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Soft Intersection Almost Tri-ideals of Semigroups

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Abstract

The notions of left (right) ideal and quasi-ideal of semigroup are generalized by the concept of tri-ideal. Likewise, the concepts of soft intersection left (right) ideal and soft intersection quasi-ideal of semigroups are generalized by the soft intersection tri-ideal. In this paper, we present and study the concept of soft intersection almost tri-ideal ideal as a further generalization of nonnull soft intersection tri-ideal. It is demonstrated that every idempotent soft intersection almost bi-ideal is a soft intersection almost tri-ideal, and vice versa. We also obtained that an idempotent soft intersection almost left (or right) tri-ideal coincides with the soft intersection almost tri-ideal. It is also shown that every idempotent soft intersection almost tri-ideal is a soft intersection almost subsemigroup. With the noteworthy result that if a nonempty subset of a semigroup is an almost tri-ideal, then its soft characteristic function is also a soft intersection almost tri-ideal, and vice versa, a number of intriguing relationships regarding minimality, primeness, semiprimeness, and strongly primeness between almost tri-ideals and soft intersection almost tri-ideals are derived.

Keywords: Soft Set; Semigroup; (Almost) Tri-ideals; Soft Intersection (Almost) Tri-ideals.

1 | Introduction

Semigroups were first formally studied in the early 1900s, and they are important in many areas of mathematics as they give the abstract algebraic basis for "memoryless" systems, which restart on every iteration. In practical mathematics, semigroups are essential models for linear time-invariant systems. Since finite semigroups naturally relate to finite automata, studying semigroups is very important in theoretical computer science. Furthermore, semigroups are related to Markov processes in probability theory.

Ideals are necessary to understand algebraic structures and their applications. Ideals were first put out by Dedekind to help in the study of algebraic numbers. Noether then extended them further by adding associative rings. Good and Hughes [1] proposed bi-ideals for semigroups for the first time in 1952. Steinfeld [2] first presented the idea of quasi-ideals for semigroups and then expanded it to rings. Many mathematicians have focused on generalizing ideals in algebraic structures.

Grosek and Satko [3] proposed the notion of almost left, right, and two-sided ideals of semigroups for the first time in 1980. Later, in 1981, Bogdanovic [4] expanded the concept of bi-ideals to almost bi-ideals in

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semigroups. By fusing the concepts of almost ideals and quasi-ideals of semigroups, Wattanatripop et al. [5] proposed almost quasi-ideals in 2018. In 2020, Kaopusek et al. [6] proposed almost interior ideals and weakly almost interior ideals of semigroups, expanding upon the ideas of almost ideals and interior ideals of semigroups and analyzing their features. The concepts of almost subsemigroups, almost bi-quasi-interior ideals, almost bi-interior ideals, and almost bi-quasi-ideals of semigroups were subsequently presented by Iampan [7] in 2022, Chinram and Nakkhasen [8] in 2022, Gaketem [9] in 2022, and Gaketem and Chinram [10] in 2023, respectively. Moreover, several types of almost fuzzy ideals of semigroups were examined in [5, 7–12].

To model uncertainty, Molodtsov [13] introduced the concept of a soft set in 1999, which has since attracted interest from various domains. The foundational operations of soft sets were explored in [14–29]. The concept of soft set was further developed and modifed by Çağman and Enginoğlu [30]. Çağman et alç [31] introduced soft intersection groups, leading to research on several soft algebraic systems. Soft sets were also applied in semigroup theory, particularly in semigroups with soft intersection left, right, and two-sided ideals, quasi-ideals, interior ideals, and generalized bi-ideals, as extensively examined in [32–33]. Sezgin and Orbay [34] classified various semigroups based on soft intersection substructures. Additionally, a variety of soft algebraic structures were investigated in [35–44]. Expanding on existing ideals, Rao [45–48] recently introduced several new types of semigroups, such as bi-interior ideals, bi-quasi-interior ideals, bi-quasi-ideals, quasi-interior ideals, and weak interior ideals. Furthermore, Baupradist et al. [49] proposed the notion of essential ideals in semigroups.

As a generalization of left (right) ideals and quasi-ideals, Rao [50] proposed the idea of a tri-ideal for semigroups. As a generalization of the semigroup's soft intersection left (right)-ideal and soft intersection quasi-ideal, the soft intersection tri-ideal was presented in [51]. In this paper, we propose the concept of soft intersection almost tri-ideal as a further generalization of nonnull soft intersection tri-ideal. It is proved that idempotent almost tri-ideals coincide with soft intersection almost bi-ideals, and idempotent soft intersection almost one sided tri-ideals and soft intersection almost tri-ideals are equivalent. Furthermore, any idempotent soft intersection almost tri-ideal was found to be a soft intersection almost subsemigroup. We prove that soft intersection almost tri-ideals of a semigroup may be used to generate a semigroup with the binary operation of soft union, but not with the operation of soft intersection. With the important result that if a nonempty subset of a semigroup is an almost tri-ideal, then its soft characteristic function is also a soft intersection almost tri-ideal, and vice versa, a number of fascinating relationships pertaining to minimality, primeness, semiprimeness, and strongly primeness between almost tri-ideals and soft intersection almost tri-ideals have been derived.

2 | Preliminary

In this section, we review several fundamental notions related to semigroups and soft sets. A semigroup S is a nonempty set with an associative binary operation, and throughout this paper, S stands for a semigroup A semigroup S is a nonempty set with an associative binary operation, and throughout this paper, S stands for a semigroup. A non-empty subset A of S is called a subsemigroup of S if $AA \subseteq A$, is called a left-ideal (right-ideal) of S if $AS \subseteq A$ $(SA \subseteq A)$, is called an ideal of S if $SA \subseteq A$ and $AS \subseteq A$, and is called an interior ideal of S if $SAS \subseteq A$. A subsemigroup A of S is called a bi-ideal of S if $ASA \subseteq A$.

Definition 2.1. [50]. A non-empty subset A of S is called a left tri-ideal (right tri-ideal) of S if A is a subsemigroup of S and $ASAA \subseteq A$ $(AASA \subseteq A)$, is called a tri-ideal of S if A is a subsemigroup of S, $ASAA \subseteq A$ and $AASA \subseteq A$.

Definition 2.2. A nonempty subset A of S is called an almost left tri-ideal (T-ideal) of S if $AsAA \cap A \neq \emptyset$ for all $s \in S$, and is called an almost right tri-ideal of S if $AAsA \cap A \neq \emptyset$ for all $s \in S$, and is called an almost tri-ideal of S if A is both an almost left tri-ideal of S and almost right tri-ideal of S.

Example 2.3. Let $S=\mathbb{Z}$ and $\emptyset\neq 2\mathbb{Z}\subseteq\mathbb{Z}$. Since $(2\mathbb{Z})s(2\mathbb{Z})(2\mathbb{Z})\cap 2\mathbb{Z}\neq\emptyset$ for all $s\in\mathbb{Z}$, $2\mathbb{Z}$ is almost tri-ideal of S.

An almost (left/right) tri-ideal A of S is called a minimal almost (left/right) tri-ideal of S if for any almost (left/right) tri-ideal B of S if whenever $B\subseteq A$, then $A=B$. An almost (left/right) tri-ideal P of S is called a prime almost (left/right) tri-ideal if for any almost (left/right) tri-ideals A and B of S such that $AB\subseteq P$ implies that $A\subseteq P$ or $B\subseteq P$. An almost (left/right) tri-ideal P of S is called a semiprime almost (left/right) tri-ideal if for any almost (left/right) tri-ideal A of S such that $AA\subseteq P$ implies that $A\subseteq P$. An almost (left/right) tri-ideal P of S is called a strongly prime almost (left/right) tri-ideal if for any almost (left/right) tri-ideals A and B of S such that $AB\cap BA\subseteq P$ implies that $A\subseteq P$ or $B\subseteq P$.

Definition 2.4. [13,30]. Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U, and $K\subseteq E$. A soft set f_K over U is a set-valued function such that $f_K: E\rightarrow P(U)$ such that for all $x\notin K, f_K(x)=\emptyset$. A soft set over U can be represented by the set of ordered pairs

$$f_K=\{(x,f_K(x)): x\in E, f_K(x)\in P(U)\}$$

Throughout this paper, the set of all the soft sets over U is designated by $S_E(U)$.

Definition 2.5. [30]. Let $f_A\in S_E(U)$. If $f_A(\mathrm{x})=\emptyset$ for all $x\in E$, then f_A is called a null soft set and denoted by $\emptyset_E$.

Definition 2.6. [30]. Let $f_A, f_B\in S_E(U)$. If $f_A(\mathrm{x})\subseteq f_B(\mathrm{x})$ for all $x\in E$, then f_A is a soft subset of f_B and denoted by $f_A\,\tilde{\subseteq}\, f_B$. If $f_A(x)=f_B(x)$ for all $x\in E$, then f_A is called soft equal to f_B and denoted by $f_A=f_B$.

Definition 2.7. [30]. Let f_A, $f_B\in S_E(U)$. The union of f_A and f_B is the soft set $f_A\,\tilde{\cup}\, f_B$, where $(f_A\,\tilde{\cup}\, f_B)(x)=f_A(x)\cup f_B(x)$, for all $x\in E$. The intersection of f_A and f_B is the soft set $f_A\,\tilde{\cap}\, f_B$, where $(f_A\,\tilde{\cap}\, f_B)(x)=f_A(x)\cap f_B(x)$, for all $x\in E$.

Definition 2.8. [18]. For a soft set f_A, the support of f_A is defined by

$$supp(f_A)=\{x\in A: f_A(x)\neq\emptyset\}$$

It is obvious that a soft set with an empty support is a null soft set, otherwise, the soft set is nonnull.

Note 2.9. [52]. If $f_A\,\tilde{\subseteq}\, f_B$, then $supp(f_A)\subseteq supp(f_B)$.

Definition 2.10. [32]. Let f_S and g_S be soft sets over the common universe U. Then, soft intersection product $f_S\circ g_S$ is defined by

$$(f_S\circ g_S)(x)=\begin{cases}\bigcup\limits_{x=yz}\{f_S(y)\cap g_S(z)\}, & if\ \exists y,z\in S\ such\ that\ x=yz\\ \emptyset, & otherwise\end{cases}$$

Theorem 2.11. [32]. Let f_S, g_S, $h_S\in S_S(U)$. Then,

i). $(f_S\circ g_S)\circ h_S=f_S\circ(g_S\circ h_S)$.

ii). $f_S\circ g_S\neq g_S\circ f_S\,, generally.$

iii). $f_S\circ(g_S\,\tilde{\cup}\, h_S)=(f_S\circ g_S)\,\tilde{\cup}\,(f_S\circ h_S)$ and $(f_S\,\tilde{\cup}\, g_S)\circ h_S=(f_S\circ h_S)\,\tilde{\cup}\,(g_S\circ h_S)$.

iv). $f_S\circ(g_S\,\tilde{\cap}\, h_S)=(f_S\circ g_S)\,\tilde{\cap}\,(f_S\circ h_S)$ and $(f_S\,\tilde{\cap}\, g_S)\circ h_S=(f_S\circ h_S)\,\tilde{\cap}\,(g_S\circ h_S)$.

v). If $f_S\,\tilde{\subseteq}\, g_S$, then $f_S\circ h_S\,\tilde{\subseteq}\, g_S\circ h_S$ and $h_S\circ f_S\,\tilde{\subseteq}\, h_S\circ g_S$.

vi). If $t_S, k_S\in S_S(U)$ such that $t_S\,\tilde{\subseteq}\, f_S$ and $k_S\,\tilde{\subseteq}\, g_S$, then $t_S\circ k_S\,\tilde{\subseteq}\, f_S\circ g_S$.

Definition 2.12. [32]. Let A be a subset of S. We denote by S_A the soft characteristic function of A and define as

$$S_A(x) = \begin{cases} U, & if\ x \in A \\ \emptyset, & if\ x \in S \backslash A \end{cases}$$

The soft characteristic function of A is a soft set over U, that is, $S_A: S \longrightarrow P(U)$.

Corollary 2.13. [52]. $supp(S_A) = A$.

Theorem 2.14. [32,52]. Let X and Y be nonempty subsets of S. Then, the following properties hold

i). $X \subseteq Y$ if and only if $S_X \tilde{\subseteq} S_Y$

ii). $S_X \tilde{\cap} S_Y = S_{X \cap Y}$ and $S_X \tilde{\cup} S_Y = S_{X \cup Y}$

iii). $S_X \circ S_Y = S_{XY}$

Definition 2.15. [53]. Let x be an element in S. We denote by S_x the soft characteristic function of x and define as

$$S_x(y) = \begin{cases} U, & if\ \ y = x \\ \emptyset, & if\ \ y \neq x \end{cases}$$

Definition 2.16. [51]. A soft set f_S over U is called a soft intersection left (resp., right) tri-ideal of S over U if $f_S(xyzt) \supseteq f_S(x) \cap f_S(z) \cap f_S(t)$ $(f_S(xyzt) \supseteq f_S(x) \cap f_S(y) \cap f_S(t)$, for all $x, y, z, t \in S$. A soft set f_S over U is called a soft intersection tri-ideal of S if it is both soft intersection left tri-ideal and right tri-ideal of S over U.

It is easy to see that if $f_S(x) = U$ for all $x \in S$, then f_S is a soft intersection (left/right) tri-ideal of S. We denote such a kind of (left/right) tri-ideal by $\tilde{\mathbb{S}}$. It is obvious that $\tilde{\mathbb{S}} = S_S$, that is, $\tilde{\mathbb{S}}(x) = U$ for all $x \in S$ [51].

Theorem 2.17. [51]. Let f_S be a soft set over U. Then, f_S is a soft intersection left (right) tri-ideal of S if and only if $f_S \circ \tilde{\mathbb{S}} \circ f_S \circ f_S \tilde{\subseteq} f_S$ $(f_S \circ f_S \circ \tilde{\mathbb{S}} \circ f_S \tilde{\subseteq} f_S)$ and f_S is a soft intersection tri-ideal of S if and only if $f_S \circ \tilde{\mathbb{S}} \circ f_S \circ f_S \tilde{\subseteq} f_S$ and $f_S \circ f_S \circ \tilde{\mathbb{S}} \circ f_S \tilde{\subseteq} f_S$.

From now on, soft intersection left (right) tri-ideal of S is denoted by SI-left (right) T-ideal.

Definition 2.18. [52-54]. A soft set f_S is called a soft intersection almost subsemigroup of S if $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, is called a soft intersection almost left (resp. right) ideal of S if $(S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ $\left((f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S\right)$, for all $x \in S$. f_S is called a soft intersection almost two-sided ideal (or briefly soft intersection almost ideal) of S if f_S is both soft intersection almost left ideal of S and soft intersection almost right ideal of S. f_S is called a soft intersection almost bi-ideal of S if $(f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$.

Regarding the probable consequences of network analysis and graph applications concerning soft sets (defined by the divisibility of determinants), and more on soft set operations, we refer to [55], and [56-71], respectively.

3 | Soft Intersection Almost Tri-ideals of Semigroups

Definition 3.1. A soft set f_S is called a soft intersection almost left tri-ideal of S if

$$(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$$

for all $x \in S$.

Definition 3.2. A soft set f_S is called a soft intersection almost right tri-ideal of S if

$$(f_S \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$$

for all $x \in S$.

Definition 3.3. A soft set f_S is called a soft intersection almost tri-ideal of S if

$(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ and $(f_S \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$

for all $x \in S$. Hereafter, for brevity, soft intersection almost left (right) tri-ideal of S is denoted by SI-almost left (right) T-ideal.

Example 3.4. Let $S = \{p, r\}$ be the semigroup with the following Cayley Table.

Table 1. Cayley table of binary operation

	p	r
p	p	r
r	r	p

Let f_S, h_S, and g_S be soft sets over $U = N$ as follows:

$$f_S = \{(p, \{1,2\}), (r, \{1,3\})\}$$

$$h_S = \{(p, \{4,5\}), (r, \{5,6\})\}$$

$$g_S = \{(p, \{1,2\}), (r, \{3,4\})\}$$

Here, f_S and h_S are both SI-almost T-ideals. Let's first show that f_S is an SI-almost left T-ideal, that is, $(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$.

Let's start with $\left(f_S \circ S_p \circ f_S \circ f_S\right) \tilde{\cap} f_S$:

$\left[\left(f_S \circ S_p \circ f_S \circ f_S\right) \tilde{\cap} f_S\right](p) = [(f_S \circ S_p) \circ (f_S \circ f_S)](p) \cap f_S(p) = \{ [\left(f_S \circ S_p\right)(p) \cap (f_S \circ f_S)(p)] \cup [\left(f_S \circ S_p\right)(r) \cap (f_S \circ f_S)(r)]\} \cap f_S(p) = \{ \{ [(f_S(p) \cap S_p(p)) \cup (f_S(r) \cap S_p(r))] \cap [(f_S(p) \cap f_S(p)) \cup (f_S(r) \cap f_S(r))]\} \cup \{ [(f_S(r) \cap S_p(p)) \cup (f_S(p) \cap S_p(r))] \cap [(f_S(r) \cap f_S(p)) \cup (f_S(p) \cap f_S(r))] \} \} \cap f_S(p) = \{ [f_S(p) \cap (f_S(p) \cup f_S(r))] \cup [f_S(r) \cap (f_S(p) \cap f_S(r))] \} \cap f_S(p) = [f_S(p) \cup (f_S(p) \cap f_S(r))] \cap f_S(p) = f_S(p) \cap f_S(p) = f_S(p)$

$\left[\left(f_S \circ S_p \circ f_S \circ f_S\right) \tilde{\cap} f_S\right](r) = [(f_S \circ S_p) \circ (f_S \circ f_S)](r) \cap f_S(r) = \{ [\left(f_S \circ S_p\right)(r) \cap (f_S \circ f_S)(p)] \cup [\left(f_S \circ S_p\right)(p) \cap (f_S \circ f_S)(r)] \} \cap f_S(r) = \{ \{ [(f_S(r) \cap S_p(p)) \cup (f_S(p) \cap S_p(r))] \cap [(f_S(p) \cap f_S(p)) \cup (f_S(r) \cap f_S(r))] \} \cup \{ [(f_S(p) \cap S_p(p)) \cup (f_S(r) \cap S_p(r))] \cap [(f_S(r) \cap f_S(p)) \cup (f_S(p) \cap f_S(r))] \} \} \cap f_S(r) = \{ [f_S(r) \cap (f_S(p) \cup f_S(r))] \cup [f_S(p) \cap (f_S(p) \cap f_S(r))]\} \cap f_S(r) = [f_S(r) \cup (f_S(p) \cap f_S(r))] \cap f_S(r) = f_S(r) \cap f_S(r) = f_S(r)$

Consequently,

$$\left(f_S \circ S_p \circ f_S \circ f_S\right) \tilde{\cap} f_S = \{(p, \{1,2\}), (r, \{1,3\})\} \neq \emptyset_S$$

Similarly,

$$(f_S \circ S_r \circ f_S \circ f_S) \tilde{\cap} f_S = \{(p, \{1\}), (r, \{1\})\} \neq \emptyset_S$$

Therefore, $(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$. Thus, f_S is an SI-almost left T-ideal. Now let's show that f_S is an SI-almost right T-ideal, that is, $(f_S \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$:

$$\left(f_S \circ f_S \circ S_p \circ f_S\right) \tilde{\cap} f_S = \{(p, \{1,2\}), (r, \{1,3\})\} \neq \emptyset_S$$

$$(f_S \circ f_S \circ S_r \circ f_S) \tilde{\cap} f_S = \{(p, \{1\}), (r, \{1\})\} \neq \emptyset_S$$

Therefore, $(f_S \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$. Hence, f_S is an SI-almost right T-ideal and so, f_S is an SI-almost T-ideal.

Similarly, h_S is an SI-almost left T-ideal and SI-almost right T-ideal, thus h_S is an SI-almost T-ideal. In fact;

$$\left(h_S \circ S_p \circ h_S \circ h_S\right) \tilde{\cap}\, h_S = \{(p,\{4,5\}),(r,\{5,6\})\} \neq \emptyset_S$$

$$(h_S \circ S_r \circ h_S \circ h_S) \tilde{\cap}\, h_S = \{(p,\{5\}),(r,\{5\})\} \neq \emptyset_S$$

Hence, h_S is an SI-almost left T-ideal and

$$\left(h_S \circ h_S \circ S_p \circ h_S\right) \tilde{\cap}\, h_S = \{(p,\{4,5\}),(r,\{5,6\})\} \neq \emptyset_S$$

$$(h_S \circ h_S \circ S_r \circ h_S) \tilde{\cap}\, h_S = \{(p,\{5\}),(r,\{5\})\} \neq \emptyset_S$$

Thus, h_S is an SI-almost right T-ideal and so, h_S is an SI-almost T-ideal. One can also show that g_S is not an SI-almost (left/right) T-ideal. In fact;

$[(g_S \circ S_r \circ g_S \circ g_S) \tilde{\cap}\, g_S](p) = [(g_S \circ S_r) \circ (g_S \circ g_S)](p) \cap g_S(p) = \{\, [\, (g_S \circ S_r)(p) \cap (g_S \circ g_S)(p)\,] \cup [\,(g_S \circ S_r)(r) \cap (g_S \circ g_S)(r)\,]\,\} \cap g_S(p) = \{\{\,[\,(g_S(p) \cap S_r(p)) \cup (g_S(r) \cap S_r(r))\,] \cap [\,(g_S(p) \cap g_S(p)) \cup (g_S(r) \cap g_S(r))\,]\,\} \cup \{\,[\,(g_S(r) \cap S_r(p)) \cup (g_S(p) \cap S_r(r))\,] \cap [\,(g_S(r) \cap g_S(p)) \cup (g_S(p) \cap g_S(r))\,]\,\}\} \cap g_S(p) = \{\,[\,g_S(r) \cap (g_S(p) \cup g_S(r))\,] \cup [\,g_S(p) \cap (g_S(p) \cap g_S(r))\,]\,\} \cap g_S(p) = [g_S(r) \cup (g_S(p) \cap g_S(r))] \cap g_S(p) = g_S(r) \cap g_S(p) = \emptyset$

$[(g_S \circ S_r \circ g_S \circ g_S) \tilde{\cap}\, g_S](r) = [(g_S \circ S_r) \circ (g_S \circ g_S)](r) \cap g_S(r) = \{\,[\,(g_S \circ S_r)(r) \cap (g_S \circ g_S)(p)\,] \cup [\,(g_S \circ S_r)(p) \cap (g_S \circ g_S)(r)\,]\,\} \cap g_S(r) = \{\{\,[\,(g_S(r) \cap S_r(p)) \cup (g_S(p) \cap S_r(r))\,] \cap [\,(g_S(p) \cap g_S(p)) \cup (g_S(r) \cap g_S(r))\,]\,\} \cup \{\,[\,(g_S(p) \cap S_r(p)) \cup (g_S(r) \cap S_r(r))\,] \cap [\,(g_S(r) \cap g_S(p)) \cup (g_S(p) \cap g_S(r))\,]\,\}\} \cap g_S(r) = \{\,[g_S(p) \cap (g_S(p) \cup g_S(r))\,] \cup [g_S(r) \cap (g_S(p) \cap g_S(r))\,]\,\} \cap g_S(r) = [g_S(p) \cup (g_S(p) \cap g_S(r))] \cap g_S(r) = g_S(p) \cap g_S(r) = \emptyset$

$(g_S \circ S_r \circ g_S \circ g_S) \tilde{\cap}\, g_S = \emptyset_S$ for $r \in S$. Thus, g_S is not an SI-almost left T-ideal. Similary,

$$(g_S \circ g_S \circ S_r \circ g_S) \tilde{\cap}\, g_S = \{(p,\emptyset),(r,\emptyset)\} = \emptyset_S$$

for $r \in S$, g_S is not an SI-almost right T-ideal. Obviously, g_S is not an SI-almost T-ideal.

From now on, the proofs are given for only SI-almost left T-ideal, since the proofs for SI-almost right T-ideal and SI-almost T-ideal can be shown similarly.

Proposition 3.5. If f_S is an SI-left (resp., right) T-ideal such that $f_S \circ S_x \circ f_S \circ f_S \neq \emptyset_S$, then f_S is an SI-almost left (resp., right) T-ideal.

Proof: Let $f_S \circ S_x \circ f_S \circ f_S \neq \emptyset_S$ be an SI-left T-ideal. By definition of SI-left T-ideal, $f_S \circ \widetilde{\mathbb{S}} \circ f_S \circ f_S \tilde{\subseteq} f_S$. We need to show that

$$(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$$

for all $x \in S$. Since $f_S \circ S_x \circ f_S \circ f_S \tilde{\subseteq} f_S \circ \widetilde{\mathbb{S}} \circ f_S \circ f_S \tilde{\subseteq} f_S$, it follows that $f_S \circ S_x \circ f_S \circ f_S \tilde{\subseteq} f_S$. Thus,

$$(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S = f_S \circ S_x \circ f_S \circ f_S \neq \emptyset_S$$

implying that f_S is an SI-almost left T-ideal.

Note that $f_S \circ S_x \circ f_S \circ f_S \neq \emptyset_S$ implies that $f_S \neq \emptyset_S$. It is obvious that $\emptyset_S$ is an SI-T-ideal, as $\emptyset_S \circ \widetilde{\mathbb{S}} \circ \emptyset_S \circ \emptyset_S \tilde{\subseteq} \emptyset_S$ and $\emptyset_S \circ \emptyset_S \circ \widetilde{\mathbb{S}} \circ \emptyset_S \tilde{\subseteq} \emptyset_S$; but it is not SI-almost T-ideal, since $(\emptyset_S \circ S_x \circ \emptyset_S \circ \emptyset_S) \tilde{\cap} \emptyset_S = \emptyset_S \tilde{\cap} \emptyset_S = \emptyset_S$, for all $x \in S$.

Here, note that if f_S is an SI-almost T-ideal, then f_S needs not be an SI-T-ideal as shown in the following example:

Example 3.6. In Example 3.4, it is shown that f_S and h_S are SI-almost T-ideals; however, f_S and h_S are not SI-T-ideals. In fact;

$(f_S \circ \widetilde{\mathbb{S}} \circ f_S \circ f_S)(p) = [(f_S \circ \widetilde{\mathbb{S}})(p) \cap (f_S \circ f_S)(p)] \cup [(f_S \circ \widetilde{\mathbb{S}})(r) \cap (f_S \circ f_S)(r)] = \{ [(f_S(p) \cap \widetilde{\mathbb{S}}(p)) \cup (f_S(r) \cap \widetilde{\mathbb{S}}(r))] \cap [(f_S(p) \cap f_S(p)) \cup (f_S(r) \cap f_S(r))]\} \cup \{ [(f_S(r) \cap \widetilde{\mathbb{S}}(p)) \cup (f_S(p) \cap \widetilde{\mathbb{S}}(r))] \cap [(f_S(r) \cap f_S(p)) \cup (f_S(p) \cap f_S(r))]\} = [((f_S(p) \cup f_S(r)) \cap (f_S(p) \cup f_S(r))] \cup [((f_S(r) \cup f_S(p)) \cap ((f_S(p) \cap f_S(r))] = [(f_S(p) \cup f_S(r)] \cup [(f_S(p) \cap f_S(r)] = [(f_S(p) \cup f_S(r)] \not\subseteq f_S(p)$

thus, f_S is not an SI-left T-ideal. Similarly,

$(f_S \circ f_S \circ \widetilde{\mathbb{S}} \circ f_S)(p) = [(f_S \circ f_S)(p) \cap (\widetilde{\mathbb{S}} \circ f_S)(p)] \cup [(f_S \circ f_S)(r) \cap (\widetilde{\mathbb{S}} \circ f_S)(r)] = \{ [(f_S(p) \cap f_S(p)) \cup (f_S(r) \cap f_S(r))] \cap [(\widetilde{\mathbb{S}}(p) \cap f_S(p)) \cup (\widetilde{\mathbb{S}}(r) \cap f_S(r))]\} \cup \{ [(f_S(r) \cap f_S(p)) \cup (f_S(p) \cap f_S(r))] \cap [(\widetilde{\mathbb{S}}(r) \cap f_S(p)) \cup (\widetilde{\mathbb{S}}(p) \cap f_S(r))]\} = [((f_S(p) \cup f_S(r)) \cap (f_S(p) \cup f_S(r))] \cup [((f_S(r) \cap f_S(p)) \cap ((f_S(p) \cup f_S(r))] = [(f_S(p) \cup f_S(r)] \cup [(f_S(p) \cap f_S(r)] = [(f_S(p) \cup f_S(r)] \not\subseteq f_S(p)$

thus, f_S is not an SI-right T-ideal; hence f_S is not an SI-T-ideal. Similarly,

$$(h_S \circ \widetilde{\mathbb{S}} \circ h_S \circ h_S)(p) = h_S(p) \cup h_S(r) \not\subseteq h_S(p)$$

thus, h_S is not an SI-left T-ideal. Similarly,

$$(h_S \circ h_S \circ \widetilde{\mathbb{S}} \circ h_S)(p) = h_S(p) \cup h_S(r) \not\subseteq h_S(r)$$

thus, h_S is not an SI-right T-ideal; hence h_S is not an SI-T-ideal.

Proposition 3.7. Let f_S be an idempotent SI-almost left (right) T-ideal. Then, f_S is an SI-almost subsemigroup.

Proof: Assume that f_S is an idempotent SI-almost left T-ideal. Then, $f_S \circ f_S = f_S$ and $(f_S \circ S_x \circ f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$. We need to show that

$$(f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$$

for all $x \in S$. Since,

$$\begin{aligned} \emptyset_S &\neq (f_S \circ S_x \circ f_S \circ f_S) \widetilde{\cap} f_S \\ &= [(f_S \circ S_x \circ f_S \circ f_S) \widetilde{\cap} f_S] \widetilde{\cap} f_S \\ &= [(f_S \circ S_x \circ f_S \circ f_S) \widetilde{\cap} (f_S \circ f_S)] \widetilde{\cap} f_S \end{aligned}$$

$\widetilde{\subseteq} (f_S \circ f_S) \widetilde{\cap} f_S$

f_S is an SI-almost subsemigroup.

Theorem 3.8. Let f_S be an idempotent soft set. Then, f_S is an SI-almost bi-ideal if and only if f_S is an SI-almost T-ideal.

Proof: Assume that f_S is an idempotent soft set. Then, $f_S \circ f_S = f_S$. By assumption, since

$$\emptyset_S \neq (f_S \circ S_x \circ f_S) \widetilde{\cap} f_S = (f_S \circ S_x \circ f_S \circ f_S) \widetilde{\cap} f_S = (f_S \circ f_S \circ S_x \circ f_S) \widetilde{\cap} f_S$$

for all $x \in S$, the rest of the proof is obvious.

Theorem 3.9. Let f_S be an idempotent soft set. Then, f_S is an SI-almost left (right) T-ideal if and only if f_S is an SI-almost T-ideal.

Proof: Assume that f_S is an idempotent soft set. Then, $f_S \circ f_S = f_S$. By assumption, since

$$\emptyset_S \neq (f_S \circ S_x \circ f_S \circ f_S) \widetilde{\cap} f_S = ((f_S \circ f_S) \circ S_x \circ f_S) \widetilde{\cap} f_S = (f_S \circ (f_S \circ S_x) \circ f_S) \widetilde{\cap} f_S$$

for all $x \in S$, the rest of the proof is obvious.

From Theorem 3.9, we can conclude that even the soft intersection product is not commutative in the set $S_E(U)$, when the soft set is idempotent; SI-almost left T-ideal implies that SI-almost right T-ideal and SI-almost right T-ideal implies that SI-almost left T-ideal; thus SI-almost one-sided T-ideal implies that SI-almost T-ideal.

Theorem 3.10. Let $f_S \tilde{\subseteq} h_S$ such that f_S is an SI-almost left (resp., right) T-ideal, then h_S is an SI-almost left (resp., right) T-ideal.

Proof: Assume that f_S is an SI-almost left T-ideal. Hence, $(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$. We need to show that $(h_S \circ S_x \circ h_S \circ h_S) \tilde{\cap} h_S \neq \emptyset_S$. In fact,

$$(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} (h_S \circ S_x \circ h_S \circ h_S) \tilde{\cap} h_S$$

Since $(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(h_S \circ S_x \circ h_S \circ h_S) \tilde{\cap} h_S \neq \emptyset_S$. This completes the proof.

Theorem 3.11. Let f_S and h_S be SI-almost left (resp., right) T-ideals. Then, $f_S \tilde{\cup} h_S$ is an SI-almost left (resp., right) T-ideal.

Proof: Since f_S is an SI-almost left T-ideal by assumption and $f_S \tilde{\subseteq} f_S \tilde{\cup} h_S$, $f_S \tilde{\cup} h_S$ is an SI-almost left T-ideal by Theorem 3.10.

Corollary 3.12. The finite union of SI-left (resp., right) T-ideals is an SI-almost left (resp., right) T-ideal.

Corollary 3.13. Let f_S or h_S be SI-almost left (resp., right) T-ideal. Then, $f_S \tilde{\cup} h_S$ is an SI-almost left (resp., right) T-ideal.

Here, note that if f_S and h_S are SI-almost left (resp., right) T-ideals, then $f_S \tilde{\cap} h_S$ needs not be an SI-almost left (resp., right) T-ideal.

Example 3.14. Consider the SI-almost T-ideals f_S and h_S in Example 3.4. Since,

$f_S \tilde{\cap} h_S = \{(p, \emptyset), (r, \emptyset)\} = \emptyset_S$

$f_S \tilde{\cap} h_S$ is not an SI-almost T-ideals.

Now, we give the relationship between almost T-ideal and SI-almost T-ideal. But first of all, we remind the following lemma in order to use it in Theorem 3.16.

Lemma 3.15 [53]. Let $x \in S$ and Y be a nonempty subset of S. Then, $S_x \circ S_Y = S_{xY}$. If X is a nonempty subset of S and $y \in S$, then $S_X \circ S_y = S_{Xy}$.

Theorem 3.16. Let A be a nonempty subset of S. Then, A is an almost left (resp., right) T-ideal if and only if S_A, the soft characteristic function of A, is an SI-almost left (resp., right) T-ideal.

Proof: Assume that $\emptyset \neq A$ is an almost left T-ideal. Then, $AxAA \cap A \neq \emptyset$, for all $x \in S$, and so there exist $t \in S$ such that $t \in AxAA \cap A$. Since,

$$\big((S_A \circ S_x \circ S_A \circ S_A) \tilde{\cap} S_A\big)(t) = (S_{AxAA} \tilde{\cap} S_A)(t) = (S_{AxAA\cap A})(t) = U \neq \emptyset$$

it follows that $(S_A \circ S_x \circ S_A \circ S_A) \tilde{\cap} S_A \neq \emptyset_S$. Thus, S_A is an SI-almost left T-ideal.

Conversely assume that S_A is an SI-almost left T-ideal. Hence, we have $(S_A \circ S_x \circ S_A \circ S_A) \tilde{\cap} S_A \neq \emptyset_S$, for all $x \in S$. In order to show that A is an almost left T-ideal of S, we should prove that $A \neq \emptyset$ and $AxAA \cap A \neq \emptyset$, for all $x \in S$. $A \neq \emptyset$ is obvious from assumption. Now,

$$\begin{aligned}\emptyset_S \neq (S_A \circ S_x \circ S_A \circ S_A) \tilde{\cap} S_A &\Rightarrow \exists n \in S\,;\ \big((S_A \circ S_x \circ S_A \circ S_A) \tilde{\cap} S_A\big)(n) \neq \emptyset \\ &\Rightarrow \exists n \in S\,;\ \big((S_{AxAA}) \tilde{\cap} S_A\big)(n) \neq \emptyset \\ &\Rightarrow \exists n \in S\,;\ (S_{AxAA\cap A})(n) \neq \emptyset\end{aligned}$$

$$\Rightarrow \ \exists n \in S \ ; \ (S_{AxAA\cap A})(n) = U$$
$$\Rightarrow \ n \in AxAA \cap A$$

Hence, $AxAA \cap A \neq \emptyset$ for all $x \in S$. Consequently, A is an almost left T-ideal.

Lemma 3.17. [52]. Let f_S be a soft set over U. Then, $f_S \tilde{\subseteq} S_{supp(f_S)}$.

Theorem 3.18. If f_S is an SI-almost left (resp., right) T-ideal, then $supp(f_S)$ is an almost left (resp., right) T-ideal.

Proof: Assume that f_S is an SI-almost left T-ideal. Thus, $(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$. In order to show that $supp(f_S)$ is an almost left T-ideal, by Theorem 3.15, it is enough to show that $S_{supp(f_S)}$ is an SI-almost left T-ideal. By Lemma 3.17,

$$(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} \left(S_{supp(f_S)} \circ S_x \circ S_{supp(f_S)} \circ S_{supp(f_S)}\right) \tilde{\cap} S_{supp(f_S)}$$

And since $(f_S \circ S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, it implies that $\left(S_{supp(f_S)} \circ S_x \circ S_{supp(f_S)} \circ S_{supp(f_S)}\right) \tilde{\cap} S_{supp(f_S)} \neq \emptyset_S$. Consequently, $S_{supp(f_S)}$ is an SI-almost left T-ideal of S and by Theorem 3.16, $supp(f_S)$ is an almost left T-ideal.

Here note that the converse of Theorem 3.18 is not true in general as shown in the following example.

Example 3.19. We know that g_S is not an SI-almost T-ideal in Example 3.4 and it is obvious that $supp(g_S) = \{p, r\}$. Since,

$$[supp(g_S)\{p\}supp(g_S)supp(g_S)] \cap supp(g_S) = \{p,r\}p\{p,r\}\{p,r\} \cap \{p,r\} = \{p,r\} \neq \emptyset$$
$$[supp(g_S)\{r\}supp(g_S)supp(g_S)] \cap supp(g_S) = \{p,r\}r\{p,r\}\{p,r\} \cap \{p,r\} = \{p,r\} \neq \emptyset$$

It is seen that $[supp(g_S)\{x\}supp(g_S)supp(g_S)] \cap supp(g_S) \neq \emptyset$, for all $x \in S$. That is to say, $supp(g_S)$ is an almost left T-ideal of S; although g_S is not an SI-almost left T-ideal. Similarly,

$$[supp(g_S)supp(g_S\{p\}supp(g_S)] \cap supp(g_S) = \{p,r\}\{p,r\}p \cap \{p,r\} = \{p,r\} \neq \emptyset$$
$$[supp(g_S)supp(g_S\{r\}supp(g_S)] \cap supp(g_S) = \{p,r\}\{p,r\}r \cap \{p,r\} = \{p,r\} \neq \emptyset$$

It is seen that $[supp(g_S)supp(g_S)\{x\}supp(g_S)] \cap supp(g_S) \neq \emptyset$, for all $x \in S$. That is to say, $supp(g_S)$ is an almost right T-ideal of S; although g_S is not an SI-almost right T-ideal. Consequently, $supp(g_S)$ is an almost T-ideal of S; although g_S is not an SI-almost T-ideal.

Definition 3.20. An SI-almost left (resp., right) T-ideal f_S is called minimal if any SI-almost left (resp., right) T-ideal h_S if whenever $h_S \tilde{\subseteq} f_S$, then $supp(h_S) = supp(f_S)$.

Theorem 3.21. Let A be a nonempty subset of S. Then, A is a minimal almost left (resp., right) T-ideal if and only if S_A, the soft characteristic function of A, is a minimal SI-almost left (resp., right) T-ideal.

Proof: Assume that A is a minimal almost left T-ideal. Thus, A is an almost left T-ideal of S, and so S_A is an SI-almost left T-ideal by Theorem 3.16. Let f_S be an SI-almost left T-ideal such that $f_S \tilde{\subseteq} S_A$. By Theorem 3.18, $supp(f_S)$ is an almost left T-ideal and by Note 2.9, and Corollary 2.13,

$supp(f_S) \subseteq supp(S_A) = A$.

Since A is a minimal almost left T-ideal, $supp(f_S) = supp(S_A) = A$. Thus, S_A is a minimal SI-almost left T-ideal by Definition 3.20.

Conversely, let S_A be a minimal SI-almost left T-ideal. Thus, S_A is an SI-almost left T-ideal of S and A is an almost left T-ideal by Theorem 3.16. Let B be an almost left T-ideal such that $B \subseteq A$. By Theorem 3.16, S_B is an SI-almost left T-ideal, and by Theorem 2.14 (i), $S_B \tilde{\subseteq} S_A$. Since S_A is a minimal SI-almost left T-ideal,

$$B = supp(S_B) = supp(S_A) = A$$

by Corollary 2.13. Thus, A is a minimal almost left T-ideal.

Definition 3.22. Let f_S, g_S, and h_S be any SI-almost left (resp., right) T-ideals. If $h_S \circ g_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-prime almost left (resp., right) T-ideal.

Definition 3.23. Let f_S and h_S be any SI-almost left (resp., right) T-ideals. If $h_S \circ h_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$, then f_S is called an SI-semiprime almost left (resp., right) T-ideal.

Definition 3.24. Let f_S, g_S, and h_S be any SI-almost (left/right) T-ideals. If $(h_S \circ g_S) \tilde{\cap} (g_S \circ h_S) \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-strongly prime almost (left/right) T-ideal.

It is obvious that every SI-strongly prime almost left (resp., right) T-ideal of S is an SI-prime almost left (resp., right) T-ideal, and every SI-prime almost left (resp., right) T-ideal of S is an SI-semiprime almost left (resp., right) T-ideal.

Theorem 3.25. If S_P, the soft characteristic function of P, is an SI-prime almost left (resp., right) T-ideal, then P is a prime almost left (resp., right) T-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-prime almost left T-ideal. Thus, S_P is an SI-almost left T-ideal of S and thus, P is an almost left T-ideal by Theorem 3.16. Let A and B be almost left T-ideals such that $AB \subseteq P$. Thus, by Theorem 3.16, S_A and S_B are SI-almost left T-ideals, and by Theorem 2.14 (i) and (iii),

$S_A \circ S_B = S_{AB} \tilde{\subseteq} S_P$.

Since S_P is an SI-prime almost left T-ideal and $S_A \circ S_B \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Therefore, by Theorem 2.14 (i), $A \subseteq P$ or $B \subseteq P$. Consequently, P is a prime almost left T-ideal.

Theorem 3.26. If S_P, the soft characteristic function of P, is an SI-semiprime left (resp., right) almost T-ideal of S, then P is a semiprime almost left (resp., right) T-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-semiprime almost left T-ideal. Thus, S_P is an SI-almost left T-ideal and thus, P is an almost left T-ideal of S by Theorem 3.16. Let A be an almost left T-ideal such that $AA \subseteq P$. Thus, by Theorem 3.16, S_A is an SI-almost left T-ideal, and by Theorem 2.14 (i) and (iii),

$S_A \circ S_A = S_{AA} \tilde{\subseteq} S_P$,

Since S_P is an SI-semiprime almost left T-ideal of S and $S_A \circ S_A \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$. Therefore, by Theorem 2.14 (i), $A \subseteq P$. Consequently, P is a semiprime almost left T-ideal.

Theorem 3.27. If S_P, the soft characteristic function of P, is an SI-strongly prime almost left (resp., right) T-ideal, then P is a strongly prime almost left (resp., right) T-ideal of S, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-strongly prime almost left T-ideal. Thus, S_P is an SI-almost left T-ideal of S and thus, P is an almost left T-ideal by Theorem 3.16. Let A and B be almost left T-ideals such that $AB \cap BA \subseteq P$. Thus, by Theorem 3.15, S_A and S_B are SI-almost left T-ideals, and by Theorem 2.14,

$$(S_A \circ S_B) \tilde{\cap} (S_B \circ S_A) = S_{AB} \tilde{\cap} S_{BA} = S_{AB \cap BA} \tilde{\subseteq} S_P$$

Since S_P is an SI-strongly prime almost left T-ideal and $(S_A \circ S_B) \tilde{\cap} (S_B \circ S_A) \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Thus, by Theorem 2.14 (i), $A \subseteq P$ or $B \subseteq P$. Therefore, P is a strongly prime almost left T-ideal.

4 | Conclusion

In this study, we introduced the concept of the soft intersection almost tri-ideal as a generalization of the nonnull soft intersection tri-ideal. We demonstrated that an idempotent soft intersection almost tri-ideal is a soft intersection almost subsemigroup, and that an idempotent soft intersection almost bi-ideals and soft intersection almost tri-ideals are equivalent. Furthermore, it is shown that idempotent soft intersection almost left (right) tri-ideals and soft intersection almost tri-ideals coincide. We also established a theorem stating that if a nonempty subset of a semigroup is an almost tri-ideal, then its soft characteristic function is also a soft intersection almost tri-ideal, and vice versa. This theorem allowed us to establish relationships between soft intersection almost tri-ideals of a semigroup and almost tri-bi-ideals of a semigroup in terms of minimality, primeness, semiprimeness, and strongly primeness. Moreover, we discovered that the binary operation of soft union can be used to construct a semigroup with the collection of soft intersection almost tri-ideals, but the soft intersection operation cannot be used for this aim. In future studies, researchers may consider examining various types of soft intersection almost ideals in semigroups.

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The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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