SciNexuses

Journal Homepage: sciencesforce.com/scin

SciNexuses Vol. 1 (2024) 75–81

Paper Type: Original Article

Results and Discussion of Dengue Model with Temperature Effects in Interval Environment

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Received: 14 Feb 2024 **Revised:** 30 Jul 2024 **Accepted:** 31 Aug 2024 **Published:** 03 Sep 2024

Abstract

We have investigated a dengue model with temperature effects under interval uncertainty in this work. This study observed the Aedes aegypti temperature-dependent entomological parameters that affect dengue illness transmission dynamics in Taiwan's subtropical zone. A vector-host transmission model was used to examine how temperature fluctuations influence the development of pre-adult mosquitoes, their egg-laying rates, adult mortality, and the incubation rate of viruses within them. This study showed that although estimations of entomological parameters were positively correlated with slow temperature rises, no such correlation was detected with mosquito mortality or maturation rates, underscoring the slow rate of maturation of pre-adult mosquitoes. The findings suggest that the dynamic modeling of vector-host interactions is significantly influenced by temperature. Additionally, our modeling indicates that a temperature range of about 32°C is ideal for dengue transmission. In the future, control measure modeling and cost-effectiveness assessments may benefit from these insights.

Keywords: Dengue, Interval Number, Stability Criteria, Numerical Results.

1 | Introduction

This study looks at a mathematical model for dengue fever, which can be adjusted to any level of complexity. We'll use data from the FongShan area in Kaohsiung, Taiwan, to figure out the relevant parameter values for this epidemic model. Between 1998 and 2010, Kaohsiung, Taiwan's second-largest city, faced a high risk of

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https://doi.org/10.61356/j.scin.2024.1372

dengue fever. In fact, from 2001 to 2003, one district in Kaohsiung experienced the worst dengue outbreak in 60 years, with 4,790 reported cases. Recently, the WHO has noted over 238,000 cases of dengue fever globally [1-8].

Given the ongoing concern about global warming, understanding how temperature impacts the spread of dengue is crucial. This topic is a major focus in fields like epidemiology, computational biology, public health, and environmental science. A recent review [9] used a meta-analysis to explore how both rainfall and temperature influence dengue outbreaks.

For public health, it's vital to develop effective strategies for predicting, managing, and treating such epidemics. Over the years, many researchers have delved into this issue, contributing to both biology and mathematics. Deterministic mathematical models for dengue transmission, like the SEI (susceptible-exposed-infectious) for mosquitoes and the SIR (susceptible-infectious-recovery) for humans, are well covered in the literature [10], with more research in [11].

To tackle uncertainties in these models, researchers have used methods like interval approaches and stochastic methods [12 - 17]. The interval approach uses interval-valued functions to describe unknown parameters. Professor Zadeh pioneered fuzzy set theory [18] and proposed fuzzy differential equations as a way to model systems with probabilistic uncertainties [19]. Additionally, Sadhukhan et al. [20] explored optimal strategies for managing a food chain model under fuzzy conditions, using a fuzzy instantaneous annual discount rate.

2 | Pre-requisite Concept

Definition: For an interval $[T_{m_1}, T_{n_1}]$ The interval-valued function can be created as $k_1(\eta) = (T_{m_1})^{1-\eta}(T_{n_1})^{\eta}$ for $\eta \in [0,1]$, which is also called parametric form in interval figure.

Properties: Let, two intervals (parametric form) as $k_1(\eta) = (T_{m_1})^{1-\eta}(T_{n_1})^{\eta}$ and $h_1(\eta) = (R_{m_1})^{1-\eta}(R_{n_1})^{\eta}$ for $\in [0,1]$, then the following operation was obtained:

i). $g_1(\eta) = k_1(\eta) + h_1(\eta) = (T_{m_1} + R_{m_1})^{1-\eta}(T_{n_1} + R_{n_1})^{\eta}$.

ii). $s_1(\eta) = k_1(\eta) - h_1(\eta) = (T_{m_1} - R_{m_1})^{1-\eta}(T_{n_1} - R_{n_1})^{\eta}$.

iii). $r_1(\eta) = k_1(\eta)h_1(\eta) =$
$(\min\{T_{m_1}R_{m_1}, T_{n_1}R_{n_1}, T_{m_1}T_{n_1}, R_{m_1}R_{n_1}\})^{1-\eta}(\max\left\{\{T_{m_1}R_{m_1}, T_{n_1}R_{n_1}, T_{m_1}T_{n_1}, R_{m_1}R_{n_1}\}\right)^{\eta}$

iv). $y\varsigma_1(\eta) = e(\eta) = y(T_{m_1})^{1-\eta}(T_{n_1})^{\eta} if y > 0,$

$= y(T_{m_1})^{1-\eta}(T_{n_1})^{\eta} if y < 0.$

v). $p_1(\alpha) = \frac{k_1(\eta)}{h_1(\eta)} = (\min\{\frac{T_{m_1}}{R_{m_1}}, \frac{R_{n_1}}{T_{n_1}}, \frac{R_{m_1}}{T_{n_1}}, \frac{T_{n_1}}{R_{m_1}}\})^{1-\eta}(\max\{\left\{\frac{R_{m_1}}{T_{m_1}}, \frac{R_{n_1}}{T_{n_1}}, \frac{T_{m_1}}{R_{n_1}}, \frac{T_{n_1}}{R_{m_1}}\right\})^{\eta}$.

Where the parametric interval-valued function is $g_1(\eta), s_1(\eta), r_1(\eta), yk_1(\eta), p_1(\alpha), e(\eta)$ for constant ς_1 and $\eta \in [0,1]$.

3 | Model Formulation

Kaohsiung [19], is a tropical city, with an average annual temperature of 25°C, a maximum temperature of 32°C, and a minimum temperature of 18°C, according to statistics from 2001 to 2010 [7]. The population is divided into three groups: pre-adult vectors, adult female mosquitoes, and human hosts. For the A. aegypti pre-adult female mosquito population, two parameters—eggs and larvae—are defined within both the susceptible (Se) and infected (Ie) populations. Sv, Ev, and Iv, which stand for the number of susceptible, infected but not yet infectious, and infectious female mosquitoes at time t, respectively, were specified as the three parameters for the adult vector (mosquito) population. Similarly, three parameters were found for the host population (humans): Sh, Ih, and Rh. With this, integer order dynamics is now used to develop the dengue model.

$$
\begin{aligned}
\frac{dS_e}{dt} &= b_v(1 - v(\frac{I_v}{S_v + E_v + I_v})) - \omega S_e, \\
\frac{dI_e}{dt} &= b_v v(\frac{I_v}{S_v+E_v+I_v}) - \omega I_e, \\
\frac{dS_v}{dt} &= \omega S_e - \beta S_v \frac{I_h}{N_h} - \mu_v S_v, \\
\frac{dE_v}{dt} &= \beta S_v \frac{I_h}{N_h} - \epsilon E_v - \mu_v E_v, \\
\frac{dI_v}{dt} &= \epsilon E_v + \omega I_e - \mu_v I_v, \\
\frac{dS_h}{dt} &= \mu_{hb} N_h - \beta S_h \frac{I_v}{N_h} - \mu_{hd} S_h, \\
\frac{dI_h}{dt} &= \beta S_h \frac{I_v}{N_h} - \gamma I_h - \mu_{hd} I_h, \\
\frac{dR_h}{dt} &= \gamma I_h - \mu_{hd} R_h.
\end{aligned}
\tag{1}
$$

These parameters were selected based on the availability of experimental or observed values (Table 1) from the literature review.

Table 1. Significance of the relevant parameters

Parameter	Significance
b_v	Rate of oviposition of the egg
v	Vertical infection rate
ω	Pre-adult mosquito maturation rate
β	Transmission biting rate
μ_v	The death rate of adult mosquito
ϵ	Rate of incubation
γ	Human recovery rate
μ_{hb}	Human birth rate
μ_{hd}	Human death rate

3.1 |Model in Imprecise Environment

The coefficients in the suggested model (1) can be treated as interval numbers in the following way to modify it in the uncertain environment:

$$
\begin{aligned}
\frac{dS_e}{dt} &= \widehat{b_v}(1 - \hat{v}(\frac{I_v}{S_v + E_v + I_v})) - \hat{\omega} S_e, \\
\frac{dI_e}{dt} &= \widehat{b_v} \hat{v}(\frac{I_v}{S_v+E_v+I_v}) - \hat{\omega} I_e, \\
\frac{dS_v}{dt} &= \hat{\omega} S_e - \hat{\beta} S_v \frac{I_h}{N_h} - \widehat{\mu_v} S_v, \\
\frac{dE_v}{dt} &= \hat{\beta} S_v \frac{I_h}{N_h} - \hat{\epsilon} E_v - \widehat{\mu_v} E_v,
\end{aligned}
\tag{2}
$$

$$\frac{dI_v}{dt} = \hat{\epsilon}E_v + \widehat{\omega}I_e - \widehat{\mu_v}I_v,$$

$$\frac{dS_h}{dt} = \widehat{\mu_{hb}}N_h - \hat{\beta}S_h\frac{I_v}{N_h} - \widehat{\mu_{hd}}S_h,$$

$$\frac{dI_h}{dt} = \hat{\beta}S_h\frac{I_v}{N_h} - \hat{\gamma}I_h - \widehat{\mu_{hd}}I_h,$$

$$\frac{dR_h}{dt} = \hat{\gamma}I_h - \widehat{\mu_{hd}}R_h.$$

Now, we take $I_l(\rho) = I_{1L}^{1-\rho}I_{1R}^{\rho}$ for $\rho \in [0,1]$ for an interval $[I_{1L}, I_{1R}]$. Then the above system (2) can be written as follows:

$$\frac{dS_e}{dt} = b_{vL}^{1-\rho}b_{vR}^{\rho}(1 - v_R^{1-\rho}v_L^{\rho}(\frac{I_v}{S_v + E_v + I_v})) - \omega_R^{1-\rho}\omega_L^{\rho}S_e,$$

$$\frac{dI_e}{dt} = b_{vL}^{1-\rho}b_{vR}^{\rho}v_L^{1-\rho}v_R^{\rho}(\frac{I_v}{S_v+E_v+I_v}) - \omega_R^{1-\rho}\omega_L^{\rho}I_e,$$

$$\frac{dS_v}{dt} = \omega_L^{1-\rho}\omega_R^{\rho}S_e - \beta_R^{1-\rho}\beta_L^{\rho}S_v\frac{I_h}{N_h} - \mu_{vR}^{1-\rho}\mu_{vL}^{\rho}S_v,$$

$$\frac{dE_v}{dt} = \beta_L^{1-\rho}\beta_R^{\rho}S_v\frac{I_h}{N_h} - \epsilon_R^{1-\rho}\epsilon_L^{\rho}E_v - \mu_{vR}^{1-\rho}\mu_{vL}^{\rho}E_v, \tag{3}$$

$$\frac{dI_v}{dt} = \epsilon_L^{1-\rho}\epsilon_R^{\rho}E_v + \omega_L^{1-\rho}\omega_R^{\rho}I_e - \mu_{vR}^{1-\rho}\mu_{vL}^{\rho}I_v,$$

$$\frac{dS_h}{dt} = \mu_{hbL}^{1-\rho}\mu_{hbR}^{\rho}N_h - \beta_R^{1-\rho}\beta_L^{\rho}S_h\frac{I_v}{N_h} - \mu_{hdR}^{1-\rho}\mu_{hdL}^{\rho}S_h,$$

$$\frac{dI_h}{dt} = \beta_L^{1-\rho}\beta_R^{\rho}S_h\frac{I_v}{N_h} - \gamma_R^{1-\rho}\gamma_L^{\rho}I_h - \mu_{hdR}^{1-\rho}\mu_{hdL}^{\rho}I_h,$$

$$\frac{dR_h}{dt} = \gamma_L^{1-\rho}\gamma_R^{\rho}I_h - \mu_{hdR}^{1-\rho}\mu_{hdL}^{\rho}R_h.$$

4 | Analysis of the System

- **The Equilibrium Points of the System**

The system has two types of disease-free equilibrium points namely trivial disease-free equilibrium $E_0 = (0,0,0,0,0,S_h{}^0,0,0)$ and biologically realistic disease-free equilibrium $E_1 = (S_e{}^*,0,S_v{}^*,0,0,S_h{}^*,0,0)$ where $S_h{}^0 = \frac{\mu_{hbL}^{1-\rho}\mu_{hbR}^{\rho}N_h}{\mu_{hdR}^{1-\rho}\mu_{hdL}^{\rho}}$, $S_e{}^* = \frac{b_{vL}^{1-\rho}b_{vR}^{\rho}}{\omega_R^{1-\rho}\omega_L^{\rho}}$, $S_v{}^* = \frac{b_{vL}^{1-\rho}b_{vR}^{\rho}}{\mu_{vR}^{1-\rho}\mu_{vL}^{\rho}}$, $S_h{}^* = \frac{\mu_{hbL}^{1-\rho}\mu_{hbR}^{\rho}N_h}{\mu_{hdR}^{1-\rho}\mu_{hdL}^{\rho}}$.

- **The Basic Reproduction Number of the System**

Using the next-generation matrix approach [21, 22] we have,

$$\mathcal{F}V^{-1} = \begin{bmatrix} 0 & \frac{\beta_L^{1-\rho}\beta_R^{\rho}}{\mu_{vR}^{1-\rho}\mu_{vL}^{\rho}} & \frac{\beta_L^{1-\rho}\beta_R^{\rho}\omega_L^{1-\rho}\omega_R^{\rho}}{\mu_{vR}^{1-\rho}\mu_{vL}^{\rho}(\omega_L^{1-\rho}\omega_R^{\rho}+\mu_{vR}^{1-\rho}\mu_{vL}^{\rho})} & \frac{\beta_L^{1-\rho}\beta_R^{\rho}}{\mu_{vR}^{1-\rho}\mu_{vL}^{\rho}} \\ 0 & v_L^{1-\rho}v_R^{\rho} & \frac{v_L^{1-\rho}v_R^{\rho}\epsilon_L^{1-\rho}\epsilon_R^{\rho}}{(v_L^{1-\rho}v_R^{\rho}+\mu_{vR}^{1-\rho}\mu_{vL}^{\rho})} & v_L^{1-\rho}v_R^{\rho} \\ \frac{\beta_L^{1-\rho}\beta_R^{\rho}N_v}{N_h(\gamma+\mu_{hd})} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Therefore, $R_0 = \frac{v_L^{1-\rho} v_R^{\rho}}{2} + \frac{1}{2}\sqrt{(v_L^{1-\rho} v_R^{\rho})^2 + \frac{4(\beta_L^{1-\rho}\beta_R^{\rho})^2 \omega_L^{1-\rho}\omega_R^{\rho} N_m}{N_h(\gamma_L^{1-\rho}\gamma_R^{\rho} + \mu_{hdR}^{1-\rho}\mu_{hdL}^{\rho})\mu_{vR}^{1-\rho}\mu_{vL}^{\rho}(\epsilon_L^{1-\rho}\epsilon_R^{\rho} + \mu_{vR}^{1-\rho}\mu_{vL}^{\rho})}}$.

5 | Numerical Study

We used the math programs Matlab (2018) and Matcont to numerically estimate the solution of our model system. Using the model parameter values listed in Table 2, we simulate the system (3) in this scenario and change the value of parameter " ρ " to four different levels (ρ = 0, 0.4, 0.6, 1). The model system (3)'s time series plot, displayed in Figure 1, for various values of parameter ρ, clearly illustrates the stability of the disease-free equilibrium point throughout the time range [0, 100].

Table 2. Shows the values of the several parameters that are used to model the system (3).

Parameters	Values (For Planer)
$\widehat{b_v}$	[0.51, 0.61]
$\hat{v}$	[0.37, 0.47]
$\hat{w}$	[0.33, 0.44]
$\hat{\beta}$	[0.066, 0.076]
N_h	0.051
$\hat{\gamma}$	[1.23, 1.33]
$\hat{\epsilon}$	[0.45, 0.55]
$\widehat{\mu_v}$	[0.57, 0.67]
$\widehat{\mu_{hb}}$	[8.81, 8.91]
$\widehat{\mu_{hd}}$	[0.87, 0.97]

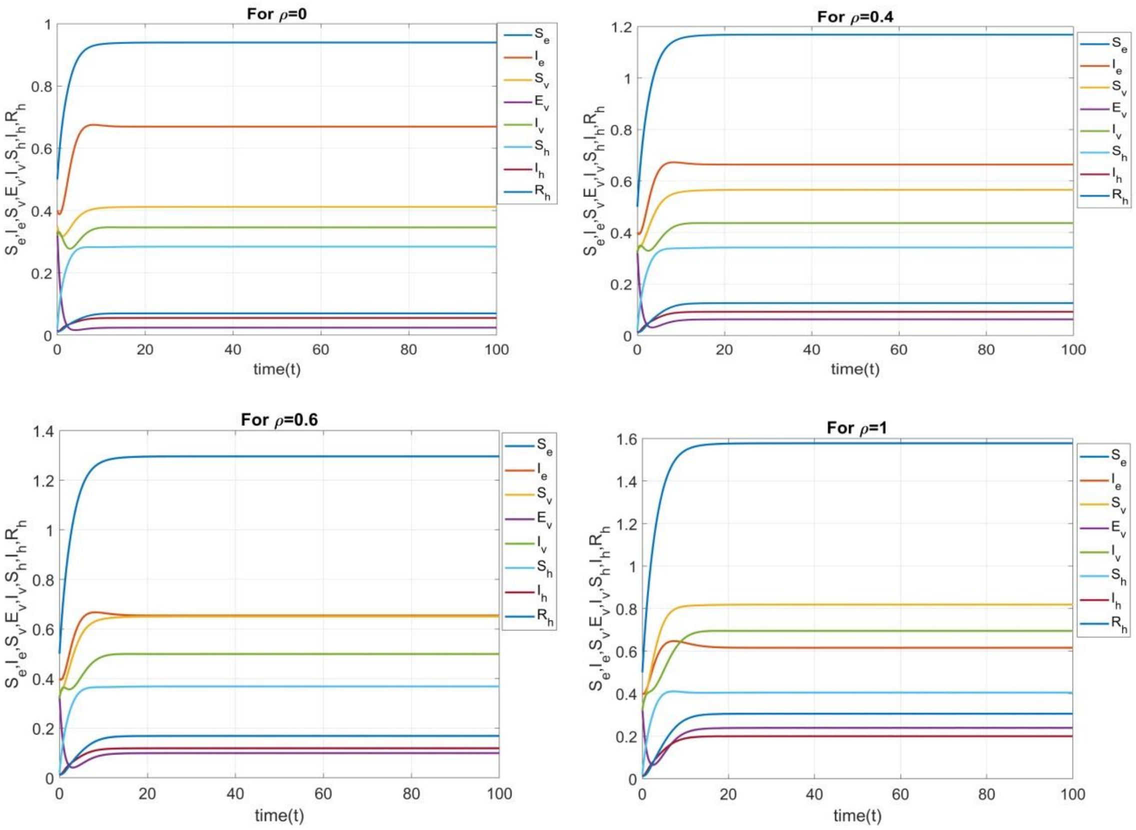

Figure 1. For various values of the parameter ρ, depicts the stable nature of the disease-free equilibrium point.

6 | Conclusion

We have investigated a dengue model with temperature effects in the presence of interval uncertainty. The behavior of the model in an uncertain environment is examined in this analysis. The temperature has an impact on dengue and other illnesses spread by mosquitoes; these effects are anticipated to vary over time and between geographical areas. Predictions regarding how these temperature effects may alter in response to climate change may also become more complex due to potential interactions with other environmental elements. Our analysis of the available data on dengue disease and temperature lends credence to the hypothesis that temperature has a nonlinear effect on ecological processes. In particular, we anticipate reduced or negative effects at extremely low and high temperatures and considerable favorable effects within certain middle. However because the information on the species that transmits disease differed throughout research, we were unable to test this notion. Compared to the simpler models that assumed a constant or linear relationship between average temperature and dengue, the flexible quadratic temperature model and the a priori marginal temperature suitability model fared much better. The authors want to use mathematical modeling methodologies and more ecological modeling with neutrosophic environments in future studies to offer new medical research models.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contributions

All authors contributed equally to this work.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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