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## Partnership of Lean Six Sigma and Digital Twin under Type 2 Neutrosophic Mystery Toward Virtual Manufacturing Environment: Real Scenario Application

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### Abstract

The correlation between Lean Six Sigma performance strategy (LSS) and contemporary innovations, including Digital Twin (DT) technology (DLSS) has been scrutinized in this study. The merger's mission is to overcome some of the organizational tensions. Environmentally speaking, it's critical to cut waste and save resources. On a social level, demonstrated boosts to labor and machine efficiency through scrutinizing analysis, simulation, and virtual-physical replication. Economically speaking, improved quality, lower prices and lead times, continuous improvement, and performance enhancement techniques are all part of the worldwide manufacturing scene. Actually, harnessing the suitable DT application amongst available applications is a noteworthy procedure. Hence, this study evaluates DT applications by constructing a decision framework. The constructed framework depends on decision methodologies of multi-criteria decision-making framework (MCDM) and the vague theory of type 2 neutrosophic sets (T2NSs). The entropy and Complex Proportional Assessment (COPRAS) of MCDM are working in cooperation with T2NSs to recommend optimal DT application which merges with LSS toward sustainability of the organization's industrial.

**Keywords:** Type 2 Neutrosophic Sets; Lean Six Sigma; Digital Twin; Sustainability; Multi-Criteria Decision-Making.

## 1 | Introduction and Background

The objectives of sustainable development (SD) quest us to embrace practices and methodologies that catalyze sustainability, especially in the field of manufacturing. As well [1] stated another catalyst where the competitive and rapidly changing business environment leads to stress and consideration which forces the organization industrial to improve their operational performance and maintain high-quality standards.

Accordingly, this study addressed Lean Six Sigma (LSS) as one of the most extensively recognized strategies for operations adopted by a variety of sectors to improve productivity and raise competitiveness [2]. Wherein



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LSS is aroused as a result of a merger between principles of lean production and Six Sigma methodologies for strengthening strategies[3], seeking to eradicate waste[4] where its foundation is the Define-Measure-Analyze-Improve-Control (DMAIC) methodology's harmony [5].

Simultaneously, LSS's reputation has increased significantly to the point where it is presently utilized by several small and major firms worldwide and can be found in every conceivable industry [6, 7]. The methodology of Lean Six Sigma (LSS) has gained popularity among leaders as a means of streamlining company operations and eliminating mistakes of all kinds. Meanwhile, companies are carefully implementing LSS to raise end-user happiness and boost profitability [8]. LSS framework for the past decade has significantly been used by various manufacturing organizations as the de-facto desolate tactic to escalate operational excellence which spurs profitable development by eliminating process variances and non-existence along with trough operating costs[9]. Antony et al. [10] held a different view notwithstanding LSS's significance and requirement in the industrial sectors where LSS strategies fail to produce the expected result due to a variety of factors, including human interaction, different regulatory regulations, ineffective recordkeeping, and data piracy.

On the other hand, Mishra et al. [11]revealed that exponential expansion in product volume, variety, and innovativeness has resulted from the Industrial Revolution's (InR) progress. In the same vein [12] stated that there has been a significant rise in the degree of connectedness among the activities since the introduction of advanced technologies including the Industry 4.0 (Ind 4.0) concept. For instance, decision-making has been monitored and performance optimization through real-time monitoring by deploying DT [13]. Due to [14] where DT possesses the capacity to construct virtual replicas of tangible systems or assets.

Numerous academics have carried out investigations to elucidate DT. For instance, Kritzinger et al. [15] stated that DT has an automated, fully integrated data flow that is bidirectional to the physical thing and is a digital depiction of the physical entity. Wherein DT is divided into three components in [16] as a physical entity and its virtual equivalent, alongside a mapping that permits co-evolution between the physical and virtual aspects of the thing. In the same vein [17, 18] described DT as a constantly learning virtual or mathematical model of the real-world object or system in the form of digital, used to simulate, analyze, predict, and/or improve the processes, interrelations, and feedback mechanisms between virtual and real models.

Accordingly, scholars attempted to capitalize on DT's competencies to embrace it in the manufacturing sector through a partnership with LSS. Hence, Performance maps were produced as a consequence of this method, which facilitated effective decision-making for the execution of strategies, when [19] used dynamic simulations to build a DT, which made it easier to choose zero-defect production techniques by adjusting parameters. Also, [20] contributed to presenting a real-time validation technique for digital twins manufactured via simulation. By guaranteeing exact alignment with real systems, this approach makes informed short-term judgments possible.

Therefore, utilizing the appropriate DT application is a crucial issue to guarantee optimal services in the manufacturing sector. This study embraced this issue and provided mathematical methodology as a means for selecting the optimal DT application through a constructed appraiser model. This model is emerging various MCDM techniques with T2NSs where each utilized technique contributed to an important role. For instance, entropy is used to generate weights for determined criteria and leverage these weights into COPRAS which in turn ranks alternatives of DT applications and recommends the optimal.

The study is structured as: Section 2 introduces the concepts, and operation formulas of T2NNs. Section 3 the proposed model is presented. Section 4 describes the case study and results of the proposed framework. Section 5 provides a Comparative analysis. Section 6 summarizes our conclusions.

## 2 | Preliminaries

This section exhibits the basic principle of uncertainty theory as T2NSs and its operations.

## 2.1 | Type-2 Neutrosophic Set [21]

**Definition 1:** A T2NN set  $\ddot{G}$  in  $\ddot{R}$  is defined by:  $\ddot{G} = \{(\ddot{u}, \varepsilon_{\ddot{G}}(\ddot{u}), \gamma_{\ddot{G}}(\ddot{u}), \xi_{\ddot{G}}(\ddot{u}) \mid \ddot{u} \in \ddot{U}\}$ , where  $\varepsilon_{\ddot{G}}(\ddot{u}): \ddot{G} \rightarrow \varepsilon[0,1], \gamma_{\ddot{G}}(\ddot{u}): \ddot{G} \rightarrow \gamma[0,1]$ , and  $\xi_{\ddot{A}}(\ddot{u}): \ddot{G} \rightarrow \xi[0,1]$ . The elements of the T2NN set can be shown as  $\varepsilon_{\ddot{G}}(\ddot{u}) = (\varepsilon_{\varepsilon_{\ddot{G}}}(\ddot{u}), \varepsilon_{\gamma_{\ddot{G}}}(\ddot{u}), \varepsilon_{\xi_{\ddot{G}}}(\ddot{u})), \gamma_{\ddot{G}}(\ddot{u}) = (\gamma_{\varepsilon_{\ddot{G}}}(\ddot{u}), \gamma_{\gamma_{\ddot{G}}}(\ddot{u}), \gamma_{\xi_{\ddot{G}}}(\ddot{u}))$ , and  $\xi_{\ddot{G}}(\ddot{u}) = (\xi_{\varepsilon_{\ddot{G}}}(\ddot{u}), \xi_{\gamma_{\ddot{G}}}(\ddot{u}), \xi_{\xi_{\ddot{G}}}(\ddot{u}))$ .  $\varepsilon_{\ddot{G}}(\ddot{u}) = (\varepsilon_{\ddot{G}}^1(\ddot{u}), \varepsilon_{\ddot{G}}^2(\ddot{u}), \varepsilon_{\ddot{G}}^3(\ddot{u})), \gamma_{\ddot{G}}(\ddot{u}) = (\gamma_{\ddot{G}}^1(\ddot{u}), \gamma_{\ddot{G}}^2(\ddot{u}), \gamma_{\ddot{G}}^3(\ddot{u}))$ , and  $\xi_{\ddot{G}}(\ddot{u}) = (\xi_{\ddot{G}}^1(\ddot{u}), \xi_{\ddot{G}}^2(\ddot{u}), \xi_{\ddot{G}}^3(\ddot{u}))$ , where  $\varepsilon_{\ddot{G}}(\ddot{u}), \gamma_{\ddot{G}}(\ddot{u})$  and  $\xi_{\ddot{G}}(\ddot{u})$  are  $\ddot{G} \rightarrow [0,1]$ . For each  $\ddot{u} \in \ddot{R}: 0 \leq \varepsilon_{\ddot{G}}^1(\ddot{u}) + \gamma_{\ddot{G}}^1(\ddot{u}) + \xi_{\ddot{G}}^1(\ddot{u}) \leq 3$  are stated.

**Definition 2:**

Let  $\ddot{G}_1 = \left\langle \left( \varepsilon_{\varepsilon_{\ddot{G}_1}}(\ddot{u}), \varepsilon_{\gamma_{\ddot{G}_1}}(\ddot{u}), \varepsilon_{\xi_{\ddot{G}_1}}(\ddot{u}) \right), \left( \gamma_{\varepsilon_{\ddot{G}_1}}(\ddot{u}), \gamma_{\gamma_{\ddot{G}_1}}(\ddot{u}), \gamma_{\xi_{\ddot{G}_1}}(\ddot{u}) \right), \left( \xi_{\varepsilon_{\ddot{G}_1}}(\ddot{u}), \xi_{\gamma_{\ddot{G}_1}}(\ddot{u}), \xi_{\xi_{\ddot{G}_1}}(\ddot{u}) \right) \right\rangle$  and  $\ddot{G}_2 = \left\langle \left( \varepsilon_{\varepsilon_{\ddot{G}_2}}(\ddot{u}), \varepsilon_{\gamma_{\ddot{G}_2}}(\ddot{u}), \varepsilon_{\xi_{\ddot{G}_2}}(\ddot{u}) \right), \left( \gamma_{\varepsilon_{\ddot{G}_2}}(\ddot{u}), \gamma_{\gamma_{\ddot{G}_2}}(\ddot{u}), \gamma_{\xi_{\ddot{G}_2}}(\ddot{u}) \right), \left( \xi_{\varepsilon_{\ddot{G}_2}}(\ddot{u}), \xi_{\gamma_{\ddot{G}_2}}(\ddot{u}), \xi_{\xi_{\ddot{G}_2}}(\ddot{u}) \right) \right\rangle$  be T2NNs in the set of real numbers. Some standard operations for T2NNs can be expressed as follows:

$$\ddot{G}_1 \oplus \ddot{G}_2 = \left\langle \left( \varepsilon_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) + \varepsilon_{\varepsilon_{\ddot{G}_2}}(\ddot{u}) - \varepsilon_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) \cdot \varepsilon_{\varepsilon_{\ddot{G}_2}}(\ddot{u}), \varepsilon_{\gamma_{\ddot{G}_1}}(\ddot{u}) + \varepsilon_{\gamma_{\ddot{G}_2}}(\ddot{u}) - \varepsilon_{\gamma_{\ddot{G}_1}}(\ddot{u}) \cdot \varepsilon_{\gamma_{\ddot{G}_2}}(\ddot{u}), \varepsilon_{\xi_{\ddot{G}_1}}(\ddot{u}) + \varepsilon_{\xi_{\ddot{G}_2}}(\ddot{u}) - \varepsilon_{\xi_{\ddot{G}_1}}(\ddot{u}) \cdot \varepsilon_{\xi_{\ddot{G}_2}}(\ddot{u}) \right), \left( \gamma_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) \cdot \gamma_{\varepsilon_{\ddot{G}_2}}(\ddot{u}), \gamma_{\gamma_{\ddot{G}_1}}(\ddot{u}) \cdot \gamma_{\gamma_{\ddot{G}_2}}(\ddot{u}), \gamma_{\xi_{\ddot{G}_1}}(\ddot{u}) \cdot \gamma_{\xi_{\ddot{G}_2}}(\ddot{u}) \right), \left( \xi_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) \cdot \xi_{\varepsilon_{\ddot{G}_2}}(\ddot{u}), \xi_{\gamma_{\ddot{G}_1}}(\ddot{u}) \cdot \xi_{\gamma_{\ddot{G}_2}}(\ddot{u}), \xi_{\xi_{\ddot{G}_1}}(\ddot{u}) \cdot \xi_{\xi_{\ddot{G}_2}}(\ddot{u}) \right) \right\rangle \tag{1}$$

$$\ddot{G}_1 \otimes \ddot{G}_2 = \left\langle \left( \varepsilon_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) \cdot \varepsilon_{\varepsilon_{\ddot{G}_2}}(\ddot{u}), \varepsilon_{\gamma_{\ddot{G}_1}}(\ddot{u}) \cdot \varepsilon_{\gamma_{\ddot{G}_2}}(\ddot{u}), \varepsilon_{\xi_{\ddot{G}_1}}(\ddot{u}) \cdot \varepsilon_{\xi_{\ddot{G}_2}}(\ddot{u}) \right), \left( \gamma_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) + \gamma_{\varepsilon_{\ddot{G}_2}}(\ddot{u}) - \gamma_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) \cdot \gamma_{\varepsilon_{\ddot{G}_2}}(\ddot{u}) \right), \left( \gamma_{\xi_{\ddot{G}_1}}(\ddot{u}) + \gamma_{\xi_{\ddot{G}_2}}(\ddot{u}) - \gamma_{\xi_{\ddot{G}_1}}(\ddot{u}) \cdot \gamma_{\xi_{\ddot{G}_2}}(\ddot{u}) \right), \left( \xi_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) + \xi_{\varepsilon_{\ddot{G}_2}}(\ddot{u}) - \xi_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) \cdot \xi_{\varepsilon_{\ddot{G}_2}}(\ddot{u}) \right), \left( \xi_{\gamma_{\ddot{G}_1}}(\ddot{u}) + \xi_{\gamma_{\ddot{G}_2}}(\ddot{u}) - \xi_{\gamma_{\ddot{G}_1}}(\ddot{u}) \cdot \xi_{\gamma_{\ddot{G}_2}}(\ddot{u}) \right), \left( \xi_{\xi_{\ddot{G}_1}}(\ddot{u}) + \xi_{\xi_{\ddot{G}_2}}(\ddot{u}) - \xi_{\xi_{\ddot{G}_1}}(\ddot{u}) \cdot \xi_{\xi_{\ddot{G}_2}}(\ddot{u}) \right) \right\rangle \tag{2}$$

$$\mathfrak{P}\ddot{G} = \left\langle \left( 1 - \left( 1 - \varepsilon_{\varepsilon_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, 1 - \left( 1 - \varepsilon_{\gamma_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, 1 - \left( 1 - \varepsilon_{\xi_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}} \right), \left( \left( \gamma_{\varepsilon_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, \left( \gamma_{\gamma_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, \left( \gamma_{\xi_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}} \right), \left( \left( \xi_{\varepsilon_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, \left( \xi_{\gamma_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, \left( \xi_{\xi_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}} \right) \right\rangle \tag{3}$$

where  $\mathfrak{P} > 0$ .

$$\ddot{G}^{\mathfrak{P}} = \left\langle \left( \left( \varepsilon_{\varepsilon_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, \left( \varepsilon_{\gamma_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, \left( \varepsilon_{\xi_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}} \right), \left( 1 - \left( 1 - \gamma_{\varepsilon_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, 1 - \left( 1 - \gamma_{\gamma_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, 1 - \left( 1 - \gamma_{\xi_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}} \right), \left( 1 - \left( 1 - \xi_{\varepsilon_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, 1 - \left( 1 - \xi_{\gamma_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}}, 1 - \left( 1 - \xi_{\xi_{\ddot{G}}}(\ddot{u}) \right)^{\mathfrak{P}} \right) \right\rangle \tag{4}$$

Where,  $\mathfrak{P} > 0$ .

**Definition 3:** The score function of  $\ddot{G}_1, S(\ddot{G}_1)$ , is shown by:

$$S(\ddot{G}_1) = \frac{1}{12} \left\langle 8 + \left( \varepsilon_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) + 2 \left( \varepsilon_{\gamma_{\ddot{G}_1}}(\ddot{u}) \right) + \varepsilon_{\xi_{\ddot{G}_1}}(\ddot{u}) \right) - \left( \gamma_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) + 2 \left( \gamma_{\gamma_{\ddot{G}_1}}(\ddot{u}) \right) + \gamma_{\xi_{\ddot{G}_1}}(\ddot{u}) \right) - \left( \xi_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) + 2 \left( \xi_{\gamma_{\ddot{G}_1}}(\ddot{u}) \right) + \xi_{\xi_{\ddot{G}_1}}(\ddot{u}) \right) \right\rangle \tag{5}$$

**Definition 4:** The accuracy function of  $\ddot{G}_1, A(\ddot{G}_1)$ , is shown by:

$$A(\ddot{G}_1) = \frac{1}{4} \left\langle 8 + \left( \varepsilon_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) + 2 \left( \varepsilon_{\gamma_{\ddot{G}_1}}(\ddot{u}) \right) + \varepsilon_{\xi_{\ddot{G}_1}}(\ddot{u}) \right) - \left( \xi_{\varepsilon_{\ddot{G}_1}}(\ddot{u}) + 2 \left( \xi_{\gamma_{\ddot{G}_1}}(\ddot{u}) \right) + \xi_{\xi_{\ddot{G}_1}}(\ddot{u}) \right) \right\rangle \tag{6}$$

**Definition 5:** suppose  $\bar{S}(\ddot{G}_i)$  and  $\bar{A}(\ddot{G}_i)$  refer to the score and accuracy functions for the T2NNs  $\ddot{G}_i (i = 1,2)$ , respectively. The following properties are valid:

If  $\bar{S}(\check{G}_1) > \bar{S}(\check{G}_2)$ , then  $\check{G}_1 > \check{G}_2$ ,

If  $\bar{S}(\check{G}_1) = \bar{S}(\check{G}_2)$  and  $\bar{A}(\check{G}_1) > \bar{A}(\check{G}_2)$ , then  $\check{G}_1 > \check{G}_2$ ,

If  $\bar{S}(\check{G}_1) = \bar{S}(\check{G}_2)$  and  $\bar{A}(\check{G}_1) = \bar{A}(\check{G}_2)$ , then  $\check{G}_1 = \check{G}_2$ .

**Definition 6:** Let  $\check{G}_1 = ((\varepsilon_1, \varepsilon_2, \varepsilon_3), (\gamma_1, \gamma_2, \gamma_3), (\xi_1, \xi_2, \xi_3))$  and  $\check{G}_2 = ((\tilde{T}_1, \tilde{T}_2, \tilde{T}_3), (\tilde{I}_1, \tilde{I}_2, \tilde{I}_3), (\tilde{F}_1, \tilde{F}_2, \tilde{F}_3))$  be T2NNs. The distance measure  $d(\check{G}_1, \check{G}_2)$  between  $\check{G}_1$  and  $\check{G}_2$  can be expressed by:

$$d(\check{G}_1, \check{G}_2) = 1 - \frac{\sum_{i=1}^3 \varepsilon_i \tilde{T}_i + \sum_{i=1}^3 \gamma_i \tilde{I}_i + \sum_{i=1}^3 \xi_i \tilde{F}_i}{(\sum_{i=1}^3 (\varepsilon_i)^2 + \sum_{i=1}^3 (\gamma_i)^2 + \sum_{i=1}^3 (\xi_i)^2) \times (\sum_{i=1}^3 (\tilde{T}_i)^2 + \sum_{i=1}^3 (\tilde{I}_i)^2 + \sum_{i=1}^3 (\tilde{F}_i)^2)} \quad (7)$$

### 3 | Development of Proposed Model

The objective of this section is to exhibit the steps of constructing the appraiser model as shown in Figure 1, based on a set of techniques that have been embraced in the following steps.

#### Step 1. Preparing process

- i). The influenced factors in the appraising process are determined in this step. Hence, the alternatives of DT application that contribute to the appraising process are determining also, the criteria on which DT applications are appraising based on it.
- ii). Decision makers (DMs) representing arbitration members have been determined.
- iii). DMs begin to present their judgments for DT applications based on determined criteria. These judgements are converted from linguistic terms into corresponding values of T2NSs which are listed in Table 1. As a result, Neutrosophic decision matrices have been constructed as formed in Eq. (8).

$$X^K = \begin{matrix} AL_1 \\ \vdots \\ AL_i \\ \vdots \\ AL_y \end{matrix} \begin{pmatrix} CR_1 & CR_j & CR_z \\ x^K_{11} & \cdots & x^K_{1j} & \cdots & x^K_{1z} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x^K_{i1} & \cdots & x^K_{ij} & \cdots & x^K_{iz} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x^K_{y1} & \cdots & x^K_{yj} & \cdots & x^K_{yz} \end{pmatrix} \quad (8)$$

Where  $x^K_{ij}$  Specify the evaluation of an  $i$ th alternative by expert  $K$  based on a specific  $j$ th criterion.

**Table 1.** T2N scale.

Linguistic term	Abbreviation	T2N
Very Bad	VB	$\langle(0.20,0.20,0.10), (0.65, 0.80, 0.85), (0.45,0.80,0.70)\rangle$
Bad	B	$\langle(0.35,0.35,0.10), (0.50,0.75,0.80), (0.50,0.75,0.65)\rangle$
Medium Bad	MB	$\langle(0.50,0.30,0.50)(0.50,0.35,0.45), (0.45,0.30,0.60)\rangle$
Medium	M	$\langle(0.40,0.45,0.50). (0.40,0.45,0.50), (0.35,0.40,0.45)\rangle$
Medium Good	MG	$\langle(0.60,0.45,0.50), (0.20,0.15,0.25), (0.10,0.25,0.15)\rangle$
Good	G	$\langle(0.70,0.75,0.30), (0.15,0.20,0.25), (0.10,0.15,0.20)\rangle$
Very Good	VG	$\langle(0.95,0.90, 0.95), (0.10,0.10,0.05), (0.05,0.05,0.05)\rangle$

#### Step 2. Getting criteria's weights

- i). The score function in Eq. (7) is utilized to convert Neutrosophic decision matrices into crisp values.
- ii). The crisp matrices are aggregated into an aggregated decision matrix based on Eq. (9).

$$Agg_{ij} = \frac{\sum_{K=1}^K X_{ij}}{K} \quad (9)$$

iii). The aggregated decision matrix is normalized based on Eq. (10) to generate the normalized matrix.

$$\text{Nor}_{ij} = \frac{\text{Agg}_{ij}}{\sum_{j=1}^n \text{Agg}_{ij}} \quad (10)$$

Where:  $\sum_{j=1}^n \text{Agg}_{ij}$  represents the sum of each criterion in an aggregated matrix per column.

iv). Normalized matrix computes its entropy by Eq. (11).

$$e_j = -h \sum_{i=1}^n \text{Nor}_{ij} \ln \text{Nor}_{ij} \quad (11)$$

$$\text{Where: } h = \frac{1}{\ln(DT)} \quad (12)$$

DTs refer to the number of alternatives.

v). Compute weight vectors through deploying Eq. (13).

$$W_j = \frac{1 - e_j}{\sum_{j=1}^n (1 - e_j)} \quad (13)$$

### Step 3. Ranking DT applications

The COMplex PROportional ASsessment (COPRAS) used under T2NSs for ranking alternatives of DT applications is as follows:

i). Leveraging the normalized matrix to obtain a weighted decision matrix based on Eq. (14).

$$\tilde{x}_{ij} = \text{Nor}_{ij} \cdot W_j \quad (14)$$

ii). The sum of the weighted decision matrix is calculated according to Eqs. (15) and (16).

$$M_{+i} = \sum_{j=1}^{g_+} \tilde{x}_{ij}; i = 1, \dots, y, \text{ for beneficial criteria} \quad (15)$$

$$M_{-i} = \sum_{j=g_++1}^{g_-} \tilde{x}_{ij}; i = 1, \dots, y, \text{ for non - beneficial criteria} \quad (16)$$

Where  $g_+$  denote the number of positive criteria and  $g_-$  represents the number of negative criteria,  $M_{+i}$  denote the maximizing indexes of  $i$ th criteria and  $M_{-i}$  describes the minimizing indexes of  $i$ th criteria.

iii). Calculate the relative significance value of each alternative through Eq. (17).

$$S_i = M_{+i} + \frac{M_{-min} \sum_{i=1}^y M_{-i}}{M_{-i} \sum_{i=1}^y (M_{-min}/M_{-i})} \quad (17)$$

iv). Calculate the quantitative utility ( $\tilde{U}_j$ ) for the  $j$ th alternative using Eq. (18). The efficiency priorities of all alternatives are compared to determine an alternative's degree of utility, which determines its rank.

$$\tilde{U}_j = \left[ \frac{S_i}{S_{max}} \right] \times 100 \quad (18)$$

where  $S_{max}$  denote the maximum relative significance value. The highest final value has the highest rank.

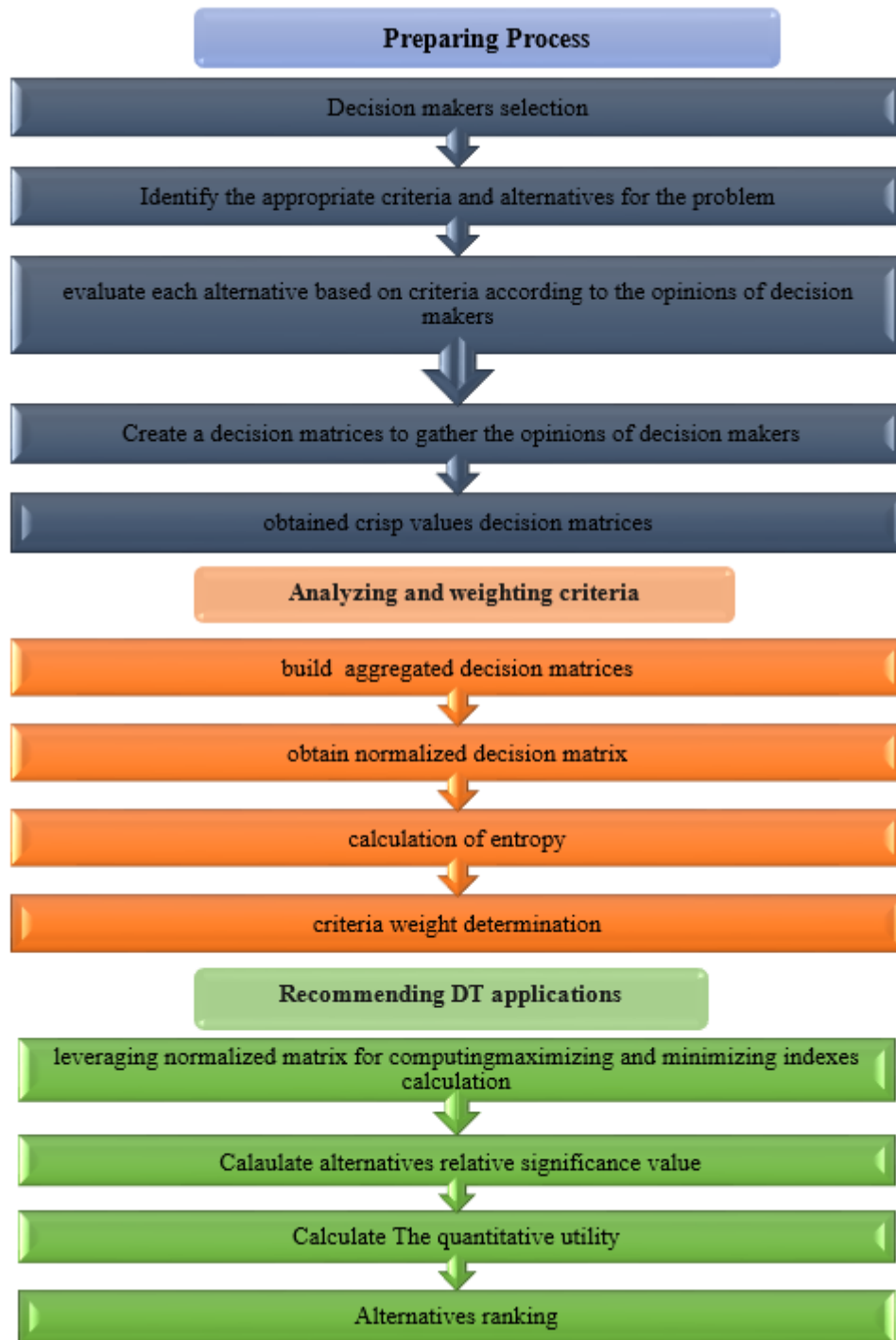


Figure 1. Steps of appraising process.

## 4 | Case Study

We applied our appraiser model in a real case study to verify the robustness of the model. Automotive company manufacturing, a leading producer of automotive components located in Cairo has embarked on a lean Six Sigma transformation journey to improve operational efficiency and product quality with the support of DT applications. As part of this initiative, the company is embracing our notion in its operation and production to simulate and optimize its manufacturing processes. Hence, the important information for starting the appraising process must be aggregated. Firstly, the model is applied to five DT applications. Secondly, the appraising of DT applications is conducted based on ten criteria that have been exhibited in Figure 2. After that, the steps of implementing the constructed model in the automotive company.

- i). Alternatives based on criteria have been appraised by four DMs through the scale mentioned in Table 1.
- ii). The constructed Neutrosophic matrices convert into crisp matrices and are integrated into an aggregated matrix based on Eq. (9) as in Table 2.
- iii). Eq. (10) utilized in the aggregated matrix to generate the normalized matrix as in Table 3.
- iv). Eqs. (11) and (12) have been utilized in the normalized matrix for computing entropy as in Table 4.
- v). Final criteria weights are generated based on Eq. (13) and represented in Figure 3.

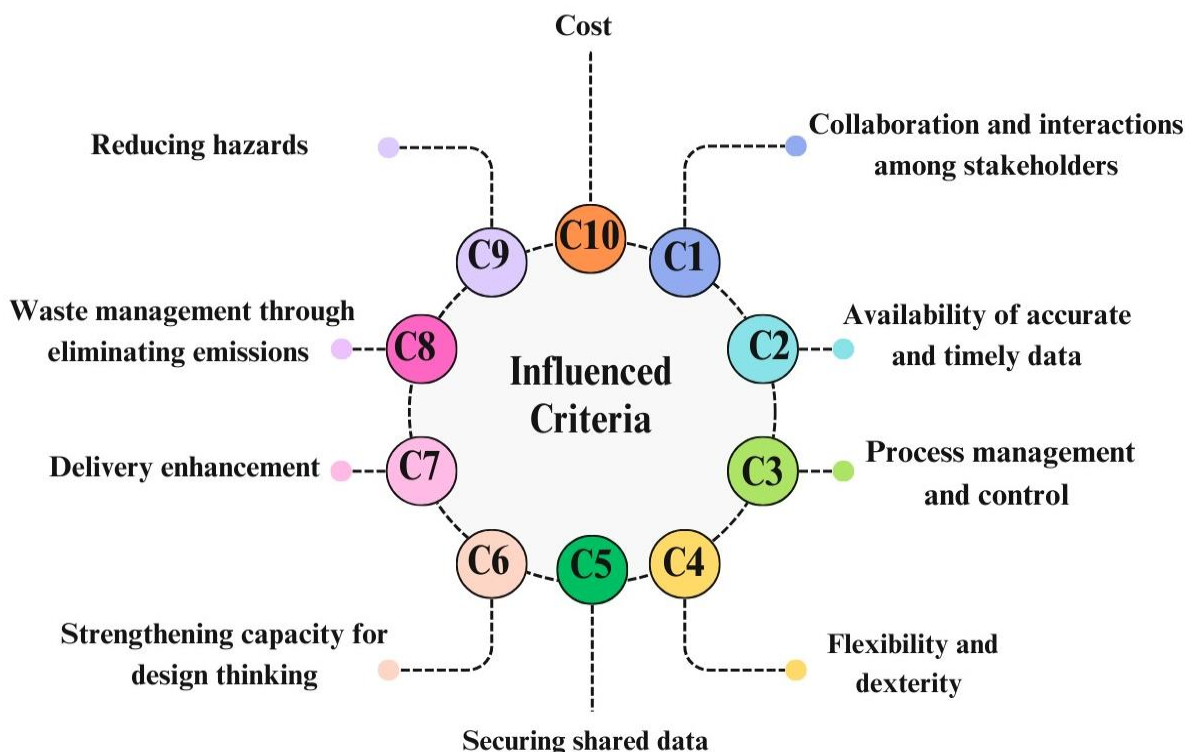


Figure 2. Influenced DLSS criteria.

Table 2. An aggregated matrix.

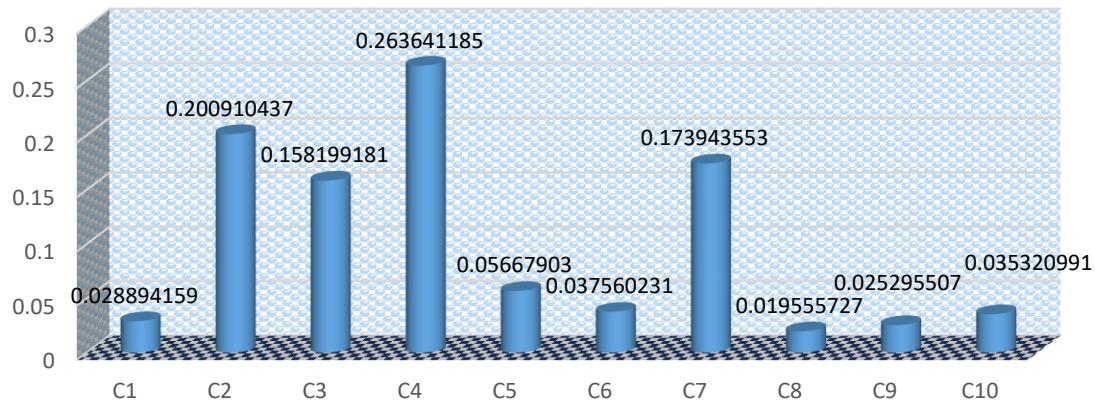
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	0.572	0.327	0.373	0.557	0.427	0.429	0.516	0.453	0.511	0.499
A2	0.529	0.675	0.383	0.733	0.442	0.475	0.525	0.517	0.575	0.533
A3	0.628	0.403	0.590	0.273	0.545	0.459	0.273	0.475	0.545	0.381
A4	0.690	0.688	0.700	0.758	0.677	0.616	0.309	0.429	0.633	0.572
A5	0.498	0.345	0.347	0.355	0.511	0.442	0.273	0.557	0.457	0.511

Table 3. Normalized decision matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	0.1961	0.1342	0.1559	0.2082	0.1641	0.1773	0.2720	0.1864	0.1879	0.1998
A2	0.1814	0.2769	0.1602	0.2739	0.1697	0.1962	0.2769	0.2125	0.2113	0.2136
A3	0.2154	0.1654	0.2464	0.1019	0.2094	0.1898	0.1440	0.1954	0.2002	0.1527
A4	0.2364	0.2821	0.2926	0.2833	0.2602	0.2543	0.1632	0.1765	0.2327	0.2290
A5	0.1707	0.1415	0.1450	0.1327	0.1966	0.1824	0.1440	0.2292	0.1680	0.2048

**Table 4.** Entropy based on normalized decision matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	-0.319	-0.269518	-0.289708	-0.326703	-0.296599	-0.30669	-0.354125	-0.313109	-0.314144	-0.321786
A2	-0.30968	-0.355574	-0.293387	-0.354707	-0.301029	-0.319544	-0.355575	-0.329128	-0.32843	-0.329724
A3	-0.33067	-0.297607	-0.345160	-0.232774	-0.327380	-0.315379	-0.27902	-0.319014	-0.321981	-0.286958
A4	-0.34096	-0.356983	-0.359580	-0.35730	-0.350311	-0.348199	-0.295835	-0.30614	-0.339269	-0.337571
A5	-0.30178	-0.27665204	-0.279972	-0.267994	-0.319759	-0.310394	-0.27902	-0.337658	-0.299682	-0.324779
e	0.028894	0.200910	0.158199	0.263641	0.05668	0.037560	0.173944	0.019556	0.025296	0.035321

**Figure 3.** Final criteria weights.

- vi). In this step, we start with the normalized decision matrix shown in Table 3. and the final criteria weights presented in Figure 3. Then, we utilize Eq. (14) to obtain the weighted normalized decision matrix which is shown in Table 5.
- vii). The sum of the weighted decision matrix is calculated according to Eqs. (15) and (16).
- viii). The relative significance value of each alternative  $S_i$  is calculated through Eq. (17) and shown in Table 6.
- ix). The final ranking for DT applications is obtained in Figure 4 based on Eq. (18). Where A4 is the optimal and A5 is the worst.

**Table 5.** Weighted normalized decision matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	0.005	0.027	0.025	0.056	0.009	0.006	0.048	0.003	0.004	0.007
A2	0.005	0.057	0.026	0.074	0.009	0.007	0.049	0.004	0.005	0.007
A3	0.006	0.034	0.039	0.027	0.012	0.007	0.025	0.003	0.005	0.005
A4	0.006	0.058	0.047	0.076	0.014	0.009	0.029	0.003	0.005	0.008
A5	0.005	0.029	0.023	0.036	0.011	0.006	0.025	0.004	0.004	0.007

**Table 6.** Relative significance.

	$M_{+i}$	$M_{-i}$	$\frac{M_{-min}}{M_{-i}}$	$S_i$
A1	0.175	0.015	0.923695375	0.191983925
A2	0.224	0.017	0.837640685	0.238610332
A3	0.149	0.014	1	0.167260003
A4	0.237	0.017	0.819206985	0.25182473
A5	0.134	0.016	0.894117682	0.15032101



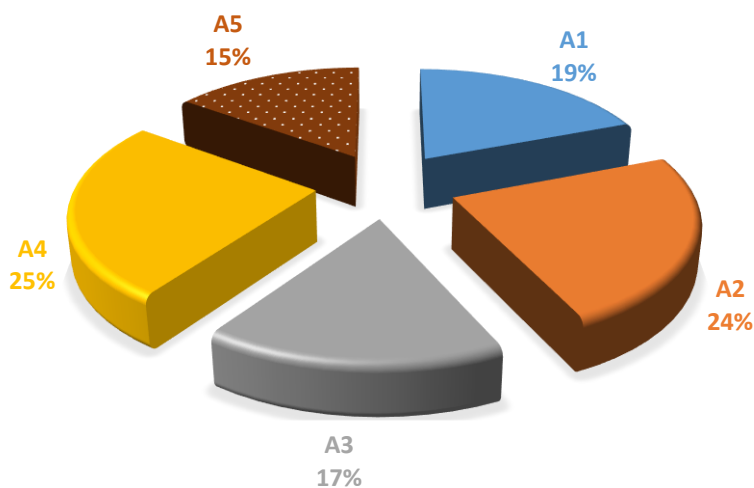


Figure 4. DT applications ranking.

### 5 | Comparative Analysis

Herein, we compared our appraiser model with other models which were constructed based on various MCDM techniques. The findings of the comparison are exhibited in Figure 5. According to Figure 5, we observed that our model and other compared models agree that DT4 (A4) is the optimal followed by DT2 (A2) otherwise DT5 (A5) is the worst.

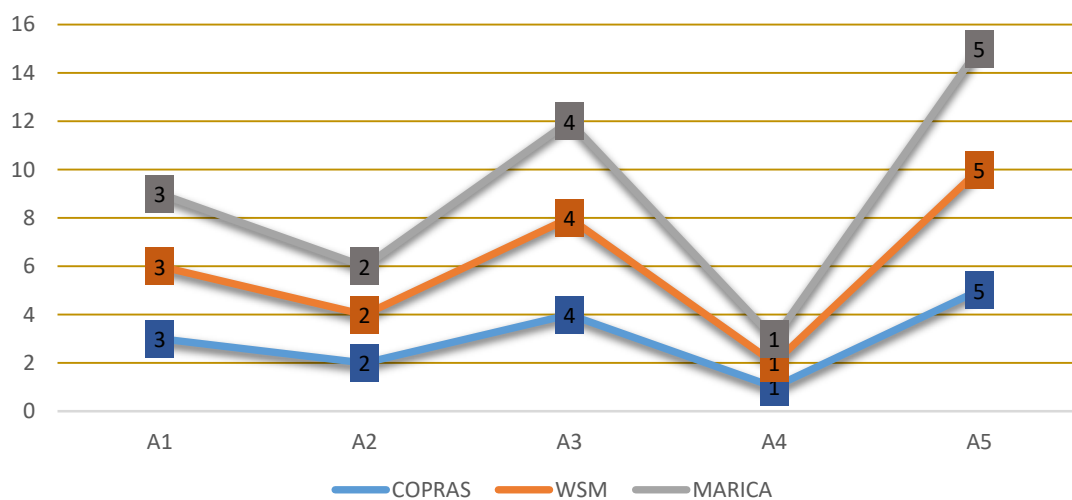


Figure 5. Ranking alternatives based on comparative analysis for various techniques.

### 6 | Conclusion

This study discussed the pressures that face any manufacturing enterprise. The advantages of LSS for any manufacturing enterprises that are seeking to enhance its performance. For instance, the growing consumer expectations for high-quality, environmentally friendly products, reduced operating expenses, and increased sustainability. All of those compelled enterprises to reorganize their business plans. One of the most effective methodologies for achieving the mentioned enterprise’s demands is LSS which is considered a well-known methodology for improving operational performance by minimizing process volatility and cutting waste. Also, known as Define-Measure-Analyze-Improve-Control (DMAIC). Also, this study embraced recently presented views which demonstrated that the adoption of Ind 4.0 has acquired significant traction due to its well-defined

idea, sincere enthusiasm for its extensive industrial applications, and capacity for simultaneous infrastructure development in related manufacturing groups.

Hence, we discussed the gained benefits from merging the contemporary technologies of Ind 4.0 with LSS toward achieving a competitive advantage for any enterprise manufacturing. This merging is DT with LSS and produces DLSS to provide real-time monitoring, research, decision-making improvement, and performance optimization in a virtual environment.

As a result of the importance of DLSS in manufacturing, implementing the appropriate and optimal DT application is an important issue. Therefore, this study attempted to solve this issue by constructing an appraiser model for comparing the set of DT applications and recommending the optimal DT based on entropy and COPRAS under T2NSs. After that, we applied our model in the real automotive enterprise where five alternatives of DT applications were contributed to the appraising process as nominees. The findings of implementing our model indicated that DT4 is the optimal, otherwise, A5 is the worst as in Fig 4. As well, we compared our model with other models and findings indicated that A4 is optimal and A5 is the worst.

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## Author Contributions

All authors contributed equally to this work.

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## Data Availability

Not applicable.

## Conflicts of Interest

The authors declare no conflict of interest.

## References

- [1] Azamfirei, V., Psarommatis, F., & Lagrosen, Y. (2023). Application of automation for in-line quality inspection, a zero-defect manufacturing approach. *Journal of manufacturing systems*, 67, 1–22.
- [2] Ferreira, L. M. D. F., Moreira, A. C., & Silva, P. (2023). Lean implementation in product development processes: a framework proposal. *Production planning & control*, 1–17.
- [3] Condé, G. C. P., Oprime, P. C., Pimenta, M. L., Sordan, J. E., & Bueno, C. R. (2023). Defect reduction using DMAIC and Lean Six Sigma: a case study in a manufacturing car parts supplier. *International journal of quality & reliability management*, 40(9), 2184–2204.
- [4] Hariyani, D., & Mishra, S. (2023). A descriptive statistical analysis of enablers for integrated sustainable-green-lean-six sigma-agile manufacturing system (ISGLSAMS) in Indian manufacturing industries. *Benchmarking: an international journal*, (ahead-of-print).
- [5] Sony, M., Naik, S., & Antony, J. (2020). Lean six sigma and social performance: a review and synthesis of current evidence. *Q Manag J* 27 (1).
- [6] Vicente, I., Godina, R., & Gabriel, A. T. (2024). Applications and future perspectives of integrating Lean Six Sigma and Ergonomics. *Safety science*, 172, 106418.
- [7] Salah, S., Rahim, A., Salah, S., & Rahim, A. (2019). The integration of six sigma and lean. An integrated company-wide management system: combining lean six sigma with process improvement, 49–93.

- [8] Rathi, R., Vakharia, A., & Shadab, M. (2022). Lean six sigma in the healthcare sector: A systematic literature review. *Materials today: proceedings*, 50, 773–781.
- [9] Hill, J., Thomas, A. J., Mason-Jones, R. K., & El-Kateb, S. (2018). The implementation of a Lean Six Sigma framework to enhance operational performance in an MRO facility. *Production & manufacturing research*, 6(1), 26–48.
- [10] Antony, J., Lizarelli, F. L., & Fernandes, M. M. (2020). A global study into the reasons for lean six sigma project failures: Key findings and directions for further research. *IEEE transactions on engineering management*, 69(5), 2399–2414.
- [11] Mishra, R., Singh, R. K., & Papadopoulos, T. (2022). Linking digital orientation and data-driven innovations: a SAP–LAP linkage framework and research propositions. *IEEE transactions on engineering management*.
- [12] Kamble, S. S., Gunasekaran, A., & Gawankar, S. A. (2018). Sustainable Industry 4.0 framework: A systematic literature review identifying the current trends and future perspectives. *Process safety and environmental protection*, 117, 408–425.
- [13] Saporiti, N., Cannas, V. G., Pozzi, R., & Rossi, T. (2023). Challenges and countermeasures for digital twin implementation in manufacturing plants: A Delphi study. *International journal of production economics*, 261, 108888.
- [14] Maheshwari, P., Kamble, S., Belhadi, A., Mani, V., & Pundir, A. (2023). Digital twin implementation for performance improvement in process industries-A case study of food processing company. *International journal of production research*, 61(23), 8343–8365.
- [15] Kritzinger, W., Karner, M., Traar, G., Henjes, J., & Sihm, W. (2018). Digital Twin in manufacturing: A categorical literature review and classification. *Ifac-papersonline*, 51(11), 1016–1022.
- [16] Wu, Y., Zhang, K., & Zhang, Y. (2021). Digital twin networks: A survey. *IEEE internet of things journal*, 8(18), 13789–13804.
- [17] Thelen, A., Zhang, X., Fink, O., Lu, Y., Ghosh, S., Youn, B. D., ... Hu, Z. (2023). A comprehensive review of digital twin—part 2: roles of uncertainty quantification and optimization, a battery digital twin, and perspectives. *Structural and multidisciplinary optimization*, 66(1), 1.
- [18] Sharma, A., Kosasih, E., Zhang, J., Brintrup, A., & Calinescu, A. (2022). Digital twins: State of the art theory and practice, challenges, and open research questions. *Journal of industrial information integration*, 30, 100383.
- [19] Psarommatis, F. (2021). A generic methodology and a digital twin for zero defect manufacturing (ZDM) performance mapping towards design for ZDM. *Journal of manufacturing systems*, 59, 507–521.
- [20] Lugaresi, G., & Matta, A. (2023). Automated digital twin generation of manufacturing systems with complex material flows: graph model completion. *Computers in industry*, 151, 103977.
- [21] Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied soft computing*, 77, 438–452. DOI:<https://doi.org/10.1016/j.asoc.2019.01.035>

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