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Recreational Mathematics: Where Is the Error?

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Abstract

This short note is a funny problem for the trigonometry students!.

Keywords: Recreational Mathematics; Mathematics; Trigonometry.

1 | Problem

Solve the following trigonometric equation:

$$\tan(x) = \cot(\pi - 2x).$$

2 | Solution 1

One has that: $\tan(x) = \cot\left(\frac{\pi}{2} - x\right)$,

whence $\cot\left(\frac{\pi}{2} - x\right) = \cot(\pi - 2x)$,

hence

$$\frac{\pi}{2} - x = \operatorname{arccot}[\cot(\pi - 2x)] + 2k\pi$$

or $\frac{\pi}{2} - x = \pi - 2x + 2k\pi$,

add *2x* on both sides,

whence $\frac{\pi}{2} + x = \pi + 2k\pi$,

subtract $\frac{\pi}{2}$ from both sides,

or $x = \frac{\pi}{2} + 2k\pi$, $k \in \mathbb{Z}$, where $\mathbb{Z}$ is the set of all negative, zero, and positive integers,

therefore, replacing *k* by all integers one gets infinitely many solutions.

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3 | Solution 2

Graph the function $f(x) = \tan(x) - \cot(\pi - 2x)$ and check where the graph intersect the x-axis.

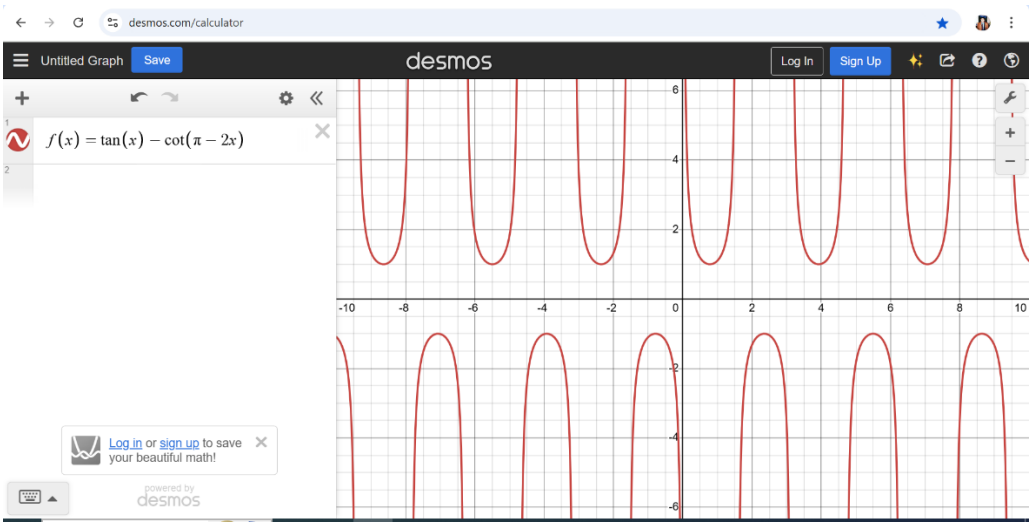

Figure 1. Function $f(x) = \tan(x) - \cot(\pi - 2x)$.

But there is no intersection point of $f(x)$ with the x-axis, therefore no solution!

4 | Proof of Error

The error is the first solution, since the solutions have to be checked if they belong or not to the domains of definition of the trigonometric functions $\tan(x)$ and respectively $\cot(\pi - 2x)$.

One has, replacing x by $\frac{\pi}{2} + 2k\pi$, that:

which is undefined, for any integer k,

similarly $\cot(\pi - 2x) = \cot\left(\pi - 2\left(\frac{\pi}{2} + 2k\pi\right)\right) = \cot(-4k\pi)$ which is also undefined for any integer k:

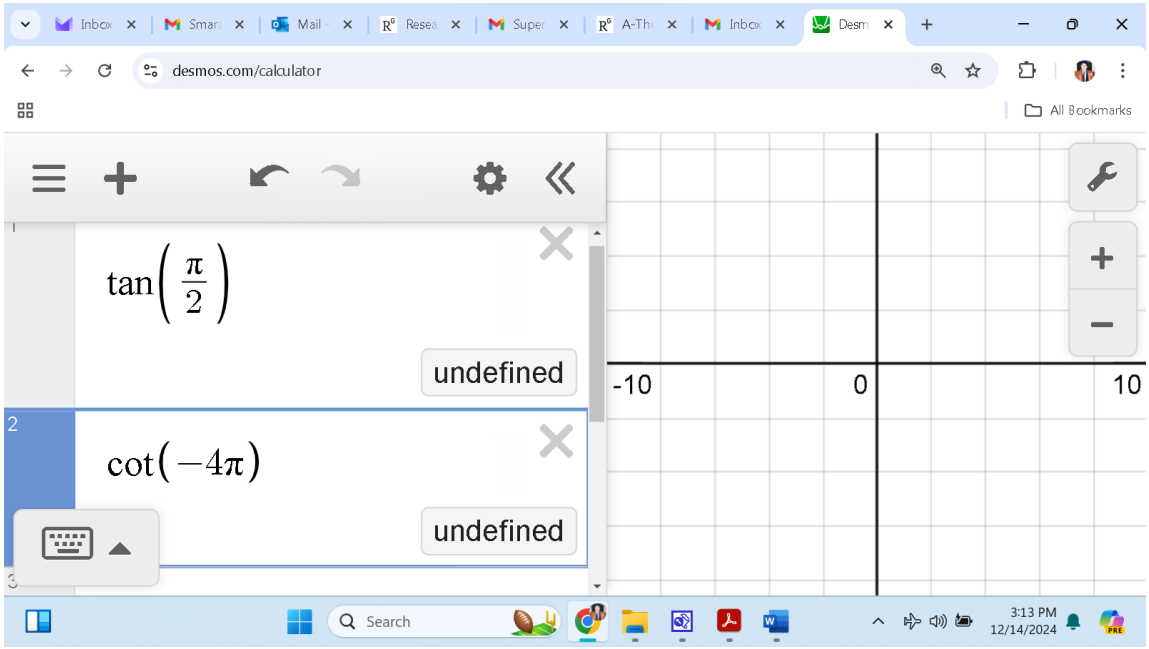

Figure 2. Undefined functions (for k =1).

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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