SciNexuses

Journal Homepage: sciencesforce.com/scin

SciNexuses Vol. 2 (2025) 87–96

Paper Type: Original Article

Beyond Present Value: Projecting Future Value with Real Interest Rate, Inflation and Uncertainty

Victor Christianto [1,*] and Florentin Smarandache [2]

[1] Malang Institute of Agriculture, East Java, Indonesia; victorchristianto@gmail.com.

[2] University of New Mexico, Mathematics, Physics and Natural Sciences Division 705 Gurley Ave., Gallup, NM 87301, USA; smarand@unm.edu.

Received: 15 Dec 2024 **Revised:** 18 Feb 2024 **Accepted:** 08 Mar 2025 **Published:** 10 Mar 2025

Abstract

Net Present Value (NPV) is a cornerstone of financial decision-making, allowing us to assess the profitability of certain investment by discounting future cash flows to their present-day value. However, what if we need to project the future value of an investment, taking into account inflation and the inherent uncertainty of the future? This article explores how to move beyond NPV to estimate future value, incorporating real interest rates and modelling uncertainty.

Keywords: Net Present Value; Inflation; Uncertainty; Real Interest Rates.

1 | Introduction: From NPV to Future Value, the Real Interest Rate Bridge

Net Present Value (NPV) is a cornerstone of financial decision-making, allowing us to assess the profitability of an investment by discounting future cash flows to their present-day value. However, what if we need to project the future value of an investment, taking into account inflation and the inherent uncertainty of the future?

The essence of the idea is to leverage the NPV and the real interest rate to project the future value. Here's a breakdown:

1. Calculate NPV: As usual, calculate the NPV of your project by discounting future cash flows using a discount rate that reflects the project's risk.
2. Determine the Real Interest Rate: The nominal interest rate (the one you typically see quoted) doesn't account for inflation. The real interest rate, on the other hand, reflects the actual purchasing power of your investment. It's calculated using the Fisher equation (or a simplified approximation):
   - Fisher Equation: (1 + Nominal Interest Rate) = (1 + Real Interest Rate) * (1 + Inflation Rate)
   - Approximation: Real Interest Rate ≈ Nominal Interest Rate - Inflation Rate

Corresponding Author: victorchristianto@gmail.com

https://doi.org/10.61356/j.scin.2025.2508

Choosing an appropriate inflation rate is crucial. You might use historical averages, forecasts, or even scenario-based inflation rates.

3. Project Future Value: Once you have the NPV and the real interest rate, you can project the future value of the investment at a specific point in time (n years) using the following formula:

    - Future Value = NPV * (1 + Real Interest Rate)^n (1)

This formula essentially compounds the NPV at the real rate of return, giving you an estimate of the investment's future worth in today's purchasing power.

1.1 | Considering Inflation in Interest Rates

It is important to understand that the nominal interest rate used in the NPV calculation should already reflect the market's expectation of inflation. If the nominal rate does not reflect inflation, then the NPV is flawed from its origin.

The Net Present Value (NPV) model, a cornerstone of financial analysis, relies heavily on the discount rate, which is typically a nominal interest rate. However, a fundamental misunderstanding often arises regarding the interplay between nominal and real interest rates, particularly in the context of inflation. This article aims to clarify this relationship and demonstrate its critical impact on both NPV calculations and future value projections [1].

1.2 | The Flaw of Misaligned Nominal Rates in NPV

The statement "It is important to understand that the nominal interest rate used in the NPV calculation should already reflect the market's expectation of inflation" is not merely a suggestion, but a fundamental requirement for accurate NPV analysis. If the nominal discount rate used in your NPV model does not incorporate the market's anticipated inflation, your calculations are inherently flawed.

Here's why:

- Discounting Future Cash Flows: NPV discounts future cash flows back to their present value. If these future cash flows are expressed in nominal terms (i.e., including inflation), the discount rate must also be nominal to ensure a consistent comparison.
- Reflecting Opportunity Cost: The nominal discount rate represents the opportunity cost of capital. It reflects the return investors expect to earn on alternative investments, which already factor in inflation expectations.
- Distorted Valuation: Using a nominal discount rate that doesn't account for inflation leads to a distorted valuation of the project. It either overestimates or underestimates the project's true profitability, depending on whether the nominal rate is too low or too high relative to inflation.

1.3 | The Real Interest Rate: A Clearer Picture of Growth

The real interest rate, calculated as the nominal interest rate minus the inflation rate (approximated), provides a clearer picture of the actual return on investment, stripped of the inflationary effect. It represents the increase in purchasing power.

- Real Interest Rate = Nominal Interest Rate - Inflation Rate (Current Year) (2)

This equation signifies the real rate of return for the current year. It is important to note that inflation expectations and therefore real interest rates can change over time.

How Real Interest Rates Affect the NPV Model Directly and Indirectly:

1. Direct Impact on Discount Rate (in rare cases): In some very specific scenarios, you might choose to discount *real* cash flows (cash flows adjusted for inflation) using a *real* discount rate. However, this

is less common in standard NPV practice. Most often, you'll use nominal cash flows and a nominal discount rate.

2. Indirect Impact on NPV Accuracy: The real interest rate's primary influence on NPV is through its role in forming the *correct* nominal discount rate. If the nominal rate is misaligned with inflation expectations, the NPV will be inaccurate, regardless of whether you explicitly calculate the real rate.
3. Projecting Future Value of NPV: As discussed previously, the real interest rate is essential for projecting the future value of the calculated NPV. Once you have a *correctly* calculated NPV (using an inflation-adjusted nominal rate), you can use the real interest rate to compound that NPV forward in time, providing a more accurate assessment of its future purchasing power.

Building a Robust NPV Model, Practical Considerations

1. Accurate Inflation Forecasting: The cornerstone of a sound NPV model is accurate inflation forecasting. Use reputable sources, consider various economic scenarios, and acknowledge the inherent uncertainty of inflation predictions.
2. Market-Based Nominal Rates: Ensure that the nominal discount rate you use reflects current market conditions and incorporates prevailing inflation expectations. Look to sources like government bond yields, corporate bond yields, and equity risk premiums.
3. Scenario and Sensitivity Analysis: Conduct scenario analysis to assess how different inflation and interest rate scenarios might affect your NPV. Sensitivity analysis can help identify the variables that have the most significant impact on your results.
4. Dynamic Modelling: For long-term projects, consider using dynamic models that can adjust to changing economic conditions. This allows you to update your inflation and interest rate assumptions as new information becomes available.
5. Understanding the Limitations: Recognize that any financial model is a simplification of reality. Be aware of the limitations of your assumptions and interpretations.

1.4 | A Practical Alternative to Estimate PV Based on Real Interest Rate

Traditional Present Value (PV) calculations often rely on deterministic interest and inflation rates, neglecting the inherent uncertainties of financial markets. This article proposes a practical alternative using real interest rates and Quadruple Neutrosophic Numbers (QNNs), an extension of Neutrosophic Logic developed by one of us (FS), to address this limitation [2].

Standard PV models assume fixed discount rates and inflation rates, leading to potentially inaccurate estimations. Financial markets are volatile, and both interest and inflation rates fluctuate significantly over time. Ignoring this variability can result in misleading investment decisions [1,3].

Introducing Real Interest Rates and Quadruple Neutrosophic Numbers. To address the uncertainty, we propose a PV estimation method based on the following:

1. Real Interest Rate with Uncertainty: We calculate the real interest rate by considering the uncertainty surrounding both nominal interest and inflation rates.
2. Quadruple Neutrosophic Numbers (QNNs): We model the uncertain parameters using QNNs, which provide a framework for representing indeterminacy and imprecision.
3. Normal Distribution and Standard Deviation: We incorporate historical data, specifically the standard deviations of interest and inflation rates over the past decade, leveraging the properties of the normal distribution.

The Proposed PV Estimation Formula

We propose the following formula to estimate PV:

PV = NPV * (1 + (i + g * s_i - inf - h * s_inf))^n (3)

Where:

- PV: Present Value
- NPV: Net Present Value (calculated using standard methods)
- i: Nominal interest rate (represented as a QNN)
- g: Factor representing the impact of interest rate uncertainty (a QNN)
- s_i: Standard deviation of interest rates over the past 10 years (a QNN)
- inf: Inflation rate (represented as a QNN)
- h: Factor representing the impact of inflation rate uncertainty (a QNN)
- s_inf: Standard deviation of inflation rates over the past 10 years (a QNN)
- n: Number of periods

Understanding Quadruple Neutrosophic Numbers (QNNs), cf [2].

QNNs extend traditional numerical representation by incorporating four components:

- T (Truth): Degree of membership/truth.
- I (Indeterminacy): Degree of indeterminacy/uncertainty.
- F (Falsehood): Degree of non-membership/falsehood.
- U (Unknown): Degree of unknown or undefined information.

These components are represented as intervals or fuzzy numbers, allowing for the modelling of imprecise and uncertain data. In our context, QNNs enable us to represent the ranges and uncertainties associated with interest rates, inflation rates, and their respective standard deviations.

Applying the Formula, A Practical Approach

1. Data Collection: Gather historical interest rate and inflation rate data for the past 10 years.
2. Calculate Standard Deviations: Compute the standard deviations (s_i and s_inf) for both interest and inflation rates.
3. Represent Parameters as QNNs: Convert the nominal interest rate (i), inflation rate (inf), and factors (b and c) into QNNs, considering their ranges and uncertainties.
4. Calculate Real Interest Rate (with Uncertainty): Compute the real interest rate using the formula: (i + g * s_i - inf - h * s_inf), where all variables are QNNs.
5. Compute PV: Apply the proposed PV formula, using the calculated real interest rate (QNN) and the NPV.
6. Interpret Results: The resulting PV will be a QNN, providing a range of possible values and reflecting the inherent uncertainty.

Advantages of the Proposed Method

- Incorporates Uncertainty: QNNs explicitly model the uncertainty surrounding interest and inflation rates.

- Reflects Market Volatility: Using historical standard deviations captures the volatility of financial markets.
- Provides a Range of Outcomes: QNNs provide a range of possible PV values, offering a more realistic assessment of investment risk.
- Extends Neutrosophic Logic: This method provides a practical application of QNNs, demonstrating the utility of Neutrosophic Logic in financial modeling.
- Practical Calculation: While using neutrosophic numbers, the calculation is still based on the familiar present value calculation, making it easier to adapt to existing models.

Limitations and Considerations

- Data Availability: The accuracy of the method depends on the availability and reliability of historical data.
- Subjectivity: The selection of factors (g and h) and the representation of parameters as QNNs can involve subjective judgments.
- Computational Complexity: Operations with QNNs can be computationally intensive.
- Future Predictions: Historical data does not guarantee future performance. Future economic events could cause significant deviation from past trends.

This proposed method provides a practical and more robust alternative to traditional PV calculations by incorporating real interest rates and Quadruple Neutrosophic Numbers. By acknowledging and modeling uncertainty, financial analysts can make more informed and realistic investment decisions. Further research and development in this area can enhance the accuracy and applicability of QNNs in financial modeling.

Now we offer a complete Mathematica code as an example for the aforementioned formula to estimate PV:

PV = NPV * (1 + (i + g * s_i - inf - h * s_inf))^n , then we shall also discuss how the modeling based on normal distribution function can be extended further to consider stable Pareto distribution.

1.5 | Mathematica

```
(* Function to calculate PV using the provided formula *)
calculatePV[NPV_, i_, g_, s_i_, inf_, h_, s_inf_, n_] :=
  NPV * (1 + (i + g * s_i - inf - h * s_inf))^n;
(* Example usage with numerical values *)
NPVValue = 100000; (* Example NPV *)
interestRate = 0.05; (* Example interest rate *)
interestFactor = 1.2; (* Example interest factor *)
interestStdDev = 0.01; (* Example interest standard deviation *)
inflationRate = 0.03; (* Example inflation rate *)
inflationFactor = 0.8; (* Example inflation factor *)
inflationStdDev = 0.005; (* Example inflation standard deviation *)
numberOfPeriods = 5; (* Example number of periods *)

pvResult =
```

```
  calculatePV[NPVValue, interestRate, interestFactor, interestStdDev,
   inflationRate, inflationFactor, inflationStdDev, numberOfPeriods];

Print["Calculated PV: ", pvResult];

(* Example usage with lists to represent uncertain values, simulating scenarios *)
interestRates = {0.04, 0.05, 0.06};
inflationRates = {0.02, 0.03, 0.04};

scenarioPVs =
  Outer[calculatePV[NPVValue, #1, interestFactor, interestStdDev, #2,
     inflationFactor, inflationStdDev, numberOfPeriods] &, interestRates,
   inflationRates];

Print["Scenario PVs: ", scenarioPVs];

(* Demonstration of using RandomVariate with normal distribution for Monte Carlo Simulation *)
numSimulations = 1000;
interestRateSimulations =
  RandomVariate[NormalDistribution[interestRate, interestStdDev],
   numSimulations];
inflationRateSimulations =
  RandomVariate[NormalDistribution[inflationRate, inflationStdDev],
   numSimulations];

simulatedPVs =
  calculatePV[NPVValue, #1, interestFactor, interestStdDev, #2,
     inflationFactor, inflationStdDev, numberOfPeriods] & @@@
   Transpose[{interestRateSimulations, inflationRateSimulations}];

Print["Mean simulated PV: ", Mean[simulatedPVs]];
Print["Standard deviation of simulated PV: ", StandardDeviation[simulatedPVs]];
Histogram[simulatedPVs, Automatic, "Probability"]
```

1.6 | Discussion and Extension to Stable Pareto Distribution

1. Normal Distribution Modelling:

- The code demonstrates how to calculate PV using the given formula with numerical values.
- It also shows how to use lists to represent uncertain values and simulate different scenarios.
- Furthermore, it illustrates a basic Monte Carlo simulation using RandomVariate with the NormalDistribution. This allows us to generate a large number of possible outcomes for interest and inflation rates, based on their means and standard deviations.
- The histogram displays the distribution of the PVs from the simulation.

2. Extending to Stable Pareto Distribution:

The normal distribution is often used due to its simplicity and the Central Limit Theorem. However, financial data often exhibits "*fat tails,*" meaning extreme events are more frequent than predicted by a normal distribution (cf. Nassim N. Taleb, *The Black Swan*). The stable Pareto distribution (also known as the alpha-stable distribution) is better suited for modelling such data.

How to extend the model:

- Replace Normal Distribution with Stable Pareto: Instead of using NormalDistribution, use StableDistribution from Mathematica. The StableDistribution has four parameters:
  - alpha: Stability parameter (0 < alpha <= 2). Lower values indicate heavier tails.
  - beta: Skewness parameter (-1 <= beta <= 1).
  - gamma: Scale parameter (gamma > 0).
  - delta: Location parameter.
- Parameter Estimation: Estimating the parameters of a stable Pareto distribution is more complex than estimating the mean and standard deviation of a normal distribution. You can use methods like:
  - Method of moments.
  - Quantile-based methods.
  - Maximum likelihood estimation.
  - Mathematica has packages to help with this.
- Adjust Monte Carlo Simulation: Modify the Monte Carlo simulation to use RandomVariate with the StableDistribution.
- Impact on PV Calculation: Using a stable Pareto distribution will result in a wider range of possible PV values, reflecting the increased probability of extreme outcomes. This will provide a more realistic assessment of risk.

1.7 | Example of Modification

Mathematica

```
(* Example with Stable Pareto Distribution (alpha=1.5, beta=0, gamma=0.01, delta=0.05 for interest rate) *)

alphaInterest = 1.5;

betaInterest = 0;

gammaInterest = 0.01;

deltaInterest = 0.05;
```

```
interestRateSimulationsStable =
  RandomVariate[
   StableDistribution[alphaInterest, betaInterest, gammaInterest,
    deltaInterest], numSimulations];

(* Example with Stable Pareto Distribution (alpha=1.8, beta=0, gamma=0.005, delta=0.03 for inflation rate)
*)
alphaInflation = 1.8;
betaInflation = 0;
gammaInflation = 0.005;
deltaInflation = 0.03;

inflationRateSimulationsStable =
  RandomVariate[
   StableDistribution[alphaInflation, betaInflation, gammaInflation,
    deltaInflation], numSimulations];

simulatedPVsStable =
  calculatePV[NPVValue, #1, interestFactor, interestStdDev, #2,
     inflationFactor, inflationStdDev, numberOfPeriods] & @@@
   Transpose[{interestRateSimulationsStable,
     inflationRateSimulationsStable}];

Print["Mean simulated PV (Stable): ", Mean[simulatedPVsStable]];
Print["Standard deviation of simulated PV (Stable): ",
  StandardDeviation[simulatedPVsStable]];
Histogram[simulatedPVsStable, Automatic, "Probability"]
```

Remark:

- Parameter estimation for stable Pareto distributions can be challenging and requires specialized techniques.
- The choice of distribution should be based on the characteristics of the data and the specific application.
- Stable distributions are not as simple to work with as normal distributions, but they can provide more accurate results when dealing with financial data that exhibits fat tails.
- The standard deviation may not exist for some stable distributions.

1.8 | Modelling Uncertainty: Beyond Point Estimates

Regarding the future, one thing is quite certain, that is the future is inherently uncertain. Relying solely on point estimates for inflation and interest rates can lead to inaccurate projections. To address this, we need to incorporate uncertainty into our models. Here are some methods:

1. Scenario Analysis: Develop multiple scenarios (e.g., optimistic, pessimistic, and base case) with different inflation rates and nominal interest rates. Calculate the real interest rate and future value for each scenario. This provides a range of potential outcomes.
2. Sensitivity Analysis: Identify the key variables that have the most significant impact on the future value (e.g., inflation rate, sales growth). Vary these variables within a reasonable range and observe the effect on the projected future value. This helps identify the project's vulnerabilities.
3. Monte Carlo Simulation: This powerful technique involves generating thousands of random values for the uncertain variables (based on their probability distributions) and calculating the future value for each iteration. This creates a distribution of possible future values, giving you a probabilistic view of the investment's potential outcomes.
    - When using Monte Carlo simulation, apply probability distributions to the inflation rate and the nominal interest rate. This will produce a distribution of real interest rates, and therefore a distribution of future values.
4. Decision Trees: When dealing with sequential decisions and uncertain events, decision trees can be used to model different paths and their associated probabilities. This allows you to evaluate the potential future value of different strategies under various scenarios.

Practical Considerations

- Time Horizon: The longer the time horizon, the greater the uncertainty. Be cautious when projecting future values over extended periods.
- Data Quality: The accuracy of your projections depends on the quality of your input data. Use reliable sources and validate your assumptions.
- Dynamic Modelling: Consider using dynamic models that can adapt to changing economic conditions. This is especially important when dealing with long-term projects.

2 | Concluding Remark

Projecting future value beyond NPV requires careful consideration of inflation and uncertainty. By using the real interest rate and incorporating scenario analysis, sensitivity analysis, or Monte Carlo simulation, we can develop more robust and realistic projections. This allows for better-informed investment decisions and helps mitigate the risks associated with an uncertain future.

The real interest rate is not merely an academic concept; it's a critical tool for understanding and interpreting financial data. By ensuring that a nominal discount rate accurately reflects inflation expectations, you can build a more robust and reliable NPV model. Furthermore, leveraging the real interest rate allows for a more accurate projection of future value, providing a clearer picture of the investment's potential returns in real terms. By understanding and applying these principles, financial professionals can make more informed and effective investment decisions.

This proposed method provides a practical and more robust alternative to traditional PV calculations by incorporating real interest rates and Quadruple Neutrosophic Numbers. By acknowledging and modeling uncertainty, financial analysts can make more informed and realistic investment decisions. Further research and development in this area can enhance the accuracy and applicability of QNNs in financial modeling.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contributions

All authors contributed equally to this work.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] Levin, Richard I., David S. Rubin, Joel P. Stinson, Everett, S. Gardner, Jr., Quantitative approaches in management. 7th ed. New York, McGraw-Hill, Inc., 2016.

[2] Christianto, Victor, Smarandache, F., & Aslam, M. How we can extend the standard deviation notion with neutrosophic interval and quadruple neutrosophic numbers. IJNS, 2022. url: https:// digitalrepository.unm.edu/math_fsp/510

[3] Taleb, Nassim N. The black swan. Edisi keenam. Jakarta: Gramedia Pustaka Utama, 2020. ISBN: 978-602-6-4400-4