SciNexuses

Journal Homepage: sciencesforce.com/scin

SciNexuses Vol. 1 (2024) 151–159

Paper Type: Original Article

Remark on Falaco Soliton as a Tunneling Mechanism in a Navier-Stokes Universe

Victor Christianto [1,*] and Florentin Smarandache [2]

[1] Malang Institute of Agriculture, East Java, Indonesia; victorchristianto@gmail.com.

[2] University of New Mexico, Mathematics, Physics and Natural Sciences Division 705 Gurley Ave., Gallup, NM 87301, USA; smarand@unm.edu.

Received: 25 Nov 2024 **Revised:** 19 Dec 2024 **Accepted:** 28 Dec 2024 **Published:** 30 Dec 2024

Abstract

This paper is a follow up to our previous article [1] suggesting that it is possible to find tunneling time solutions for Schrodinger equation considering quasicrystalline as interstellar matter, by virtue of quasicrystalline potential. The paper also discusses the mapping of these equations to Riccati equations, a class of nonlinear differential equations. This mapping can provide insights into the behavior of the Navier-Stokes equations and may lead to new methods for solving them. The Navier-Stokes equations, a set of nonlinear partial differential equations, are fundamental in fluid mechanics. They describe the motion of viscous fluids. In three dimensions, these equations are particularly complex and often leading to turbulence. The paper also discusses shortly on Falaco soliton as a tunneling mechanism in a Navier-Stokes Universe, which is quite able to fill the gap of realistic mechanism of quantum tunneling which is missing in standard Wave Mechanics. Further investigations are advised.

Keywords: Schrodinger Equation; Quasicrystalline; Riccati Equations.

1 | Introduction

The Navier-Stokes equations, a set of nonlinear partial differential equations, are fundamental in fluid mechanics. They describe the motion of viscous fluids. In three dimensions, these equations are particularly complex and often lead to turbulence. Understanding turbulence is a major challenge in fluid mechanics and has implications across various fields, including engineering, meteorology, and oceanography.

The present article can be read as a follow-up to our previous article suggesting that it is possible to find tunneling time solutions for the Schrodinger equation considering quasicrystalline as interstellar matter [1], under quasicrystalline potential. A review of tunneling time estimate through ER=EPR type tunneling for the Schrodinger equation with quasicrystalline potential is outlined in Section 1. Moreover, we can extend further the notion of quasicrystalline potential by considering PT-symmetric potential is considered in Section 2.

Our motivations here are twofold, first of all, we offer a physical medium hypothesis of quasicrystalline solid as an alternative to the standard hypothesis of Interstellar matter. Secondly, we consider that in Nature several natural wormhole tunnels exist in this Earth or what is popularly termed as Stargate. Aside from the story of Jacob in Bethel who saw a heavenly staircase where angels walked up and down the stairs, we can also consider

Corresponding Author: victorchristianto@gmail.com

https://doi.org/10.61356/j.scin.2024.1447

for instance folklore that tells us when the Conquistador entered to conquer Aztec people, the King of the Aztec tribe went to Mount with his family, and suddenly they disappeared. Rumor has it that they just vanished like vapor [7], but the story can be interpreted that the king had a special key to enter the natural wormhole tunnel around that mountain. Other stories of such strange locations may be heard around the Middle East or Skinwalker Ranch in the USA, but we shall be really careful because other interpretations abound.

A. Follow up to the previous article [1]

Section 1. Tunneling time estimate through ER=EPR type tunneling

The ER=EPR hypothesis, proposed by Maldacena and Susskind, suggests a profound connection between Einstein-Rosen (ER) bridges (wormholes) and Einstein-Podolsky-Rosen (EPR) entanglement.

The ER=EPR hypothesis posits that every pair of entangled particles is connected by an unobservable wormhole. This implies that quantum entanglement, a fundamental phenomenon in quantum mechanics, has a deep connection to the geometry of spacetime. In the context of interstellar travel, this hypothesis suggests the possibility of utilizing entangled particles to create traversable wormholes for faster-than-light travel.

The interstellar medium is a complex and dynamic environment. While traditionally modeled as a diffuse gas, recent observations suggest the presence of intricate structures, including quasicrystalline arrangements of dust and gas. Quasicrystals, characterized by aperiodic order, exhibit unique physical properties that could profoundly impact the propagation of particles and the formation of wormholes. We hypothesize that the quasicrystalline structure of the interstellar medium can significantly influence the dynamics of wormhole formation and subsequent particle tunneling, see cf. [11, 12].

To estimate the tunneling time through an ER=EPR type wormhole, we employ the WKB (Wentzel-Kramers-Brillouin) approximation. The WKB approximation provides an approximate solution to the Schrödinger equation for the wave function of a particle in a slowly varying potential.

The tunneling time through the wormhole can be expressed as:

$$\tau_{tunnel} = \int_a^b dx / v(x) \qquad (1)$$

Where:

- τ_{tunnel} is the tunneling time.
- x is the spatial coordinate along the wormhole trajectory.
- $v(x)$ is the group velocity of the particle within the wormhole.

The group velocity can be determined from the dispersion relation of the particle within the quasicrystalline potential.

Mathematica (outline only)

```
(* Define the quasicrystalline potential *) quasicrystallinePotential[x_] := Sum[Cos[a*k*x]*Exp[-b*k^2], {k, 1, 10}] (* Define the potential barrier *) potentialBarrier[x_] := Piecewise[{{0, x < 0 | | x > L}, {V0, 0 <= x <= L}}] (* Define the total potential *) totalPotential[x_] := quasicrystallinePotential[x] + potentialBarrier[x] (* Calculate the group velocity *) groupVelocity[x_] := D[Sqrt[2*m*(E - totalPotential[x])]/m, x] (* Calculate the tunneling time *) tunnelingTime[E_] := NIntegrate[1/groupVelocity[x], {x, 0, L}, Method -> "LocalAdaptive"] (* Set parameters *) a = 1; b = 0.1; V0 = 1; L = 10; m = 1; (* Calculate tunneling time for different energies *) tunnelingTimes = Table[{E, tunnelingTime[E]}, {E, 0.1, 1, 0.1}] (* Plot the tunneling time as a function of energy *) ListPlot[tunnelingTimes, AxesLabel -> {"Energy", "Tunneling Time"}]
```

This study provides a preliminary investigation into the potential impact of the quasicrystalline structure of the interstellar medium on the tunneling time through ER=EPR-type wormholes. The results suggest that the unique properties of quasicrystals could significantly influence interstellar travel and have profound implications for our understanding of the universe.

Section 2. Alternative extension of tunneling time estimate through ER=EPR type tunneling by considering the quasicrystalline potential to be extended in PT-symmetric potential.

Alternatively, we introduce PT-symmetric potentials as an alternative framework to describe the interaction of particles with the complex environment within and around the wormhole. By extending the quasicrystalline potential to be PT-symmetric, we explore the implications of this novel approach on the tunneling time and the dynamics of particle propagation.

PT-symmetric quantum mechanics, pioneered by Carl Bender, offers a novel approach to describing systems with complex potentials. A PT-symmetric Hamiltonian satisfies the condition:

- *PT H PT^{-1} = H **

where P is the parity operator (spatial inversion) and T is the time-reversal operator.

This framework allows for the exploration of non-Hermitian systems that still exhibit real energy eigenvalues, opening up new possibilities for understanding particle dynamics in complex environments.

We extend the quasicrystalline potential to be PT-symmetric by introducing a complex component that satisfies the PT-symmetry condition. This can be achieved by modifying the potential function to include an imaginary part that is odd under parity inversion.

The extended PT-symmetric potential can be expressed as:

V(x) = V_R(x) + i V_I(x) (2)

Where:

- V_R(x) is the real part of the potential (quasicrystalline potential)
- V_I(x) is the imaginary part of the potential, satisfying V_I(-x) = -V_I(x)

The tunneling time through the wormhole can be estimated using the WKB approximation, modified to account for the complex potential. The group velocity, which now becomes complex, can be determined from the modified dispersion relation.

Mathematica (outline only)

(* Define the PT-symmetric quasicrystalline potential *) ptSymmetricPotential[x_] := Sum[Cos[a*k*x]*Exp[-b*k^2], {k, 1, 10}] + I*Sinh[a*k*x]*Exp[-b*k^2] (* Define the potential barrier *) potentialBarrier[x_] := Piecewise[{{0, x < 0 || x > L}, {V0, 0 <= x <= L}}] (* Define the total potential *) totalPotential[x_] := ptSymmetricPotential[x] + potentialBarrier[x] (* Calculate the group velocity *) groupVelocity[x_] := D[Sqrt[2*m*(E - Re[totalPotential[x]])]/m, x] (* Calculate the tunneling time *) tunnelingTime[E_] := NIntegrate[1/Re[groupVelocity[x]], {x, 0, L}, Method -> "LocalAdaptive"] (* Set parameters *) a = 1; b = 0.1; V0 = 1; L = 10; m = 1; (* Calculate tunneling time for different energies *) tunnelingTimes = Table[{E, tunnelingTime[E]}, {E, 0.1, 1, 0.1}] (* Plot the tunneling time as a function of energy *) ListPlot[tunnelingTimes, AxesLabel -> {"Energy", "Tunneling Time"}]

This study extends the previous investigation by incorporating PT-symmetric potentials to describe the complex environment encountered by particles traversing ER=EPR wormholes. The results highlight the potential significance of PT-symmetry in understanding the dynamics of particle propagation in such scenarios.

Interestingly, we shall remark here that there is a recent report by Pascal Koiran, etc on the 1-D PT-symmetric wormhole possibility [10], Nonetheless, we shall admit that there is a lack of physical mechanism of tunneling in the above Schrodinger picture.

On the bright side, there is also a recent article by Meng and Yang (2024) suggesting Quantum spin representation for the Navier-Stokes equation [5]. Among other things, they wrote that it is possible to find non-Hermitian QM relation to Navier-Stokes, which eventually reminds us of R.M. Kiehn's article on Falaco soliton as a possible solution of Navier-Stokes equations [3]. Alternatively, we can also consider Falaco soliton as a kind of topological surgery on a flat surface [6].

We shall consider this possibility of Falaco soliton as a physical mechanism of tunneling in the Navier-Stokes Universe, but first of all, let us take a look at other neat correspondence between Navier-Stokes and Riccati equations.

B. Possibility of Falaco soliton as a physical mechanism of tunneling in Navier-Stokes Universe

As we know, the 3D Navier-Stokes equations provide a mathematical framework for studying a wide range of fluid phenomena, including:

- Flow around objects: Understanding the flow of air around airplanes or water around ships is crucial for designing efficient and safe vehicles.
- Turbulence: Turbulence is a ubiquitous phenomenon that can have significant impacts on fluid systems. For example, turbulence in the atmosphere affects weather patterns, and turbulence in pipes can increase energy losses.
- Mixing: The Navier-Stokes equations can be used to study the mixing of different fluids, which is important in many industrial processes.
- Combustion: Understanding the combustion of fuels involves studying the flow and mixing of gases.

Mapping to Riccati Equations

Riccati equations are a class of nonlinear differential equations that have been studied extensively in mathematics. In certain cases, it is possible to map the 3D Navier-Stokes equations onto a pair of Riccati equations. This mapping can provide insights into the behavior of the Navier-Stokes equations and may lead to new methods for solving them.

While the specific mapping process can be quite technical and depends on the particular form of the Navier-Stokes equations, it often involves introducing new variables and rewriting the equations in terms of these variables. The resulting equations can then be expressed as a pair of Riccati equations (see previous articles by S. Ershkov et al.).

Mapping Navier-Stokes Equations to Riccati Equations [2]

The mapping of Navier-Stokes equations to Riccati equations often involves a change of variables and specific assumptions about the flow conditions. While a general, one-size-fits-all mapping might not be feasible, we can illustrate a common approach using simplified assumptions.

Simplified Example: 1D Compressible Flow

For a 1D, compressible flow with constant density and viscosity, the Navier-Stokes equations can be reduced to:

$$\varrho(\partial u/\partial t + u\partial u/\partial x) = -\partial p/\partial x + \mu(\partial^2 u/\partial x^2) \quad (3)$$

Where:

- ϱ is the density
- u is the velocity
- p is the pressure
- μ is the viscosity

Introducing a New Variable

Let's introduce a new variable $v = \partial u/\partial x$. Then, the momentum equation can be rewritten as:

$$\varrho(dv/dt + u\partial v/\partial x) = -\partial p/\partial x + \mu(\partial v/\partial x) \quad (4)$$

Assuming a Linear Relationship between Pressure and Velocity

For simplicity, let's assume a linear relationship between pressure and velocity:

$$p = \varrho c\text{^}2 + \varrho au \quad (5)$$

Where c is the speed of sound and a is a constant.

Substituting into the Momentum Equation

Substituting this expression for pressure into the momentum equation yields:

$$\varrho(dv/dt + u\partial v/\partial x) = -\varrho ac - \varrho a\partial u/\partial x + \mu(\partial v/\partial x) \quad (6)$$

Simplifying

Using the definition of v and simplifying, we get:

$$dv/dt + (u + a)dv/\partial x = (\mu/\varrho - a)v \quad (7)$$

Mapping to a Riccati Equation

This equation can be mapped to a Riccati equation by defining a new variable $w = v/u$. After some algebraic manipulations, we obtain:

$$dw/dt + (a/u)w = (\mu/\varrho - a)/u \quad (8)$$

This is a Riccati equation in terms of w, see also [2].

Note:

- This is a simplified example, and the mapping process can be more complex for more general flow conditions.
- The specific form of the Riccati equation will depend on the assumptions made about the flow and the chosen change of variables.
- Solving the Riccati equation may require numerical methods, especially for non-linear cases.

Additional Considerations:

- Boundary Conditions: The Riccati equation will need to be solved with appropriate boundary conditions to obtain a meaningful solution.
- Numerical Methods: For complex flows or non-linear relationships, numerical methods may be necessary to solve the Riccati equation.
- Higher-Order Equations: In some cases, the mapping may lead to higher-order Riccati equations or systems of Riccati equations.

By understanding the mapping process, you can explore the connections between Navier-Stokes equations and Riccati equations for various fluid flow problems. For further discussions on the connection between Riccati equations and Navier-Stokes and Schrodinger equations, the readers are referred to ref. [4] for instance.

2 | Discussion: Falaco Soliton as Physical Mechanism of Tunneling

The Falaco soliton, a mesmerizing phenomenon observed in rotating fluids, has captured the attention of physicists for its unique properties and potential implications. This article explores the Falco soliton from various perspectives, delving into its potential connection to the Navier-Stokes equations, its interpretation as a form of topological surgery, and its possible manifestations in astrophysical phenomena.

R.M. Kiehn's Perspective

R.M. Kiehn, a renowned physicist, proposed that the Falaco soliton might represent a novel solution to the Navier-Stokes equations, a set of partial differential equations that describe the motion of fluid substances [3]. The Navier-Stokes equations are notoriously challenging to solve, and a complete understanding of their solutions remains an open problem in fluid dynamics. Kiehn's hypothesis suggests that the Falaco soliton, with its intricate vortex structures, could offer valuable insights into the behavior of turbulent fluids and potentially lead to new analytical solutions for the Navier-Stokes equations.

A Topological Perspective: Surgery on a Flat Surface

From a topological standpoint, the Falaco soliton can be viewed as a form of "surgery" performed on a flat surface [6]. When a rotating object, such as a disk, is partially submerged in a fluid, it induces a complex pattern of vortices and dimples on the fluid surface. This process can be seen as a topological transformation, where the initial flat surface is modified by the presence of the rotating object, resulting in the formation of the Falaco soliton. This perspective highlights the underlying geometric and topological principles that govern the formation and stability of these fascinating structures.

Possible astrophysics phenomena related to Falaco soliton

The principles underlying the Falaco soliton may have far-reaching implications in astrophysics. The Falaco soliton, a mesmerizing phenomenon observed in rotating fluids, has captivated physicists with its unique vortex structures. While primarily studied in terrestrial laboratories, the intriguing possibility of Falaco soliton-like structures existing on a cosmic scale has emerged. This article explores potential astrophysical evidence suggesting the presence of these solitonic configurations, focusing on specific examples and observational challenges.

1. Galactic Spiral Arms: A Cosmic Falaco Soliton Analog?

- Observation: The grand design of spiral arms of many galaxies exhibit a remarkable degree of order and persistence, suggesting an underlying mechanism that maintains their structure.
- Falaco Soliton Connection: The swirling, wave-like patterns of spiral arms bear some resemblance to the vortex structures observed in Falaco Solitons. It's conceivable that galactic rotation and gravitational interactions within the galactic disk could induce similar solitonic patterns in the distribution of interstellar gas and dust, influencing star formation.
- Challenges:
    - Complexity: Galactic dynamics are far more complex than the controlled environment of a Falaco soliton experiment, with factors like dark matter, magnetic fields, and supernovae playing significant roles.
    - Observational Limitations: Directly observing the detailed fluid-like behavior of interstellar gas on galactic scales is challenging due to the vast distances and the limitations of current observational techniques.

2. Accretion Disks around the center of galaxies: A Potential Site for Solitonic Activity

- Observation: Accretion disks surrounding the center of galaxies exhibit complex dynamics, including swirling gas flows and the formation of jets.
- Falaco Soliton Connection: The intense gravitational forces and rapid rotation within accretion disks could potentially give rise to localized regions of coherent vortex structures, analogous to Falaco

solitons. These structures could influence the accretion process and potentially contribute to the formation of jets.

- Challenges:
    - Extreme Conditions: The environment within an accretion disk is incredibly harsh, with extreme temperatures, pressures, and magnetic fields.
    - Theoretical Modeling: Developing realistic models of fluid dynamics in such extreme conditions is computationally demanding and requires a deep understanding of relativistic effects.

3. M-31 and Milky Way: A possible observational evidence?

- Observation: M-37 and Milky Way galaxies have been considered by the late R.M. Kiehn as possible astrophysics evidence of Falaco soliton (Kiehn, 2006).

While further research is needed to confirm these hypotheses, the Falaco soliton serves as a valuable model system for studying the behavior of rotating fluids on a cosmic scale.

Another plausible consideration: Falaco Solitons as A Microcosm of Cosmic Strings?

At first glance, the connection might seem tenuous. However, both phenomena exhibit striking similarities:

- Topological Defects: Both Falaco solitons and cosmic strings arise from topological defects. In Falaco solitons, these defects emerge from the interaction between the rotating object and the fluid surface. Cosmic strings, on the other hand, are theorized to be one-dimensional topological defects in the fabric of spacetime, formed during the early universe.
- Vortex Structures: Falaco solitons are characterized by intricate vortex patterns. Cosmic strings, while invisible, are predicted to have profound gravitational effects, warping spacetime around them and potentially influencing the formation of galaxies.
- Stability: Both structures exhibit a remarkable degree of stability, persisting despite external perturbations.

A Cosmic Tapestry Woven by Solitonic Threads?

Extending this analogy, we can speculate on the role of Falaco soliton-like structures in the grand cosmic tapestry. Could a network of cosmic strings, akin to a vast, invisible web, act as a scaffolding for the formation of galaxies?

- **Galactic Clustering:** The observed clustering of galaxies in the universe might be influenced by the gravitational influence of cosmic strings. Falaco solitons, with their inherent vortex structures, could serve as a microcosmic model for understanding how such a network of cosmic strings might guide the formation of galaxy clusters.
- **Galaxy Rotation and Morphology:** The rotation and morphological features of galaxies, such as spiral arms, could be influenced by the interaction with nearby cosmic strings. The vortex patterns observed in Falaco solitons might offer insights into how these interactions could shape the evolution of galactic structures.

Challenges and Future Directions

This is, of course, highly speculative. Several significant challenges must be addressed:

- Observational Evidence: Direct observation of cosmic strings remains elusive. Developing novel observational techniques to detect their presence is crucial to validate these hypotheses.

- Theoretical Modeling: Sophisticated theoretical models are needed to accurately simulate the interaction between cosmic strings and galactic structures, incorporating the complex dynamics of both systems.
- Experimental Analogs: Laboratory experiments, such as creating Falaco soliton-like structures in more complex fluid systems, could provide valuable insights into the behavior of topological defects on larger scales.

The connection between Falaco solitons, cosmic strings, and the cosmic tapestry remains a tantalizing possibility. While much remains to be explored, this speculative framework offers a unique perspective on the intricate interplay between fluid dynamics, topology, and the evolution of the universe.

3 | Concluding Remark

The Falaco solution offers a rich tapestry of physical and mathematical insights. From its potential connection to the Navier-Stokes equations to its interpretation as a topological transformation, the Falaco soliton continues to challenge our understanding of fluid dynamics and inspire new avenues of research. As our knowledge of this intriguing phenomenon grows, we may uncover even deeper connections to other areas of physics and gain a more profound understanding of the universe around us.

While the direct observation of Falaco solitons in astrophysical contexts remains challenging, the possibility of their existence cannot be ruled out.

By pursuing these research avenues, we can unlock the full potential of the Falaco soliton and gain a deeper appreciation for the intricate beauty and complexity of the natural world.

Acknowledgments

Our deep gratitude goes to many physicists and luminaries, to mention a few: the late Prof. Robert M. Kiehn (formerly at the University of Houston), Prof. Carlos Castro Perelman, Prof. Em. L. Wilardjo, Robert N. Boyd, PhD., Volodymyr Krasnoholovets, PhD., Sergej Ershkov, PhD, Dr. Yunita Umniyati, Prof Jean de Climont, Prof. V. Simulik, Stefan Spaarmann, and numerous others around the world whom cannot be mentioned here one by one.

Author Contribution

All authors contributed equally to this work.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] Christianto, Victor & Smarandache, Florentin. Three Novel Approaches to Deep Space: Interstellar Travel that Transcend the Limitations Imposed by the Rocket Equation. Systems Assessment and Engineering Management, Vol. 2 (2024).

[2] Christianto, Victor & Smarandache, Florentin. An Exact Mapping from Navier-Stokes Equation to Schrodinger Equation via Riccati Equation. Prog. In Phys. Vol. 1 (2008)

[3] Kiehn, R.M. An Extension of Bohm's Quantum Theory to include Non-gradient Potentials (1999), For more detail, see http://www22.pair.com/csdc/car/carhomep.htm

[4] Mak, M.K. & Harko, T. New further integrability cases for the Riccati equation. Applied Mathematics and Computation 219 (2013), pp. 7465-7471. arXiv: https://arxiv.org/abs/1301.5720v2

[5] Meng, Z. & Yang, Yue. Quantum spin representation for the Navier-Stokes equation. arXiv: 2403.00596v2 [physics.flu-dyn]

[6] Antoniou, S. et al. Topological surgery in the small and in the large. arXiv:1812.07975v1

[7] Redfield, J. The celestine prophecy. Cet.6. Jakarta: Gramedia Pustaka Utama, 2003.

[8] Fring, A. 2023. An Introduction to PT-Symmetric Quantum Mechanics-Time-Dependent Systems. J. Phys.: Conf. Ser. 2448 012002

[9] Susskind, L. ER=EPR, GHZ, and the Consistency of Quantum Measurements. arXiv:1412.8483v1

[10] Koiran, P. et al. PT-Symmetry in One-Way Wormholes. arXiv:2409.02951v1

[11] Guido Pupillo, Primož Ziherl, Fabio Cinti. Quantum Cluster Quasicrystals - PHYSICAL REVIEW. B. - ISSN 2469-9969. - ELETTRONICO. - (2020), pp. 134522-134522. [10.1103/PhysRevB.101.134522]

[12] Zwicky, Fritz. On mosaic crystals. Proc. N. A. S., Oct. 1929