to be risk-averse, favoring a certain outcome (e.g., saving 200 lives). Under the negative frame, the decision-maker becomes risk-seeking, preferring uncertain options to avoid losses.

Theorem 183 (Framing-Induced Preference Reversal)**.** *Let F_1 and F_2 represent two frames of the same decision problem with identical A, O, and P, but different valuations V_1 and V_2. Then:*

$$U_1(a_i) \neq U_2(a_i) \text{ for some } a_i \in A \implies \text{preference reversal.}$$

Proof: The utility U depends on V, the valuation of outcomes:

$$U_k(a_i) = \sum_{o_j \in O} P(o_j|a_i) \cdot V_k(o_j), \quad k = 1, 2.$$

If $V_1(o_j) \neq V_2(o_j)$ for at least one $o_j \in O$, then:

$$U_1(a_i) \neq U_2(a_i).$$

This difference in utilities alters the decision-maker's ranking of actions A, leading to a preference reversal. □

Definition 184 (Neutrosophic Framing)**.** *Neutrosophic Framing* is a mathematical representation of a decision problem where uncertainty, ambiguity, and contradiction are explicitly incorporated into the evaluation of outcomes. A Neutrosophic frame F_N is defined as:

$$F_N = (A, O, P, V_N, U_N),$$

where:

- $A = \{a_1, a_2, \dots, a_n\}$: The set of available *actions* or *choices*.
- $O = \{o_1, o_2, \dots, o_m\}$: The set of possible *outcomes*.
- $P : A \times O \to [0, 1]$: The *probability function*, which assigns a probability $P(o_j|a_i)$ to each outcome $o_j \in O$ given action $a_i \in A$. The function satisfies:
$$\forall a_i \in A, \sum_{o_j \in O} P(o_j|a_i) = 1.$$
- $V_N : O \to [0, 1]^3$: The *neutrosophic valuation function*, which assigns a triple $V_N(o_j) = (T_{o_j}, I_{o_j}, F_{o_j})$ to each outcome $o_j \in O$, where:
  - T_{o_j}: The degree of truth (positive evaluation) of the outcome o_j.
  - I_{o_j}: The degree of indeterminacy (uncertainty or ambiguity) of the outcome o_j.
  - F_{o_j}: The degree of falsity (negative evaluation) of the outcome o_j.

  - $T_{o_j} + I_{o_j} + F_{o_j} \leq 1$: Consistency condition ensuring the total evaluation remains bounded.

- $U_N : A \to [0,1]^3$: The *neutrosophic utility function*, defined for each action $a_i \in A$ as:

$$U_N(a_i) = \left(T_{a_i}, I_{a_i}, F_{a_i}\right),$$

  where:

$$T_{a_i} = \sum_{o_j \in O} P(o_j|a_i) \cdot T_{o_j}, \quad I_{a_i} = \sum_{o_j \in O} P(o_j|a_i) \cdot I_{o_j}, \quad F_{a_i} = \sum_{o_j \in O} P(o_j|a_i) \cdot F_{o_j}.$$

The decision-maker selects the action $a^* \in A$ that maximizes the *truth utility* T_{a_i} while considering the indeterminacy I_{a_i} and falsity F_{a_i}:

$$a^* = \arg\max_{a_i \in A} T_{a_i}.$$

Remark 185 (Neutrosophic Valuation and Decision-Making)**.** *Neutrosophic framing allows for a richer evaluation of decision problems by incorporating:*

- *Positive outcomes (T) that contribute directly to utility.*
- *Uncertain or ambiguous outcomes (I), which reflect incomplete or unclear information.*
- *Negative outcomes (F) that reflect losses or contradictions.*

This framework can model real-world scenarios where outcomes are not purely true or false but lie within a range of truth, uncertainty, and falsity.

Remark 186 (Neutrosophic framing)**.** *Fuzzy framing is a special case of Neutrosophic framing where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic framing is notable for its ability to generalize both Neutrosophic and Fuzzy framing.*

Example 187 (Neutrosophic Framing in Decision-Making)**.** Consider a decision-maker choosing between two investment options $A = \{a_1, a_2\}$ with uncertain outcomes $O = \{o_1, o_2\}$.

- **Action** a_1 leads to outcome o_1 with:

$$V_N(o_1) = (T_{o_1}, I_{o_1}, F_{o_1}) = (0.7, 0.2, 0.1), \quad P(o_1|a_1) = 0.8.$$

- **Action** a_2 leads to outcome o_2 with:

$$V_N(o_2) = (T_{o_2}, I_{o_2}, F_{o_2}) = (0.6, 0.3, 0.1), \quad P(o_2|a_2) = 0.9.$$

The neutrosophic utilities for each action are calculated as:

$$U_N(a_1) = \left(T_{a_1}, I_{a_1}, F_{a_1}\right) = (0.8 \cdot 0.7, 0.8 \cdot 0.2, 0.8 \cdot 0.1) = (0.56, 0.16, 0.08),$$

$$U_N(a_2) = \left(T_{a_2}, I_{a_2}, F_{a_2}\right) = (0.9 \cdot 0.6, 0.9 \cdot 0.3, 0.9 \cdot 0.1) = (0.54, 0.27, 0.09).$$

The decision-maker compares the truth utilities:

$$T_{a_1} = 0.56, \quad T_{a_2} = 0.54.$$

Since $T_{a_1} > T_{a_2}$, the decision-maker selects a_1 as the optimal action.

Theorem 188. *The Neutrosophic Frames exhibits the structure of a Neutrosophic Set.*

Proof: The result follows directly from the definition. □

Theorem 189. *The Neutrosophic Frames exhibits the structure of a Classic Frames.*

Proof: The result follows directly from the definition. □

Theorem 190 (Preference Reversal in Neutrosophic Frames). *Let F_N^1 and F_N^2 be two Neutrosophic frames of the same decision problem with identical A, O, and P but different neutrosophic valuations V_N^1 and V_N^2. Then:*

$$U_N^1(a_i) \neq U_N^2(a_i) \text{ for some } a_i \in A \implies \text{preference reversal.}$$

Proof: The neutrosophic utility U_N depends on the valuation V_N. If $V_N^1(o_j) \neq V_N^2(o_j)$ for at least one $o_j \in O$, then:

$$U_N^1(a_i) \neq U_N^2(a_i).$$

This change in utility leads to a different ranking of actions A, resulting in a preference reversal. □

4.4.2 | Hyperframing

Additionally, we introduce the concepts of Hyperframing and Superhyperframing, which incorporate hierarchical structures into traditional framing. While these concepts are currently at the conceptual stage, their definitions are outlined below.

We hope that future research will explore and develop these frameworks further.

Definition 191 (Hyperframing). *Hyperframing* extends the classical framing concept into a hyperstructure framework, allowing multi-level relationships between actions, outcomes, and utilities. A Hyperframe F_H is defined as:

$$F_H = (\mathcal{P}(A), \mathcal{P}(O), P_H, V_H, U_H),$$

where:

- $\mathcal{P}(A)$: The powerset of the set of available actions $A = \{a_1, a_2, \dots, a_n\}$, representing multi-level or grouped actions.
- $\mathcal{P}(O)$: The powerset of the set of outcomes $O = \{o_1, o_2, \dots, o_m\}$, representing interconnected or combined outcomes.
- $P_H : \mathcal{P}(A) \times \mathcal{P}(O) \to [0, 1]$: The *hyperprobability function*, which assigns probabilities to outcomes $X \subseteq O$ given hyper-actions $Y \subseteq A$, satisfying:
$$\forall Y \in \mathcal{P}(A), \sum_{X \in \mathcal{P}(O)} P_H(X|Y) = 1.$$
- $V_H : \mathcal{P}(O) \to \mathbb{R}$: The *hypervaluation function*, which assigns numerical values to subsets of outcomes $X \in \mathcal{P}(O)$.
- $U_H : \mathcal{P}(A) \to \mathbb{R}$: The *hyperutility function*, defined as:
$$U_H(Y) = \sum_{X \in \mathcal{P}(O)} P_H(X|Y) \cdot V_H(X), \quad Y \in \mathcal{P}(A).$$

The decision-maker selects the hyper-action $Y^* \in \mathcal{P}(A)$ that maximizes the hyperutility:

$$Y^* = \arg \max_{Y \in \mathcal{P}(A)} U_H(Y).$$

Remark 192 (Hyperstructure in Hyperframing). *Hyperframing incorporates multiple layers of choices and outcomes, where actions and outcomes are represented as subsets rather than individual elements. This allows for a more flexible and interconnected decision-making process.*

Example 193 (Hyperframing in a Project Management Context). Consider a project with two main tasks $A = \{a_1, a_2\}$ and two outcomes $O = \{o_1, o_2\}$. The hyperstructure allows grouping of actions and outcomes as subsets:

$$\mathcal{P}(A) = \{\{a_1\}, \{a_2\}, \{a_1, a_2\}\}, \quad \mathcal{P}(O) = \{\{o_1\}, \{o_2\}, \{o_1, o_2\}\}.$$

Suppose:

$$P_H(\{o_1\}|\{a_1, a_2\}) = 0.7, \quad V_H(\{o_1\}) = 10.$$

The hyperutility is:

$$U_H(\{a_1, a_2\}) = P_H(\{o_1\}|\{a_1, a_2\}) \cdot V_H(\{o_1\}) = 0.7 \cdot 10 = 7.$$

Definition 194 (n-Superhyperframing)**.** *n-Superhyperframing* is a higher-order generalization of hyperframing using n-th powersets, enabling multi-level hierarchies of actions, outcomes, and utilities. An n-Superhyperframe F_{SH}^n is defined as:

$$F_{SH}^n = (\mathcal{P}^n(A), \mathcal{P}^n(O), P_{SH}^n, V_{SH}^n, U_{SH}^n),$$

where:

- $\mathcal{P}^n(A)$: The n-th powerset of the set of available actions A, capturing n-level groupings of actions.
- $\mathcal{P}^n(O)$: The n-th powerset of the set of outcomes O, capturing n-level interdependencies of outcomes.
- $P_{SH}^n : \mathcal{P}^n(A) \times \mathcal{P}^n(O) \to [0,1]$: The *$n$-superhyperprobability function*, satisfying:

$$\forall Y \in \mathcal{P}^n(A), \sum_{X \in \mathcal{P}^n(O)} P_{SH}^n(X|Y) = 1.$$

- $V_{SH}^n : \mathcal{P}^n(O) \to \mathbb{R}$: The *$n$-superhypervaluation function*, assigning a value to $X \in \mathcal{P}^n(O)$.
- $U_{SH}^n : \mathcal{P}^n(A) \to \mathbb{R}$: The *$n$-superhyperutility function*, defined as:

$$U_{SH}^n(Y) = \sum_{X \in \mathcal{P}^n(O)} P_{SH}^n(X|Y) \cdot V_{SH}^n(X), \quad Y \in \mathcal{P}^n(A).$$

The decision-maker selects the n-superhyperaction $Y^* \in \mathcal{P}^n(A)$ that maximizes the n-superhyperutility:

$$Y^* = \arg \max_{Y \in \mathcal{P}^n(A)} U_{SH}^n(Y).$$

Example 195 (n-Superhyperframing in Complex Decision-Making)**.** Consider three actions $A = \{a_1, a_2, a_3\}$ and outcomes $O = \{o_1, o_2, o_3\}$. The 2-Superhyperframe includes:

$$\mathcal{P}^2(A) = \{\{\{a_1\}\}, \{\{a_1, a_2\}, \{a_3\}\}\}, \quad \mathcal{P}^2(O) = \{\{\{o_1\}\}, \{\{o_2, o_3\}\}\}.$$

Suppose the probabilities and valuations are:

$$P_{SH}^2(\{\{o_1\}\}|\{\{a_1, a_2\}\}) = 0.8, \quad V_{SH}^2(\{\{o_1\}\}) = 15.$$

The 2-superhyperutility is:

$$U_{SH}^2(\{\{a_1, a_2\}\}) = P_{SH}^2(\{\{o_1\}\}|\{\{a_1, a_2\}\}) \cdot V_{SH}^2(\{\{o_1\}\}) = 0.8 \cdot 15 = 12.$$

4.5 | New Mentoring Method

4.5.1 | Neutrosophic Mentoring

Mentoring is a structured process where an experienced mentor guides, supports, and transfers knowledge to a less experienced protégé for skill and personal development[255, 233, 97, 156].

Definition 196 (Mentoring)**.** *Mentoring* is a structured knowledge transfer process between two agents, defined as a tuple:

$$M = (E, P, K, T, G),$$

where:

- $E = \{e_1, e_2\}$: A set of agents where e_1 is the mentor (knowledge provider) and e_2 is the protege (knowledge receiver), such that $e_1 \neq e_2$.
- $K = \{k_1, k_2, \dots, k_n\}$: A finite set of knowledge components shared in the mentoring process.
- $P : E \times K \times T \to [0,1]$: The *knowledge transfer function*, where $P(e_1, k_i, t)$ represents the degree of knowledge $k_i \in K$ transferred from e_1 to e_2 at time $t \in T$, satisfying:

$$\sum_{k_i \in K} P(e_1, k_i, t) \leq 1, \quad \forall t \in T.$$

- $T = \{t_0, t_1, \dots, t_m\}$: A finite or infinite set of discrete or continuous time steps during which mentoring occurs.

- $G : K \to \mathbb{R}^+$: The *goal attainment function*, mapping knowledge k_i to a measurable value g_i indicating the protege's learning progress.

The total knowledge gained by the protege e_2 at time t_m is:

$$K_{\text{gain}}(e_2, t_m) = \int_{t_0}^{t_m} \sum_{k_i \in K} P(e_1, k_i, t) \cdot G(k_i)\, dt.$$

The mentoring process is considered successful if:

$$K_{\text{gain}}(e_2, t_m) \geq K_{\text{target}},$$

where K_{target} is a predefined learning threshold.

Example 197 (Mentoring: Software Development Training)**.** Consider a senior software engineer e_1 mentoring a junior developer e_2 over $T = [0, 10]$ days. The knowledge components K include:

$$K = \{\text{Algorithms}, \text{Debugging}, \text{Coding Standards}\}.$$

The mentor transfers knowledge P at time t, such that:

$$P(e_1, \text{Algorithms}, t) = 0.3, \quad P(e_1, \text{Debugging}, t) = 0.5, \quad P(e_1, \text{Coding Standards}, t) = 0.2.$$

The goal attainment function G assigns weights based on importance:

$$G(\text{Algorithms}) = 1.5, \quad G(\text{Debugging}) = 2.0, \quad G(\text{Coding Standards}) = 1.0.$$

The total knowledge gained by e_2 at $t = 10$ is:

$$K_{\text{gain}}(e_2, 10) = \int_0^{10} [0.3 \cdot 1.5 + 0.5 \cdot 2.0 + 0.2 \cdot 1.0]\, dt = 10 \cdot 1.6 = 16.$$

If $K_{\text{target}} = 15$, the mentoring process is successful.

Definition 198 (Neutrosophic Mentoring)**.** *Neutrosophic Mentoring* extends traditional mentoring by incorporating uncertainty, indeterminacy, and falsity into the knowledge transfer process. It is defined as a tuple:

$$M_N = (E, P^N, K, T, G),$$

where:

- $E = \{e_1, e_2\}$: A set of agents where e_1 is the mentor (knowledge provider) and e_2 is the protege (knowledge receiver), with $e_1 \neq e_2$.
- $K = \{k_1, k_2, \ldots, k_n\}$: A finite set of knowledge components shared in the mentoring process.
- $P^N : E \times K \times T \to [0, 1]^3$: The *neutrosophic knowledge transfer function*, where $P^N(e_1, k_i, t) = (T_{k_i}, I_{k_i}, F_{k_i})$ represents the truth (T), indeterminacy (I), and falsity (F) degrees of knowledge k_i transferred at time t.
- $T = \{t_0, t_1, \ldots, t_m\}$: A finite or infinite set of discrete or continuous time steps during which mentoring occurs.
- $G : K \to \mathbb{R}^+$: The *goal attainment function*, mapping knowledge k_i to a measurable value g_i, representing the protege's learning progress.

The total neutrosophic knowledge gained by the protege e_2 at time t_m is:

$$K^N_{\text{gain}}(e_2, t_m) = \int_{t_0}^{t_m} \sum_{k_i \in K} (T_{k_i} - F_{k_i}) \cdot G(k_i)\, dt.$$

The mentoring process is considered successful if:

$$K^N_{\text{gain}}(e_2, t_m) \geq K^N_{\text{target}},$$

where K^N_{target} is a predefined neutrosophic learning threshold.

Remark 199. *Fuzzy Mentoring is a special case of Neutrosophic Mentoring where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Mentoring is notable for its ability to generalize both Neutrosophic and Fuzzy Mentoring.*

Example 200 (Neutrosophic Mentoring: Uncertain Knowledge Transfer)**.** Consider a scenario where e_1 mentors e_2 on the same topics K. The neutrosophic transfer function P^N is:

$$P^N(e_1, \text{Algorithms}, t) = (0.7, 0.2, 0.1), \quad P^N(e_1, \text{Debugging}, t) = (0.6, 0.3, 0.1), \quad P^N(e_1, \text{Coding Standards}, t) = (0.8, 0.1, 0.1).$$

The goal attainment function G remains the same:

$$G(\text{Algorithms}) = 1.5, \quad G(\text{Debugging}) = 2.0, \quad G(\text{Coding Standards}) = 1.0.$$

The neutrosophic knowledge gained by e_2 over $T = [0, 10]$ is:

$$K^N_{\text{gain}}(e_2, 10) = \int_0^{10} [(0.7 - 0.1) \cdot 1.5 + (0.6 - 0.1) \cdot 2.0 + (0.8 - 0.1) \cdot 1.0] \, dt.$$

Simplifying:

$$K^N_{\text{gain}}(e_2, 10) = 10 \cdot [0.6 \cdot 1.5 + 0.5 \cdot 2.0 + 0.7 \cdot 1.0] = 10 \cdot 2.95 = 29.5.$$

If $K^N_{\text{target}} = 25$, the mentoring process is successful despite uncertainty.

4.5.2 | HyperMentoring

We define Hypermentoring and Superhypermentoring as extensions of traditional mentoring by incorporating hyperstructure and superhyperstructure frameworks. Although these concepts remain at the conceptual stage, we anticipate that future research will advance their understanding and application.

Definition 201 (Hypermentoring)**.** *Hypermentoring* extends traditional mentoring by incorporating higher-order relationships and multi-level knowledge structures among agents. It is formally defined as a tuple:

$$H_M = (\mathcal{P}(E), \mathcal{P}(K), P_H, T, G_H),$$

where:

- $\mathcal{P}(E)$: The powerset of agents E, where each element represents subsets of mentors and protégés. Higher-order mentoring involves multiple mentors or protégés simultaneously.
- $\mathcal{P}(K)$: The powerset of knowledge $K = \{k_1, k_2, \ldots, k_n\}$, where subsets of knowledge components are shared in the mentoring process.
- $P_H : \mathcal{P}(E) \times \mathcal{P}(K) \times T \to [0, 1]$: The *hyper knowledge transfer function*, where $P_H(E', K', t)$ represents the degree of knowledge transfer among subsets $E' \subseteq E$ and $K' \subseteq K$ at time $t \in T$, satisfying:
$$\sum_{K' \subseteq K} P_H(E', K', t) \leq 1, \quad \forall t \in T.$$
- $T = \{t_0, t_1, \ldots, t_m\}$: A discrete or continuous set of time steps.
- $G_H : \mathcal{P}(K) \to \mathbb{R}^+$: The *hyper goal attainment function*, mapping subsets of knowledge K' to measurable values indicating cumulative learning progress.

The total knowledge gained in a hypermentoring process by a protégé subset $E_2 \subseteq E$ at time t_m is:

$$K^H_{\text{gain}}(E_2, t_m) = \int_{t_0}^{t_m} \sum_{K' \subseteq K} P_H(E_1, K', t) \cdot G_H(K') \, dt,$$

where $E_1 \subseteq E$ are the mentors.

The hypermentoring process is successful if:

$$K^H_{\text{gain}}(E_2, t_m) \geq K^H_{\text{target}},$$

where K^H_{target} is a predefined hypermentoring threshold.

Example 202 (Hypermentoring in Research Collaboration)**.** Consider a research collaboration program involving senior researchers (mentors) and junior researchers (protégés). The Hypermentoring process is structured as follows:

- $E = \{e_1, e_2, e_3, e_4, e_5\}$: A set of agents where:
  - e_1, e_2: Senior researchers (mentors).
  - e_3, e_4, e_5: Junior researchers (protégés).
- $\mathcal{P}(E)$: Powerset of E, including subsets of mentors and protégés:
$$\mathcal{P}(E) = \{\{e_1\}, \{e_2\}, \{e_3, e_4\}, \{e_1, e_2, e_5\}, \dots\}.$$
- $K = \{k_1, k_2, k_3\}$: Knowledge components shared during the mentoring process:
  - k_1: Advanced research methodologies.
  - k_2: Statistical modeling techniques.
  - k_3: Paper writing and publishing skills.
- $\mathcal{P}(K)$: Powerset of K, including combinations of knowledge components:
$$\mathcal{P}(K) = \{\{k_1\}, \{k_2\}, \{k_1, k_3\}, \{k_1, k_2, k_3\}, \dots\}.$$
- $P_H : \mathcal{P}(E) \times \mathcal{P}(K) \times T \to [0, 1]$: The hyper knowledge transfer function. For example:
$$P_H(\{e_1, e_2\}, \{k_1, k_2\}, t) = 0.6, \quad P_H(\{e_3, e_4\}, \{k_3\}, t) = 0.8.$$
Here, mentors e_1 and e_2 transfer knowledge k_1 and k_2 to protégés with 60% effectiveness, while protégés e_3 and e_4 focus on learning k_3 with 80% effectiveness.
- $G_H : \mathcal{P}(K) \to \mathbb{R}^+$: The hyper goal attainment function. For example:
$$G_H(\{k_1\}) = 10, \quad G_H(\{k_1, k_2\}) = 25, \quad G_H(\{k_1, k_2, k_3\}) = 40.$$
- $T = \{t_0, t_1, t_2, t_3\}$: Time steps over which mentoring occurs.

The total knowledge gained by protégés $\{e_3, e_4, e_5\}$ at time t_3 is:
$$K^H_{\text{gain}}(\{e_3, e_4, e_5\}, t_3) = \int_{t_0}^{t_3} \sum_{K' \subseteq K} P_H(\{e_1, e_2\}, K', t) \cdot G_H(K')\, dt.$$

Substituting values:
$$K^H_{\text{gain}}(\{e_3, e_4, e_5\}, t_3) = (0.6 \cdot 25) + (0.8 \cdot 15) = 15 + 12 = 27.$$

If the hypermentoring threshold $K^H_{\text{target}} = 25$, the process is successful because:
$$K^H_{\text{gain}} = 27 \geq K^H_{\text{target}}.$$

Definition 203 (n-Superhypermentoring)**.** *n-Superhypermentoring* generalizes hypermentoring to n-levels of powersets and interactions, capturing higher-order complexities across agents and knowledge structures. It is defined as a tuple:
$$SH^n_M = (\mathcal{P}^n(E), \mathcal{P}^n(K), P^n_{SH}, T, G^n_{SH}),$$
where:

- $\mathcal{P}^n(E)$: The n-th powerset of E, representing hierarchical and multi-level subsets of agents.
- $\mathcal{P}^n(K)$: The n-th powerset of K, representing higher-order groupings of knowledge components.
- $P^n_{SH} : \mathcal{P}^n(E) \times \mathcal{P}^n(K) \times T \to [0, 1]$: The *n-superhyper knowledge transfer function*, where $P^n_{SH}(E', K', t)$ measures the degree of knowledge transfer among n-th level subsets $E' \subseteq \mathcal{P}^n(E)$ and $K' \subseteq \mathcal{P}^n(K)$ at time t.
- $T = \{t_0, t_1, \dots, t_m\}$: A set of time steps during which mentoring occurs.

- $G_{SH}^n : \mathcal{P}^n(K) \to \mathbb{R}^+$: The *n-superhyper goal attainment function*, mapping higher-order subsets $K' \subseteq \mathcal{P}^n(K)$ to cumulative learning values.

The total knowledge gained in an n-superhypermentoring process by $E_2^n \subseteq \mathcal{P}^n(E)$ at time t_m is:

$$K_{\text{gain}}^{SH^n}(E_2^n, t_m) = \int_{t_0}^{t_m} \sum_{K' \subseteq \mathcal{P}^n(K)} P_{SH}^n(E_1^n, K', t) \cdot G_{SH}^n(K')\, dt,$$

where $E_1^n \subseteq \mathcal{P}^n(E)$ are the mentor subsets at n-levels.

The n-superhypermentoring process is successful if:

$$K_{\text{gain}}^{SH^n}(E_2^n, t_m) \geq K_{\text{target}}^{SH^n},$$

where $K_{\text{target}}^{SH^n}$ is the predefined n-superhyper mentoring threshold.

Example 204 (n-Superhypermentoring in Research Collaboration)**.** Consider a collaborative research environment with hierarchical mentoring:

- $E = \{e_1, e_2, e_3\}$: Senior mentor e_1, mid-level mentor e_2, and junior protégé e_3.
- $\mathcal{P}^2(E) = \{\{e_1, e_2\}, \{e_2, e_3\}, \{e_1, e_2, e_3\}\}$.
- $K = \{k_1, k_2\}$: Research knowledge components.
- $\mathcal{P}^2(K) = \{\{k_1\}, \{k_2\}, \{k_1, k_2\}\}$.
- P_{SH}^2: Knowledge transfer function for second-level subsets:

$$P_{SH}^2(\{e_1, e_2\}, \{k_1\}, t) = 0.8, \quad P_{SH}^2(\{e_2, e_3\}, \{k_2\}, t) = 0.6.$$

The total knowledge gained by $\{e_2, e_3\}$ at t_m is:

$$K_{\text{gain}}^{SH^2}(\{e_2, e_3\}, t_m) = \int_{t_0}^{t_m} \left(0.6 \cdot G_{SH}^2(\{k_2\})\right) dt.$$

If $K_{\text{target}}^{SH^2} = 1.0$, the mentoring process's success depends on achieving this cumulative threshold.

4.6 | New Storytelling Definition

4.6.1 | Neutrosophic Storytelling

Storytelling is the process of conveying information, values, or experiences through structured narratives, fostering emotional engagement and facilitating knowledge transfer [111, 276, 209, 47, 169]. This concept is extended using Neutrosophic Logic, leading to the development of Neutrosophic Storytelling. The definitions and associated concepts are provided below.

Definition 205 (Storytelling)**.** *Storytelling* is the process of transmitting knowledge or values through structured narratives, defined as a tuple:

$$S = (N, R, V, A, T, C),$$

where:

- $N = \{n_1, n_2, \dots, n_m\}$: A sequence of narrative events n_i, where each n_i represents a discrete element of the story.
- $R : N \times N \to \mathcal{R}$: The *relation function*, mapping pairs of events (n_i, n_j) to a set of relationships $\mathcal{R}$ such as causality, sequence, or thematic links.
- $V : N \to \mathbb{R}^+$: The *value function*, assigning a positive weight v_i to each narrative event n_i, representing its importance or impact in the story.
- $A : E \times N \to [0, 1]$: The *audience comprehension function*, where $A(e, n_i)$ measures the degree of understanding or emotional response of audience member e to event n_i.

- $T = \{t_1, t_2, \dots, t_p\}$: A time sequence over which the narrative is delivered.
- $C : N \to K$: The *knowledge content function*, mapping each event n_i to a knowledge element $k \in K$, where K represents the set of transferable knowledge.

The total impact I of storytelling for an audience E is defined as:

$$I = \sum_{n_i \in N} \sum_{e \in E} A(e, n_i) \cdot V(n_i) \cdot C(n_i).$$

The storytelling process is deemed effective if:

$$I \geq I_{\text{target}},$$

where I_{target} is the minimum desired impact threshold.

Example 206 (Storytelling: Leadership Training)**.** A manager shares a story with employees about overcoming challenges in a previous project:

- $N = \{n_1 : \text{Initial failure}, n_2 : \text{Team collaboration}, n_3 : \text{Successful outcome}\}$.
- R: Events are causally related, with $n_1 \to n_2 \to n_3$.
- $V(n_1) = 2.0, V(n_2) = 3.0, V(n_3) = 5.0$.
- $A(e, n_i)$: Audience comprehension for e_1 and e_2:

$$A(e_1, n_1) = 0.8, A(e_1, n_2) = 0.9, A(e_1, n_3) = 1.0.$$

- $C(n_1) = 0.5, C(n_2) = 1.0, C(n_3) = 1.5$.

The total impact I is:

$$I = \sum_{n_i \in N} A(e_1, n_i) \cdot V(n_i) \cdot C(n_i).$$

Calculating:

$$I = (0.8 \cdot 2.0 \cdot 0.5) + (0.9 \cdot 3.0 \cdot 1.0) + (1.0 \cdot 5.0 \cdot 1.5) = 0.8 + 2.7 + 7.5 = 11.0.$$

If $I_{\text{target}} = 10$, the storytelling process is effective.

Definition 207 (Neutrosophic Storytelling)**.** *Neutrosophic Storytelling* extends traditional storytelling by integrating neutrosophic logic into the narrative process, capturing uncertainty, indeterminacy, and falsity in audience comprehension and value transmission. It is defined as a tuple:

$$S_N = (N, R, V^N, A^N, T, C^N),$$

where:

- $N = \{n_1, n_2, \dots, n_m\}$: A sequence of narrative events n_i, where each n_i represents a discrete element of the story.
- $R : N \times N \to \mathcal{R}$: The *relation function*, mapping pairs of events (n_i, n_j) to a set of relationships $\mathcal{R}$, such as causality, sequence, or thematic links.
- $V^N : N \to [0,1]^3$: The *neutrosophic value function*, assigning a triplet $V^N(n_i) = (T_{n_i}, I_{n_i}, F_{n_i})$ to each event n_i, representing its truth (T), indeterminacy (I), and falsity (F).
- $A^N : E \times N \to [0,1]^3$: The *neutrosophic audience comprehension function*, where $A^N(e, n_i) = (T_{e,n_i}, I_{e,n_i}, F_{e,n_i})$ measures the audience member e's degree of understanding, uncertainty, and misunderstanding for event n_i.
- $T = \{t_1, t_2, \dots, t_p\}$: A time sequence over which the narrative is delivered.
- $C^N : N \to K$: The *neutrosophic knowledge content function*, mapping each event n_i to a knowledge element $k \in K$, with truth, indeterminacy, and falsity components.

The total neutrosophic impact I^N of storytelling for an audience E is defined as:

$$I^N = \sum_{n_i \in N} \sum_{e \in E} \left(T_{e,n_i} - F_{e,n_i} \right) \cdot V^N(n_i) \cdot C^N(n_i).$$

The storytelling process is deemed effective if:

$$I^N \geq I^N_{\text{target}},$$

where I^N_{target} is the minimum desired neutrosophic impact threshold.

Remark 208. *Fuzzy Storytelling is a special case of Neutrosophic Storytelling where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Storytelling is notable for its ability to generalize both Neutrosophic and Fuzzy Storytelling.*

Example 209 (Neutrosophic Storytelling: Uncertain Leadership Communication)**.** Suppose a manager narrates a project story with uncertainty:

- $V^N(n_1) = (0.7, 0.2, 0.1), V^N(n_2) = (0.6, 0.3, 0.1), V^N(n_3) = (0.9, 0.05, 0.05)$.
- $A^N(e_1, n_1) = (0.8, 0.1, 0.1), A^N(e_1, n_2) = (0.7, 0.2, 0.1), A^N(e_1, n_3) = (0.9, 0.05, 0.05)$.
- $C^N(n_1) = 0.5, C^N(n_2) = 1.0, C^N(n_3) = 1.5$.

The total neutrosophic impact is:

$$I^N = \sum_{n_i \in N} \left(T_{e_1,n_i} - F_{e_1,n_i} \right) \cdot T_{n_i} \cdot C^N(n_i).$$

Simplifying:

$$I^N = (0.8 - 0.1) \cdot 0.7 \cdot 0.5 + (0.7 - 0.1) \cdot 0.6 \cdot 1.0 + (0.9 - 0.05) \cdot 0.9 \cdot 1.5.$$

Calculating:

$$I^N = 0.49 + 0.36 + 1.1475 = 1.9975.$$

If $I^N_{\text{target}} = 1.8$, the storytelling process is effective.

4.6.2 | Hyper Storytelling

Hyper Storytelling and SuperHyper Storytelling are concepts extended using Hyperstructure and SuperHyperstructure frameworks. The related definitions and concepts are outlined below.

Definition 210 (Hyper Storytelling)**.** *Hyper Storytelling* extends traditional storytelling by incorporating higher-order relationships and multi-level narrative structures. It is formally defined as a tuple:

$$H_S = (\mathcal{P}(N), \mathcal{P}(R), V_H, A_H, T, C_H),$$

where:

- $\mathcal{P}(N)$: The powerset of narrative events $N = \{n_1, n_2, \dots, n_m\}$, where each subset represents a higher-level narrative structure composed of individual events n_i.
- $\mathcal{P}(R)$: The powerset of relationships $R : N \times N \to \mathcal{R}$, where $\mathcal{R}$ represents relationships such as causality, sequence, and thematic links between subsets of events.
- $V_H : \mathcal{P}(N) \to \mathbb{R}^+$: The *hyper value function*, assigning a positive weight to subsets of narrative events $N' \subseteq N$, representing their collective importance or impact.
- $A_H : \mathcal{P}(E) \times \mathcal{P}(N) \to [0, 1]$: The *hyper audience comprehension function*, where $A_H(E', N')$ measures the degree of understanding or emotional response of audience subsets $E' \subseteq E$ to narrative subsets $N' \subseteq N$.
- $T = \{t_1, t_2, \dots, t_p\}$: A time sequence over which the narrative is delivered.
- $C_H : \mathcal{P}(N) \to \mathcal{P}(K)$: The *hyper knowledge content function*, mapping subsets of events $N' \subseteq N$ to subsets of knowledge K, where $K = \{k_1, k_2, \dots, k_n\}$ represents transferable knowledge.

The total hyper impact I_H of storytelling for an audience E is defined as:

$$I_H = \sum_{N' \subseteq \mathcal{P}(N)} \sum_{E' \subseteq \mathcal{P}(E)} A_H(E', N') \cdot V_H(N') \cdot C_H(N').$$

The storytelling process is deemed effective if:

$$I_H \geq I_{\text{target}}^H,$$

where I_{target}^H is the minimum desired hyper impact threshold.

Example 211 (Hyper Storytelling in Educational Training)**.** Consider a company implementing a multi-level educational training program using Hyper Storytelling to transfer knowledge effectively. The elements of Hyper Storytelling are defined as follows:

- $N = \{n_1, n_2, n_3, n_4\}$: A set of narrative events, where:
  - n_1: Introduction to project management principles.
  - n_2: A real-life case study of a successful project.
  - n_3: A failure analysis of a previous project.
  - n_4: A simulated project task for participants.
- $\mathcal{P}(N)$: The powerset of N, including:

$$\mathcal{P}(N) = \{\{n_1\}, \{n_2\}, \{n_3\}, \{n_4\}, \{n_1, n_2\}, \{n_2, n_3, n_4\}, \dots\}.$$

- $\mathcal{P}(R)$: The powerset of relationships, where higher-level relationships represent thematic and causal links:
  - $R(\{n_1\}, \{n_2\})$: The introduction (n_1) prepares the audience for the case study (n_2).
  - $R(\{n_2\}, \{n_3\})$: The success story (n_2) contrasts with the failure analysis (n_3).
  - $R(\{n_1, n_2\}, \{n_4\})$: The combined knowledge from n_1 and n_2 is applied in the simulation task n_4.
- $V_H : \mathcal{P}(N) \to \mathbb{R}^+$: The hyper value function assigns weights to subsets of narrative events:

$$V_H(\{n_1\}) = 0.3, \quad V_H(\{n_2\}) = 0.5, \quad V_H(\{n_3\}) = 0.4, \quad V_H(\{n_4\}) = 0.8.$$

- $A_H : \mathcal{P}(E) \times \mathcal{P}(N) \to [0, 1]$: The hyper audience comprehension function measures understanding for subsets of the audience E:

$$A_H(\{e_1, e_2\}, \{n_1, n_2\}) = 0.7, \quad A_H(\{e_2, e_3\}, \{n_2, n_3, n_4\}) = 0.8.$$

- $C_H : \mathcal{P}(N) \to \mathcal{P}(K)$: The hyper knowledge content function maps subsets of events to subsets of knowledge:

$$C_H(\{n_1, n_2\}) = \{k_1, k_2\}, \quad C_H(\{n_2, n_3, n_4\}) = \{k_2, k_3, k_4\}.$$

  Here, $K = \{k_1 : \text{Project Principles}, k_2 : \text{Case Study Insights}, k_3 : \text{Failure Lessons}, k_4 : \text{Simulation Skills}\}$.

The total hyper impact I_H is calculated as:

$$I_H = \sum_{N' \subseteq \mathcal{P}(N)} \sum_{E' \subseteq \mathcal{P}(E)} A_H(E', N') \cdot V_H(N') \cdot C_H(N').$$

For example, considering $N' = \{n_2, n_3, n_4\}$ and $E' = \{e_2, e_3\}$:

$$I_H = A_H(\{e_2, e_3\}, \{n_2, n_3, n_4\}) \cdot V_H(\{n_2, n_3, n_4\}) \cdot |C_H(\{n_2, n_3, n_4\})|.$$

Substitute values:

$$I_H = 0.8 \cdot (0.5 + 0.4 + 0.8) \cdot 3 = 0.8 \cdot 1.7 \cdot 3 = 4.08.$$

If the threshold $I_{\text{target}}^H = 4.0$, the hyper storytelling process is deemed effective.

Definition 212 (n-Superhyper Storytelling)**.** *n-Superhyper Storytelling* generalizes hyper storytelling to n-levels of powersets and interactions, capturing higher-order complexities across narrative structures, relationships, and audience responses. It is defined as a tuple:

$$SH_S^n = (\mathcal{P}^n(N), \mathcal{P}^n(R), V_{SH}^n, A_{SH}^n, T, C_{SH}^n),$$

where:

- $\mathcal{P}^n(N)$: The n-th powerset of $N = \{n_1, n_2, \dots, n_m\}$, representing n-level narrative groupings and higher-order event structures.
- $\mathcal{P}^n(R)$: The n-th powerset of relationships $R : N \times N \to \mathcal{R}$, where higher-level relationships describe interactions among subsets of events across multiple levels.
- $V_{SH}^n : \mathcal{P}^n(N) \to \mathbb{R}^+$: The *n-superhyper value function*, assigning positive weights to n-level narrative subsets.
- $A_{SH}^n : \mathcal{P}^n(E) \times \mathcal{P}^n(N) \to [0,1]$: The *n-superhyper audience comprehension function*, measuring the understanding or emotional response of audience subsets $E' \subseteq \mathcal{P}^n(E)$ to n-level narrative subsets $N' \subseteq \mathcal{P}^n(N)$.
- $T = \{t_1, t_2, \dots, t_p\}$: A time sequence over which the narrative unfolds.
- $C_{SH}^n : \mathcal{P}^n(N) \to \mathcal{P}^n(K)$: The *n-superhyper knowledge content function*, mapping n-level narrative subsets to n-level knowledge components.

The total n-superhyper impact I_{SH}^n for an audience E is defined as:

$$I_{SH}^n = \sum_{N' \subseteq \mathcal{P}^n(N)} \sum_{E' \subseteq \mathcal{P}^n(E)} A_{SH}^n(E', N') \cdot V_{SH}^n(N') \cdot C_{SH}^n(N').$$

The n-superhyper storytelling process is deemed effective if:

$$I_{SH}^n \geq I_{\text{target}}^{SH^n},$$

where $I_{\text{target}}^{SH^n}$ is the predefined n-superhyper impact threshold.

Example 213 (n-Superhyper Storytelling in Training Programs)**.** Consider a corporate training program that uses multi-level storytelling to transfer knowledge:

- $N = \{n_1, n_2, n_3\}$: Three narrative events n_1 (introductory session), n_2 (case study), and n_3 (simulation exercise).
- $\mathcal{P}^2(N) = \{\{n_1, n_2\}, \{n_2, n_3\}, \{n_1, n_2, n_3\}\}$: Second-level narrative groupings.
- V_{SH}^2: Narrative value function:

$$V_{SH}^2(\{n_1, n_2\}) = 0.8, \quad V_{SH}^2(\{n_2, n_3\}) = 0.9.$$

- A_{SH}^2: Audience comprehension function:

$$A_{SH}^2(\{e_1, e_2\}, \{n_1, n_2\}) = 0.7, \quad A_{SH}^2(\{e_2, e_3\}, \{n_2, n_3\}) = 0.8.$$

The total second-level superhyper impact I_{SH}^2 is:

$$I_{SH}^2 = \sum_{N' \subseteq \mathcal{P}^2(N)} \sum_{E' \subseteq \mathcal{P}^2(E)} A_{SH}^2(E', N') \cdot V_{SH}^2(N') \cdot C_{SH}^2(N').$$

If the desired threshold $I_{\text{target}}^{SH^2}$ is met, the program achieves its storytelling objectives.

4.6.3 | Neutrosophic Work-Life Balance

Work-Life Balance refers to the effective management of time and energy between professional responsibilities and personal life to ensure well-being and productivity [306, 307, 145, 146, 144, 71, 32]. When mathematically defined and extended using Neutrosophic Logic, it is formalized as follows. Since this concept remains in the conceptual stage, further refinements and research into its applications are anticipated as necessary.

Definition 214 (Work-Life Balance)**.** *Work-Life Balance (WLB)* is a mathematical framework that models the allocation of time, resources, and energy between professional responsibilities (work) and personal priorities (life) to optimize overall well-being and sustainability. It is formally defined as:

$$\mathcal{WLB} = (W, L, T, U, C, R, \mathcal{S}),$$

where:

- $W = \{w_1, w_2, \dots, w_n\}$: A set of work-related activities, where w_i represents specific professional tasks or obligations.
- $L = \{l_1, l_2, \dots, l_m\}$: A set of life-related activities, where l_j includes personal, social, or recreational activities.
- $T : W \cup L \to \mathbb{R}^+$: The *time allocation function*, where $T(x)$ represents the time allocated to activity $x \in W \cup L$, subject to:
$$\sum_{x \in W \cup L} T(x) = T_{\text{total}},$$
where T_{total} is the total available time.
- $U : W \cup L \to \mathbb{R}^+$: The *utility function*, which quantifies satisfaction, productivity, or benefit derived from activity x.
- $C : W \cup L \to \mathbb{R}^+$: The *cost function*, representing physical, mental, or emotional burdens associated with activity x.
- $R : W \cup L \to \mathbb{R}$: The *recovery function*, where:
$$R(x) > 0 \implies \text{recovery (e.g., rest, relaxation)}, \quad R(x) < 0 \implies \text{depletion (e.g., fatigue, stress)}.$$
- $\mathcal{S} = (S_W, S_L, \Omega)$: The *sustainability state*, where S_W and S_L measure cumulative work and life balance, and Ω represents overall equilibrium.

The work-life balance condition is achieved if:

$$\mathcal{S} = S_W + S_L, \quad \text{where } \Omega \in [\Omega_{\min}, \Omega_{\max}],$$

and:

$$S_W = \sum_{w_i \in W} [U(w_i) - C(w_i)], \quad S_L = \sum_{l_j \in L} [U(l_j) + R(l_j) - C(l_j)].$$

Work-Life Imbalance. Work-life imbalance occurs when:

$$\mathcal{S} \notin [\Omega_{\min}, \Omega_{\max}],$$

indicating that costs outweigh benefits or recovery is insufficient.
Optimal Work-Life Balance. The optimal balance maximizes overall utility while maintaining sustainability:

$$\mathcal{WLB}^* = \arg \max_{\{T(w), T(l)\}} [S_W + S_L],$$

subject to:

$$\sum_{x \in W \cup L} T(x) = T_{\text{total}}, \quad \mathcal{S} \in [\Omega_{\min}, \Omega_{\max}].$$

Example 215 (Work-Life Balance Scenario)**.** A software engineer allocates time in a 24-hour day as follows:

- Work activities $W = \{w_1 : \text{coding}, w_2 : \text{meetings}\}$ with $T(w_1) = 6$ hours and $T(w_2) = 2$ hours.

- Life activities $L = \{l_1 : \text{exercise}, l_2 : \text{family time}, l_3 : \text{sleep}\}$ with $T(l_1) = 1$ hour, $T(l_2) = 2$ hours, and $T(l_3) = 8$ hours.

The recovery values R are:

$$R(l_3) = 10\,(\text{high recovery}),\; R(l_1) = 5\,(\text{moderate recovery}),\; R(w_1) = -3\,(\text{work fatigue}).$$

If sleep (l_3) is reduced to 4 hours, $R(l_3)$ decreases significantly, leading to imbalance:

$$\mathcal{S} \notin [\Omega_{\min}, \Omega_{\max}],$$

indicating increased risk of burnout.

Definition 216 (Neutrosophic Work-Life Balance)**.** *Neutrosophic Work-Life Balance (NWLB)* is a generalized mathematical model for assessing work-life equilibrium by incorporating truth, indeterminacy, and falsity components into the evaluation of time allocation, utility, and recovery. It is formally defined as:

$$\mathcal{NWLB} = (W, L, T^N, U^N, C^N, R^N, \mathcal{S}^N),$$

where:

- $W = \{w_1, w_2, \dots, w_n\}$: The set of work activities (e.g., meetings, projects).
- $L = \{l_1, l_2, \dots, l_m\}$: The set of life activities (e.g., family, exercise, sleep).
- $T^N : (W \cup L) \to [0,1]^3$: The *neutrosophic time allocation function*, defined as:

$$T^N(x) = (T_T(x), T_I(x), T_F(x)),$$

  where:
  - $T_T(x)$: Truth degree of time allocated to activity x.
  - $T_I(x)$: Indeterminacy degree of time allocation for x.
  - $T_F(x)$: Falsity degree of time allocated to x.
- $U^N : (W \cup L) \to \mathbb{R}^3$: The *neutrosophic utility function*, where:

$$U^N(x) = (U_T(x), U_I(x), U_F(x)),$$

  representing the truth, indeterminacy, and falsity components of utility derived from activity x.
- $C^N : (W \cup L) \to \mathbb{R}^3$: The *neutrosophic cost function*, quantifying the burden of activity x as:

$$C^N(x) = (C_T(x), C_I(x), C_F(x)),$$

  where truth, indeterminacy, and falsity components reflect perceived and uncertain costs.
- $R^N : (W \cup L) \to \mathbb{R}^3$: The *neutrosophic recovery function*, representing the recovery (restoration of mental/physical energy) as:

$$R^N(x) = (R_T(x), R_I(x), R_F(x)).$$

- $\mathcal{S}^N = (S^N_W, S^N_L, \Omega^N)$: The *neutrosophic sustainability state*, where:
  - S^N_W: Cumulative neutrosophic balance for work activities.
  - S^N_L: Cumulative neutrosophic balance for life activities.
  - Ω^N: Overall neutrosophic sustainability threshold.

Neutrosophic Work-Life Balance Condition. Work-life balance is achieved if the following holds:

$$\mathcal{S}^N = S^N_W + S^N_L, \quad \text{where } \Omega^N \in [\Omega^N_{\min}, \Omega^N_{\max}],$$

and:

$$S^N_W = \sum_{w_i \in W} \left[U_T(w_i) - C_T(w_i)\right], \quad S^N_L = \sum_{l_j \in L} \left[U_T(l_j) + R_T(l_j) - C_T(l_j)\right].$$

Neutrosophic Work-Life Imbalance. Work-life imbalance occurs when:

$$\Omega^N \notin [\Omega^N_{\text{min}}, \Omega^N_{\text{max}}],$$

indicating that the perceived utility, time allocation, and recovery are insufficient to offset work burdens. Optimal Neutrosophic Work-Life Balance. The optimal neutrosophic balance maximizes overall neutrosophic utility while accounting for indeterminacy and falsity:

$$\mathcal{NWLB}^* = \arg \max_{\{T^N(w), T^N(l)\}} \left[S^N_W + S^N_L \right],$$

subject to:

$$\sum_{x \in W \cup L} T_T(x) = T_{\text{total}} \quad \text{and} \quad \Omega^N \in [\Omega^N_{\text{min}}, \Omega^N_{\text{max}}].$$

Remark 217 (Neutrosophic Work-Life Balance)**.** *Fuzzy Work-Life Balance is a special case of Neutrosophic Work-Life Balance where both indeterminacy and falsity are set to zero. Furthermore, Plithogenic Work-Life Balance is notable for its ability to generalize both Neutrosophic and Fuzzy Work-Life Balance.*

Example 218 (Neutrosophic Work-Life Balance Scenario)**.** A manager allocates their time as follows in a 24-hour day:

- Work activities: $W = \{w_1 : \text{emails}, w_2 : \text{meetings}\}$, with neutrosophic time $T^N(w_1) = (0.8, 0.1, 0.1)$ and $T^N(w_2) = (0.7, 0.2, 0.1)$.
- Life activities: $L = \{l_1 : \text{exercise}, l_2 : \text{family}, l_3 : \text{sleep}\}$, with:

$$T^N(l_1) = (0.6, 0.2, 0.2),\ T^N(l_2) = (0.9, 0.05, 0.05),\ T^N(l_3) = (0.95, 0.03, 0.02).$$

The recovery values R^N and costs C^N are:

$$R^N(l_3) = (0.9, 0.05, 0.05), \quad C^N(w_1) = (0.7, 0.2, 0.1).$$

If $T_T(l_3)$ decreases to 0.5 (e.g., reduced sleep), recovery becomes insufficient, leading to imbalance:

$$\Omega^N \notin [\Omega^N_{\text{min}}, \Omega^N_{\text{max}}],$$

indicating stress accumulation and unsustainability.

Theorem 219. *The Neutrosophic Work-Life Balance exhibits the structure of a Neutrosophic Set.*

Proof: The result follows directly from the definition. □

Theorem 220. *The Neutrosophic Work-Life Balance exhibits the structure of a Classic Work-Life Balance.*

Proof: The result follows directly from the definition. □

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Data Availability

This paper does not involve any data analysis.

Ethical Approval

This article does not involve any research with human participants or animals.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Disclaimer

This study primarily focuses on theoretical aspects, and its application to practical scenarios has not yet been validated. Future research may involve empirical testing and refinement of the proposed methods. The authors have made every effort to ensure that all references cited in this paper are accurate and appropriately attributed. However, unintentional errors or omissions may occur. The authors bear no legal responsibility for inaccuracies in external sources, and readers are encouraged to verify the information provided in the references independently. Furthermore, the interpretations and opinions expressed in this paper are solely those of the authors and do not necessarily reflect the views of any affiliated institutions.

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