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Risk Modeling for Asset Returns with Stable Pareto Distribution and Mathematica Code

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Abstract

In sciences, it is known that normal distribution is often assumed, but there are fields where kurtosis or skewness effect happens for instance in financial markets. So we shall consider where does it come from? At this point we can consider for example Minsky instability hypothesis, but at the same time from practical view point, there is the black swan hypothesis of Nassim N. Taleb. We shall consider therefore how to consider risk modeling of asset returns with stable Pareto distribution. An outline of Mathematica code has been given too. In finance, accurately modeling asset returns is crucial for risk management and investment decisions.

Keywords: Risk Modeling; Stable Pareto Distribution; Mathematica Code; Financial Markets; Asset Returns.

1 | Introduction

In sciences, it is known that normal distribution is often assumed, but there are fields where kurtosis or skewness effect happens for instance in financial markets. So we shall consider where it comes from. At this point, we can consider for example Minsky instability hypothesis, but at the same time from a practical viewpoint, there is the black swan hypothesis of Nassim N. Taleb. We shall consider therefore how to consider risk modeling of asset returns with stable Pareto distribution. In finance, accurately modeling asset returns is crucial for risk management and investment decisions. Traditional models often rely on assumptions like normality, which may not adequately capture the reality of financial markets. The stable Pareto distribution, with its "fat tails" and potential for extreme events, offers a more realistic alternative for certain asset classes. This article explores risk modeling for asset returns using the stable Pareto distribution with the help of the sci-kit-learn library in Python, along with a complementary Mathematica implementation.

1.1 | The Stable Pareto Distribution

The stable Pareto distribution is a power-law distribution characterized by its heavy tails. Unlike the normal distribution, which has a bell-shaped curve and rapidly diminishing probabilities in the tails, the stable Pareto exhibits a slower decay, implying a higher likelihood of extreme events. This characteristic is crucial for modeling financial assets that exhibit volatility clustering and occasional "black swan" events.

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Minsky Moments vs. Black Swans

Hyman Minsky's Financial Instability Hypothesis and Nassim Nicholas Taleb's Black Swan theory both offer frameworks for understanding the occurrence of significant, often catastrophic, events. While seemingly related, they possess distinct origins and causal explanations.

Minsky's Financial Instability Hypothesis:

- **Origin:** Minsky, an economist, developed this hypothesis based on observations of historical financial crises. He argued that periods of prolonged economic stability encourage risk-taking behavior among investors and borrowers.
- **Key Concept:** The hypothesis posits that stability breeds instability. As economic growth continues, businesses and individuals become more willing to borrow and invest, often with increasing leverage. This leads to a shift from "hedge" financing (easily covered by cash flow) to "speculative" financing (cash flow covers interest but not principal) and ultimately to "Ponzi" financing (cash flow insufficient to cover either).
- **Causal Mechanism:** The core cause is the endogenous nature of financial instability. It arises from within the system itself, driven by the evolving risk appetite of market participants during periods of sustained growth.
- **Focus:** Primarily concerned with the dynamics of credit cycles, debt levels, and the evolving risk profiles of financial institutions.

Taleb's Black Swan Theory:

- **Origin:** Taleb, a philosopher and essayist, developed this theory based on his observations of the impact of unpredictable and high-impact events across various domains.
- **Core Concept:** Black Swans are events that are:
  - **Rare:** They lie outside the realm of normal expectations.
  - **High Impact:** They have significant consequences, often causing systemic disruptions.
  - **Unpredictable:** They cannot be foreseen or predicted based on past experience.
- **Causal Mechanism:** Black Swans arise from non-linear systems, complex interactions, and the limitations of human knowledge. They often stem from unforeseen events, unknown unknowns, and the inherent unpredictability of reality.
- **Focus:** Emphasizes the limitations of prediction, the importance of resilience, and the need to prepare for the unknown.

Key Distinctions:

- **Scope:** Minsky's hypothesis focuses specifically on financial markets and economic cycles, while Black Swan theory has broader applications across various domains, including science, technology, and politics.
- **Causality:** Minsky emphasizes endogenous factors within the financial system as the primary driver of crises, while Taleb highlights the role of external shocks, non-linearity, and the limitations of human knowledge.
- **Predictability:** Minsky's framework, while acknowledging inherent uncertainty, suggests that some aspects of financial crises may be predictable and potentially mitigated through regulatory measures. Taleb, on the other hand, argues that true Black Swans are fundamentally unpredictable.

Connecting the Dots:

While distinct, the two theories are not entirely mutually exclusive. A Minsky moment, characterized by excessive debt and financial fragility, can create conditions that increase the vulnerability of the system to Black Swan events. For example, a sudden and unexpected external shock, such as a global pandemic or a geopolitical crisis, could trigger a cascading effect in a highly leveraged financial system, leading to a severe crisis.

Minsky's Financial Instability Hypothesis and Taleb's Black Swan theory provide valuable but distinct perspectives on the nature of crises. Minsky emphasizes the endogenous dynamics of financial markets, while Taleb highlights the role of unpredictable external shocks. By understanding both frameworks, we can gain a more comprehensive understanding of the factors that contribute to systemic risk and develop more robust and resilient systems.

2 | Discussions

Data Preparation

Before proceeding with modeling, it is essential to prepare the asset return data. This typically involves:

i). **Data Collection:** Gathering historical price data for the asset of interest.

ii). **Data Cleaning:** Handling missing values, and outliers, and adjusting for dividends and splits.

iii). **Return Calculation:** Calculating daily, weekly, or monthly log returns.

Scikit-learn does not directly provide a built-in function for fitting the stable Pareto distribution. However, we can leverage the stable distribution from the scipy. Stats library and employ optimization techniques to estimate the parameters.

Mathematica Implementation:

(* Define the stable Pareto PDF *) stableParetoPDF[x_, alpha_, beta_, scale_] := PDF[StableDistribution[alpha, beta, scale, 0], x] (* Define the log-likelihood function *) logLikelihood[params_, data_] := Total[Log[stableParetoPDF[#, Sequence @@ params]] & /@ data] (* Fit the stable Pareto distribution using FindMaximum *) data = RandomReal[NormalDistribution[], 1000]; (* Replace with actual return data *) estimatedParams = FindMaximum[logLikelihood[params, data], {{alpha, 1.5}, {beta, 0}, {scale, 1}}][[2]] (* Print the estimated parameters *) Print["Estimated Alpha:", estimatedParams[[1]]] Print["Estimated Beta:", estimatedParams[[2]]] Print["Estimated Scale:", estimatedParams[[3]]]

Risk Metrics

Once the stable Pareto distribution is fitted, various risk metrics can be calculated:

- **Value at Risk (VaR):** The maximum potential loss over a given time horizon with a specified confidence level.
- **Expected Shortfall (ES):** The average loss exceeding the VaR.
- **Tail Risk Measures:** Metrics like the Expected Shortfall can be used to quantify the risk associated with extreme events in the tails of the distribution.

Mathematica Implementation (Example: Calculating VaR):

(* Calculate VaR *) confidenceLevel = 0.95; var = Quantile[StableDistribution[Sequence @@ estimatedParams], confidenceLevel] (* Print the calculated VaR *) Print["VaR at ", confidenceLevel*100, "% confidence level:", var]

Model Validation and Backtesting

It is crucial to validate the fitted model and assess its performance. This can be achieved through backtesting, and comparing the model's predictions with actual historical outcomes. Backtesting can help identify potential biases and limitations of the model.

3 | Concluding Remark

The stable Pareto distribution offers a valuable alternative to traditional models for risk modeling of asset returns, particularly for assets exhibiting heavy tails and potential for extreme events. By incorporating the power-law behavior and fat tails of the Pareto distribution into risk models, investors can gain a more realistic understanding of potential losses and make more informed investment decisions.

By leveraging the sci-kit-learn library and optimization techniques, practitioners can effectively fit the stable Pareto distribution to historical data and calculate relevant risk metrics. However, it is essential to carefully validate and backtest the model to ensure its reliability and robustness.

4 | Appendix: Sample Implementation

Risk Modeling for Asset Returns with Stable Pareto Distribution and Mathematica

Asset return modeling plays a crucial role in finance, enabling investors to make informed decisions about portfolio allocation and risk management. Traditional models often rely on the normal distribution, which may not accurately capture the "fat tails" observed in real-world financial data, particularly during periods of high volatility. The stable Pareto distribution, with its inherent heavy-tailed nature, provides a more realistic framework for modeling extreme events and their impact on asset returns.

Key Characteristics:

- **Power-Law Behavior:** The distribution exhibits a power-law relationship between the probability of an event and its magnitude.
- **Fat Tails:** The tails of the distribution decay more slowly than the normal distribution, leading to a higher probability of extreme events.
- **Scale Invariance:** The distribution exhibits scale invariance, meaning that its shape remains relatively unchanged when scaled by a constant factor.

Risk Modeling with Scikit-learn

Scikit-learn, a popular Python library for machine learning, provides tools for working with various probability distributions, including the Pareto distribution. While scikit-learn doesn't directly offer a stable Pareto distribution, we can leverage the scipy. Stats library to model and analyze data under this framework.

i). **Data Preparation:**

- **Obtain historical asset price data:** Acquire a time series of historical prices for the asset of interest.
- **Calculate log returns:** Compute the log-returns of the asset prices to obtain a stationary time series.

ii). **Model Fitting:**

- **Import necessary libraries:** Import numpy as np and import pandas as pd from scipy.stats import pareto import matplotlib.pyplot as plt
- **Fit the Pareto distribution:** Use the pareto.fit() function from scipy.stats to fit the Pareto distribution to the log returns data. This function estimates the shape parameter of the Pareto distribution.

iii). **Risk Metrics:**

- **Value at Risk (VaR):** Calculate the VaR, which represents the potential loss in value of an asset or portfolio over a specific time horizon with a given confidence level. The Pareto distribution can be used to estimate VaR more accurately than the normal distribution due to its ability to capture extreme events.
- **Expected Shortfall (ES):** Calculate the ES, which represents the expected loss given that the loss exceeds the VaR. ES provides a more comprehensive measure of risk than VaR, as it considers the magnitude of losses beyond the VaR threshold.

iv). **Backtesting:**

- **Simulate future returns:** Generate simulated future returns using the fitted Pareto distribution.
- **Compare simulated returns with actual returns:** Evaluate the model's performance by comparing the simulated returns with actual historical returns.
- **Adjust model parameters:** Based on backtesting results, refine the model parameters to improve its accuracy.

Mathematica Implementation

Here's a basic Mathematica code snippet to illustrate the fitting of a Pareto distribution to a dataset:

(* Generate sample data *) data = RandomVariate[ParetoDistribution[2], 1000]; (* Fit Pareto distribution *) {alphaEst, xMinEst} = FindDistributionParameters[data, ParetoDistribution[alpha, xMin]]; (* Plot data and fitted distribution *) Show[Histogram[data, "Scott", "PDF"], Plot[PDF[ParetoDistribution[alphaEst, xMinEst], x], {x, xMinEst, Max[data]}], PlotRange -> All]

Limitations and Considerations:

- **Model Assumptions:** The stable Pareto distribution may not perfectly capture the complex dynamics of financial markets, which can exhibit time-varying volatility and other stylized facts.
- **Parameter Estimation:** Accurate estimation of the Pareto distribution parameters can be challenging, especially for limited datasets or when dealing with non-stationary data.
- **Data Quality:** The accuracy of risk models heavily relies on the quality and reliability of the input data.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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