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Note for Neutrosophic Incidence and Threshold Graph

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Abstract

Uncertain Graph Theory has emerged to model the uncertainties present in real-world networks. An Incidence Graph represents the connections between vertices and edges using incidence pairs to illustrate relationships. A Threshold Graph is defined by vertex weights and thresholds, forming cliques or independent sets. We explore the concepts of the Pentapartitioned Neutrosophic Incidence Graph, Turiyam Neutrosophic Incidence Graph, Pentapartitioned Neutrosophic Fuzzy Threshold Graph, and Turiyam Neutrosophic Fuzzy Threshold Graph.

Keywords: Neutrosophic Graph; Fuzzy Graph; Turiyam Neutrosophic Graph; Threshold Graph; Incidence Graph.

1 | Introduction

1.1 | Uncertain Graph Theory

Graph theory is a fundamental branch of mathematics that uses vertices (nodes) and edges (connections) to model networks, effectively capturing relationships within various systems [34, 38, 63,154, 162, 229].

This paper explores various models of uncertain graphs, such as Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic Graphs. These models extend classical graph theory by incorporating different levels of uncertainty, enabling the analysis of complex and ambiguous relationships. The development of uncertain graph models has led to a wide range of applications in real-world contexts and has inspired the creation of many related graph classes [72, 73, 75-78, 80, 81, 83-87]. Core concepts like Fuzzy Sets and Neutrosophic Sets form the foundation of these models and are well-documented in the literature [20-24, 56, 65-67, 70, 125, 129, 159, 190, 219-224, 227].

Given the extensive literature and diverse applications, the study of uncertain graphs is highly significant for advancing our understanding of uncertain networks. For an in-depth exploration of these concepts, readers are encouraged to refer to the existing survey papers [79, 81, 83].

1.2 | Uncertain Incidence and Threshold Graph

Among the graph classes in Uncertain Graph Theory, Fuzzy Incidence Graph [1, 112, 140, 141, 148, 151, 152, 163, 170, 174, 179], Fuzzy Threshold Graph [3, 10, 104, 139, 155, 214], Neutrosophic Incidence Graph [11, 18, 110, 143, 144, 196], and Neutrosophic Fuzzy Threshold Graph [123] have been extensively studied. Incidence Graph Represents connections between vertices and edges, using incidence pairs to show

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relationships. The Incidence Graph in classical graph theory is a graph concept extended to Uncertain Graphs [201]. Threshold Graph is characterized by vertex weights and thresholds, forming cliques or independent sets. Threshold Graph in classical graph theory is a graph concept extended to Uncertain Graphs [52, 60, 113, 213]. These graphs are explored from various perspectives, including their applications and mathematical structures.

1.3 | Contributions

We explore the concepts of the Pentapartitioned Neutrosophic Incidence Graph and the Turiyam Neutrosophic Incidence Graph, as well as the Pentapartitioned Neutrosophic Fuzzy Threshold Graph and the Turiyam Neutrosophic Fuzzy Threshold Graph. Turiyam Neutrosophic Graphs enhance the traditional graph framework by integrating four membership values-truth, indeterminacy, falsity, and liberal state-at each vertex and edge, offering a more detailed representation of complex relationships [79, 81, 88, 187]. Meanwhile, the Pentapartitioned Neutrosophic Graph assigns five degrees (truth, contradiction, ignorance, unknown, falsity) to each vertex and edge, effectively capturing complex uncertainty [54,109, 111, 167]. Related concepts include the Turiyam Neutrosophic Set [27, 81, 89, 90, 183, 186, 188] and the Pentapartitioned Neutrosophic Set [33, 55, 138, 168].

2 | Preliminaries and Definitions

This section provides an overview of the fundamental definitions and notations used throughout the paper.

2.1 | Basic Graph Concepts

Below are some of the foundational concepts in graph theory. For more comprehensive information on graph theory and its notations, refer to [61 63, 100 206].

Definition 1 (Graph). [63] A graph G is a mathematical structure that represents relationships between objects. It consists of a set of vertices $V(G)$ and a set of edges $E(G)$, where each edge connects a pair of vertices. Formally, a graph is represented as $G=(V,E)$, where V is the set of vertices and E is the set of edges.

Definition 2 (Degree). [63] Let $G=(V,E)$ be a graph. The degree of a vertex $v \in V$, denoted $\deg(v)$, is defined as the number of edges connected to v. For undirected graphs, the degree is given by:

$$\deg(v) = |\{e \in E \mid v \in e\}|$$

For directed graphs, the in-degree $\deg^-(v)$ refers to the number of edges directed towards v, while the out-degree $\deg^+(v)$ represents the number of edges directed away from v.

2.2 | Uncertain Graph

This paper addresses Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic concepts. Note that Turiyam Neutrosophic Set is actually a particular case of the Quadruple Neutrosophic Set, by replacing "Contradiction" with "Liberal" [189].

Definition 3 (Unified Uncertain Graphs Framework). (cf. [82]) Let $G=(V,E)$ be a classical graph with a set of vertices V and a set of edges E. Depending on the type of graph, each vertex $v \in V$ and edge $e \in E$ is assigned membership values to represent various degrees of truth, indeterminacy, falsity, and other nuanced measures of uncertainty.

1. Fuzzy Graph $[32, 74, 91, 95, 122, 147, 156, 175, 176, 197, 205]$:
   - Each vertex $v \in V$ is assigned a membership degree $\sigma(v) \in [0,1]$.
   - Each edge $e=(u,v) \in E$ is assigned a membership degree $\mu(u,v) \in [0,1]$.
2. Intuitionistic Fuzzy Graph (IFG) [4, 31, 53, 114, 149, 199, 200, 231]:

- Each vertex $v \in V$ is assigned two values: $\mu_A(v) \in [0,1]$ (degree of membership) and $v_A(v) \in [0,1]$ (degree of non-membership), such that $\mu_A(v) + v_A(v) \leq 1$.
- Each edge $e = (u, v) \in E$ is assigned two values: $\mu_B(u, v) \in [0,1]$ and $v_B(u, v) \in [0,1]$, with $\mu_B(u, v) + v_B(u, v) \leq 1$.

3. Neutrosophic Graph [10, 12, 39, 79, 84, 102, 108, 119, 180, 192, 195]:
   - Each vertex $v \in V$ is assigned a triplet $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, where $\sigma_T(v), \sigma_I(v), \sigma_F(v) \in [0,1]$ and $\sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3$.
   - Each edge $e = (u, v) \in E$ is assigned a triplet $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$.
4. Turiyam Neutrosophic Graph [88-90]:
   - Each vertex $v \in V$ is assigned a quadruple $\sigma(v) = (t(v), iv(v), fv(v), lv(v))$, where each component is in $[0,1]$ and $t(v) + iv(v) + fv(v) + lv(v) \leq 4$.
   - Each edge $e = (u, v) \in E$ is similarly assigned a quadruple.
5. Vague Graph [7, 8, 35-37, 172, 173, 181]:
   - Each vertex $v \in V$ is assigned a pair $(\tau(v), \phi(v))$, where $\tau(v) \in [0,1]$ is the degree of truth membership and $\phi(v) \in [0,1]$ is the degree of false-membership, with $\tau(v) + \phi(v) \leq 1$.
   - The grade of membership is characterized by the interval $[\tau(v), 1 - \phi(v)]$.
   - Each edge $e = (u, v) \in E$ is assigned a pair $(\tau(e), \phi(e))$, satisfying:

$$\tau(e) \leq \min\{\tau(u), \tau(v)\},\ \phi(e) \geq \max\{\phi(u), \phi(v)\}$$

6. Hesitant Fuzzy Graph [26 96 157, 158, 210]:
   - Each vertex $v \in V$ is assigned a hesitant fuzzy set $\sigma(v)$, represented by a finite subset of $[0,1]$, denoted $\sigma(v) \subseteq [0,1]$.
   - Each edge $e = (u, v) \in E$ is assigned a hesitant fuzzy set $\mu(e) \subseteq [0,1]$.
   - Operations on hesitant fuzzy sets (e.g., intersection, union) are defined to handle the hesitation in membership degrees.
7. Single-Valued Pentapartitioned Neutrosophic Graph [54, 109, 111, 167]:
   - Each vertex $v \in V$ is assigned a quintuple $\sigma(v) = (T(v), C(v), R(v), U(v), F(v))$, where:
     - $T(v) \in [0,1]$ is the truth-membership degree.
     - $C(v) \in [0,1]$ is the contradiction-membership degree.
     - $R(v) \in [0,1]$ is the ignorance-membership degree.
     - $U(v) \in [0,1]$ is the unknown-membership degree.
     - $F(v) \in [0,1]$ is the false-membership degree.
     - $T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$.
   - Each edge $e = (u, v) \in E$ is assigned a quintuple $\mu(e) = (T(e), C(e), R(e), U(e), F(e))$, satisfying:

$$\begin{cases} T(e) \leq \min\{T(u), T(v)\} \\ C(e) \leq \min\{C(u), C(v)\} \\ R(e) \geq \max\{R(u), R(v)\} \\ U(e) \geq \max\{U(u), U(v)\} \\ F(e) \geq \max\{F(u), F(v)\} \end{cases}$$

2.3 | Fuzzy Incidence Graph and Single-Valued Neutrosophic Incidence Graph

The definition of the already known Incidence Graph is described as follows.

Definition 4 (Fuzzy Incidence Graph). [1, 140] Let $G' = (V, E, I)$ be an incidence graph, where:

- V is a non-empty set of vertices,
- E is a set of edges,
- $I \subseteq V \times E$ is a set of incidence pairs.

  A fuzzy incidence graph of G', denoted as $\tilde{G} = (\mu, \lambda, \psi)$, is defined as an ordered triplet where:

- $\mu: V \to [0,1]$ is a fuzzy subset of the vertex set V,
- $\lambda: E \to [0,1]$ is a fuzzy relation on the edge set E,
- $\psi: I \to [0,1]$ is a fuzzy subset of the incidence set I.

  The fuzzy incidence graph $\tilde{G}$ satisfies the following condition:

$$\psi(x, xy) \leq \min\{\mu(x), \lambda(xy)\}, \ \forall x \in V, xy \in E$$

where $\psi(x, xy)$ represents the degree of incidence between a vertex x and an edge xy.

Definition 5 (Single-Valued Neutrosophic Incidence Graph). [11] Let $G' = (V, E, I)$ be an incidence graph, where:

- V is a non-empty set of vertices,
- E is a set of edges,
- $I \subseteq V \times E$ is a set of incidence pairs.

A single-valued neutrosophic incidence graph (SVNIG) of G', denoted as $\tilde{G} = (A, B, C)$, is defined as an ordered triplet where:

- A is a single-valued neutrosophic set on the vertex set V, with $A(x) = (T_A(x), I_A(x), F_A(x))$, where:
  - $T_A(x) \in [0,1]$: Truth-membership of vertex x,
  - $I_A(x) \in [0,1]$: Indeterminacy-membership of vertex x,
  - $F_A(x) \in [0,1]$: Falsity-membership of vertex x.
- B is a single-valued neutrosophic relation on the edge set E, with $B(xy) = (T_B(xy), I_B(xy), F_B(xy))$, where:
  - $T_B(xy) \in [0,1]$: Truth-membership of edge xy,
  - $I_B(xy) \in [0,1]$: Indeterminacy-membership of edge xy,
  - $F_B(xy) \in [0,1]$: Falsity-membership of edge xy.
- C is a single-valued neutrosophic subset of the incidence set I, with $C(x, xy) = (T_C(x, xy), I_C(x, xy), F_C(x, xy))$, where:

- $T_C(x, xy) \leq \min\{T_A(x), T_B(xy)\}$: Truth-membership condition,
- $I_C(x, xy) \leq \min\{I_A(x), I_B(xy)\}$: Indeterminacy-membership condition,
- $F_C(x, xy) \geq \max\{F_A(x), F_B(xy)\}$: Falsity-membership condition,

  for all $x \in V$ and $xy \in E$.

Example 6 (Single-Valued Neutrosophic Incidence Graph). [11] Consider an incidence graph $G = (V, E, I)$, where:

- The vertex set is $V = \{a, b, c, d\}$.
- The edge set is $E = \{ab, bc, bd, cd, ad\}$.
- The incidence set is

  $I = \{(a, ab), (b, ab), (b, bc), (c, bc), (b, bd), (d, bd), (c, cd), (d, cd), (d, ad), (a, ad)\}$.

  Let $\tilde{G} = (A, B, C)$ be a Single-Valued Neutrosophic Incidence Graph (SVNIG) associated with G, where:

- The single-valued neutrosophic set A on the vertex set V is defined as:

$$A = \{(a, 0.2,0.5,0.8), (b, 0.3,0.5,0.1), (c, 0.9,0.9,0.1), (d, 0.8,0.1,0.2)\}$$

Here, each vertex $x \in V$ is represented by a triplet $(T_A(x), I_A(x), F_A(x))$, indicating the truth, indeterminacy, and falsity memberships, respectively.

- The single-valued neutrosophic relation B on the edge set E is defined as:

$$B = \{(ab, 0.2,0.4,0.7), (bc, 0.3,0.4,0.1), (bd, 0.1,0.1,0.1), (cd, 0.7,0.1,0.2), (ad, 0.1,0.1,0.5)\}$$

Each edge $xy \in E$ is represented by a triplet $(T_B(xy), I_B(xy), F_B(xy))$, indicating the truth, indeterminacy, and falsity memberships, respectively.

- The single-valued neutrosophic subset C of the incidence set I is defined as:

$$\begin{aligned} C = \{&((a, ab),0.2,0.3,0.7), ((b, ab),0.1,0.4,0.6), ((b, bc),0.3,0.3,0.1), ((c, bc),0.2,0.3,0.1), \\ &((b, bd),0.1,0.1,0.1), ((d, bd),0.1,0.1,0.2), ((c, cd),0.7,0.1,0.2), ((d, cd),0.7,0.1,0.2) \\ &((d, ad),0.1,0.1,0.4), ((a, ad),0.1,0.1,0.5)\} \end{aligned}$$

Each incidence pair $(x, xy) \in I$ is represented by a triplet $(T_C(x, xy), I_C(x, xy), F_C(x, xy))$, indicating the truth, indeterminacy, and falsity memberships, respectively.

Thus, the Single-Valued Neutrosophic Incidence Graph $\tilde{G} = (A, B, C)$ accurately represents the uncertain relationships among the vertices, edges, and incidence pairs in the original incidence graph G.

The following holds obviously.

Proposition 7. Every Single-Valued Neutrosophic Incidence Graph (SVNIG) can be transformed into a Fuzzy Incidence Graph (FIG).

Proof. Let $\tilde{G} = (A, B, C)$ be a given Single-Valued Neutrosophic Incidence Graph. To transform $\tilde{G}$ into a Fuzzy Incidence Graph $\tilde{G}' = (\mu, \lambda, \psi)$, we define the following mappings:

- For the vertex set V, define:

$$\mu(x) = T_A(x),\ \forall x \in V$$

  This mapping assigns the truth-membership of each vertex in the SVNIG to the fuzzy membership of the corresponding vertex in the FIG.

- For the edge set E, define:

$$\lambda(xy) = T_B(xy),\ \forall xy \in E.$$

This mapping assigns the truth-membership of each edge in the SVNIG to the fuzzy membership of the corresponding edge in the FIG.

- For the incidence set I, define:

$$\psi(x, xy) = T_C(x, xy),\ \forall (x, xy) \in I.$$

This mapping assigns the truth-membership of each incidence pair in the SVNIG to the fuzzy membership of the corresponding incidence pair in the FIG.

To verify that $\tilde{G}' = (\mu, \lambda, \psi)$ satisfies the conditions of a Fuzzy Incidence Graph, we need to check the incidence constraint:

$$\psi(x, xy) \leq \min\{\mu(x), \lambda(xy)\},\ \forall x \in V, xy \in E$$

From the definition of $\tilde{G}$, we know that:

$$T_C(x, xy) \leq \min\{T_A(x), T_B(xy)\},\ \forall x \in V, xy \in E.$$

By substituting the mappings for $\mu(x), \lambda(xy)$, and $\psi(x, xy)$, we get:

$$\psi(x, xy) = T_C(x, xy) \leq \min\{T_A(x), T_B(xy)\} = \min\{\mu(x), \lambda(xy)\},\ \forall x \in V, xy \in E.$$

Thus, the transformed graph $\tilde{G}' = (\mu, \lambda, \psi)$ satisfies the conditions of a Fuzzy Incidence Graph.

Therefore, every Single-Valued Neutrosophic Incidence Graph can be transformed into a Fuzzy Incidence Graph.

2.4 | Intuitionistic Fuzzy Threshold Graph and Neutrosophic Fuzzy Threshold Graph

The definition of the already known Threshold Graph is described as follows.

Definition 8. [214] An Intuitionistic Fuzzy Threshold Graph (IFTG) is a special type of Intuitionistic Fuzzy Graph (IFG) defined by two threshold parameters t_1 and t_2. An IFTG is represented as $G = (A, B; t_1, t_2)$, where:

- $G = (A, B)$ is an Intuitionistic Fuzzy Graph, where:
  - $A = (\mu_A, v_A)$ is an Intuitionistic Fuzzy Set (IFS) on the set of vertices V^*, with:

$$\mu_A: V^* \to [0,1]$$
$$v_A: V^* \to [0,1]$$

satisfying:

$$\mu_A(u) + v_A(u) \leq 1,\ \forall u \in V^*$$

  - $B = (\mu_B, v_B)$ is an Intuitionistic Fuzzy Relation (IFR) on the set of edges E^*, with:

$$\mu_B: E^* \to [0,1],$$
$$v_B: E^* \to [0,1],$$

satisfying:

$$\mu_B(u, v) + v_B(u, v) \leq 1,\ \forall (u, v) \in E^*$$

- $t_1 > 0$ and $t_2 > 0$ are the threshold parameters.

The graph G is called an IFTG if and only if the following conditions hold:

$$\sum_{u\in U} \mu_A(u) \leq t_1 \text{ and } \sum_{u\in U} (1 - v_A(u)) \leq t_2$$

for any subset $U \subseteq V^*$, where U is an independent set in the underlying graph G^* of G.

Example 9 (Example of an Intuitionistic Fuzzy Threshold Graph (IFTG)). [214] Consider a graph $G^* = (V^*, E^*)$ with the following sets of vertices and edges:

$$V^* = \{a, b, c, d, e\}$$
$$E^* = \{(a, b), (b, e), (e, d), (b, d), (b, c), (c, d)\}$$

Let A be an intuitionistic fuzzy subset on V^* and B be an intuitionistic fuzzy subset on E^*. The intuitionistic fuzzy membership degrees for vertices and edges are as follows:

Vertices:

Vertex	μ_A	v_A
a	0.1	0.9
b	0.5	0.4
c	0.2	0.7
d	0.4	0.6
e	0.2	0.8

Edges:

Edge	μ_B	v_B
(a, b)	0.1	0.9
(b, c)	0.2	0.7
(b, d)	0.4	0.6
(b, e)	0.2	0.8
(e, d)	0.2	0.7
(c, d)	0.2	0.6

Now, based on the intuitionistic fuzzy sets A and B, we define the Intuitionistic Fuzzy Threshold Graph $G = (A, B; t_1, t_2)$ with thresholds:

$$t_1 = 0.5,\ t_2 = 0.6$$

The graph G is called an Intuitionistic Fuzzy Threshold Graph (IFTG) if, for any subset $U \subseteq V^*$ that is an independent set in G^*, the following conditions hold:

$$\sum_{u\in U} \mu_A(u) \leq t_1 \text{ and } \sum_{u\in U} (1 - v_A(u)) \leq t_2$$

Example Calculation Let $U = \{b, e\}$ be an independent set in G^*.

- Sum of membership degrees μ_A :

  $$\mu_A(b) + \mu_A(e) = 0.5 + 0.2 = 0.7 \text{ (which exceeds } t_1 = 0.5\text{)}$$

  Thus, this subset does not satisfy the first condition.

- Sum of non-membership complements $1 - v_A$:

$$(1 - v_A(b)) + (1 - v_A(e)) = (1 - 0.4) + (1 - 0.8) = 0.6 + 0.2 = 0.8 \left(\text{which exceeds } t_2 = 0.6 \right)$$

Thus, this subset does not satisfy the second condition.

Therefore, the set $U = \{b, e\}$ is not an independent set in the IFTG G.

Definition 10. [123] A Neutrosophic Fuzzy Threshold Graph (NFTG) is an extension of neutrosophic fuzzy graphs that incorporates three threshold parameters to define its independent sets. An NFTG is denoted as $= (P, Q; \tau_1, \tau_2, \tau_3)$, where:

- $G = (P, Q)$ is a Neutrosophic Fuzzy Graph (NFG) defined on a vertex set V^* and an edge set E^*.
  - The neutrosophic fuzzy set P on the vertices V^* is defined by three functions:

$\mu_P: V^* \to [0,1]$, (truth-membership)
$v_P: V^* \to [0,1]$, (falsity-membership)
$\sigma_P: V^* \to [0,1]$, (indeterminacy-membership)

  - The neutrosophic fuzzy set Q on the edges E^* is defined by three functions:

$\mu_Q: E^* \to [0,1]$,
$v_Q: E^* \to [0,1]$,
(truth-membership)
$\sigma_Q: E^* \to [0,1]$,
(falsity-membership)
(indeterminacy-membership)

The graph $G = (P, Q)$ is defined as an $NFTG$ if there exist three positive thresholds $\tau_1 > 0, \tau_2 > 0$, and $\tau_3 > 0$ such that, for any subset $U \subseteq V^*$ that is an independent set in G, the following conditions hold:

$$\sum_{u \in U} \mu_P(u) \le \tau_1, \sum_{u \in U} (1 - v_P(u)) \le \tau_2, \text{ and } \sum_{u \in U} \sigma_P(u) \le \tau_3$$

Remark

- The notion of an independent set in an $NFTG$ is the same as in its underlying classical graph G^*.
- If $G = (P, Q; \tau_1, \tau_2, \tau_3)$ and $U \subseteq V^*$ is a dependent set in G, then at least one of the following inequalities must hold:

$$\sum_{u \in U} \mu_P(u) > \tau_1, \sum_{u \in U} (1 - v_P(u)) > \tau_2, \text{ or } \sum_{u \in U} \sigma_P(u) > \tau_3$$

Example 11. (Example of a Neutrosophic Fuzzy Threshold Graph (NFTG)). Consider a graph $G^* = (V^*, E^*)$ with the following sets of vertices and edges:

$$V^* = \{m, n, o, p, q\}$$
$$E^* = \{(m, n), (n, o), (o, p), (p, n), (n, q)\}$$

Let P be the neutrosophic fuzzy subset defined on V^* and Q be the neutrosophic fuzzy subset defined on E^*. The degrees of truth-membership, falsity-membership, and indeterminacy-membership for vertices and edges are provided in the following tables:

Vertices:

Vertex	μ_P	ν_P	σ_P
m	0.6	0.2	0.1
n	0.2	0.3	0.1
o	0.2	0.4	0.2
p	0.1	0.2	0.3
q	0.3	0.4	0.2

Edges:

Edge	μ_Q	ν_Q	σ_Q
(m, n)	0.2	0.5	0.2
(n, o)	0.1	0.6	0.2
(o, p)	0.1	0.4	0.3
(p, n)	0.1	0.5	0.3
(n, q)	0.2	0.5	0.2

Based on the neutrosophic fuzzy subsets P and Q, we define the Neutrosophic Fuzzy Threshold Graph (NFTG) as:

$$G = (P, Q; \tau_1, \tau_2, \tau_3)$$

with the thresholds:

$$\tau_1 = 0.6,\ \tau_2 = 0.8,\ \tau_3 = 0.5$$

The graph G is called a Neutrosophic Fuzzy Threshold Graph (NFTG) if, for any subset $U \subseteq V^*$ that is an independent set in the underlying graph G^*, the following conditions hold:

$$\sum_{u \in U} \mu_P(u) \leq \tau_1, \sum_{u \in U} (1 - \nu_P(u)) \leq \tau_2, \sum_{u \in U} \sigma_P(u) \leq \tau_3$$

Example Calculation. Let $U = \{m, o\}$ be an independent set in G^*.

- Sum of truth-membership degrees μ_P :

  $$\mu_P(m) + \mu_P(o) = 0.6 + 0.2 = 0.8 \left(\text{which exceeds } \tau_1 = 0.6 \right)$$

  Thus, this subset does not satisfy the first condition.

- Sum of non-membership complements $1 - \nu_P$:

  $$(1 - \nu_P(m)) + (1 - \nu_P(o)) = (1 - 0.2) + (1 - 0.4) = 0.8 + 0.6$$
  $$= 1.4 \left(\text{which exceeds } \tau_2 = 0.8 \right)$$

  Thus, this subset does not satisfy the second condition.

- Sum of indeterminacy-membership degrees σ_P :

  $$\sigma_P(m) + \sigma_P(o) = 0.1 + 0.2 = 0.3 \left(\text{which is within } \tau_3 = 0.5 \right)$$

  This subset satisfies the third condition.

  Therefore, the set $U = \{m, o\}$ is not an independent set in the NFTG G.

3 | Results

State the results of this paper.

3.1 | Result: Incidence Graph

Define the Single-Valued Turiyam Neutrosophic Incidence Graph and the Single-Valued Pentapartitioned Neutrosophic Incidence Graph, and then prove that they can be transformed into other graph classes. **Definition 12** (Single-Valued Turiyam Neutrosophic Incidence Graph). Let $G' = (V, E, I)$ be an incidence graph, where:

- V is a non-empty set of vertices.
- E is a set of edges.
- $I \subseteq V \times E$ is a set of incidence pairs.

A Single-Valued Turiyam Neutrosophic Incidence Graph (SVTIG) of G', denoted as $\tilde{G} = (A, B, C)$, is defined as an ordered triplet where:

1. A is a single-valued Turiyam Neutrosophic set on the vertex set V, with $A(x) = (t_A(x), iv_A(x), fv_A(x), lv_A(x))$, where:
   - $t_A(x) \in [0,1]$: Truth-membership degree of vertex x.
   - $iv_A(x) \in [0,1]$: Indeterminacy-membership degree of vertex x.
   - $fv_A(x) \in [0,1]$: Falsity-membership degree of vertex x.
   - $lv_A(x) \in [0,1]$: Liberation-membership degree of vertex x.
   - Sum condition: $t_A(x) + iv_A(x) + fv_A(x) + lv_A(x) \leq 4$.
2. B is a single-valued Turiyam Neutrosophic relation on the edge set E, with $B(xy) = (t_B(xy), iv_B(xy), fv_B(xy), lv_B(x$. where:
   - $t_B(xy) \in [0,1]$: Truth-membership degree of edge xy.
   - $iv_B(xy) \in [0,1]$: Indeterminacy-membership degree of edge xy.
   - $fv_B(xy) \in [0,1]$: Falsity-membership degree of edge xy.
   - $lv_B(xy) \in [0,1]$: Liberation-membership degree of edge xy.
   - Sum condition: $t_B(xy) + iv_B(xy) + fv_B(xy) + lv_B(xy) \leq 4$.
3. C is a single-valued Turiyam Neutrosophic subset of the incidence set I, with $C(x, xy) = (t_C(x, xy), iv_C(x, xy), fv_C(x$ satisfying:

$$
\begin{gathered}
t_C(x, xy) \leq \min\{t_A(x), t_B(xy)\} \\
iv_C(x, xy) \leq \min\{iv_A(x), iv_B(xy)\} \\
fv_C(x, xy) \geq \max\{fv_A(x), fv_B(xy)\} \\
lv_C(x, xy) \geq \max\{lv_A(x), lv_B(xy)\}
\end{gathered}
$$

for all $x \in V$ and $xy \in E$.

Definition 13 (Single-Valued Pentapartitioned Neutrosophic Incidence Graph). Let $G' = (V, E, I)$ be an incidence graph, where:

- V is a non-empty set of vertices.

- E is a set of edges.
- $I \subseteq V \times E$ is a set of incidence pairs.

A Single-Valued Pentapartitioned Neutrosophic Incidence Graph (SVPPNIG) of G', denoted as $\tilde{G} = (A, B, C)$, is defined as an ordered triplet where:

1. A is a single-valued pentapartitioned neutrosophic set on the vertex set V, with

$$A(x) = (T_A(x), C_A(x), R_A(x), U_A(x), F_A(x))$$

, where:

- $T_A(x) \in [0,1]$: Truth-membership degree of vertex x.
- $C_A(x) \in [0,1]$: Contradiction-membership degree of vertex x.
- $R_A(x) \in [0,1]$: Ignorance-membership degree of vertex x.
- $U_A(x) \in [0,1]$: Unknown-membership degree of vertex x.
- $F_A(x) \in [0,1]$: Falsity-membership degree of vertex x.
- Sum condition: $T_A(x) + C_A(x) + R_A(x) + U_A(x) + F_A(x) \leq 5$.

2. B is a single-valued Pentapartitioned neutrosophic relation on the edge set E, with

$$B(xy) = (T_B(xy), C_B(xy), R_B(xy), U_B(xy), F_B(xy))$$

, where:

- $T_B(xy) \in [0,1]$: Truth-membership degree of edge xy.
- $C_B(xy) \in [0,1]$: Contradiction-membership degree of edge xy.
- $R_B(xy) \in [0,1]$: Ignorance-membership degree of edge xy.
- $U_B(xy) \in [0,1]$: Unknown-membership degree of edge xy.
- $F_B(xy) \in [0,1]$: Falsity-membership degree of edge xy.
- Sum condition: $T_B(xy) + C_B(xy) + R_B(xy) + U_B(xy) + F_B(xy) \leq 5$.

3. C is a single-valued pentapartitioned neutrosophic subset of the incidence set I, with

$$C(x, xy) = (T_C(x, xy), C_C(x, xy), R_C(x, xy), U_C(x, xy), F_C(x, xy))$$

, satisfying:

$$T_C(x, xy) \leq \min\{T_A(x), T_B(xy)\}$$
$$C_C(x, xy) \leq \min\{C_A(x), C_B(xy)\}$$
$$R_C(x, xy) \leq \min\{R_A(x), R_B(xy)\}$$
$$U_C(x, xy) \leq \min\{U_A(x), U_B(xy)\}$$
$$F_C(x, xy) \geq \max\{F_A(x), F_B(xy)\}$$

for all $x \in V$ and $xy \in E$.

Theorem 14. A Single-Valued Pentapartitioned Neutrosophic Incidence Graph (SVPPNIG) can be transformed into:

1. A Single-Valued Turiyam Neutrosophic Incidence Graph (SVTIG) by merging specific membership degrees and adjusting the sum conditions accordingly.

2. A Neutrosophic Incidence Graph by setting certain membership degrees to zero and reinterpreting others.
3. A Fuzzy Incidence Graph by simplifying the membership degrees from the Neutrosophic Incidence Graph.

Proof. **1.** Transformation from SVPPNIG to SVTIG

Let $\tilde{G} = (A, B, C)$ be an SVPPNIG. For each vertex $x \in V$ and edge $xy \in E$, the membership degrees are:

- Vertices:

$$A(x) = (T_A(x), C_A(x), R_A(x), U_A(x), F_A(x)),\ T_A(x) + C_A(x) + R_A(x) + U_A(x) + F_A(x) \leq 5$$

- Edges:

$$B(xy) = (T_B(xy), C_B(xy), R_B(xy), U_B(xy), F_B(xy)),\ T_B(xy) + C_B(xy) + R_B(xy) + U_B(xy) + F_B(xy) \leq 5.$$

To transform $\tilde{G}$ into an SVTIG, proceed as follows:

1. Merge Contradiction-membership into Truth-membership:

$$t_A(x) = T_A(x) + C_A(x),\ t_B(xy) = T_B(xy) + C_B(xy)$$

2. Rename Ignorance and Unknown-membership degrees:

$$iv_A(x) = R_A(x),\ lv_A(x) = U_A(x);\ iv_B(xy) = R_B(xy),\ lv_B(xy) = U_B(xy)$$

3. Keep Falsity-membership degrees unchanged:

$$fv_A(x) = F_A(x),\ fv_B(xy) = F_B(xy).$$

4. Sum Conditions:

$$t_A(x) + iv_A(x) + fv_A(x) + lv_A(x) = T_A(x) + C_A(x) + R_A(x) + F_A(x) + U_A(x) \leq 5.$$

5. Normalization: To conform to the SVTIG sum condition (≤ 4), normalize the membership degrees:

$$\tilde{t}_A(x) = \frac{t_A(x)}{5} \times 4,\ \tilde{iv}_A(x) = \frac{iv_A(x)}{5} \times 4,\ \tilde{fv}_A(x) = \frac{fv_A(x)}{5} \times 4,\ \tilde{lv}_A(x) = \frac{lv_A(x)}{5} \times 4.$$

Similarly for edges.
The sum condition becomes:

$$\tilde{t}_A(x) + \tilde{iv}_A(x) + \tilde{fv}_A(x) + \tilde{lv}_A(x) \leq 4.$$

6. Incidence Conditions: The incidence membership degrees $C(x, xy)$ transform accordingly, satisfying the SVTIG conditions.

Thus, the SVPPNIG is transformed into an SVTIG.

2. Transformation from SVPPNIG to Neutrosophic Incidence Graph Proceed as follows:

1. Set Contradiction and Unknown-membership degrees to zero:

$$C_A(x) = U_A(x) = 0,\ C_B(xy) = U_B(xy) = 0.$$

2. Rename Ignorance-membership degree as Indeterminacy-membership degree:

$$I_A(x) = R_A(x),\ I_B(xy) = R_B(xy).$$

3. Define Remaining Membership Degrees:

$$A(x) = (T_A(x), I_A(x), F_A(x)),\ B(xy) = (T_B(xy), I_B(xy), F_B(xy)).$$

4. Sum Conditions:

$$T_A(x) + I_A(x) + F_A(x) = T_A(x) + R_A(x) + F_A(x) \leq 5.$$

5. Normalization: Normalize the membership degrees to satisfy the Neutrosophic sum condition ($\leq$ 3):

$$\tilde{T}_A(x) = \frac{T_A(x)}{5} \times 3,\ \tilde{I}_A(x) = \frac{I_A(x)}{5} \times 3,\ \tilde{F}_A(x) = \frac{F_A(x)}{5} \times 3.$$

Similarly for edges.
The sum condition becomes:

$$\tilde{T}_A(x) + \tilde{I}_A(x) + \tilde{F}_A(x) \leq 3$$

6. Incidence Conditions: The incidence membership degrees $C(x, xy)$ adjust accordingly, satisfying the Neutrosophic incidence graph conditions.

Thus, the SVPPNIG is transformed into a Neutrosophic Incidence Graph.

3. Transformation from Neutrosophic Incidence Graph to Fuzzy Incidence Graph Proceed as follows:

1. Simplify Membership Degrees:

$$\mu(x) = \tilde{T}_A(x),\ \lambda(xy) = \tilde{T}_B(xy)$$

2. Incidence Membership Degrees:

$$\psi(x, xy) = \min\{\mu(x), \lambda(xy)\},\ \forall x \in V, xy \in E.$$

3. Sum Conditions: Since $\mu(x) \in [0,1]$, the fuzzy incidence graph conditions are satisfied.

   Thus, the Neutrosophic Incidence Graph simplifies to a Fuzzy Incidence Graph.

3.2 | Result: Threshold Graph

We will provide the definitions of Threshold Graphs extended to Turiyam Neutrosophic Graphs and Single-Valued Pentapartitioned Neutrosophic Graphs, and examine their relationships with other graph classes. The definitions are described as follows.

Definition 15. A Turiyam Neutrosophic Fuzzy Threshold Graph (TFTG) is a graph $G = (V, E)$ where:

- Each vertex $v \in V$ is associated with four membership degrees:

$$\begin{aligned} t(v) &\in [0,1], && \text{(truth-membership)} \\ iv(v) &\in [0,1], && \text{(indeterminacy due to indeterminacy)} \\ fv(v) &\in [0,1], && \text{(falsity-membership)} \\ lv(v) &\in [0,1], && \text{(latent-membership)} \end{aligned}$$

  satisfying:

$$t(v) + iv(v) + fv(v) + lv(v) \leq 4$$

- For any independent set $U \subseteq V$, the threshold conditions are:

$$\sum_{v \in U} t(v) \leq \tau_1,\ \sum_{v \in U} iv(v) \leq \tau_2,\ \sum_{v \in U} fv(v) \leq \tau_3,\ \sum_{v \in U} lv(v) \leq \tau_4,$$

  where $\tau_1, \tau_2, \tau_3, \tau_4 > 0$.

Definition 16. A Pentapartitioned Neutrosophic Fuzzy Threshold Graph (PNFTG) is a graph $G = (V, E)$ where:

- Each vertex $v \in V$ is associated with five membership degrees:

$$\begin{aligned} &T(v) \in [0,1], (\text{truth} - \text{membership}) \\ &C(v) \in [0,1], (\text{contradiction} - \text{membership}) \\ &R(v) \in [0,1], (\text{ignorance} - \text{membership}) \\ &U(v) \in [0,1], (\text{unknown} - \text{membership}) \\ &F(v) \in [0,1], (\text{falsity} - \text{membership}) \end{aligned}$$

satisfying:

$$T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$$

- For any independent set $U \subseteq V$, the threshold conditions are:

$$\sum_{v \in U} T(v) \leq \theta_1, \sum_{v \in U} C(v) \leq \theta_2, \sum_{v \in U} R(v) \leq \theta_3, \sum_{v \in U} U(v) \leq \theta_4, \sum_{v \in U} F(v) \leq \theta_5,$$

  where $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5 > 0$.

Theorem 17. Every Turiyam Neutrosophic Fuzzy Threshold Graph can be transformed into a Neutrosophic Fuzzy Threshold Graph.

Proof. We define the mappings:

$$\begin{aligned} \mu_P(v) &= t(v) \\ \sigma_P(v) &= iv(v) + lv(v) \\ v_P(v) &= fv(v) \end{aligned}$$

Since:

$$\mu_P(v) + \sigma_P(v) + v_P(v) \leq 4$$

We normalize:

$$\begin{aligned} \mu_P'(v) &= \frac{\mu_P(v)}{4} \times 3 \\ \sigma_P'(v) &= \frac{\sigma_P(v)}{4} \times 3 \\ v_P'(v) &= \frac{v_P(v)}{4} \times 3 \end{aligned}$$

Adjust the thresholds:

$$\tau_i' = \frac{\tau_i}{4} \times 3,\ i = 1,2,3$$

Thus, we obtain a Neutrosophic Fuzzy Threshold Graph $G' = (P', Q'; \tau_1', \tau_2', \tau_3')$.

Theorem 18. Every Pentapartitioned Neutrosophic Fuzzy Threshold Graph can be transformed into a Neutrosophic Fuzzy Threshold Graph and an Intuitionistic Fuzzy Threshold Graph.

Proof. We define the mappings:

$$\begin{aligned} \mu_P(v) &= T(v) \\ \sigma_P(v) &= C(v) + R(v) + U(v) \\ v_P(v) &= F(v) \end{aligned}$$

Since:

$$\mu_P(v) + \sigma_P(v) + v_P(v) \leq 5$$

We normalize:

$$\mu'_P(v) = \frac{\mu_P(v)}{5} \times 3$$
$$\sigma'_P(v) = \frac{\sigma_P(v)}{5} \times 3$$
$$v'_P(v) = \frac{v_P(v)}{5} \times 3$$

Adjust the thresholds:

$$\tau'_i = \frac{\theta_i}{5} \times 3,\ i = 1,2,3$$

To transform into an Intuitionistic Fuzzy Threshold Graph, we map:

$$\mu_A(v) = \frac{T(v)}{T(v) + F(v)}$$
$$v_A(v) = \frac{F(v)}{T(v) + F(v)}$$

This ensures $\mu_A(v) + v_A(v) = 1$.

Theorem 19. Every Pentapartitioned Neutrosophic Fuzzy Threshold Graph can be transformed into a Turiyam Neutrosophic Fuzzy Threshold Graph.

Proof. Define the mappings for each vertex $\in V$:

$$\begin{aligned} t(v) &= T(v) \\ iv(v) &= C(v) + R(v) \\ fv(v) &= F(v) \\ lv(v) &= U(v) \end{aligned}$$

Then,

$$\begin{aligned} t(v) + iv(v) + fv(v) + lv(v) &= T(v) + [C(v) + R(v)] + F(v) + U(v) \\ &= T(v) + C(v) + R(v) + U(v) + F(v) \\ &\leq 5 \end{aligned}$$

Normalize the membership degrees:

$$t'(v) = \frac{t(v)}{5} \times 4$$
$$iv'(v) = \frac{iv(v)}{5} \times 4$$
$$fv'(v) = \frac{fv(v)}{5} \times 4$$
$$lv'(v) = \frac{lv(v)}{5} \times 4$$

Adjust the thresholds:

$$\tau_i = \frac{\theta_i}{5} \times 4, \text{ for } i = 1,2,3,4$$

Thus, we obtain a Turiyam Neutrosophic Fuzzy Threshold Graph $G' = (V, E)$.

Theorem 20. In a Pentapartitioned Neutrosophic Fuzzy Threshold Graph G, the sum of all membership degrees for any independent set $U \subseteq V$ satisfies:

$$\sum_{v \in U} [T(v) + C(v) + R(v) + U(v) + F(v)] \leq 5 \times |U|$$

Proof. For each $\in U$:

$$T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$$

Summing over :

$$\sum_{v \in U} [T(v) + C(v) + R(v) + U(v) + F(v)] \leq 5 \times |U|$$

Theorem 21. The maximum cardinality of an independent set $U \subseteq V$ in a Pentapartitioned Neutrosophic Fuzzy Threshold Graph G is bounded by:

$$|U| \leq \min\left\{\frac{\theta_1}{\delta_T}, \frac{\theta_2}{\delta_C}, \frac{\theta_3}{\delta_R}, \frac{\theta_4}{\delta_U}, \frac{\theta_5}{\delta_F}\right\}$$

where $\delta_i = \min\{d_i(v) \mid v \in V\}$ for $i = T, C, R, U, F$.

Proof. Since $d_i(v) \geq \delta_i$ for all $\in U$:

$$\delta_i \times |U| \leq \theta_i \Longrightarrow |U| \leq \frac{\theta_i}{\delta_i}$$

Thus,

$$|U| \leq \min\left\{\frac{\theta_1}{\delta_T}, \frac{\theta_2}{\delta_C}, \frac{\theta_3}{\delta_R}, \frac{\theta_4}{\delta_U}, \frac{\theta_5}{\delta_F}\right\}.$$

Theorem 22. A Pentapartitioned Neutrosophic Fuzzy Threshold Graph does not contain any Pentapartitioned neutrosophic fuzzy alternating 4-cycles.

Proof. Assuming the existence of such a cycle leads to a violation of the threshold conditions due to the cumulative membership degrees exceeding the thresholds. This contradicts the definition of a PNFTG.

Theorem 23. If G is a Pentapartitioned Neutrosophic Fuzzy Threshold Graph, then its complement $\bar{G}$ is also a Pentapartitioned Neutrosophic Fuzzy Threshold Graph under complementary membership degrees.

Proof. Define:

$$\begin{gathered} T'(v) = F(v) \\ C'(v) = U(v) \\ R'(v) = R(v) \\ U'(v) = C(v) \\ F'(v) = T(v) \end{gathered}$$

Since:

$$T'(v) + C'(v) + R'(v) + U'(v) + F'(v) = 5$$

Adjust thresholds θ_i' to maintain the PNFTG conditions in $\bar{G}$.

Theorem 24. In a Pentapartitioned Neutrosophic Fuzzy Threshold Graph, the vertex set can be partitioned into a clique and an independent set based on the membership degrees and thresholds.

Proof. Using the threshold conditions, vertices can be partitioned into a clique K and an independent set I, satisfying the PNFTG properties.

4 | Future Tasks: Some Uncertain Graph and Linguistic Graphs

Future research aims to extend the aforementioned graphs to hypergraphs [40, 71, 97-99] and superhypergraphs [105, 191-194]. In the context of Uncertain Graphs, hypergraphs and superhypergraphs are considered generalizations of traditional graphs and have been studied extensively [5, 6, 9, 153].

And I would like to study Some Uncertain Graph and Linguistic Graphs. Although it is still at the conceptual stage, I aim to provide a clear definition, including related concepts.

4.1 | Z-Graph

We plan to extend the concepts of Z-Number [16, 25, 120, 121, 142, 150, 212, 225, 226] and Z-Numbers Soft Set [128, 230] to graph theory in the future. Z-Number and Z-Numbers Soft Set are well-studied in areas like Uncertain Set Theory. Additionally, the Soft Set [13, 14, 136, 215] is known as a related concept of the Z-Numbers Soft Set. Although it is still at the conceptual stage, the definitions are described as follows.

Definition 25. [226] A Z-number is an ordered pair $Z = (A, B)$, where:

- A is a fuzzy restriction on the possible values of a real-valued uncertain variable X.
- B is a measure of reliability or certainty of the information described by A.

In other words, A represents the uncertain value of X, and B quantifies the confidence in that uncertainty. Z-numbers provide a formal framework to handle both the uncertainty and the reliability of information simultaneously.

Definition 26. Let $G = (V, E)$ be a classical graph, where:

- V is the set of vertices.
- $E \subseteq V \times V$ is the set of edges.

A Z-Graph is a graph where each vertex and/or edge is associated with a Z-number to model uncertainty and reliability in the graph's structure. Formally, a Z-Graph $G_Z = (V, E, \sigma_V, \sigma_E)$ is defined as:

- $\sigma_V: V \to \mathcal{Z}$, a function assigning a Z-number to each vertex.
- $\sigma_E: E \to \mathcal{Z}$, a function assigning a Z-number to each edge.

Here, $\mathcal{Z}$ denotes the set of all possible Z-numbers. For each vertex $v \in V$ and edge $\in E$:

$$\sigma_V(v) = Z_v = (A_v, B_v),$$
$$\sigma_E(e) = Z_e = (A_e, B_e),$$

where:

- A_v and A_e are fuzzy restrictions representing uncertainty about vertex v and edge e, respectively.
- B_v and B_e are measures of reliability for A_v and A_e, respectively.

Definition 27. [230] Let U be a universe of discourse, E be a set of parameters, and $A \subseteq E$ be a non-empty set of attributes.

A Z-Numbers Soft Set over U is a pair $(\tilde{F}, A)$, where:

- $\tilde{F}: A \to \mathcal{P}_Z(U)$ is a mapping from parameters to the power set of Z-numbers over U.

  For each parameter $e \in A, \tilde{F}(e)$ is defined as:

$$\tilde{F}(e) = \{(x, Z_{x,e}) \mid x \in U\}$$

where $Z_{x,e} = (A_{x,e}, B_{x,e})$ is a Z-number associated with the element x under the parameter e :

- $A_{x,e}$ is the fuzzy restriction representing the degree to which x satisfies the parameter e.
- $B_{x,e}$ is the reliability of the information $A_{x,e}$.

Definition 28. A Z-Numbers Soft Graph is a graph that incorporates Z-numbers soft sets to model uncertainty and reliability in both its vertices and edges. Formally, a Z-Numbers Soft Graph $G_{ZSS} = (V, E, \tilde{F}_V, \tilde{F}_E)$ consists of:

- V, a non-empty set of vertices.
- $E \subseteq V \times V$, the set of edges.
- $\tilde{F}_V: A \to \mathcal{P}_Z(V)$, a mapping assigning Z-numbers soft sets to vertices.
- $\tilde{F}_E: A \to \mathcal{P}_Z(E)$, a mapping assigning Z-numbers soft sets to edges.

  For each parameter $\in A$:

$$\tilde{F}_V(e) = \{(v, Z_{v,e}) \mid v \in V\}$$
$$\tilde{F}_E(e) = \{(e', Z_{e',e}) \mid e' \in E\}$$

where:

- $Z_{v,e} = (A_{v,e}, B_{v,e})$ represents the fuzzy restriction and reliability for vertex v under parameter e.
- $Z_{e',e} = (A_{e',e}, B_{e',e})$ represents the fuzzy restriction and reliability for edge e' under parameter e.

A related concept is the Linguistic Z-Graph [135], which is defined as follows. Additionally, concepts like the Linguistic Set [48, 116, 164] are known to be related to the Linguistic Z-Graph.

Definition 29. [135] Let V be a non-empty set and R be a relation on $V \times V$. A Linguistic Z-Graph $G = (V, \sigma, \mu)$ is defined as follows:

- V is the set of vertices.
- $\sigma: V \to \theta(z)$ is a function that maps each vertex to a linguistic Z-number.
- $\mu: V \times V \to \theta(z)$ is a function that maps each pair of vertices to a linguistic Z-number.

Here, $\theta(z)$ represents a set of linguistic Z-numbers, where each Z-number is denoted as $z = (A, B) = (h_\alpha, g_\beta)$. The membership value $\sigma(x)$ of a vertex x is given by $\theta(z_x) = (h_\alpha, g_\beta)$, and the edge membership value $\mu(x, y)$ for an edge (x, y) is calculated as:

$$\mu(x, y) = \sigma(x) * \sigma(y) = (h_r, g_\gamma)$$

where:

- $h_r \leq \min\{h_\alpha, h'_\alpha\}$, for all $x, y \in V$,
- $\beta \leq \gamma \leq \beta'$, and
- $\sigma(x) = \theta(z_x) = (h_\alpha, g_\beta), \sigma(y) = \theta(z_y) = (h'_\alpha, g'_\beta)$.

4.2 | Neutrosophic Linguistic Graph

One of the future prospects is to define the concepts of Single-Valued Neutrosophic Linguistic Set [116, 165, 204, 217], Interval-Valued Neutrosophic Linguistic Set [47], and Multi-Valued Neutrosophic Linguistic Set [118] extended to graphs. This will involve examining their mathematical structures, graph parameters, and various applications. Although it is still at the conceptual stage, the definitions are described as follows. **Definition 30.** Let U be a universe of discourse, and $V \subseteq U$ be a non-empty set of vertices. Let $E \subseteq V \times V$ be the set of edges. Let Θ be an ordered set of linguistic terms.

A Single-Valued Neutrosophic Linguistic Graph $(SVNLG)$ is a quadruple $G = (V, E, \sigma_V, \sigma_E)$, where:

- $\sigma_V: V \to \Theta \times [0,1]^3$ assigns to each vertex $v \in V$ a linguistic term $G_q(v) \in \Theta$ and a neutrosophic triplet $\left(t_Q(v), d_Q(v), l_Q(v)\right)$, with $t_Q(v), d_Q(v), l_Q(v) \in [0,1]$ satisfying:

$$0 \leq t_Q(v) + d_Q(v) + l_Q(v) \leq 3$$

  Here, $t_Q(v)$ is the degree of truth-membership, $d_Q(v)$ is the degree of indeterminacy-membership, and $l_Q(v)$ is the degree of falsity-membership for vertex v.

- $\sigma_E: E \to \Theta \times [0,1]^3$ assigns to each edge $e = (u, v) \in E$ a linguistic term $G_q(e) \in \Theta$ and a neutrosophic triplet $\left(t_Q(e), d_Q(e), l_Q(e)\right)$, satisfying:

$$0 \leq t_Q(e) + d_Q(e) + l_Q(e) \leq 3$$

Definition 31. Let U be a universe of discourse.

An Interval-Valued Neutrosophic Linguistic Graph $(IVNLG)$ is a quadruple $G = (V, E, \sigma_V, \sigma_E)$, where:

- $V \subseteq U$ is the set of vertices.
- $E \subseteq V \times V$ is the set of edges.
- $\sigma_V: V \to \Theta \times [0,1]^6$ assigns to each vertex $v \in V$ a linguistic term $G_q(v) \in \Theta$ and interval-valued neutrosophic triplets:

$$\left([t_Q^-(v), t_Q^+(v)], [d_Q^-(v), d_Q^+(v)], [l_Q^-(v), l_Q^+(v)]\right)$$

  where $t_Q^-(v), t_Q^+(v), d_Q^-(v), d_Q^+(v), l_Q^-(v), l_Q^+(v) \in [0,1]$ satisfy:

$$0 \leq t_Q^-(v) + d_Q^-(v) + l_Q^-(v) \leq 3,\ 0 \leq t_Q^+(v) + d_Q^+(v) + l_Q^+(v) \leq 3$$

- $\sigma_E: E \to \Theta \times [0,1]^6$ assigns interval-valued neutrosophic linguistic values to edges similarly.

Definition 32. Let U be a universe of discourse, and $\Theta = \{G_0, G_1, \ldots, G_t\}$ be an ordered set of linguistic terms, where t is an odd integer.

A Multi-Valued Neutrosophic Linguistic Graph $(MVNLG)$ is a quadruple $G = (V, E, \sigma_V, \sigma_E)$, where:

- $V \subseteq U$ is the set of vertices.
- $E \subseteq V \times V$ is the set of edges.
- $\sigma_V: V \to \Theta \times [0,1]^3$ assigns to each vertex $v \in V$ a linguistic term $G_q(v) \in \Theta$ and a multi-valued neutrosophic triplet $\left(\tilde{t}_Q(v), \tilde{d}_Q(v), \tilde{l}_Q(v)\right)$, with $\tilde{t}_Q(v), \tilde{d}_Q(v), \tilde{l}_Q(v) \in [0,1]$ satisfying:

$$0 \leq \tilde{t}_Q(v) + \tilde{d}_Q(v) + \tilde{l}_Q(v) \leq 3$$

- $\sigma_E: E \to \Theta \times [0,1]^3$ assigns multi-valued neutrosophic linguistic values to edges similarly.

4.3 | Linguistic Soft Graph

One of the future prospects is to define the concepts of Fuzzy Linguistic Soft Set [2, 137], Intuitionistic Fuzzy Linguistic Soft Set [101], and Multi-Valued Neutrosophic Linguistic Soft Set [118] extended to graphs. This will involve examining their mathematical structures, graph parameters, and various applications. Although it is still at the conceptual stage, the definitions are described as follows.

Definition 33. Let U be a universe of discourse, and $\Theta = \{G_0, G_1, \dots, G_t\}$ be a linguistic assessment set. Let $\mathrm{FLSS}(U)$ denote the set of all fuzzy subsets of U.

A Fuzzy Linguistic Soft Set (FLSS) over U is a pair (K, A), where:

- $A \subseteq \Theta$ is a non-empty set of parameters.
- $K: A \to \mathrm{FLSS}(U)$ assigns to each linguistic term $G_g \in A$ a fuzzy subset of U.

A Fuzzy Linguistic Soft Graph (FLSG) is a graph $G = (V, E, K_V, K_E)$, where:

- V is the set of vertices.
- E is the set of edges.
- $K_V: A \to \mathrm{FLSS}(V)$ assigns fuzzy linguistic soft sets to vertices.
- $K_E: A \to \mathrm{FLSS}(E)$ assigns fuzzy linguistic soft sets to edges.

Definition 34. Let U be a universe of discourse, and $\Theta = \{G_0, G_1, \dots, G_t\}$ be a linguistic assessment set. Let $\mathrm{IFLSS}\ (U)$ denote the set of all intuitionistic fuzzy subsets of U.

An Intuitionistic Fuzzy Linguistic Soft Set (IFLSS) over U is a pair (P, A), where:

- $A \subseteq \Theta$ is a non-empty set of parameters.
- $P: A \to \mathrm{IFLSS}(U)$ such that for each $G_g \in A$:

$$P(G_g) = \{\langle m_P(G_g)(y), n_P(G_g)(y) \rangle \mid y \in U\},$$

where $m_P(G_g)(y), n_P(G_g)(y) \in [0,1]$ and $m_P(G_g)(y) + n_P(G_g)(y) \le 1$.

An Intuitionistic Fuzzy Linguistic Soft Graph (IFLSG) is a graph $G = (V, E, P_V, P_E)$, where:

- V is the set of vertices.
- E is the set of edges.
- $P_V: A \to \mathrm{IFLSS}(V)$ assigns intuitionistic fuzzy linguistic soft sets to vertices.
- $P_E: A \to \mathrm{IFLSS}(E)$ assigns intuitionistic fuzzy linguistic soft sets to edges.

Definition 35. Let U be a universe of discourse, and $\Theta = \{G_0, G_1, \dots, G_t\}$ be an ordered set of linguistic terms. A Multi-Valued Neutrosophic Linguistic Soft Set (MVNLSS) over U is a pair (Q, A), where:

- $A \subseteq \Theta$ is a non-empty set of parameters.
- $Q: A \to \mathrm{MVNLS}(U)$, with $\mathrm{MVNLS}(U)$ denoting the set of all multi-valued neutrosophic linguistic subsets of U.
- For each $G_q \in A$:

$$Q(G_q) = \{\langle G_q(y), (\tilde{t}_Q(y), \tilde{d}_Q(y), \tilde{l}_Q(y)) \rangle \mid y \in U\},$$

where $\tilde{t}_Q(y), \tilde{d}_Q(y), \tilde{l}_Q(y) \in [0,1]$ satisfy:

$$0 \le \tilde{t}_Q(y) + \tilde{d}_Q(y) + \tilde{l}_Q(y) \le 3.$$

A Multi-Valued Neutrosophic Linguistic Soft Graph (MVNLSG) is a graph $G = (V, E, Q_V, Q_E)$, where:

- V is the set of vertices.
- E is the set of edges.
- $Q_V: A \rightarrow \mathrm{MVNLS}(V)$ assigns multi-valued neutrosophic linguistic soft sets to vertices.
- $Q_E: A \rightarrow \mathrm{MVNLS}(E)$ assigns multi-valued neutrosophic linguistic soft sets to edges.

4.4 | Extending Other Sets to Graph Theory

In set theory, many other sets and related concepts are known. In the future, we plan to explore the mathematical characteristics of these extended graph concepts. For example, we would like to consider the following concepts (cf. [80]).

- Extend Genuine Sets [57, 106] to Genuine graph.
- Extend Tolerance Rough Fuzzy Sets [17, 228] to Tolerance Rough Fuzzy graph.
- Extend Hybrid Fuzzy Sets [44, 161] to Hybrid Fuzzy graph.
- Extend Level Fuzzy Sets [131, 169] to Level Fuzzy graph.
- Extend the Bell-Shaped Fuzzy Set [49, 51] to Bell-Shaped Fuzzy Graph.
- Extend the Hyperbolic Fuzzy Set [64, 68, 69] to Hyperbolic Fuzzy Graph.
- Extend the Probabilistic Fuzzy Set [43, 103, 115, 133] to Probabilistic Fuzzy Graph.
- Extend Conditional Fuzzy Set [203] to graph theory.
- Extend the Hexagonal Fuzzy Set [45, 46, 145, 182] to Hexagonal Fuzzy Graph.
- Extend the Sigmoid Fuzzy Set [59] to Sigmoid Fuzzy Graph.
- Extend the Convex Fuzzy Set [124, 132, 178] to Convex Fuzzy Graph.
- Extend Atanassov intuitionistic fuzzy sets [15, 30, 92, 146] to graph theory.
- Extend the Gray Fuzzy Set [19, 107, 198, 211] to Gray Fuzzy Graph.
- Extend the Granular Fuzzy Set [127, 130, 134, 177, 202, 209, 216] to Granular Fuzzy Graph.
- Extend the Continuous Fuzzy Set [126, 171, 218] to Continuous Fuzzy Graph.
- Extend Symmetric Fuzzy Set [28, 166] to graph theory.
- Extend shadowed fuzzy set [41, 42, 160, 207] to graph theory.
- Extend Stochastic Fuzzy Set [93] to graph theory.
- Extend Fuzzy Power Set [29, 58, 208] to Fuzzy Power graph.
- Extend Hyperfuzzy Sets [94, 117] to graph theory.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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