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Logical Pluralism and Neutrosophy: Reflections on the Nature of Truth

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Abstract

When students first encounter logic, their journey often begins with classical logic, a foundational framework widely applied in fields such as mathematics, computer science, and philosophy. The classical logic is a rigid structure with binary distinctions between true and false, frequently regarded as the default system of reasoning. However, deeper exploration into symbolic logic reveals a more complex landscape, where no single, universally accepted system prevails. Instead, an array of logical frameworks emerges, each offering multifaceted perspectives on truth, validity, or inference.

Keywords: Logical pluralism, Neutrosophic Logic, Truth, Validity, Non-classical logics, Philosophy of Logic, Classical Logic, Intuitionistic Logic, Paraconsistent Logic, Dialetheism, Fuzzy Logic, Indeterminacy, Vagueness, Contradictions, Triadic reasoning, Beyond binary reasoning, Neutralities in logical systems, Liar Paradox, Decision-making under uncertainty, Multi-valued logics.

1 | Introduction

The theory of logical pluralism [1] suggests that there is no single one true logic. As Susan Haack argues in Philosophy of Logics [2], validity is not a singular, unambiguous concept; it is inherently vague [Haack] Different logical systems, by making this vague idea precise in different ways, provide distinct but equally legitimate ways of understanding reasoning. This view has been expanded by philosophers like J. C. Beall and Greg Restall in their Logical Pluralism [Beall&Restall]. They argue that multiple systems of logic can each offer a valid account of reasoning, according to the Generalized Tarski Schema of logical validity.[1] This pluralistic perspective resonates closely with the ideas behind neutrosophy—which, again, is a framework that goes beyond classical boundaries and encompassing ambiguity and contradiction.

[1] Hodges, Wilfrid, "Tarski's Truth Definitions", *The Stanford Encyclopedia of Philosophy* (Winter 2022 Edition), Edward N. Zalta & Uri Nodelman (eds.). https://plato.stanford.edu/archives/ win2022/entries/tarski-truth.

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2 | Multiple Paths to Understanding Validity

At the heart of logical pluralism lies the idea that different logical systems offer different methods for determining the truth or validity of statements. Each system might be better suited to certain types of problems or contexts, suggesting that no one (single) system can fully capture the complexity of reasoning. In the work of Beall and Restall, the concept of validity is decoupled from any single logical system. By extending the Tarski Schema, which states that a logical system is valid if its conclusions are true whenever its premises are true, they allow for different interpretations of truth and validity, depending on the system in question. This idea echoes Haack's view that the concept of validity is inherently vague, and different systems offer different precisions of this vagueness. For instance, intuitionistic logic, paraconsistent logic, and dialetheic logic each provide different ways to deal with the limits of classical reasoning. Are these systems in competition, each vying for the title of the ultimate logic, or are they better understood as tools that serve distinct purposes depending on the problem at hand?

2.1 | Object Language and Metalanguage

When defining truth for a language L (the *object language*), the definition must be framed in another language M (the *metalanguage*). M must:

- Include a copy of L, allowing anything expressible in L to also be stated in M.
- Be capable of discussing L's sentences and syntax.
- Incorporate set-theoretic concepts and a unary predicate, {True}, which denotes "is a true sentence of L."

The purpose of the metalanguage is to formalize statements about L, supported by axioms in M that justify the truth definition. Tarski stipulated that {True} should be defined using syntax, set theory, and L's notions, avoiding semantic terms like "denote" or "mean" unless they are part of L.

2.2 | Formal Correctness

The truth definition for L must be *formally correct*, i.e. {True} is:

$\forall x, \text{True}(x) \Leftrightarrow \phi(x)$,

where {True} does not appear in ϕ. Alternatively, the definition must be provably equivalent [Figure 1] to such a statement using axioms of M that exclude {True}. Definitions of this form are called *explicit*, or "normal" in Tarski's terminology. [Tarski 1933]

Let us emphasize in a chart the necessary conditions for formal correctness in Tarski's framework:

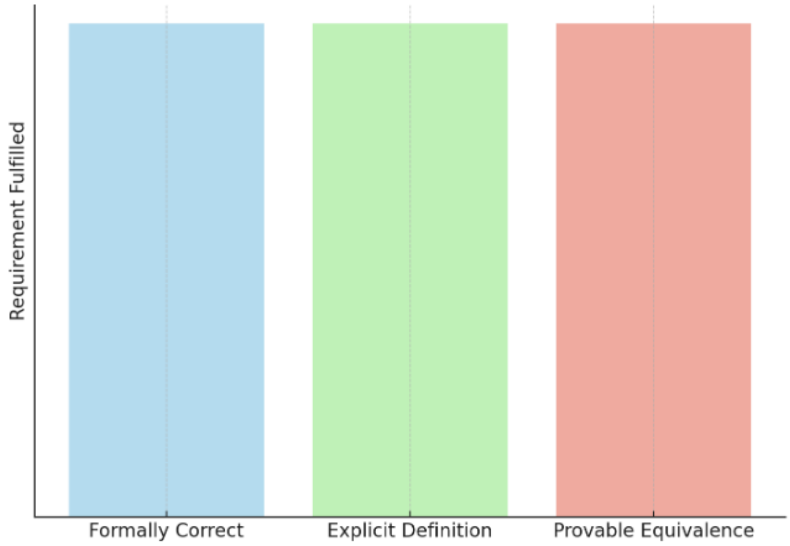

Figure 1. Requirements for a formally correct truth definition.

Each category represents a key condition:

1. Formally Correct: {True} True must be defined as $\forall x$;True$(x) \Leftrightarrow \phi(x)$.
2. Explicit Definition: ϕ must exclude {True}.
3. Provable Equivalence: The definition must be provably equivalent to the explicit form using axioms of M that do not include {True}.

2.3 | Material Adequacy

The truth definition must also be *materially adequate* (or "accurate"), meaning ϕ should precisely capture the sentences intuitively considered true in L. This must be provable using M's axioms.

At first glance, this seems paradoxical: proving material adequacy appears to assume an already adequate definition of truth, risking infinite regress. Tarski avoided this by requiring M to formalize infinitely many equivalences of the form:

$\phi(s) \Leftrightarrow \psi$,

where s is the name of an L-sentence S, and ψ is S's counterpart in M. The challenge lies in identifying a single formula ϕ to derive all such equivalences from M's axioms, thus defining {True}.

Tarski [11, 12] formalized this requirement as *Convention T*, a cornerstone of his semantic conception of truth. However, when L can discuss its own semantics, Convention T leads to the liar paradox. Tarski concluded that M must be stronger than L.

2.4 | Implications for Mathematics

In mathematics [3], first-order *Zermelo-Fraenkel set theory* (ZF) is considered the standard for correctness. However, Tarski's results [4] imply that a truth definition for ZF cannot be given within ZF itself. The usual workaround is to define truth informally in a natural language like English.

Limited formal truth definitions for ZF exist. For instance, Azriel Levy showed that for any natural number n, a Σ_n formula identifies precisely the true Σ_n sentences of set theory. Key properties of Σ_n formulas include:

- Any set-theoretic sentence is equivalent to a Σ_n sentence for sufficiently large n.
- Σ_n formulas are closed under existential quantifiers but not universal quantifiers.

- Σ_n formulas are not closed under negation, which avoids Tarski's paradox.

Similar techniques underpin Jaakko Hintikka's internal truth definitions for independence-friendly logic, which shares these characteristics with Σ_n classes [Hintikka].

3 | Dialetheism and the Case for Logical Pluralism

One of the most radical challenges to classical logic comes from dialetheism, a position championed by Graham Priest [5, 6]. Dialetheism asserts that some propositions can be both true and false at the same time, directly violating the classical law of non-contradiction. Priest's works [Priest 1987; 2002] explore the possibility that contradictions are not just logical anomalies to be dismissed but could be coherent and meaningful in certain contexts.

Let us go into further detail. A *dialetheia* is a sentence A for which both A and its negation $\neg A$ are true simultaneously. If we accept that falsity is the truth of negation, a dialetheia represents a situation where a sentence is both true and false at the same time. This paradoxical scenario results in what is known as a *truth-value glut*, where a sentence possesses two conflicting truth values. This stands in contrast to a *truth-value gap*, where a sentence is neither true nor false.[1] This definition of a dialetheia can easily be extended to other truth-bearers such as propositions or statements. The choice of truth-bearer is not crucial, as the key concept remains the same: a sentence or equivalent entity that embodies a simultaneous truth and falsity.

For example, the liar paradox,[2] in which a statement refers to itself as false, may be resolved by accepting that the statement is both true and false simultaneously. This paradoxical situation challenges the very foundation of classical logic, suggesting that reasoning may sometimes need to accommodate contradictions rather than reject them outright.[3]

Logic proves ineffective in addressing paradoxes [7-9], as resolving inconsistencies necessitates choosing between alternatives to abandon certain premises.[4] According to Rescher, logic itself is value-neutral, and managing paradoxes demands resources that extend beyond the scope of logic. [Rescher] Consequently, the challenge lies in identifying appropriate and practical concepts capable of realizing his intuitions.

Dialetheic logic is a system designed specifically to handle such contradictions, allowing for the simultaneous truth and falsity of propositions. As such, it embodies a logical pluralist approach, where the classical logic of true/false oppositions is inadequate.

4 | Neutrosophic Logic: Beyond True and False

Building on fuzzy logic and intuitionistic logic, neutrosophic logic introduces a third value—indeterminacy—in addition to truth and falsity. In neutrosophic logic, every proposition is assigned a degree of truth (T), a degree of falsity (F), and a degree of indeterminacy (I), each of which can range from 0 to 1.

This triadic approach proved to be particularly useful in contexts where information is incomplete, contradictory, or vague, such as in decision-making under uncertainty or quantum mechanics. [Smarandache]

[1] Priest, Graham, Francesco Berto, and Zach Weber, "Dialetheism", *The Stanford Encyclopedia of Philosophy* (Summer 2024 Edition), Edward N. Zalta & Uri Nodelman (eds.), https://plato.stanford.edu/archives/sum2024/entries/dialetheism/.

[2] Beall, Jc, Michael Glanzberg, and David Ripley, "Liar Paradox", *The Stanford Encyclopedia of Philosophy* (Winter 2023 Edition), Edward N. Zalta & Uri Nodelman (eds.), https://plato.stanford.edu/archives/win2023/entries/liar-paradox/.

[3] Smith, J. M. (1991). [Review of *In Contradiction, A Study of the Transconsistent.*, by G. Priest]. *Noûs*, *25*(3), 380–383. https://doi.org/10.2307/2215513.

[4] Cantini, A. (2004). [Review of *Paradoxes: Their Roots, Range, and Resolution*, by N. Rescher]. *Studia Logica: An International Journal for Symbolic Logic*, *76*(1), 135–142. http://www.jstor.org/stable/20016577.

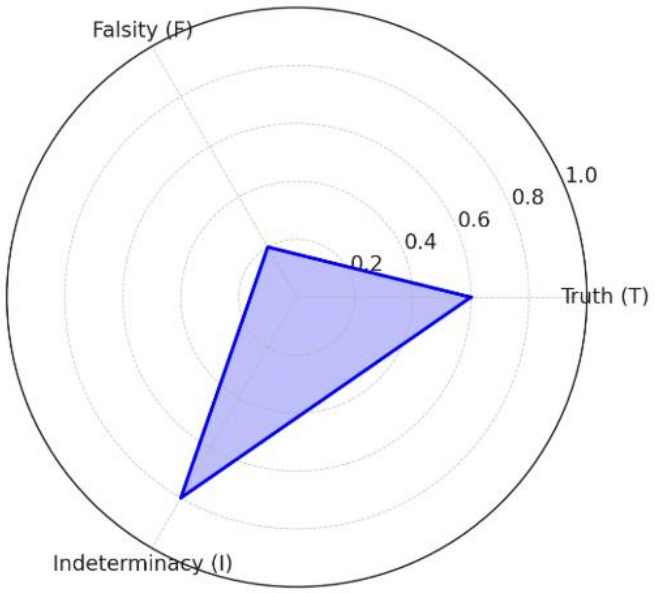

Figure 2. Degrees of Truth (T), Falsity (F), and Indeterminacy (I).

This is a radar chart visualizing the degrees of truth (T), falsity (F), and indeterminacy (I) in neutrosophic logic.

The values (T = 0.6, F = 0.2, I = 0.8) are illustrative and represent how neutrosophic logic embraces a spectrum of possibilities.

The introduction of indeterminacy allows neutrosophic logic to embrace the *neutralities* that are essential to the concept of neutrosophy itself [Figure 2]. Like logical pluralism, neutrosophic logic does not insist on a single, absolute truth but instead recognizes that truth is often a matter of degree and context.

Neutrosophic logic, by accommodating indeterminacy, also aligns with the views of Haack and Beall and Restall, who argue that logical systems should not be seen as competing for dominance but as addressing different aspects of reasoning.

In neutrosophic logic, indeterminacy is not a flaw or a problem to be resolved but a fundamental aspect of reasoning, especially in contexts where uncertainty is an inherent feature of the system being modeled.

5 | The Philosophical Implications: Tools or Rivals?

The debate over whether different logical systems are rivals or tools for different contexts touches on deeper philosophical questions about the nature of truth and validity. For *logical pluralists* like Beall and Restall, the diversity of logical systems reflects the inherent complexity of reasoning. There is no one-size-fits-all solution to the problems of truth and inference; rather, different systems offer different ways to navigate the vagueness and contradictions present in the world.

This pluralistic view challenges the idea of a One True Logic, suggesting instead that logic is a flexible, context-sensitive tool. Just as different tools are suited for different tasks, different logical systems are suited to different types of reasoning. *Dialetheic logic* excels at resolving paradoxes, *intuitionistic logic* offers insights into the nature of constructive proof, and *neutrosophic logic* is particularly effective in contexts of indeterminacy and uncertainty.

6 | Conclusion: A Pluralistic Understanding of Logic and Truth

The exploration of logical pluralism, dialetheism, and neutrosophic logic reveals a deep philosophical shift in how we understand the nature of truth and logic. Rather than seeking a single, ultimate logic, we are encouraged to recognize the diversity of logical systems as reflecting the multifaceted nature of reality. Whether dealing with contradictions, uncertainties, or vague concepts, these systems offer complementary tools for navigating the complexities of reasoning. Just as neutrosophy embraces contradiction and indeterminacy, so too does logical pluralism encourage us to expand our understanding of logic beyond the confines of classical thinking. Through this lens, the search for a single "one true logic" becomes less important than the recognition of the diverse tools available to us in the pursuit of knowledge.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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