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Rough Fermatean Neutrosophic Sets and its Applications in Medical Diagnosis

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Abstract: This paper introduces the concept of rough fermatean neutrosophic sets and investigates their properties. Additionally, a cosine similarity measure between these sets is proposed. By applying this measure to a medical diagnosis example, the paper illustrates how the method can be used in practical situations, highlighting its effectiveness in complex decision-making scenarios. This innovation holds promise for improving decision-making processes, especially in critical areas like medical diagnosis, where making accurate assessments amidst uncertainty is crucial.

Keywords: Neutrosophic Sets; Rough Neutrosophic Sets; Fermatean Sets; Rough Fermatean Sets; Rough Fermatean Neutrosophic Sets; Cosine Similarity Measure.

1. Introduction

The concept of neutrosophic sets [14] originated from the new branch of philosophy called neutrosophy, which means knowledge of neutral thought and this neutral represents the main distinction between fuzzy and intuitionistic fuzzy logic and set. It is a logic in which each proposition is estimated to have a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F) respectively, and which lies between [0, 1]. The neutrosophic set generalizes the classical set or crisp set proposed by Cantor, the fuzzy set proposed by Zadeh [20], the interval-valued fuzzy set proposed independently by Zadeh [21], Grattan-Guiness [6], the intuitionistic fuzzy set proposed by Atanassov [1], and interval-valued intuitionistic fuzzy set proposed by Atanassov and Gargov [2].

Authors explored neutrosophic sets & SVNS across fields like decision-making, image processing, medical diagnosis, and more[4,5,7-11,13, 15-19]. Senapati and Yager[22] discussed a numerical case to validate the rationality of the concept of fermatean fuzzy sets. It is also important to mention that the class of this type of fuzzy set has more ability to capture the uncertainties as compared to intuitionistic fuzzy sets and Pythagorean fuzzy sets. Fermatean neutrosophic sets are studied by C. Antony Crispin Sweety et al. [3].

Rough set theory, introduced by Pawlak [12] indeed offers a valuable framework for handling imprecise and uncertain information, which is common in many real-world scenarios. It extends the traditional crisp set theory to accommodate this kind of data, making it particularly useful in the study of intelligent systems where information may be incomplete or ambiguous.

The recent development of rough neutrosophic sets adds another layer of sophistication to this field. The fusion of rough set theory with neutrosophic sets in the form of rough neutrosophic sets offers a powerful mathematical tool for handling incomplete information. In rough neutrosophic sets, the rating of alternatives is expressed using upper and lower approximation operators, capturing the uncertainty inherent in the data. Moreover, the characterization of sets by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree allows for a nuanced representation of incomplete information, making rough neutrosophic sets a versatile tool for decision-making in uncertain environments.

The paper consists of four sections. The first two sections serve as the introduction and provide preliminary information. In the third section, we define rough fermatean neutrosophic sets and establish several operations associated with them. Section four presents the introduction of cosine similarity measures for rough fermatean neutrosophic sets. Lastly, a numerical example is solved to demonstrate the practicality, relevance, and effectiveness of the proposed methodologies.

2. Preliminaries

This section consists of the basic results of this paper, refer to [1-22].

3. Rough Fermatean Neutrosophic Sets

In this section, we have to introduce the Rough Fermatean Neutrosophic (RFN)set.

Definition 3.1. Let K be the universal set and Θ be an equivalence relation on K. Let F be the fermatean neutrosophic set of K. The lower and upper approximations of F in the approximation space (K, Θ) are defined as follows:

$$\Theta_{\blacksquare}(F) = \langle \big(g, \Theta_{\blacksquare}(F_t)(g), \Theta_{\blacksquare}(F_i)(g), \Theta_{\blacksquare}(F_f)(g) \big), g \in K \rangle$$
$$\Theta^{\blacksquare}(F) = \langle \big(g, \Theta^{\blacksquare}(F_t)(g), \Theta^{\blacksquare}(F_i)(g), \Theta^{\blacksquare}(F_f)(g) \big), g \in K \rangle$$

Where $\Theta_{\blacksquare}(F_t)(g) = \wedge_{s \in [g]_\Theta} F_t(s)$
$$\Theta_{\blacksquare}(F_i)(g) = \vee_{s \in [g]_\Theta} F_i(s)$$
$$\Theta_{\blacksquare}(F_f)(g) = \vee_{s \in [g]_\Theta} F_f(s)$$

Also $\Theta^{\blacksquare}(F_t)(g) = \vee_{s \in [g]_\Theta} F_t(s)$
$$\Theta^{\blacksquare}(F_i)(g) = \wedge_{s \in [g]_\Theta} F_i(s)$$
$$\Theta^{\blacksquare}(F_f)(g) = \wedge_{s \in [g]_\Theta} F_f(s)$$

Where $0 \le (\Theta_{\blacksquare}(F_t)(g))^3 + (\Theta_{\blacksquare}(F_i)(g))^3 + (\Theta_{\blacksquare}(F_f)(g))^3 \le 2$ and
$$0 \le (\Theta^{\blacksquare}(F_t)(g))^3 + (\Theta^{\blacksquare}(F_i)(g))^3 + (\Theta^{\blacksquare}(F_f)(g))^3 \le 2$$

Example 3.2. Let $U = \{a, b, c, d\}$ be the universal set. Let F_1 be the FN set defined by $\left\{\frac{a}{0.6,0.1,0.7}, \frac{b}{0.5,0.8,0.4}, \frac{c}{0.7,0.5,0.3}, \frac{d}{0.4,1,0.8}\right\}$. Let Θ be a congruence relation on P such that congruence classes are the subsets given by $\{\{a\},\{b,c,d\}\}$. Then the lower and upper approximations of F_1 are given by,

$$\Theta_{\blacksquare}(F_1)(x) = \left\{\frac{a}{0.6,0.1,0.7}, \frac{b}{0.4,1,0.8}, \frac{c}{0.4,1,0.8}, \frac{d}{0.4,1,0.8}\right\}, for\ all\ x \in U$$

and

$$\Theta^{\blacksquare}(F_1)(x) = \left\{\frac{a}{0.6,0.1,0.7}, \frac{b}{0.7,0.5,0.4}, \frac{c}{0.7,0.5,0.4}, \frac{d}{0.7,0.5,0.4}\right\}, for\ all\ x \in U$$

Example 3.3. Let $U = \{a, b, c, d\}$ be the universal set. Let F_2 be the fermatean Neutrosophic set defined by,

$$\left\{\frac{a}{0.3,0.7,0.5}, \frac{b}{0.4,0.6,0.7}, \frac{c}{0.8,0.3,0.7}, \frac{d}{0.7,0.2,0.4}\right\}$$

Let Θ be congruence relations on P such that congruence classes are the subsets given by $\{\{a,b,c\},\{d\}\}$. Then the lower and upper approximations of F_2 are given by,

$$\Theta_{\blacksquare}(F_2)(x) = \left\{\frac{a}{0.3,0.7,0.7}, \frac{b}{0.3,0.7,0.7}, \frac{c}{0.3,0.7,0.7}, \frac{d}{0.7,0.2,0.4}\right\}, for\ all\ x \in U$$

and

$$\Theta^{\blacksquare}(F_2)(x) = \left\{\frac{a}{0.8,0.3,0.5}, \frac{b}{0.8,0.3,0.5}, \frac{c}{0.8,0.3,0.5}, \frac{d}{0.7,0.2,0.4}\right\}, for\ all\ x \in U$$

Definition 3.4. Let F be the RFN fuzzy set. Then the complement of F, F^c is defined as follows:

$$\Theta_{\blacksquare}(F^c)(g) = \{\Theta_{\blacksquare}(F_f)(g), 1 - \Theta_{\blacksquare}(F_i)(g), \Theta_{\blacksquare}(F_t)(g)\}$$

And
$$\Theta^{\blacksquare}(F^c)(g) = \{\Theta^{\blacksquare}(F_f)(g), 1 - \Theta^{\blacksquare}(F_i)(g), \Theta^{\blacksquare}(F_t)(g)\}$$

For all $g \in F$.

Definition 3.6. Let $\Theta(F_1)$ and $\Theta(F_2)$ be two RFN fuzzy sets. Then $\Theta(F_1) \subseteq \Theta(F_2)$ if and only if the following conditions hold:

$$\Theta_{\blacksquare}(F_{1t})(g) \leq \Theta_{\blacksquare}(F_{2t})(g)$$
$$\Theta_{\blacksquare}(F_{1i})(g) \geq \Theta_{\blacksquare}(F_{2i})(g)$$
$$\Theta_{\blacksquare}(F_{1f})(g) \geq \Theta_{\blacksquare}(F_{2f})(g)$$

and

$$\Theta^{\blacksquare}(F_{1t})(g) \leq \Theta^{\blacksquare}(F_{2t})(g)$$
$$\Theta^{\blacksquare}(F_{1i})(g) \geq \Theta^{\blacksquare}(F_{2i})(g)$$
$$\Theta^{\blacksquare}(F_{1f})(g) \geq \Theta^{\blacksquare}(F_{2f})(g)$$

Definition 3.7. Let $\Theta(F_1)$ and $\Theta(F_2)$ be two RFN fuzzy sets. Then $\Theta(F_1) \cup \Theta(F_2)$ is defined as follows.

$$(\Theta_{\blacksquare}(F_{1t}) \cup \Theta_{\blacksquare}(F_{2t}))(h) = max\{\Theta_{\blacksquare}(F_{1t}), \Theta_{\blacksquare}(F_{2t})\}$$
$$(\Theta_{\blacksquare}(F_{1i}) \cup \Theta_{\blacksquare}(F_{2i}))(h) = min\{\Theta_{\blacksquare}(F_{1i}), \Theta_{\blacksquare}(F_{2i})\}$$
$$(\Theta_{\blacksquare}(F_{1f}) \cup \Theta_{\blacksquare}(F_{2f}))(h) = min\{\Theta_{\blacksquare}(F_{1f}), \Theta_{\blacksquare}(F_{2f})\}$$

and

$$(\Theta^{\blacksquare}(F_{1t}) \cup \Theta^{\blacksquare}(F_{2t}))(h) = max\{\Theta^{\blacksquare}(F_{1t}), \Theta^{\blacksquare}(F_{2t})\}$$
$$(\Theta^{\blacksquare}(F_{1i}) \cup \Theta^{\blacksquare}(F_{2i}))(h) = min\{\Theta^{\blacksquare}(F_{1i}), \Theta^{\blacksquare}(F_{2i})\}$$
$$(\Theta^{\blacksquare}(F_{1f}) \cup \Theta^{\blacksquare}(F_{2f}))(h) = min\{\Theta^{\blacksquare}(F_{1f}), \Theta^{\blacksquare}(F_{2f})\}$$

Example 3.7 Consider the RFN sets in Example 2.2 and 2.3. Then the union is given by,

$$(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2))(a) = (0.6,0.7,0.7)$$
$$(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2))(b) = (0.4,0.7,0.7)$$
$$(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2))(c) = (0.4,0.7,0.7)$$
$$(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2))(d) = (0.7,0.2,0.4)$$

and

$$(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2))(a) = (0.8,0.3,0.5)$$
$$(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2))(b) = (0.8,0.3,0.4)$$
$$(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2))(c) = (0.8,0.3,0.4)$$
$$(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2))(d) = (0.7,0.2,0.4)$$

Definition 3.8. Let $\Theta(F_1)$ and $\Theta(F_2)$ be two RFN fuzzy sets. Then $\Theta(F_1) \cap \Theta(F_2)$ is defined as follows.

$$(\Theta_{\blacksquare}(F_{1t}) \cap \Theta_{\blacksquare}(F_{2t}))(h) = min\{\Theta_{\blacksquare}(F_{1t}), \Theta_{\blacksquare}(F_{2t})\}$$
$$(\Theta_{\blacksquare}(F_{1i}) \cap \Theta_{\blacksquare}(F_{2i}))(h) = max\{\Theta_{\blacksquare}(F_{1i}), \Theta_{\blacksquare}(F_{2i})\}$$
$$(\Theta_{\blacksquare}(F_{1f}) \cap \Theta_{\blacksquare}(F_{2f}))(h) = max\{\Theta_{\blacksquare}(F_{1f}), \Theta_{\blacksquare}(F_{2f})\}$$

and

$$(\Theta^{\blacksquare}(F_{1t}) \cap \Theta^{\blacksquare}(F_{2t}))(h) = min\{\Theta^{\blacksquare}(F_{1t}), \Theta^{\blacksquare}(F_{2t})\}$$
$$(\Theta^{\blacksquare}(F_{1i}) \cap \Theta^{\blacksquare}(F_{2i}))(h) = max\{\Theta^{\blacksquare}(F_{1i}), \Theta^{\blacksquare}(F_{2i})\}$$
$$(\Theta^{\blacksquare}(F_{1f}) \cap \Theta^{\blacksquare}(F_{2f}))(h) = max\{\Theta^{\blacksquare}(F_{1f}), \Theta^{\blacksquare}(F_{2f})\}$$

Example 3.9. Consider the RFN set in Example 2.2 and 2.3. Then the intersection is given by,

$$(\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2))(a) = (0.3,1,0.7)$$
$$(\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2))(b) = (0.3,1,0.8)$$
$$(\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2))(c) = (0.3,1,0.8)$$
$$(\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2))(d) = (0.4,1,0.8)$$

and

$$(\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2))(a) = (0.6,1,0.7)$$
$$(\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2))(b) = (0.7,0.5,0.5)$$
$$(\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2))(c) = (0.7,0.5,0.5)$$
$$(\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2))(d) = (0.7,0.5,0.4)$$

Definition 3.10. If F_1 and F_2 be two RFN sets. Then we define the following

1. $\Theta(F_1) = \Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1) = \Theta_{\blacksquare}(F_2)$ and $\Theta^{\blacksquare}(F_1) = \Theta^{\blacksquare}(F_2)$.
2. $\Theta(F_1) \subseteq \Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1) \subseteq \Theta_{\blacksquare}(F_2)$ and $\Theta^{\blacksquare}(F_1) \subseteq \Theta^{\blacksquare}(F_2)$.
3. $\Theta(F_1) \cup \Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2)$and $\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2)$.
4. $\Theta(F_1) \cap \Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2)$ and $\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2)$.
5. $\Theta(F_1) + \Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1) + \Theta_{\blacksquare}(F_2)$ and $\Theta^{\blacksquare}(F_1) + \Theta^{\blacksquare}(F_2)$.
6. $\Theta(F_1)^{\circ}\Theta(F_2)$if and only if $\Theta_{\blacksquare}(F_1)^{\circ}\Theta_{\blacksquare}(F_2)$ and $\Theta^{\blacksquare}(F_1)^{\circ}\Theta^{\blacksquare}(F_2)$.

Proposition 3.11. If $\Theta(F_2), \Theta(F_2)$ and $\Theta(F_3)$are RFN sets. Then the following are straightforward from the definitions.

1. $\sim\Theta(F_1)\big(\sim\Theta(F_1)\big) = \Theta(F_1)$
2. $\Theta(F_1) \cup \Theta(F_2) = \Theta(F_2) \cup \Theta(F_1), \Theta(F_1) \cap \Theta(F_2) = \Theta(F_2) \cap \Theta(F_1)$
3. $\big(\Theta(F_1) \cup \Theta(F_2)\big) \cup \Theta(F_3) = \Theta(F_1) \cup \big(\Theta(F_2) \cup \Theta(F_3)\big)$ and $\big(\Theta(F_1) \cap \Theta(F_2)\big) \cap \Theta(F_3) = \Theta(F_1) \cap \big(\Theta(F_2) \cup \Theta(F_3)\big)$
4. $\big(\Theta(F_1) \cup \Theta(F_2)\big) \cap \Theta(F_3) = \big(\Theta(F_1) \cup \Theta(F_3)\big) \cap \big(\Theta(F_2) \cup \Theta(F_3)\big)$ and $\big(\Theta(F_1) \cup \Theta(F_2)\big) \cap \Theta(F_3) = \big(\Theta(F_1) \cup \Theta(F_3)\big) \cap \big(\Theta(F_2) \cup \Theta(F_1)\big)$

Proposition 3.12. If$\Theta(F_1) and \Theta(F_2)$are RFN sets. Then the following are satisfied.

1. $\sim\big(\Theta(F_1) \cup \Theta(F_2)\big) = \big(\sim\Theta(F_1)\big) \cap \big(\sim\Theta(F_2)\big)$
2. $\sim\big(\Theta(F_1) \cap \Theta(F_2)\big) = \big(\sim\Theta(F_1)\big) \cup \big(\sim\Theta(F_2)\big)$

Proof:

(i) $\sim\big(\Theta(F_1) \cup \Theta(F_2)\big) = \sim\big\{\big(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2)\big), \big(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2)\big)\big\}$

$$= \big\{\sim\big(\Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2)\big), \sim\big(\Theta^{\blacksquare}(F_1) \cup \Theta^{\blacksquare}(F_2)\big)\big\}$$

$$= \big\{\sim\big(\Theta_{\blacksquare}(F_1) \cap \Theta_{\blacksquare}(F_2)\big), \sim\big(\Theta^{\blacksquare}(F_1) \cap \Theta^{\blacksquare}(F_2)\big)\big\}$$

$$= \big(\sim\Theta(F_1)\big) \cap \big(\sim\Theta(F_2)\big)$$

(ii) Similarly, we prove this part.

Proposition 3.13. Let F_1 and F_2be two RFN sets. Then

(i) $F_1 \cup F_2 \supseteq \mathrm{F}_1 \cup \mathrm{F}_2$

(ii) $F_1 \cap F_2 \subseteq \mathrm{F}_1 \cap \mathrm{F}_2$

Proof:

$$(\Theta_{\blacksquare}(F_{1t}) \cup \Theta_{\blacksquare}(F_{2t}))(h) = max\{\Theta_{\blacksquare}(F_{1t}), \Theta_{\blacksquare}(F_{2t})\}$$
$$(\Theta_{\blacksquare}(F_{1i}) \cup \Theta_{\blacksquare}(F_{2i}))(h) = min\{\Theta_{\blacksquare}(F_{1i}), \Theta_{\blacksquare}(F_{2i})\}$$
$$(\Theta_{\blacksquare}(F_{1f}) \cup \Theta_{\blacksquare}(F_{2f}))(h) = min\{\Theta_{\blacksquare}(F_{1f}), \Theta_{\blacksquare}(F_{2f})\}$$

Consider$\Theta_{\blacksquare}(F_{1t} \cup (\Theta_{\blacksquare}(F_{1t}) \cup \Theta_{\blacksquare}(F_{2t}))(h) = max\{\Theta_{\blacksquare}(F_{1t}), \Theta_{\blacksquare}(F_{2t})\}$

$$(\Theta_{\blacksquare}(F_{1i}) \cup \Theta_{\blacksquare}(F_{2i}))(h) = min\{\Theta_{\blacksquare}(F_{1i}), \Theta_{\blacksquare}(F_{2i})\}$$
$$(\Theta_{\blacksquare}(F_{1f}) \cup \Theta_{\blacksquare}(F_{2f}))(h) = min\{\Theta_{\blacksquare}(F_{1f}), \Theta_{\blacksquare}(F_{2f})\}$$
$$(F_{1t} \cup F_{2t})(g) = \wedge_{s\in[g]_\Theta} F_{1t} \cup F_{2t}(s)$$
$$= \wedge_{s\in[g]_\Theta} (max\{F_{1t}, F_{2t}\})$$
$$\geq max\{\wedge_{s\in[g]_\Theta} F_{1t}(s), \wedge_{s\in[g]_\Theta} F_{2t}(s)\}$$
$$\geq max\{\wedge_{s\in[g]_\Theta} F_{1t}(s), \quad \wedge_{s\in[g]_\Theta} F_{2t}(s)\}$$

$$= max\{\Theta_{\blacksquare}(F_{1t})(g), \Theta_{\blacksquare}(F_{2t})(g)\}$$
$$= \Theta_{\blacksquare}(F_{1t})(g) \cup \Theta_{\blacksquare}(F_{2t})(g)$$

Similarly,

$$\Theta_{\blacksquare}(F_{1i} \cup F_{2i})(g) \leq \Theta_{\blacksquare}(F_{1i})(g) \cup \Theta_{\blacksquare}(F_{2i})(g)$$
$$\Theta_{\blacksquare}(F_{1i} \cup F_{2i})(g) \leq \Theta_{\blacksquare}(F_{1i})(g) \cup \Theta_{\blacksquare}(F_{2i})(g)$$

Thus, $\Theta_{\blacksquare}(F_1 \cup F_2) \supseteq \Theta_{\blacksquare}(F_1) \cup \Theta_{\blacksquare}(F_2)$

In the same way, we prove for upper approximation.

Hence, $F_1 \cup F_2 \supseteq \mathrm{F}_1 \cup \mathrm{F}_2$.

(iii) The proof is similar to the proof (i).

4. Application of Rough Fermatean Neutrosophic Sets

In this section, we introduce the application of rough fermatean neutrosophic sets. Also, study the cosine similarity measure of rough fermatean neutrosophic sets. Moreover, medical diagnosis problems are discussed for establishing the proposed model.

4.1 Cosine Similarity Measure of Rough Fermatean Neutrosophic Sets

Definition 4.1.1. $\Theta(F_1)$ *and* $\Theta(F_2)$ are RFN sets in $X = \{x_1, x_2 \dots \dots x_n\}$. A cosine similarity measure between $\Theta(F_1)$ *and* $\Theta(F_2)$ is defined as follows:

$$COS_{RFN}(\Theta(F_1), \Theta(F_2)) = \frac{1}{n}\sum_{i=1}^{n} \frac{\left(\delta\Theta(F_{1t})(x_i)\delta\Theta(F_{2t})(x_i) + \delta\Theta(F_{1i})(x_i)\delta\Theta(F_{2i})(x_i) + \delta\Theta(F_{1f})(x_i)\delta\Theta(F_{2f})(x_i)\right)}{\sqrt{(\delta\Theta(F_{1t})(x_i))^2 + (\delta\Theta(F_{1t})(x_i))^2 + (\delta\Theta(F_{1f})(x_i))^2}\sqrt{(\delta\Theta(F_{2t})(x_i))^2 + (\delta\Theta(F_{2i})(x_i))^2 + (\delta\Theta(F_{2f})(x_i))^2}}$$

Where

$$\delta\Theta(F_{1t})(x_i) = \frac{\left(\Theta_{\blacksquare}(F_{1t})(x_i) + \Theta^{\blacksquare}(F_{1t})(x_i)\right)}{2}$$

$$\delta\Theta(F_{1i})(x_i) = \frac{\left(\Theta_{\blacksquare}(F_{1i})(x_i) + \Theta^{\blacksquare}(F_{1i})(x_i)\right)}{2}$$

$$\delta\Theta(F_{1f})(x_i) = \frac{\left(\Theta_{\blacksquare}(F_{1f})(x_i) + \Theta^{\blacksquare}(F_{1f})(x_i)\right)}{2} \text{and}$$

$$\delta\Theta(F_{2t})(x_i) = \frac{\left(\Theta_{\blacksquare}(F_{2t})(x_i) + \Theta^{\blacksquare}(F_{2t})(x_i)\right)}{2}$$

$$\delta\Theta(F_{2i})(x_i) = \frac{\left(\Theta_{\blacksquare}(F_{2i})(x_i) + \Theta^{\blacksquare}(F_{2i})(x_i)\right)}{2}$$

$$\delta\Theta(F_{2f})(x_i) = \frac{\left(\Theta_{\blacksquare}(F_{2f})(x_i) + \Theta^{\blacksquare}(F_{2f})(x_i)\right)}{2}$$

Proposition 4.1.2. A RFN fuzzy cosine similarity measure between $\Theta(F_1)$ *and* $\Theta(F_2)$ satisfies the following properties:

1. $0 \leq C_{RFN}(\Theta(F_1), \Theta(F_2)) \leq 1$
2. $C_{RFN}(\Theta(F_1), \Theta(F_2)) = 1 \Leftrightarrow \Theta(F_1) = \Theta(F_2)$
3. $C_{RFN}(\Theta(F_1), \Theta(F_2)) = C_{RFN}(\Theta(F_1), \Theta(F_2))$

If we consider the weight ω_i of each element, x_i, a weighted RFNcosine similarity measure between RFN sets $\Theta(F_1)$ *and* $\Theta(F_2)$ is defined as follows:

$COS_{RFN}\big(\Theta(F_1)\,,\Theta(F_2)\big) =$

$$\frac{1}{n}\sum_{i=1}^{n}\omega_i\frac{\big(\delta\Theta(F_{1t})(x_i)\delta\Theta(F_{2t})(x_i)+\delta\Theta(F_{1i})(x_i)\delta\Theta(F_{2i})(x_i)+\delta\Theta(F_{1f})(x_i)\delta\Theta(F_{2f})(x_i)\big)}{\sqrt{(\delta\Theta(F_{1t})(x_i))^2+(\delta\Theta(F_{1t})(x_i))^2+(\delta\Theta(F_{1f})(x_i))^2}\sqrt{(\delta\Theta(F_{2t})(x_i))^2+(\delta\Theta(F_{2i})(x_i))^2+(\delta\Theta(F_{2f})(x_i))^2}}$$

$\omega_i \in [0,1], i = 1,2,3 \dots n$ and $\sum_{i=1}^{n}\omega_i = 1$. If we take $\omega_i = \frac{1}{n}, i = 1,2, \dots . n$ then$C_{WRFN}\big(\Theta(F_1)\,,\Theta(F_2)\big) = C_{RFN}\big(\Theta(F_1)\,,\Theta(F_2)\big)$.

The weighted RFN cosine similarity measure between two RFN sets $\Theta(F_1)\ and\ \Theta(F_2)$ also satisfies the following properties:

Proposition 4.1.3.

1. $0 \le C_{WRFN}\big(\Theta(F_1)\,,\Theta(F_2)\big) \le 1$
2. $C_{WRFN}\big(\Theta(F_1)\,,\Theta(F_2)\big) = 1 \Leftrightarrow \Theta(F_1) = \Theta(F_2)$
3. $C_{WRFN}\big(\Theta(F_1)\,,\Theta(F_2)\big) = C_{WRFN}\big(\Theta(F_1)\,,\Theta(F_2)\big)$

5. Methodology

This section explores the application of RFN sets in the realm of medical diagnosis. Specifically, within a given medical scenario, F represents the set of symptoms, D denotes the array of diseases, and P signifies the cohort of patients manifesting symptoms in S.

Consider Q as the RFN relation mapping patients to symptoms (P → S), and R as a RFN relation mapping symptoms to diseases (S → D). The methodology comprises three primary tasks:

1. Identifying symptoms.
2. Formulating medical insights using RFN sets.
3. Establishing diagnoses.

5.1 Algorithm for RFN Cosine Similarity Measure

In this section, we present an algorithm of cosine similarity measure in RFN environment to diagnose the disease of the patient. Let $F = \{f_1, f_2 \dots \dots . f_n\}$ be set of symptoms and $P = \{p_1, p_2 \dots \dots . p_n\}$be set of patients and $D = \{d_1, d_2 \dots \dots . d_n\}$be set of disease.

The procedure unfolds as follows:

1. Gather symptoms exhibited by patients, concurrently establishing the patient-symptom relationship denoted as Q.
2. Derive the relationship R between symptoms and diseases.
3. Perform computations.
4. Choose the highest cosine similarity measure value.
5. Determine that the disease D affects the patients in P.

6. Illustrative Example

In this section, we provide an illustrative example demonstrating the application of cosine similarity measures for RFN sets.

6.1 Example of Rough Fermatean Cosine Similarity Measure

We consider a practical perspective on a medical diagnosis scenario to clarify the proposed approach. Within the field of medical science, the primary objective is the diagnosis of diseases. Therefore, medical diagnosis is highly valued as an art dedicated to identifying an individual's pathological conditions of the body from all the available symptoms.

Let D = $\{d_1, d_2 \dots \dots . d_n\}$ represent the set of diseases, F = $\{f_1, f_2 \dots \dots . f_n\}$ denote the symptoms, and P = $\{p_1, p_2 \dots \dots . p_n\}$ represent the set of patients exhibiting symptoms in F. The relationship between symptoms and diseases is described in the form of RNF sets.

Table 1. Relationship between patients and diseases in the form of RNF sets.

	d_1	d_2	d_3	d_4
p_1	(0.8,0.8,0.8), (0.8,0.8,0.8)	(0.7,0.9,0.8), (0.8,0.7,0.7)	(0.7,0.9,0.8), (0.8,0.7,0.7)	(0.9,0.7,0.8), (0.9,0.7,0.8)
p_2	(0.7,0.8,0.8), (0.7,0.8,0.8)	(0.8,0.9,0.7), (0.85,0.9,0.65)	(0.8,0.9,0.7), (0.85,0.9,0.65)	(0.8,0.95,0.7), (0.8,0.95,0.7)
p_3	(0.7,0.8,0.9), (0.7,0.8,0.6)	(0.9,0.8,0.5), (0.9,0.8,0.5)	(0.5,0.4,0.9), (0.5,0.4,0.9)	(0.7,0.8,0.9) (0.7,0.8,0.6)
p_4	(0.6,0.8,0.9), (0.9,0.6,0.4)	(0.7,0.7,0.8), (0.7,0.7,0.8)	(0.9,0.9,0.7), (0.9,0.9,0.7)	(0.6,0.8,0.9) (0.9,0.6,0.4)

Table 2. Relationship between patients and symptoms in the form of RNF sets.

	d_1	d_2	d_3	d_4
f_1	(0.6,1,0.7) (0.6,1,0.7)	(0.5,0.8,0.4) (0.7,0.5,0.3)	(0.5,0.8,0.4) (0.7,0.5,0.3)	(0.4,0.1,0.8) (0.4,0.1,0.8)
f_2	(0.4,0.9,0.6) (0.4,0.9,0.6)	(0.3,1,0.8) (0.8,0.7,0.8)	(0.3,1,0.8) (0.8,0.7,0.8)	(0.3,1,0.8) (0.8,0.7,0.8)
f_3	(0.3,0.7,0.7) (0.8,0.2,0.4)	(0.7,0.2,0.4) (0.7,0.2,0.4)	(0.3,0.7,0.7) (0.8,0.2,0.4)	(0.4,0.6,0.7) (0.4,0.6,0.7)
f_4	(0.3,0.8,0.7) (0.7,0.2,0.4)	(0.6,0.5,0.8) (0.6,0.5,0.8)	(0.3,0.7,0.7) (0.7,0.2,0.4)	(0.3,0.7,0.7) (0.7,0.2,0.4)

Table 3. Final results.

	d_1	d_2	d_3	d_4
p_1	0.5983	0.8877	0.9927	0.9990
p_2	0.9834	0.9771	0.9771	0.9826
p_3	0.9902	0.8405	0.9571	0.9838
p_4	0.9951	0.9910	0.9876	0.9951

From Table 3 we conclude that patient $p_1$affected byd_4, p_2 affected by d_1, $p_3$affected by d_1 and p_4 affected by d_1 and d_4.

7. Conclusions

The concept of uncertainty plays a vital role in all science and engineering problems. Especially, Fuzzy theory, Intuitionistic fuzzy theory and then Neutrosophic theory are the most valuable tools for finding the optimum solution to medical diagnosis problems. In this work, we include one more concept called RFN sets in the list which has Pythagorean Neutrosophic, Single Valued Neutrosophic, and Bipolar Neutrosophic graphs. We also apply this new type of set in a decision-making problem. We are extending our research on this new concept to introduce rough Interval-valued Fermatean Neutrosophic sets and their application in our future work.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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Conflict of interest

The authors declare that there is no conflict of interest in the research.

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